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Understanding the dynamic behaviour of a tennis racket under play conditions

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Abstract

The ‘feel’ of tennis rackets is of increasing importance to manufacturers seeking product differentiation in a context where further performance enhancements are prevented by a combination of mechanical limits and regulations imposed to protect the integrity of the sport. Vibrations excited during a shot contribute greatly to the perception of ‘feel’. Previous studies have been reported but none has covered the full set of mode families or the frequency range in this study. In-plane vibrations associated with the routine use of topspin shots in modern tennis have not been documented so far in the literature. To consider modal behaviour, multiple measurements during play conditions are required but this is practically impossible. This paper proposes an alternative approach and successfully relates a comprehensive modal analysis on a freely suspended racket to vibration measurements under play conditions. This is achieved through an intermediate stage comprising a necessarily more limited modal analysis on a hand-gripped racket and use of the mass modification modal analysis tool. This stage confirmed the prevailing view that hand-gripping can be considered as a mass modification distributed along the handle of the freely suspended racket but the associated mass was much lower than that of an actual hand and the hand also increased the damping ratio of frame modes significantly. Furthermore, in frame vibration measurements during forehand groundstrokes, a greater reduction in bending mode frequencies was observed, consistent with a mass-loading of around 25% of the actual hand as a consequence of the tighter grip. In these play tests, the first two bending modes, the first torsional mode, the first eight stringbed modes, the first three hoop modes and the third in-plane bending mode were identified, with the stringbed modes being particularly prominent.

Introduction

During the second half of the twentieth century, tennis rackets changed dramatically due to improvements in manufacturing techniques and the introduction of new materials [1]. The

International Tennis Federation (ITF) was concerned that these equipment changes could have a detrimental effect on the nature of the sport and reduce its market appeal [2]. As a consequence, the ITF introduced rules and regulations [3] which have made it harder for manufacturers to enhance racket performance with innovative designs, for example to develop rackets with movable masses to enable tuning of racket static and dynamic characteristics. In view of this, tennis racket manufacturers and other sports equipment companies now seek to design products that are superior to their competitors' products in areas other than simply performance, often characterised by parameters such as coefficient of restitution, stiffness, moment of inertia etc..

Barrass *et al.* [4] theorized that, in order for players to perform to their full potential, they must 'feel' comfortable with their equipment. Hocknell *et al.* [5] defined 'feel' as the "physical and psychological feedback" experienced by a player in hitting a shot; a player receives *feedback* from the position of their limbs as well as visual, tactile and auditory sensations. Roberts *et al.* [6] suggested that the sensations received during the shot by the player's tactile and auditory receptors were the most important for evaluating the perceived quality of the item of sports equipment. It is proposed, therefore, that tennis racket manufacturers should be concerned with developing rackets with vibration and sound qualities regarded as desirable by the user. Since significant components of the sound originate with the mechanical vibrations of the racket, this paper will explore the dynamic mechanical behaviour of tennis rackets.

Analysis of the vibrations excited in tennis rackets and sports equipment is not a new endeavour. In 1976, Hatze [7] used strain gauges mounted onto a wooden racket to measure the effect of grip tightness on racket vibrations post-impact. Later, Brody [8] identified that the first bending frequency of graphite tennis racket frames is typically between 120 and 200 Hz, while the strings vibrate at higher frequencies due to their lower mass. It is generally accepted that the fundamental frequency of the frame is responsible for the discomfort associated with unwanted vibrations due to its higher amplitude modal response relative to other natural frequencies [9] and the relatively greater human sensitivity to vibrations with frequencies in the 50-200 Hz range [10,11]. Hennig [12] investigated the transmission of vibration from a racket into the player's hand and forearm and found that the frequency of the measured vibrations on the hand and arm closely matched the racket's first bending frequency. Kawazoe [13] has developed models to predict vibration magnitude experienced by the player; the models, which are based on Experimental Modal Analysis (EMA) data, suggest that impacts using light modern rackets cause vibrations of greater amplitude at the gripping location due to the fact that the lighter rackets are relatively head-heavy [14]. The result of such revised mass distribution is that the region in which the nodal lines of the significant vibration modes, i.e. that in which there is a reduced local vibration level, shifts up the handle of the racket, away from the gripping location.

The effect of the ball impact location on the vibrational response of a racket has been investigated by numerous authors [15,24,25]. Brody [24] suggests that an impact in the centre of the stringbed generates the smallest vibrations in the racket frame and that these vibrations become progressively more significant as the impact location is moved further away from the stringbed centre while remaining on the vertical centre-line of the racket. Stroede [15] indicates that this is due to the location of the node line of the fundamental out-of-plane mode being close to the centre of the stringbed. Barrass [25] went a stage further by analysing the frequency content of the vibrations generated; the results indicated that the player "excited little or no vibration" at the frequency of the first bending mode when the impact was in the centre of the racket.
While the majority of research to date has focused on the racket's fundamental frequency, investigations using accelerometers placed on a racket frame during a typical tennis impact have revealed that frequencies up to 1500 Hz are excited during a typical forehand drive [15,16]. Given that humans are able to sense vibrations at frequencies up to 1000 Hz through tactile receptors [10] and can hear frequencies between a maximum range of 20 – 20000 Hz [17] it is important to analyse the dynamic behaviour of a racket beyond the fundamental frequency. Several studies have been conducted where the mode shapes of higher natural frequencies have been identified; Vethecan *et al.* [18] and Iwatsubo *et al.* [19] each identified the 2nd bending mode and 1st torsional mode. Korte *et al.* [20] investigated the differences between conventional midsize rackets and the, then new, *wide body* rackets and found that the wide body racket's bending and torsional natural frequencies were far higher than those of the midsize racket. More recently, Timme *et al.* [21] used electronic speckle-pattern interferometry to identify twelve mode shapes of a clamped tennis racket up to 1500 Hz, including seven stringbed modes, the first of which was found to be at 562 Hz. Computational modal analyses of finite element models of tennis rackets have been performed by a number of researchers [22,23] and, while the mode shapes of the lower frequency

modes correlate well with experimental data, far more mode shapes have been identified up to a frequency of 745 Hz.

Previous EMA on tennis rackets has only considered vibration modes with predominantly out-of-plane motion. However, with modern tennis being dominated by aggressive topspin shots [26], in which work is done on the ball by a non-normal interaction between ball and stringbed, *in-plane* modes of vibration are being excited more readily than ever before. Previous research [18,19] has shown that it is possible to damp *out-of-plane* vibrations once the mode shapes of the unwanted frequencies are identified using strategically placed tuned dampers. If the mode shapes of the in-plane vibrations are also known then similar devices could be used to damp these.

All of the EMA investigations referenced thus far have either been performed on grip-clamped rackets or freely suspended rackets. Whilst Timme *et al*. may have conducted the most comprehensive modal analysis of a racket to date, a clamped racket was a necessity for the optical measurement system and, as they acknowledged, this will have significantly affected the vibrational behaviour compared to a hand-held racket. Kotze *et al*. [27] state that neither fixed-free nor free-free boundary conditions exactly represent that of a hand-gripped racket; of the two, a freely suspended racket provides a much better representation of a hand-gripped racket but its fundamental frequency is approximately 10% greater and the nodes of the vibration mode shapes are shifted slightly away from the end-points of the racket [28]. Both Carsolo *et al*. [22] and Kawazoe and Yoshinari [13] have attempted to simulate the effect of the hand in their mathematical models by adding mass at the grip. Kawazoe and Yoshinari demonstrated the influence of adding a 1.0 kg mass to the handle on the shock acceleration at the grip for different impact locations but didn't explain why this particular mass had been used or whether it had been optimised using experimental data. Casolo *et al*. [22] reported the effect of adding a lumped mass at the handle on the first five mode shapes of a tennis racket using finite element analysis;the lumped mass, which wasn't specified but stated to be "equivalent to that of a player", was found to reduce the frequencies of the frame modes by circa 10 %. The effective mass of the hand is, therefore, not clearly understood, nor is how the hand affects the frequencies and mode shapes of higher order modes that are excited during a tennis impact as the position of the hand relative to node lines is different for each mode.

Banwell *et al.* [10] used EMA data from a freely suspended racket to identify the mode shapes associated with the frequencies excited during a tennis shot measured using accelerometers placed on the racket frame. All frequencies excited by the ball impact, apart from one, were attributed to racket mode shapes, but the authors could not comment on the correlation between mode shapes of a hand-gripped racket and a freely suspended racket.

The ultimate motivation for this study is, therefore, to identify the modes associated with vibrations measured from a racket during normal tennis play. This requires investigation of modes in three dimensions and across a frequency range up to 1500 Hz; both of these aspects represent advances relative to previous studies. A number of novel steps are required to achieve this aim. Multipoint measurements during play conditions are not practical and so this paper shows how a limited set of in-play measurements can be combined with modal analysis to provide the necessary insight. The modal analysis is itself challenging; the hand is known to affect racket modal behaviour and so this paper will look at modal analysis on both freely suspended and hand-gripped rackets and then simulate the addition of mass at the handle to compare a mass modified version of the freely suspended racket modal analysis with a hand-gripped racket modal analysis. Stringing is an important factor and so the variability in racket behaviour attributable to differences between nominally identical strings is also considered. Furthermore, consideration is given to experimental arrangements suited to the racket tests with the different support conditions.

Experimental Modal Analysis

Although frequencies up to 3000 Hz are excited in the frame of the racket by an impact between a tennis racket and a ball, the majority of energy is concentrated in the region below 1500 Hz [16]. For this reason, this study was designed to investigate modes up to at least 1500 Hz.

Freely suspended racket experimental arrangement

The racket frame used throughout this study was a HEAD AirFlow 7, with an unstrung mass of 228 g and a headsize of 740 cm^2. The racket was strung with HEAD RIP Control strings at a tension of 245 N (55 lbs). The RIP Control string is described as a multifilament string with a core of flexible fibres encased by a stiffer protective cover.

The racket geometry was discretised by positioning small (3 mm diameter) circular markers (adhesive paper with white points on a black background) on the racket frame at the chosen response locations. The circular markers were to be used directly as the measurement points thereby removing any discrepancy between the position of the points in the modal model and the actual measurement points. 38 markers were placed around the frame and 87 markers directly on the stringbed intersections as shown in Fig. 1, which also illustrates the wireframe model and the XYZ global axes. Experience was used in this study to determine the preferred locations of the markers and the number of points provided sufficient spatial resolution to investigate mode shapes up to 1500 Hz. A two-dimensional (2D) wire-frame model was subsequently created from the "marked-up" frame using an optical coordinate measurement technique (GOM Tritop [29]).

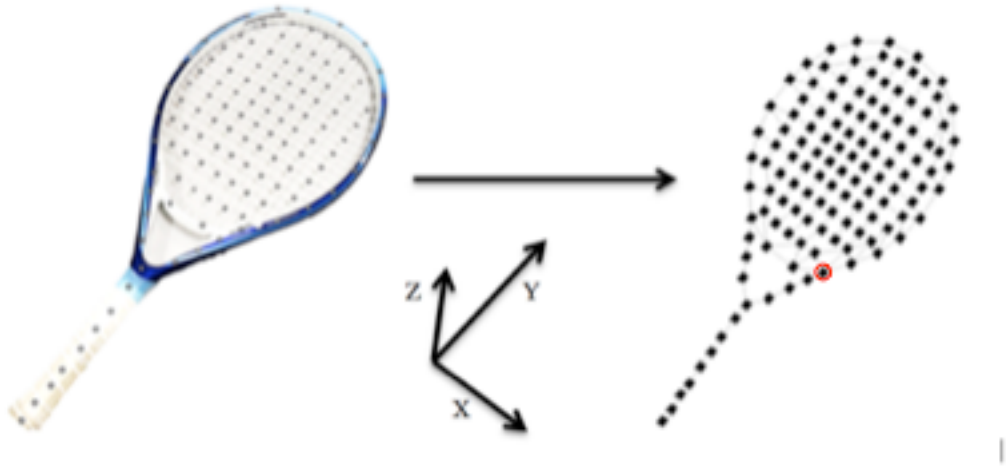

Fig. 1. Tennis racket with optical markers and corresponding wireframe model with global axis system; shaker excitation location highlighted

Initially the racket was excited with a modally tuned impact hammer (Bruel & Kjaer Type 8206) and the response measured with a lightweight accelerometer using the roving excitation technique. It soon became apparent, however, that the impact hammer used was not capable of adequately exciting the stringbed up to 1500 Hz due to the lower combined stiffness of the strings and hammer tip compared to that of the frame and hammer tip.

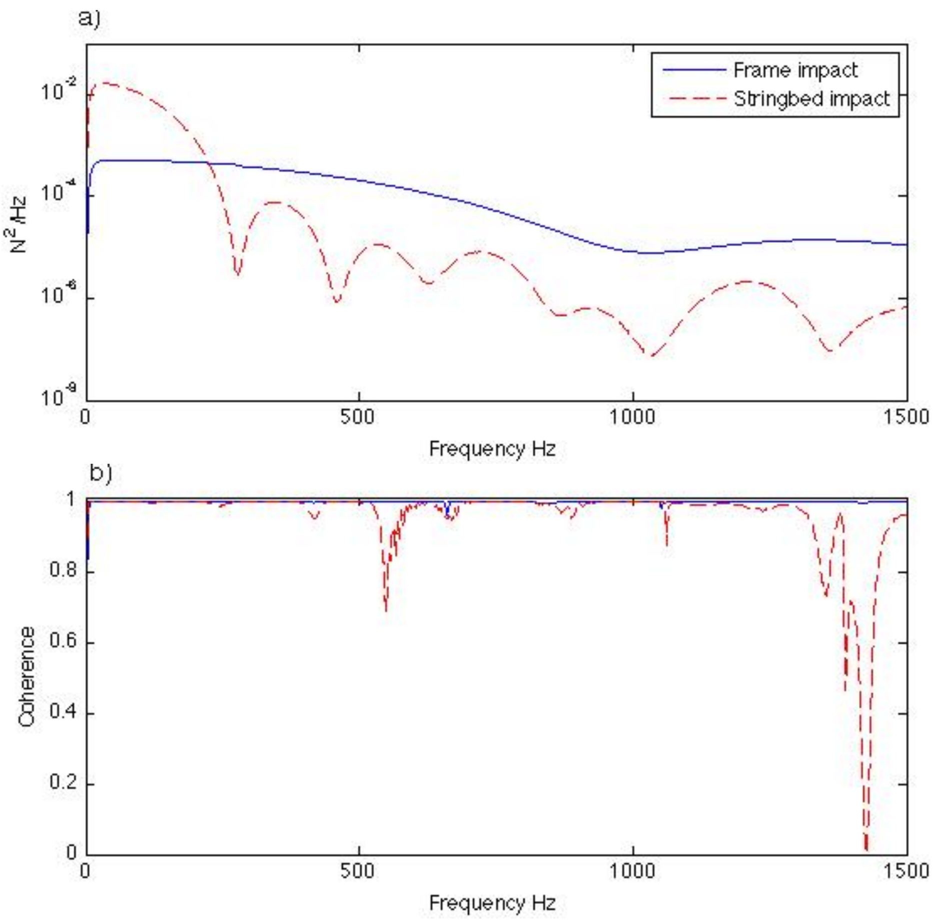

Fig. 2. Sum autopower spectra a) and average Coherence function b) from impacts on frame and on stringbed

Fig.2a) illustrates the autopower spectra of impacts with the frame and the stringbed and Fig 2b) shows the corresponding coherence plots, which indicates that the coherence from the stringbed impacts decreases dramatically as the frequency increases towards 1500 Hz. Obtaining repeatable excitation of the stringbed with the hammer also proved difficult due to the discontinuous nature of the stringbed surface. To enable more consistent excitation in the full survey, an electromagnetic shaker attached to the racket via a stinger and a scanning laser Doppler vibrometer (SLDV) were used to excite and measure the response of the racket respectively using the roving response measurement technique. This setup significantly decreased the total time taken to complete the measurements since the SLDV can be programmed to move its beam between the response locations automatically. Care must be taken because velocity in the direction of the incident laser beam is measured and, for large measurement regions located close to the instrument, this direction can vary significantly from point-to-point [30].

Although the results from impact hammer excitation were not used in the final analysis, these data were processed and revealed the first eight natural frequencies and their mode shapes. This information was useful i) for determining a suitable driving point for the electromagnetic shaker (to ensure that all natural frequencies of interest would be sufficiently excited) and ii) for direct comparison with the data captured during the full survey (to ensure that the attachment of the shaker did not introduce significant local mass loading and/or stiffening effects.)

The shaker was configured to exert a force normal to the plane of the stringbed (Z direction in Fig. 1) at the frame position where the yoke joins the frame, as shown in Fig. 3. The racket was freely suspended with nylon threads from an extruded section of the aluminium structure, which was designed to allow the racket to be suspended in vertical or horizontal orientations. This enabled the response at various measurement locations on the frame to be readily measured in three orthogonal directions without changing the location of the SLDV. The response of the stringbed could only be measured normal to the stringbed plane as the frame obstructed the line of sight of the laser when measuring *in-plane.* While the response of the racket was measured both *in-plane* and *out-of-plane*, the excitation force was always normal to the stringbed (i.e. out-of-plane). Preliminary testing confirmed that the in-plane modes were also excited by this method.

The optical markers used to define the racket discretised geometry were also used as the measurement points as they provided the SLDV with a suitably reflective surface. The markers on the stringbed were not used for measurements, however, since there was discrepancy between the marker and string velocity. A sufficiently strong SLDV signal was obtained with the laser beam focused directly on the stringbed intersections without any surface treatment.

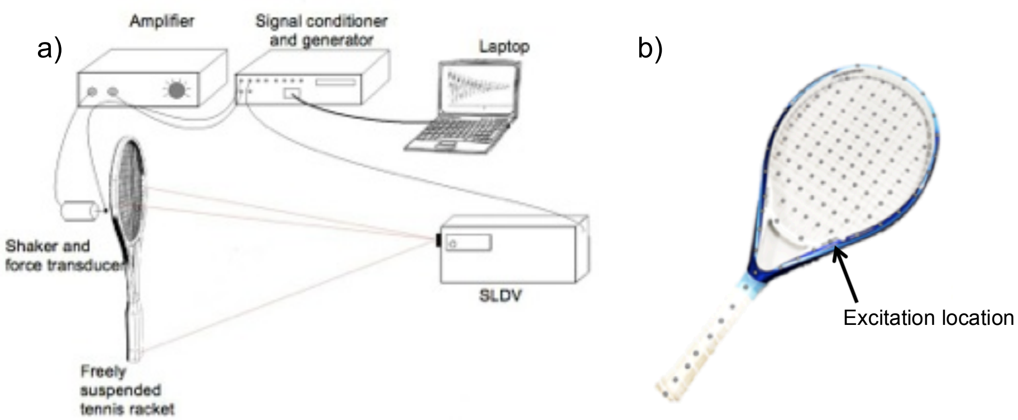

Fig. 3. Experimental arrangement a) schematic diagram and b) excitation location

Freely suspended racket experimental results

LMS Test.Lab software was used with an LMS SCADAS Mobile acquisition system to acquire the data as well as to act as a signal generator. Linear averaged frequency response functions (FRF's) were calculated from 20 individual measurements (no overlap, sample length of 1.3 sec, sample frequency 6.4 kHz). Burst random (white noise) excitation with a 0.05 sec ramp time was used to excite the racket for 50% of sample length.

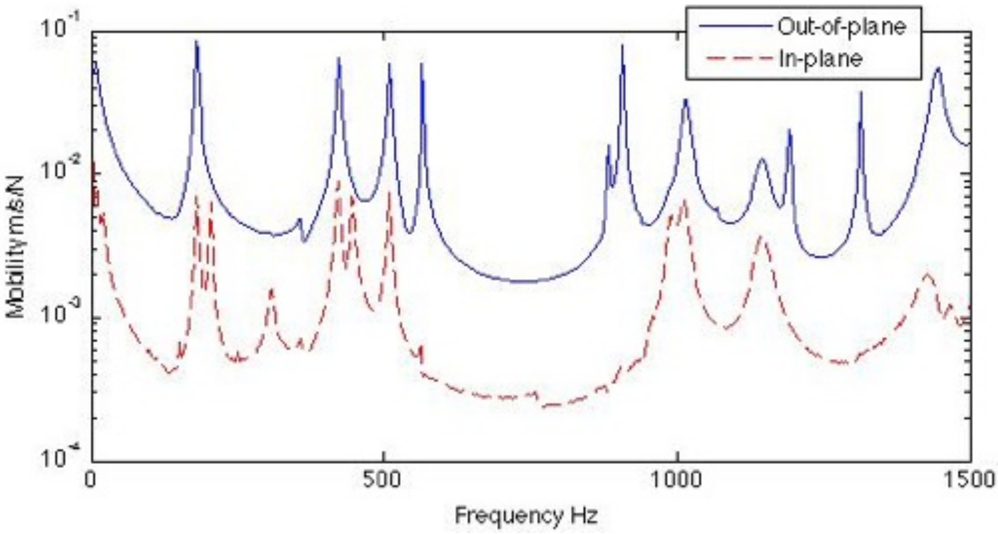

Fig. 4. Sum FRF's measured out-of-plane and in-plane from AirFlow 7 strung with Sonic Pro strings at 245 N (55 lbs.)

The sum FRF's of the AirFlow 7 (AF7) strung with Sonic Pro strings at a tension of 245 N are displayed in Fig. 4. The level of the out-of-plane FRF is approximately an order of magnitude greater than the in-plane equivalent because the intended racket excitation was in the out-of-plane direction. Even though the signal-to-noise ratio is lower in the in-plane data than in the out-of-plane data, in-plane modes of vibration appear clearly, for example at 203, 442 and 974 Hz. In addition to the content specifically due to the in-plane modes, it is possible to see that the out-of-plane modes are also evident in the in-plane FRF. This is due to the fact that all of the mode shapes, whether in- or out-of-plane involve motion in all three direction; the descriptions in- and out-of-plane is simply chosen in accordance with the motion direction(s) that dominate.

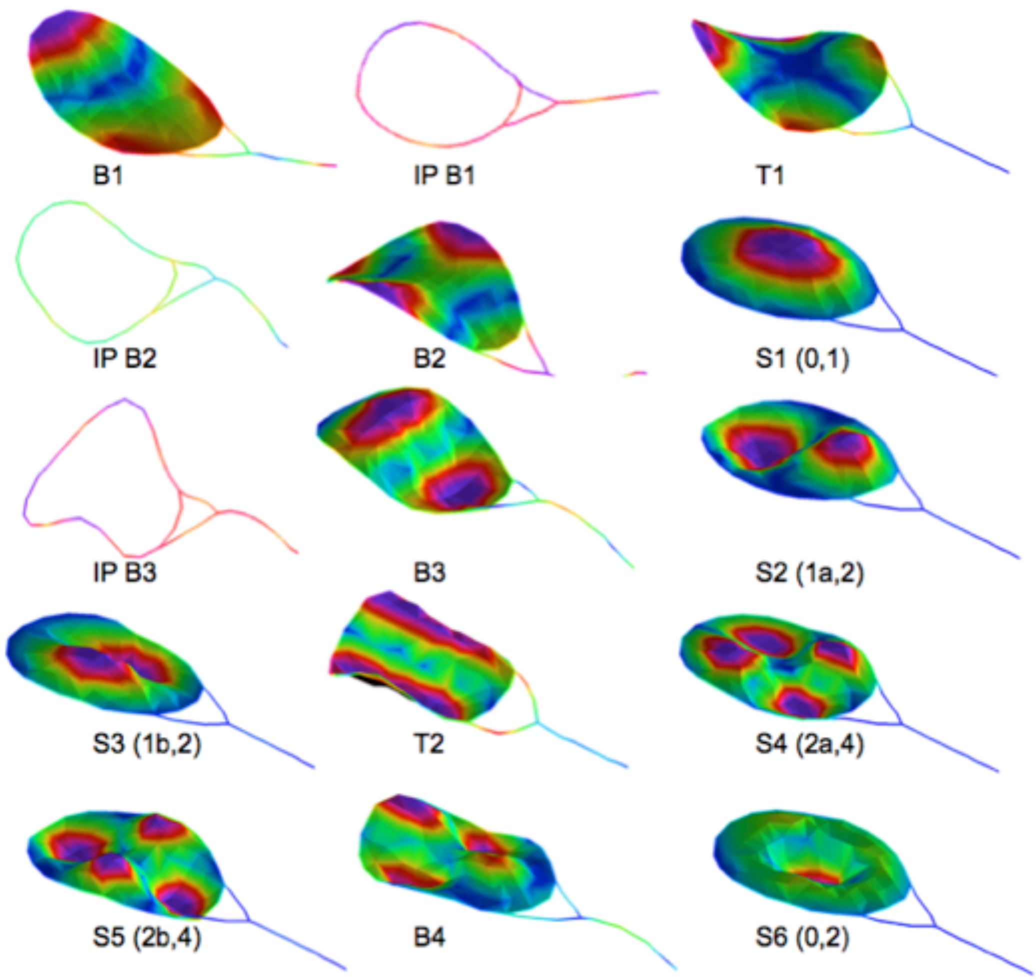

Fig. 5. Mode shapes of the tennis racket up to ≈ 1600 Hz

The modes in Fig. 5 are labelled **B**, **IP B**, **T** or **S** identifying (out-of-plane) **B**ending, **I**n-**P**lane **B**ending**,** **T**orsional or **S**tringbed modes. The bending and torsional modes are relatively easy to visualize, as the racket behaves in a similar manner to a freely suspended vibrating beam. For stringbed mode shapes, the labelling method used by Timme *et al.* [21] has been adopted. The nomenclature for labelling is (x,y), where x is the number of nodal lines and y is the number of nodal circles. Since the stringbed area is not circular, there will be mode shape pairs with the same number of nodal lines and circles but slightly different frequencies; these modes are differentiated by letters a and b to show the lower and higher frequency modes in each pair.

In total 15 modes were identified below 1600 Hz: nine frame modes and six stringbed modes. The frame mode family variants include out-of-plane bending, in-plane bending and torsional modes. The in-plane modes, being less well excited, were only readily identifiable once the out-of-plane velocity measurements were omitted from the data matrix within the software. Since this data set was to be used to identify the modes excited during an impact with the same racket restrung with the same string at the same tension, there was therefore a need to investigate the variation associated with nominally identical stringing. Consequently a subsequent EMA experiment was performed that used the same racket and was restrung with the same string at the same tension, the experiment followed the exact same protocol as previously described however in-plane modes were not measured. The differences between the natural frequencies of the two rackets are shown in Table I. The maximum percentage difference recorded was for T1 at 1.44 %, while S1 was determined to be of the same frequency (0 decimal places). This shows that the dynamic behaviour of the racket will remain relatively consistent if it is strung at the same tension.

Table I. Natural frequencies and damping ratios of the first 15 modes from two nominally identically strung rackets. Natural frequencies displayed in bold and damping ratios displayed in italics.

	Stringing 1		**Stringing 2**		**Freq. Diff. %**
Mode	***f.* Hz**	***ζ.* %**	***f.* Hz**	***ζ.* %**	***f.* %**
B1	**179**	*0.44*	**178**	*0.72*	0.56
B2	**511**	*0.37*	**511**	*0.45*	0.00
B3	**1000**	*0.58*	**1004**	*0.55*	0.40
B4	**1499**	*1.25*	**1501**	*1.35*	0.13
IP B1	**203**	*0.84*	**-**	-	-
IP B2	**442**	*0.61*	**-**	-	-
IP B3	**974**	*0.73*	**-**	-	-
T1	**416**	*0.54*	**422**	*0.52*	1.44
T2	**1152**	*0.79*	**1152**	*0.94*	0.00
S1	**667**	*0.09*	**667**	*0.11*	0.00
S2	**1065**	*0.16*	**1065**	*0.20*	0.00
S3	**1069**	*0.14*	**1073**	*0.16*	0.37
S4	**1395**	*0.18*	**1408**	*0.27*	0.93
S5	**1416**	*0.19*	**1414**	*0.18*	0.14
S6	**1567**	*0.13*	**1559**	*0.14*	0.51

Hand-gripped racket experimental arrangement

One of the main difficulties associated with EMA of a hand-gripped racket is the inherent variability of the boundary condition that the hand-grip represents. Changes in the grip location and pressure are inevitable; the effect of these variables has not been investigated in this paper. Additionally, a human will be incapable of holding the racket sufficiently still to prevent variation in mass or stiffness loading from the shaker assembly or to allow the SLDV to reliably address the measurement locations.

For these reasons, two lightweight (0.6 g) charge accelerometers (B&K Type 4517-C) were used in this experiment to simultaneously measure the response of the racket in two directions (one out-of-plane and one in-plane) at 38 points around the racket frame as shown in Fig. 6. Response measurements were not measured from the stringbed as even these lightweight accelerometers significantly alter the modal properties of the stringbed, however the frequencies of the stringbed modes could be measured from the frame mounted accelerometers. Far fewer measurement points were used but those selected were a subset of the points from the freely suspended racket experiment. The accelerometer used to capture the in-plane response was always mounted at a tangent to the frame and, as such, its measurements included varying proportions of the two in-plane components according to its location. To accommodate this, Euler angles were defined at each of the measurement locations and during subsequent software data processing, the measured FRF's were resolved into each of the two global coordinate directions. The racket was excited using an instrumented modally tuned impact hammer (B&K Type 8206) at three locations to adequately excite out-of-plane, in-plane and stringbed modes (Fig. 6), providing 228 FRF's. During this investigation one experienced subject was used to hold the racket as if they were about to play a forehand shot and to try to maintain a consistent pressure throughout the entire experimental procedure.

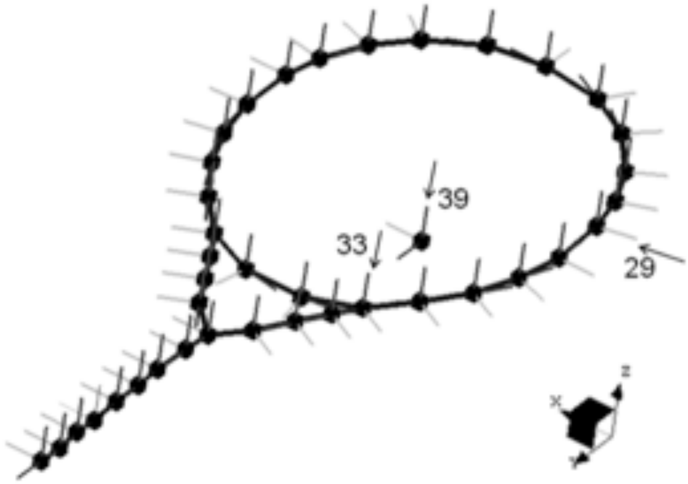

Fig. 6. Discretised geometry of the hand-gripped racket with the three excitation points and arrows indicating direction of impact defined.

Hand-gripped racket experimental results

FRF's were the linear average of five individual measurements (sampling frequency 6.4 kHz, sample length 0.64 sec with a 0.12 sec pre-trigger). Recorded signals had decayed to zero by the end of the acquisition period. Despite the variability of the hand-gripped boundary condition, the LMS Test.Lab Modal Analysis software identified numerous stable modes. Fig. 7a shows the sum of the FRF's collected. Equivalent data from the freely suspended racket indicated that there was high modal density, particularly around 1000 Hz. To improve the identification of the various modes, the data were split into three subsets, one for each impact position. The sums of each of these subsets are shown in Fig. 7b; using this processing it was possible to identify many more modes of vibration than would otherwise have been possible.

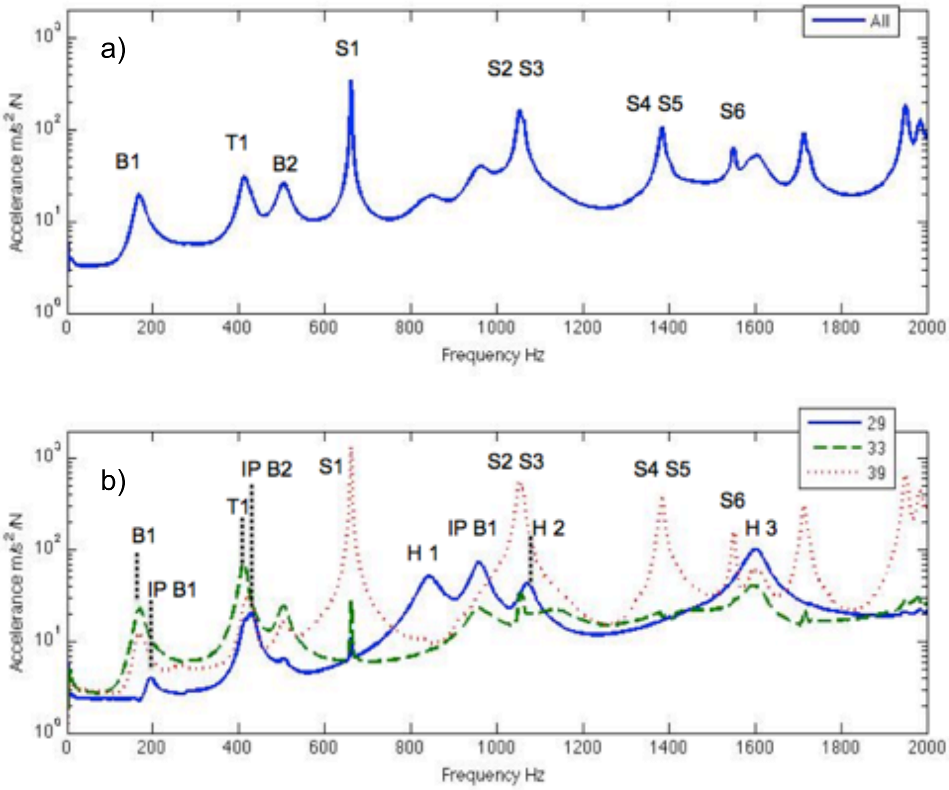

Fig. 7. Sum of a) all FRF's and b) FRF's for each impact location.

From visual inspection, the mode shapes obtained for the hand-gripped racket were very similar to those of the freely suspended racket and examples are shown in Fig. 8. All modes for the freely suspended racket were also identified for the hand-gripped racket. Three additional mode shapes were, however, identified in the hand-gripped racket data; they are annotated H1, H2 and H3 in Fig. 7 and illustrated in Fig. 9. These modes have only in-plane motion and were excited to the greatest extent with impact at point 29. They differ from in-plane bending modes as they are symmetrical around the y-axis; the handle does not vibrate and the head vibrates in a similar manner to hoop modes found in cylinders. For this reason they have been annotated as 'H'. These modes were not identifiable in the data from impact at location 33, which is the location used for shaker excitation of the freely suspended racket.

Fig. 8. Example mode shapes for a hand-gripped racket left to right: B1, T1 and S1

Fig. 9. Illustration of hand-gripped hoop modes left to right: “hoop modes” H1, H2, H3

Comparison of Modal Analysis Results

Table II. Comparison of the modal properties of a freely suspended racket and a hand-gripped racket.

Boundary Condition	Hand-gripped		Freely suspended		Freely suspended		Frequency difference hand-gripped and freely suspended / hammer %
Excitation	Hammer		Shaker		Hammer		
Mode	*f. Hz*	*ζ. %*	*f. Hz*	*ζ. %*	*f. Hz*	*ζ. %*	
B1	167	*5.83*	179	*0.44*	180	*0.65*	*7.22*
B2	502	*2.46*	511	*0.37*	510	*0.41*	*1.57*
B3	980	*3.36*	1000	*0.58*	1000	*0.60*	*2.00*
IP B1	192	*5.97*	203	*0.84*	206	*0.76*	*6.80*
IP B2	428	*2.38*	442	*0.61*	439	*0.58*	*2.51*
IP B3	962	*1.41*	974	*0.73*	969	*0.76*	*0.72*
T1	413	*1.59*	416	*0.54*	416	*0.58*	*0.72*
T2	1130	*3.93*	1152	*0.79*	1146	*0.88*	*1.40*
S1	664	*0.12*	667	*0.09*	664	*0.14*	*0.00*
S2	1051	*0.50*	1065	*0.16*	1060	*0.28*	*0.85*
S3	1060	*0.44*	1069	*0.14*	1063	*0.21*	*0.28*
S4	1387	0.37	1395	0.18	1384	0.20	0.22
S5	1402	*0.19*	1416	*0.19*	1407	*0.18*	*0.36*
S6	1551	*0.23*	1567	*0.13*	1556	*0.18*	*0.32*
H1	837	*2.60*	-	-	853	*1.61*	*1.87*
H2	1069	*1.41*	-	-	1083	*1.04*	*1.29*
H3	1600	*1.44*	-	-	1606	*1.01*	*0.37*

Table II compares the modal frequencies and damping ratios for freely suspended and hand-gripped rackets. Comparison of the natural frequencies identified for the freely suspended racket with the two different excitation techniques reveals good correlation; less than 1 % difference in all modes apart from IP B1 which is decreased by 1.5 % with the shaker experimental design. The damping ratios are also similar, suggesting that the shaker attachment method did not add significant damping to the system. Table II reveals the effect of the hand on the racket; the first out-of-plane and in-plane bending mode frequencies are reduced the most (7.2 & 6.8 %, respectively). The effect of the hand on the damping ratios is far greater. For example the damping ratio of B1 is increased from 0.44 to 5.83. The effect of the hand is greater on the frame modes than the stringbed modes. The location of the hand relative to a node line of a mode shape influences the effect of the hand on that particular mode's frequency and damping ratio.

Although the mode shapes were labelled by visually comparing the mode shapes, a Modal Assurance Criterion (MAC) was performed to establish the effect of the hand on the mode shapes and whether mode switching had occurred.

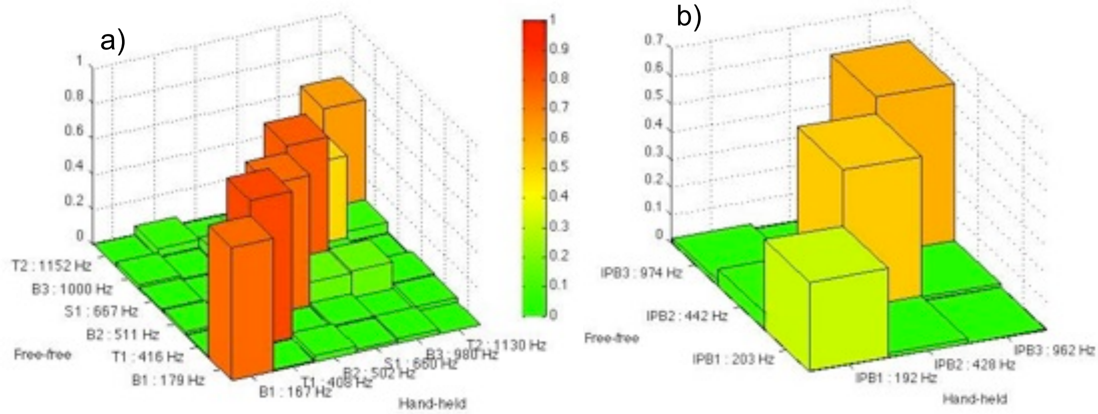

Fig 10: Modal Assurance Criterion analysis of freely suspended and hand-gripped racket a) out-of-plane modes and b) in-plane modes

Fig. 10a illustrates the MAC for the out-of-plane modes with good agreement for all modes (above 0.7), except B3 which was calculated to be 0.498. The plot also shows that mode switching has not occurred. The in-plane modes are more poorly correlated, as shown in Fig. 10b, with values of 0.32, 0.52 and 0.57 for IP B1, IP B2 and IP B3 respectively. This lower correlation may be due to the hand having a larger effect on in-plane than out-of-plane mode shapes but poor excitation of in-plane modes with the chosen shaker orientation is believed to be the main factor.

Vibrations measured from an Impact

The final stage of the experimental investigation was to compare the two modal analysis data sets from the freely suspended and hand-gripped racket with vibrations excited during a typical tennis shot. A male tennis player was recruited from Loughborough University's tennis team to participate in the study (the same subject as used for the hand-gripped modal analysis experiment). It was important that the player was of a high standard to improve the consistency in the data from shot to shot. The subject was instructed to hit 10 forehand flat shots (normal impact), aiming at a target marked onto protective netting. Tennis players normally intentionally impact the ball at non-normal angles to impart angular velocity to the ball, but in this study it was paramount that the excitation of the racket from the ball should be as consistent as possible in terms of impact location and angle of impact.

The vibrations excited in the racket frame were measured with the same accelerometer arrangement as used in the hand-gripped modal analysis experiment. The measurements were triggered with a positive slope in the signal from the out-of-plane accelerometer (sample frequency 25600 Hz, sample time 0.08 s with a 0.008 s pre-trigger).

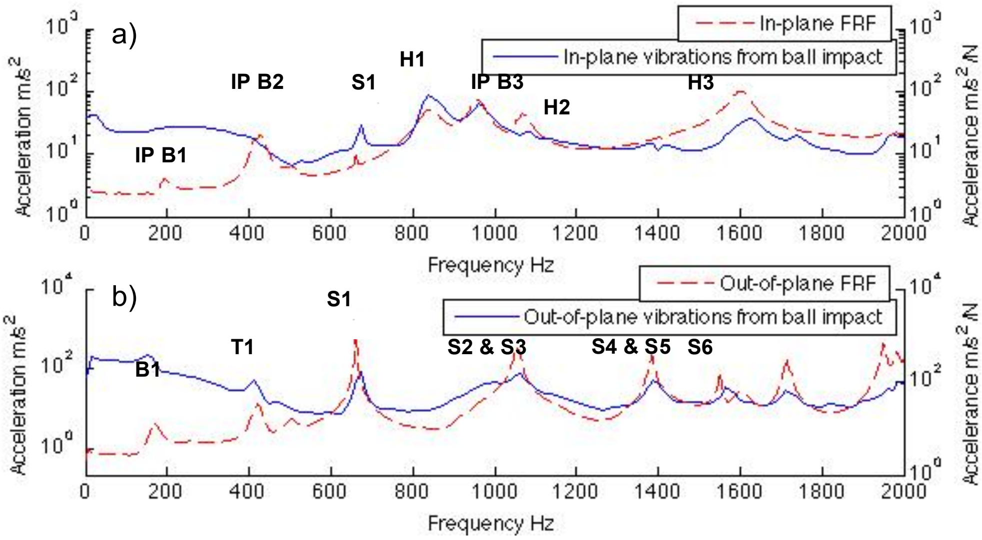

Fig. 11. Sum of FRF's and mean frequency responses of the racket frame during 10 forehand shots in the a) in-plane and b) out-of-plane direction.

The acquired data was averaged in the frequency domain across 10 impacts so that the mean response of each accelerometer could be compared with the sum of the FRF's from the hand-gripped modal analysis (Fig. 11). The frequencies excited during the shot can be attributed to discrete mode shapes. Although the amplitude of the fundamental out-of-plane bending mode (B1) does not dominate the spectrum as might be thought with a contact time of 5 ms, this result is consistent with data presented by [25]: a ball impact nearer the tip of the racket would result in greatly increased excitation of the B1 mode. Fig. 11 provides a visual comparison of the data while Table III compares the exact frequencies. The difference in the frequency of the stringbed mode is likely to be caused by inconsistencies in the stringing process rather than any differences in the boundary conditions. The frequency of B1 and B2 was lower in the hand-gripped impacts data than from the hand-gripped modal analysis testing, although this could partly due to the 12.5 Hz resolution in the impact data, which is considerably higher than the uncertainty in the modal frequency for which curve fitting has been performed. This frequency resolution may also be a reason why it was not possible to distinguish between the pairs of stringbed modes in the ball impact data (S2 and S3 as well as S4 and S6).

Although the grip force was not measured, the same subject was used for both investigations. However as Brody [8] suggests that the grip force is increased during an impact due to the recoil of the racket, it is assumed that the grip force acting on the racket could be greater during the tennis impacts than during the modal analysis. The results presented in this paper so far have quantified how the hand adds damping to the modal frequencies of the frame as well as reducing their frequencies. It is believed that it is the mass of the hand that reduces the frequencies of the structure but the hand is a complex mass comprised of many different interconnected bodies each with its own mass and damping properties. The data suggest, therefore, that the effective mass of the hand changes depending on the grip force. Literature states that the mass of a human hand is circa 0.61 % of the total mass of the body [31]; for the subject this equated to 0.61 kg.

To investigate the mass required to reduce the natural frequencies of the freely suspended racket to the values recorded for hand-gripped rackets, the structural modification tool within LMS Test.Lab was used. Mass was added to the handle of the racket by evenly distributing lumped mass across the nodes which were contacted by the hand during the hand-gripped testing. The mass was increased incrementally until the desired frequency of B1 was achieved. A total mass of 40 g was required to reduce the frequency of B1 to the frequency of first bending mode recorded from the hand-gripped modal analysis, whereas 140 g was needed to reduce the frequency by a further 19 Hz to the frequency of the first mode recorded from the hand-gripped impact data. Table III details the effect of the added mass on the other modes. IP B and H modes are not shown in the mass modification data as these modes were weakly excited in the shaker modal test and so unlikely to be reliably predicted in the mass modification calculation. This result seems to be in accordance with Cross's [32] experiment where a lumped mass of 184 g attached to the butt of the racket had the same effect on the natural frequency of the racket as a "firm" grip.

Table III. Comparison of frequencies from a freely suspended racket, hand-gripped modal and hand-gripped tennis shot with the predicted mass needed to reduce the frequency

Boundary Condition	Freely suspended Shaker	Hand-gripped hammer	Hand-gripped ball impact	40 g Mass modification	140 g Mass modification
Mode	***f. Hz***	***f. Hz***	***f. Hz***	***f. Hz***	***f. Hz***
B1	179	169	150	169	150
B2	511	502	475	503	479
B3	1000	980	-	968	856
IP B 1	203	192	-	-	-
IP B 2	442	428	-	-	-
IP B3	974	962	962	-	-
T1	416	413	412	417	417
T2	1152	1130		1147	1145
S1	667	664	675	667	667
S2	1065	1051	-	1065	1065
S3	1069	1060	1062	1069	1069
S4	1395	1387	1388	1395	1395
S5	1416	1402	-	1416	1416
S6	1567	1551	1562	1567	1567
H1	-	837	837	-	-
H2	-	1069	1088	-	-
H3	-	1600	1612	-	-

Discussion and Conclusions

Experimental modal analysis has been conducted on freely suspended and hand-gripped rackets with both frame and stringbed modes identified in three dimensions and up to 1.5 kHz. An impact hammer and accelerometer combination was best suited to the hand-gripped racket but a scanning laser Doppler vibrometer and electromagnetic shaker combination was preferred for the freely suspended racket enabling measurements directly from the stringbed. Impact hammer / accelerometer data captured from a freely suspended racket verified that the shaker attachment had minimal effect on the modal behaviour of the racket.

In all, 4 bending, 3 in-plane bending, 2 torsional, 6 stringbed and 3 hoop modes were identified in the range between 178 Hz (first bending) and 1559 Hz (sixth stringbed). Variations in modal frequencies associated with nominally identical stringing were found to be generally less than 1%. Hand-gripping of the racket resulted in the expected reductions in the frame modal frequencies but stringbed modes were largely unaffected. The results suggested that all the modes of the freely suspended racket are identifiable in hand-gripped rackets. Using MAC, the predominantly out-of-plane modes correlated well (>0.7) between the freely suspended and hand-gripped rackets while the in-plane modes, which were not well excited in the freely suspended racket tests, were less well correlated (0.3 – 0.6). For the freely suspended racket, frame modes typically had damping ratios of circa 0.5 % rising significantly in the hand-gripped condition. Stringbed modes were closer to 0.1 % and unaffected by support condition.

Mass modification to the modal model of the freely suspended racket confirmed the widely-held view that the hand-gripped racket can be considered as a freely suspended racket with mass addition around the handle. However, the added mass required to match the experimental first bending mode of the hand-gripped racket was significantly lower (by a factor of around 15) than the mass of the actual hand.

In play tests, the first two bending modes, the first torsional mode, the first two stringbed modes, the first two hoop modes and the third in-plane bending mode were identified in the frequency range from 150 Hz to just over 1 kHz. While stringbed modes are dominated by stringbed deflection, there is also motion of the frame, particularly for S1, and so these lightly damped modes feature prominently in frame measurements taken under play conditions. Further reductions in the frequency of the bending modes was observed, equivalent to a mass addition at the handle three to four times greater than that required to match the bending mode frequencies in the hand-gripped racket modal tests, buts still only some 25% of the actual hand.

Together, these data show that play-test vibration data can be reliably associated with vibration modes through a comprehensive modal analysis on a freely suspended racket and the use of the mass modification modal analysis tool. Such intimate knowledge of the modes excited under play conditions is the basis from which innovative structural modifications, in terms of mass, stiffness or damping, can be implemented to develop rackets, and other sports equipment, with enhanced ‘feel’ characteristics.

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