Impacts of Drying-Wetting and Loading-Unloading Cycles on Small-Strain Shear Modulus of Unsaturated Soils

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ABSTRACT

The small strain shear modulus (G_{max}) is an important parameter in geodynamic problems. In order to predict G_{max} of unsaturated soils which are normally subjected to complex drying, wetting processes, effect of hydraulic hysteresis needs to be evaluated. Although several equations have been proposed in recent years, limitations still exist, requiring more research studies in this field. In this study, G_{max} was investigated in a multi-stage test during several drying-wetting cycles and a loading-unloading cycle of net stress. The results revealed four key factors that directly influence the magnitude of G_{max}: the void ratio, the net stress, matric suction and degree of saturation. While variations of the void ratio, net stress, and matric suction cause persistent responses of G_{max} (i.e. if all other factors remain unchanged, G_{max} would then be reversely proportional to the void ratio and directly proportional to the net stress and matric suction), variations in the degree of saturation result in different responses. A decrease in the degree of saturation may induce a reduction or growth of G_{max} since on the one hand it reduces the effect of matric suction, while on the other hand it increases the total effect of van der Waals attractions and electric double layer repulsions. At the same stress state, a reverse trend, induced by an increase in the degree of saturation, will occur with a growth in the effect of matric suction and a reduction in the combined effect of van der Waals attractions and electric double layer repulsions. An analysis of the results showed that hydraulic hysteresis occurred in all the stress loops; and it directly influenced the response of G_{max}. The effect of hydraulic hysteresis can only be captured if the van der Waals attractions and electric double layer repulsions are considered. A model to estimate G_{max} while incorporating the van der Waals attractions and electric double layer repulsions has been proposed and it provided a good agreement with the experimental measurements.

KEYWORDS: unsaturated soil, small-strain modulus, degree of saturation, matric suction, net stress, hydraulic hysteresis

Introduction

Small strain shear modulus denotes the shear stiffness of soils at small strain amplitude (0.001% or less), hence determining its value is very important when predicting the response of a soil-structure system exposed to dynamic loading, and when investigating the quality of earth works using a spectral analysis of surface waves. Comparisons between the magnitude of G_{max} determined in the field and laboratory also show the degree to which soil samples are disturbed (Jung et al. 2012). For saturated soils, numerous experimental data has proved that G_{max} is a function of the void ratio, the over consolidation ratio (*OCR*) of soil, and the mean effective stress (Hardin 1978; Richart et al. 1970). However, recent studies revealed that the mean effective stress and void ratio will indicate the state of a saturated soil (Jamiolkowski et al. 1994; Santagata et al. 2005; Shibuya et al. 1992). With unsaturated soils, the mean effective stress that results from normal inter-particle forces is due to net stress, the matric suction, van der Waals attractions, and electric double layer repulsions (Khalili and Khabbaz 1998; Lu and Likos 2006). Here, net stress is the difference between the total stress and the pore air pressure (i.e., $\sigma_n=\sigma-u_a$).

Matric suction is the difference between the pore air pressure and the pore water pressure (i.e., $\psi=u_a-u_w$), van der Waals attractions are the intermolecular forces between atoms of adjacent particles, and electric double layer repulsions are the forces between cations in the electric double layers in the vicinity of particle surfaces. The net stress can directly influence the effective stresses for the full range of degree of saturation, whereas matric suction, van der Waals attractions and electric double layer repulsions produce inter-particle forces in different patterns so their influence depends very much on the degree of saturation (Khalili and Khabbaz 1998; Lu and Likos 2006).

The results of many experiments available in literature indicate how the water content affects G_{max} during an air-drying process. They have shown G_{max} constantly increases as the drying process begins, while the tests start from the saturated state. However, G_{max} responds differently at low degrees of saturation when it is close to the dry state:

1. G_{max} peaked at an optimum degree of saturation before consistently declining until it reached a dry state, as reported in tests on clean sands by Wu et al. (1984) and Qian et al. (1993).
2. G_{max} consistently increased during the drying process, as Cho and Santamarina (2001) observed for granite powder, Sandboil sand, and a mixture consisting of 80% glass bead and 20% kaolinite; and by Dong and Lu (2016) for Bonny silt and BALT silt.
3. G_{max} consistently increased at the beginning, plateaued in the middle, and then experienced a sharp growth near the dry state, as Inci et al. (2003) observed for soils with plasticity indices from 7% to 55%; and by Dong and Lu (2016) for Georgia kaolinite and Hopi silt.

In these tests, only the relationship between G_{max} and degree of saturation (or water content) was investigated. Therefore, in order to study and assess the effects of matric suction on G_{max}, other researchers have modified conventional equipment, such as triaxial cells, oedometer cells, and Rowe cells into suction-control apparatus by replacing a common porous stone with a high air-entry porous disk (*HAEPD*) to separate the air and water phases to maintain a desired matric suction. The responses of G_{max} were studied during drying and wetting by increasing and decreasing the matric suction complying with the axis translation technique (Hilf 1956; Vanapalli et al. 2008); during these tests the void ratio and the degree of saturation could be evaluated at any specific stress states. Mancuso et al. (2002) investigated the G_{max} of a compacted silty sand during three drying processes using a suction-control resonant column device in which matric suction increased from 0 to 400 kPa under constant net stresses of 100 kPa, 200 kPa, and 400 kPa. The results showed S-shaped curves for variations of G_{max} with matric suction. Mancuso et al. (2002) believed that during the drying process, G_{max} gradually shifted from a free water-dominated zone at low matric suction to a menisci water-dominated zone at relatively high matric suction. Sawangsuriya et al. (2006) developed a suction-control triaxial apparatus equipped with two horizontal bender elements to study the G_{max} of clayey sand compacted at different moisture contents during drying tests under a constant net confining pressure of 34.5 kPa. They noted the G_{max} was reversely proportional to compaction moisture content and the semi-log plot of G_{max} versus matric suction showed consistent growth with increasing matric suction. Similar trends were also witnessed during drying tests using different types of suction-controlled devices in experiments carried out by Hoyos et al. (2015) for silty sand.

Responses of G_{max} when hydraulic hysteresis presents were reported during drying-wetting cycles and loading-unloading cycles of net stress. Khosravi (2011) investigated effects of void ratio, degree of saturation, matric suction and net stress on G_{max} of statically compacted Bonny silt during drying-wetting cycles under three different confining net pressures of 100 kPa, 150 kPa, and 200 kPa. The experiment used a fixed-free Stokoe-type resonant column device that was modified to include suction and saturation control. It was observed that G_{max} was proportional to net stress, and at the same levels of net stress and matric suction, G_{max} was higher in wetting processe than in the previous drying. A similar observation was also reported by Ng et al. (2009) in their study on recompacted completely decomposed granite samples, subjected to different net stresses of 110 kPa and 300 kPa, using a suction-controlled triaxial system with built-in bender elements. Biglari et al. (2012) measured G_{max} of Zenoz kaolin (a lean clay from a mine in northwest Iran) during loading and unloading stages under a constant matric suction of 300 kPa, using a fixed–free resonant column-torsional shear device. For a given net stress, higher values of G_{max} were reported during unloading process in comparison to the corresponding values during loading.

A number of equations for predicting the G_{max} of unsaturated soils were established and verified based on the results available in the literature. In these equations the variation of G_{max} during drying and wetting processes under a constant net stress was attributed to changes in the matric suction, while ignoring other factors in the predictive models such as the van der Waals attractions and electric double layer repulsions, both of which contribute to inter-particle forces, and can result in disparities between predictions and measurements of G_{max} of unsaturated soils when they are subjected to several cycles of wetting and drying (i.e. hydraulic hysteresis).

Recently, Khosravi et al. (2018) proposed a semi-empirical equation to predict G_{max} of unsaturated sand during hydraulic hysteresis. The model adopted the concepts of suction stress proposed by Lu and Likos (2006) to capture the effect of hydraulic hysteresis. Six parameters were used to determine suction stress and control effects of effective stress and hydraulic hysteresis. According to Lu and Likos (2006), suction stress represents all local inter-particle forces in unsaturated soils including van der Waals attractions, electric double layer repulsions and force induce by matric suction. However, suction stress itself is not capable of capturing hydraulic hysteresis as it is based on the corresponding soil water characteristic curve (SWCC) which keeps changing during drying-wetting cycles. Indeed, to consider hydraulic hysteresis effects, different parameters corresponding to the suction stress equation would be required (Lu et al. 2010). Hence, the prediction of G_{max} when hydraulic hysteresis presents requires SWCCs and parameters related to the suction stress equation to be redetermined repeatedly. It seems that the combination of the inter-particle forces induced by matric suction, van der Waals attractions and electric double layer repulsions which possibly follow different patterns with variations of degree of saturation makes suction stress incapable of capturing hydraulic hysteresis intrinsically.

In this study, an array of laboratory experiments were carried out to investigate the magnitude of G_{max} during several drying-wetting cycles, and a loading-unloading cycle of net stress. These experiments also set out to establish a predictive model for G_{max} evaluating the combined contribution of van der Waals attractions and electric double layer repulsions separately from the contributions of matric suction in order to capture the effects of hydraulic hysteresis on G_{max}.

Laboratory Experiments

Unsaturated Rowe cell apparatus with bender elements

A modified Rowe cell was developed for testing unsaturated soil by adopting the axis-translation technique to study the G_{max} with a high degree of accuracy (Fig. 1). The Rowe cell apparatus was connected with three pressure/volume controllers using water or air, to simultaneously apply an upper chamber pressure p_u (the external vertical pressure), the air pressure p_a on top of the sample, and back water pressure p_b at the bottom of the sample. The two water controllers were for applying and controlling p_u and p_b, while the air controller was for applying p_a. With this system, the volume change of sample and the volume of water entering or exiting the sample can be computed at any time during the tests. The desired matric suction (i.e. the difference between p_a and p_b) was maintained with a *HAEPD* installed at the bottom of the sample, as shown in Fig. 1c, with an air entry value of 1500 kPa. However, even at matric suctions below the air entry value of *HAEPD*, some air could still diffuse through and accumulate under the disk (Padilla et al. 2006), which would result in a slight overestimation in the variation of the pore water volume from the volumetric readings of the back controller. To address this issue, water was periodically flushed from the bottom of the *HAEPD* by another water controller kept at 3 kPa lower than the corresponding back pressure; this pressure would avoid reducing the back pressure during flushing. This apparatus is also equipped with two bender elements with a maximum frequency of 100 kHz, built into the top and bottom of the sample, to measure the shear wave velocity during different applications of stress. The tip-to-tip distance between two bender elements was calculated from the readings of a linear variable differential transformer (*LVDT*), placed at the top of the sample during the test.

Determining the shear wave velocity and selecting the input wave

Two bender elements, made from piezoelectric ceramic bimorphs, were embedded into the top and bottom of the sample (Figs. 1b and 1c). One is a transmitter to generate a shear wave when excited by a single pulse, while the other is a receiver to pick up the wave after it has propagated through the sample. Measuring the propagation distance (L) (i.e. the tip-to-tip distance between two bender elements) and the propagation time (Δt) (i.e. the time difference between the transmitted and received wave) will give the shear wave velocity (V_s):

$$V_s = \frac{L}{\Delta t} \qquad (1)$$

The propagation distance can be computed directly and precisely from readings of the *LVDT* (see Fig.1a). However, the propagation time is affected by the method used to identify the arrival time of the received wave. Three approaches, currently used, are the cross correlation (Lee and Santamarina 2005), the frequency domain (Greening and Nash 2004), and the time domain (Rees et al. 2013). The first method evaluates the cross correlation between transmitted and received waves to determine the maximum amplitude, which indicates the arrival time of the first peak. This method assumes that the frequencies of the transmitted and received waves are identical, but this assumption is improper in most cases, because the receiver measures a complex interaction of incident and reflected waves (Arulnathan et al. 1998). In the time domain method, Δt is the difference in time between the starting point or the first peak point of the transmitted wave and the corresponding point in the received wave. Since the surrounding noises and near-field effect induced by the early arrival of the pressure wave frequently obscure the starting point of the real arrival, especially for short propagation distances, the first peak point is now preferred in recent studies (e.g. Ogino et al. 2015). The propagation time can also be determined by the frequency domain method based on a phase analysis of the cross-power spectrum between transmitted and received waves to avoid the subjective visual inspection of the two waves. The cross-power spectrum is obtained using a Fast Fourier Transform, and the propagation time is then calculated from the slope of the unwrapped phase angle function using a linear regression across a defined frequency window. However, there are more scattered measurements in this method, as Ogino et al. (2015) and Camacho-Tauta et al. (2015) have reported. For these reasons, the peak-to-first peak technique was used in this study, as shown in Fig. 2.

The shear wave velocity is affected by the characteristics of the input wave, particularly the wave form and wave frequency. Leong et al. (2005) examined how the two most common waveforms (i.e. square and sinusoidal waves) affect the received signal and concluded that the latter causes less distortion of the output signal than the former. Wave frequency plays even a more important role. The best output signals occur for the input frequencies near the resonant frequency of the sample-bender element system, while for lower or higher input frequencies, amplitude of the output signal decreases until it cannot be detected at extremely low or high ranges (Lee and Santamarina 2005). The frequency of the input wave should also be chosen to minimize the sample size effect.

The effects of sample size in determining the shear wave velocity using bender elements has been reported by a number of researchers (Arroyo 2001; Leong et al. 2009; Sanchez-Salinero et al. 1986). According to (Sanchez-Salinero et al. 1986), coupling of the shear wave and compressive wave components occurs at short distances (around 2 times of the wavelength) from the source. This near-field effect causes wrong determination of the time of arrival. Leong et al. (2009) conducted bender element test on compacted residual soil samples using triaxial apparatus and observed that the near-field effect was negligible when the ratio of propagation distance to wavelength was greater than 3.33. In addition, the amplitude of transmitted wave decreases with increasing distance due to the dissipation of elastic energy within the material and the spreading of wave energy from the source point (Sanchez-Salinero et al. 1986). The attenuation effect can potentially blur the received wave, making it more difficult to identify the arrival time. To minimise the attenuation effect, Sanchez-Salinero et al. (1986) suggested using ratios of propagation distance to wavelength less than 4. Boundary effect also contributes to distortion of the received wave in cases of a thin cylindrical sample. When a wave arrives at the horizontal boundary of the sample, part of its energy is reflected, forming a composite wave, thus veils the real transmitted wave. As recommended in the ASTM Standard D2845-08, the ratio of the propagation distance to the minimum lateral dimension of the sample must not exceed 5.

In this experiment, the initial sample size was 31 mm in height and 100 mm in diameter with the corresponding tip-to-tip distance between two bender elements being 25 mm. The initial ratio of the propagation distance to the diameter of the sample was 0.25, much smaller than 5, thus the lateral boundary should have insignificant effect on the determination of shear wave velocity. To minimize the near-field and attenuation effects, single sinusoidal wave with frequency of 50 kHz was selected so as the ratio of propagation distance to wavelength was approximately 4 for the first measurements at low stress levels, considering decreases of the ratio at higher stress levels. In fact, the ratio of propagation distance to wavelength varied in the range from 2.4 to 4.1, indicating minor near-field and attenuation effects. For each determination of the propagation time, a minimum of 10 output signals were manually stacked to minimise electrical noises.

Determining the small strain shear modulus and effective dynamic mass density

The magnitude of G_{max} is determined using the following equation:

$$G_{max}=\rho_{eff}.V_s^2 \qquad (2)$$

where ρ_{eff} is the effective dynamic mass density of soil and V_s is the shear wave velocity. For one-phase materials, ρ_{eff} is equal to the static mass density, but determining the dynamic mass density of water-solid composites is challenging. Even though pore water cannot transmit a shear wave, it still affects the magnitude of shear wave velocity due to the mass of a thin viscous boundary layer attached to the surface of solid particles that moves in unison as a single phase during shear wave propagation (Biot 1956; Wu et al. 2012). The thickness of the viscous boundary layer (l_{vis}) is influenced by the wave frequency, thus as l_{vis} approaches zero at extremely high wave frequencies, there is no mass coupling and ρ_{eff} would be equal to the dry mass density ρ_d (Wu et al. 2012). However, l_{vis} approaches infinity at very low wave frequencies, which leads to full mass coupling, so ρ_{eff} would be equal to the static mass density. It is rational to believe that for mid-range wave frequencies, the degree of mass coupling in two-phase materials such as saturated soils would depend on the size of the maximum pore; hence full mass coupling would occur only when the maximum pore size is smaller than $2l_{vis}$. In case the maximum pore size is greater than $2l_{vis}$, there would be relative motions between the pore water and soil particles. For partially saturated soils, which are three-phase materials, the water body may be smaller than the pore size, thus pore size cannot be used to check the mass coupling effect. To date there are no correlations for estimating the ρ_{eff} for three-phase composites, but in practice the ρ_{eff} of unsaturated soils is the total mass density (ρ_t) (e.g. Biglari et al. 2012, Dong & Lu 2016). In this study, the maximum effective dynamic mass density ($\rho_{eff,max}$) corresponding to the full thickness of the viscous boundary layer (or full mass coupling) was computed for a given porosity, using Eq. (3) for two-phase composites at low-frequency range, where the wavelength is larger than the typical pore sizes in the composite (Martin et al. 2010; Mei et al. 2007):

$$\frac{\rho_{eff,max}}{\rho_s} = \frac{\rho_s + \rho_w - n(\rho_s - \rho_w)}{\rho_s + \rho_w + n(\rho_s - \rho_w)} \quad (3)$$

where ρ_s is the density of the solid particles (ρ_s =2636 kg/m^3 in this study), ρ_w is the density of water (ρ_w =998 kg/m^3 at 22^{o}C), and n is the soil porosity. It is assumed that the viscous layer of pore water is only removed during drainage when all the external water has been pushed out, and during imbibition, formation of the viscous layer occurs first. During the test when $\rho_{eff,max} \leq \rho_t$, full mass coupling occurs corresponding to the full thickness of the viscous boundary layer and thus $\rho_{eff} = \rho_{eff,max}$. However, when $\rho_{eff,max} > \rho_t$, partial mass coupling occurs corresponding to thinner thickness of the viscous boundary layer and thus $\rho_{eff} = \rho_t$. Therefore, ρ_{eff} can always be taken as the minimum of $\rho_{eff,max}$ (Eq. (3)) and the corresponding ρ_t (measured during the test) for both saturated and unsaturated conditions.

Sample preparation and calculating the initial physical properties

A mixture of 75% fine sand and 25% kaolin clay was used for the soil sample, and its particle size distribution curve is shown in Fig. 3. Laboratory tests provided the specific gravity, plastic limit, and liquid limit of 2.636, 8.9%, and 18.7% of the soil sample, respectively. According to the Unified Soil Classification System the soil is classified as clayey sand (SC). While the sample was being prepared, the dry sand and kaolin were mixed with a spatula until it became homogeneous, and then a predetermined amount of distilled water was gradually sprayed into the mixture. The final mix of fine sand, kaolin, and water was then stored in an air-tight plastic bag, and left for 24 hours for moisture stabilization at a room temperature of 21.5^{o}C. The moisture content of the soil was determined before the mix was placed into the unsaturated Rowe cell. The initial mass of soil placed in the Rowe cell was also measured, and since its initial volume was known (i.e. 31 mm in height and 100 mm in diameter), the initial density and unit weight could be readily calculated. From three known quantities, including the gravimetric water content, specific gravity and unit weight, every other physical property at the initial state, such as void ratio and degree of saturation could be determined.

Procedure for unsaturated tests

Before applying a vertical net stress (i.e. $\sigma_{vn}=p_u-u_a$) and matric suction, the sample was saturated by simultaneously increasing the back pressure (p_b) and upper chamber pressure (p_u) so that the vertical effective stress remained constant at 50 kPa until the Skempton's pore pressure parameter (*B-value*) of 0.95 or a greater value was obtained. The initial vertical net stress of 20 kPa and initial matric suction of 30 kPa were chosen so that the corresponding vertical effective stress remained unchanged in comparison to the stress state at the end of the saturation stage. The initial matric suction was also expected to be smaller than the air entry value (*AEV*) of the sample to obtain the first drying *SWCC* from a saturated state. In each stage the change in pore volume (i.e., the change in volume of the upper chamber), the volumetric change of pore water (i.e., the change of back volume), and the axial displacement were measured every 10 seconds. The shear wave velocity was evaluated using the built-in bender elements to monitor the development of G_{max} due to changes in the matric suction and the vertical net stress, as presented in Table 1.

In a suction-control apparatus, an application of a matric suction on the soil sample is accomplished by imposing predetermined values of pore air and pore water pressures at the top and bottom of the soil sample. The determination of the state at which the suction reaches equalisation along the soil sample is of importance in computing the volume changes of pore water and voids during a drying or wetting process. However, there is still not a unified criterion to determine this state. Some researchers assumed the stabilization of matric suction based on the change in the volume of pore water when the rate of water content change becomes less than 0.04 %/day (Mancuso et al., 2002; Rampino et al., 1999; Sivakumar, 1993), or the specific water volume change rate drops below 0.001/day (Wheeler and Sivakumar, 1995). In order to avoid effect of temperature fluctuations on the measurement of water volume, some researchers proposed criteria for matric suction equalisation based on the volumetric strain rate computed from the reading of a vertical displacement gauge with the limit of 0.025%/day (Romero et al., 2003). In the experiments conducted in this study using the suction-control Rowe cell apparatus, the presence of the top bender element prevents the use of a stiff porous stone, thus the volumetric strain rate could not be obtained from the change of vertical displacement. Matric suction equalisation was assumed to be reached when the water content change and the volumetric strain rates became less than 0.04%/day and 0.025%/day, respectively. Unchanged shear wave velocity over a day was also considered as an additional criterion. The effect of temperature fluctuations on the measurement of pore water volume was diminished by taking the readings at the same room temperature. It was observed that the determination of matric suction equalisation was governed by the criterion used for rate of water content change in the current experiments. It should be noted that the time required for matric suction equalisation varied from 3 to 10 days and increased with increasing matric suction.

In this test, the upper chamber pressure (p_u) remained constant, while the back pressure (p_b) and pore air pressure (p_a) were changed to control the vertical net stress and matric suction. Therefore, only p_b changed in the drying-wetting cycles, but in the loading-unloading cycle of vertical net stress both p_b and p_a were altered.

Results and Discussion

Variation of void ratio and SWCC during drying-wetting cycles

Changes in the void ratio had a direct effect on the stiffness and physical properties of soil such as its porosity, density, and degree of saturation. Variations in the void ratio due to variations in the vertical net stress and/or matric suction can be assessed using the unsaturated Rowe cell setup. Fig. 4 shows the variations in the void ratio (e) with matric suction (ψ) during the three drying-wetting cycles measured in this study**.** The irreversible reductions in the void ratio during the drying-wetting cycles revealed the plastic volume changes, such that the first drying caused a 3.23% reduction in the void ratio, whereas in the following wetting, the swellings were negligible, and there was a further reduction in volume (collapse) when the matric suction decreased to small values. Put simply, an 0.3% reduction in the void ratio occurred in the first wetting, whereas the induced drying (wetting) reductions in the void ratio in the second and the third cycles were 0.73% (0.43%) and 1.53% (0.2%), respectively. The reduction in the void ratio in the third cycle was higher than the second cycle probably due to previous collapses that rearranged the particles and facilitated compression. Clearly, apart from the plastic strain that accumulated within the drying-wetting cycles, collapse also contributed to the irreversible volumetric compressions.

The soil water characteristic curve (*SWCC*) can generally be presented in terms of the gravimetric water content (ψ-w), the volumetric water content (ψ-θ), or the degree of saturation (ψ-S_r). Since the gravimetric and volumetric water contents of deformable soils may alter as the matric suction changes, even though the soils are still saturated, ψ-S_r is a better way to estimate the *AEV* (Fredlund and Houston 2013; Pasha et al. 2015). The variations of overall and pore water volumes were taken as variations of the upper chamber and back volumes, and since their values at the saturated state were known, these volumes and the corresponding degree of saturation could be computed at any stage of the test.

The semi-log plots of S_r-ψ shown at different drying-wetting cycles in Fig. 5 indicate that the degree of saturation in wetting at a given matric suction is constantly lower than that in the previous drying process. In fact, comparisons of the drying curves show a tendency to decrease in degrees of saturation when the number of cycles increases, especially between the first and second cycles; a similar tendency occurred in the wetting curves.

Ma et al. (2015) investigated the *SWCC* of unsaturated silt during drying-wetting cycles. It was reported that effects of the stress state on the *SWCC* are only attributed to changes of the void ratio. However, as can be observed in Fig. 6, the degree of saturation at the end of a drying-wetting cycle was smaller than the corresponding value in the previous cycle at the same matric suction (i.e. 30 kPa), even though the void ratio decreased. The phenomenon can be explained by possible alterations of pore network induced by drying, wetting processes as observed in study by Nowamooz et al. (2016) which might be insignificant for silt used in the experiments reported by Ma et al. (2015).

Variations of effective dynamic mass density during drying-wetting cycles

Variations of effective dynamic mass density (ρ_{eff}) with matric suction (ψ) during three drying-wetting cycles are depicted in Fig. 7. Persistent increases of ρ_{eff} at low matric suctions reached an upper bound in the middle range of matric suction before beginning to fall at higher matric suctions, but as the void ratio and degree of saturation decreased during drying (see Figs. 4 and 5), the void ratio dominates at low matric suctions and the degree of saturation dominates at higher matric suctions. However, Fig. 7 shows that a reduction of the matric suction during wetting caused a growth of ρ_{eff}.

The magnitude of effective dynamic mass density, ρ_{eff}, directly depends on the void ratio and the water content, changing considerably during the drying-wetting cycles, as reported in Figs. 4 and 5. Since mass density of soil is inversely proportional to the specific volume $(1+e)$, the relationship between $\rho_{eff}(1+e)$ and the degree of saturation is shown in Fig. 8 in an attempt to investigate how the degree of saturation affects ρ_{eff} while excluding the void ratio. Obviously, a threshold of the degree of saturation $S_r \approx 0.5$ exists and it divides the range of degree of saturation into high and low ranges. Fig. 8 shows that at the high range ($S_r > 0.5$), variations of the degree of saturation had no or negligible effect on $\rho_{eff}(1+e)$, which indicates that changes of ρ_{eff} should be induced mainly by variations of the void ratio. In other words, the amount of viscous layer coupling with soil particles should be constant regardless of variations in the water content. However, at the low range ($S_r < 0.5$), variations in the degree of saturation contributed directly to changes of $\rho_{eff}(1+e)$, which indicates variations in the thickness of the viscous layers.

Effects of matric suction and degree of saturation on G_{max} during drying-wetting cycles

The three drying-wetting cycles enable the investigation of effects of matric suction as well as degree of saturation on G_{max} as shown in Fig. 9, where $G_{max}/f(e)$ was used instead of G_{max} to exclude the effect of the void ratio, and where $f(e)=1/(0.3+0.7e^2)$ is the void ratio function proposed by Hardin (1978). As expected, $G_{max}/f(e)$ increased with increase in matric suction at a given degree of saturation. According to Wheeler et al. (2003), and Ng and Zhou (2014), an increase in the matric suction within the free water and meniscus water (i.e. pore water forms between the particles as capillary bridges) will generate extra normal inter-particle forces which enhance the soil mechanical properties. Therefore, the soil stiffness should be proportional to matric suction for the full range of degree of saturation, except for the dried state.

Furthermore, matric suction generates inter-particle forces through the contact area between pore water and particle surfaces (Alonso et al. 2010; Han and Vanapalli 2016; Vanapalli et al. 1996), and since the total contact area between the pore water and particle surfaces is proportional to the amount of water phase, the enhanced mechanical properties of soil induced by matric suction are proportional to the degree of saturation (Alonso et al. 2010; Gray and Schrefler 2001; Öberg and Sällfors 1997). However, Fig. 9 shows that at a given matric suction, the values of $G_{max}/f(e)$ are higher at lower degrees of saturation. This indicates there are other influencing factors in addition to void ratio, degree of saturation, and matric suction, whose contributions accelerated as the degree of saturation decreased. Two factors that have minor effects on mechanical properties of saturated soils, but possibly play vital roles in the response of unsaturated soils, are the van der Waals attraction and electric double layer repulsion (Lu and Likos 2006; Santamarina 2002).

Effects of van der Waals attraction and electric double layer repulsion on G_{max} during drying-wetting cycles

The van der Waals attraction (the attractive intermolecular force) and electric double layer repulsion (the repulsive force that exist among the cations in the electric double layers at the soil/water interface) between two spheres are estimated using Eq (4) and Eq (5), respectively (Israelachvili 2011):

$$F_{vdW} = \frac{A}{6D^2}\frac{(R_1 R_2)}{(R_1 + R_2)} \quad (4)$$

$$F_{ddl} = \kappa Z e^{-\kappa D}\frac{(R_1 R_2)}{(R_1 + R_2)} \quad (5)$$

where $R_1$and R_2 are radii of the two spheres, D is the separation between particle surfaces, A is the Hamaker constant (A is proportional to the density of interacting particles and depends on the medium between the particle surfaces), Z is the interaction constant, which is a function of the electrolyte valency and the properties of particle surfaces, and κ^{-1} is the Debye length, which is a function of the solution and on the geometry and separation. As the van der Waals attraction follows a power-law decay with separation and the electric double layer repulsion decreases exponentially, the former decays slower at short distances but faster at larger distances than the latter. According to Tan (2010) the attractive forces dominate between particles at distances smaller than 20 Å, whereas the repulsive forces dominate for particles at lager distances. Since the porosity and pore size distribution determine the distance between particles in a soil body, an increase in porosity would enhance the repulsive force and diminish the attractive force.

Even though the two forces are proportional to particle diameter as showed in Eq (4) and Eq (5), the corresponding inter-particle stresses generated within a soil body were reported to inversely proportional to particle diameter as showed in Fig. 10a (Ingles 1962). The inter-particle stress generated is significant for clay particle while it is minor for silt and especially for sand particles. Hence, effect of the two forces on the mechanical properties of soils would be proportional to the proportion of clay particle.

Apart from the clay content and porosity, variations in the degree of saturation also influence the van der Waals and electric double layer forces. Fig. 10b illustrates the dependence of inter-particle stresses, generated by the two forces, on the degree of saturation. According to Visser (1995), the van der Waals attraction between clay (hectorite) particles in water is greatly smaller than in air with the Hamaker constant of 0.49×10^{-20} J in water and 7.4×10^{-20} J in air.

At the saturated state, water is the only medium existing among the particles; this corresponds to the thickest electric double layers and the minimum value of the Hamaker constant, and therefore inter-particle stresses induced by the van der Waals attraction and electric double layer repulsion reach their lower and upper bound values, respectively, as reported by Lu and Likos (2006). Effect of the attractive force on the mechanical properties of soil might dominate in case separations shorter than 20 Å hold the highest fraction (e.g. extremely dense saturated soils), whereas in most other cases effect of the repulsive forces dominate and lead to almost no attraction or even repulsive inter-particle stresses that adversely influence the G_{max} of saturated soils.

During the drying process, the water phase decreased and the air phase increased, which led to the evolution of van der Waals attraction-induced inter-particle stress and degradation of electric double layer repulsion-induced inter-particle stress; this facilitates the aggregation of fine particles and growth in the combined contribution of the two forces on G_{max}. This phenomenon continues until the dry state is reached where the absence of water phase denotes the disappearance of electric double layer repulsion-induced inter-particle stress and the upper bound of van der Waals attraction-induced inter-particle stress. An opposing phenomenon occurs during the wetting process as water is added, cations in the electric double layers are hydrated and the electric double layers become thicker; these actions reduce the van der Waals attractions and increase the electric double layer repulsions. Therefore, the overall contribution of the forces induces a decline of G_{max} with increasing degrees of saturation.

By considering the combined effect of van der Waals attractions and electric double layer repulsions on G_{max} during drying and wetting processes, rises of $G_{max}/f(e)$ with decreasing degrees of saturation at constant matric suctions during three drying-wetting cycles shown in Fig. 9 can now be explained. A reduction in the degree of saturation led to a decline in the inter-particle stresses induced by matric suction, but it also led to an increase in the resultant attractive stresses induced by van der Waals attractions and electric double layer repulsions. The result is that G_{max} for a given stress state and void ratio experienced growth instead of a fall with a decrease in the degree of saturation.

Effects of drying-wetting cycles on G_{max}

The responses of G_{max} during three drying-wetting cycles are shown in Fig. 11. Similar to observations reported in the literature, G_{max} was always larger during wetting than in the previous drying at the same level of matric suction in the first drying-wetting cycle. This trend also presented itself in the second and third cycles due to lower void ratios and degrees of saturation during the wetting processes, as shown in Fig. 4 and Fig. 5. As expected, G_{max} increased with increasing number of cycles, especially at low range of matric suctions, reflecting the accumulation in irreversible changes of void ratio and degree of saturation. Therefore, the effect that the drying-wetting cycles have on G_{max} can be captured by evaluating effects of the void ratio and degree of saturation.

Variations of void ratio during loading-unloading of net stress

Apart from drying-wetting cycles, irreversible compressions also occurred in the loading-unloading cycle of vertical net stress, as shown in Fig. 12. This observation demonstrated similar trends in the variation of void ratio with net stress as those in literature (Alonso et al. 2005; Ho et al. 1992). It is therefore expected that due to irreversible reductions of the void ratio, the soil sample also experienced hydraulic hysteresis during the loading-unloading cycle of net tress.

Hydraulic hysteresis behaviour during loading-unloading of net stress

While investigating hydraulic hysteresis in terms of the degree of saturation, it is of importance to consider variations of void ratio and gravimetric water content. As depicted in Fig. 13**,** during the loading stage, the void ratio dominates with consistent increases in the degree of saturation even though the gravimetric water content consistently decreased. However, during unloading, the gravimetric water content was more dominant because the degree of saturation continued to grow. Nonetheless, the plastic volumetric strain that occurred in the loading-unloading cycle of vertical net stress (see Fig. 12), contributed to the differences between degrees of saturation for a given vertical net stress, as shown in Fig. 13.

Observations of hydraulic hysteresis under a loading-unloading cycle of net stress at constant suctions were reported by Zakaria (1995) and Gallipoli et al. (2003) for compacted speswhite kaolin tested in suction-controlled triaxial tests, and by Pham et al. (2004) for slurry processed silt in pressure plate tests. In these tests, the loading/unloading stage was conducted by increasing/decreasing the external stress (i.e. cell pressure in triaxial tests and vertical pressure for the pressure plate tests), while p_a and p_b remained unchanged. The degree of saturation of speswhite kaolin and processed silt increased considerably in the loading stage but not as much in the unloading stages, whereas the gravimetric water content decreased in the loading stages and increased in the unloading stages. The hydraulic hysteresis that occurred in the loading-unloading cycle of net stress probably resulted from the plastic volumetric strains that accumulated within the cycle, which caused irreversible changes in the pore size distribution and corresponding soil water retention, even at a constant suction. Farulla and Rosone (2012) and Rostami et al. (2013) reported that in the loading/unloading stages of net stress, the micro-pore and macro-pore volumes altered, but most changes occurred in the latter.

Variations of effective dynamic mass density during loading-unloading of net stress

Fig. 14 shows that ρ_{eff} grew consistently with the vertical net stress, but at reducing rates during loading. The opposite trend occurred during unloading where ρ_{eff} decreased at an increasing rate. The effective dynamic mass density is a function of the void ratio and gravimetric water content, so the relationship between ρ_{eff} and vertical net stress revealed that the void ratio dominated over the gravimetric water content at the stress levels investigated. Variations of ρ_{eff} (Fig. 14) were obviously associated with changes in the void ratio during loading and unloading (Fig. 12).

Variations of G_{max} during loading-unloading of net stress

Fig. 15 shows the variations of G_{max} with vertical net stress at a constant matric suction; here G_{max} kept increasing with the vertical net stress during loading, and during the corresponding unloading G_{max} was less for a given vertical net stress. This response of G_{max} is the opposite of that reported by other researchers (e.g. Fioravante et al. 2013 and Zeng & Ni 1998) for saturated soils where G_{max} during unloading was higher than during loading at a given stress. For a saturated soil, at a given stress level, G_{max} depends reversely on the void ratio (Hardin 1978; Richart et al. 1970; Vucetic and Dobry 1991), therefor, the plastic volume change accumulated during a loading-unloading cycle results in higher values of G_{max} during unloading.

For an unsaturated soil, the response of G_{max} during a loop of loading and unloading of net stress depends on variations of both the void ratio and degree of saturation which follow opposite trends as shown in Fig. 12 and Fig. 13. Hence, G_{max} during an unloading process might have higher or lower values than in the previous loading process depending on which effect (i.e. reduction in void ratio and increase in degree of saturation) outweighs the other. That explains different responses of G_{max} during the unloading process between the soil investigated in this study and Zenoz kaolin reported by Biglari et al. (2012). After a stress loop, it was reported that the decrease in void ratio and increase in degree of saturation for Zenoz kaolin were 20% and 24.3%, and for the soil in this study were 6.3% and 10.6%, respectively. It is rational to believe that for Zenoz kaolin, the large decrease of void ratio (i.e. 20%) led to the dominant effect of void ratio over the effect of degree of saturation on G_{max}; on the contrary, for the soil investigated in this study, effects of degree of saturation outweighed influence of the void ratio (small decrease of 6.3%) on G_{max}.

Fig. 16 depicts the variations of G_{max} with void ratio and degree of saturation in the loading-unloading cycle investigated. The decrease in G_{max} due to a decrease in the void ratio and an increase in the degree of saturation, as shown in Fig. 16, indicates how the degree of saturation affected G_{max} at the stress levels investigated. Under a constant matric suction $\psi = 30$ kPa, and at a vertical net stress $\sigma_{vn} = 100$ kPa, after a stress loop, G_{max} decreased 17.5% (from 218MPa to 180MPa) when the corresponding void ratio decreased by 6.2%, and corresponding degree of saturation increased by 16.2%. At higher vertical net stresses (i.e. 200 kPa and 400 kPa), after a stress loop, variations in the void ratio, degree of saturation and resulting G_{max} were smaller.

To investigate effect that the degree of saturation has on G_{max} while excluding the influence of void ratio, variations of $G_{max}/f(e)$ with vertical net stress and degree of saturation during the loading-unloading cycle are depicted in Fig. 17. Under a constant matric suction $\psi = 30$ kPa, at a given vertical net stress and after a stress loop of the loading-unloading cycle, $G_{max}/f(e)$

decreased as the degree of saturation increased. The highest reduction of $G_{max}/f(e)$ of 19.2% occurred at vertical net stresses of 100 kPa; this corresponds to an increase of 16.2% in the degree of saturation.

As discussed earlier, an increase in the degree of saturation leads to a rise in the contribution of the matric suction investigated (due to an increase in the contact area between pore water and soil particles) and a fall in the combined effects of van der Waals attractions and electric double layer repulsions (due to a decrease in the resultant attractive inter-particle forces). Fig. 17 shows that the latter had a greater effect during the unloading stage so the effects of the van der Waals attractions and electric double layer repulsions should be captured in order to predict the G_{max} in partially saturated soils.

Relationship for G_{max} Incorporating the Hydraulic Hysteresis Effect

Model establishment

The relationship developed to predict G_{max} for reconstituted unsaturated soils in this study was inspired by the Hardin-style equations for saturated soils (Hardin 1978; Hardin and Blandford 1989) where the void ratio function is used as an alternative to the overconsolidation ratio as shown in Eq. (6):

$$\frac{G_{max}}{p_r} = f(e)a\left(\frac{p'}{p_r}\right)^n \qquad \text{for saturated soil} \qquad (6)$$

where a and n are material constants; p' is the mean confining effective stress; $f(e)$ is the void ratio function presenting the properties of packing and density, $f(e)=1/(0.3+0.7e^2)$ is for saturated sands and clays (Hardin 1978), and p_r is the reference pressure (atmospheric pressure).

The inter-particle forces for unsaturated soils are attributed to the net stress, matric suction, van der Waals attractions, and electric double layer repulsions. It is believed that the contributions of a given net stress to the mechanical properties of unsaturated soil, including the small strain shear stiffness, are independent of the water content (Bishop 1959; Khalili et al. 2004; Khalili and Khabbaz 1998; Lu and Likos 2006; Vanapalli et al. 1996), whereas contributions from a given matric suction are proportional to the degree of saturation (Alonso et al. 2010; Gray and Schrefler 2001; Öberg and Sällfors 1997). Thus, it is recommended to evaluate the effects of variations in net stress and matric suction on mechanical properties of unsaturated soils separately (Fredlund et al. 1993; Ho and Fatahi 2015; Ho and Fatahi 2016; Ho et al. 2016). In addition, Figs. 9 and 17 show that at a given void ratio, net stress, and matric suction, an increase in the degree of saturation led to a decrease in G_{max} despite of an increase in the contribution of matric suction. This indicates a decrease of the combined contribution of van der Waals attractions and electric double layer repulsions that dominates over the contribution of matric suction at the stress states investigated. Therefore, an equation to predict G_{max} while incorporating all the above mentioned contributions is proposed in this study.

The proposed Eq. (7) indicates that the contribution of net stress follows a power law that is similar to saturated soils, and is independent of the degree of saturation. Effects of degree of saturation on G_{max} are evaluated in the contribution of matric suction and the combined contribution of van der Waals attractions and electric double layer repulsions. It is noted that the two contributions follow different patterns with variation of degree of saturation. During a drying process beginning from saturated state, the combined contribution of van der Waals attractions and electric double layer repulsions increases persistently with decreasing degree of saturation, whereas the contribution of matric suction follows a rapid growth in the early stage due to fast increases of matric suction, before commencing to decrease to zero when approaching the dry state due to the loss of pore water. The reverse patterns occur during a wetting process. In addition, at the same stress state, while the former is proportional to the degree of saturation, the latter is reversely proportional to the degree of saturation, as Figs. 9 and 17 show. Thus, the two contributions need to be evaluated separately to capture hysteresis. In Eq. (7), variations of the two contributions with degree of saturation are captured through parameters m and k, while parameters b and c are for evaluating their magnitudes. The proposed equation can be presented as follows:

$$G_{max} = p_r f(e)\left[a\left(\frac{\sigma_n}{p_r}\right)^n + b\left(\frac{\psi S_r}{p_r}\right)^m + c(1-S_r)^k\right] \quad (7)$$

where a, n, b, m, c and k are the material constants, p_r = 100 kPa is the reference pressure, $\sigma_n=p-u_a$ is the net stress, $\psi=u_a-u_w$ is the matric suction, and S_r is the degree of saturation. All the material constants can be determined using the nonlinear least-square data fitting method with at least six observations. At saturated state, when $u_a=u_w$, $\psi=0$, and S_r=1, Eq. (7) returns to Eq. (6) which is for saturated soils.

Model calibration

Six model parameters (i.e. a, n, b, m, c and k) were determined by using the least-squares data fitting method (Coleman and Li 1996; Marquardt 1963). This determination was based on the values of two independent variables (i.e. the net stress and the matric suction) and two dependent variables (i.e. the void ratio and the degree of saturation) during the three drying-wetting cycles and the loading-unloading cycle of net stress. The objective function $y(x)$ of the least squares problems is the sum of the squares of residuals that represent the absolute differences between the measured and corresponding computed values of G_{max}. By solving the minimisation problem presented in Eq (8), the model parameters which are variables of the function could be obtained:

$$min_x y(x) = min_x \sum_{i=1}^{j} (G_{maxi} - G_{maxi}(x))^2 \quad (8)$$

where x =[x_1, x_2, x_3, x_4, x_5, x_6] = [a, n, b, m, c, k] is the vector of the model parameters, G_{maxi} and $G_{maxi}(x)$ are the i^{th} measurement and corresponding prediction of G_{max}, and j is the number of observation/measurement points.

This mathematical model is nonlinear in terms of the parameters n, m, and k, and therefore determining the model parameters requires a set of initial values and an effective algorithm for iterations. The trust-region reflective optimisation algorithm utilised in this study has strong convergence properties proven to provide reliable and robust solutions (Yuan 2000). One advantage of the trust-region reflective algorithm is its ability to reduce the number of iterations (Coleman and Li 1996). This algorithm was coded in the MATLAB software package using the "lsqnonlin" function. Table 2 is a summary of the calibrated model parameters.

In order to assess reliability of the newly proposed models, the coefficient of determination R^2 is widely used in the literature. However, R^2 tends to increase with the addition of a new variable and its accompanying parameters into the model, even if the new variable is not very relevant (Cornell and Berger 1987; Craven and Islam 2011). Hence, it is recommended to use the adjusted coefficient of determination, $\mathbf{R^2_{adj}}$, which is a modified version of R^2 to account for the number of model parameters and the number of observations, along with R^2 to measure the reliability of models (Cornell and Berger 1987; Ostertagová 2012; Rossiter 2009). The $\mathbf{R^2_{adj}}$ will increase if the new variable improves the model quality and decrease if the new variable is not relevant. Thus, in this study, $\mathbf{R^2_{adj}}$ was used in assessing the reliability of the proposed model in fitting data measured along with evaluating the agreement between predicted and measured values of G_{max}.

As shown in Figs. 18a-18d, the predicted G_{max} values adopting the calibrated model parameters were in good agreement with the measurements during the three drying-wetting cycles and the loading-unloading cycle of vertical net stress. The highest $\mathbf{R^2_{adj}}$ of 0.97 was obtained for fitting data measured from the first drying-wetting cycle with 16 observations. Even at the numbers of observations slightly larger than the number of parameters in the third drying-wetting cycle and the loading-unloading cycle of vertical net stress, high values of $\mathbf{R^2_{adj}}$ i.e. 0.93 and 0.85 were obtained, respectively.

5.1. Model verification via laboratory tests with complex loading paths

To verify the proposed model for the investigated soil, further experiments were conducted in the laboratory considering three mixed applications of matric suction and net stress, and a drying process as demonstrated in Table 1. Variations of the void ratio and degree of saturation with matric suction and net stress are shown in Figs. 19a and 19b. During the mix applications, responses of the void ratio and degree of saturation reveal different effects of the two stress state variables. During the stress loop with changes of 200 kPa of both matric suction and vertical net stress, while the response of void ratio shows dominant effect of net stress over matric suction as reported by Bagherieh et al. (2009) and Mašín (2010), response of degree of saturation shows dominant effect of matric suction. As a result, G_{max} was observed to vary in accordance with variations in matric suction instead of vertical net stress (see Fig. 20). The observation confirms the finding of Leong and Cheng (2016) that matric suction can induce greater influence on shear wave velocity in comparison with an equal-magnitude isotropic effective stress, and can be explained by the dominant effect of the degree of saturation over the void ratio. During the drying process, the void ratio, the degree of saturation and G_{max} varied similarly to what observed in the other drying processes for the soil investigated. By adopting the calibrated model parameters reported in Table 2, good agreements between the measured data and the predicted results were obtained, as can clearly be observed in Fig. 20. The measured and predicted values of G_{max} for the soil sample, investigated in this study, are shown in Fig. 21.

5.2. Further verification using data available in literature

The proposed equation was then validated based on data available in literature reported by Biglari et al. (2012), Khosravi (2011) and Ng et al. (2009) for three types of soils, including Zenoz clay, statically compacted Bonny silt, and recompacted completely decomposed granite, respectively. Variations of the void ratio and degree of saturation under different stages of stress were reported for these soils in the above mentioned references.

Table 3 summarises the optimised model parameters following the procedure reported in the previous section. Fig. 22 shows the measured and predicted values of G_{max} for these reported studies. There were good agreements between the measurements and predictions of G_{max} for different types of soil during drying-wetting cycles and loading-unloading stages of net stress, confirming the ability of the proposed model to capture the response of G_{max} while incorporating hydraulic hysteresis.

1. Conclusions

A suction-control Rowe cell apparatus with two built-in bender elements at the top and bottom of the sample was developed to measure the shear wave velocities and corresponding small strain shear modulus (G_{max}) of an unsaturated soil. In this study, the soil sample was subjected to different types of stress loops in a multi-stage test including three drying-wetting cycles and a loading-unloading cycle of vertical net stress, allowing the investigation of effects of matric suction and degree of saturation on G_{max} simultaneously.

To avoid overestimation of G_{max} determination from shear wave velocity, the effective dynamic mass density (ρ_{eff}) was used instead of the total mass density. It was assumed that during drying processes, the thin viscous layers of pore water moving together with particle surfaces during shear wave propagation reduces only when all the external water has been removed, while the reverse procedure occurs during wetting processes. Therefore, the effective dynamic mass density was determined as the minimum of the total mass density and the maximum effective dynamic mass density, which corresponds to the full thickness of the viscous boundary layer. Investigation of the relationship between $\rho_{eff}(1+e)$ and degree of saturation during the three drying-wetting cycles shows a threshold of degree of saturation that separates the low range corresponding to partial mass coupling and the high range corresponding to full mass coupling. At a given void ratio, ρ_{eff} is proportional to the degree of saturation only at the low range while at the high range degree of saturation has no or minor effect on ρ_{eff}.

G_{max} was observed to reflect the irreversible changes of void ratio and degree of saturation that occurred in all the stress loops investigated, especially during the drying-wetting cycles where these changes accumulated with increasing number of cycles. It is noted that the hydraulic hysteresis had a huge effect on the measured G_{max}. The growth of $G_{max}/f(e)$ resulting from decreases of degree of saturation at a given stress state, during cycles of drying-wetting and loading-unloading of net stress, is clear evidence for the impact of van der Waals attractions and electric double layer repulsions on G_{max} of unsaturated soils. Thus, when hydraulic hysteresis presents, a decrease in the degree of saturation leads to a decrease in the contribution of matric suction and an increase in the combined contribution of van der Waals attractions and electric double layer repulsions to G_{max}. Indeed, as these two contributions follow different patterns with variations of degree of saturation, they need to be evaluated separately to capture effect of hydraulic hysteresis on G_{max}, An equation has been proposed for predicting G_{max} of unsaturated soils incorporating the effects of the void ratio, net stress, matric suction and degree of saturation. This equation was verified for the investigated soil in further experiments including a stress loop with mixed variations of matric suction and net stress, and a drying process. Apart from a good agreement between measurements and predictions for the entire data set, the proposed equation successfully captures the greater effect induced by matric suction compared to an equal-magnitude net stress during the stress loop. Verifications for measurements available in literature were also conducted for different soils in different testing conditions during cycles of drying-wetting and loading-unloading. The proposed equation provided good predictions proving its ability to capture the effect of hydraulic hysteresis on G_{max} of unsaturated soils. For practical issues, this equation allows the determination of G_{max} of unsaturated soils based on the stress state, void ratio, and degree of saturation regardless of the stress and drying-wetting history, thus improving the accuracy of capturing response of G_{max} in complex loading-unloading and drying-wetting cycles.

Notation

The following symbols are used in this paper:

a	Material constant
b	Material constant
c	Material constant
e	Void ratio
$f(e)$	Void ratio function
G_{max}	Maximum small strain shear modulus
k	Material constant
L	Propagation distance

m Material constant

n Material constant

OCR Over consolidation ratio

p_a Air pressure

p_b Back water pressure

p_r Reference pressure

p_u Upper chamber pressure

p' Mean effective stress

S_r Degree of saturation

t Propagation time

u_a Pore air pressure

u_w Pore water pressure

V_s Shear wave velocity

σ_n Net stress

σ_v Vertical stress

σ_{vn} Vertical net stress

ρ_{eff} Effective dynamic mass density

ψ Matric suction

AEV Air entry value

$HAEPD$ High air entry value porous disk

$LVDT$ Linear variable differential transformer

$SWCC$ Soil water characteristic curve

References

Alonso, E., Romero, E., Hoffmann, C., and García-Escudero, E. (2005). "Expansive bentonite–sand mixtures in cyclic controlled-suction drying and wetting." *Engineering geology*, 81(3), 213-226.

Alonso, E. E., Pereira, J.-M., Vaunat, J., and Olivella, S. (2010). "A microstructurally based effective stress for unsaturated soils." *Géotechnique*, 60(12), 913-925.

Arroyo, M. (2001). "Pulse tests in soil samples." PhD Thesis, University of Bristol, UK.

Arulnathan, R., Boulanger, R. W., and Riemer, M. F. (1998). "Analysis of bender element tests." *Geotechnical Testing Journal*, 21(2), 120-131.

Bagherieh, A., Khalili, N., Habibagahi, G., and Ghahramani, A. (2009). "Drying response and effective stress in a double porosity aggregated soil." *Engineering Geology*, 105(1-2), 44-50.

Biglari, M., d'Onofrio, A., Mancuso, C., Jafari, M. K., Shafiee, A., and Ashayeri, I. (2012). "Small-strain stiffness of Zenoz kaolin in unsaturated conditions." *Canadian Geotechnical Journal*, 49(3), 311-322.

Biot, M. A. (1956). "Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range." *The Journal of the acoustical Society of america*, 28(2), 168-178.

Bishop, A. W. (1959). "The principle of effective stress." *Teknisk ukeblad*, 39, 859-863.

Camacho-Tauta, J., Cascante, G., Viana da Fonseca, A., and Santos, J. (2015). "Time and frequency domain evaluation of bender element systems." *Géotechnique*, 65(7), 548-562.

Cho, G. C., and Santamarina, J. C. (2001). "Unsaturated particulate materials—particle-level studies." *Journal of geotechnical and geoenvironmental engineering*, 127(1), 84-96.

Coleman, T. F., and Li, Y. (1996). "A reflective Newton method for minimizing a quadratic function subject to bounds on some of the variables." *SIAM Journal on Optimization*, 6(4), 1040-1058.

Cornell, J., and Berger, R. (1987). "Factors that influence the value of the coefficient of determination in simple linear and nonlinear regression models." *Phytopathology*, 77(1), 63-70.

Craven, B., and Islam, S. (2011). *Ordinary least-squares regression*, Sage Publications.

Dong, Y., and Lu, N. (2016). "Dependencies of Shear Wave Velocity and Shear Modulus of Soil on Saturation." *Journal of Engineering Mechanics*, 142(11), 04016083.

Farulla, C. A., and Rosone, M. "Microstructure Characteristics of Unsaturated Compacted Scaly Clay." *In* Unsaturated Soils: Research and Applications. *Edited by* C. Mancuso, C. Jommi, and F. D'Onza. Springer Berlin Heidelberg. 123-130.

Fioravante, V., Giretti, D., and Jamiolkowski, M. (2013). "Small strain stiffness of carbonate Kenya Sand." *Engineering Geology*, 161, 65-80.

Fredlund, D., and Houston, S. "Interpretation of soil-water characteristic curves when volume change occurs as soil suction is changed." *Proc., The 1st Pan-American conference on unsaturated soils*, 15-31.

Fredlund, D. G., Rahardjo, H., and Rahardjo, H. (1993). *Soil mechanics for unsaturated soils*, John Wiley & Sons.

Gallipoli, D., Gens, A., Sharma, R., and Vaunat, J. (2003). "An elasto-plastic model for unsaturated soil incorporating the effects of suction and degree of saturation on mechanical behaviour." *Géotechnique.*, 53(1), 123-136.

Gray, W. G., and Schrefler, B. A. (2001). "Thermodynamic approach to effective stress in partially saturated porous media." *European Journal of Mechanics-A/Solids*, 20(4), 521-538.

Greening, P. D., and Nash, D. F. (2004). "Frequency domain determination of G 0 using bender elements." *Geotechnical Testing Journal*, 27(3), 288-294.

Han, Z., and Vanapalli, S. (2016). "Stiffness and shear strength of unsaturated soils in relation to soil-water characteristic curve." *Géotechnique*, 66(8), 627-647.

Hardin, B. O. "The nature of stress-strain behavior for soils." *Proc., ASCE Geotechnical Engineering Division Specialty Conference on Earthquake Engineering and Soil Dynamics*, 3-39.

Hardin, B. O., and Blandford, G. E. (1989). "Elasticity of particulate materials." *Journal of Geotechnical Engineering*, 115(6), 788-805.

Hilf, J. W. (1956). *An Investigation of Pore-water Pressure in Compacted Cohesive Soils*, United States. Bureau of Reclamation. Design and Construction Division. Tech. Memo. 654.

Ho, D., Fredlund, D., and Rahardjo, H. (1992). "Volume change indices during loading and unloading of an unsaturated soil." *Canadian Geotechnical Journal*, 29(2), 195-207.

Ho, L., and Fatahi, B. (2015). "One-dimensional consolidation analysis of unsaturated soils subjected to time-dependent loading." *International Journal of Geomechanics*, 16(2), 04015052.

Ho, L., and Fatahi, B. (2016). "Axisymmetric consolidation in unsaturated soil deposit subjected to time-dependent loadings." *International Journal of Geomechanics*, 17(2), 04016046.

Ho, L., Fatahi, B., and Khabbaz, H. (2016). "Analytical solution to axisymmetric consolidation in unsaturated soils with linearly depth-dependent initial conditions." *Computers and Geotechnics*, 74, 102-121.

Hoyos, L. R., Suescún-Florez, E. A., and Puppala, A. J. (2015). "Stiffness of intermediate unsaturated soil from simultaneous suction-controlled resonant column and bender element testing." *Engineering Geology*, 188(7), 10-28.

Inci, G., Yesiller, N., and Kagawa, T. (2003). "Experimental investigation of dynamic response of compacted clayey soils." *Geotechnical Testing Journal*, 26(2), 125-141.

Ingles, O. "Bonding forces in soils, Part 3: A theory of tensile strength for stabilized and naturally coherent soils." *Proc., The 1st Conference of the Australian Road Research Board*, 1025-1047.

Israelachvili, J. N. (2011). *Intermolecular and surface forces*, Academic press.

Jamiolkowski, M., Lancellotta, R., DCF, L. P., and Pallara, O. V. "Stiffness of Toyoura sand at small and intermediate strain." *Proc., XIII ICSMFE*, A.A. Balkema. 169-172.

Jung, Y.-H., Finno, R. J., and Cho, W. (2012). "Stress–strain responses of reconstituted and natural compressible Chicago glacial clay." *Engineering geology*, 129-130, 9-19.

Khalili, N., Geiser, F., and Blight, G. (2004). "Effective stress in unsaturated soils: Review with new evidence." *International journal of Geomechanics*, 4(2), 115-126.

Khalili, N., and Khabbaz, M. (1998). "A unique relationship for χ for the determination of the shear strength of unsaturated soils." *Geotechnique*, 48(5), 681-687.

Khosravi, A. (2011). "Small strain shear modulus of unsaturated, compacted soils during hydraulic hysteresis." Doctoral dissertation, University of Colorado at Boulder.

Khosravi, A., Shahbazan, P., and Pak, A. (2018). "Impact of hydraulic hysteresis on the small strain shear modulus of unsaturated sand." *Soils and Foundations*, 58(2), 344-354.

Lee, J.-S., and Santamarina, J. C. (2005). "Bender elements: performance and signal interpretation." *Journal of geotechnical and geoenvironmental engineering*, 131(9), 1063-1070.

Leong, E., and Cheng, Z. (2016). "Effects of confining pressure and degree of saturation on wave velocities of soils." *International Journal of Geomechanics*, 16(6), D4016013.

Leong, E. C., Cahyadi, J., and Rahardjo, H. (2009). "Measuring shear and compression wave velocities of soil using bender–extender elements." *Canadian geotechnical journal*, 46(7), 792-812.

Leong, E. C., Yeo, S. H., and Rahardjo, H. (2005). "Measuring shear wave velocity using bender elements." *Geotechnical Testing Journal*, 28(5), 488-498.

Lu, N., Godt, J. W., and Wu, D. T. (2010). "A closed-form equation for effective stress in unsaturated soil." *Water Resources Research*, 46(5), W05515.

Lu, N., and Likos, W. J. (2006). "Suction stress characteristic curve for unsaturated soil." *Journal of geotechnical and geoenvironmental engineering*, 132(2), 131-142.

Ma, T., Wei, C., Wei, H., and Li, W. (2015). "Hydraulic and mechanical behavior of unsaturated silt: Experimental and theoretical characterization." *International Journal of Geomechanics*, 16(6), D4015007.

Mancuso, C., Vassallo, R., and d'Onofrio, A. (2002). "Small strain behavior of a silty sand in controlled-suction resonant column torsional shear tests." *Canadian Geotechnical Journal*, 39(1), 22-31.

Marquardt, D. W. (1963). "An algorithm for least-squares estimation of nonlinear parameters." *Journal of the society for Industrial and Applied Mathematics*, 11(2), 431-441.

Martin, P., Maurel, A., and Parnell, W. (2010). "Estimating the dynamic effective mass density of random composites." *The Journal of the Acoustical Society of America*, 128(2), 571-577.

Mašín, D. (2010). "Predicting the dependency of a degree of saturation on void ratio and suction using effective stress principle for unsaturated soils." *International Journal for Numerical and Analytical Methods in Geomechanics*, 34(1), 73-90.

Mei, J., Liu, Z., Wen, W., and Sheng, P. (2007). "Effective dynamic mass density of composites." *Physical Review B*, 76(13), 134205.

Ng, C. W. W., Xu, J., and Yung, S. (2009). "Effects of wetting–drying and stress ratio on anisotropic stiffness of an unsaturated soil at very small strains." *Canadian Geotechnical Journal*, 46(9), 1062-1076.

Ng, C. W. W., and Zhou, C. (2014). "Cyclic behaviour of an unsaturated silt at various suctions and temperatures." *Géotechnique*, 64(9), 709-720.

Nowamooz, H., Jahangir, E., Masrouri, F., and Tisot, J.-P. (2016). "Effective stress in swelling soils during wetting drying cycles." *Engineering Geology*, 210, 33-44.

Öberg, A., and Sällfors, G. (1997). "Determination of shear strength parameters of unsaturated silts and sands based on the water retention curve." *Geotechnical Testing Journal*, 20(1), 40-48.

Ogino, T., Kawaguchi, T., Yamashita, S., and Kawajiri, S. (2015). "Measurement deviations for shear wave velocity of bender element test using time domain, cross-correlation, and frequency domain approaches." *Soils and foundations*, 55(2), 329-342.

Ostertagová, E. (2012). "Modelling using polynomial regression." *Procedia Engineering*, 48, 500-506.

Padilla, J., Perera, Y., Houston, W., Perez, N., and Fredlund, D. "Quantification of air diffusion through high air-entry ceramic disks." *Proc., The 4th International Conference on Unsaturated Soils*, 1852-1863.

Pasha, A. Y., Khoshghalb, A., and Khalili, N. (2015). "Pitfalls in interpretation of gravimetric water content–based soil-water characteristic curve for deformable porous media." *International Journal of Geomechanics*, 16(6), D4015004.

Pham, Q., Fredlund, D., and Padilla, J. "Use of the GCTS apparatus for the measurement of soilwater characteristic curves." *Proc., The 57th Canadian Geotechnical Conference*, 1-6.

Qian, X., Gray, D. H., and Woods, R. D. (1993). "Voids and granulometry: effects on shear modulus of unsaturated sands." *Journal of Geotechnical Engineering*, 119(2), 295-314.

Rees, S., Le Compte, A., and Snelling, K. "A new tool for the automated travel time analyses of bender element tests." *Proc., The 18th International Conference on Soil Mechanics and Geotechnical Engineering*, 2843-2846.

Richart, F. E., Hall, J. R., and Woods, R. D. (1970). *Vibrations of soils and foundations*, N.J. Prentice-Hall Englewood Cliffs.

Romero, E., A. Gens, and A. Lloret. 2003. “Suction effects on a compacted clay under non-isothermal conditions.” G eotechnique 53 (1): 65–81. https://doi.org/10.1680/geot.2003.53.1.65.

Rossiter, D. (2009). "Technical note: Curve fitting with the R Environment for Statistical Computing." Enschede (NL): 17, International Institute for Geo-information Science & Earth Observations.

Rostami, A., Habibagahi, G., Ajdari, M., and Nikooee, E. (2013). "Pore network investigation on hysteresis phenomena and influence of stress state on the SWRC." *International Journal of Geomechanics*, 15(5), 04014072.

Sanchez-Salinero, I., Roesset, J. M., Stokoe, I., and Kenneth, H. (1986). "Analytical studies of body wave propagation and attenuation." *Geotechnical Engineering Report GR86-15*, The University of Texas at Austin.

Santagata, M., Germaine, J. T., and Ladd, C. C. (2005). "Factors affecting the initial stiffness of cohesive soils." *Journal of Geotechnical and Geoenvironmental Engineering*, 131(4), 430-441.

Santamarina, J. C. "Soil behavior at the microscale: particle forces." *Proc., The Symposium on Soil Behavior and Soft Ground Construction*, ASCE Geotechnical Special Publication. 25-56.

Sawangsuriya, A., Edil, T., Bosscher, P., and Wang, X. "Small-strain stiffness behavior of unsaturated compacted subgrade." *Proc., The 4th International Conference on Unsaturated Soils 2006*, ASCE, Reston, VA. 1121-1132.

Shibuya, S., Tatsuoka, F., Teachavorasinskun, S., Kong, X. J., Abe, F., Kim, Y., and Park, C. (1992). "Elastic deformation properties of geomaterials." *Soils and Foundations*, 32(3), 26-46.

Sivakumar, V. 1993. “A critical state framework for unsaturated soil.” Ph.D. thesis,Univ. of Sheffield.

Tan, K. H. (2010). *Principles of soil chemistry*, CRC press.

Vanapalli, S., Fredlund, D., Pufahl, D., and Clifton, A. (1996). "Model for the prediction of shear strength with respect to soil suction." *Canadian Geotechnical Journal*, 33(3), 379-392.

Vanapalli, S. K., Nicotera, M., and Sharma, R. S. (2008). "Axis translation and negative water column techniques for suction control." *Geotechnical and Geological Engineering*, 26(6), 645.

Visser, J. (1995). "Particle adhesion and removal: A review." *Particulate science and technology*, 13(3-4), 169-196.

Vucetic, M., and Dobry, R. (1991). "Effect of soil plasticity on cyclic response." *Journal of geotechnical engineering*, 117(1), 89-107.

Wheeler, S., and V. Sivakumar. 1995. “An elasto-plastic critical state framework for unsaturated soil.” *Geotechnique* 45 (1): 35–53. https://doi.org/10.1680/geot.1995.45.1.35.

Wheeler, S., Sharma, R., and Buisson, M. (2003). "Coupling of hydraulic hysteresis and stress–strain behaviour in unsaturated soils." *Géotechnique*, 53(1), 41-54.

Wu, S., Gray, D. H., and Richart Jr, F. (1984). "Capillary effects on dynamic modulus of sands and silts." *Journal of Geotechnical Engineering*, 110(9), 1188-1203.

Wu, Y., Mei, J., and Sheng, P. (2012). "Anisotropic dynamic mass density for fluid–solid composites." *Physica B: Condensed Matter*, 407(20), 4093-4096.

Yuan, Y.-x. "A review of trust region algorithms for optimization." *Proc., The fourth international congress on industrial and applied mathematics*, Oxford University Press. 271-282.

Zakaria, I. (1995). "Yielding of unsaturated soil." PhD thesis, University of Sheffield.

Zeng, X., and Ni, B. (1998). "Application of bender elements in measuring G max of sand under K 0 condition." *Geotechnical Testing Journal*, 21(3), 251-263.

Figure Captions

Fig. 1. Modified Rowe cell apparatus with bender elements: a) schematic diagram of the Rowe cell setup, b) top loading system, and c) bottom pedestal

Fig. 2. Determination method for the shear wave propagation time in the Rowe cell equipped with two bender elements

Fig. 3. Particle size distribution curve for the sand and kaolin mixture

Fig. 4. e–ψ relationship during three drying-wetting cycles

Fig. 5. S_r–ψ (SWCC) in three drying-wetting cycles

Fig. 6. S_r–e relationship during three drying-wetting cycles

Fig. 7. ρ_{eff}–ψ relationship during three drying-wetting cycles

Fig. 8. S_r–$\rho_{eff}(1+e)$ relationship during three drying-wetting cycles

Fig. 9. $G_{max}/f(e)$–S_r relationship during three drying-wetting cycles

Fig. 10. Dependence of inter-particle stress generated by van der Waals and electric double layer forces on a) particle diameter (Ingles 1962); and b) degree of saturation (adapted from Lu and Likos 2006, © ASCE).

Fig. 11. G_{max}–ψ relationship during the three drying-wetting cycles

Fig. 12. σ_{vn}–e relationship in the loading-unloading cycle

Fig. 13. Hydraulic hysteresis in a loading-unloading cycle

Fig. 14. σ_{vn}–ρ_{eff} relationship in a loading-unloading cycle

Fig. 15. σ_{vn}–G_{max} relationship in a loading-unloading cycle

Fig. 16. Variation of G_{max} with e and S_r in loading-unloading cycle

Fig. 17. Variation of $G_{max}/f(e)$ with σ_{vn} and S_r in loading-unloading cycle

Fig. 18. Measured and predicted G_{max} values during the a) 1st, b) 2nd, and c) 3rd drying-wetting cycles, and d) loading-unloading cycle.

Fig. 19. Variations of a) e and b) S_r during model test stages applying different ψ and σ_{vn}.

Fig. 20. Measured and predicted G_{max} values during model variation test stages.

Fig. 21. Entire set of measured and predicted G_{max} values in this study.

Fig. 22. Prediction of G_{max} for a) Bonny silt and b) completely decomposed granite during different drying-wetting cycles against ψ; and c) Zenoz kaolin during a loading-unloading cycle against σ_n.

Table 1. Stress states of the Rowe cell unsaturated test with shear wave velocity measurement

Test stage	Net stress (kPa)	Suction (kPa)
Drying-wetting cycles	20	30, 50, 70, 90, 110, 130, 150, 170, 190, 210, 230, 330, 430, 530, 630, 730
	20	530, 430, 330, 90, 50, 30
	20	90, 630, 730, 830, 930
	20	630, 90, 30
	20	90, 330, 630, 930
	20	630, 330, 30
Loading-unloading cycle of net stress	50, 100, 200, 400, 800	30
	400, 200, 100	30
Verification exercise		
Mixed applications of net stress and matric suction	400	200
	200	400
	400	200
Drying process	5	400, 800, 970

Table 2. Calibrated model parameters for the soil tested in this study

Model parameter	Value
a	296
n	0.4
b	493
m	0.5
c	2307
k	2.7

Table 3. Model parameters for data available in literature

Model parameter	Bonny silt	Completely decomposed granite	Zenoz kaolin
a	225	308	381
n	0.5	1.08	0.57
b	54	42	26
m	3.27	0.14	61
c	64	695	802
k	0.5	0.59	4