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*Research Article*

# Optimal Container Routing in Liner Shipping Networks Considering Repacking 20 ft Containers into 40 ft Containers

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The volume of a 40 ft container is twice as large as that of a 20 ft container. However, the handling cost (loading, unloading, and transshipment) of a 40 ft container is much lower than twice the corresponding handling cost of two 20 ft containers. Enlightened by this observation, we propose a novel container routing with repacking problem in liner shipping, where two 20 ft containers can be repacked to a 40 ft container in order to reduce the handling cost. We develop a mixed-integer linear programming model that formulates the routing decisions and the repacking decisions in a holistic manner. An illustrative example is reported to demonstrate the applicability of the model. Results show that the benefit of repacking is the most significant when containers are transshipped several times.

## 1. Introduction

Maritime transportation is the backbone of international trade. Around 80 per cent of global trade by volume and over 70 per cent by value are carried by sea. Among all the sea cargos, about half in monetary terms are containerized [1]. Containers are transported by shipping lines on regularly serviced ship routes. At the port of origin, containers are loaded onto ships by quay cranes; and at the port of destination, containers are discharged from ships by quay cranes. Containers may also be transshipped between their origin ports and destination ports. In fact, transshipment is a common operation in container shipping. As reported by [1], the total container trade volumes amounted to 160 million twenty-foot equivalent units (TEUs) in 2013, whereas world container port throughput was estimated at 651 million TEUs. These numbers mean that on average a container was transshipped $(651 - 2 \times 160)/160 \approx 2$ times (the throughput data for ports also include empty containers.).

Container routing determines how to transport containers from their origins to their destinations in a liner shipping network. Take Figure 1 as an example, which shows a liner shipping network consisting of three ship routes. Containers from Singapore to Hong Kong can be transported on either ship route 1 or ship route 2. If there are many containers to be transported from Singapore to Jakarta, then containers from Singapore to Hong Kong should be transported on ship route 2 to reserve the capacity on ship route 1 for containers from Singapore to Jakarta. In addition to different ship routes on which containers can be transported from origin to destination, another complicating factor is transshipment. For instance, containers from Hong Kong to Colombo can be transported on ship route 2, or they can be transported on ship route 1 to Singapore and transshipped to ship route 2 and then transported to Colombo. The choice of direct shipment on ship route 2 is preferable because otherwise it would involve a high transshipment cost at Singapore. However, if there are many containers to be transported from Hong Kong to Xiamen or from Xiamen to Singapore, then the choice of transshipment at Singapore from ship route 1 to ship route 2 has to be adopted. Consequently, it is not an easy task to determine the optimal container routing.

Table 1: Laden container handling cost (USD/container) at three ports (source: [11]).

| Port | Type | Loading | Discharge | Transshipment |
|---|---|---|---|---|
| A (a major transshipment port in Europe) | D20 | 248 | 324 | 183 |
| | D40 | 256 | 332 | 198 |
| | Ratio | 1.03 | 1.02 | 1.08 |
| B (a major transshipment port in Southeast Asia) | D20 | 118 | 118 | 145 |
| | D40 | 148 | 148 | 145 |
| | Ratio | 1.25 | 1.25 | 1 |
| C (an export-driven port in Southeast Asia) | D20 | 110 | 110 | 71 |
| | D40 | 156 | 156 | 106 |
| | Ratio | 1.42 | 1.42 | 1.49 |

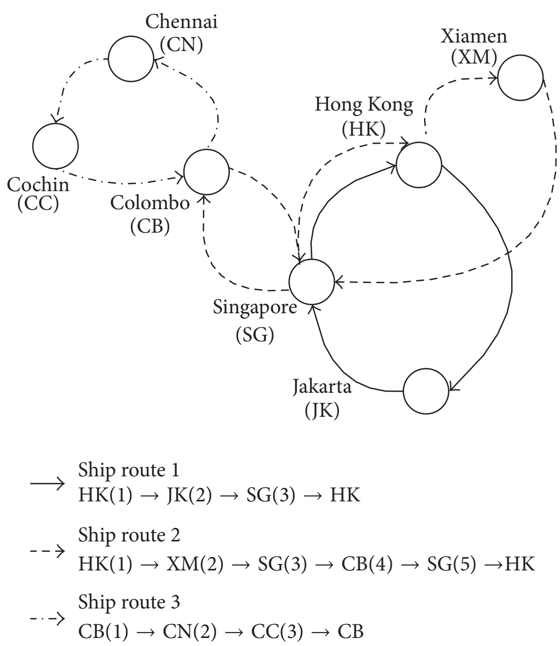



Figure 1: An illustrative liner shipping network [8].

Container routing determines the container handling cost. Table 1 shows the handling costs for two types of laden containers at three ports: D20 means dry 20 ft container, and D40 is dry 40 ft container. In terms of cargo capacity, a D40 is equivalent to two D20s. However, Table 1 clearly indicates in the three rows "Ratio" that the ratio of the cost of handling a D40 and that of handling a D20 is strictly less than 2. In fact, all the ratios in Table 1 are less than 1.5, and some ratios are even 1 or very close to 1. This is because both the handling of a D20 and the handling of a D40 involve one quay crane move (we note that nowadays some quay cranes can handle one D40 or two D20s in each move.). Therefore, to reduce container handling costs, a shipping line should try to transport more D40s instead of D20s as a D40 can hold as much cargos as two D20s.

*1.1. Container Repacking.* As the handling cost of a D40 is much lower than that of two D20s, it might be advantageous to unpack two D20s and repack them to one D40. In the sequel, we use "TEU" and "D20" interchangeably and use "forty-foot equivalent unit (FEU)" and "D40" interchangeably. The load, transshipment, and discharge cost (USD/container) of a TEU at a port $p$ is denoted by $\hat{c}_p^T$, $\bar{c}_p^T$, and $\tilde{c}_p^T$, respectively. The load, transshipment, and discharge cost (USD/container) of an FEU at port $p \in \mathscr{P}$ is denoted by $\hat{c}_p^F$, $\bar{c}_p^F$, and $\tilde{c}_p^F$, respectively. We further let $c_p^{2T\to F}$ be the cost of repacking two TEUs into one FEU and $c_p^{F\to 2T}$ be the cost of unpacking one FEU to two TEUs (Since multiple rehandling of containers would increase the risk for damage and therefore may increase insurance costs, we can include in $c_p^{2T\to F}$ and $c_p^{F\to 2T}$ the extra insurance costs. Moreover, repacking requires consent from shippers and we can include in the rehandling cost the component of discount for shippers who agree for their cargos to be repacked.).

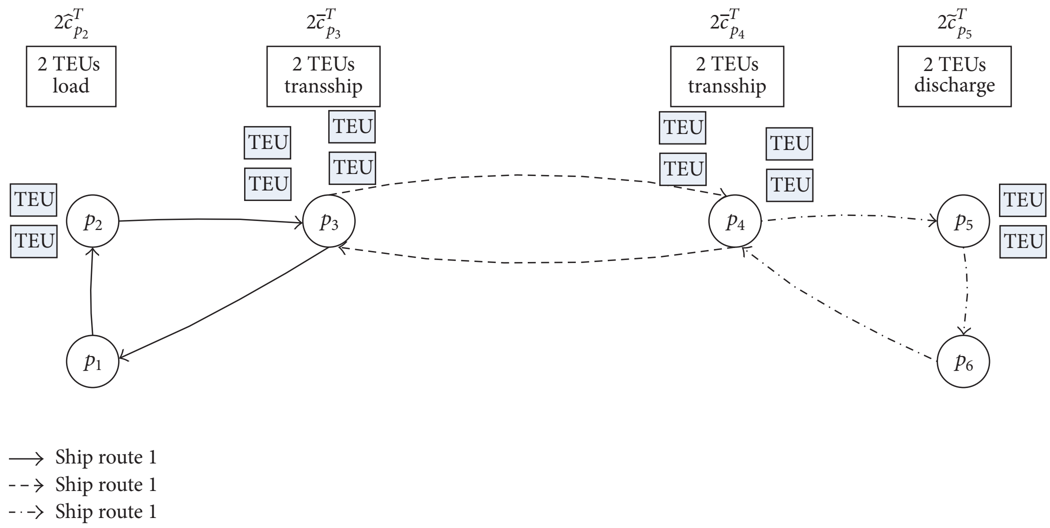
Figure 2 shows an example of transporting two TEUs from port $p_2$ to port $p_5$. The two TEUs need to be transshipped twice. If they are transported as two TEUs, as shown in Figure 2(a), then, at the port of origin, that is, $p_2$, two TEUs are loaded; at $p_3$, two TEUs are transshipped; at $p_4$, two TEUs are transshipped; and, at the destination port $p_5$, two TEUs are discharged. Therefore, the total container handling cost is

$$2\hat{c}_{p_2}^T + 2\bar{c}_{p_3}^T + 2\bar{c}_{p_4}^T + 2\tilde{c}_{p_5}^T. \tag{1}$$

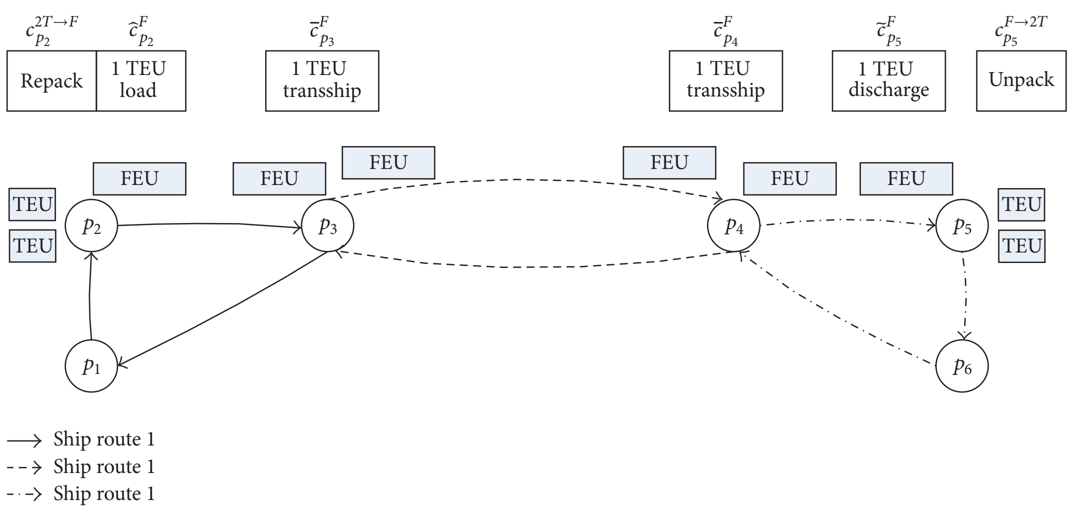
If these two TEUs are repacked into an FEU at the origin port and unpacked at the destination port, as shown in Figure 2(b), then, at the port of origin, that is, $p_2$, one FEU is loaded; at $p_3$, one FEU is transshipped; at $p_4$, one FEU is transshipped; and, at the destination port $p_5$, one FEU is discharged. Moreover, container repacking and unpacking costs are incurred at $p_2$ and $p_5$, respectively (The FEU is unpacked at $p_5$ into two TEUs because the two TEUs from port $p_2$ to port $p_5$ actually have different inland destinations.). Therefore, the total container handling and packing cost is

$$c_{p_2}^{2T\to F} + \hat{c}_{p_2}^F + \bar{c}_{p_3}^F + \bar{c}_{p_4}^F + \tilde{c}_{p_5}^F + c_{p_5}^{F\to 2T}, \tag{2}$$

which may be considerably smaller than (1).




(a) Two TEUs from $p_2$ to $p_5$ without repacking




(b) Two TEUs from $p_2$ to $p_5$ with repacking at $p_2$ and unpacking at $p_5$

Figure 2: Container packing with the same origin and destination.

Not only can TEUs with the same origin and destination ports be repacked into FEUs, as shown in Figure 2, TEUs with different origins and destinations can also be repacked into FEUs. For example, in Figure 3, one TEU is from $p_2$ to $p_5$ and the other TEU is from $p_1$ to $p_6$. If they are transported as TEUs throughout their trips, as shown in Figure 3(a), the total handling cost is

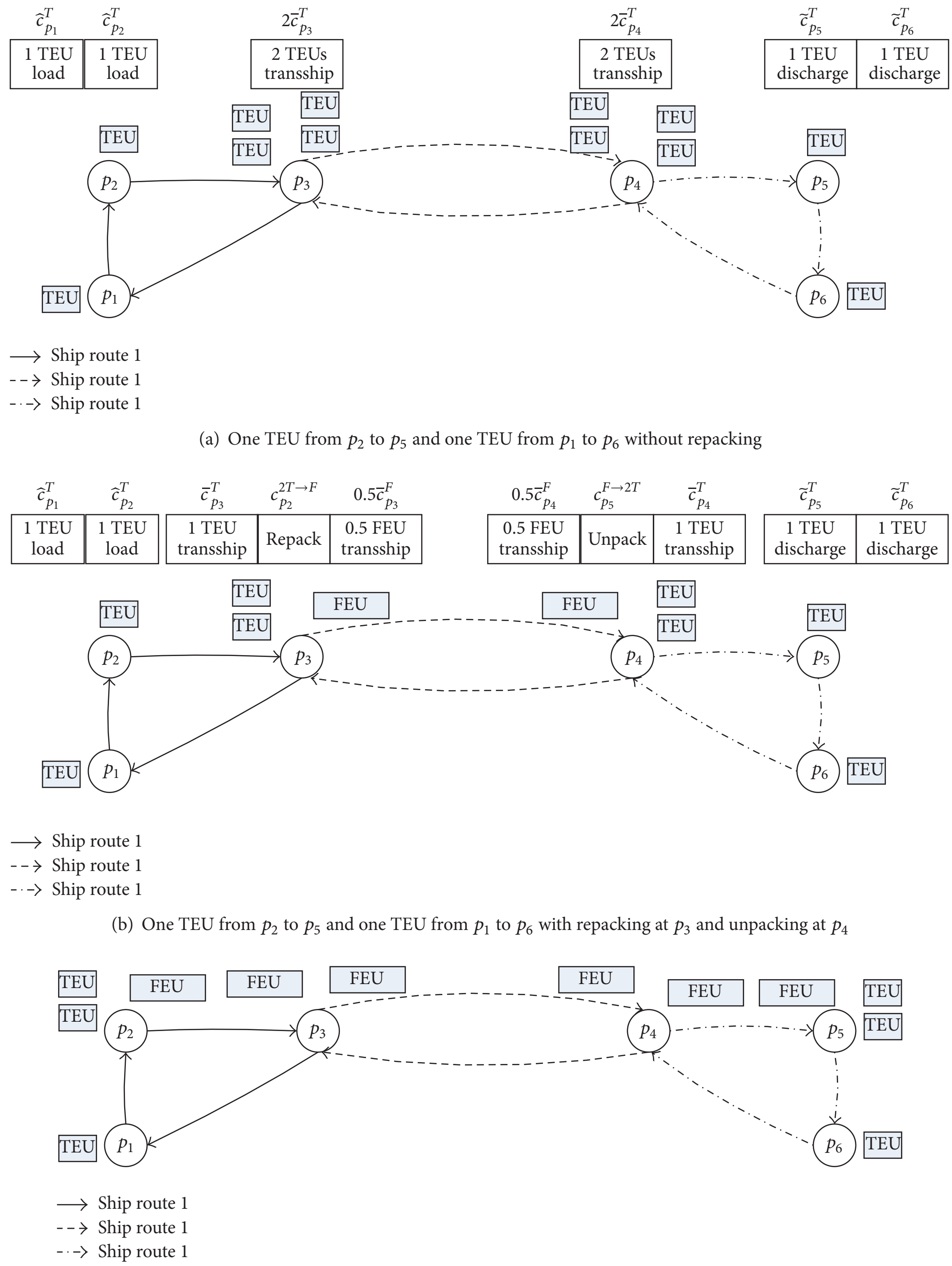
$$\tilde{c}^T_{p_1} + \tilde{c}^T_{p_2} + 2\bar{c}^T_{p_3} + 2\bar{c}^T_{p_4} + \tilde{c}^T_{p_5} + \tilde{c}^T_{p_6}. \tag{3}$$

They may be repacked into an FEU at the transshipment port $p_3$ and unpacked as two TEUs at transshipment port $p_4$, as shown in Figure 3(b). At $p_3$, since two TEUs are unloaded and one FEU is loaded in the transshipment process, the total transshipment cost is calculated as half of the sum of the transshipment cost of two TEUs and the transshipment cost of one FEU, that is, $\bar{c}^T_{p_3} + 0.5\bar{c}^F_{p_3}$. Similarly, the transshipment cost at $p_4$ is calculated as $\bar{c}^T_{p_4} + 0.5\bar{c}^F_{p_4}$. Therefore, the total container handling and packing cost is

$$\begin{aligned} &\tilde{c}^T_{p_1} + \tilde{c}^T_{p_2} + \underbrace{\bar{c}^T_{p_3} + 0.5\bar{c}^F_{p_3} + c^{2T\to F}_{p_3}}_{\text{Cost at } p_3} + \underbrace{\bar{c}^T_{p_4} + 0.5\bar{c}^F_{p_4} + c^{F\to 2T}_{p_4}}_{\text{Cost at } p_4} \\ &\quad + \tilde{c}^T_{p_5} + \tilde{c}^T_{p_6}, \end{aligned} \tag{4}$$

which may be smaller than (3) if the container packing cost is small. Of course, the benefit of repacking TEUs into FEUs in Figure 3(b) is not as significant as Figure 2.

Container packing may also affect how the containers are transported. For example, in Figure 3(c), the container shipment demand (the number of containers to be transported from one port to another in a week) is the same as




(a) One TEU from $p_2$ to $p_5$ and one TEU from $p_1$ to $p_6$ without repacking



(b) One TEU from $p_2$ to $p_5$ and one TEU from $p_1$ to $p_6$ with repacking at $p_3$ and unpacking at $p_4$



(c) One TEU from $p_2$ to $p_5$ and one TEU from $p_1$ to $p_6$ with repacking at $p_2$ and unpacking at $p_5$

FIGURE 3: Container packing with different origins and destinations.

Figures 3(a) and 3(b). Containers are transported as follows in Figure 3(c): one TEU is transported from $p_1$ to $p_2$; at $p_2$, the TEU is unloaded and repacked with another TEU whose origin is $p_2$ into an FEU; the FEU is loaded/transshipped at $p_2$ and transported to $p_5$; at $p_5$, the FEU is unpacked to two TEUs, one of which has arrived at its destination and the other should be transported to its destination $p_6$. To compute the handling cost of the two TEUs at $p_2$, we assume that there are two TEUs with origin $p_1$ and another two TEUs with origin $p_2$ that are repacked into two FEUs at $p_2$ (hence, the handling cost is doubled). The two TEUs with origin $p_1$ are actually transshipped at $p_2$, and therefore the handling cost is

$(2\bar{c}_{p_2}^T + \bar{c}_{p_2}^F)/2$. The two TEUs with origin $p_2$ are actually loaded at $p_2$, and therefore the handling cost is $\hat{c}_{p_2}^F$. Consequently, the total handling cost at $p_2$ in the example of Figure 3(c) is

$$\frac{\left(2\bar{c}_{p_2}^T + \bar{c}_{p_2}^F\right)/2 + \hat{c}_{p_2}^F}{2} = \frac{\bar{c}_{p_2}^T}{2} + \frac{\bar{c}_{p_2}^F}{4} + \frac{\hat{c}_{p_2}^F}{2}. \quad (5)$$

Similar arguments apply to the handling cost at $p_5$. Therefore, the total container handling and packing cost in Figure 3(c) is

$$\begin{aligned} &\hat{c}_{p_1}^T + \underbrace{\frac{\bar{c}_{p_2}^T}{2} + \frac{\bar{c}_{p_2}^F}{4} + \frac{\hat{c}_{p_2}^F}{2} + c_{p_2}^{2T\to F}}_{\text{Cost at } p_2} + \bar{c}_{p_3}^F + \bar{c}_{p_4}^F \\ &\quad + \underbrace{\frac{\bar{c}_{p_5}^T}{2} + \frac{\bar{c}_{p_5}^F}{4} + \frac{\hat{c}_{p_5}^F}{2} + c_{p_5}^{F\to 2T}}_{\text{Cost at } p_5} + \hat{c}_{p_6}^T. \end{aligned} \quad (6)$$

In some cases, it is still possible that (6) is smaller than (3).

*1.2. Literature Review, Objectives, and Organization.* Quantitative research on container liner shipping can be classified into three categories: strategic, tactical, and operational problems [2, 3]. Strategic problems include ship fleet planning [4] and alliance formation [5]. Strategic problems involve decisions that have effects for many years, and hence it is very difficult to predict the container demand. As a result, the demand is usually simplified; for instance, containers are formulated as either TEUs, where a D40 is considered as two TEUs, or FEUs, where a D20 is considered as half an FEU. Tactical-level decisions include network design [6, 7], fleet deployment [8], speed optimization [9], and schedule design [10]. Tactical-level decisions are usually made taking into account how containers are transported in shipping networks. Nevertheless, in almost all of these studies on tactical-level problems, similar to the ones on strategic level problems, containers are formulated as either just TEUs or just FEUs. Wang [11] examined containership fleet deployment with both TEUs and FEUs; however, the TEUs are not allowed to be repacked to FEUs.

Quantitative research on container terminal operations can be divided into studies on sea-side operations and studies on land-side operations. Sea-side decisions are mainly on berth allocation and quay crane assignment [12–19]. Land-side problems are mainly yard storage area planning and allocation [20–24]. There are also studies considering both container terminal operations planning and vessel scheduling [25–27]. None of the above studies related to container terminal operations has investigated the problem of repacking TEUs into FEUs.

The objective of our study is to investigate how a container shipping line can transport containers in an efficient manner while accounting for the possibility of repacking two TEUs into one FEU to reducing the handling costs. We systematically examine this problem and develop a holistic model that incorporates both container routing and container repacking. Hence, we address a practical problem that is significant for container shipping industry.

The remainder of the paper is organized as follows: Section 2 describes the problem. Section 3 proposes a mixed-integer linear programming model that captures both container routing and container repacking. Section 4 reports a case study. Section 5 concludes.

## 2. Problem Description

Consider a set $\mathscr{R}$ of ship routes, regularly serving a group of ports denoted by set $\mathscr{P}$. Ship route $r \in \mathscr{R}$ can be expressed as

$$p_{r1} \longrightarrow p_{r2} \longrightarrow \cdots \longrightarrow p_{rN_r} \longrightarrow p_{r1}, \quad (7)$$

where $N_r$ is the number of ports of call and $p_{ri}$ is the $i$th port of call, $i = 1, 2, \ldots, N_r$. Define $\mathscr{I}_r := \{1, 2, \ldots, N_r\}$. The voyage from port of call $i$ to port of call $i+1$ is called leg $i$ and leg $N_r$ is the voyage from port of call $N_r$ to the first port of call. In Figure 1, three ship routes are shown: ship route 1 has three legs, ship route 2 has five legs, and ship route 3 has three legs. Each ship route has a weekly service frequency, which means that each port of call is visited on the same day every week. A string of homogeneous ships with a capacity of $V_r$ (TEUs) is deployed on ship route $r$ to maintain the weekly frequency. The liner services are similar to bus services [28].

Represent by $\mathscr{W}$ the set of origin-to-destination (OD) port pairs, which is a subset of $\mathscr{P} \times \mathscr{P}$. There are two types of containers to ship: TEUs ($T$) and FEUs ($F$). The demand for OD pair $(o, d) \in \mathscr{W}$ is denoted by $n_{od}^T$ and $n_{od}^F$ (containers/week) for TEUs and FEUs, respectively. Containers can be transshipped at any port from their origins to their destinations. The loading, unloading, and transshipment costs should be included in making container routing and repacking decisions.

As aforementioned, two TEUs may be repacked into an FEU to reduce handling cost. From the practical point of view, we assume that a TEU can be repacked and unpacked at most once. The repacking and unpacking costs should also be included in making container routing and repacking decisions.

It should be noted that two TEUs can only be repacked into an FEU at container yards. In other words, the repacking activity could not be carried out on ships. For example, in Figure 3(c), the TEU with origin $p_1$ must be unloaded from ships at $p_2$ so as to be repacked with the TEU from $p_2$ into an FEU (and hence handling cost of the TEU with origin $p_1$ at $p_2$ is incurred).

The container routing with repacking problem aims to determine where to repack TEUs into FEUs and where to unpack the FEUs, and how to transport both TEUs and FEUs in order to minimize the total handling and packing cost, while allowing containers to be transshipped at any port in a liner shipping network.

## 3. Mixed-Integer Linear Programming Model

*3.1. Demand Reformulation for Container Repacking.* For each OD pair $(o, d) \in \mathscr{W}$, we define $y_{od}^{pq,T}$ as the number of

TEUs that are repacked at port $p \in \mathscr{P}$ and unpacked at port $q \in \mathscr{P}$. Let $\mathbb{Z}^+$ be the set of nonnegative integers. We have

$$y_{od}^{pq,T} \in \mathbb{Z}^+, \quad \forall (o,d) \in \mathscr{W},\ \forall p \in \mathscr{P},\ \forall q \in \mathscr{P}. \tag{8}$$

Therefore, the number of TEUs that are transported for OD pair $(o,d) \in \mathscr{W}$ without repacking, denoted by $n_{od}^{T4}$, is

$$n_{od}^{T4} = n_{od}^{T} - \sum_{p\in\mathscr{P}}\sum_{q\in\mathscr{P}} y_{od}^{pq,T}, \quad \forall (o,d) \in \mathscr{W} \tag{9}$$

$$n_{od}^{T4} \geq 0, \quad \forall (o,d) \in \mathscr{W}. \tag{10}$$

Note that (8) and (9) imply that $n_{od}^{T4}$ is an integer. The total loading and unloading cost of the $n_{od}^{T4}$ TEUs for all OD pairs is

$$C\left(n_{od}^{T4}\right) = \sum_{(o,d)\in\mathscr{W}} \left(\widehat{c}_o^{T} + \widetilde{c}_d^{T}\right) n_{od}^{T4}. \tag{11}$$

The transshipment cost of these $n_{od}^{T4}$ TEUs is not a constant as it depends on container routing.

The number of TEUs with origin port $o \in \mathscr{P}$ and repacked into FEUs at port $p \in \mathscr{P}$, denoted by $n_{op}^{T1}$, is

$$n_{op}^{T1} = \sum_{d\in\mathscr{P}}\sum_{q\in\mathscr{P}} y_{od}^{pq,T}, \quad \forall o \in \mathscr{P},\ \forall p \in \mathscr{P}. \tag{12}$$

In (12), $o = p$ means that the TEU is repacked at its origin port. Hence, for this TEU, the loading cost is equal to half the loading cost of an FEU at port $o$. If $o \neq p$, the loading cost of a TEU at port $o$, half the transshipment cost of a TEU, and a quarter of the transshipment cost of an FEU at port $p$ should be considered. In either case, the repacking cost at port $p$ should be considered. Therefore, the sum of the total handling cost excluding the transshipment cost between port $o$ and port $p$ (these TEUs may be transshipped when they are transported from port $o$ to port $p$) and the repacking cost for these $n_{op}^{T1}$ TEUs is

$$\begin{aligned} C\left(n_{od}^{T1}\right) = {} & \sum_{o\in\mathscr{P}} \frac{\widehat{c}_o^{F}}{2} n_{oo}^{T1} \\ & + \sum_{o\in\mathscr{P}}\sum_{p\in\mathscr{P}\setminus\{o\}} \left(\widehat{c}_o^{T} + \frac{\overline{c}_p^{T}}{2} + \frac{\overline{c}_p^{F}}{4}\right) n_{op}^{T1} \\ & + \sum_{o\in\mathscr{P}}\sum_{p\in\mathscr{P}} \frac{c_p^{2T\to F}}{2} n_{op}^{T1}. \end{aligned} \tag{13}$$

The number of TEUs that are unpacked from FEUs at port $q \in \mathscr{P}$ and transported from port $q \in \mathscr{P}$ to their destination port $d \in \mathscr{P}$, denoted by $n_{qd}^{T3}$, is

$$n_{qd}^{T3} = \sum_{o\in\mathscr{P}}\sum_{p\in\mathscr{P}} y_{od}^{pq,T}, \quad \forall q \in \mathscr{P},\ \forall d \in \mathscr{P}. \tag{14}$$

If $q = d$, the TEU is unpacked at its destination port. Hence, the discharge cost of half an FEU at port $d$ should be considered. Otherwise, the discharge cost of a TEU at port $d$, half the transshipment cost of a TEU, and a quarter of the transshipment cost of an FEU at port $q$ should be included. In either case, the unpacking cost at port $q$ should be incorporated. Therefore, the sum of the total handling cost excluding the transshipment cost between port $q$ and port $d$ (these TEUs may be transshipped when they are transported from port $q$ to port $d$) and the unpacking cost for these $n_{qd}^{T3}$ TEUs is

$$\begin{aligned} C\left(n_{od}^{T3}\right) = {} & \sum_{d\in\mathscr{P}} \frac{\widetilde{c}_d^{F}}{2} n_{dd}^{T3} \\ & + \sum_{d\in\mathscr{P}}\sum_{q\in\mathscr{P}\setminus\{d\}} \left(\widetilde{c}_d^{T} + \frac{\overline{c}_q^{T}}{2} + \frac{\overline{c}_q^{F}}{4}\right) n_{qd}^{T3} \\ & + \sum_{q\in\mathscr{P}}\sum_{d\in\mathscr{P}} \frac{c_q^{F\to 2T}}{2} n_{qd}^{T3}. \end{aligned} \tag{15}$$

The last reformulated demand is the transportation of the repacked containers, which are FEUs. The number of FEUs to be transported from port $p \in \mathscr{P}$ where they are repacked to port $q \in \mathscr{P}$ where they are unpacked, denoted by $n_{pq}^{F2}$, is

$$\begin{aligned} & n_{pq}^{F2} = \frac{\sum_{o\in\mathscr{P}}\sum_{d\in\mathscr{P}} y_{od}^{pq,T}}{2}, \quad \forall p \in \mathscr{P},\ \forall q \in \mathscr{P}, \\ & n_{pq}^{F2} \in \mathbb{Z}^+, \quad \forall p \in \mathscr{P},\ \forall q \in \mathscr{P}. \end{aligned} \tag{16}$$

Note that, for the $n_{pq}^{F2}$ FEUs, the handling costs at port $p$ and port $q$ are already included in the calculation in (13) and (15). The transshipment cost between port $p$ and port $q$ (if any) depends on container routing.

FEUs in the original demand $n_{od}^{F}$ do not need further processing for modeling. The total loading and unloading cost of $n_{od}^{F}$ FEUs is

$$C\left(n_{od}^{F}\right) = \sum_{(o,d)\in\mathscr{W}} \left(\widehat{c}_o^{F} + \widetilde{c}_d^{F}\right) n_{od}^{F}. \tag{17}$$

The transshipment cost of these $n_{od}^{F}$ containers is not a constant as it also depends on container routing.

We use $\widehat{n}_{od}^{T}$ and $\widehat{n}_{od}^{F}$ (containers/week) to represent the reformulated demand of TEUs and FEUs for OD pair $(o,d) \in \mathscr{W}$, respectively. The total reformulated demand is

$$\begin{aligned} \widehat{n}_{od}^{T} &= \begin{cases} 0, & o = d \\ n_{od}^{T1} + n_{od}^{T3} + n_{od}^{T4}, & \forall (o,d) \in \mathscr{W},\ o \neq d, \end{cases} \\ \widehat{n}_{od}^{F} &= \begin{cases} 0, & o = d \\ n_{od}^{F} + n_{od}^{F2}, & \forall (o,d) \in \mathscr{W},\ o \neq d. \end{cases} \end{aligned} \tag{18}$$

We define vector $\mathbf{n}$ of decision variables for formulating container repacking:

$$\begin{aligned} \mathbf{n} := \Big( & n_{od}^{T1}, n_{od}^{T3}, n_{od}^{T4}, \widehat{n}_{od}^{T}, n_{od}^{F2}, \widehat{n}_{od}^{F},\ \forall (o,d) \\ & \in \mathscr{W};\ y_{od}^{pq,T},\ \forall (o,d) \in \mathscr{W},\ \forall p \in \mathscr{P},\ \forall q \in \mathscr{P}\Big). \end{aligned} \tag{19}$$

The domain of $\mathbf{n}$, represented by set $N$, is all vectors $\mathbf{n}$ in (19) that satisfy (8), (9), (10), (12), (14), (16), and (18).

*3.2. Mathematical Model for Container Routing with Repacking.* We have already characterized container repacking decisions using vector $\mathbf{n} \in N$. To formulate container routing, we simply consider the new container shipment demand shown in (18). The decision variables for formulating container routing are as follows. $\widehat{z}_{ri}^{od,T}$ and $\widetilde{z}_{ri}^{od,T}$ are the number of TEUs per week from $(o,d) \in \mathscr{W}$ loaded and discharged at port of call $i$ on ship route $r$, respectively (note that when calculating $\widehat{z}_{ri}^{od,T}$ and $\widetilde{z}_{ri}^{od,T}$, a transshipped TEU is considered as being discharged once and being loaded once); $f_{ri}^{od,T}$ is the number of TEUs per week from $(o,d) \in \mathtt{W}$ flowing on leg $i$ on ship route $r$ (we define $f_{r0}^{od,T} := f_{rN_r}^{od,T}$); $\overline{z}_p^T$ is the total number of TEUs from all OD pairs transshipped at port $p \in \mathscr{P}$ per week. $\widehat{z}_{ri}^{od,F}$, $\widetilde{z}_{ri}^{od,F}$, $f_{ri}^{od,F}$, and $\overline{z}_p^F$ are defined correspondingly for FEUs. We define the vector of container routing decision variables:

$$\mathbf{x} := \Big(\widehat{z}_{ri}^{od,T}, \widetilde{z}_{ri}^{od,T}, f_{ri}^{od,T}, \widehat{z}_{ri}^{od,F}, \widetilde{z}_{ri}^{od,F}, f_{ri}^{od,F}, \ \forall r \in \mathscr{R}, \ \forall i \in \mathscr{I}_r, \ \forall (o,d) \in \mathscr{W}; \ \overline{z}_p^T, \overline{z}_p^F, \ \forall p \in \mathscr{P}\Big). \tag{20}$$

The container routing with repacking problem, with decisions in (19)-(20), can be formulated as an integer linear optimization model [29, 30]:

$$\min_{\mathbf{n}\in N,\mathbf{x}} \sum_{p\in\mathscr{P}} \left(\overline{c}_p^T \overline{z}_p^T + \overline{c}_p^F \overline{z}_p^F\right) + C\left(n_{od}^{T1}\right) + C\left(n_{od}^{T3}\right) + C\left(n_{od}^{T4}\right) + C\left(n_{od}^{F}\right), \tag{21}$$

subject to

$$f_{r,i-1}^{od,T} + \widehat{z}_{ri}^{od,T} = f_{ri}^{od,T} + \widetilde{z}_{ri}^{od,T}, \quad \forall r \in \mathscr{R}, \ \forall i \in \mathscr{I}_r, \ \forall (o,d) \in \mathscr{W} \tag{22}$$

$$\overline{z}_p^T = \sum_{r\in\mathscr{R}} \sum_{i\in\mathscr{I}_r, p_{ri}=p} \sum_{(o,d)\in\mathscr{W}} \widehat{z}_{ri}^{od,T} - \sum_{d\in\mathscr{P}} \widehat{n}_{pd}^T, \quad \forall p \in \mathscr{P} \tag{23}$$

$$\sum_{r\in\mathscr{R}} \sum_{i\in\mathscr{I}_r, p_{ri}=p} \left(\widehat{z}_{ri}^{od,T} - \widetilde{z}_{ri}^{od,T}\right) = \begin{cases} \widehat{n}_{od}^T, & p = o \\ -\widehat{n}_{od}^T, & p = d, \ \forall (o,d) \in \mathscr{W}, \ \forall p \in \mathscr{P} \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

$$f_{r,i-1}^{od,F} + \widehat{z}_{ri}^{od,F} = f_{ri}^{od,F} + \widetilde{z}_{ri}^{od,F}, \quad \forall r \in \mathscr{R}, \ \forall i \in \mathscr{I}_r, \ \forall (o,d) \in \mathscr{W} \tag{25}$$

$$\overline{z}_p^F = \sum_{r\in\mathscr{R}} \sum_{i\in\mathscr{I}_r, p_{ri}=p} \sum_{(o,d)\in\mathscr{W}} \widehat{z}_{ri}^{od,F} - \sum_{d\in\mathscr{P}} \widehat{n}_{pd}^F, \quad \forall p \in \mathscr{P} \tag{26}$$

$$\sum_{r\in\mathscr{R}} \sum_{i\in\mathscr{I}_r, p_{ri}=p} \left(\widehat{z}_{ri}^{od,F} - \widetilde{z}_{ri}^{od,F}\right) = \begin{cases} \widehat{n}_{od}^F, & p = o \\ -\widehat{n}_{od}^F, & p = d, \ \forall (o,d) \in \mathscr{W}, \ \forall p \in \mathscr{P} \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

$$\sum_{(o,d)\in\mathscr{W}} \left(f_{ri}^{od,T} + 2 f_{ri}^{od,F}\right) \le V_r, \quad \forall r \in \mathscr{R}, \ \forall i \in \mathscr{I}_r \tag{28}$$

$$\widehat{z}_{ri}^{od,T} \in \mathbb{Z}^+, \quad \widetilde{z}_{ri}^{od,T} \in \mathbb{Z}^+, \quad f_{ri}^{od,T} \in \mathbb{Z}^+, \quad \forall r \in \mathscr{R}, \ \forall i \in \mathscr{I}_r, \ \forall (o,d) \in \mathscr{W} \tag{29}$$

$$\widehat{z}_{ri}^{od,F} \in \mathbb{Z}^+, \quad \widetilde{z}_{ri}^{od,F} \in \mathbb{Z}^+, \quad f_{ri}^{od,F} \in \mathbb{Z}^+, \quad \forall r \in \mathscr{R}, \ \forall i \in \mathscr{I}_r, \ \forall (o,d) \in \mathscr{W}. \tag{30}$$

The objective function (21) minimizes the total handling and repacking and unpacking cost. Constraint (22) is the TEU flow conservation equation. Constraint (23) defines the total number of transshipped TEUs at each port. Constraint (24) requires that the reformulated TEU demand is fulfilled. Equations (25)–(27) define the corresponding constraints for FEUs. Constraint (28) imposes ship capacity constraint on each leg of each ship route. Constraints (29)-(30) enforce the integrality of the number of containers.

Note that as packing and unpacking containers take time, we can further incorporate constraints into the model related to the maximum number of containers that can be packed or unpacked. For instance, if ship A arrives at a port on Monday, ship B arrives at the port on Wednesday, and ship C arrives at the port on Friday, then there are two days' time to pack ships A and B's 20 ft containers into 40 ft containers to be loaded onto ship C. Taking into account the packing efficiency of the port, the maximum number of containers that can be packed can be calculated.

## 4. An Illustrative Example

We use the shipping network in Figure 4 to demonstrate the applicability of the proposed model. There are three ship routes, and all ship routes are deployed with ships of a capacity of 5000 TEUs. There are three OD pairs, where all containers are TEUs. The container handling costs are assumed to be the same at all ports: $\widehat{c}_p^T = \widetilde{c}_p^T = 100$, $\overline{c}_p^T = 150$, $\widehat{c}_p^F = \widetilde{c}_p^F = 120$, and $\overline{c}_p^F = 160$. We consider four groups of repacking and unpacking costs: group 1 with $c_p^{2T\to F} = c_p^{F\to 2T} = 50$, group 2 with $c_p^{2T\to F} = c_p^{F\to 2T} = 85$, group 3 with $c_p^{2T\to F} = c_p^{F\to 2T} = 200$, and group 4 with $c_p^{2T\to F} = c_p^{F\to 2T} = 250$. We also consider four possible packing decisions: in decision 1, no container is

Table 2: Results.

| Total handling and packing cost (×1000 USD) | $c_p^{2T \to F} = c_p^{F \to 2T}$ | | | |
|---|---|---|---|---|
| | 50 USD | 85 USD | 200 USD | 250 USD |
| No repacking | 1900 | 1900 | 1900 | **1900** |
| Containers from Shanghai to Colombo repacked | 1560 | 1630 | **1860** | 1960 |
| Containers from Shanghai to Colombo and from Xiamen to Singapore repacked | 1360 | **1500** | 1960 | 2160 |
| All containers repacked | **1330** | 1505 | 2080 | 2330 |

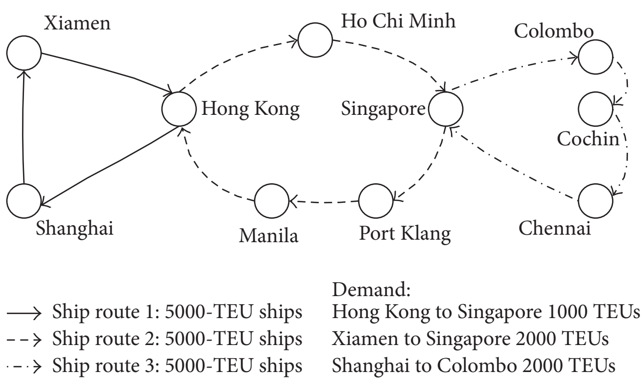



Figure 4: Another illustrative liner shipping network.

repacked and unpacked; in decision 2, only containers from Shanghai to Colombo are repacked and unpacked; in decision 3, containers from Shanghai to Colombo and containers from Xiamen to Singapore are repacked and unpacked; and, in decision 4, all containers are repacked and unpacked.

The results are shown in Table 2, and the minimum total cost in each repacking and unpacking cost group is highlighted in bold. We can see that when the repacking and unpacking costs are low (i.e., 50 USD), all TEUs should be repacked and unpacked. When the repacking and unpacking costs are higher, for example, when they are 200 USD, only TEUs from Shanghai to Colombo should be repacked and unpacked because they are transshipped twice and can take the largest advantage of handling FEUs instead of TEUs. When the repacking and unpacking costs are extremely high (i.e., 250 USD), it is no longer viable to repack TEUs into FEUs. Interestingly, those TEUs that are transshipped many times usually originate from a remote small port and are destined for another remote small port, and there are not many shipping services available. As a result, the shippers may have no choice but to agree to allow their cargos to be repacked during the trip from origin to destination. Another implication is that the packing costs may affect the routing of containers. If a port has very low packing costs, shipping companies may transship containers at this port for the sake of unpacking and repacking containers, and this will bring business to the port.

## 5. Conclusions

This paper has proposed a novel container routing with repacking problem in liner shipping. When routing containers, TEUs can be repacked into FEUs to reduce the handling cost as the handling cost of an FEU is much lower than twice the handling cost of a TEU. To the best of our knowledge, this is a new research topic that has not been dealt with in the literature. We developed a mixed-integer linear programming model that formulates the routing decisions and the repacking decisions in a holistic manner. The model could help container shipping lines to transport containers more efficiently. An illustrative example was reported to demonstrate the applicability of the model. Results show that the benefit of repacking two TEUs into an FEU is the most significant when containers are transshipped several times.

Container routing not only is significant to liner shipping companies as an independent problem but also serves as a subproblem in a number of tactical-level decision problems such as network alteration and fleet deployment. How to address tactical-level decision problems while considering container repacking is an interesting future research direction.

## Competing Interests

The authors declare that they have no competing interests.

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