# Investigation of Dynamic Performance of On-road Vehicles with Random System Parameters

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**Abstract**

Ride comfort, suspension working space and road holding are three important indices to evaluate the dynamic performance of vehicles. The object of this paper is to develop the analytical model and numerical solution procedure of dynamic performance indices of vehicles with uncertainty in system parameters and road conditions. A quarter car model is used to describe the dynamic behavior of vehicles running on randomly profiled roads. The sprung mass, unsprung mass, suspension damping, suspension and tire stiffness are considered as random variables. The road irregularity is considered as a Gaussian random process and modeled by means of a simple exponential power spectral density. The mean value, standard deviation and variation coefficient of the vehicle's natural frequencies and mode shapes are obtained by using the Monte-Carlo simulation method. The computational expressions for the numerical characteristics of vehicle's dynamic performance indices in frequency domain are developed by means of the random variable's functional moment method and algebra synthesis method. The influences of the vehicle's parameters on the vehicle's dynamic performance indices are investigated using a practical example, and some useful conclusions are given.

***Key words***: Dynamic performance indices, Vehicles, Uncertainty, Quarter car model, Random variables, Random responses

## 1. Introduction

Analysis of dynamic performance of vehicles is essential to vehicle design. Ride comfort, working space and road holding are three important indices to assess the performance of vehicles and vehicle suspension systems. Ride comfort (discomfort) is dependent on the acceleration of vehicle body or sprung mass. Working space (suspension travel) is the relative displacement between the wheel and vehicle body. Road holding is the function of the distance between the wheel and the road surface, in other words, road holding depends on the tire deflection. Therefore, the analysis of vehicle's dynamic responses is the base of vehicle performance assessment and vehicle design. Vehicle dynamic response analysis has been a hot research topic for many years. Numerous papers about the theoretical and experimental investigation on the dynamic behavior of passively and actively suspended road vehicles have been published [1-3]. The quarter-car model [2-6], half-car model [7-9] and full-vehicle model [10-12] have been developed with researches related to the dynamic behavior of vehicle and its vibration control. The simplest representation of a ground vehicle is a quarter-car model with a spring and a damper connecting the body to a single wheel which is in turn connected to the ground via the tire spring. The mass of the body usually is described as sprung mass, the mass representing the wheel, tire, brakes and part of the suspension linkage mass is referred to as the unsprung mass.

Actually, the spring stiffness and damping rate may vary with respect to the nominal value due to production tolerances and/or wear, ageing... etc. The vehicle body mass and the tire radial stiffness can have stochastic variations due to the variety of possible vehicle loading conditions and to the uncertainty of the inflating pressure of poorly maintained tires [13]. In cars and buses, weight and placements of passengers often exhibit significant variability. In addition, even same brand and type vehicles leaving the production line may have uncertainties in size, mass and performance and so on.

In this paper, a two-degree-of-freedom quarter-car model is used to investigate the dynamic responses of cars with uncertainty. The vehicle's parameters are considered as random variables. The dynamic performance indices of vehicles are investigated by using the Monte-Carlo simulation method (MCSM) [14], random variable's functional moment method (RVFMM) [15] and algebra synthesis method (ASM) [16].

## 2. Interval seismic vibration of shear beam structures

A two-degree-of-freedom quarter-car model is shown in Fig. 1. In this model, the sprung and unsprung masses corresponding to the one corner of the vehicle are denoted respectively by $m_s$ and $m_u$. The suspension system is represented by a linear spring of stiffness $k_s$ and a linear damper with a damping rate $c_s$, while the tire is modeled by a linear spring of stiffness $k_t$. The excitation comes from the road irregularity $x_r$. The model is generally reputed to be sufficiently accurate for capturing the essential features related to discomfort, road holding and working space [13]. The linear equations of motions of the system model are

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{P\} \tag{1}$$

where

$$[M]=\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix},\quad [C]=\begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix},\quad [K]=\begin{bmatrix} k_s & -k_s \\ -k_s & k_s+k_t \end{bmatrix}$$
$$\{P\}=\begin{Bmatrix} 0 \\ k_t x_r \end{Bmatrix},\quad \{X\}=\begin{Bmatrix} x_s \\ x_u \end{Bmatrix} \tag{2}$$

Using Rayleigh's quotient, the $j^{th}$ natural frequency of the vehicle $\omega_j$ can be expressed as

$$\omega_j^2=\frac{\{\phi_j\}^T[K]\{\phi_j\}}{\{\phi_j\}^T[M]\{\phi_j\}} \tag{3}$$

where $\{\phi_j\}$ is the $j^{th}$ mode shape.

The $j^{th}$ modal damping of the vehicle $\zeta_j$ can be obtained from the following equation [15]

$$\zeta_j = \frac{1}{2\omega_j}\frac{\{\phi_j\}^T[C]\{\phi_j\}}{\{\phi_j\}^T[M]\{\phi_j\}} \quad (4)$$

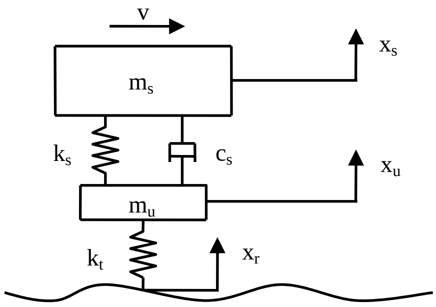



Fig. 1 The quarter-car model of the vehicle.

The displacement $x_r$ (road irregularity) may be represented by a random variable defined by a stationary and ergodic stochastic process with zero mean value. The power spectral density of the process may be determined on the basis of experimental measurements and in the literature there are many different formulations for it. In this paper for sake of simplicity, the following spectrum [13] is considered

$$S_{x_r}(\omega) = \frac{A_b v}{\omega^2} \quad (5)$$

where $A_b$ is the road irregularity parameter.

From equations (2) and (5), the power spectral density matrix $[S_P(\omega)]$ of $\{P\}$ can be obtained

$$[S_P(\omega)] = \begin{bmatrix} 0 & 0 \\ 0 & k_t^2 S_{x_r}(\omega) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & k_t^2 \dfrac{A_b v}{\omega^2} \end{bmatrix} \quad (6)$$

Using the theory of structural random vibration [15,17], the power spectral density of the *kth* degree of freedom of the vehicle's displacement $S_{x_k}(\omega)$ and acceleration responses $S_{\ddot{x}_k}(\omega)$ in frequency domain can be developed as

$$S_{x_k}(\omega) = \phi_k [H(\omega)][\phi]^T [S_P(\omega)][\phi][H^*(\omega)]\phi_k^T \qquad k = 1,2 \quad (7)$$

$$\begin{aligned} S_{\ddot{x}_k}(\omega) = \phi_k \Big\{ & \frac{\partial^2}{\partial\omega^2}[H(\omega)]\cdot[\phi]^T[S_P(\omega)][\phi]\cdot\frac{\partial^2}{\partial\omega^2}[H^*(\omega)] \\ & + 2\cdot\frac{\partial^2}{\partial\omega^2}[H(\omega)]\cdot[\phi]^T\cdot\frac{\partial}{\partial\omega}[S_P(\omega)]\cdot[\phi]\cdot\frac{\partial}{\partial\omega}[H^*(\omega)] \\ & + \frac{\partial^2}{\partial\omega^2}[H(\omega)]\cdot[\phi]^T\cdot\frac{\partial^2}{\partial\omega^2}[S_P(\omega)]\cdot[\phi][H^*(\omega)] \\ & + 2\cdot\frac{\partial}{\partial\omega}[H(\omega)]\cdot[\phi]^T\cdot\frac{\partial}{\partial\omega}[S_P(\omega)]\cdot[\phi]\cdot\frac{\partial^2}{\partial\omega^2}[H^*(\omega)] \\ & + 4\cdot\frac{\partial}{\partial\omega}[H(\omega)]\cdot[\phi]^T\cdot\frac{\partial^2}{\partial\omega^2}[S_P(\omega)]\cdot[\phi]\cdot\frac{\partial}{\partial\omega}[H^*(\omega)] \end{aligned}$$

$$+2\cdot\frac{\partial}{\partial\omega}[H(\omega)]\cdot[\phi]^T\cdot\frac{\partial^3}{\partial\omega^3}[S_P(\omega)]\cdot[\phi][H^*(\omega)]$$
$$+[H(\omega)][\phi]^T\cdot\frac{\partial^2}{\partial\omega^2}[S_P(\omega)]\cdot[\phi]\cdot\frac{\partial^2}{\partial\omega^2}[H^*(\omega)]$$
$$+2[H(\omega)][\phi]^T\cdot\frac{\partial^3}{\partial\omega^3}[S_P(\omega)]\cdot[\phi]\cdot\frac{\partial}{\partial\omega}[H^*(\omega)]$$
$$+[H(\omega)][\phi]^T\cdot\frac{\partial^4}{\partial\omega^4}[S_P(\omega)]\cdot[\phi][H^*(\omega)]\Big\}\phi_k^T \qquad k=1,2 \qquad (8)$$

where $\phi_k$ is the *kth* line vector of the matrix $[\phi]$.

Equation (8) can also be concisely expressed as follows

$$S_{\ddot{X}_k}(\omega)=f\left(\phi_k,[H(\omega)],[\phi]^T,[S_P(\omega)],[\phi],[H^*(\omega)],\phi_k^T\right) \qquad k=1,2 \qquad (9)$$

Integrating power spectral density within the frequency domain, the mean square value of the vehicle's displacement and acceleration responses, that is $\psi_{X_k}^2$ and $\psi_{\ddot{X}_k}^2$, can be obtained

$$\psi_{X_k}^2=\int_{-\infty}^{\infty}S_{X_k}(\omega)d\omega=\int_{-\infty}^{\infty}\phi_k[H(\omega)][\phi]^T[S_P(\omega)][\phi][H^*(\omega)]\phi_k^T d\omega \qquad k=1,2 \qquad (10)$$

$$\psi_{\ddot{X}_k}^2=\int_{-\infty}^{\infty}S_{\ddot{X}_k}(\omega)d\omega=\int_{-\infty}^{\infty}f\left(\phi_k,[H(\omega)],[\phi]^T,[S_P(\omega)],[\phi],[H^*(\omega)],\phi_k^T\right)d\omega \qquad k=1,2 \qquad (11)$$

Furthermore, the root mean square value of vehicle's random displacement and acceleration responses can be expressed as

$$\psi_{X_k}=\left\{\int_{-\infty}^{\infty}\phi_k[H(\omega)][\phi]^T[S_P(\omega)][\phi][H^*(\omega)]\phi_k^T d\omega\right\}^{1/2} \qquad k=1,2 \qquad (12)$$

$$\psi_{\ddot{X}_k}=\left\{\int_{-\infty}^{\infty}f\left(\phi_k,[H(\omega)],[\phi]^T,[S_P(\omega)],[\phi],[H^*(\omega)],\phi_k^T\right)d\omega\right\}^{1/2} \qquad k=1,2 \qquad (13)$$

## 3. Dynamic performance indices of vehicles

Based on the random displacement and acceleration analysis of vehicles, the expressions of dynamic performance indices in frequency domain can be obtained. Ride comfort index can be described by the root mean square sprung mass acceleration as follows

$$\psi_{\ddot{X}_s}=\left\{\int_{-\infty}^{\infty}f\left(\phi_1,[H(\omega)],[\phi]^T,[S_P(\omega)],[\phi],[H^*(\omega)],\phi_1^T\right)d\omega\right\}^{1/2}, \quad \phi_1=[\phi_{11} \quad \phi_{12}] \qquad (14)$$

Road holding index (tire deflection) can be expressed as

$$\psi_{X_u} = \left\{ \int_{-\infty}^{\infty} \phi_2 [H(\omega)][\phi]^T [S_P(\omega)][\phi][H^*(\omega)]\phi_2^T d\omega \right\}^{1/2}, \quad \phi_2 = [\phi_{21} \quad \phi_{22}] \tag{15}$$

Working space index (suspension travel) can be given by

$$\psi_{X_u} - \psi_{X_s} = \left\{ \int_{-\infty}^{\infty} \phi_2 [H(\omega)][\phi]^T [S_P(\omega)][\phi][H^*(\omega)]\phi_2^T d\omega \right\}^{1/2} - \left\{ \int_{-\infty}^{\infty} \phi_1 [H(\omega)][\phi]^T [S_P(\omega)][\phi][H^*(\omega)]\phi_1^T d\omega \right\}^{1/2} \tag{16}$$

## 3. Numerical characteristics analysis of vehicle random responses

The following parameters corresponding to $m_s$, $m_u$, $k_s$, $c_s$ and $k_t$ are considered as random variables. The randomness of vehicle's parameters will result in randomness of the matrices $[M]$ and $[K]$ and $[C]$, and consequently the natural frequencies $\omega_j$, mode matrix $[\phi]$ and modal damping $\zeta_j$. The random variables are each given a mean value ($\mu$) and standard deviation ($\sigma$). A further parameter used in this paper is the variation coefficient $\nu$, defined by the ratio of the standard deviation to the mean value, that is $\nu = \sigma / \mu$. In this paper, all uncertain vehicle's parameters are normal random variables and they are independent each other. For the two-degree-freedom system, the computational effort is acceptable for the analysis of the power spectral density, mean value and standard deviation of vehicle's dynamic characteristics. By using the MCSM, $\mu_{\omega_j}$, $\sigma_{\omega_j}$, $\mu_{[\phi]}$, $\sigma_{[\phi]}$, $\mu_{\zeta_j}$ and $\sigma_{\zeta_j}$ can be obtained.

A combination of uncertainty in the structural dynamic characteristics and stochastic excitation will lead to randomness in dynamic performance indices. From equation (14) and by means of the ASM [16], the mean value $\mu_{\psi_{\ddot{X}_k}}$, standard deviation $\sigma_{\psi_{\ddot{X}_k}}$ and variation coefficient $\nu_{\psi_{\ddot{X}_k}}$ of the ride comfort index can be determined

$$\mu_{\psi_{\ddot{X}_s}} = \left( \sqrt{\mu^2_{\psi^2_{\ddot{X}_s}} - \frac{1}{2}\sigma^2_{\psi^2_{\ddot{X}_s}}} \right)^{1/2}, \sigma_{\psi_{\ddot{X}_s}} = \left( \mu_{\psi^2_{\ddot{X}_s}} - \sqrt{\mu^2_{\psi^2_{\ddot{X}_s}} - \frac{1}{2}\sigma^2_{\psi^2_{\ddot{X}_s}}} \right)^{1/2}, \nu_{\psi_{\ddot{X}_s}} = \frac{\sigma_{\psi_{\ddot{X}_s}}}{\mu_{\psi_{\ddot{X}_s}}} \tag{17}$$

where

$$\mu_{\psi^2_{\ddot{X}_s}} = \int_{-\infty}^{\infty} f\left(\mu_{\phi_1}, \mu_{[H(\omega)]}, \mu_{[\phi]^T}, \mu_{[S_P(\omega)]}, \mu_{[\phi]}, \mu_{[H^*(\omega)]}, \mu_{\phi_1^T}\right) d\omega \tag{18}$$

$$\sigma^2_{\psi^2_{\ddot{X}_s}} = \int_{-\infty}^{\infty} \sigma^2_{S_{\ddot{X}_s}(\omega)} d\omega \tag{19}$$

By using the RVFMM [15], $\sigma^2_{S_{\ddot{X}_s}(\omega)}$ can be expressed as

$$\sigma^2_{S_{\ddot{X}_s}(\omega)} = \left\{ \frac{\partial f}{\partial \phi_1} \sigma_{\phi_1} \right\}^2 + \left\{ \frac{\partial f}{\partial [H(\omega)]} \sigma_{[H(\omega)]} \right\}^2 + \left\{ \frac{\partial f}{\partial [\phi]^T} \sigma_{[\phi]^T} \right\}^2$$

$$+\left\{\frac{\partial f}{\partial[S_P(\omega)]}\sigma_{[S_P(\omega)]}\right\}^2+\left\{\frac{\partial f}{\partial[\phi]}\sigma_{[\phi]}\right\}^2+\left\{\frac{\partial f}{\partial[H^*(\omega)]}\sigma_{[H^*(\omega)]}\right\}^2+\left\{\frac{\partial f}{\partial\phi_1^T}\sigma_{\phi_1^T}\right\}^2 \quad (20)$$

In the following, only the $\left\{\frac{\partial f}{\partial\phi_1}\sigma_{\phi_1}\right\}^2$ is given in detail

$$\sigma^2_{\psi^2_{\ddot{X}_s}}=\int_{-\infty}^{\infty}\sigma^2_{S_{\ddot{X}_s}(\omega)}d\omega$$

$$\begin{aligned}\left\{\frac{\partial f}{\partial\phi_1}\sigma_{\phi_1}\right\}^2=&\left\{\sigma_{\phi_1}\left\{\frac{\partial^2}{\partial\omega^2}\mu_{[H(\omega)]}\cdot\mu_{[\phi]^T}\mu_{[S_P(\omega)]}\mu_{[\phi]}\cdot\frac{\partial^2}{\partial\omega^2}\mu_{[H^*(\omega)]}\right.\right.\\
&+2\cdot\frac{\partial^2}{\partial\omega^2}\mu_{[H(\omega)]}\cdot[\phi]^T\cdot\frac{\partial}{\partial\omega}\mu_{[S_P(\omega)]}\cdot[\phi]\cdot\frac{\partial^2}{\partial\omega^2}\mu_{[H^*(\omega)]}\\
&+\frac{\partial^2}{\partial\omega^2}\mu_{[H(\omega)]}\cdot[\phi]^T\cdot\frac{\partial^2}{\partial\omega^2}\mu_{[S_P(\omega)]}\cdot[\phi]\mu_{[H^*(\omega)]}\\
&+2\cdot\frac{\partial}{\partial\omega}\mu_{[H(\omega)]}\cdot[\phi]^T\cdot\frac{\partial}{\partial\omega}\mu_{[S_P(\omega)]}\cdot[\phi]\cdot\frac{\partial^2}{\partial\omega^2}\mu_{[H^*(\omega)]}\\
&+4\cdot\frac{\partial}{\partial\omega}\mu_{[H(\omega)]}\cdot[\phi]^T\cdot\frac{\partial^2}{\partial\omega^2}\mu_{[S_P(\omega)]}\cdot[\phi]\cdot\frac{\partial}{\partial\omega}\mu_{[H^*(\omega)]}\\
&+2\cdot\frac{\partial}{\partial\omega}\mu_{[H(\omega)]}\cdot[\phi]^T\cdot\frac{\partial^3}{\partial\omega^3}\mu_{[S_P(\omega)]}\cdot[\phi]\mu_{[H^*(\omega)]}\\
&+\mu_{[H(\omega)]}[\phi]^T\cdot\frac{\partial^2}{\partial\omega^2}\mu_{[S_P(\omega)]}\cdot[\phi]\cdot\frac{\partial^2}{\partial\omega^2}\mu_{[H^*(\omega)]}\\
&+2\cdot\mu_{[H(\omega)]}[\phi]^T\cdot\frac{\partial^3}{\partial\omega^3}\mu_{[S_P(\omega)]}\cdot[\phi]\cdot\frac{\partial}{\partial\omega}\mu_{[H^*(\omega)]}\\
&\left.\left.+\mu_{[H(\omega)]}[\phi]^T\cdot\frac{\partial^4}{\partial\omega^4}\mu_{[S_P(\omega)]}\cdot[\phi]\mu_{[H^*(\omega)]}\right\}\mu_{\phi_1^T}\right\}^2\end{aligned} \quad (21)$$

Similarly, from equation (15) and by means of the ASM, the mean value $\mu_{\psi_{X_u}}$ and standard deviation $\sigma_{\psi_{X_u}}$ and variation coefficient $v_{\psi_{X_u}}$ of road holding index can be obtained

$$\mu_{\psi_{X_u}}=\left(\sqrt{\mu^2_{\psi^2_{\ddot{X}_u}}-\frac{1}{2}\sigma^2_{\psi^2_{\ddot{X}_u}}}\right)^{1/2},\ \sigma_{\psi_{X_u}}=\left(\mu_{\psi^2_{\ddot{X}_u}}-\sqrt{\mu^2_{\psi^2_{\ddot{X}_u}}-\frac{1}{2}\sigma^2_{\psi^2_{\ddot{X}_u}}}\right)^{1/2},\ v_{\psi_{X_u}}=\frac{\sigma_{\psi_{X_u}}}{\mu_{\psi_{X_u}}} \quad (19)$$

Likely, from equation (16) and by means of the ASM, the mean value $\mu_{\psi_{X_u}-\psi_{X_s}}$ and standard deviation $\sigma_{\psi_{X_u}-\psi_{X_s}}$ and standard deviation $v_{\psi_{X_u}-\psi_{X_s}}$ of working space index can be obtained

$$\mu_{\psi_{X_u}-\psi_{X_S}}=\left(\frac{1}{2}\sqrt{4\mu^2_{\psi^2_{\ddot{X}_u}}-2\sigma^2_{\psi^2_{\ddot{X}_u}}}\right)^{1/2}-\left(\frac{1}{2}\sqrt{4\mu^2_{\psi^2_{\ddot{X}_S}}-2\sigma^2_{\psi^2_{\ddot{X}_S}}}\right)^{1/2} \quad (20)$$

$$\sigma_{\psi_{X_u}-\psi_{X_S}} = \left\{\left(\mu_{\psi^2_{\ddot{X}_u}} - \frac{1}{2}\sqrt{4\mu^2_{\psi^2_{\ddot{X}_u}} - 2\sigma^2_{\psi^2_{\ddot{X}_u}}}\right) + \left(\mu_{\psi^2_{\ddot{X}_s}} - \frac{1}{2}\sqrt{4\mu^2_{\psi^2_{\ddot{X}_s}} - 2\sigma^2_{\psi^2_{\ddot{X}_s}}}\right)\right\}^{1/2} \tag{21}$$

$$\nu_{\psi_{X_u}-\psi_{X_S}} = \frac{\sigma_{\psi_{X_u}-\psi_{X_S}}}{\mu_{\psi_{X_u}-\psi_{X_S}}} \tag{22}$$

## 4. Numerical examples

Mean values of vehicle's parameters for this study are given in Table 1, which are typical for a lightly damped passenger car [6]. In the following simulations, $A_b = 1.4e-5(m)$ and $v = 50(m/s)$ are taken into consideration.

Table 1. The mean values of vehicle system parameters

| Parameters | Mean values |
|---|---|
| Sprung mass $m_s$ | $\mu_{m_s}$ =240kg |
| Unsprung mass $m_u$ | $\mu_{m_u}$ =36kg |
| Suspension damping coefficient $c_s$ | $\mu_{c_s}$ =980Ns/m |
| Suspension stiffness $k_s$ | $\mu_{k_s}$ =16,000N/m |
| Tire stiffness $k_t$ | $\mu_{k_t}$ =160,000N/m |

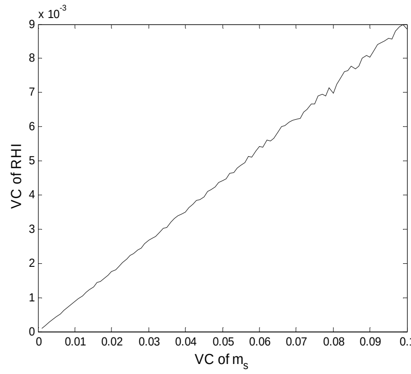
In order to investigate the effect of the uncertainty of random vehicle parameters on the vehicle's performance indices, the values of variation coefficients of random vehicle's parameters are respectively taken as different groups. According to the preceding method and expressions, the corresponding computational programs are designed. The computational results of ride comfort index are given in Table 2. The varying curves of the relationship between the variation coefficient (VC) of the road holding index (RHI) and the VC of vehicle's parameters are shown in Fig. 2, as well as the relationship between the VC of working space index (WCI) and vehicle's parameters are given in Fig. 3. In addition, the ride comfort indices obtained by the MCSM are also given in Table 2 in which 50000 simulations are used.

From Table 2, it can be seen that the standard deviation and variation coefficient of vehicle's ride comfort index from the method presented in this paper are bigger than the

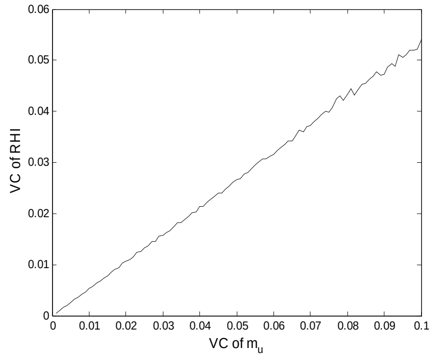
Table 2. The computational results of ride comfort index (*MCSM, unit: mm/s$^2$)

| Model | $\mu_{\psi_{\ddot{X}s}}$ | $\sigma_{\psi_{\ddot{X}s}}$ | $\nu_{\psi_{\ddot{X}s}}$ |
|---|---|---|---|
| $\nu_{m_s}$ =0.05 $\nu_{m_u}=\nu_{c_s}=\nu_{k_s}=\nu_{k_t}$ =0 | 331.0523 | 7.4195 | 0.0224 |
| $\nu_{m_u}$ =0.05 $\nu_{m_s}=\nu_{c_s}=\nu_{k_s}=\nu_{k_t}$ =0 | 331.0158 | 4.6864 | 0.0142 |
| $\nu_{c_s}$ =0.05 $\nu_{m_s}=\nu_{m_u}=\nu_{k_s}=\nu_{k_t}$ =0 | 331.0706 | 1.8388 | 0.0055 |
| $\nu_{k_s}$ =0.05 $\nu_{m_s}=\nu_{m_u}=\nu_{c_s}=\nu_{k_t}$ =0 | 331.2349 | 7.4001 | 0.0223 |
| $\nu_{k_t}$ =0.05 $\nu_{m_s}=\nu_{m_u}=\nu_{c_s}=\nu_{k_s}$ =0 | 331.1527 | 1.8800 | 0.0057 |
| $\nu_{m_s}=\nu_{m_u}=\nu_{c_s}=\nu_{k_s}=\nu_{k_t}$ =0.5 | 332.4216 | 21.8568 | 0.0657 |
| $\nu_{m_s}=\nu_{m_u}=\nu_{c_s}=\nu_{k_s}=\nu_{k_t}$ =0.1 | 336.1735 | 48.0215 | 0.1428 |
| * $\nu_{m_s}=\nu_{m_u}=\nu_{c_s}=\nu_{k_s}=\nu_{k_t}$ =0.1 | 336.0366 | 47.7869 | 0.1422 |

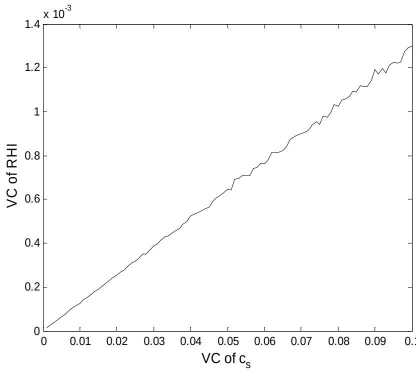
results obtained from the MCSM, that is, the results obtained by the RVFMM and ASM are more conservative. The uncertainty of sprung mass and suspension stiffness produces significant effect on the ride comfort index. Comparing with the case that only one of the uncertainty of sprung mass, unsprung mass, suspension damping, suspension and tyre stiffness is taken into account, the change of the vehicle's ride comfort index is greater when their uncertainty are considered simultaneously. Along with the increase of the variation coefficients of vehicle's parameters, the uncertainty of vehicle's ride comfort index will increase.



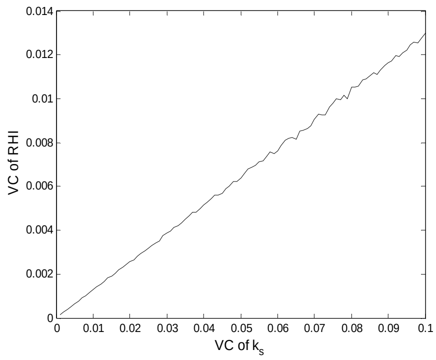

(a) Relationship between RHI and $m_s$



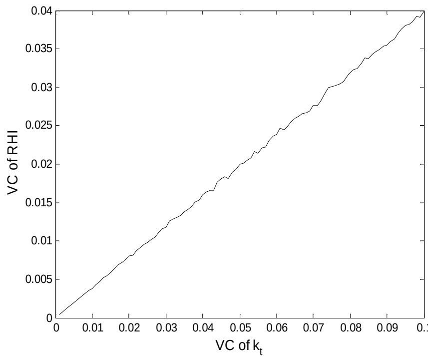

(b) Relationship between RHI and $m_u$



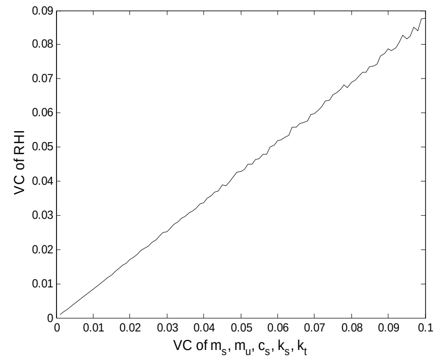

(c) Relationship between RHI and $c_s$




(d) Relationship between RHI and $k_s$



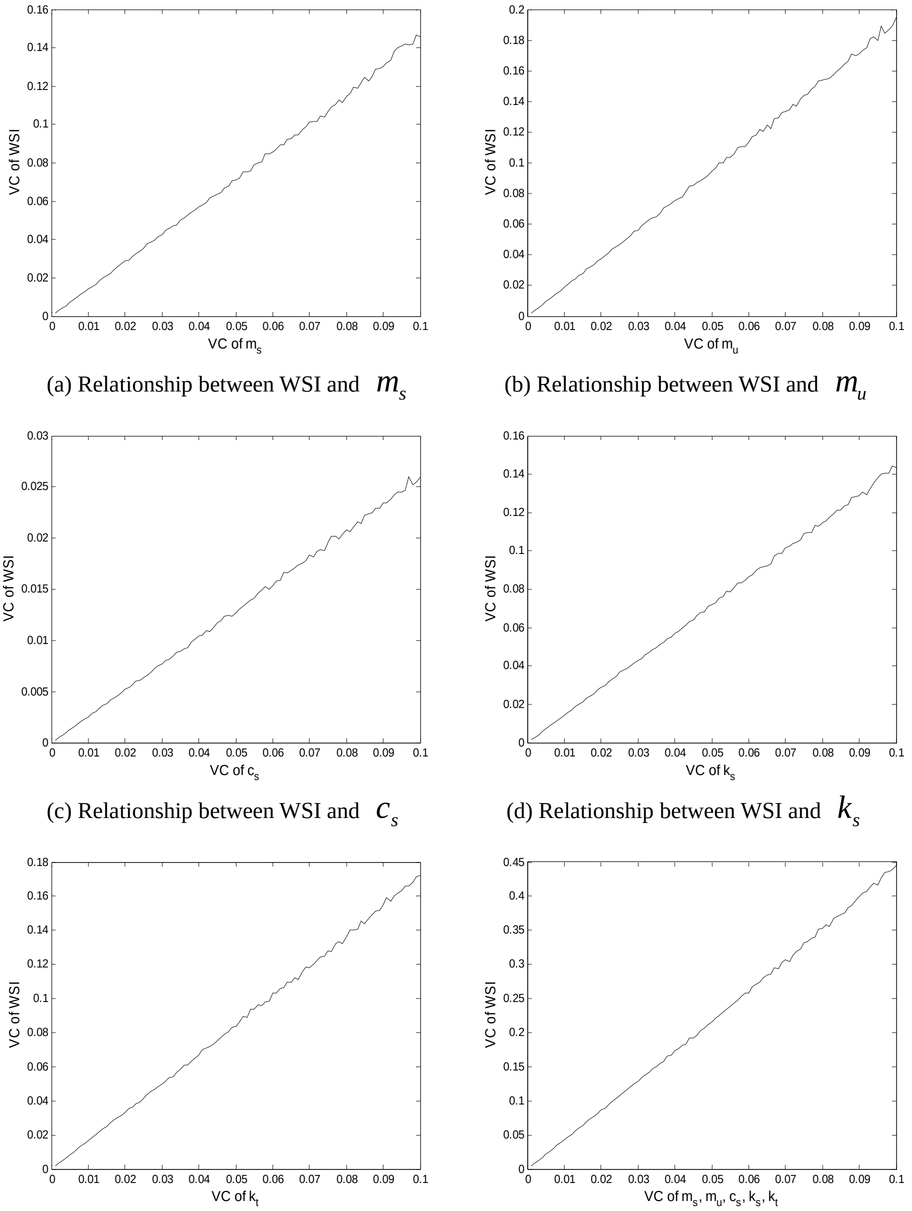

(e) Relationship between RHI and $k_t$




(f) Relationship between RHI and all parameters

Fig. 2  Computational results of road holding index (tire deflection).

Fig. 2 shows that the uncertainties of unsprung mass and tire stiffness affect the vehcile's road holding index significantly. However, the change of the RHI is almost independent of the sprung mass and suspension damping. Fig. 3 shows that the randomness of sprung mass, unsprung mass, suspension and tire stiffness produce similar effect on uncertainty of the working space index. Again, the change of the vehicle's dynamic displacement responses is greater when the uncertainty of all vehicle's parameters are considered simultaneously. In general, the uncertainty of the RHI and WSI will increase gradually along with the increase of the variation coefficients of vehicle's parameters, but the relationships between the variation coefficient of performance indices and parameters are nonlinear.




(a) Relationship between WSI and $m_s$ (b) Relationship between WSI and $m_u$

(c) Relationship between WSI and $c_s$ (d) Relationship between WSI and $k_s$

(e) Relationship between WSI and $k_t$ (f) Relationship between WSI and all parameters

Fig. 3 Computational results of working space index (suspension travel).

## 5. Conclusions

In this paper, a random quarter-car model is used to investigate the dynamic performance indices of cars with uncertainty. Computational expressions of the mean value, standard deviation and variation coefficient of the ride comfort, road holding and working space indices have been developed. The random responses of stochastic vehicles are obtained expediently. This method will also be applied to the dynamic performance analysis of random vehicles by using stochastic half-car and full-car models.

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