“© 2018 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.”

A Low-Complexity Energy Minimization Based SCMA Detector and Its Convergence Analysis

Weijie Yuan, Nan Wu, Chaoxing Yan, Yonghui Li, *Senior Member, IEEE,*
Xiaojing Huang, *Senior Member, IEEE* and Lajos Hanzo, *Fellow, IEEE*

***Abstract*—Sparse code multiple access (SCMA) has emerged as a promising non-orthogonal multiple access (NOMA) technique for the next generation wireless communication systems. Since the signal of multiple users is mapped to the same resources in SCMA, its detection imposes a higher complexity than that of the orthogonal schemes, where each resource slot is dedicated to a single user. In this paper, we propose a low-complexity receiver for SCMA systems based on the radical variational free energy framework. By exploiting the pairwise structure of the likelihood function, the Bethe approximation is utilized for estimating the data symbols. The complexity of the proposed algorithm only increases linearly with the number of users, which is much lower than that of the maximum *a posteriori* (MAP) detector associated with exponentially increased complexity. Furthermore, the convergence of the proposed algorithm is analyzed and its convergence conditions are derived. Simulation results demonstrate that the proposed receiver is capable of approaching the error probability performance of the conventional message passing based receiver.**

***Index Terms*—Sparse code multiple access, variational free energy, Bethe approximation, convergence analysis**

I. Introduction

The next generation wireless communication systems aim for an increased spectral efficiency and low latency [1], [2]. In this context, sparse code multiple access (SCMA) has recently attracted substantial attention, since it is capable of supporting large-scale connectivity and improved coverage [3].

SCMA techniques can be regarded as a generalized form of low density signature based code division multiple access [4]. In SCMA, the bit-to-symbol mapping and spreading operation are merged and bit streams of different users are directly mapped to sparse codewords. With the aid of appropriate sparse codebook design, SCMA achieves an improved performance. However, due to the non-orthogonal resource allocation of the SCMA system, the optimal maximum a posteriori (MAP) detectors impose a high complexity. By exploiting the sparsity of the codewords, several factor graph and message passing algorithm (MPA) [5] based receivers have been developed [6]–[9]. Nevertheless, the rank-deficient SCMA system results in a factor graph having short cycles, for which the message passing algorithm may not be able to converge. Therefore, it is important to investigate the convergence of iterative SCMA receivers.

Hence, we propose a low-complexity receiver based on Bayesian inference [10]. Considering the low density of non-zero elements in the SCMA codewords, we represent the joint distribution of data symbols as the product of several clique potentials. Then based on the Bethe approximation [11], we derive the corresponding variational free energy (VFE). By minimizing the VFE, the marginal distributions of symbols are determined. We show that the complexity of the proposed receiver only increases linearly with the number of users. Moreover, since the proposed scheme is an iterative one, we analyze its convergence. In summary, the main contributions of this paper are as follows,

- We develop a low-complexity iterative receiver for our SCMA system. In contrast to the conventional variational inference method [12], we use the Bethe approximation which explicitly considers the conditional dependencies of users for improving the performance. Furthermore, a belief damping scheme is employed to improve the performance.
- We prove that the variance of symbol marginals is guaranteed to converge. For its mean, we conceive the necessary and sufficient condition to guarantee its convergence.

Finally, our simulation results show that the proposed algorithm approaches the optimal MAP detector's performance despite significantly reducing its complexity.

The remainder of this paper is organized as follows. The model of our SCMA system is introduced in Section II. In Section III, the proposed energy minimization based receiver is presented, while in Section IV, we analyze the convergence of the proposed algorithm. Simulation results are discussed in Section V. Finally, our conclusions are provided in Section VI.

Notations: We use a boldface capital letter to denote a matrix while lowercase letter to denote a vector. C denotes a constant number; $|\cdot|$ denotes the modulus of a complex number or the cardinality of a set; $\|\cdot\|$ denotes a ℓ^2 matrix norm; $\propto$ represents equality up to a constant normalization factor; (i,j) denotes the corresponding variables are included in a pairwise potential.

II. System Model

We consider the SCMA system supporting J users multiplexed onto K orthogonal resources, as shown in Fig. 1.

W. Yuan is with the the School of Information and Electronics, Beijing Institute of Technology, China and also with Faculty of Engineering and Information Technology, University of Technology Sydney, NSW, Australia (e-mail:wjyuan,@bit.edu.cn).

N. Wu is with the School of Information and Electronics, Beijing Institute of Technology, China (e-mail:wunan@bit.edu.cn).

C. Yan and L. Hanzo are with the School of Electronics and Computer Science, University of Southampton, SO17 1BJ, UK (e-mail:{c.yan, lh}@ecs.soton.ac.uk).

Y. Li is with the School of Electrical and Information Engineering, University of Sydney, NSW, Australia (e-mail:yonghui.li@sydney.edu.au).

X. Huang is with Faculty of Engineering and Information Technology, University of Technology Sydney, NSW, Australia (e-mail:xiaojing.huang@uts.edu.au).

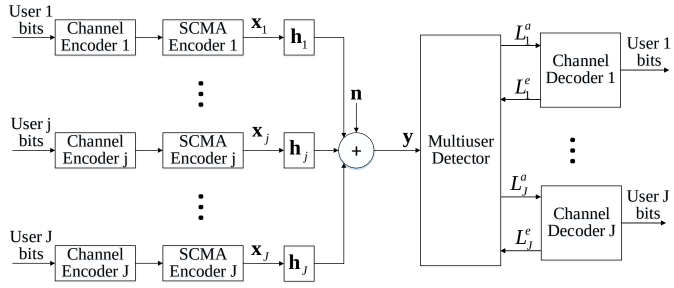

Fig. 1. Block diagram of the SCMA system.

In orthogonal multiple access (OMA), $J \leq K$ is satisfied to ensure that each user is allocated to an orthogonal resource. By contrast, in the SCMA system, the number of users is larger than the number of resource blocks, which leads to a rank-deficient system. The ratio $\lambda = \frac{J}{K} > 1$ is referred to as the normalized user-load. For each user, the bits $\mathbf{b}_j$ are mapped to a K-component SCMA codeword $\mathbf{x}_j$. The mapping function for the jth user is $\mathbf{x}_j = \phi(\mathbf{b}_j)$, $\phi : \mathbb{B}^{\log_2 M} \to \boldsymbol{\chi}$, where $\boldsymbol{\chi} \in \mathbb{C}^K$ and $|\boldsymbol{\chi}| = M$.

The SCMA codeword of the jth user is selected from a predefined codebook. Let $\mathbf{x}_j = [x_{j,1}, ..., x_{j,K}]$ be the transmitted symbols of user j. Given the sparsity of SCMA codewords, there are only $D < K$ nonzero entries in $\mathbf{x}_j$.

The spread signal of user j is then transmitted through the corresponding channel $\mathbf{h}_j = [h_{j,1}, ..., h_{j,K}]^T$. Assuming perfect synchronization between the base station and users, the received signal is expressed as

$$\mathbf{y} = \sum_{j=1}^{J} \text{diag}(\mathbf{h}_j)\mathbf{x}_j + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is the Gaussian noise having a power spectral density of N_0.

III. Proposed Low-complexity Receiver

Generally, utilizing message passing algorithm (MPA) based receiver can provide the optimal maximum *a posteriori* (MAP) estimate. Nevertheless, MPA has exponential complexity which limits its implementation in practice. In this section, we propose a low-complexity receiver for SCMA system based on the variational free energy framework.

A. The Proposed Algorithm

Under the Gaussian assumption of $\mathbf{n}$, we have the likelihood function of the received signals $\mathbf{y}$ conditioned on the user's transmitted symbols $\mathbf{x}_j$, formulating as

$$p(\mathbf{y}|\mathbf{x}) \propto \exp\left(-\frac{1}{N_0}\|\mathbf{y} - \sum_{j=1}^{J}\text{diag}(\mathbf{h}_j)\mathbf{x}_j\|^2\right). \tag{2}$$

According to the classic Bayesian theorem, the *a posteriori* distribution can be expressed as

$$p(\mathbf{x}|\mathbf{y}) \propto p(\mathbf{x})p(\mathbf{y}|\mathbf{x}), \tag{3}$$

where $p(\mathbf{x})$ is the joint *a priori* distribution of all users' source symbols, which is expressed as

$$p(\mathbf{x}) \propto \prod_{j=1}^{J} \mathcal{N}(\mathbf{x}_j; \mathbf{m}^0_{x_j}, \mathbf{V}^0_{x,j}). \tag{4}$$

In (4), $\mathbf{m}^0_{x_j} = [m^0_{x_{j,1}}, ..., m^0_{x_{j,K}}]^T$ and $\mathbf{V}^0_{x,j} = \text{diag}([v^0_{x_{j,1}}, ..., v^0_{x_{j,K}}])$ are calculated based on the extrinsic information gleaned from the channel decoder.

Since the SCMA source symbols and received signal samples are independent, (3) is factorized as

$$p(\mathbf{x}|\mathbf{y}) \propto \prod_{k=1}^{K}\prod_{j}\phi^k_j\prod_{(i,j)}\phi^k_{i,j}, \tag{5}$$

where

$$\phi^k_j = p(x_{j,k})\exp\left(-\frac{h^2_{j,k}x^2_{j,k} - 2h_{j,k}y_k x_{j,k}}{N_0}\right) \tag{6}$$

$$\phi^k_{i,j} = \phi^k_{j,i} = \exp\left(-\frac{2h^2_{j,k}x_{i,k}x_{j,k}}{N_0}\right) \tag{7}$$

are referred to as the self-potential and pairwise potential derived in [13], respectively. Usually, we aim for finding the estimate of every data symbol, i.e. $\hat{x}_{j,k}$, which is equivalent to obatain the *a posteriori* probability $p(x_{j,k}|\mathbf{y})$. Direct marginalization of $p(\mathbf{x}|\mathbf{y})$ imposes a high complexity. Motivated by the energy minimization framework, we propose to find an appropriate trail distribution $b(\mathbf{x})$ that can be readily marginalized to approximate $p(\mathbf{x}|\mathbf{y})$.

The variational free energy is defined as [14]

$$F = F_H + \int b(\mathbf{x})\ln\frac{b(\mathbf{x})}{p(\mathbf{x}|\mathbf{y})}\text{d}\mathbf{x}, \tag{8}$$

where $F_H = -\ln Z$ is termed as *Helmholtz free energy* [14]. In our problem, Z denotes the normalization factor of $p(\mathbf{x}|\mathbf{y})$. The simplest form for $b(\mathbf{x})$ is the mean-field (MF) approach [13], which factorizes $b(\mathbf{x})$ as $b_{\text{MF}}(\mathbf{x}) = \prod_{j,k} b^k_j(x_{j,k})$, where $b^k_j(x_{j,k})$ is a marginalized trail distribution ('belief') over the single variable $x_{j,k}$. Accordingly, we can readily compute the free energy and then obtain an MF approximation for the beliefs $b^k_j(x_{j,k})$. However, the main problem of the factorized MF approximation is that it assumes all variables in $\mathbf{x}$ to be conditionally independent of each other, even though actually they are not. This motivates us to find a more accurate approximation than the MF approximation. The Bethe method has been recognized as an efficient tool in probabilistic problems, since it considers the conditional dependencies amongst the variables as follows:

$$b(\mathbf{x}) = \prod_{k}\prod_{j} b^k_j(x_{j,k})\prod_{(i,j)}\frac{b^k_{i,j}(x_{j,k}, x_{i,k})}{b^k_j(x_{j,k})b^k_i(x_{i,k})}. \tag{9}$$

Substituting (9) into (8) yields

$$F = \sum_{k}\left(\sum_{(i,j)}\int b^k_{i,j}(x_{j,k}, x_{i,k})\ln\frac{b^k_{i,j}(x_{j,k}, x_{i,k})}{\phi^k_{i,j}}\text{d}x_{j,k}\text{d}x_{i,k}\right.$$
$$\left. + (J-1)\sum_{j}\int b^k_j(x_{j,k})\ln\frac{b^k_j(x_{j,k})}{\phi^k_j}\text{d}x_{j,k}\right) + C. \tag{10}$$

The variational free energy is constrained by the normalization and consistency constraints of:

$$\int b_j^k(x_{j,k})\mathrm{d}x_{j,k} = 1 \tag{11}$$

$$\int b_{i,j}^k(x_{j,k}, x_{i,k})\mathrm{d}x_{i,k} = b_j^k(x_{j,k}). \tag{12}$$

We invoke the classic Lagrangian multipliers λ_j for the normalization constraint and $\lambda_{j,i}(x_{j,k})$ for the consistency constraints. Thus the Lagrangian is constructed as

$$\begin{aligned} L =& F + \sum_k \left(\sum_i \lambda_j \left(1 - \int b_j^k(x_{j,k})\mathrm{d}x_{j,k} \right) + \right. \\ & \left. \sum_{(i,j)} \int \lambda_{i,j}(x_{j,k}) \left(\int b_{i,j}^k(x_{j,k}, x_{i,k})\mathrm{d}x_{i,k} - b_j^k(x_{j,k}) \right) \mathrm{d}x_{j,k} \right). \end{aligned} \tag{13}$$

According to the calculus of variations, setting $\delta(L) = 0$ gives the beliefs $b_{i,j}^k(x_{j,k}, x_{i,k})$ and $b_j^k(x_{j,k})$ at stationary points, which yield

$$\begin{aligned} b_{i,j}^k(x_{j,k}, x_{i,k}) =& C\phi_{i,j}^k b_i^k(x_{i,k}) b_j^k(x_{j,k}) \\ & \times \exp\left[-\lambda_{j,i}(x_{j,k}) - \lambda_{i,j}(x_{i,k})\right] \end{aligned} \tag{14}$$

$$b_j^k(x_{j,k}) = C\phi_j^k \exp\left(\sum_{(i,j)} \lambda_{i,j}(x_{j,k}) \right). \tag{15}$$

By substituting (15) into (14), we arrive at

$$\begin{aligned} & b_{i,j}^k(x_{j,k}, x_{i,k}) \propto \phi_j^k \phi_i^k \phi_{i,j}^k \\ & \times \exp\left(\sum_{(j,l),l\neq i} \lambda_{j,l}(x_{j,k}) + \sum_{(i,m),m\neq j} \lambda_{i,m}(x_{i,k}) \right). \end{aligned} \tag{16}$$

For simplicity, we introduce the shorthand

$$b_{i\backslash j}^k(x_{i,k}) \propto \phi_i^k \exp\left(\sum_{(i,m),m\neq j} \lambda_{i,m}(x_{i,k}) \right). \tag{17}$$

After integrating $b_{i,j}^k(x_{j,k}, x_{i,k})$ over $x_{i,k}$ and comparing it to (15), finally we infer

$$\lambda_{j,i}(x_{j,k}) = \ln\left(\int \phi_{i,j}^k b_{i\backslash j}^k(x_{i,k})\mathrm{d}x_{i,k} \right). \tag{18}$$

Without loss of generality, we denote the mean and variance of $b_{i\backslash j}^k(x_{i,k})$ by $m_{i\backslash j}^k$ and $v_{i\backslash j}^k$. Then $\lambda_{j,i}(x_{j,k})$ is expressed as a quadratic polynomial

$$\begin{aligned} \lambda_{j,i}(x_{j,k}) &= \frac{h_{j,k}^4 v_{i\backslash j}^k}{N_0^2} x_{j,k}^2 - 2\frac{h_{j,k}^2 m_{i\backslash j}^k}{N_0} x_{j,k} + C \\ &= -\beta_{j,i}^k x_{j,k}^2 + 2\gamma_{j,i}^k x_{j,k} + C. \end{aligned} \tag{19}$$

Based on (15) and (19), we can now calculate the mean and variance of the approximate marginal $b_j^k(x_{j,k})$

$$m_{x_{j,k}} = v_{x_{j,k}} \left(\frac{m_{x_{j,k}}^0}{v_{x_{j,k}}^0} + \frac{2h_{j,k}y_k}{N_0} + \sum_{(i,j)} \gamma_{j,i}^k \right) \tag{20}$$

$$v_{x_{j,k}} = \left(\frac{1}{v_{x_{j,k}}^0} + \frac{h_{j,k}^2}{N_0} + \sum_{(i,j)} \beta_{j,i}^k \right)^{-1}. \tag{21}$$

Since the terms $\frac{1}{v_{x_{j,k}}^0} + \frac{h_{j,k}^2}{N_0}$ and $\frac{m_{x_{j,k}}^0}{v_{x_{j,k}}^0} + \frac{2h_{j,k}y_k}{N_0}$ do not change during the iterations, we use ρ_j^k and ϱ_j^k to denote those two terms for simplicity. We further consider a damping scheme which is expected to improve the performance in a high density connected network [15]. Denoting the belief obtained at the lth iteration as $b_j^k(l)$, the damped belief is computed as

$$\tilde{b}_j^k(l) = (b_j^k(l))^{\alpha}(b_j^k(l-1))^{(1-\alpha)}, \tag{22}$$

where $0 < \alpha < 1$ is the damping factor. That is to say the updating of the belief is based on a combination of the new belief and the old one. The mean and variance of the damped belief are given as

$$\tilde{m}_{x_{j,k}}(l) = \tilde{v}_{x_{j,k}}(l) \left(\frac{\alpha m_{x_{j,k}}(l)}{v_{x_{j,k}}(l)} + \frac{(1-\alpha) m_{x_{j,k}}(l-1)}{v_{x_{j,k}}(l-1)} \right) \tag{23}$$

$$\tilde{v}_{x_{j,k}}(l) = \left(\frac{\alpha}{v_{x_{j,k}}(l)} + \frac{1-\alpha}{v_{x_{j,k}}(l-1)} \right)^{-1}. \tag{24}$$

Given the mean and variance of $\tilde{b}_j^k(x_{j,k})$, we can calculate the log likelihood ratios (LLRs) L_j^a fed to the channel decoder. Note that updating $m_{x_{j,k}}$ and $v_{x_{j,k}}$ relies on other variables, hence the extrinsic information corresponding to different data symbols is updated iteratively. After decoding, the output LLRs L_j^e are converted to $p(x_{j,k})$ and we start the next turbo iteration. The details of the proposed algorithm is summarized in Algorithm **1**.

Algorithm 1 Energy Minimization Based Turbo Detector for SCMA System

---

Initialization:
The initial *a priori* distribution of user's source symbol is set as Gaussian distribution with zero mean and infinite variance;
for iter=1:N_{Iter} (number of iterations) **do**
    For all users $\forall j$
    Determine $\beta_{j,i}^k$ and $\gamma_{j,i}^k$ according to (19);
    Compute $b_{i\backslash j}^k(x_{i,k})$ and $b_j^k(x_{j,k})$ according to (17) and (20), (21);
    Update the mean and variance of the damped belief according to (23), (24);
    Calculate LLR L_j^a based on $\tilde{b}_j^k(x_{j,k})$ and feed them to the channel decoder;
    Perform standard BP channel decoding;
    Calculate $m_{x_{j,k}}^0$ and $v_{x_{j,k}}^0$ based on the output extrinsic information from the channel decoder ;
end for

B. Computational Complexity

Note that the complexity of the proposed algorithm is dominated by the integration in (18). For the symbol $x_{j,k}$ of user j, there are a total of $(D-1)$ interfering symbols. Note that with the beliefs being Gaussian, when obtaining the marginal of symbol $x_{j,k}$, only simple addition and multiplication calculations are involved. As a result, a complexity on the order of $\mathcal{O}[(D-1)])$ is imposed and the complexity of the proposed algorithm is then $\mathcal{O}[J(D-1)]$. The original MPA receiver requires an optimal MAP detector, having a complexity order of $\mathcal{O}(|\boldsymbol{\chi}|^D)$. The Max-log based SCMA detector of [7] still has a complexity order of $|\boldsymbol{\chi}|^D$, although the number of operations has been significantly reduced. By contrast, the complexity of the proposed algorithm only increases linearly with the number of users. Compared to the method of [9], the proposed algorithm has the same complexity order. In summary, the different algorithms' complexities summarized in Table I.

TABLE I
COMPLEXITY COMPARISON

Algorithm Name	Computational Complexity
Conventional MPA	$\mathcal{O}(J\|\boldsymbol{\chi}\|^D)$
Max-log based MPA	$\mathcal{O}(J\|\boldsymbol{\chi}\|^D)$
Modified MPA [9]	$\mathcal{O}[J\|\boldsymbol{\chi}\|(D-1)]$
The proposed algorithm	$\mathcal{O}[J\|\boldsymbol{\chi}\|(D-1)]$

IV. CONVERGENCE ANALYSIS

The convergence is a key issue for an iterative algorithm. In this section, we derive the realistic conditions that guaranteed the convergence of the proposed energy minimization based receiver.

According to (17), at the lth iteration, $m_{i\backslash j}^k$ and $v_{i\backslash j}^k$ are updated based on the parameters determined in the previous iteration, following

$$v_{i\backslash j}^k(l) = \left(\rho_i^k + \sum_{(i,m),m\neq j} \beta_{i,m}^k(l-1)\right)^{-1} \tag{25}$$

$$m_{i\backslash j}^k(l) = v_{i\backslash j}^k(l)\left(\varrho_i^k + \sum_{(i,m),m\neq j} \gamma_{i,m}^k(l-1)\right). \tag{26}$$

Proposition 1 *The variance $\tilde{v}_{x_{j,k}}$ of the belief is guaranteed to converge, satisfying*

$$\tilde{v}_{x_{j,k}}(l) \le \tilde{v}_{x_{j,k}}(l-1).$$

Proof: See Appendix A. ∎

Next let's analyze the convergence of $\tilde{m}_{x_{j,k}}$, which is equivalent to proving that the difference between $m_{x_{j,k}}$ obtained in two consecutive iterations becomes smaller as l becomes larger, yielding,

$$|\tilde{m}_{x_{j,k}}(l+1) - \tilde{m}_{x_{j,k}}(l)| \le |\tilde{m}_{x_{j,k}}(l) - \tilde{m}_{x_{j,k}}(l-1)|. \tag{27}$$

Provided that the number of iterations is large enough, we can assume that the parameters, $v_{x_{j,k}}$ and $v_{i\backslash j}^k$ converge to v^* and $\bar{v}^*$ for all j, respectively. According to (24), $\tilde{v}_{x_{j,k}}$ also converges to v^*. Thus (20) and (23) can be rewritten as

$$m_{x_{j,k}}(l) = v^*\left(\varrho_j^k + \sum_{(i,j)} \gamma_{j,i}^k(l)\right) \tag{28}$$

$$\tilde{m}_{x_{j,k}}(l) = \alpha m_{x_{j,k}}(l) + (1-\alpha) m_{x_{j,k}}(l-1). \tag{29}$$

Then we have

$$\begin{aligned}\tilde{m}_{x_{j,k}}(l+1) - \tilde{m}_{x_{j,k}}(l) &= \alpha(m_{x_{j,k}}(l+1) - m_{x_{j,k}}(l)) \\ &\quad + (1-\alpha)(m_{x_{j,k}}(l) - m_{x_{j,k}}(l-1)) \\ &= v^* \sum_{(i,j)} \left[\alpha\left(\gamma_{j,i}^k(l+1) - \gamma_{j,i}^k(l)\right)\right. \\ &\quad \left.+ (1-\alpha)\left(\gamma_{j,i}^k(l) - \gamma_{j,i}^k(l-1)\right)\right],\end{aligned} \tag{30}$$

which implies that the convergence of $m_{x_{j,k}}$ is related to that of $\gamma_{j,i}^k$.

Substituting $m_{i\backslash j}^k$ from (26) into (19) yields

$$\gamma_{i,m}^k(l) = -b\left(\varrho_i^k + \sum_{(i,m),m\neq j} \gamma_{i,m}^k(l-1)\right) \tag{31}$$

with $b = \frac{h_{j,k}^2 \bar{v}^*}{N_0}$. Similar to (35) in Appendix A, we obtain

$$\gamma_{i,m}^k(l+1) - \gamma_{i,m}^k(l) = b \sum_{(i,m),m\neq j} \left[\gamma_{i,m}^k(l-1) - \gamma_{i,m}^k(l)\right]. \tag{32}$$

Again, by stacking all γ having the index k as a vector, we have the following equation

$$\boldsymbol{\gamma}^k(l+1) - \boldsymbol{\gamma}^k(l) = b\mathbf{A}\left[\boldsymbol{\gamma}^k(l) - \boldsymbol{\gamma}^k(l-1)\right]. \tag{33}$$

Proposition 2 *The mean $m_{x_{j,k}}$ of the symbol belief is guaranteed to converge, if and only if the spectral radius of $\mathbf{A}$ satisfies $\rho(\mathbf{A}) < \frac{1}{b}$.*

Proof: See Appendix B. ∎

V. SIMULATION RESULTS

We consider a pair of SCMA systems using parameters of: a) $J = 6$ and $K = 4$, $\lambda = 150\%$, where the corresponding codebook is defined in [16]; b) $J = 12$ and $K = 6$, $\lambda = 200\%$, where the corresponding codebook is defined in [17]. A rate-5/7 LDPC code is employed and a flat-fading Rayleigh channel associated with perfect channel state information (CSI) is used. The number of iterations between the channel decoder and the multiuser detector is set to $N_{\text{Iter}} = 10$.

The bit error rate (BER) performance of the proposed algorithm is compared to that of the state-of-the-art methods in Fig. 2 and Fig. 3. The damping factor of the proposed algorithm is $\alpha = 0.3$.[1] It can be seen from both figures that the proposed algorithm matches the performance of the MPA receiver, but the complexity of the conventional MPA receiver increases exponentially with the number of users. The variational inference method has a low complexity as a benefit of the mean-field approximation. Nevertheless, due to ignoring the conditional dependencies amongst data symbols, the variational inference method leads to a significant performance loss. When $\lambda = 150\%$, the modified MPA method of [9] has a similar performance to that of the proposed algorithm. However, when $\lambda = 200\%$, the modified MPA [9] experiences performance loss. This is because the factor graph has more cycles due to the more severe interference, which will result in convergence problems for the modified MPA.

[1] There are several methods proposed to find the optimal damping factor under different criteria. Here we use the value in [15].

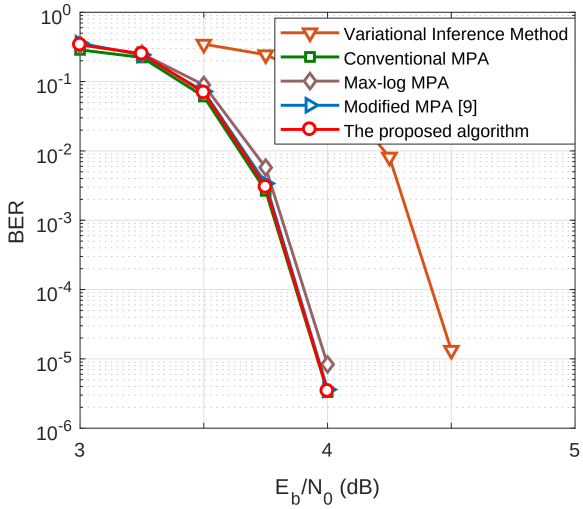

Fig. 2. BER performance of different algorithms. ($\lambda = 150\%$)

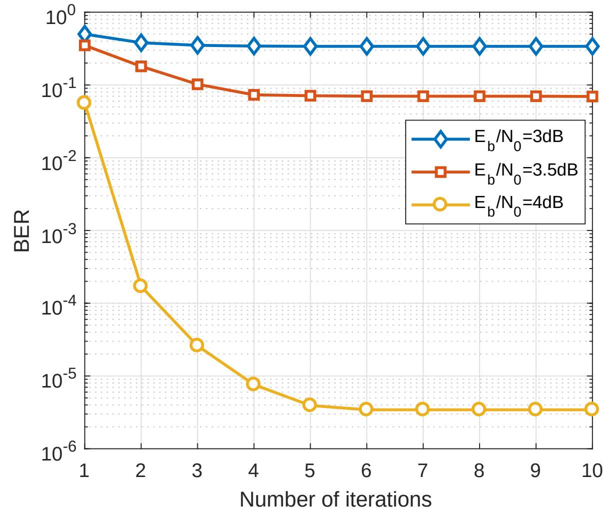

Fig. 4. BER performance versus the number of iterations. ($\lambda = 150\%$)

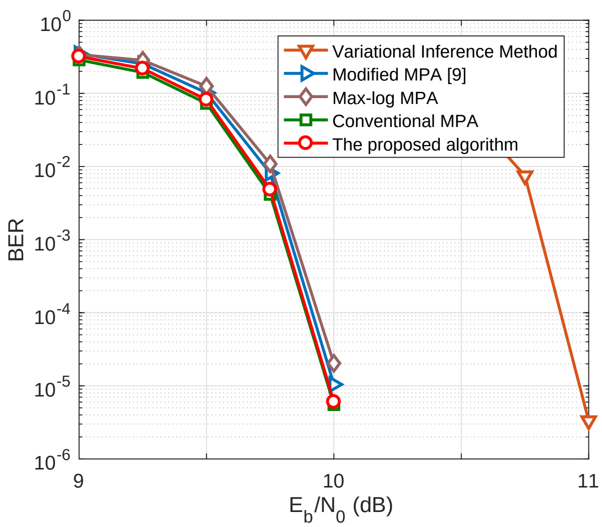

Fig. 3. BER performance of different algorithms. ($\lambda = 200\%$)

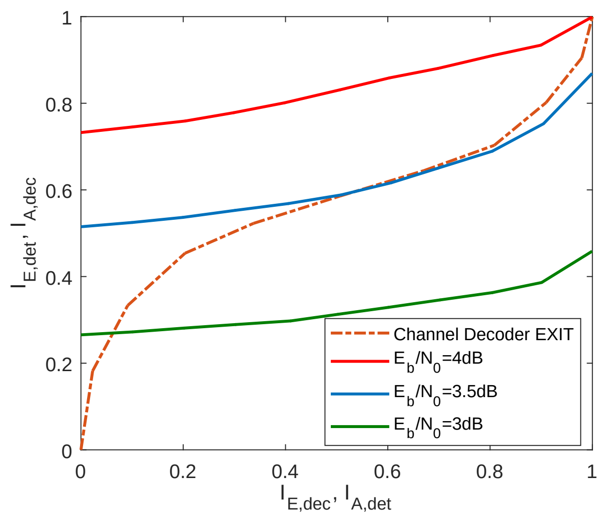

Fig. 5. EXIT chart between the SCMA detector and channel decoder. ($\lambda = 150\%$)

To evaluate the convergence of the proposed algorithm, in Fig. 4 we depict its BER performance for a single user versus the number of iterations at different values of E_b/N_0. An SCMA system associated with $\lambda = 150\%$ is considered. We can observe that increasing the number of iterations improves the performance of the proposed algorithm. Moreover, at $E_b/N_0 = 3$ dB, the performance improvement becomes marginal, as the number of iterations increases. When E_b/N_0 becomes higher, more iterations are required to guarantee convergence. We additionally invoked the extrinsic information transfer (EXIT) chart to reveal the mutual information (MI) convergence property between the channel decoder and the proposed SCMA detector, which is shown in Fig. 5. We use $I_{A,dec}$ to denote the MI between the transmitted bits and the LLRs fed to the channel decoder and $I_{E,dec}$ to denote the MI between the bits and the extrinsic LLRs output by the channel decoder. Similar definitions of $I_{A,det}$ and $I_{E,det}$ are used for the SCMA detector. We can see from Fig. 5 that at $E_b/N_0 = 4dB$, an open tunnel is attained and both curves reach the (1,1) point of perfect convergence to a vanishingly low BER. Hence, it shows that the proposed algorithm is expected to converge.

VI. Conclusions

In this paper, we proposed an energy minimization based low-complexity iterative receiver for SCMA systems. By factorizing the joint distribution into the product of several potentials, we used the Bethe approximation to derive the marginal of data symbols. The complexity of the proposed algorithm only increases linearly with the number of users, instead of the exponential complexity of the optimal MAP detector. We further analyzed the convergence of the proposed algorithm and derived its convergence conditions. Our simulation results for two SCMA systems with normalized user-load $\lambda = 150\%$ and $\lambda = 200\%$, respectively, showed that the low-complexity energy minimization based algorithm closely approaches the performance of the conventional MPA scheme and outperforms both the modified MPA and the variational inference methods.

Appendix A
Proof of Proposition 1

According to (19) and (25), the update equation of $\beta_{j,i}^k(l)$ is rewritten as

$$\beta_{j,i}^k(l) = -a\left(\rho_i^k + \sum_{(i,m),m\neq j}\beta_{i,m}^k(l-1)\right)^{-1}, \quad (34)$$

with $a = h_{j,k}^4/N_0^2$. Note that if $\beta_{i,j}^k \leq 0$, we can derive the following inequality

$$\beta_{j,i}^k(l+1) - \beta_{j,i}^k(l) = \frac{a\sum_{(i,m),m\neq j}\left[\beta_{i,m}^k(l) - \beta_{i,m}^k(l-1)\right]}{\left(\rho_i^k + \sum_{(i,m),m\neq j}\beta_{i,m}^k(l-1)\right)\left(\rho_i^k + \sum_{(i,m),m\neq j}\beta_{i,m}^k(l)\right)} \geq \frac{a}{(\rho_i^k)^2}\sum_{(i,m),m\neq j}\left[\beta_{i,m}^k(l) - \beta_{i,m}^k(l-1)\right]. \quad (35)$$

By stacking all β values with respect to the resource index k to form $\boldsymbol{\beta}^k$, the above inequality can be expressed in a vectorial form as

$$\begin{aligned}\boldsymbol{\beta}^k(l+1) - \boldsymbol{\beta}^k(l) &\geq \frac{a}{(\rho_i^k)^2}\mathbf{A}\left[\boldsymbol{\beta}^k(l) - \boldsymbol{\beta}^k(1-1)\right] \\ &\geq \frac{a^l}{(\rho_i^k)^{2l}}\mathbf{A}^l\left[\boldsymbol{\beta}^k(1) - \boldsymbol{\beta}^k(0)\right], \quad (36)\end{aligned}$$

where $\mathbf{A}$ is an adjacent matrix with $\mathbf{A}_{ij} = 1$ if and only if user i and j are interfering with each other. Since the symbols are initialized with $v_{x_{j,k}}(0) = \infty$, which indicates that $v_{x_{j,k}}(1) \leq v_{x_{j,k}}(0)$, therefore $\beta_{j,i}^k(1) \geq \beta_{j,i}^k(0)$ holds and we arrive at $\boldsymbol{\beta}^k(l+1) - \boldsymbol{\beta}^k(l) \geq \mathbf{0}$, which shows that the parameter $\beta_{j,i}^k(l+1)$ is monotonically increasing. According to (21), we have $v_{x_{j,k}}(l+1) < v_{x_{j,k}}(l)$, which proves that $v_{x_{j,k}}$ is guaranteed to converge.

Considering the belief damping, to prove $\tilde{v}_{x_{j,k}}(l+1) < \tilde{v}_{x_{j,k}}(l)$ is equivalent to prove

$$\frac{\alpha}{v_{x_{j,k}}(l)} + \frac{1-\alpha}{v_{x_{j,k}}(l+1)} \geq \frac{\alpha}{v_{x_{j,k}}(l-1)} + \frac{1-\alpha}{v_{x_{j,k}}(l)}. \quad (37)$$

Obviously, with the conclusion drawn above, $\frac{\alpha}{v_{x_{j,k}}(l)} \geq \frac{\alpha}{v_{x_{j,k}}(l-1)}$ and $\frac{1-\alpha}{v_{x_{j,k}}(l+1)} \geq \frac{1-\alpha}{v_{x_{j,k}}(l)}$ always hold. Therefore the variance of damped belief is also guaranteed to converge.

Appendix B
Proof of Proposition 2

• *Necessary Condition*: Based on (33), we can express $\boldsymbol{\gamma}^k(l+1) - \boldsymbol{\gamma}^k(l)$ as

$$\boldsymbol{\gamma}^k(l+1) - \boldsymbol{\gamma}^k(l) = b^l\mathbf{A}^l\left[\boldsymbol{\gamma}^k(1) - \boldsymbol{\gamma}^k(0)\right]. \quad (38)$$

Deriving the limits of both sides of (38) yields

$$\lim_{l\to\infty}\left[\boldsymbol{\gamma}^k(l+1) - \boldsymbol{\gamma}^k(l)\right] = \lim_{l\to\infty}(b\mathbf{A})^l\left[\boldsymbol{\gamma}^k(1) - \boldsymbol{\gamma}^k(0)\right]. \quad (39)$$

If $\tilde{m}_{x_{j,k}}$ converges, we have

$$\begin{aligned}&\lim_{l\to\infty}\left[\alpha\left(\boldsymbol{\gamma}^k(l+1) - \boldsymbol{\gamma}^k(l)\right) + (1-\alpha)\left(\boldsymbol{\gamma}^k(l) - \boldsymbol{\gamma}^k(l-1)\right)\right] \\ &= \lim_{l\to\infty}\left[\alpha(b\mathbf{A})^l + (1-\alpha)(b\mathbf{A})^{l-1}\right]\cdot\left[\boldsymbol{\gamma}^k(1) - \boldsymbol{\gamma}^k(0)\right] = 0 \quad (40)\end{aligned}$$

which in turn requires $\lim_{l\to\infty}(b\mathbf{A})^l = 0$. Assuming that λ and $\boldsymbol{\nu}$ are the eigenvalue and eigenvector of $\mathbf{A}$, we have

$$\begin{aligned}\boldsymbol{\nu}\lim_{l\to\infty}(b\mathbf{A})^l &= \lim_{l\to\infty}\left[(b\mathbf{A})^l\boldsymbol{\nu}\right] = \lim_{l\to\infty}\left[(b\lambda)^l\boldsymbol{\nu}\right] \\ &= \boldsymbol{\nu}\lim_{l\to\infty}(b\lambda)^l. \quad (41)\end{aligned}$$

Since $\boldsymbol{\nu} \neq \mathbf{0}$, $\lim_{l\to\infty}(b\lambda)^l = 0$ must be satisfied. Therefore $|\lambda| < \frac{1}{b}$ holds for any eigenvalue of the matrix $\mathbf{A}$. Consequently, $\rho(\mathbf{A}) < \frac{1}{b}$.

• *Sufficient Condition*: According to the matrix theorem of [18], we have

$$\begin{aligned}\|\boldsymbol{\gamma}^k(l+1) - \boldsymbol{\gamma}^k(l)\| &= \|b\mathbf{A}\left[\boldsymbol{\gamma}^k(l) - \boldsymbol{\gamma}^k(l-1)\right]\| \\ &\leq b\cdot\rho(\mathbf{A})\|\boldsymbol{\gamma}^k(l) - \boldsymbol{\gamma}^k(l-1)\|. \quad (42)\end{aligned}$$

If $\rho(\mathbf{A}) < \frac{1}{b}$, then $\|\boldsymbol{\gamma}^k(l+1) - \boldsymbol{\gamma}^k(l)\| < \|\boldsymbol{\gamma}^k(l) - \boldsymbol{\gamma}^k(l-1)\|$ holds and therefore $\tilde{m}_{x_{j,k}}$ is convergent.

References

[1] Z. Ding, X. Lei, G. K. Karagiannidis, R. Schober, J. Yuan, and V. K. Bhargava, "A survey on non-orthogonal multiple access for 5G networks: Research challenges and future trends," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 10, pp. 2181–2195, Oct 2017.

[2] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. Soong, and J. C. Zhang, "What will 5G be?" *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1065–1082, Jun 2014.

[3] M. Moltafet, N. M. Yamchi, M. R. Javan, and P. Azmi, "Comparison study between NOMA and SCMA," *IEEE Trans. Veh. Technol.*, early access.

[4] M.-K. Tsay, Z.-S. Lee, and C.-H. Liao, "Fuzzy power control for downlink CDMA-based LMDS network," *IEEE Trans. Veh. Technol.*, vol. 57, no. 6, pp. 3917–3921, Nov 2008.

[5] F. R. Kschischang, B. J. Frey, and H.-A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 498–519, Feb 2001.

[6] Y. Du, B. Dong, Z. Chen, J. Fang, and L. Yang, "Shuffled multiuser detection schemes for uplink sparse code multiple access systems," *IEEE Commun. Lett.*, vol. 20, no. 6, pp. 1231–1234, June 2016.

[7] F. Wei and W. Chen, "Low complexity iterative receiver design for sparse code multiple access," *IEEE Trans. Commun.*, vol. 65, no. 2, pp. 621–634, Feb 2017.

[8] J. Dai, K. Niu, C. Dong, and J. Lin, "Improved message passing algorithms for sparse code multiple access," *IEEE Trans. Veh. Technol.*, vol. 66, no. 11, pp. 9986–9999, Nov 2017.

[9] X. Meng, Y. Wu, Y. Chen, and M. Cheng, "Low complexity receiver for uplink SCMA system via expectation propagation," in *Proc. IEEE Wireless Commun. Networking Conf.* IEEE, 2017, pp. 1–5.

[10] G. E. Box and G. C. Tiao, *Bayesian inference in statistical analysis*. John Wiley & Sons, 2011.

[11] F. Ricci-Tersenghi, "The Bethe approximation for solving the inverse Ising problem: a comparison with other inference methods," *J. Statis. Mechan.: Theory, Experiment*, vol. 2012, no. 08, pp. 1–23, Aug 2012.

[12] F. Li, Z. Xu, and S. Zhu, "Variational-inference-based data detection for OFDM systems with imperfect channel estimation," *IEEE Trans. Veh. Technol.*, vol. 62, no. 3, pp. 1394–1399, Mar 2013.

[13] W. Yuan, N. Wu, H. Wang, and J. Kuang, "Variational inference-based frequency-domain equalization for faster-than-Nyquist signaling in doubly selective channels," *IEEE Signal Process. Lett.*, vol. 23, no. 9, pp. 1270–1274, Sept 2016.

[14] K. Friston, J. Mattout, N. Trujillo-Barreto, J. Ashburner, and W. Penny, "Variational free energy and the Laplace approximation," *Neuroimage*, vol. 34, no. 1, pp. 220–234, Jan 2007.

[15] P. Som, T. Datta, N. Srinidhi, A. Chockalingam, and B. S. Rajan, "Low-complexity detection in large-dimension MIMO-ISI channels using graphical models," *IEEE J. Sel. Topics Signal Process.*, vol. 5, no. 8, pp. 1497–1511, Aug 2011.

[16] S. Zhang, K. Xiao, B. Xiao, Z. Chen, B. Xia, D. Chen, and S. Ma, "A capacity-based codebook design method for sparse code multiple access systems," in *Proc. 8th Int. Conf. Wireless Commun. Signal Process.* IEEE, 2016, pp. 1–5.

[17] M. Taherzadeh, H. Nikopour, A. Bayesteh, and H. Baligh, "SCMA codebook design," in *Proc. IEEE 80th Veh. Technol. Conf.* IEEE, 2014, pp. 1–5.

[18] R. A. Horn and C. R. Johnson, *Matrix analysis*. Cambridge University Press, 2012.