Erratum: Linear Optical Quantum Metrology with Single Photons: Exploiting Spontaneously Generated Entanglement to Beat the Shot-Noise Limit [Phys. Rev. Lett. 114, 170802 (2015)]

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(Received 13 January 2017; published 17 February 2017)

DOI: 10.1103/PhysRevLett.118.079901

The ordinal resource counting (ORC) method as presented in the Supplemental Material suggests a misleading comparison between the architecture in question and the Heisenberg and shot-noise limits. The Heisenberg and shot-noise limits should represent the phase sensitivity of the best possible quantum or classical strategy, respectively, given a fixed amount of resources. We now provide an example where an architecture can beat the Heisenberg limit as generated by the ORC method, showing that the limit provided by this method is not as intended.

Let $\hat{U} = \hat{V} \cdot \hat{\Phi} \cdot \hat{\Theta} \cdot \hat{V}^{\dagger}$ as in Eq. (2) of the Letter, where $\hat{V}$, $\hat{V}^{\dagger}$ are defined as before in Eq. (3), and $\hat{\Theta}$ is again, for simplicity, the identity operator. In contrast to the Letter, however, consider the case where the phase gradient grows exponentially,

$$\hat{\Phi}_{j,k} = \delta_{j,k} \exp[i(2^{j-1} - 1)\varphi]. \tag{1}$$

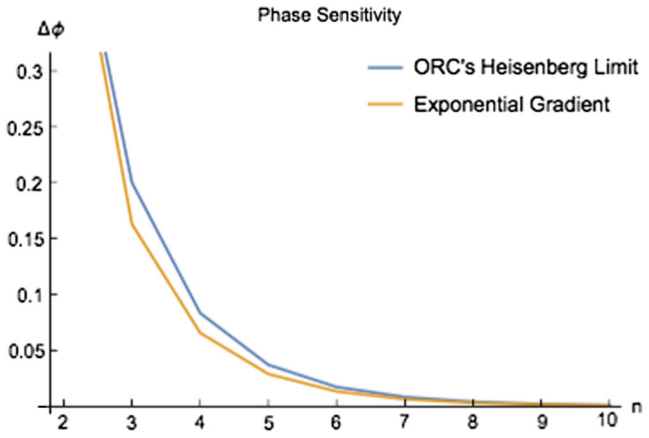

FIG. 1. Comparison between the phase sensitivity of the exponential gradient architecture and the ORC Heisenberg limit.

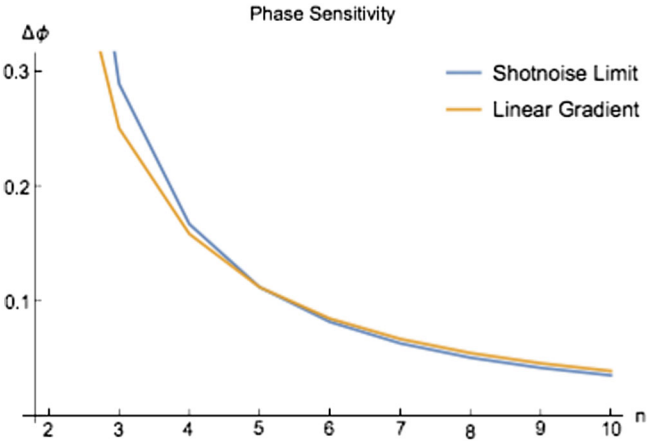

FIG. 2. Comparison between the phase sensitivity of the linear gradient architecture against the shot-noise limit as defined in Ref. [1].

Using Eq. (15) of the ORC method in the Supplemental Material, the Heisenberg-limited sensitivity would be counted as

$$\Delta\varphi_{\mathrm{HL}} = \frac{1}{N} = \frac{1}{1 + \sum_{j=1}^{n}(2^{j-1} - 1)} = \frac{1}{2^n - n}. \tag{2}$$

However, by directly calculating the phase sensitivity of the exponential gradient architecture using the error propagation formula, one can see that it outperforms the ORC's Heisenberg limit (see Fig. 1). This suggests that the ORC method has little value for comparing the phase sensitivity of interferometers of the type presented in the Letter. It follows that ORC also does not fairly account for the resources in its comparison with the shot-noise limit.

If we instead compare the linear gradient as proposed in the Letter against the shot-noise limit using the technique presented in Ref. [1] (namely, the parallel classical-classical strategy), we see that the architecture is sub-shot-noise only for $n < 5$ (see Fig. 2). The conclusions of the Letter should then be understood to hold only for interferometers with a small number of modes.

[1] V. Giovannetti, S. Lloyd, and L. Maccone, Phys. Rev. Lett. **96**, 010401 (2006).