Efficient Sensor Deployments for Spatio-Temporal Environmental Monitoring

Linh V. Nguyen, *Member, IEEE,* Guoqiang Hu, *Member, IEEE,* and Costas J. Spanos, *Fellow, IEEE*

***Abstract*—The paper addresses the problem of efficiently deploying sensors in spatial environments, e.g. buildings, for the purposes of monitoring spatio-temporal environmental phenomena. By modelling the environmental fields using spatio-temporal Gaussian processes, a new and efficient optimality-cost function of minimizing prediction uncertainties is proposed to find the best sensor locations. Though the environmental processes spatially and temporally vary, the proposed approach of choosing sensor positions is proven not to be affected by time variations, which significantly reduces computational complexity of the optimization problem. The sensor deployment optimization problem is then solved by a practical and feasible polynomial algorithm, where its solutions are theoretically proven to be guaranteed. The proposed method is also theoretically and experimentally compared with the existing works. The effectiveness of the proposed algorithm is demonstrated by implementation in a real tested space in a university building, where the obtained results are highly promising.**

***Index Terms*—Environmental monitoring, spatio-temporal model, sensor network, smart building.**

I. Introduction

RECENTLY, wireless sensor networks [1], [2] increasingly play a crucial role in monitoring environments [3]. The applications include exploring ecosystem change in ocean and on land [4], monitoring air quality and pollution [5]–[10], detecting forest fires [11] and observing indoor environmental parameters [12]–[16]. Nonetheless, in the context of deploying sensors in the spatial environments, multiple sensors can be co-located within the vicinity of a phenomenon and generate similar data samples, which apparently produce sizable redundancy in the measured data. The redundant measurements have an adverse influence on effectively using the sensor networks since they do not provide any additional information about the observed spatial fields. Moreover, the expendable samples result in many issues for the network of sensors in terms of collecting and analyzing data, particularly in long-term monitoring. For instance, constraints including initial installations, higher energy consumption, elevated maintenance burden, and increased complexity of computational cost are very practically expensive. On the other hand, in monitoring the spatial environments, resources are truly constrained. That is, the number of sensors to be utilized in sensing tasks is limited. Therefore, efficiently deploying sensors in monitoring spatio-temporal phenomena is of theoretical importance and practical relevance.

Up to now, to the best of our knowledge, the conditional entropy and mutual information based methods are two frequently-used techniques in sensor deployment for environmental monitoring. There are two major disadvantages with those methods. Firstly, these two approaches indirectly represent the prediction uncertainties via information theory based concepts. It may be improved if a direct criterion is formulated. Secondly, the information-theoretic criteria have been demonstrated only in applications of monitoring purely spatial processes, excluding time variations. In other words, if considered in spatio-temporal fields, they are prohibitively expensive when the number of measurements increases over time. Therefore, in this work, we first propose to consider the sensor deployments in scenarios of monitoring spatio-temporal environmental fields. We then develop a direct cost function of minimizing the prediction errors to find the optimal sensor positions. The proposed approach is illustrated to outperform the existing methods.

The contributions of this paper are fourfold as follows.

- Formulate a new and efficient cost function to find the best sensor locations in the spatio-temporal environments, which is only dependent on spatial elements, though measurements are gathered through time. The optimization problem is then solved by a practical and feasible algorithm in polynomial time.
- Prove that the solutions of optimizing the sensor locations in spatio-temporal environments are theoretically guaranteed by a bound.
- Comparisons between the proposed technique and the existing methods are theoretically and experimentally analyzed to demonstrate the effectiveness of the proposed methods.

The rest of the paper is organized as follows. Section II reviews the related works while Section III introduces the problem of finding the optimal environmental sensor locations in spatio-temporal scenarios. An approach to solve the optimization problem is proposed in Section IV. Section V theoretically analyses the performances of the spatio-temporal sensor deployments as well as compares the proposed technique with the existing methods. The experimental results are discussed

This work was supported by the National Research Foundation, Prime Ministers Office, Singapore under the Energy Innovation Research Programme (EIRP) for Building Energy Efficiency Grant Call, administered by the Building and Construction Authority (NRF2013EWT-EIRP004-051).

Linh V. Nguyen is with Centre for Autonomous Systems, University of Technology Sydney, New South Wales 2007, Australia (e-mail: vanlinh.nguyen@uts.edu.au).

Guoqiang Hu is with School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore (e-mail: gqhu@ntu.edu.sg).

Costas J. Spanos is with Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA 94720, United States (e-mail: spanos@berkeley.edu).

in Section VI before conclusions are drawn in Section VII.

II. Related Works

In literature, regarding deploying sensors in a static network, there is a rich library on choosing sensor locations for different purposes such as coverage [17], [18], target tracking [19], localization [20], navigation [21], and detection/surveillance [22]. For instance, in terms of parameter estimation for linear models, Joshi *et al.* [23] proposed a heuristic method based on convex optimization for the sensor selection problem. The heuristic approach in [23] utilizes a relaxation technique to convert a discrete optimization problem of sensor selection into a continuous optimization problem.

In the context of environmental monitoring, there exist some related works addressing sensor placements in the environments, e.g., indoor spaces. In [24], performance metrics based on joint conditional entropy and values of information were proposed to allocate the optimal sensing locations in the civil infrastructure systems. Brunelli *et al.* [14] positioned sensors at working areas in the university department to observe the indoor environments. However, the sensor locations are not optimal and the obtained results do not present spatial distributions of the indoor environmental parameters in the whole space. The work [13] proposed to separate sensor nodes into clusters where sensors have the same output signals, and one sensor in each cluster then is representative of the whole cluster. In [7], Wang *et al.* proposed to employ the genetic algorithm to optimal design of air quality monitoring stations. Likewise, the work [10] proposed to utilize the genetic programming to design a network of wells to identify unknown pollution sources in aquifers. To efficiently monitor air quality, while [5] designs a network of stations so that variance of gauged pollution concentrations is maximized, the optimal site network in [6] is relied upon ratio of an individual station concentration to the total of the network. A number of issues to design a network of monitoring extreme values are addressed in [25].

Generally, in aforementioned existing works, the metrics of finding sensor locations do not address the quality of sensing as well as the prediction accuracy. It should be noted that the sensing quality is defined by a maximum level of prediction uncertainties at locations without measurements. The less the prediction uncertainties are, the better the sensing quality is. Considering sensing quality in monitoring spatial fields in environments, the objective of the sensor deployment becomes to maximize the accuracy of predictions at unobserved locations of interest, after the observations are made. The deploying criteria are then formulated into an experimental design problem [26], [27]. In this case, the sensor deployment is very challenging. One can simply process all direct enumerations of possible choices and pick the best subset of sensor positions out of potential ones. However, this straightforward approach is not scalable when the network size increases.

Chadalavada *et al.* in both their works [8], [9] took designing a network of wells to monitor a spatial field of groundwater pollution into account. To find the optimal sampling locations, they proposed to minimize uncertainties in unmonitored simulated concentrations by the use of the genetic algorithm. Furthermore, information-theoretic optimality criteria based methods [24], [28]–[32] have employed information theory based concepts of conditional entropy and mutual information [33]–[35] to examine prediction uncertainties of random variables at unmeasured positions in the environments. Particularly, the authors of [24], [28], [29] proposed to find the optimal sensor locations in a given space by minimizing the conditional entropy of all the predicted environmental values at unobserved locations in the whole space. While the conditional entropy describes uncertainty of a random variable, the mutual information measures dependence between two random variables. As a result, to achieve the optimal sensor deployment, the works [30]–[32] developed the optimization criteria that maximize the mutual information between random variables at chosen locations and those at unselected positions. Moreover, these optimization criteria for sensor deployments have been theoretically proved to be combinatorial NP-hard problems [28], [36]. Nevertheless, fortunately, near-optimal solutions of the NP-hard problems can be practically obtained by a greedy yet efficient algorithm.

III. Spatio-temporal Sensor Deployment Problem

Throughout this paper, let $\mathbb{R}$ denote the set of real numbers. The norm of a vector in the Euclidean space is also denoted by $\|\cdot\|$. We let $\mathbb{E}$ define the expectation operator while $|\cdot|$ defines the absolute value of a scalar. For a matrix A, its transpose and trace are denoted as A^T and $tr(A)$, respectively. If we have a set $\mathcal{B}$, then $card(\mathcal{B})$ denotes its cardinality. Other notations will be explained as and when they occur.

A. A Spatio-temporal Model for Sensor Measurements

We consider a network of n sensors whose locations are denoted as $\mathbf{s} = (\mathbf{s}_1^T, \mathbf{s}_2^T, \cdots, \mathbf{s}_n^T)^T \in \mathbb{R}^{d\times n}$. In this study, we suppose that at each of collecting instants, all sensors can take measurements of indoor environmental phenomena. Hence, we define the time at which sensors take measurements as $\mathbf{t} = (t_1, t_2, \cdots, t_m)^T \in \mathbb{R}^m$. The collective measurements gathered by the network up to current time t_m are denoted by $Y(\mathbf{s},\mathbf{t}) = (y(\mathbf{s}_1, t_1), \cdots, y(\mathbf{s}_n, t_m))^T \in \mathbb{R}^{n\times m}$, which can be delineated by

$$Y(\mathbf{s},\mathbf{t}) = Z'(\mathbf{s},\mathbf{t}) + \epsilon(\mathbf{s},\mathbf{t}), \tag{1}$$

where $\epsilon(\mathbf{s},\mathbf{t}) \in \mathbb{R}^{n\times m}$ is normally distributed with a zero mean and an unknown variance, and $Z'(\mathbf{s},\mathbf{t}) \in \mathbb{R}^{n\times m}$ are the random/latent variables at $(\mathbf{s},\mathbf{t})$. The spatio-temporal sensor readings $Y(\mathbf{s},\mathbf{t})$ are proposed to follow a multivariate Gaussian distribution as follows,

$$Y(\mathbf{s},\mathbf{t}) \sim \mathcal{N}(\mu, \Sigma), \tag{2}$$

where the mean vector is defined by $\mu = \mathbb{E}[Y(\mathbf{s},\mathbf{t})]$. Σ is the $nm \times nm$ covariance matrix of $Y(\mathbf{s},\mathbf{t})$, where its elements can be computed by a space-time covariance function, e.g. $cov((\mathbf{s}_i, t_j), (\mathbf{s}_k, t_l))$ for any two pairs of spatio-temporal locations $(\mathbf{s}_i, t_j)$ and $(\mathbf{s}_k, t_l)$ on $\mathbb{R}^d \times \mathbb{R}$, $i, k = \{1, \cdots, n\}$ and $j, l = \{1, \cdots, m\}$. Note that the covariance function spatially temporally represents dependence of observations.

It is noteworthy that though the number of sensors is limited, knowing the whole environment is needed for most monitoring purposes. Hence, we let $\mathbf{s_N} = (\mathbf{s'}_1^T, \mathbf{s'}_2^T, \cdots, \mathbf{s'}_N^T)^T \in \mathbb{R}^{d\times N}$ and $\mathbf{t_M} = (t'_1, t'_2, \cdots, t'_M)^T \in \mathbb{R}^M$ denote unobserved locations of interest in the space and specific instants in the time, respectively, where and when the environmental phenomenon is required to be predicted. Note that locations of interest are subject to applications; for example, interested locations are on a dense grid if a map of the environmental field is expected to be created. Then $N \gg n$. The random/latent variables $Z(\mathbf{s}_N, \mathbf{t}_M)$ at predicted spatio-temporal locations $(\mathbf{s}_N, \mathbf{t}_M)$ and the observations $Y(\mathbf{s}, \mathbf{t})$ have a joint distribution. Since $Y(\mathbf{s}, \mathbf{t})$ is normally distributed as (2), the marginalization property of the Gaussian distribution [37] yields

$$\begin{bmatrix} Y(\mathbf{s}, \mathbf{t}) \\ Z(\mathbf{s}_N, \mathbf{t}_M) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu \\ \mu_Z \end{bmatrix}, \begin{bmatrix} \Sigma & \Sigma_{YZ} \\ \Sigma_{YZ}^T & \Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)} \end{bmatrix} \right), \quad (3)$$

where μ_Z and $\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)}$ are the mean vector and the covariance matrix of $Z(\mathbf{s}_N, \mathbf{t}_M)$. Σ_{YZ} is the cross-covariance matrix representing the dependence between $Y(\mathbf{s}, \mathbf{t})$ and $Z(\mathbf{s}_N, \mathbf{t}_M)$. From (3), we can now infer the conditional distribution of $Z(\mathbf{s}_N, \mathbf{t}_M)$, given the measurements $Y(\mathbf{s}, \mathbf{t})$, by taking the following form.

$$Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s}, \mathbf{t}) \sim \mathcal{N}(\mu_{Z|Y(\mathbf{s},\mathbf{t})}, \Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}), \quad (4)$$

where

$$\mu_{Z|Y(\mathbf{s},\mathbf{t})} = \mu_Z + \Sigma_{YZ}^T \Sigma^{-1}(Y(\mathbf{s}, \mathbf{t}) - \mu), \quad (5)$$

$$\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})} = \Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)} - \Sigma_{YZ}^T \Sigma^{-1} \Sigma_{YZ}. \quad (6)$$

It can be seen that the uncertainties at predicted spatio-temporal points $(\mathbf{s}_N, \mathbf{t}_M)$ are on the diagonal line of the covariance matrix $\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}$. Therefore, the problem of sensor deployment becomes finding n locations in the environment for deploying n sensors so that the uncertainties at $(\mathbf{s}_N, \mathbf{t}_M)$ are minimized.

It is noticed that all the mean parameters and hyperparameters of the Gaussian distribution (2), which are also employed to compute the mean vector, the covariance matrix and the cross-covariance matrices in (5) and (6), are unknown. Nonetheless, these parameters can be directly estimated based on the measurements $Y(\mathbf{s}, \mathbf{t})$ by the use of the maximum likelihood method as presented in [29], [38].

B. Problem Statement

As discussed, understanding environments is a paramount task that is required to be completed before any control strategies can be carried out. What people usually are concerned about in environmental monitoring is the sensing quality. In other words, there are two major and challenging questions arising when implementing the sensor deployments: (i) where to locate a given number of sensors in a space to observe environmental parameters in order to minimize prediction uncertainties at all unmeasured positions, and (ii) how many sensors are needed so that prediction accuracy is guaranteed by a desired threshold. The problems are formulated as follows.

In this paper, it is proposed to compute a total of the variances at all predicted spatio-temporal locations $(\mathbf{s}_N, \mathbf{t}_M)$. That is, the formal formula of the proposed optimality-cost function is to calculate the trace of $\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}$. Let us define $\mathcal{P}$ a set of all possible locations where environmental sensors can be deployed to observe the physical fields in the space, where $card(\mathcal{P}) = p$ is the cardinality of $\mathcal{P}$. The sensor deployment is to address the problem of choosing a subset $\mathcal{C} \subseteq \mathcal{P}$, where $card(\mathcal{C}) = n$, so that if n sensors are positioned at n locations in $\mathcal{C}$ then the corresponding measurements $Y(\mathbf{s}, \mathbf{t})$ allow the total of the variances at $(\mathbf{s}_N, \mathbf{t}_M)$ to be minimized. Mathematically, the sensor deployment problem is initially formulated as follows,

$$\mathcal{C}^{opt} = \underset{\substack{\mathcal{C} \subseteq \mathcal{P} \\ card(\mathcal{C}) = n}}{\mathbf{argmin}} \quad tr(\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}), \quad (7)$$

where $\mathcal{C}^{opt}$ is the optimal set of sensor locations. Since $\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)}$ in (6) is the covariance matrix of the random variables $Z(\mathbf{s}_N, \mathbf{t}_M)$ at unobserved spatio-temporal locations $(\mathbf{s}_N, \mathbf{t}_M)$, it is not dependent on $\mathcal{C}$, a set of observed sensor locations. Consequently, the problem in (7) can be simplified as

$$\mathcal{C}^{opt} = \underset{\substack{\mathcal{C} \subseteq \mathcal{P} \\ card(\mathcal{C}) = n}}{\mathbf{argmax}} \quad tr(\Sigma_{YZ}^T \Sigma^{-1} \Sigma_{YZ}). \quad (8)$$

Note that (8) is a combinatorial optimization problem. Choosing a subset $\mathcal{C}$ out of a possible set $\mathcal{P}$ in the combinatorial optimization problem is always NP-hard [28]. Nevertheless, up to now, the NP-hard problem can be near-optimally solved by an approximate polynomial algorithm called the greedy algorithm. Let us consider how the greedy method can deal with the NP-hard problem in (8). Obviously, computing $\Sigma_{YZ}^T \Sigma^{-1} \Sigma_{YZ}$ requires $\mathcal{O}(N^2M^2nm + n^3m^3)$ operations. Moreover $N \gg n$, which leads to the fact that the optimality-cost function in (8) is computationally costly. Thus, further simplification is needed. Since $tr(AB) = tr(BA)$, the problem of sensor deployments can eventually be defined as

$$\mathcal{C}^{opt} = \underset{\substack{\mathcal{C} \subseteq \mathcal{P} \\ card(\mathcal{C}) = n}}{\mathbf{argmax}} \quad tr(\Sigma_{YZ} \Sigma_{YZ}^T \Sigma^{-1}), \quad (9)$$

where the complexity of computing $\Sigma_{YZ} \Sigma_{YZ}^T \Sigma^{-1}$ is $\mathcal{O}(NMn^2m^2 + n^3m^3)$.

It is apparent that if the problem in (9) is comprehensively solved, the question (i) in the first paragraph of Section III-B is answered. Furthermore, the solutions of the problem in (9) imply that by varying the number of sensors until the total of the variances at all predicted spatio-temporal locations $(\mathbf{s}_N, \mathbf{t}_M)$ satisfies a predefined requirement, the number of sensors in that case is an answer to the question (ii) stated in the same paragraph.

IV. A Separable Approach

The problem stated in (9) can be practically resolved in a small-scale sensor network with a small data set. The problem definitely becomes intractable as it is applied to a large-scale network (n is large), where measurements are collected within a long period of time (m is large). In this section, we present a separable method to reduce the complexity of the problem of sensor deployments.

In recent research, there are discussions about types of covariance functions for space-time models [39], [40]. There are two major sorts of spatio-temporal covariance models that are separable and non-separable, respectively. The non-separable covariance functions well known in the literature are Gneiting models [41], Porcu and Mateu mixture-based models [42] and Integrated product and product-sum models [43]. Theoretically, the spatio-temporal non-separable covariance models have been thought to better capture possible space-time interactions. Nevertheless, due to computational complexity of the spatio-temporal non-separable covariance functions, especially when applied in large data sets, in this work, it is proposed to choose a spatio-temporal separable covariance function for the space-time environmental field model. If dependence between the data is separable in terms of space and time, the covariance matrix Σ of the collective measurements $Y(\mathbf{s},\mathbf{t})$ can be represented by block structures. In other words, Σ can be delineated by a Kronecker product as

$$\Sigma = \Sigma^{(s)} \otimes \Sigma^{(t)}, \tag{10}$$

where $\Sigma^{(s)}$ is an $n \times n$ covariance matrix of purely spatial covariance values, and $\Sigma^{(t)}$ is an $m \times m$ covariance matrix of purely temporal covariance values.

Due to restructuring the covariance matrix of all available observations, the optimality-cost function for the problem of sensor deployments in buildings in (9) can be simplified as in the following theorem.

Theorem 1: If a space-time covariance function of a spatio-temporal environmental field model is separable, the problem of optimally deploying sensors in environments to observe spatio-temporal fields is only dependent on space variations, not time variations.

Proof: Since spatio-temporal correlation function is separable, the cross-covariance matrix Σ_{YZ} between $Y(\mathbf{s},\mathbf{t})$ and $Z(\mathbf{s}_N,\mathbf{t}_M)$ in (6) can be also re-specified by

$$\Sigma_{YZ} = \Sigma_{YZ}^{(s)} \otimes \Sigma_{YZ}^{(t)}, \tag{11}$$

where $\Sigma_{YZ}^{(s)}$ and $\Sigma_{YZ}^{(t)}$ are $n \times N$ purely spatial and $m \times M$ purely temporal cross-covariance matrices of $Y(\mathbf{s},\mathbf{t})$ and $Z(\mathbf{s}_N,\mathbf{t}_M)$, respectively.

Under properties of the Kronecker product, the expression in the trace function in (9) is rewritten as

$$\begin{aligned}
&\Sigma_{YZ}\Sigma_{YZ}^{T}\Sigma^{-1} = \qquad\qquad (12)\\
&\left(\Sigma_{YZ}^{(s)} \otimes \Sigma_{YZ}^{(t)}\right)\left((\Sigma_{YZ}^{(s)})^{T} \otimes (\Sigma_{YZ}^{(t)})^{T}\right)\left((\Sigma^{(s)})^{-1} \otimes (\Sigma^{(t)})^{-1}\right)\\
&= \left(\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^{T} \otimes \Sigma_{YZ}^{(t)}(\Sigma_{YZ}^{(t)})^{T}\right)\left((\Sigma^{(s)})^{-1} \otimes (\Sigma^{(t)})^{-1}\right)\\
&= \left(\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^{T}(\Sigma^{(s)})^{-1}\right) \otimes \left(\Sigma_{YZ}^{(t)}(\Sigma_{YZ}^{(t)})^{T}(\Sigma^{(t)})^{-1}\right).
\end{aligned}$$

Thus, the function in (9) can be rewritten as

$$\begin{aligned}
&tr(\Sigma_{YZ}\Sigma_{YZ}^{T}\Sigma^{-1}) = \qquad\qquad (13)\\
&= tr\left(\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^{T}(\Sigma^{(s)})^{-1}\right) tr\left(\Sigma_{YZ}^{(t)}(\Sigma_{YZ}^{(t)})^{T}(\Sigma^{(t)})^{-1}\right).
\end{aligned}$$

Since $\Sigma_{YZ}^{(t)}(\Sigma_{YZ}^{(t)})^{T}(\Sigma^{(t)})^{-1}$ is a purely temporal covariance matrix, if sensor locations spatially vary then $tr\left(\Sigma_{YZ}^{(t)}(\Sigma_{YZ}^{(t)})^{T}(\Sigma^{(t)})^{-1}\right)$ will not change. This completes the proof. ■

The optimality-cost function of the sensor deployment problem can be now spatially stated as

$$\mathcal{C}^{opt} = \underset{\substack{\mathcal{C} \subseteq \mathcal{P} \\ card(\mathcal{C}) = n}}{\textbf{argmax}} \; tr\left(\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^{T}(\Sigma^{(s)})^{-1}\right). \tag{14}$$

Complexity of computing $\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^{T}(\Sigma^{(s)})^{-1}$ in (14) is $\mathcal{O}(Nn^2+n^3)$, which is significantly scaled down as compared with that in (9). In the following, we present how the greedy algorithm near-optimally addresses the problem in (14). Let us define a near-optimal subset corresponding to $\mathcal{C}^{opt}$ as $\mathcal{C}^{n-opt}$.

It is assumed that at the beginning, the near-optimal subset is empty, $\mathcal{C}^{n-opt} = \oslash$. The algorithm randomly chooses a point $\mathbf{s}_i$, $i = 1, \cdots, p$, from the possible set $\mathcal{P}$, $\mathbf{s}_i \in \mathcal{P}$. The corresponding measurement at $\mathbf{s}_i$ is denoted as $y(\mathbf{s}_i)$. In addition, $Y(\mathbf{s},\mathbf{t}) = Y(\mathbf{s}) = \{y(\mathbf{s}_i)\}$ and $Z(\mathbf{s}_N,\mathbf{t}_M) = Z(\mathbf{s}_N)$ as time is no longer involved in the calculation. It computes $\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^{T}(\Sigma^{(s)})^{-1}$, and then it iterates the computations for each other location $\mathbf{s}_i \in \mathcal{P}$. Each calculation returns one real value. A sequence of obtained values consequently corresponds to the possible set $\mathcal{P}$. Choosing the maximum value from this sequence, it can find the corresponding location from $\mathcal{P}$. This chosen point is the first near-optimal sensor location, denoted as $\mathbf{s}_1^{n-opt}$, and $\mathcal{C}^{n-opt} = \{\mathbf{s}_1^{n-opt}\}$. Correspondingly, $Y(\mathbf{s})$ now firmly has $y(\mathbf{s}_1^{n-opt})$, $Y(\mathbf{s}) = \{y(\mathbf{s}_1^{n-opt})\}$. The chosen location $\mathbf{s}_1^{n-opt}$ is now removed from $\mathcal{P}$. In the next step, the algorithm again chooses a location $\mathbf{s}_i$, $i = 1, \cdots, p-1$, from the remaining $\mathcal{P}$ and temporally adds it into $\mathcal{C}^{n-opt}$, $\mathcal{C}^{n-opt} = \{\mathcal{C}^{n-opt}, \mathbf{s}_i\}$. Likewise, the corresponding measurement $y(\mathbf{s}_i)$ is also temporally added into $Y(\mathbf{s})$, $Y(\mathbf{s}) = \{Y(\mathbf{s}), y(\mathbf{s}_i)\}$. Every $\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^{T}(\Sigma^{(s)})^{-1}$ in the second step, corresponding to one $\mathbf{s}_i \in \mathcal{P}$, is a 2×2 matrix, and their traces create another sequence. Finding the sequence's maximal value and choosing its corresponding location in $\mathcal{P}$, one has the second near-optimal sensor location, which is then firmly moved from $\mathcal{P}$ to $\mathcal{C}^{n-opt}$. $Y(\mathbf{s})$ also has a second permanent element. Iteratively, the algorithm runs this iteration until cardinality of $\mathcal{C}^{n-opt}$ reaches n. In each iteration, a new near-optimal sensor location obtained is *greedily* added into $\mathcal{C}^{n-opt}$. The greedy algorithm addressing the sensor deployment problem is illustratively summarized in Algorithm 1.

V. Performance Analysis on Sensor Deployment

A. A Solution Bound

Though the greedy algorithm has been empirically demonstrated to be very efficient in tackling NP-hard problems [44], it can only provide near-optimal solutions. That is, it is essential to guarantee the near-optimal solutions obtained in the NP-hard problem in (14). In the following, we discuss the bound of the approximate solutions obtained in the problem of optimally deploying sensors in spatio-temporal environments.

Algorithm 1 Approximation algorithm for sensor deployments in smart buildings

Input:

1) Set of possible sensor locations $\mathcal{P}$
2) A learned model to generate corresponding measurements $y(\mathbf{s}_i)$
3) Number of sensors n

Output:

1) Near-optimal set of sensor locations $\mathcal{C}^{n-opt}$

At the start, do
$\mathcal{C}^{n-opt} \leftarrow \oslash$

for i = 1 to p **do**
  $\mathbf{s}_i \in \mathcal{P}$
  $Y(\mathbf{s}) = \{y(\mathbf{s}_i)\}$
  Compute $\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^T(\Sigma^{(s)})^{-1}$
end for
$\mathbf{s}_1^{n-opt} \leftarrow \underset{\mathbf{s}_i \in \mathcal{P}}{\mathbf{argmax}}\, \Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^T(\Sigma^{(s)})^{-1}$
$\mathcal{C}^{n-opt} = \{\mathbf{s}_1^{n-opt}\}$
$Y(\mathbf{s}) = \{y(\mathbf{s}_1^{n-opt})\}$
$\mathcal{P} \leftarrow \mathcal{P} \setminus \mathbf{s}_1^{n-opt}$
for k = 2 to n **do**
  $\mathcal{C}_{tmp}^{n-opt} = \mathcal{C}^{n-opt}$
  $Y_{tmp}(\mathbf{s}) = Y(\mathbf{s})$
  for i = 1 to cardinality of $\mathcal{P}$ **do**
    $\mathbf{s}_i \in \mathcal{P}$
    $\mathcal{C}^{n-opt} = \{\mathcal{C}_{tmp}^{n-opt}, \mathbf{s}_i\}$
    $Y(\mathbf{s}) = \{Y_{tmp}(\mathbf{s}), y(\mathbf{s}_i)\}$
    Compute $tr\left(\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^T(\Sigma^{(s)})^{-1}\right)$
  end for
  $\mathbf{s}_k^{n-opt} \leftarrow \underset{\mathbf{s}_i \in \mathcal{P}}{\mathbf{argmax}}\, tr\left(\Sigma_{YZ}^{(s)}(\Sigma_{YZ}^{(s)})^T(\Sigma^{(s)})^{-1}\right)$
  $\mathcal{C}^{n-opt} = \{\mathcal{C}_{tmp}^{n-opt}, \mathbf{s}_k^{n-opt}\}$
  $Y(\mathbf{s}) = \{Y_{tmp}(\mathbf{s}), y(\mathbf{s}_k^{n-opt})\}$
  $\mathcal{P} \leftarrow \mathcal{P} \setminus \mathbf{s}_k^{n-opt}$
end for

For the purpose of generalization, we consider the bound of the solutions of the general sensor deployment problem defined in (9), which is subject to both spatial and temporal terms. Before providing the results, we first introduce a preliminary basis in the following lemma.

Lemma 2: The matrix $\Sigma_{YZ}\Sigma_{YZ}^T\Sigma^{-1}$ in (9) is positive definite.

Proof: Generally speaking, every covariance matrix is positive semi-definite. In this case, since the covariance matrix Σ is invertible, it is positive definite. It is noteworthy that Σ is symmetric, from spectral theorem [45] we have

$$\Sigma = UDU^T, \tag{15}$$

where U is an orthogonal matrix, and D is a diagonal matrix. Because the eigenvalues of Σ are positive, so are those of D, $\lambda_i(D) > 0$, where $\lambda_i(D)$ is an i^{th} eigenvalue of D. We can also present Σ^{-1} in a spectral decomposition form as

$$\Sigma^{-1} = UD^{-1}U^{-1} = UD^{-1}U^T. \tag{16}$$

Since the eigenvalues of D^{-1} are positive, $\frac{1}{\lambda_i(D)} > 0$, which indicates that Σ^{-1} is positive definite.

On the other hand, Σ_{YZ} measures dependence among the measurements Y and the random variables Z, which represent a spatio-temporal process at different locations; that is, all rows of Σ_{YZ} are independent, or rank of Σ_{YZ} is n. Therefore, rank of $\Sigma_{YZ}\Sigma_{YZ}^T$ is n. In equivalent words, since Σ_{YZ} is real and has full rank of n, $\Sigma_{YZ}\Sigma_{YZ}^T$ is positive definite [46].

Now, we denote

$$W = \left(\left(\Sigma_{YZ}\Sigma_{YZ}^T\right)^{\frac{1}{2}}\right)^{-1}\left(\Sigma_{YZ}\Sigma_{YZ}^T\Sigma^{-1}\right)\left(\Sigma_{YZ}\Sigma_{YZ}^T\right)^{\frac{1}{2}}. \tag{17}$$

Here, $\left(\Sigma_{YZ}\Sigma_{YZ}^T\right)^{\frac{1}{2}}$ is non-singular since $\Sigma_{YZ}\Sigma_{YZ}^T$ is non-singular. Therefore, W and $\Sigma_{YZ}\Sigma_{YZ}^T\Sigma^{-1}$ are similar.

Moreover, W can be represented as

$$W = \left(\Sigma_{YZ}\Sigma_{YZ}^T\right)^{\frac{1}{2}}\Sigma^{-1}\left(\Sigma_{YZ}\Sigma_{YZ}^T\right)^{\frac{1}{2}}. \tag{18}$$

As mentioned above, Σ^{-1} is positive definite, hence W is also positive definite. We recall that when W and $\Sigma_{YZ}\Sigma_{YZ}^T\Sigma^{-1}$ are similar, they have the same eigenvalues [47]. In other words, $\Sigma_{YZ}\Sigma_{YZ}^T\Sigma^{-1}$ is positive definite, which completes the proof. ∎

We now mathematically write the problem in (9) in the set function form as follows,

$$\mathcal{F}(\mathcal{C}) = tr(\Sigma_{YZ}\Sigma_{YZ}^T\Sigma^{-1}), \tag{19}$$

where $\mathcal{F}(\mathcal{C})$ is a set function on a set $\mathcal{C}$ and Σ is the covariance matrix of spatio-temporal measurements $Y(\mathbf{s},\mathbf{t})$ gathered at n locations in $\mathcal{C}$. Then, the bound of the solutions of the sensor deployment problem can be stated by the following theorem.

Theorem 3: The near-optimal solutions $\mathcal{F}(\mathcal{C}^{n-opt})$ for the spatio-temporal sensor deployment problem obtained by the greedy algorithm presented in Section IV, which correspond to near-optimal sensor locations, are guaranteed by a $1 - \left(1 - \frac{1}{n}\right)^n$ level of optimal performances, where n is the number of sensors to be deployed.

Proof: Since $\Sigma_{YZ}\Sigma_{YZ}^T\Sigma^{-1}$ is positive definite, $\mathcal{F}(\mathcal{C})$ is monotonic increasing. In equivalent words, if we add one sensor location $\mathbf{s}_i \in \mathcal{P} \setminus \mathcal{C}$ to $\mathcal{C}$, then the number of the eigenvalues of the matrix $\Sigma_{YZ}\Sigma_{YZ}^T\Sigma^{-1}$ is increased by one. Thus, $\mathcal{F}(\mathcal{C} \cup \mathbf{s}_i) > \mathcal{F}(\mathcal{C})$. On the other hand, the theorem 2 in [48] states that given a monotonic increasing set function (19), a greedy algorithm can address the NP-hard problem (9) by an

$$1 - \left(\frac{\gamma + \cdots + \gamma^{n-1}}{1 + \gamma + \cdots + \gamma^{n-1}}\right)^n \tag{20}$$

approximation as compared with the optimum. Here, γ is elemental curvature computed by

$$\gamma = \max_{\substack{\mathcal{C} \subset \mathcal{P} \\ \mathbf{s}_i, \mathbf{s}_j \in \mathcal{P} \setminus \mathcal{C}}} \frac{\mathcal{F}(C \cup \mathbf{s}_i \cup \mathbf{s}_j) - \mathcal{F}(C \cup \mathbf{s}_i)}{\mathcal{F}(C \cup \mathbf{s}_i) - \mathcal{F}(C)}. \tag{21}$$

Let us investigate a specific property of the set function $\mathcal{F}(\mathcal{C})$. If we have $\mathcal{C}' = \mathcal{C} \cup \mathbf{s}_i$, $\mathbf{s}_i \in \mathcal{P} \setminus \mathcal{C}$, then $\mathcal{C}' \cup \mathcal{C} = \mathcal{C}'$ and $\mathcal{C}' \cap \mathcal{C} = \mathcal{C}$. Consequently,

$$\mathcal{F}(\mathcal{C}) + \mathcal{F}(\mathcal{C}') = \mathcal{F}(\mathcal{C}' \cap \mathcal{C}) + \mathcal{F}(\mathcal{C}' \cup \mathcal{C}). \tag{22}$$

As illustrated in [49], $\mathcal{F}(\mathcal{C})$ is a modular function. Furthermore, the set function $\mathcal{F}(\mathcal{C})$ also holds another property. For every $\mathcal{C} \subset \mathcal{C}' \subset \mathcal{P}$ and $\mathbf{s}_j \in \mathcal{P} \setminus \mathcal{C}'$, one has

$$\mathcal{F}(\mathcal{C} \cup \mathbf{s}_j) - \mathcal{F}(\mathcal{C}) = \mathcal{F}(\mathcal{C}' \cup \mathbf{s}_j) - \mathcal{F}(\mathcal{C}'). \tag{23}$$

Therefore, it is derived that $\gamma = 1$, which completes the proof. ■

It is remarked that the bound for the near-optimal solutions of the sensor deployment problem is only dependent on the number of sensors to be located in the environments. That is, if one needs to find a small number of optimal sensor locations, the proposed approach can return approximate solutions that are very close to the optimal ones.

B. Comparisons with Existing Methods

In the literature, there are some works relating to the sensor deployments for the purposes of monitoring environments [24], [28]–[32]. In these works, all the authors proposed the criteria for finding sensor locations so as to maximize sensing quality, e.g. minimizing uncertainties of prediction results. Nonetheless, the authors only consider applications of observing purely spatial fields, where time variations are excluded. As a consequence, this subsection discusses whether the existing approaches are feasible to apply for spatio-temporal environmental processes.

1) Conditional Entropy: The authors in [24], [28], [29] introduced the optimization for optimal sensor deployments in spatial environments, which is based on conditional entropy [33]. The conditional entropy based cost function is extensively stated in the space-time environmental sensor deployments as

$$\mathcal{C}^{opt} = \underset{\substack{\mathcal{C} \subseteq \mathcal{P} \\ card(\mathcal{C}) = n}}{\textbf{argmin}} \log\det\left(\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}\right), \tag{24}$$

where $\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}$ is computed by (6). For the purpose of comparisons, we also choose a separable covariance function for the spatio-temporal environmental model. Therefore, elements in $\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}$ are presented by (10), (11) and

$$\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)} = \Sigma_{Z(\mathbf{s}_N)} \otimes \Sigma_{Z(\mathbf{t}_M)}, \tag{25}$$

where $\Sigma_{Z(\mathbf{s}_N)}$ and $\Sigma_{Z(\mathbf{t}_M)}$ are $N \times N$ purely spatial and $M \times M$ purely temporal covariance matrices of $Z(\mathbf{s}_N, \mathbf{t}_M)$, respectively.

The problem (24) is clearly NP-hard and can be resolved by the greedy algorithm [24], [28], [29], whose complexity is illustrated by the following theorem.

Theorem 4: The approximation solution of the conditional entropy based spatio-temporal environmental sensor deployment problem (24) can be achieved by a greedy algorithm in $\mathcal{O}((N^3M^3 + n^3m^3)np)$ operations.

Proof: The complexity of computing $\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}$ is $\mathcal{O}(N^2M^2nm + n^3m^3)$ as shown in Section III. On the other hand, the size of $\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}$ is $N \times M$. Therefore, $\log\det\left(\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)|Y(\mathbf{s},\mathbf{t})}\right)$ can be calculated in time $\mathcal{O}(N^3M^3)$ [50]. Note that n locations of the set $\mathcal{C}$ are chosen from p possible positions available in the set $\mathcal{P}$. Then, it is required to compute this logarithm of the determinant in $\mathcal{O}(p)$ times for each time of choosing one location for $\mathcal{C}$. Moreover, since the cardinality of $\mathcal{C}$ is n, the solution of the combinatorial optimization problem (24) can be obtained in running time $\mathcal{O}((N^3M^3 + n^3m^3)np)$ ■

2) Mutual Information: Another information-theoretic method for the sensor deployments in monitoring spatial environments is mutual information (MI) [30]–[32], which measures the dependency between two random variables [34], [35]. As presented in [31], Krause *et al.* proposed to find the optimal sensor locations in the spatial environments by maximizing the mutual information between random variables at chosen sensor locations and at unselected positions. Note that the set of possible locations is a joint of chosen sensor locations and unselected positions. Though the method in [31] does not mention locations of interest, this paper merges locations of interest into the set of unselected positions. Therefore, the mutual information based approach is now extended for the problem of optimally deploying sensors in monitoring spatio-temporal environments as

$$\mathcal{C}^{opt} = \underset{\substack{\mathcal{C} \subseteq \mathcal{P} \\ card(\mathcal{C}) = n}}{\textbf{argmax}} \left(\log\det(\Sigma_{\mathcal{P}Z\backslash\mathcal{C}}) - \log\det(\Sigma_{\mathcal{P}Z\backslash\mathcal{C}|\mathcal{C}})\right), \tag{26}$$

where $\mathcal{P}Z$ is an emerging set of $\mathcal{P}$ and the locations of $Z(\mathbf{s}_N, \mathbf{t}_M)$. $card(\mathcal{P}Z) \leq N+p$ since some possible locations and interested positions may be overlapped. In worst cases, $\Sigma_{\mathcal{P}Z\backslash\mathcal{C}}$ and $\Sigma_{\mathcal{P}Z\backslash\mathcal{C}|\mathcal{C}}$ are $(N+p-n)M \times (N+p-n)M$ covariance matrices of random variables at $\mathcal{P}Z\backslash\mathcal{C}$ and random variables at $\mathcal{P}Z\backslash\mathcal{C}$ given observations at $\mathcal{C}$, respectively. Here,

$$\Sigma_{\mathcal{P}Z\backslash\mathcal{C}|\mathcal{C}} = \Sigma_{\mathcal{P}Z\backslash\mathcal{C}} - \Sigma_{\mathcal{P}Z\backslash\mathcal{C},\mathcal{C}}\Sigma^{-1}\Sigma^T_{\mathcal{P}Z\backslash\mathcal{C},\mathcal{C}}, \tag{27}$$

where $\Sigma_{\mathcal{P}Z\backslash\mathcal{C},\mathcal{C}}$ is a $(N+p-n)M \times nm$ cross-covariance matrix between random variables at $\mathcal{P}Z \setminus \mathcal{C}$ and observations at $\mathcal{C}$.

Regardless of the types of spatio-temporal covariance functions chosen, approximate solutions of the combinatorial NP-hard optimization problem (26) can be obtained by a greedy algorithm with computational complexity as below.

Theorem 5: The mutual information based combinatorial NP-hard optimization (26) for sensor deployments in spatio-temporal environments can be addressed by a greedy approach in running time of $\mathcal{O}(((N+p-n)^3M^3 + n^3m^3)np)$.

Proof: The proof is similar to that of *Theorem 4*. ■

3) The Proposed Approach: Comparing with two well-known existing methods, the proposed approach significantly reduces computational complexity in addressing the spatio-temporal sensor deployments as demonstrated in the following.

Theorem 6: Algorithm 1 can resolve the proposed optimality-cost function (14) for space-time sensor deployments in $\mathcal{O}((Nn^2 + n^3)np)$ operations.

Proof: The proof is similar to that of *Theorem 4* with reference to Section IV. ■

Remark 7: There are some conclusions regarding comparisons of the proposed approach with the others in literature, for optimally deploying sensors in spatio-temporal environments, as follows.

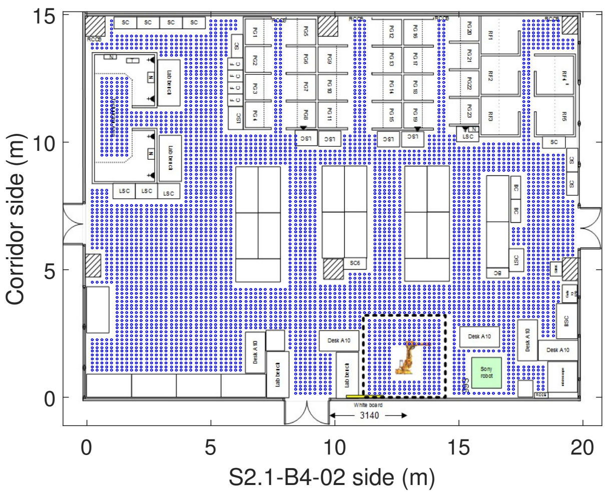

Fig. 1: Possible sensor locations (blue circles) in room S2.1-B4-01, Nanyang Technological University

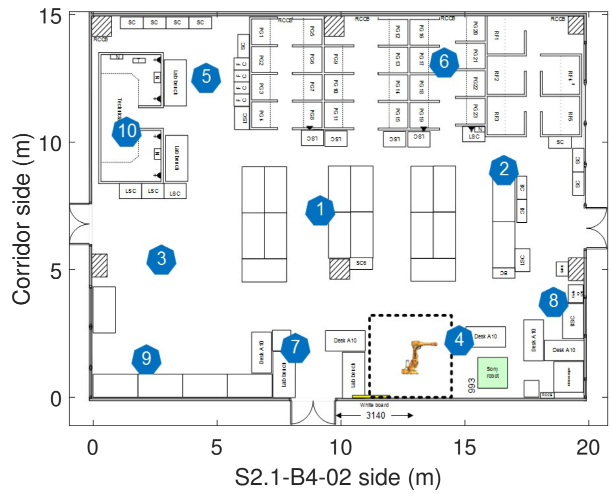

Fig. 2: 10 near-optimal sensor locations, ordered from most to least informative locations, in room S2.1-B4-01, Nanyang Technological University

- It can be clearly seen that complexity of the proposed approach is not dependent on M or m. That is, computing time of the conditional entropy and mutual information based methods increases when collected times m and predicted times M go up, while running time of our algorithm is consistent.
- Both conditional entropy and mutual information based methods are indirect. That is, they do not directly maximize the sensing quality. The optimal sensor locations are found by optimizing indirect information-theoretic cost functions. Nevertheless, our technique proposes optimal sensor deployments by directly minimizing prediction errors.
- The functions of conditional entropy and mutual information based cost functions in (24) and (26) are not completely monotonic [29], [31]. Under conditions for monotonicity, these well-known methods can provide a bound for their near-optimal solutions, which is $1-\frac{1}{e}$ of the optimal performance. Meanwhile, the function of the optimality-cost in our approach is theoretically proven to be comprehensively monotone. Moreover, the bound of the solutions in the proposed technique is $1-\left(1-\frac{1}{n}\right)^n$ as compared with the optimum. It can be clearly seen that the new bound is practically better than that in the other two methods, given a limited number of sensors.
- The memory complexity in the conditional entropy and mutual information based approaches are $\mathcal{O}(N^2M^2)$ and $\mathcal{O}((N+p-n)^2M^2)$, respectively, while that of the proposed algorithm is $\mathcal{O}(N^2)$.

VI. Experiments in Buildings

In this section, we present the results of applying the proposed approach for deploying the wireless sensors at the best locations in the tested room S2.1-B4-01, Nanyang Technological University campus.

A. Indoor Experimental Description

We conducted experiments at the room S2.1-B4-01 in the Nanyang Technological University campus, Singapore, which is sized 19.80 m in length and 14.86 m in width, shown in Fig. 2. The experiments were carried out during the 4 week time from 25 April to 22 May 2016. In the experiments, we utilized two wireless networks of 10 Libelium temperature sensors and 10 Monnit temperature sensors. These sensor nodes located randomly, after measuring indoor temperatures, send the measurements directly to the network routers via a one-hop routing structure. The data can be accessed from any internet connected devices. To provide good feedback for strategies of controlling indoor environments, which is aimed to increase human comfort, in this work we deliberately positioned all sensors at sitting levels.

Note that the sensors were set to take environmental measurements every 2 hours. That is, each sensor could gather 84 measurements at its location every week. Hence, over 4 weeks, 6720 temperature values were collected by the 20 sensors. A spatio-temporal model as discussed in Section III was then learned by using these 6720 measurements. We call this model as $model_1$. Note that, as proposed, in the implementation we chose a spatio-temporal separable covariance function as given by

$$cov((\mathbf{s}_i,t_j),(\mathbf{s}_k,t_l)) = \sigma^2 \exp\left(-\frac{\parallel \mathbf{s}_i-\mathbf{s}_k \parallel}{\psi_s} - \frac{|t_j-t_l|}{\psi_t}\right), \tag{28}$$

where σ^2 is a marginal variance; ψ_s and ψ_t are two positive scale parameters in terms of space and time, respectively. ψ_s and ψ_t are referred to as reduction rates of the dependence between two random variables $z(\mathbf{s}_i,t_j)$ and $z(\mathbf{s}_k,t_l)$ at two spatio-temporal locations $(\mathbf{s}_i,t_j)$ and $(\mathbf{s}_k,t_l)$ when $\parallel \mathbf{s}_i-\mathbf{s}_k \parallel$ or $|t_j-t_l|$ increases.

B. Best Sensor Locations

We first discretized the room into a 100×100 grid; thus, each small spatial area of the grid is approximately sized $20cm \times 15cm$, which is empirically reasonable to locate our available Libelium and Monnit sensor nodes. According to the layout map of the room, all unavailable areas of cubicles, lab benches, personal computer tables, occupants' desk stations and experimental facilities are identified. We then removed all cells on the grid, which correspondingly overlap the unavailable areas, from the set of the possible points. The cells remaining on the grid are all the possible locations that can be utilized to deploy the wireless sensors. In fact, there are 4683 possible sensor locations visually illustrated by the blue circle points in Fig. 1.

It is now supposed that we would find 10 best locations in the room to position the temperature wireless sensors for future data collection. As discussed in Section IV, one can near-optimally find those best locations by implementing the Algorithm 1 into the collective dataset. After running the algorithm with $model_1$, the 10 near-optimal locations for effectively deploying the temperature sensors were found and shown in Fig. 2. It is to be noted that they are numbered from most to least informative positions. In equivalent words, if we have only 6 temperature sensors, we will locate them at the positions numbered from 1 to 6.

For the purpose of comparisons, in this illustrative implementation, we also conducted the conditional entropy and mutual information based methods for finding their own best sensor positions. In the setting, we assumed that locations of interest are on a 150×150 grid; that is, $N = 22500$. Moreover, from the experimental measurements, we observed that the indoor temperature is dynamic yet approximately weekly iterative. Thus, we defined $M = 84$. In other words, we would like the model to predict the indoor environment in a whole week at every 2 hours. Unfortunately, under this setting, the spatio-temporal covariance matrix $\Sigma_{Z(\mathbf{s}_N, \mathbf{t}_M)}$ is sized 1890000×1890000, which requires a memory of 26614 GB. This requirement is not practically feasible. Consequently, we reset the locations of interest on a 10×10 grid, and then $N = 100$. Note that the more locations of interest are validated, the more accurate are the results obtained. Then the final setting-up parameters for evaluating the methods are $N = 100$, $M = 84$, $p = 4683$, $n = 10$, $m = 84$. The computing time of each approach for finding the best 10 sensor locations is summarized in Table I, where the numbers were recorded when the methods, which were implemented on R V3.0, run on a PC of 3.1GHz Intel Core i5-2400 Processor. It can be clearly seen that our approach can achieve the 10 best locations for deploying the 10 temperature sensors in the tested room S2.1-B4-01 in 29 seconds, while the two other methods in literature returned the solutions in approximate 4 months and 2 months, respectively.

C. Prediction Results

For more evaluations of the proposed method, we now present and compare the results of predictions in this subsection. Due to limited number of sensors, we separated the 20 available temperature wireless sensors into three groups. More specifically, we first used 6 sensors to locate at 6 locations numbered from 1 to 6 in Fig. 2. We also utilized two other sets of the 6 temperature sensors each to place at the best locations found by the two existing methods based on conditional entropy and mutual information. These sensors then measured temperature in the tested room in another week, from 23 to 29 May 2016. By using the same settings of the wireless network configurations, each sensor also sampled the indoor temperature every two hours. Hence, each group of 6 wireless sensors collected 504 temperature values in the studied week. The 504 measurements in each sensor group were employed to statistically learn space-time models of the heat in the room. For the three approaches used to be compared, we have three different models, respectively. We subsequently named them as $model_{proposed}$ for the proposed method, $model_{MI}$ and $model_{entropy}$ for the others. All the three temperature models $model_{proposed}$, $model_{MI}$ and $model_{entropy}$ could be then employed to predict and estimate the heat at any time and any locations in the experimental room. For instance, to validate the predictions, we utilized these three space-time models to predict the temperature fields in the whole test room at 16:45 on 26 May 2016, when there were no measurements carried out. The resulting maps are demonstrated in Figures 3c to 3h. More importantly, for the purpose of comparisons, the maps of the temperature in the room at the same time were also predicted by the use of the 4 week measurement based model $model_1$, which are illustrated in Figures 3a and 3b. It is noticed that interested readers may be referred to our previous work [51] for more details about effectiveness of the spatial-temporal prediction method employed in this work.

TABLE I: COMPARISON OF RUNNING TIME FOR FINDING 10 BEST SENSOR LOCATIONS IN ROOM S2.1-B4-01

Number of possible locations	Running time of the methods		
	The proposed	MI	Entropy
4683	29 seconds	110 days	59 days

Illustratively, the predicted heat map shown in Fig. 3c created by the proposed method with the measurements gathered by 6 wireless sensors is comparable with that illustrated in Fig. 3a created by a 20 sensor network, which is considered as a ground truth. However, 6 sensors located at positions obtained by the mutual information and conditional entropy based methods could not gain much information in the furthest corner of the room, which leads to qualitative contrast in terms of the prediction at the top right corners of Figures 3e and 3g, respectively, as compared with the ground truth in Fig. 3a. More importantly, comparability and contrast can be qualitatively seen in the prediction standard deviation error surfaces, as demonstrated in Figures 3b, 3d, 3f and 3h. The prediction error variances at points obtained by the model $model_{proposed}$ learned from the spatio-temporal measurements of the 6 near-optimal locations based sensors in Fig. 3d are comparable with those at corresponding positions in Fig. 3b, which were created by the space-time model $model_1$ using all the 6720 sensor readings gathered in the 4 weeks by the

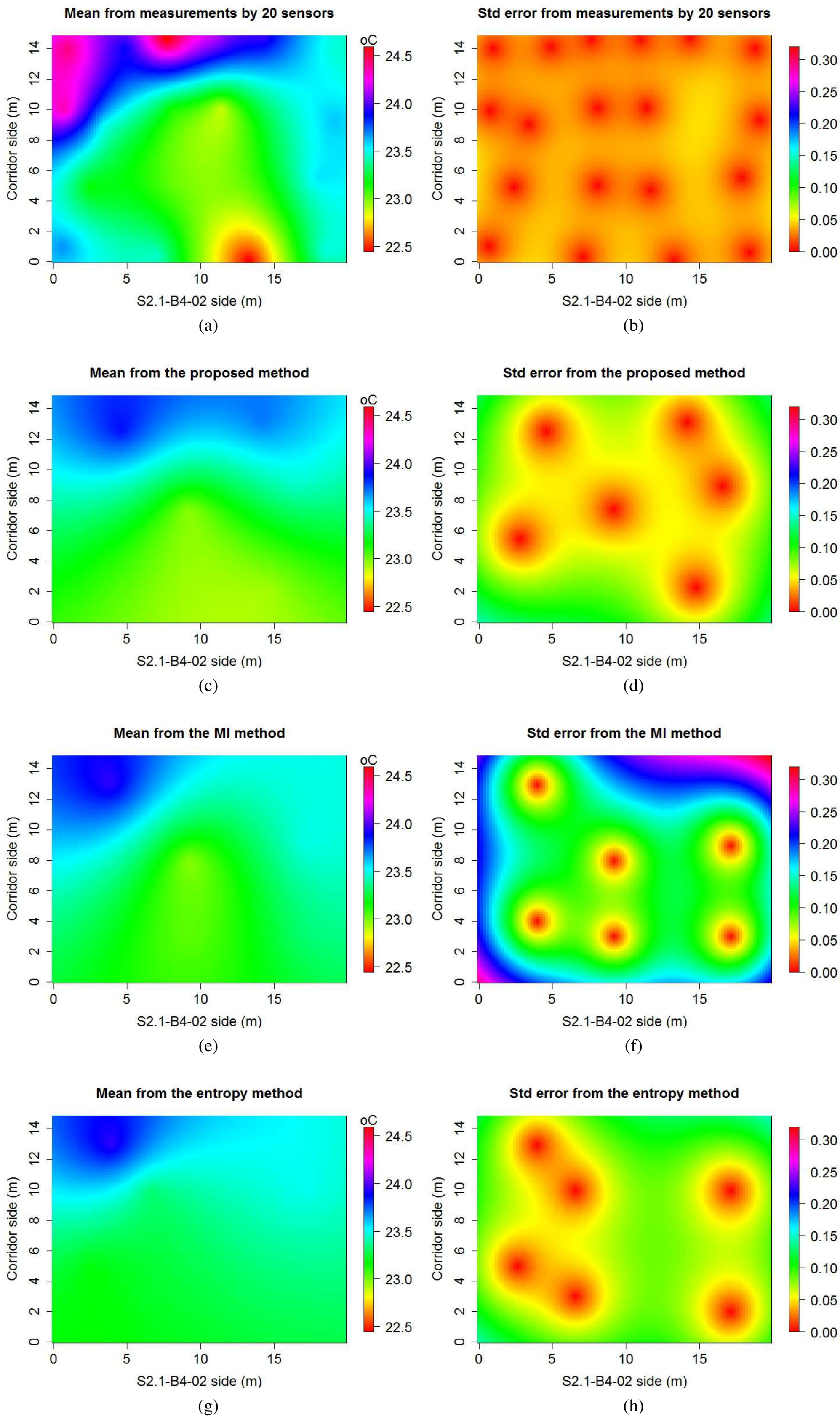

Fig. 3: Predicted temperature fields (left column) and prediction standard deviation errors (right column) at 16:45 on 26 May 2016, obtained by the model learned by measurements collected by 20 sensors as shown in (a) and (b), and by spatio-temporal models learned by the use of different sets of measurements gathered by: 6 sensors at locations found by the proposed algorithm as shown in (c) and (d); 6 sensors at positions found by the MI method as shown in (e) and (f); and 6 sensors at locations found by the entropy method. Ranges of fields and standard deviation errors are demonstrated in color bars.

TABLE II: ROOT MEAN SQUARE ERRORS BETWEEN THE PREDICTION FIELDS OBTAINED BY 6 AND 20 SENSORS

Root mean square errors		
The proposed	MI	Entropy
0.199	0.266	0.285

network of 20 wireless temperature sensors. Nevertheless, the prediction standard deviation errors in Fig. 3f obtained by the model $model_{MI}$, which was trained by the observations of the temperature sensors positioned at the positions obtained by the mutual information based technique, are much higher than those in Figures 3b and 3d, compared at equivalent positions. To quantify the comparisons, we computed root mean square errors between the prediction fields illustrated in Figures 3c, 3e and 3g and the ground truth shown in Fig. 3a. The errors are summarized in Table II.

It is noted that since prediction error at a sensor location is zero, the sensor locations found by the mutual information and conditional entropy based algorithms can be seen in the standard deviation error maps in Figures 3f and 3h. In other words, 6 sensors utilized in the mutual information and conditional entropy based techniques are located in the middles of 6 red round marks on the standard deviation error maps in Figures 3f and 3h, respectively.

In general, even though there is a network of only 6 wireless sensors deployed at the locations found by our proposed approach to be utilized in taking the temperature measurements in a week, the results of its spatio-temporal model in terms of both the predicted field and the prediction standard deviation errors are highly reasonable as compared with those obtained by the model of a 20 wireless sensor network with 4 week collections. More particularly, the better results of the prediction accuracy as shown in Fig. 3 practically consolidate the direct criterion of finding optimal sensor locations in our proposed approach to outperform the indirect criteria in the two other information-theoretic methods.

In another aspect of evaluating the performances of the proposed algorithm, we considered the bound for the solutions obtained in the real experiments conducted. As presented in *Theorem 3*, the near-optimal solutions for the sensor deployments in the tested room, S2.1-B4-01 at Nanyang Technological University campus obtained by our method are definitely guaranteed. In these conducted experiments, we intended to find 6 best locations for deploying the 6 temperature sensors in the space. That is, the results are at least bounded by a level of 67.23% as compared with the optimal performance. This guarantee level is better than the bound level of $1 - \frac{1}{e} \simeq 63.21\%$ obtained by the information theory based approaches of the conditional entropy [28], [29] and the mutual information [30]–[32].

VII. Conclusions

The paper has considered the problem of optimally deploying environmental sensors for monitoring spatio-temporal phenomena. Based on space-time Gaussian processes, a separable optimization approach to efficiently find the best environmental sensor locations in the spaces has been developed, which is only dependent on spatial variations, though measurements are gathered over time. The optimality problem of sensor deployments is near-optimally resolved by an approximation algorithm, yet its performances are guaranteed by a level of $1 - \left(1 - \frac{1}{n}\right)^n$ as compared with the optimum, where n is the number of sensors. The proposed approach is also theoretically and practically demonstrated to outperform the existing methods. The efficiency of the proposed technique has been extensively evaluated in a real tested spatio-temporal environment in a university building.

References

[1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, “Wireless sensor networks: A survey,” *Computer Networks*, vol. 38, pp. 393–422, 2002.

[2] A. Rogers, E. David, and N. R. Jennings, “Self-organized routing for wireless microsensor networks,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 35(3), pp. 349–359, 2005.

[3] P. Corke, T. Wark, R. Jurdak, W. Hu, P. Valencia, and D. Moore, “Environmental wireless sensor networks,” *Proceedings of the IEEE*, vol. 98(11), pp. 1903–1917, 2010.

[4] N. E. Leonard, D. Paley, F. Lekien, R. Sepulchre, D. M. Fratantoni, and R. Davis, “Collective motion, sensor networks and ocean sampling,” *Proceedings of the IEEE*, vol. 95(1), pp. 48–74, 2007.

[5] F. Nejadkoorki, K. Nicholson, and K. Hadad, “The design of long-term air quality monitoring networks in urban areas using a spatiotemporal approach,” *Environmental Monitoring and Assessment*, vol. 172, p. 215223, 2011.

[6] K. ZoroufchiBenis, E. Fatehifar, J. Ahmadi, and A. Rouhi, “Optimal design of air quality monitoring network and its application in an oil refinery plant,” *Health Promotion Perspectives*, vol. 5(4), pp. 269–279, 2015.

[7] K. Wang, H. Zhao, Y. Ding, T. Li, L. Hou, and F. Sun, “Optimization of air pollutant monitoring stations with constraints using genetic algorithm,” *Journal of High Speed Networks*, vol. 21, p. 141153, 2015.

[8] S. Chadalavada and B. Datta, “Dynamic optimal monitoring network design for transient transport of pollutants in groundwater aquifers,” *Water Resources Management*, vol. 22, pp. 651–670, 2008.

[9] S. Chadalavada, B. Datta, and R. Naidu, “Uncertainty based optimal monitoring network design for a chlorinated hydrocarbon contaminated site,” *Environmental Monitoring and Assessment*, vol. 173, pp. 929–940, 2011.

[10] O. Prakash, “Optimal monitoring network design and identification of unknown pollutant sources in polluted aquifers,” Ph.D. dissertation, James Cook University, Australia, 2014.

[11] M. Mysorewala, D. Popa, and F. Lewis, “Multi-scale adaptive sampling with mobile agents for mapping of forest fires,” *Journal of Intelligent and Robotic Systems*, vol. 54(4), pp. 535–565, 2009.

[12] S. Wu and D. Clements-Croome, “Understanding the indoor environment through mining sensory data - A case study,” *Energy and Buildings*, vol. 39, pp. 1183–1191, 2007.

[13] J. Yun and J. Kim, “Deployment support for sensor networks in indoor climate monitoring,” *International Journal of Distributed Sensor Networks*, pp. 1–10, 2013.

[14] D. Brunelli, I. Minakov, R. Passerone, and M. Rossi, “POVOMON: An ad-hoc wireless sensor network for indoor environmental monitoring,” in *Proc. IEEE Workshop on Environmental Energy and Structural Monitoring Systems*, Naples, Italy, September 2014, pp. 1–6.

[15] M. Alirezaie and A. Loutfi, “Reasoning for sensor data interpretation: An application to air quality monitoring,” *Journal of Ambient Intelligence and Smart Environments*, vol. 7(4), pp. 579–597, 2015.

[16] P. Zhou, G. Huang, L. Zhang, and K. Tsang, “Wireless sensor network based monitoring system for a large-scale indoor space: data process and supply air allocation optimization,” *Energy and Buildings*, vol. 103, pp. 365–374, 2015.

[17] S. Temel, N. Unaldi, and O. Kaynak, “On development of wireless sensors on 3-D terrains to maximize sensing coverage by utilizing cat swarm optimization with wavelet transform,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 44(1), pp. 111–120, 2014.

[18] J. Gedeon and I. Schweizer, "Understanding spatial and temporal coverage in participatory sensor networks," in *Proc. IEEE 40th Local Computer Networks Conference Workshops*, Clearwater Beach, FL, USA, October 2015, pp. 699–707.

[19] Z. Guo, M. Zhou, and G. Jiang, "Adaptive sensor placement and boundary estimation for monitoring mass objects," *IEEE Transactions on Cybernetics*, vol. 38(1), pp. 222–232, 2008.

[20] I. Vlasenko, I. Nikolaidis, and E. Stroulia, "The smart-condo: Optimizing sensor placement for indoor localization," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 45(3), pp. 436–453, 2015.

[21] C. Lin and C. King, "Sensor-deployment strategies for indoor robot navigation," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 40(2), pp. 388–398, 2010.

[22] A. Ababnah and B. Natarajan, "Optimal control-based strategy for sensor deployment," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 41(1), pp. 97–104, 2011.

[23] S. Joshi and S. Boyd, "Sensor selection via convex optimization," *IEEE Transaction on Signal Processing*, vol. 57, pp. 451–462, 2009.

[24] C. Malings and M. Pozzi, "Conditional entropy and value of information metrics for optimal sensing in infrastructure systems," *Structural Safety*, vol. 60, pp. 77–90, 2016.

[25] H. Chang, A. Q. Fu, N. D. Le, and J. V. Zidek, "Designing environmental monitoring networks to measure extremes," *Environmental and Ecological Statistics*, vol. 14(3), p. 301321, 2007.

[26] J. V. Zidek, W. Sun, and N. D. Le, "Designing and integrating composite networks for monitoring multivariate gaussian pollution fields," *Journal of the Royal Statistical Society, Series C - Applied Statistics*, vol. 49(1), pp. 63–79, 2000.

[27] F. Pukelsheim, *Optimal design of experiments*. Society for Industrial and Applied Mathematics, 2006.

[28] C. Ko, J. Lee, and M. Queyranne, "An exact algorithm for maximum entropy sampling," *Operations Research*, vol. 43, pp. 684–691, 1995.

[29] L. V. Nguyen, S. Kodagoda, R. Ranasinghe, and G. Dissanayake, "Information-driven adaptive sampling strategy for mobile robotic wireless sensor network," *IEEE Transaction on Control Systems Technology*, vol. 24(1), pp. 372–379, 2016.

[30] W. F. Caselton and J. V. Zidek, "Optimal monitoring network designs," *Statistics and Probability Letters*, vol. 2, pp. 223–227, 1984.

[31] A. Krause, A. Singh, and C. Guestrin, "Near-optimal sensor placements in Gaussian processes: Theory, efficient algorithm and empirical studies," *Journal of Machine Learning Research*, vol. 9(Feb), pp. 235–284, 2008.

[32] L. V. Nguyen, S. Kodagoda, and R. Ranasinghe, "Spatial sensor selection via Gaussian Markov random fields," *IEEE Transaction on Systems, Man, and Cybernetics: Systems*, vol. 46(9), pp. 1226–1239, 2016.

[33] T. M. Cover and J. A. Thomas, *Elements of information theory*. John Wiley and Sons, Lnc, Hoboken, New Jersey, 2006.

[34] B. Guo and M. S. Nixon, "Gait feature subset selection by mutual information," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 39(1), pp. 36–46, 2009.

[35] W. Liao, Q. Li, and W. A. Wallace, "Approximate nonmyopic sensor selection via submodularity and partitioning," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 39(4), pp. 782–794, 2009.

[36] F. Bian, D. Kempe, and R. Govindan, "Utility based sensor selection," in *Proc. 17th International Conference on Information Processing Sensor Networks*, Tennessee, USA, April 2006, pp. 11–18.

[37] C. E. Rasmussen and C. K. I. Williams, *Gaussian processes for machine learning*. The MIT Press, Cambridge, Massachusetts, London, England, 2006.

[38] P. J. Diggle and P. J. Ribeiro, *Model-based geostatistics*. Springer, New York, USA, 2007.

[39] N. Cressie and C. K. Wikle, *Statistics for spatio-temporal data*. Wiley, 2011.

[40] J. Montero, G. Fernández-Avilés, and J. Mateu, *Spatial and Spatio-temporal geostatistical modeling and krigning*. Wiley, 2015.

[41] T. Gneiting, "Nonseparable, stationary covariance functions for space-time data," *Journal of the American Statistical Association*, vol. 97(458), pp. 590–600, 2002.

[42] E. Porcu, J. Mateu, and G. Christakos, "Quasi-arithmetic means of covariance functions with potential applications to space-time data," *Journal of Multivariate Analysis*, vol. 8(100), pp. 1830–1844, 2009.

[43] S. D. Iaco, D. Myers, and T. Posa, "Nonseparable space-time covariance models: Some parametric families," *Mathematical Geology*, vol. 34, pp. 23–42, 2002.

[44] D. Sharma, A. Deshpande, and A. Kapoor, "On greedy maximization of entropy," in *Proc. International Conference on Machine Learning*, Lille, France, July 2015, pp. 1330–1338.

[45] M. Aigner and G. M. Ziegler, *The proofs from the book*. Springer, 2014.

[46] G. Strang, *Linear algebra and its applications*. Thomson, 1988.

[47] R. A. Horn and C. R. Johnson, *Matrix analysis*. Cambridge University Press, 1985.

[48] Z. Wang, B. Moran, X. Wang, and Q. Pan, "Approximation for maximizing monotone non-decreasing set functions with a greedy method," *Journal of Combinatorial Optimization*, vol. 31(1), pp. 29–43, 2016.

[49] A. Schrijver, *Combinatorial Optimization: Polyhedra and Efficieny*. Springer, 2003.

[50] M. L. Stein, Z. Chi, and L. J. Welty, "Approximating likelihood for large spatial data sets," *Journal of the Royal Statistical Society (Series B)*, vol. 66, pp. 275–296, 2004.

[51] L. V. Nguyen, G. Hu, and C. J. Spanos, "Spatio-temporal environmental monitoring for smart buildings," in *Proc. IEEE International Conference on Control and Automation*, Ohrid, Macedonia, July 2017, pp. 277–282.

Linh V. Nguyen received a PhD degree in Robotics from the University of Technology Sydney, Australia in March 2015. He then work at the same university as a Postdoctoral Research Associate until September 2015. From January to July 2016, he was as Research Fellow at the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. He rejoined the Centre for Autonomous Systems at the University of Technology Sydney as a Research Fellow in August 2016. His research interest includes robotics, internet of things, sensor placement, artificial intelligence, machine learning, signal processing, embedded systems and non-destructive testing.

Guoqiang Hu joined the School of Electrical and Electronic Engineering at Nanyang Technological University, Singapore in 2011, and is currently a tenured Associate Professor and the Director of the Centre for System Intelligence and Efficiency. He was an Assistant Professor at Kansas State University from 2008 to 2011. He received the B.Eng. degree in Automation from the University of Science and Technology of China in 2002, the M.Phil. degree in Automation and Computer-Aided Engineering from the Chinese University of Hong Kong in 2004, and the Ph.D. degree in Mechanical Engineering from the University of Florida in 2007. His research focuses on analysis, control, design and optimization of distributed intelligent systems, with applications to cooperative robotics and smart city systems. He was a recipient of the Best Paper in Automation Award in the 14th IEEE International Conference on Information and Automation, and a recipient of the Best Paper Award (Guan Zhao-Zhi Award) in the 36th Chinese Control Conference. He serves as Associate Editor for IEEE Transactions on Control Systems Technology, Technical Editor for IEEE/ASME Transactions on Mechatronics, Associate Editor for IEEE Transactions on Automation Science and Engineering, and Subject Editor for International Journal of Robust and Nonlinear Control.

Costas J. Spanos received the EE Diploma from the National Technical University of Athens, Greece, and the M.S. and Ph.D. degrees in ECE from Carnegie Mellon University. In 1988 he joined the department of Electrical Engineering and Computer Sciences (EECS) at the University of California, Berkeley, where he is now the Andrew S. Grove Distinguished Professor and the Director of the Center for Information Technology Research in the Interest of Society (CITRIS), an Information Technology Research Institute across UC Berkeley, UC Davis, UC Merced and UC Santa Cruz. He is also the Founding Director and CEO of the Berkeley Education Alliance for Research in Singapore (BEARS), and the Lead Investigator of a large research program on smart buildings based in California and Singapore. Prior to that, Professor Spanos has been the Chair of EECS at UC Berkeley, the Associate Dean for Research in the College of Engineering at UC Berkeley, and the Director of the UC Berkeley Microfabrication Laboratory. Dr. Spanos has co-founded two successful companies in the Semiconductor Manufacturing space. Dr. Spanos research focuses on Sensing, Data Analytics, Modeling and Machine Learning, with broad applications in semiconductor technologies, and cyber-physical systems. He has authored or co-authored more than 300 papers, 15 patents, and a textbook. He has supervised more than 40 Ph.D. recipients who now hold key positions in industry and academia. In 2000 he was elected Fellow of the Institute of Electrical and Electronic Engineers for contributions and leadership in semiconductor manufacturing.