A comprehensive velocity sensitivity model for scanning and tracking laser Doppler vibrometry on rotating structures

Ben Halkon* and Steve Rothberg,
Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University

ABSTRACT

Recent work set out a comprehensive analysis of the velocity sensed by a single laser Doppler vibrometer beam incident in an arbitrary direction on a target that is of substantial interest in engineering – a rotating shaft requiring three translational and three rotational co-ordinates to describe its vibratory motion fully. Six separate "vibration sets", each a combination of motion parameters, appeared in the full expression for vibration velocity sensitivity and the difficulties associated with resolving individual vibration components within a complex motion were highlighted. The velocity sensitivity model can incorporate time dependent beam orientation and this is described in this paper with reference to scanning laser Doppler vibrometry. Continuously scanning strategies, in which the laser beam orientation is a continuous function of time, have recently received considerable attention, including a tracking profile in which the probe laser beam remains fixed on a single point on a target such as a rotating disc. Typically, one beam deflection mirror is driven using a cosine function whilst the other is driven with a sine function, resulting in a slightly elliptical beam trajectory. This and other more significant issues such as the effects of misalignment are easily accommodated in the velocity sensitivity model and a thorough analysis of their influence on the measured vibration signal is reported in this paper.

Keywords: laser Doppler vibrometry, scanning, tracking, vibration measurement, rotating machinery

1. INTRODUCTION

A laser Doppler vibrometer (LDV) measures target velocity in the direction of the incident laser beam; interpretation of the measurement in terms of the various target velocity components is essential. For rotating targets, pure axial vibration measurements are obtained by careful alignment of the laser beam with the rotation axis. Provided consideration is given for the laser speckle effect[1], the measurement can be obtained in the same way as for on-axis translational surface vibration. For radial vibration measurements, however, the presence of a velocity component due to the rotation itself generates significant cross-sensitivities to rotation speed fluctuation (including torsional oscillation) and motion components perpendicular to the intended measurement.

The velocity sensed by a single laser beam incident on a rotating shaft vibrating in all six degrees of freedom is made up of six separate vibration "sets", each an inseparable combination of motion parameters. By using a single LDV it is possible to isolate the translational vibration sets – two radial and one axial – but it has been shown not to be possible using a single LDV to isolate the three rotational vibration sets – pitch and yaw (including bending vibration) and torsional oscillation (including whole body roll and/or speed fluctuation)[2]. It is possible to measure the rotational vibration sets by making use of multiple laser beam configurations, with parallel beam arrangements being particularly useful[3].

This paper contains an overview of the recently developed comprehensive velocity sensitivity model for LDV measurements on rotating targets and subsequently a detailed discussion of the application of this mathematical model to circular scanning and tracking LDV measurements. In particular, the model will be used to predict the significant additional LDV output components that result in circular scanning measurements on rotating targets and an alternative scanning method that reduces the severity of these components will be introduced.

* b.j.halkon@lboro.ac.uk; phone +44 (0) 1509 227589/7566; fax +44 (0) 1509 227648; http://www.lboro.ac.uk/departments/mm/; Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University, Loughborough, LEICS., LE11 3TU, U.K.

2. VELOCITY MEASURED BY A SINGLE LASER BEAM INCIDENT ON A ROTATING SHAFT

With reference to Figure 1, the case considered is that of an axial element of a shaft of arbitrary cross-section, rotating about its spin axis whilst undergoing arbitrary, six degree of freedom vibration but this theory is equally applicable to any non-rotating, vibrating structure. A translating reference frame, *xyz*, maintains its direction at all times and has its origin, O, fixed to a point on the shaft spin axis with the undeflected shaft rotation axis defining the direction and position of the *z* axis. The time dependent unit vector $\hat{z}_R$ defines the changing direction of the spin axis, which deviates from the *z* axis as the shaft tilts. P is the instantaneous point of incidence of the laser beam on the shaft and is identified by the time dependent position vector $\vec{r}_P$.

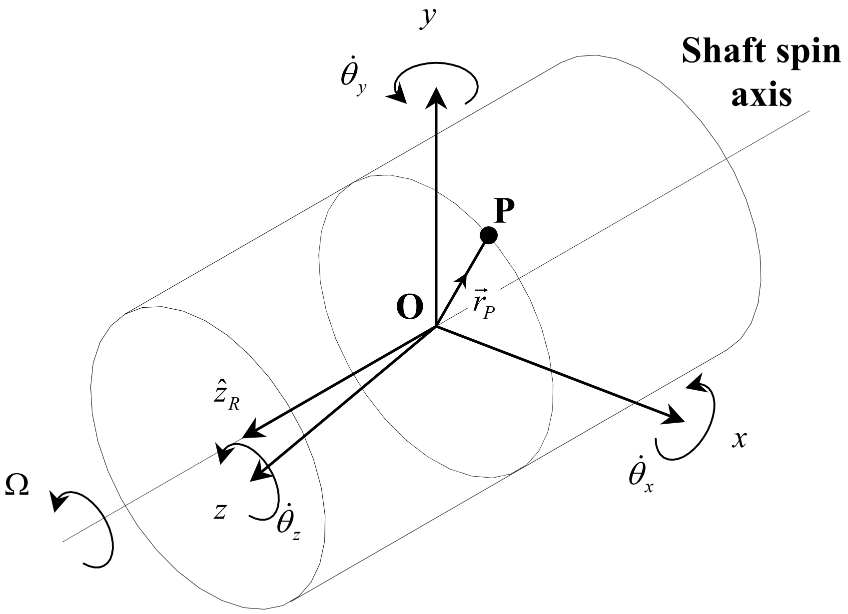


Figure 1 – Definition of axes and the point P on a vibrating and rotating structure

In the usual configuration, an LDV measures target velocity at the point of incidence in the direction of the laser beam, described by the unit vector $\hat{b}$, which, if orientated according to the angles α and β as shown in Figure 2, is given by:

$$\hat{b} = (\cos\beta\cos\alpha)\hat{x} + (\cos\beta\sin\alpha)\hat{y} - (\sin\beta)\hat{z}\,. \tag{1}$$

Provided that the illuminated axial element of the shaft can be assumed to be of rigid cross-section, the velocity measured by a laser beam, incident on the shaft surface, is given by[2]:

$$\begin{aligned} U_m = {} & \cos\beta\cos\alpha\left[\dot{x} + \left(\dot{\theta}_z + \Omega\right)y - \left(\dot{\theta}_y - \Omega\theta_x\right)z\right] \\ & + \cos\beta\sin\alpha\left[\dot{y} - \left(\dot{\theta}_z + \Omega\right)x + \left(\dot{\theta}_x + \Omega\theta_y\right)z\right] \\ & - \sin\beta\left[\dot{z} - \left(\dot{\theta}_x + \Omega\theta_y\right)y + \left(\dot{\theta}_y - \Omega\theta_x\right)x\right] \\ & - \left(y_0\sin\beta + z_0\cos\beta\sin\alpha\right)\left[\dot{\theta}_x + \Omega\theta_y\right] \\ & + \left(z_0\cos\beta\cos\alpha + x_0\sin\beta\right)\left[\dot{\theta}_y - \Omega\theta_x\right] \\ & + \left(x_0\cos\beta\sin\alpha - y_0\cos\beta\cos\alpha\right)\left[\dot{\theta}_z + \Omega\right], \end{aligned} \tag{2}$$

where $\dot{x}$, $\dot{y}$, $\dot{z}$ and x, y, z are the translational vibration velocities and displacements of the origin, O, in the x, y, z directions, Ω is the total rotation speed of the axial shaft element (combining rotation speed and any torsional oscillation), θ_x, θ_y, $\dot{\theta}_x$, $\dot{\theta}_y$, $\dot{\theta}_z$ are the angular vibration displacements and velocities of the shaft around the x, y, z axes, referred to as pitch, yaw and roll, respectively, and (x_0, y_0, z_0) is the position of an arbitrary known point that lies along the line of the beam.

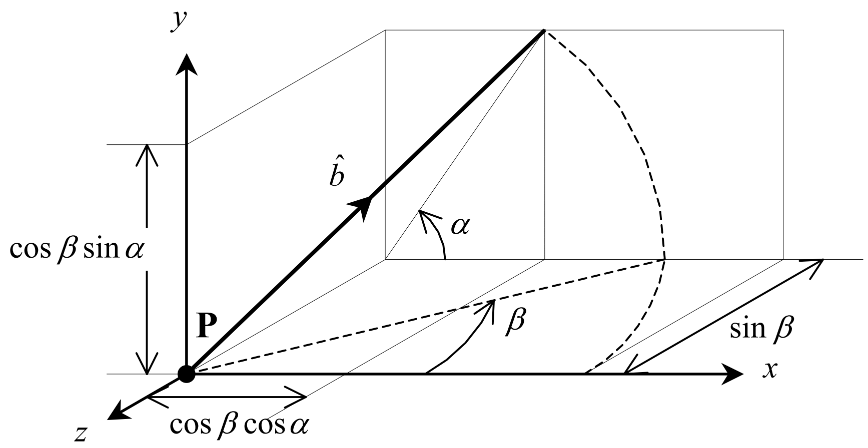


Figure 2 – Laser beam orientation, defining angles α and β

The original derivation[2] of this important equation showed, more generally than in any previous study, that the velocity sensed by an LDV incident on a vibrating target is unaffected by the shape of the target, even when the axial and radial position of the incident beam on the target alters significantly. This immunity gives the instrument significant advantages over, for example, proximity probes, and the same immunity is obviously found for targets with less complex motions. The analysis is sufficiently versatile to give the velocity sensitivity in applications where the laser beam is scanned[4,6] or where a single point on the target is tracked[5,6].

3 APPLICATION TO CIRCULAR SCANNING LASER DOPPLER VIBROMETRY

Scanning LDV measurements are typically performed via the introduction of some form of laser beam deflection around two orthogonal axes. A circular scan profile can be achieved by deflecting the laser beam around the two axes simultaneously through suitable angles, typically by driving the mirror controllers with cosine and sine functions.

With reference to Figures 3, 4 and 5, the scanning system optical axis is defined as being the line along which the laser beam is directed towards the target when there is “zero” beam deflection. In the usual configuration, the scanning system and target reference frames are collinear such that the scanning system optical axis lies on the z axis and the two orthogonal axes about which the beam is deflected during scanning are in the x and y directions. The effect of such beam deflection is easily accounted for in the velocity sensitivity model by temporal variation of one or both of the beam orientation angles, and, in many cases, temporal variation of the arbitrary known point that lies along the line of the laser beam[6].

3.1 The idealised scanning system

In the idealised scanning system, the laser beam deflection is performed by a single optical element, i.e. a mirror that can be rotated simultaneously about the x and y axes as shown schematically in Figure 3. In such a system, the known point (x_0, y_0, z_0) can be defined as the incidence point of the LDV beam on the scanning mirror. Clearly, the position of this point remains constant in time and scanning can be conveniently accounted for in the velocity sensitivity model by defining β as a constant and α as a function of time:

$$\beta = \frac{3\pi}{2} - \varepsilon \tag{3a}$$

and

$$\alpha(t) = \Omega_S t + \phi_S , \tag{3b}$$

where ε is determined by the stand-off distance (between the target and the LDV) and the desired scan radius, Ω_S is the scan rotational angular frequency and ϕ_S is the scan initial phase angle. In this idealised configuration, application of the velocity sensitivity model to a circular scan profile is particularly straightforward.

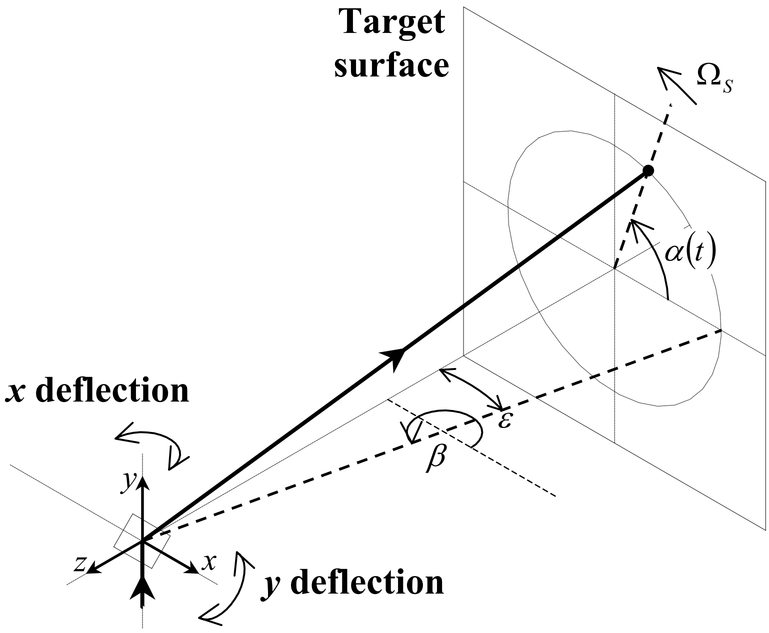


Figure 3 – The idealised scanning arrangement

3.2 The dual mirror scanning system – analysis of the laser beam orientation angles

In commercially available scanning LDV's, laser beam deflection is performed by the introduction of two orthogonally aligned mirrors, separated by some distance d_S, into the beam path. With reference to Figure 4, it can be seen that when the laser beam is traced back to the point from which it appears to originate, the position of this point does not remain constant but scans back and forth along the rotation axis of the y deflection mirror. The velocity sensitivity model is, however, sufficiently versatile to be able to account for this time dependency in the known point position. In this case, scanning is accommodated in the velocity sensitivity model by time dependency in x_0, β and α; x_0 varies at the scan frequency whilst β and α contain components that vary at twice the scan frequency.

Equations (3a&b) must be rewritten to incorporate the time variation in β and α necessary to scan a perfect circle, i.e.:

$$\beta(t) = \frac{3\pi}{2} - \varepsilon(t), \tag{4a}$$

and

$$\alpha(t) = \Omega_S t + \phi_S + \delta(t), \tag{4b}$$

where time dependency in $\varepsilon(t)$ is the result of time dependency in x_0. As shown in Figure 4, $\delta(t)$ is the small difference between $\alpha(t)$ and $\Omega_S t + \phi_S$ and is determined by the probe laser beam position in the target plane and the time dependency in x_0. In a previous publication[6] $\alpha(t)$ was set equal to $\Omega_S t$ (setting $\phi_S = 0$) but this paper offers a refinement to this approximation. Clearly, these parameters can be defined algebraically but it is more appropriate to continue the discussion in terms of the beam deflection mirror scan angles, since it is these, not the laser beam orientation angles, that are controlled in real scanning LDV systems. This paper gives this new development in the velocity sensitivity model for the first time.

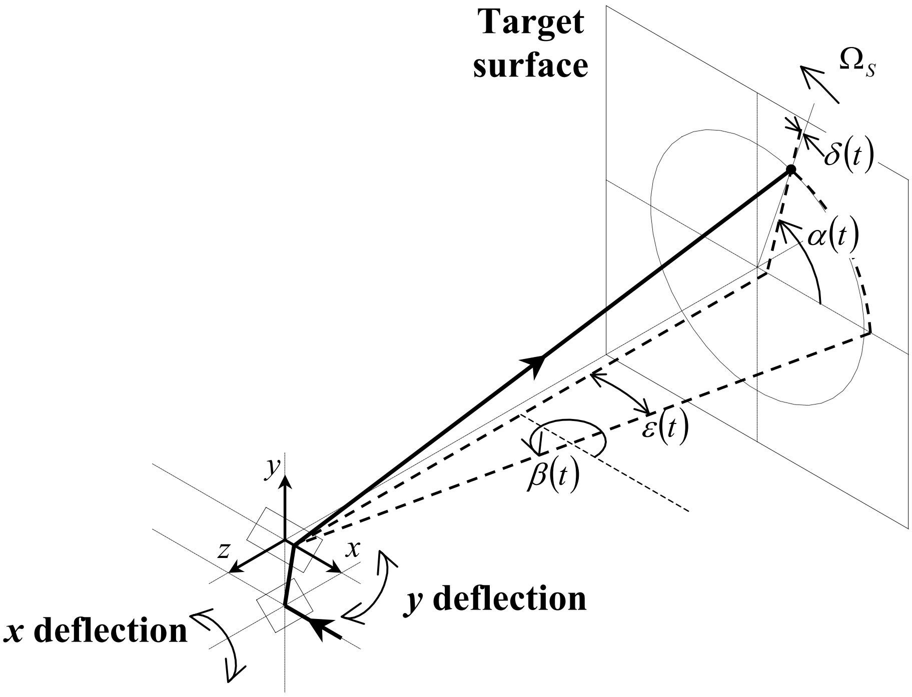


Figure 4 – The dual mirror scanning arrangement incorporating two orthogonally aligned mirrors, shown in terms of laser beam orientation angles

3.3 The dual mirror scanning system – analysis of the deflection mirror scan angles

With reference to Figures 5 and 6, the "zero" positions of the *x* and *y* deflection mirrors which result in deflection of the laser beam along the *z* axis are both 45° (to the *y* direction). The mirror scan angles, $\theta_{Sx}(t)$ and $\theta_{Sy}(t)$, are defined as positive if anticlockwise about the *z* direction and *x* axis respectively and can be described by the unit vectors $\hat{u}_{nx}(t)$ and $\hat{u}_{ny}(t)$ which are normal to the mirror reflective surface.

With reference to Figure 6, it is possible to express $\hat{u}_{nx}(t)$ and $\hat{u}_{ny}(t)$ in terms of the principal unit vectors, $\hat{x}$, $\hat{y}$ and $\hat{z}$, as follows:

$$\hat{u}_{nx}(t) = \sin(45 - \theta_{Sx}(t))\hat{x} + \cos(45 - \theta_{Sx}(t))\hat{y} \tag{5a}$$

and

$$\hat{u}_{ny}(t) = -\sin(45 - \theta_{Sy}(t))\hat{y} - \cos(45 - \theta_{Sy}(t))\hat{z}\,. \tag{5b}$$

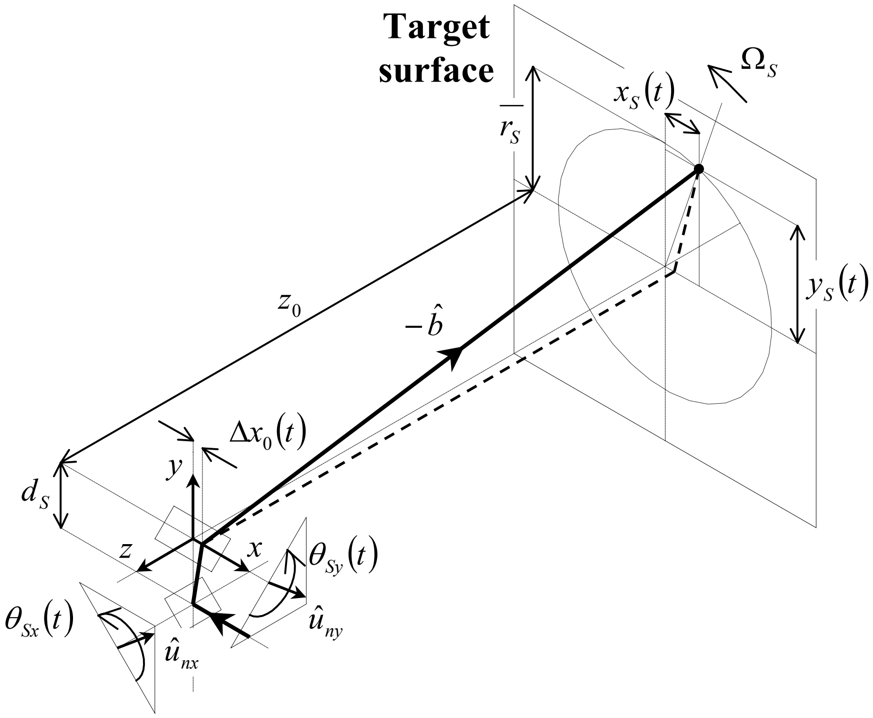


Figure 5 – The dual mirror scanning arrangement incorporating two orthogonally aligned mirrors, shown in terms of laser beam deflection mirror scan angles

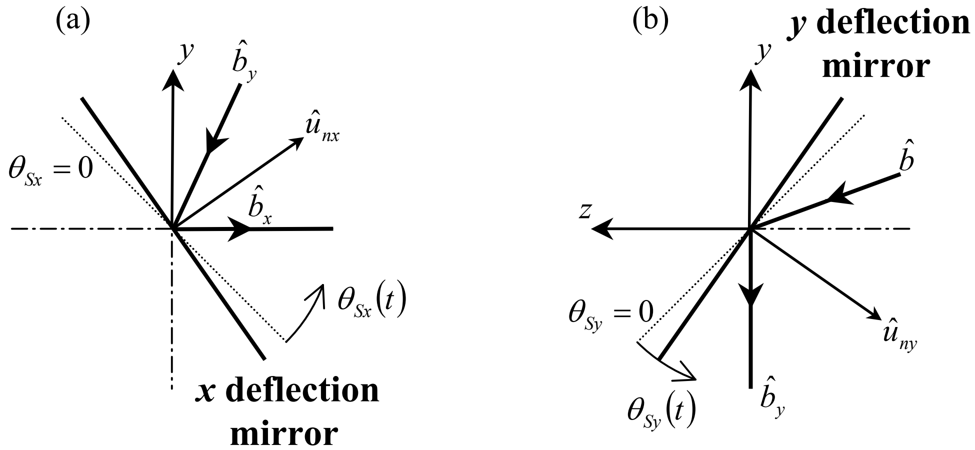


Figure 6 – Laser beam reflections at (a) the *x* deflection mirror and (b) the *y* deflection mirror

Let $\hat{b}_x$ be the direction of the laser beam before reflection at the *x* deflection mirror, $\hat{b}_y$ be the direction of the laser beam before reflection at the *y* deflection mirror and recall that $\hat{b}$ is the laser beam direction before incidence on the target. The convention used is that the direction of the unit vectors is from the target to the LDV (along the beam path), as illustrated in Figure 6.

Figure 6a shows the view of the reflection at the x deflection mirror in the negative z direction, illustrating that:

$$\hat{b}_y = \hat{b}_x - 2\left(\hat{b}_x \cdot \hat{u}_{nx}\right)\hat{u}_{nx} = \hat{x} - 2\left(\hat{x} \cdot \hat{u}_{nx}\right)\hat{u}_{nx}, \tag{6}$$

since in this configuration $\hat{b}_x = \hat{x}$. Similarly, Figure 6b shows the view of the reflection at the y deflection mirror in the negative x direction, illustrating that:

$$\hat{b} = \hat{b}_y - 2\left(\hat{b}_y \cdot \hat{u}_{ny}\right)\hat{u}_{ny} = \hat{x} - 2\left(\hat{x} \cdot \hat{u}_{nx}\right)\hat{u}_{nx} - 2\left(\left(\hat{x} - 2\left(\hat{x} \cdot \hat{u}_{nx}\right)\hat{u}_{nx}\right) \cdot \hat{u}_{ny}\right)\hat{u}_{ny}. \tag{7}$$

Since, as can be seen in equation (5b), $\hat{u}_{ny}$ is always perpendicular to $\hat{x}$, equation (7) can be re-written as:

$$\hat{b} = \hat{x} - 2\left(\hat{x} \cdot \hat{u}_{nx}\right)\hat{u}_{nx} + 4\left(\left(\hat{x} \cdot \hat{u}_{nx}\right)\hat{u}_{nx} \cdot \hat{u}_{ny}\right)\hat{u}_{ny}. \tag{8}$$

Making use of the fact that (from equations (5a&b)):

$$\hat{x} \cdot \hat{u}_{nx} = \sin\left(45 - \theta_{Sx}\right) \tag{9a}$$

and

$$\hat{u}_{nx} \cdot \hat{u}_{ny} = -\cos\left(45 - \theta_{Sx}\right)\sin\left(45 - \theta_{Sy}\right), \tag{9b}$$

equation (8) can be re-written as:

$$\hat{b} = \left(\sin 2\theta_{Sx}\right)\hat{x} - \left(\cos 2\theta_{Sx} \sin 2\theta_{Sy}\right)\hat{y} + \left(\cos 2\theta_{Sx} \cos 2\theta_{Sy}\right)\hat{z}. \tag{10}$$

Equation (10) is of great significance since it defines the incident laser beam direction for any combination of deflection mirror scan angles.

3.4 Derivation of mirror scan angles for arbitrary scan profiles

With reference to Figures 5 and 6, the probe laser beam position in the target plane, described by $x_S(t)$ and $y_S(t)$, can be defined in terms of $\theta_{Sx}(t)$ and $\theta_{Sy}(t)$ by consideration of the time dependent positions of the mirror incidence points and the target incidence point.

Since the time dependency in the known point position x coordinate, $\Delta x_0(t)$, is given by:

$$\Delta x_0(t) = -d_S \tan 2\theta_{Sx}, \tag{11}$$

the probe laser beam position in the target plane can be evaluated as follows:

$$x_S(t) = \Delta x_0(t) - n\left(\hat{b} \cdot \hat{x}\right) = -d_S \tan 2\theta_{Sx} - n \sin 2\theta_{Sx} \tag{12a}$$

and

$$y_S(t) = -n\left(\hat{b}.\hat{y}\right) = n \cos 2\theta_{Sx} \sin 2\theta_{Sy}, \tag{12b}$$

$$z_0 = n\left(\hat{b}.\hat{z}\right) = n \cos 2\theta_{Sx} \cos 2\theta_{Sy}, \tag{12c}$$

where n is the distance between the y deflection mirror and the target along the line of the laser beam. Substitution for n in equations (12a&b) results in a totally general description of the probe laser beam position in the target plane for any combination of mirror scan angles:

$$x_S(t) = -\tan 2\theta_{Sx}\left(d_S + \frac{z_0}{\cos 2\theta_{Sy}}\right) \tag{13a}$$

and

$$y_S(t) = z_0 \tan 2\theta_{Sy}\,. \tag{13b}$$

Whilst equation 13b can be rearranged such that the y deflection mirror scan angle can be obtained for any y_S, it can be seen from equation 13a that x_S is not a simple function of the x deflection mirror scan angle. This is particularly important when attempting to obtain a circular scan profile via the simultaneous modulation of the x and y deflection mirror scan angles.

4 CIRULAR SCAN PROFILE AND LDV MEASUREMENT ANALYSIS

As illustrated in Figure 5, a circular scan profile in the target plane, with scan angular frequency Ω_S and initial phase ϕ_S, requires that $x_S(t)$ and $y_S(t)$ are cosine and sine functions, respectively, which can be written as:

$$x_S(t) = \overline{r_S}\cos(\Omega_S t + \phi_S) \tag{14a}$$

and

$$y_S(t) = \overline{r_S}\sin(\Omega_S t + \phi_S), \tag{14b}$$

where $\overline{r_S}$ is the desired scan radius. Substituting for $x_S(t)$ and $y_S(t)$ in equations (13a) and (13b) results in two equations which must be rearranged for the deflection mirror scan angles if such a scan profile is to be achieved. This rearrangement is not possible for the first equation, the consequence of which is that a perfect circular scan cannot be achieved using basic functions to drive the deflection mirrors.

4.1 Typical deflection mirror scan angles

In real circular scanning LDV systems[4,5], cosine and sine functions of equal amplitude are used to perform a "circular" scan profile, i.e.:

$$\theta_{Sx}(t) = -\Theta_{Sx}\cos(\Omega_S t + \phi_S) \tag{15a}$$

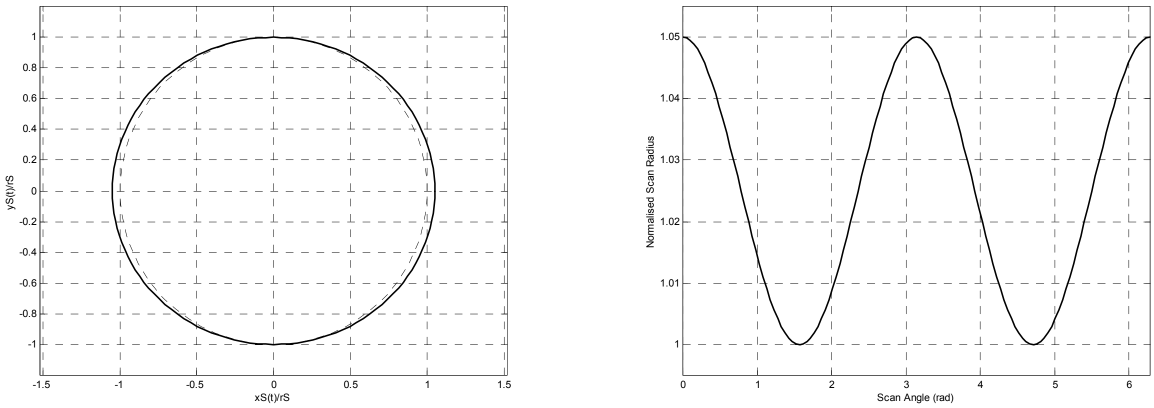
and

$$\theta_{Sy}(t) = \Theta_{Sy}\sin(\Omega_S t + \phi_S), \tag{15b}$$

where

$$\Theta_{Sx} = \Theta_{Sy} = 0.5\tan^{-1}\left(\frac{\overline{r_S}}{z_0}\right). \tag{15c}$$

The slightly elliptical trajectory[5,6] which results can clearly be observed by substituting equations (15a,b&c) into equations (13a&b) and is shown, normalised to the desired scan radius, in Figure 7a for the case where d_S = 50 mm, $\overline{r_S}$ = 100 mm and z_0 = 1 m. Figure 7b shows the normalised actual scan radius as a function of scan angle.



(a) (b)

Figure 7 – Normalised scan profile which results from equal amplitude cosine and sine mirror drive signals

Figure 7a clearly shows the inherent problem that occurs when employing such equal amplitude mirror drive signals, i.e. the fact that the probe laser beam does not follow the desired path. For this particular combination of mirror separation, desired scan radius and LDV stand-off, the maximum absolute error in the actual scan radius is in the order of 5%, as illustrated in Figure 7b. The effect of such a probe laser beam position error on the validity of the measurement is clearly structure dependent but, in some cases, there may be a significant difference between the amplitude and phase of the velocity at the intended and actual measurement points.

4.2 Additional measurement components resulting from typical mirror scan angles

In addition to this effect, and arguably more importantly, is the influence that such variation in laser beam orientation has on the LDV measurement. Re-writing equation (2) in the following form:

$$\begin{aligned} U_m &= \cos\beta\cos\alpha\left[\dot{x} - \left(\dot{\theta}_z + \Omega\right)\left(y_0 - y\right) + \left(\dot{\theta}_y - \Omega\theta_x\right)\left(z_0 - z\right)\right] \\ &+ \cos\beta\sin\alpha\left[\dot{y} + \left(\dot{\theta}_z + \Omega\right)\left(x_0 - x\right) - \left(\dot{\theta}_x + \Omega\theta_y\right)\left(z_0 - z\right)\right] \\ &\sin\beta\left[\dot{z} + \left(\dot{\theta}_x + \Omega\theta_y\right)\left(y_0 - y\right) - \left(\dot{\theta}_y - \Omega\theta_y\right)\left(x_0 - x\right)\right] \end{aligned} \tag{16a}$$

and replacing the laser beam orientation angles for the mirror scan angles by equating equations (1) and (10) and evaluating the principal unit vector coefficients, enables the measured vibration to be expressed as:

$$\begin{aligned} U_m &= \sin 2\theta_{Sx}\left[\dot{x} - \left(\dot{\theta}_z + \Omega\right)\left(y_0 - y\right) + \left(\dot{\theta}_y - \Omega\theta_x\right)\left(z_0 - z\right)\right] \\ &- \cos 2\theta_{Sx}\sin 2\theta_{Sy}\left[\dot{y} + \left(\dot{\theta}_z + \Omega\right)\left(x_0(t) - x\right) - \left(\dot{\theta}_x + \Omega\theta_y\right)\left(z_0 - z\right)\right] \\ &+ \cos 2\theta_{Sx}\cos 2\theta_{Sy}\left[\dot{z} + \left(\dot{\theta}_x + \Omega\theta_y\right)\left(y_0 - y\right) - \left(\dot{\theta}_y - \Omega\theta_y\right)\left(x_0(t) - x\right)\right], \end{aligned} \tag{16b}$$

where x_0 is time dependent and, using equation (11), is given by:

$$x_0(t) = x_0 + \Delta x_0(t) = x_0 - d_S \tan 2\theta_{Sx}. \tag{17}$$

The additional measurement components which result from using the equal amplitude mirror scan angles can therefore be quantified by substituting equations (15a,b&c) into equation (16b) and setting the translational and angular vibration component parameters to zero. The system arrangement is as discussed in section 3, i.e. the scanning system and target reference frames are collinear (no translational or angular misalignment), such that the measured LDV signal per unit rotation speed for this "no target vibration" case is given by:

$$\frac{U_m}{\Omega} = -\cos\left(\tan^{-1}\left(\frac{\overline{r_S}}{z_0}\right)\cos(\Omega_S t + \phi_S)\right)\sin\left(\tan^{-1}\left(\frac{\overline{r_S}}{z_0}\right)\sin(\Omega_S t + \phi_S)\right)\left[\frac{d_S \overline{r_S}}{z_0}\cos(\Omega_S + \phi_S)\right]. \quad (18)$$

The additional information that exists in the measured LDV signal occurs at twice and six times the scan frequency, as shown in Figure 8. For typical rotation frequencies, the level of the component at six times the scan frequency is well below the noise floor that results from the laser speckle effect (generally greater than 10^{-2} mm/s[1]) and is therefore insignificant. The component at twice the scan frequency is, however, of great significance since typical levels are of the order tens of mm/s. This component has been observed previously[5] but its origin was unknown until recently[6]. It is due to additional measured "vibration" components such as this one that care must be taken when interpreting vibration information obtained from such measurements. The theory presented here provides the vibration engineer with the ability to predict the amplitude of the additional components that exist in the measured LDV signal and therefore make measurements with confidence.

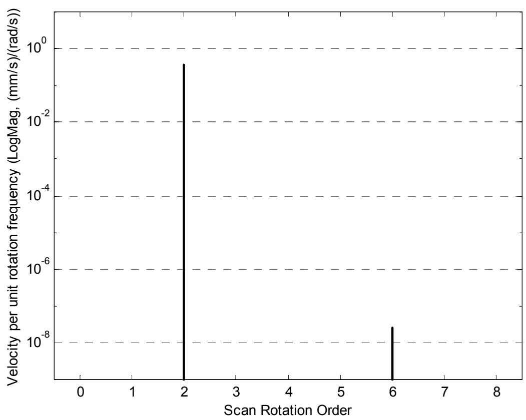


Figure 8 – Additional measurement components that occur in a dual-mirror circular scan when employing equal amplitude cosine and sine mirror drive signals

Theoretical component amplitudes show good agreement with those obtained from initial experimentation and with those that have been previously reported[5].

4.3 Elliptical scan trajectory correction

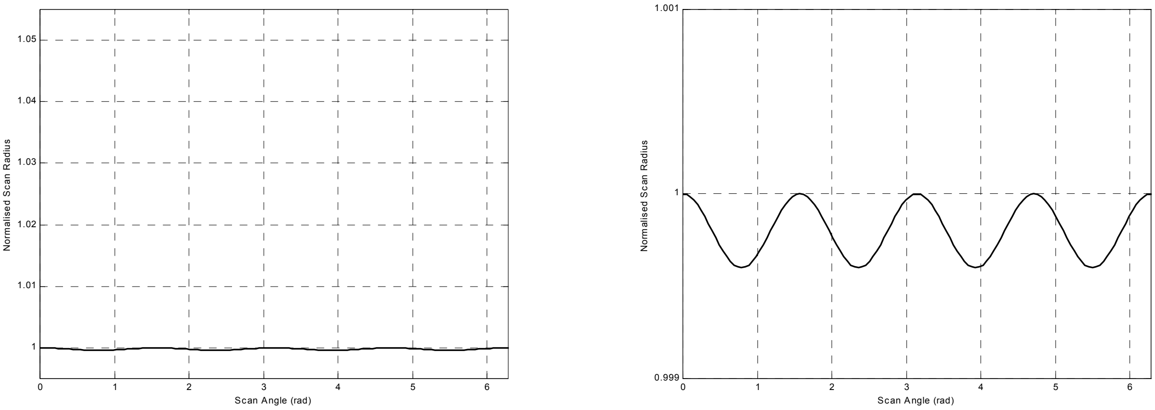
The elliptical shape resulting from the scan trajectory may be overcome to an extent by using "corrected" mirror drive signals that take into account the difference between the target to *x* deflection mirror and target to *y* deflection mirror distances. With reference to Figure 5, the amplitudes of the cosine and sine functions (equations (15a,b)) used to drive the deflection mirrors should not be equal, i.e.:

$$\Theta_{Sx} = 0.5\tan^{-1}\left(\frac{\overline{r_S}}{z_0 + d_S}\right) \quad (19a)$$

and

$$\Theta_{Sy} = 0.5\tan^{-1}\left(\frac{\overline{r_S}}{z_0}\right). \quad (19b)$$

Substitution of equations (15a&b) and (19a&b) into equations (13a&b) results in a scanning profile that is much closer to the required circular path. Figure 9 shows the normalised scan radius as a function of scan angle for this corrected mirror drive signal case (d_S = 50 mm, $\overline{r_S}$ = 100 mm and z_0 = 1 m), with the scale on the abscissa the same as for the typical mirror drive signal case shown in Figure 9a. As illustrated in Figure 9b, the maximum absolute error in the actual scan radius is reduced to much less than 0.1% by employing mirror drive signals with unequal amplitudes.



(a) (b)

Figure 9 – Scan profile which results from corrected amplitude cosine and sine mirror drive signals

4.4 Additional measurement components resulting from corrected mirror scan angles

It is possible to quantify the influence that this reduced variation in laser beam orientation has on the additional measurement components by substituting equations (15a&b) and (19a&b) into equation (16b) and setting the translational and angular vibration component parameters to zero as before. The measured LDV signal per unit rotation speed for this "corrected, no target vibration" case, with no translational or angular misalignment, is given by:

$$\frac{U_m}{\Omega} = -\cos\left(\tan^{-1}\left(\frac{\overline{r_S}}{z_0 + d_S}\right)\cos(\Omega_S t + \phi_S)\right)\sin\left(\tan^{-1}\left(\frac{\overline{r_S}}{z_0}\right)\sin(\Omega_S t + \phi_S)\right)\left[\frac{d_S \overline{r_S}}{z_0 + d_S}\cos(\Omega_S + \phi_S)\right]. \quad (20)$$

The additional information that exists in the measured LDV signal now occurs at twice, four and six times the scan frequency, as shown in Figure 10a. Again, only the component at twice the scan frequency is of significance but in this case its level is approximately 40% lower than that which occurs when using equal amplitude mirror drive signals. The reduction in the deviation of the probe laser beam from the desired point is clearly advantageous in many cases and, when performing tracking LDV measurements, the resulting minimisation of pseudo-random noise generated by the laser speckle effect[1] is also desirable.

4.5 Additional measurement components resulting from scanning system and target misalignment

Any translational and/or angular misalignment between the scanning system and target reference frames results in additional components at multiples of the target rotation frequency as illustrated in Figure 10b. One source of the component at synchronous frequency has previously been correctly attributed to angular misalignment[4,5] but there had been no analysis of the origins of the other first order components and the mechanism by which the higher order components occur until recently[6]. Whilst this paper has concentrated on the additional measurement components that occur as a result of the dual mirror arrangement used in typical scanning systems, the velocity sensitivity model presented here is well suited to the analysis of the effects of translational and angular misalignments and this is the subject of ongoing research.

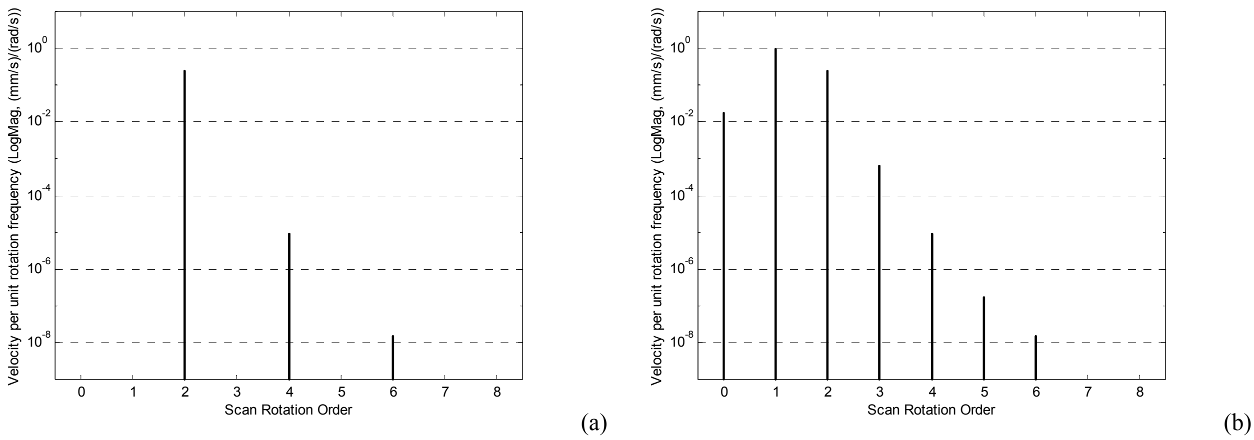


Figure 10 – Additional measurement components that occurs in a dual-mirror circular scan when employing corrected amplitude cosine and sine mirror drive signals for (a) zero misalignment and (b) 5 mrad angular and 5 mm translational misalignment

CONCLUSIONS

The use of LDV's incorporating some form of manipulation of the laser beam orientation, typically using two orthogonally aligned mirrors, has become increasingly popular in recent years. Considerable attention has been given to the operation of such scanning LDV's in continuous scanning mode in which the laser beam orientation is a continuous function of time, making it possible to track a single point on a moving target such as a rotor. This paper has investigated the application of a previously developed velocity sensitivity model to this particularly challenging measurement technique. A novel development is the reformulation of the original model to make use of mirror scan angles, rather than laser beam orientation angles, since it is these angles that an operator would seek to control in practice.

The revised velocity sensitivity model has been applied in this paper to show how the common use of a pair of orthogonally aligned scanning mirrors leads to a significant yet predictable additional component in the LDV output at twice the scan frequency in rotating target measurements. In addition, it has been shown how the combination of this mirror configuration and equal amplitude cosine and sine mirror drive functions leads to an elliptical beam profile. An alternative scheme has been proposed in which the scan mirror drive signals are "corrected" to produce a more circular profile as intended. The velocity sensitivity model has been used to indicate how, for rotating target measurements, this scheme would bring about a useful reduction in the additional component at twice the scan frequency, in addition to other benefits. Furthermore, the model has been used to predict the additional components in a circular scanning LDV output resulting from the presence of small misalignments, both translational and angular, that inevitably occur in such measurements.

The theory presented in this paper provides the vibration engineer with the capacity to calculate the amplitude of all of the additional components that exist in the measured LDV signal in order that such measurements can be made with confidence.

REFERENCES

1. S.J. Rothberg, J.R. Baker and N.A. Halliwell, "Laser vibrometry: pseudo-vibrations.", *Journal of Sound and Vibration*, **135**(3), pp. 516-522, 1989.

2. J.R. Bell and S.J. Rothberg, "Laser vibrometers and contacting transducers, target rotation and six degree-of-freedom vibration: what do we really measure?", *Journal of Sound and Vibration*, **237**(2), pp. 245-261, 2000.

3. J.R. Bell and S.J. Rothberg, "Rotational vibration measurements using laser vibrometry: comprehensive theory and practical application.", *Journal of Sound and Vibration*, **238**(4), pp. 673-690, 2000.
4. A.B. Stanbridge and D.J. Ewins, "Measurement of translational and angular vibration using a scanning laser Doppler vibrometer.", *First International Conference on Vibration Measurements by Laser Techniques, Ancona, Italy*, E.P. Tomasini, **2358**, pp. 37-47, International Society for Optical Engineering, Bellingham, WA 98227, 1994.
5. P. Castellini and N. Paone, "Development of the tracking laser vibrometer: performance and uncertainty analysis." *Review of Scientific Instruments*, **71**(12), pp. 4639-4647, 2000.
6. B.J. Halkon and S.J. Rothberg, "Comprehensive Velocity Sensitivity Model for Scanning and Tracking Laser Vibrometry.", *Proceedings of IMAC XX: A Conference on Structural Dynamics, Los Angeles, California*, **II**, pp. 1166-1170, Society for Experimental Mechanics, Bethel, CT 06801, 2002.