A review on the influence of degree of saturation on small strain shear modulus of unsaturated soils

Une revue de l'influence du degré de saturation sur le module de cisaillement de petites contraintes de sols non saturés

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ABSTRACT: Small-strain shear modulus (G_{max}) is an important parameter in the analysis and design of structures resting on liquefiable soils, particularly under dynamic loads such as earthquakes. In real condition, soil layers near the ground surface consistently undergo variation of degree of saturation (S_r) due to the change of weather or loading-unloading processes that lead to the variation of G_{max}. To date, this area has received limited attention and still encounters difficulties in evaluating the influence of S_r on G_{max} as well as capturing the effect of hysteresis on water retention behaviour. This study concentrates on the relationship between S_r and G_{max} based on available experimental data in literature. The results of the analysis show that S_r plays an important role in the magnitude of G_{max} for both cohesionless and cohesive unsaturated soils, while it has a greater influence on the latter. In order to predict G_{max} for cohesive soils within the full range of degree of saturation, apart from the influence of S_r on the contribution of matric suction (ψ_m), the influence of S_r on the contribution of plastic fines, salt concentration and van der Waals attraction should be additionally included.

RÉSUMÉ: Le module de cisaillement à petites contraintes (G_{max}) est un paramètre important dans l'analyse et la conception des structures reposant sur les sols, en particulier dans les analyses dynamiques telles que le séisme et la liquéfaction. En conditions réelles, les couches de sol proches de la surface du sol subissent constamment une variation de degré de saturation (S_r) due au changement de temps ou aux processus de chargement-déchargement qui conduisent à la variation de G_{max}. À ce jour, cette zone a reçu une attention limitée et rencontre toujours des difficultés à évaluer l'influence du S_r sur G_{max} ainsi que la capture de l'hystérésis dans le comportement de rétention d'eau. Cette étude se concentre sur la relation entre le S_r et G_{max} basé sur les données expérimentales disponibles dans la littérature. Les résultats de l 'analyse montrent que le S_r joue un rôle important dans l' importance de G_{max} pour les sols insaturés sans cohésion et cohésifs alors qu'il a une influence plus grande sur ces derniers. Pour prédire la G_{max} pour les sols cohésifs dans toute la gamme de S_r, en dehors de l'influence du S_r sur la contribution de l'aspiration matricielle (ψ_m), l'influence du S_r sur la contribution des fines de plastique, le sel La concentration et l'attraction de van der Waals devraient être ajoutés en plus.

KEYWORDS: Unsaturated soil, Small-strain modulus, Degree of saturation, Matric suction, Net stress

1. INTRODUCTION

Small-strain shear modulus is one of the parameters influencing the dynamic response of soils and plays a key role in engineering issues relating to vibration such as foundations for machines, vibration isolation, liquefaction and dynamic soil-structure interaction. Experimental investigation of G_{max} was conducted, using resonant column or consolidation apparatus integrated with bender elements, which enables the study of soil stiffness without disturbance. Most of existing equations used to predict the magnitude of G_{max} for unsaturated soils are established from these experimental tests and based on two stress state variables which are net stress ($\sigma^{net}=\sigma-u_a$) and matric suction ($\psi_m=u_a-u_w$), where σ is the total stress, u_a is the pore air pressure and u_w is the pore water pressure. Many of them simply ignored the influence of S_r (e.g. Mancuso et al. 2002; Khosravi and MacCartney 2009). However, Wheeler et al. (2003), based on experimental data, pointed out that two samples of the same soil with the same void ratio might exhibit different mechanical behaviours under the same stress state conditions if they have different values of degree of saturation. According to Wheeler et al. (2003), irreversible changes of S_r occurring during wetting-drying cycles influence the stress-strain behaviour of unsaturated soils. In the effort to couple the hydro and mechanical behaviours, some equations employ S_r (or a function of S_r) as a weighting parameter for matric suction with an assumption that the contribution of matric suction to G_{max} is considered to be reversely proportional to S_r (e.g. Sawangsuriya 2006; Khosravi and MacCartney 2012; Heitor et al. 2013). This assumption implies that G_{max} consistently decreases as S_r approaches zero under a constant net stress. The trend was proved based on many tests conducted on cohesionless soils but for tests on cohesive soils, an inverse relationship with consistent increases of G_{max} was observed (e.g. Cho and Santamarina 2001). So far, the hardening behaviour of G_{max} with decreasing S_r, particularly near dry state, has not been formulated correctly. The main objectives of this paper are to analyze and interpret previously published experimental test results, carried out on different types of soils, in light of unsaturated soil mechanics to investigate the contribution of S_r on G_{max}.

2. EXPERIMENTAL STUDIES ON THE RELATIONSHIP BETWEEN G_{max} & S_r

Wu et al. (1984) investigated the variation of G_{max} from saturated to dry state using resonant column device under three confining pressures. Five fine grained cohesionless soils were studied in the test with different effective grain sizes (D_{10}), varying from 0.0024 mm to 0.17 mm, and different shapes of particles (namely, angular, flaky, angular to sub-rounded). Results of all the tests showed that the magnitudes of G_{max} at completely saturated state and dry state are approximately the same, while an optimum degree of saturation ($S_{r,opt}$) was observed, at which the combination of S_r and corresponding matric suction results in the maximum value of G_{max} (Figure 1).

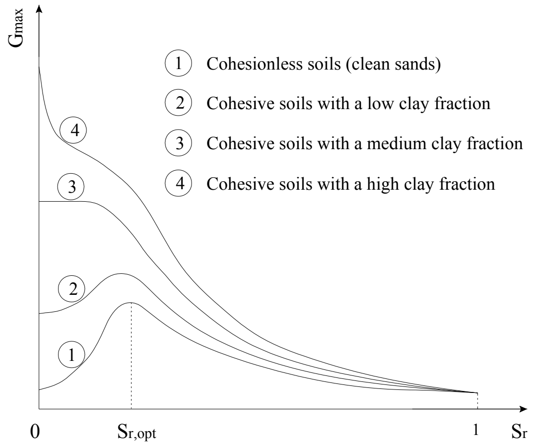

Figure 1. Possible S_r-G_{max} relationships with different clay fractions during a drying process

This pattern can be explained by considering the contribution of matric suction to the small strain stiffness of soils. At matric suctions smaller than the air entry value (AEV), it can readily be assumed that there are still only two phases with pores being completely filled with bulk water and the mechanical behaviour of the soil still complies with saturated soil mechanic. As a result, an increase of matric suction can be considered as an increase of effective stress, resulting in an increase of G_{max} which can be determined by equations for saturated soils. This behaviour of small strain stiffness was observed in tests for a range of soils conducted by Mancuso et al. (2002), Sawangsuriya et al. (2008) and Heitor et al. (2013). At matric suctions greater than AEV, air commences occupying larger space causing progressive reductions of contact area between bulk water and particle surfaces resulting in reductions of the growth rate of G_{max} with increasing matric suction. As the fraction of air volume grows, meniscus water starts forming between particles as capillary bridges (Figure 2).

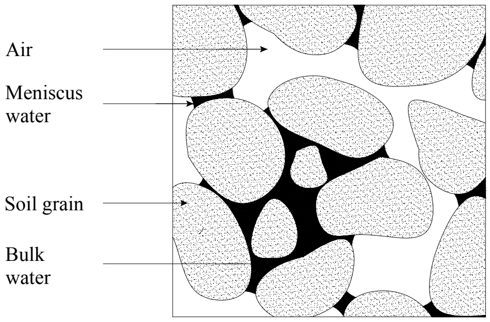

Figure 2. Bulk water and meniscus water within an RVE of unsaturated soils

Despite meniscus water and bulk water both affect the soil stiffness, their influences to the mechanical behaviour of soils are different. Variation of matric suction within bulk water influences both normal and tangential inter-particle forces, while variation of matric suction within meniscus water affects only normal inter-particle forces (Karube et al. 1996; Wheeler et al. 2003; Ng and Zhou 2014). Here, the normal inter-particle forces can be considered as confining pressure, which contributes to the increase in the small strain stiffness of soils. An increment of matric suction within bulk water contributes to an increase of normal inter-particle forces as an isotropic pressure does, whereas the contribution caused by the same increment of matric suction within meniscus water can be calculated from equations associated with the capillary effect. The total magnitude of normal inter-particle forces within an investigated representative volume element (RVE) is proportional to the magnitude of matric suction and the total contact area between pore water and the particle surfaces (Vanapalli et al. 1996; Lu 2008; Alonso et al. 2010; Han and Vanapalli 2016). Since it is difficult to determine the total contact area, its variation during drying and wetting processes can be represented by the volumetric variation of pore water which is tightly associated with the variation of S_r. The early stages of a drying process experience reductions of bulk water volume along with the development of meniscus water. There is a gradual transition of the soil response from bulk-water to meniscus-water regulated behaviour (Mancuso et al. 2002). For higher range of matric suction at which bulk water is nearly dried up, the amount of water loss at the same increment of matric suction will reduce significantly leading to a higher growth rate of G_{max} with decreasing S_r. This trend continues until the optimum degree of saturation is reached. As desiccation progresses, the contribution of matric suction to G_{max} commences falling down due to the loss of meniscus water and finally becomes zero when meniscus water has been completely dried up at a low value of S_r. There are different interpretations and methods to determine this degree of saturation. Fredlund and Raharjo (1993) defined it as residual degree of saturation, thus can be evaluated from the soil water characteristic curve (SWCC). Residual degree of saturation corresponds to the unsaturated stage which has negligible effect on the mechanical behavior of soil, representing the state below which the liquid flow ceases, and vapour flow dominates. Alonso et al. (2010) named this degree of saturation as a microscopic degree of saturation (S_r^m), corresponding to the state at which water is hold only in micropores and capillary effect is negligible. S_r^m can be experimentally determined by mercury intrusion porosimetry method and for cohesionless soils, its value is approximately equal to zero. From the test results, Wu et al. (1984) also proposed an empirical equation for the optimum degree of saturation:

$$S_{r,opt}(\%) = -6.5\log_{10}(D_{10}) + 1.5 \tag{1}$$

where, $S_{r,opt}$ is the optimum degree of saturation and D_{10} is the effective grain size. Equation (1) indicates that for investigated soils an increase of D_{10}, due to an increase of pore size, causes a decrease in $S_{r,opt}$. This behavior is attributed to the fact that the capillary effect decreases with increasing pore size. For high values of D_{10}, the contribution of matric suction to G_{max} notably reduces and depends only on the variation of bulk water. Qian et al. (1993) conducted the same tests on thirteen cohessionless soils with different grain shapes and grain size distributions. Four of them were natural sands, namely, Glazier Way, mortar, Ottawa F-125 and Agsco. The presence of $S_{r,opt}$ was observed in all tests showing that $S_{r,opt}$ is proportional to void ratio and the portion of grains smaller than 400 sieve size (i.e. 38μm). The authors also reported that angular sands have higher values of

$S_{r,opt}$ compared with that of subrounded sands with the same average grain size. The above findings lead to a general conclusion that $S_{r,opt}$ is proportional to the fraction of small pores, which are the ideal environment for the capillary attraction.

Cho and Santamarina (2001) used a modified oedometer cell to explore the variation of shear wave velocity (V_s) against S_r, reducing from a fully saturated condition to a dry condition. The cell was put inside an incubator under a constant temperature of 50°C. Results of four tests on both cohesionless and cohesive specimens showed different relationships between S_r and V_s. From the magnitude of shear wave velocity, G_{max}, can be calculated using the following equation:

$$G_{max} = \rho . V_s^2 \qquad (2)$$

where, ρ is the soil mass density. As the variation of ρ during the test is small, the S_r–V_s relationship and S_r–G_{max} relationship can be considered to follow the same pattern. For clean glass bead, considered as a cohessionless specimen, an optimum degree of saturation corresponding to the maximum V_s was observed, similar to observations reported by Wu et al. (1984). For mixture consisting of 80% glass bead and 20% kaolinite (a cohesive specimen), a different pattern was noted with a consistent increase of V_s during the drying process. Two significant increments in the slope of S_r–V_s curve are noted at two degrees of saturation including 40% and the nearly dry stage. The same trend was observed for granite powder and Sandboil sand specimens and the magnitude of V_s at the dry condition was almost proportional to the fraction of the fine content. This trend cannot be attributed to the contribution of the increasing matric suction during drying as matric suction contribution becomes insignificant when meniscus water was completely desiccated at low degrees of saturation.

Three factors that possibly contribute to the stiffness of soil during drying would be cementation effects due to salt precipitation, van der Waals attraction between particles and the plastic fine content. As S_r decreases, the ionic concentration in meniscus water increases until reaching saturation resulting in the formation of salt precipitation at particle contacts (bridge connection), significantly contributing to the stiffness of the soil specimen (Cho and Santamarina 2001; Truong et al 2012; He and Chu 2016). The results of tests conducted by Truong et al (2012) with a wide range of salt concentration reveal that remarkable contributions of cementation induced by salt in the soil to G_{max} occurs mainly near the dry state and are proportional to the salt concentration. Van der Waals forces resulting from electromagnetic field interaction between atoms of adjacent particle surfaces are the other factor that can cause the observed increase in the soil stiffness. According to Lu and Likos (2006), van der Waals attraction is active and more significant for particles smaller than 10μm (i.e., clay and silt particles) and depends on the properties of the medium between particle surfaces (e.g., pore water) and become greater at closer distances. At saturated state, van der Waals attraction is smallest, which is corresponding to the largest inter-particle distance, while at lower degrees of saturation, the distance is shorten and van der Waals attraction increases exponentially until reaching the upper bound at the dry state. The total magnitude of van der Waals forces is known to be remarkably affected by the grain size, type of mineral and fraction of plastic fines. During the drying process, fines accumulate at the contacts of larger particles increasing the number of interparticle contacts. Thus the smaller the grain size and the higher fraction of plastic fines, the higher total van der Waals forces. In addition, since the stiffness of plastic fines increases with reducing water content, the plastic fine content is expected to enhance the stiffness of the soil during drying with the highest contribution occurring at the dry state. Figure 3 schematically represents contributions of matric suction, van der Waals attraction, plastic fines and salt precipitation to G_{max} based on experimental results in literature. As the contribution of van der Waals attraction is influenced by plastic fines, these two effects are combined and can be considered as only one factor. From the above discussion, it can be concluded that matric suction controls the small strain stiffness of unsaturated soils within the high range of S_r, whereas at low values of S_r, the contribution of matric suction approaches zero, along with increases of the contributions of other factors (Figure 3). Consequently, the small strain stiffness may vary in different ways with degrees of saturation at low range. Specifically, G_{max} may persistently increase as reported by Cho and Santamarina (2001), or remain constant, or persistently decrease mainly depending on the salt concentration, the mineral and fraction of plastic fines (Figure 1).

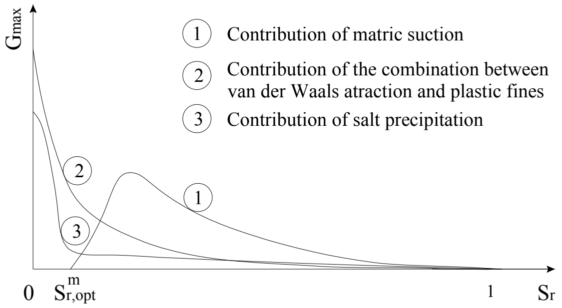

Figure 3. Schematic contributions of different factors influencing G_{max} during drying process

A number of other experimental tests using different types of suction-control devices have been conducted to explore the variations of G_{max} during drying and wetting processes. However, the difficulties in creating and controlling matric suction at high range and the long required time for matric suction equalization at each stress stage, restrict those investigations only to low range of matric suctions (i.e., high degrees of saturation) with the maximum reported value of approximate 1000 kPa by Sawangsuriya (2006), while others tested with lower matric suctions (e.g. 400 kPa by Mancuso et al. 2002 and Hoyos et al. 2011, 200 kPa by Ng and Yung 2008, 100 kPa by Khosravi and McCartney 2012). Thus, the contribution of matric suction to G_{max} dominates the results in those tests and conceals the contribution of other factors. From the measured values of G_{max} corresponding to varying matric suction and S_r, a number of empirical equations for G_{max} prediction, considering only the contribution of net stress and matric suction, were proposed (Sawangsuriya 2006; Sawangsuriya et al. 2008; Khosravi 2011; Heitor et al. 2013).

However, these equations fail capturing the effect of hysteresis on water retention behaviour of unsaturated soils during wetting-drying cycles and loading-unloading stages. During a drying-wetting cycle, a decrease of S_r at the same magnitude of matric suction causes a decrease of contribution of suction, but at the same time it induces increments in the contributions of salt precipitation, van der Waals attraction and plastic fines. When the hydraulic hysteresis behaviour occurs, the errors caused by omitting the contributions of salt precipitation, van der Waals attraction and the plastic fines might significantly accumulate as number of cycles increase. The accuracy of predictions can be improved by capturing not only the contribution of matric suction but also the contribution of other possible factors in the empirical equations. A challenging issue remaining is that there are limited experimental studies on relationships between S_r and those factors during drying-wetting cycles. In addition, it should be noted that different responses of excess pore air and pore water pressures during suction equalizations, as observed by Ho & Fatahi (2015a, 2015b), Ho et al. (2015), might affect the contributions of those factors. Thus, further studies in those fields are recommended.

3. CONCLUSIONS

Experimental tests exploring the S_r-G_{max} relationship during the entire drying process from the saturated state to dry state have been analysed in light of unsaturated soil mechanics. The analysis exposed that water content (i.e. degree of saturation) plays a key role and has to be included in estimating G_{max}. Where the hydraulic hysteresis behaviour is present, a decrease of degree of saturation, under a constant matric suction, may cause a decrease or an increase of the small strain stiffness depending on the influential level of S_r on the contributions of matric suction, salt precipitation, van der Waals attraction and the plastic fines. For cohesionless clean sands the influence of S_r on G_{max} can be captured through the contribution of only matric suction. However, for cohesive soils the contribution of matric suction to G_{max} at high degrees of saturation surpasses the contribution of other factors, while at low degrees of saturation, the opposite trend dominates. As the contribution of matric suction approaches zero, the contributions of salt precipitation, van der Waals attraction and the plastic fines reach their maximums at the dry state. Therefore, consideration of the influence of S_r only indirectly in the contribution of matric suction can result in acceptable prediction of G_{max} in low degrees of saturation only, while to cover the full range of S_r and capture the hydraulic hysteresis, the contributions of van der Waals forces, salt precipitation and plastic fine content have to be additionally included.

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