

Development of Finite Element Analysis Package for Induction Machines

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Abstract—In this paper, a software package of finite element (FE) analysis for induction machines is presented. The back-EMF in windings is expressed by magnetic vector potential, which is used to couple with transient finite element analysis. The equations of drive circuit and rotor bars are described by loop circuit equations. The pre-processing and post-processing of this package are developed based on AutoCAD. The FE model and some machine parameters can be input by friendly graphical user interfaces. The calculated magnetic field distribution and transient results can be shown in AutoCAD or graphical interfaces.

I. INTRODUCTION

A friendly and practical package for finite element analysis is desirable for designer to analyse the performances of electrical machines in high precision and adaptability. Induction machine, a popular kind of electrical machines, has been widely applied in many industry fields, such as traffic transport, wind power generations and household appliances, and so on. This paper aims to present a developed FE analysis package for induction machines, including three phase and single phase induction machines.

The field-circuit coupled simulation, which can deal with voltage driving problems [1], is developed in this package. Instead of iterating between circuit and magnetic field solutions, the circuit equations of the armature windings were coupled directly with the finite element equations so that the currents could be computed simultaneously. Further development allowed arbitrary connected circuits to be coupled with magnetic fields. For establishing the circuit equations some authors preferred using the nodal method [2-3] and others preferred the loop method [3-9]. Some literatures are available for the handling of stranded windings [2], [5-7] and solid conductors [10].

In this paper, some pre- and post-processing modules have been developed based on AutoCAD environment because AutoCAD provides several powerful second-development tools and more motor designer use AutoCAD expertly. In this developed package, the object definition, boundary condition definition, meshing control and the display of calculated magnetic field distribution are all realized in AutoCAD. Other graphical interfaces, e.g. exciting voltage, management of material properties, parameters of motor winding, parameters of rotor solid bars, are all coded with Visual Basic.

II. FUNDAMENTAL EQUATIONS

A two-dimensional (2D) finite-element method is used for the induction machines, where the magnetic vector potential has only the z -component.

A. Electromagnetic Field Equations

The Maxwell's equations are applied to the air gap and iron core regions by the following diffusion equation [10]:

$$\nabla \cdot (\nu \nabla A) = 0 \quad (1)$$

where A is component of magnetic vector potential in the z axis, ν is the reluctivity of materials.

The field equation in the stranded windings can be written as:

$$\nabla \cdot (\nu \nabla A) + \frac{d_f N_f}{S_f a p} i_f = 0 \quad (2)$$

where i_f is the winding current, S_f is the total cross-sectional area of the region occupied by this winding in the solution domain, p is the symmetry multiplier which is defined as the ratio of the original full cross-sectional area to the solution area, d_f is the polarity to represent forward path or return path, a is the number of parallel branches, and N_f is the total conductor number.

The magnetic field equation in the region of solid conductors can be expressed as:

$$\nabla \cdot (\nu \nabla A) - \sigma \frac{\partial A}{\partial t} + \frac{d_f a \sigma}{N_f l} u_{bar} = 0 \quad (3)$$

where u_{bar} is the voltage difference between the terminals of solid conductor winding, and l is the model depth in the z axis.

B. Branch Equation of Stranded Windings and Solid Conductors

The branch equations of the stranded windings and solid conductors are coupled with field equations. The branch equation of the stranded windings is described as:

$$\frac{d_f N_f l}{S_f a} \iint_{\Omega} \frac{\partial A}{\partial t} d\Omega + R_f i_f = u_{sf} \quad (4)$$

where u_{sf} is the voltage on the two terminals of the winding, l is the model depth in z axis, R_f is the total DC resistance of the winding.

For a generator, (4) may be rewritten as

$$\frac{d_f N_f l}{S_f a} \iint_{\Omega} \frac{\partial A}{\partial t} d\Omega - R_f i_f = u_{sf}$$

The branch equation of solid conductors is shown as

$$i_{\text{bar}} = -\sigma \iint_{S_{\text{bar}}} \frac{\partial A}{\partial t} d\Omega + \frac{1}{R_{\text{bar}}} u_{\text{bar}} \quad (5)$$

where S_{bar} is the cross-sectional area of the conductor and R_{bar} is the DC resistance of this one conductor bar. u_{bar} is the voltage on the two terminals of the conductor bar.

C. Loop Equations

The loop method is applied to describe the circuit equations of the stranded windings, the solid conductors and the external circuits. The currents flowed in stranded winding and solid conductors are regarded as branch currents. In loop method, the variables are chosen as loop current.

The relationship between the branch current column vector I_b and the loop current column vector I_l is

$$I_b = B_{lb}^T I_l \quad (6)$$

where B_{lb} is the loop-to-branch incidence matrix.

The Kirchhoff's voltage law can be expressed as,

$$B_{lb} U_b = 0 \quad (7)$$

where U_b is the branch voltage column vector.

From above (1)-(7), the field-circuit coupled equation can be established. The magnetic vector potentials of the nodes and the loop currents are chosen as unknowns.

If capacitor exists in the external circuit of machine, such as capacitor-running single-phase induction motor, the voltage-current relationship of capacitor shown as follows need to be considered. The capacitor voltage u_c is regarded as unknowns in coupled equations.

$$i_c = C \frac{du_c}{dt} \quad (8)$$

D. Mechanical Equations

At each time step the electromagnetic torque T_{em} is calculated by using the Maxwell stress tensor. The angular

speed and the rotor displacement at the k th time step can be determined by the following equations:

$$\omega_k = \omega_{k-1} + \frac{1}{J} (T_{em} - \alpha \omega_{k-1} - T_L) \Delta t \quad (9)$$

$$\theta_k = \theta_{k-1} + \omega_k \Delta t \quad (10)$$

For a generator, (9) may be rewritten as

$$\omega_k = \omega_{k-1} + \frac{1}{J} (T_L - T_{em} - \alpha \omega_{k-1}) \Delta t$$

In equation (9), ω is angular velocity, α is the damping coefficient, T_{em} is the electromagnetic torque, T_L is the load torque, and J is the total inertia. In equation (10), θ is the angular displacement of rotor. The rotor is then turned and the movement is accounted.

E. Eddy Loss Equations

Iron loss is computed by a traditional loss calculation approach of iron. Because the current density is normally not uniform over the cross-section of rotor bar, the loss of rotor bar is expressed as

$$p = q l \rho \int_S \left(-\sigma \frac{\partial A}{\partial t} + \sigma \frac{u_{\text{bar}}}{l} \right)^2 dS \quad (11)$$

where S is the full cross-sectional area of one rotor bar, ρ is the resistivity of the bar, and q is the ratio of the solution area to full cross-sectional area.

The back-Euler method and Newton-Raphson iteration are used to solve the transient nonlinear field-circuit coupled equations.

III. DEVELOPMENT OF FINITE ELEMENT PACKAGE

As a finite element package, pre-processor, analysis and post-processor should be developed to provide the users a convenient tool for the performance analysis of induction machines.

In this paper, some pre- and post-processing modules have been developed based on AutoCAD environment because AutoCAD provides powerful second-development tools, such as AutoCAD ARX, VBA for AutoCAD and AutoLISP.

In AutoCAD, the object definition, boundary condition definition and mesh controller are developed and the magnetic field distribution may be shown. The user may navigate the friendly interfaces to establish the finite element model and acquire the results of magnetic field from the field distribution.

Fig.1, 2 and 3 show the object definition interface, mesh controller and magnetic distribution in AutoCAD, respectively.

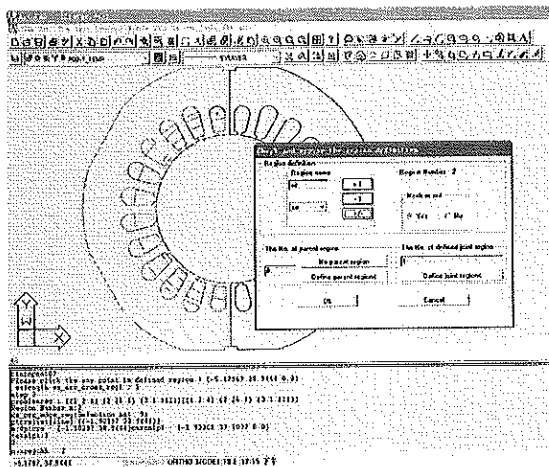


Figure 1. Object definition in AutoCAD.

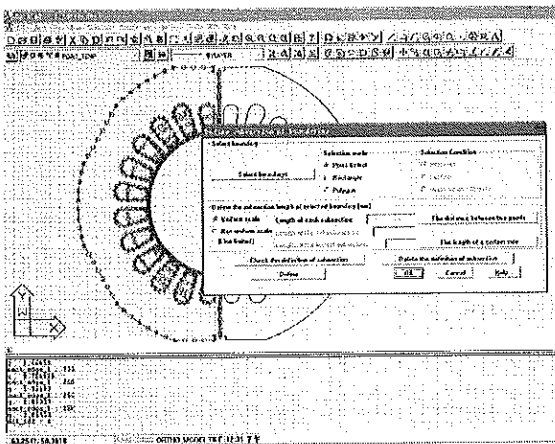


Figure 2. Mesh Controller in AutoCAD.

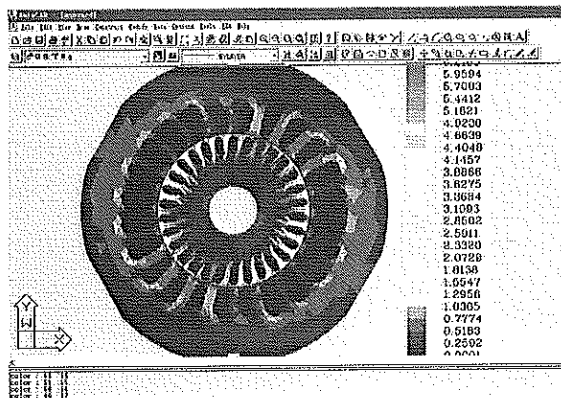


Figure 3. Distribution of magnetic flux density shown in AutoCAD.

The main interface and some other interfaces are developed using Visual Basic. The users may employ all of the provided functions, such as material definition, parameter definition of induction machine, FE computation and performance waveform browser, as well as running AutoCAD. Fig.4, 5 and

6 show the main interface, B-H curve definition interface and transient winding currents interface, respectively.

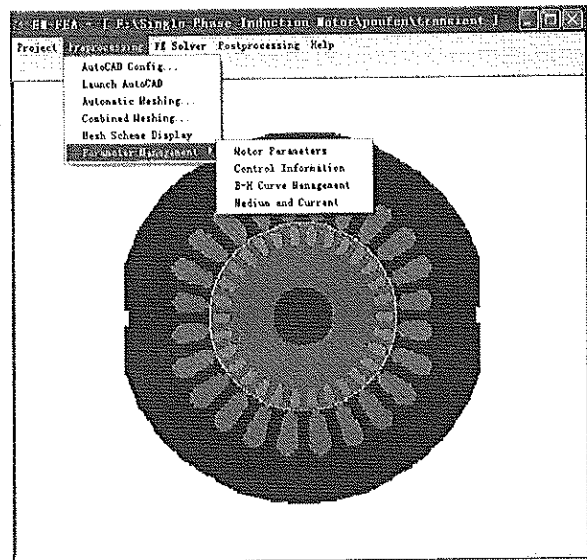


Figure 4. Main interface.

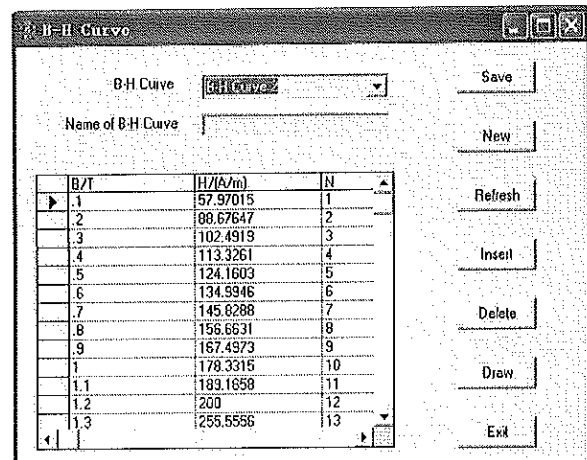


Figure 5. B-H curve definition interface

The developed FE analysis package may provide user transient performance computation of induction machines, such as transient currents in windings and rotor bars, electromagnetic torque, speed, conducting current loss, iron loss and efficiency.

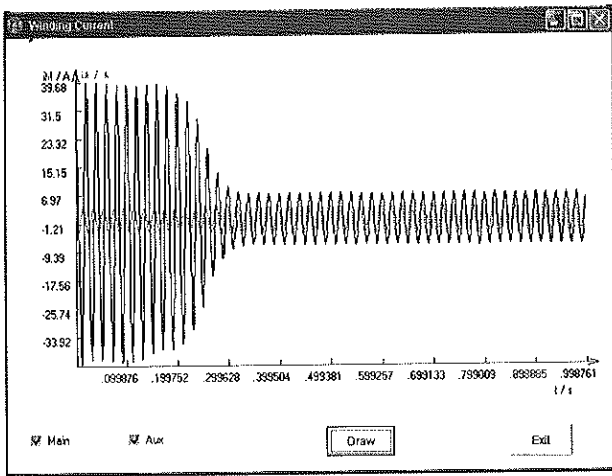


Figure 6. Transient winding currents interface

IV. APPLICATION EXAMPLES

A. Single Phase Induction Motor

A single-phase two-pole induction motor for compressor is used to show the functions of this package. The motor is fed by a 50 Hz sinusoidal voltage, and presents a different number of turns of main and auxiliary windings in each slot, and a capacitor is connected in the auxiliary winding. The electric circuit of stator windings is shown in Fig. 7.

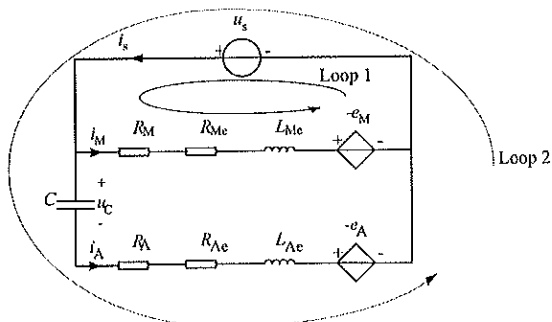


Figure 7. Electric circuit of stator windings

In Fig. 7, e_M and e_A are the back EMF of the main and the auxiliary windings, R_M and R_A are resistances of the main and the auxiliary windings, and R_{Me} , R_{Ae} , L_{Me} and L_{Ae} are the end leakage resistances and end leakage inductances of the main and the auxiliary windings, respectively.

The field distribution calculated by using this package is shown in Fig. 8, and the speed is given in Fig. 9.

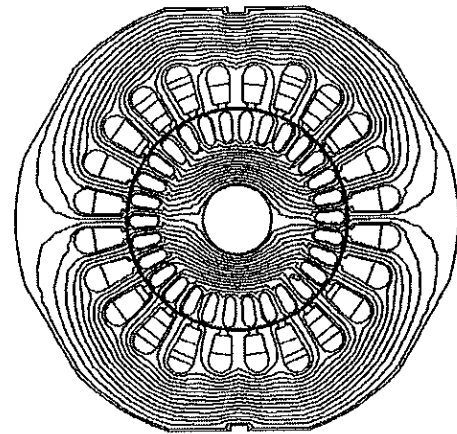


Figure 8. Flux distribution in steady state of the single-phase induction motor

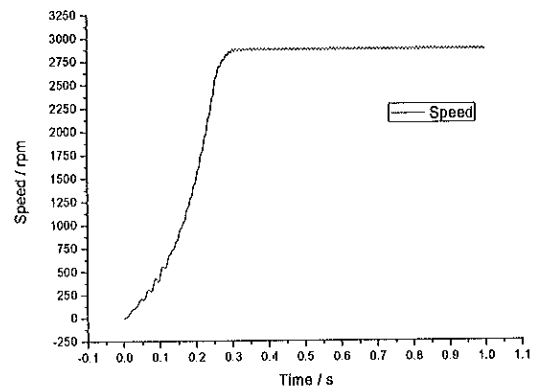


Figure 9. Instantaneous speed

Instantaneous electromagnetic torque is given in Fig. 10. Currents in the main and auxiliary windings are shown in Fig. 6.

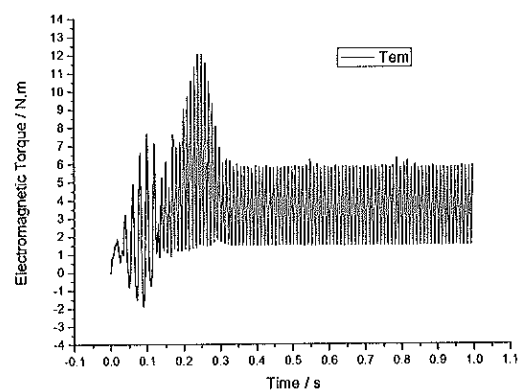


Figure 10. Electromagnetic torque

B. Asynchronous Generator

This example is a 3-phase four-pole asynchronous generator, a kind of induction machines used in a wind generator. The rated output power is 750 kW, output voltage is 690 V and the frequency is 50 Hz. The finite element mesh is shown as Fig. 11.

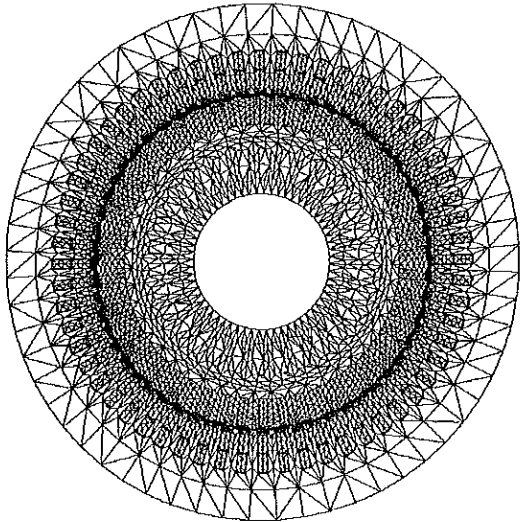


Figure 11. Finite element mesh of an asynchronous generator

One phase current of stator winding is given in Fig. 12. The phase currents have correlation with speed which is shown in Fig. 13. The starting-up current is very large before the rotor reaches the synchronous speed.

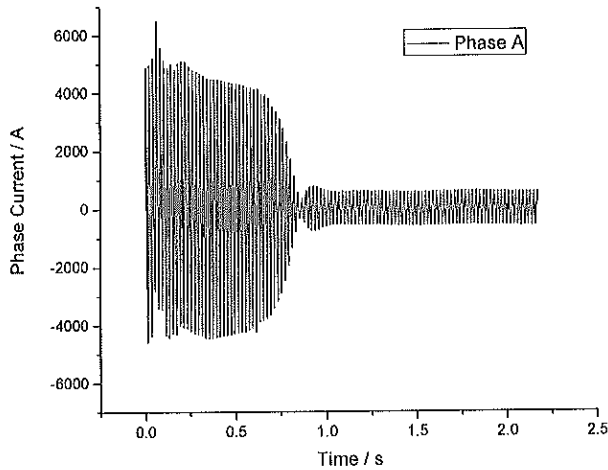


Figure 12. One phase current of stator winding

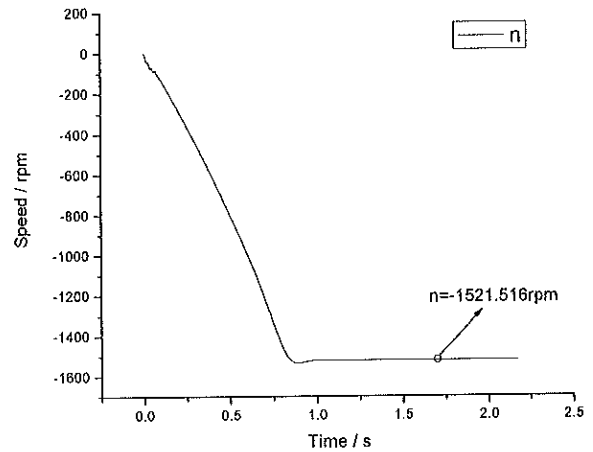


Figure 13. Instantaneous speed

The current of one rotor bar are shown in Fig. 14. The change of slip ratio is reflected by the current in rotor bar, and when the generator is at the steady state condition, the current frequency is low.

The field distributions of starting and steady states, respectively shown in Fig. 15 and 16, are different due to skin effect. The loss density distribution at the steady state condition is given in Fig. 17.

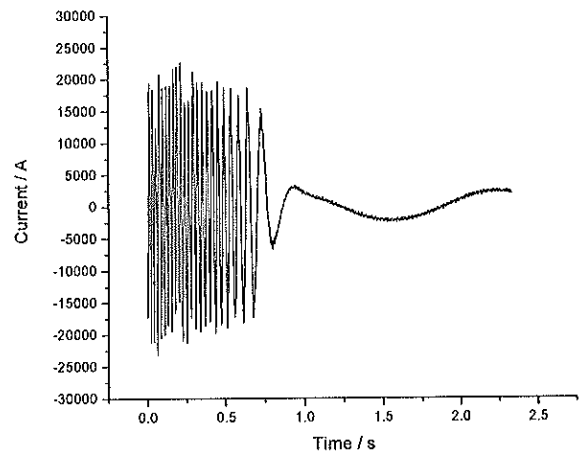


Figure 14. Current flowed in one rotor bar

V. CONCLUSION

A developed finite element package of transient analysis of induction machines is presented. A directly-coupled field-circuit method for solid conductors and stranded windings has been realized to calculate the motor performance. The pre- and post-processing have been developed with user-friendly interfaces based on AutoCAD. The package has been employed in some industry applications, such as compressor motor and wind generator. Considering the equivalent models of permanent magnets, different drive and control strategies, the package may be expanded to analyse the performances of adjust-speed and line-start synchronous motors, BLDC motors, DC motor, and as well as switched reluctance motors and some other new types of electrical machines.

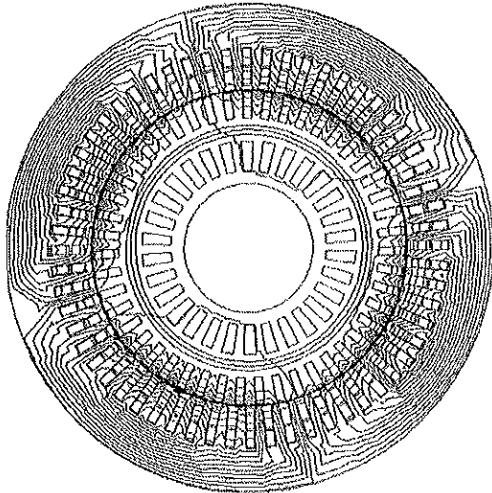


Figure 15. Field distribution in starting state

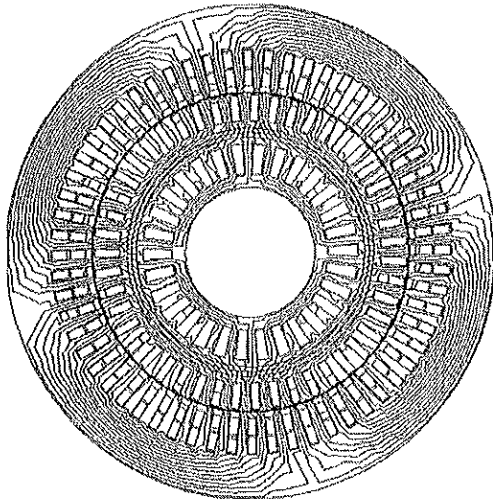


Figure 16. Field distribution in steady state

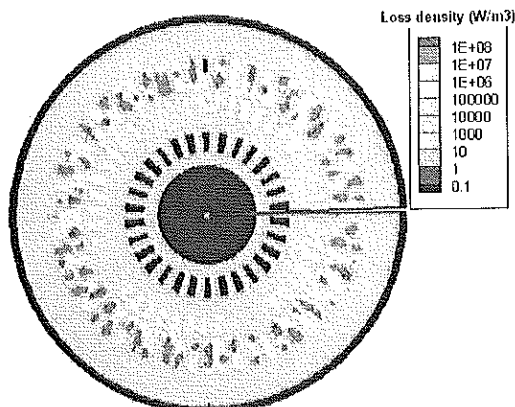


Figure 17. Loss density distribution in the generator

REFERENCES

- [1] S. L. Ho and W. N. Fu, "Review and Further Application of Finite Element Methods in Induction Motors," *Electric Machines and Power Systems*, Taylor & Francis, Vol. 26, No. 2, 1998, pp. 111-125.
- [2] M. C. Costa, S. I. Nabeta, and J. R. Cardoso, "Modified nodal analysis applied to electric circuits coupled with FEM in the simulation of a universal motor," *IEEE Trans. Magn.*, vol. 36, pp. 1431-1434, July, 2000.
- [3] J.-S. Wang, "A nodal analysis approach for 2D and 3D magnetic-circuit coupled problems," *IEEE Trans. Magn.*, vol. 32, pp. 1074-1077, May, 1996.
- [4] N. M. Abe and J. R. Cardoso, "Coupling electric circuit and 2D-FEM model with Dommel's approach for transient analysis," *IEEE Trans. Magn.*, vol. 34, pp. 3487-3490, Sept. 1998.
- [5] T. Renyuan, H. Yan, L. Zhanhong, Y. Shiyu, and M. Lijie, "Computation of transient electromagnetic torque in a turbo-generator under the cases of different sudden short circuits," *IEEE Trans. Magn.*, vol. 26, pp. 1042-1045, Mar. 1993.
- [6] F. Piriou and A. Razek, "Finite element analysis in electromagnetic systems accounting for electric circuits," *IEEE Trans. Magn.*, vol. 29, pp. 1669-1675, Mar. 1993.
- [7] P. Zhou, W. N. Fu, D. Lin, S. Stanton, Z. J. Cendes, and L. Xu, "Numerical modeling of electrical machines and its application," in *IEEE Industry Applications Soc. 37th Annu. Meeting*, Pittsburgh, PA, Oct. 13-18, 2002.
- [8] I. A. Tsukerman, A. Konrad, and J. D. Lavers, "A method for circuit connections in time-dependent eddy current problems," *IEEE Trans. Magn.*, vol. 28, pp. 1299-1302, Mar. 1993.
- [9] P. Lombard and G. Meunier, "A general method for electric and magnetic coupled problem in 2D and magnetodynamic domain," *IEEE Trans. Magn.*, vol. 28, pp. 1291-1294, Mar. 1993.
- [10] W. N. Fu, P. Zhou, D. Lin, S. Station, and Z. J. Cendes, "Modeling of solid conductors in two-dimensional transient finite-element analysis and its application to electric machines," *IEEE Trans. Magn.* vol. 40, no. 2, pp.426-434, Mar. 2004.