Designing the bandwidth of single-photon sources with classical antenna techniques

Iñigo Liberal[1,2], Iñigo Ederra[1,2], Richard W. Ziolkowski[3],
[1]Department of Electrical and Electronic Engineering, Public University of Navarre, Pamplona, Spain, inigo.liberal@unavarra.es
[2]Institute of Smart Cities, Public University of Navarre, Pamplona, Spain
[3]Global Big Data Technologies Centre, University of Technology Sydney, Ultimo, Australia

***Abstract*—We discuss the role of the classical electromagnetic theory concept of reactive interactions on determining the bandwidth of a single-photon source. Typically, quantum emitters operate in the weak-coupling regime where the bandwidth of the emission spectrum is simply proportional to their decay rates. However, we introduce a first-order correction to the emission spectrum, demonstrating that its bandwidth is also directly affected by the dispersion properties of the reactive interactions of the quantum emitter with its environment. This correction is particularly important in the intermediate region bridging the weak and strong coupling regimes. As an example of the applicability of this theory, we study the behaviour of a quantum emitter decaying through a coupled two-cavity system. Our results suggests that this setup could be utilized for the design of efficient, but narrowband single-photon sources.**

***Index Terms*— antenna theory, quantum optics, reactive power, quantum emitters.**

I. Introduction

Controlling and designing the bandwidth of single-photon sources (quantum dots, color centers, atoms,...) is of general interest for the development of quantum communication and sensing technologies [1], [2], [3]. Most state-of-the-art single-photon sources operate within the weak coupling regime, where their 3dB bandwidths simply equal their decay rates. Interestingly, reactive interactions do not have any impact on the bandwidth within this regime. They only lead to a small frequency shift; i.e., the well-known Lamb shift [4]. This scenario limits the possibilities of controlling the bandwidth, and it is in stark contrast with the design of electrically small antennas. For (classical) electrically small antennas, managing the energy stored in their reactive fields is essential in determining their bandwidth and establishing their physical bounds of performance in addition to developing design and optimization strategies [5], [6], [7]. We remark that typical size scales of single-photon sources are much more deeply subwavelength than those usually associated with electrically small antennas. From this perspective, it is even more surprising that reactive interactions appear not to play any role in the determination of their bandwidths.

However, some works have pointed out similarities between antennas and quantum emitters, as well as the possibility of transferring ideas between both fields. Successful examples include quantum antenna arrays [8] (where interference in an ensemble of quantum emitters enables shaping the angular distribution of photon statistics of any order), as well as isotropic single-photon sources [9] (a hybrid between classical antennas and thermal sources that emits isotropic and unpolarized, but temporally coherent single photons). With this motivation and following our previous work [10], we investigate the role of reactive interactions on the bandwidth of single-photon sources. In particular, we will demonstrate that the bandwidth does become affected by the dispersion properties of reactive interactions when the first-order correction to the weak-coupling regime is properly taken into account. While this result provides a better understanding of the bandwidth of quantum emitters, it also provides an important link to classical antenna theory.

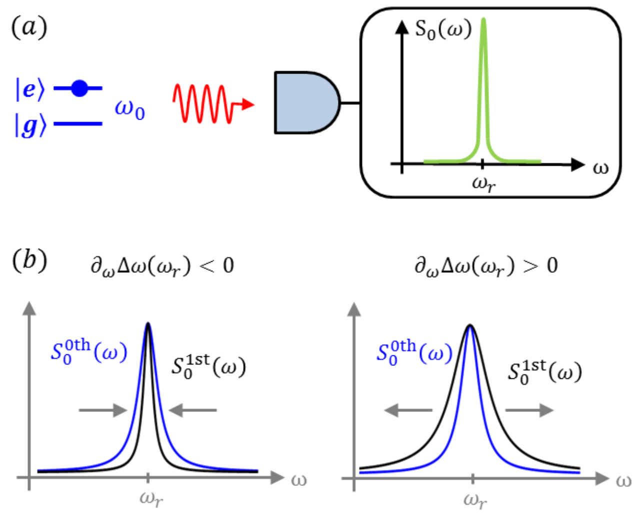

Fig. 1. Sketch of the quantum emitter system. (a) An initially excited two-level system, with transition frequency ω_0, decays emitting an single photon and measurement of the emission spectrum $S_0(\omega)$. (b) Zeroth $S_0^{0\text{th}}(\omega)$ and first-order $S_0^{1\text{st}}(\omega)$ approximations to the emission spectrum according to the sign of the frequency derivative of the dispersive frequency shift $\partial_\omega \Delta\omega$.

II. Bandwidth of a Single-Photon Source

As schematically depicted in Fig. 1(a), we study the emission spectrum of a quantum emitter. It is modelled as a two-level system, $\{|e\rangle, |g\rangle\}$, whose transition frequency is ω_0. We assume that the system is initially excited and the photonic environment is simply the vacuum state, $|\psi(t=0)\rangle = |e,\{0\}\rangle$, and that the two-level system decays by emitting a single photon into the photonic environment. The spectral components of this photon can be characterized by using selective filters in front of a broadband photon counter. In this manner, the probability of measuring the emitted photon in a given frequency range is recovered. In our previous work it was demonstrated within the rotating wave and one-photon correlation approximations that the measured emission spectrum can be compactly written as: [10]

$$S_0(\omega) = \frac{1}{(\omega - \omega_0 - \triangle\omega(\omega))^2 + \frac{1}{4}(\Gamma_0 + \Gamma(\omega))^2} \quad (1)$$

Here, Γ_0 is the intrinsic decay rate of the emitter, taking into account both the nonradiative decay channels and the radiative channels not contained in our description of the system. We have also introduced a dispersive frequency shift $\triangle\omega(\omega)$ and decay rate $\Gamma(\omega)$, that account for the reactive and radiative interactions with the photonic environment, respectively. They can be represented as:

$$\triangle\omega(\omega) - i\,\frac{\Gamma(\omega)}{2} = $$

$$= -\frac{\mu_0}{\hbar}\int d^3\mathbf{r}\int d^3\mathbf{r}'\;\mathbf{j}_{ge}^*(\mathbf{r})\cdot\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\cdot\mathbf{j}_{ge}(\mathbf{r}') \quad (2)$$

where $\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)$ stands for the dyadic Green's function of the photonic environment, while $\mathbf{j}_{ge}(\mathbf{r}) = \langle g|\,\widehat{\mathbf{j}}(\mathbf{r})\,|e\rangle$ is the transition current distribution, with $\widehat{\mathbf{j}}(\mathbf{r}) = q/(2m)\,|\mathbf{r}\rangle\langle\mathbf{r}|\,\widehat{\mathbf{p}} + h.c.$ being the current density operator. This current-oriented formulation enables establishing analogies with classical antenna theory. In particular, the dispersive decay rate and frequency shift can be directly linked to the supplied and reactive powers of a classical current with the same distribution, $\mathbf{j}_{ge}(\mathbf{r})$, as: $\triangle\omega(\omega) = 2/\omega\, P_{\mathrm{reac}}(\omega)$ and $\Gamma(\omega) = 4/\omega\, P_{\mathrm{sup}}(\omega)$.

Most single-photon sources operate within the weak-coupling regime. In our formulation, this approximation can be recovered by neglecting the dispersion properties of the interaction terms around the resonant frequency obtained as a solution to the implicit equation $\omega_r = \omega_0 + \triangle\omega(\omega_r)$. This exercise leads to the following emission spectrum:

$$S_0^{\mathrm{0th}}(\omega) = \frac{1}{(\omega-\omega_r)^2 + \frac{1}{4}(\Gamma_0 + \Gamma(\omega_r))^2} \quad (3)$$

It is clear from (3) that the emission spectrum is characterized by a Lorentzian lineshape whose bandwidth is given by the value of the decay rate at the transition frequency:

$$BW^{\mathrm{0th}} = \Gamma_0 + \Gamma(\omega_r) \quad (4)$$

Importantly, this implies that the reactive interactions do not play any role in the bandwidth of the quantum emitter; they only result in a frequency shift $\triangle\omega(\omega_r)$. However, this behavior is expected to change as the coupling strength of between the quantum emitter and the photonic environment increases. In order to clarify this point, we introduce a first-order correction to the emission spectrum by including the first-order term in the Taylor series expansion of the dispersive frequency shift. This leads to the expression:

$$S_0^{\mathrm{1st}}(\omega) = A\,\frac{1}{(\omega-\omega_r)^2 + \frac{1}{4}\left(\frac{\Gamma_0+\Gamma(\omega_r)}{1-\partial_\omega\triangle\omega(\omega_r)}\right)^2} \quad (5)$$

with $A = (1-\partial_\omega\triangle\omega(\omega_r))^{-2}$. Equation (5) demonstrates that as we move away from the weak-coupling regime, the first-order correction preserves a spectrum with a Lorentzian line, but whose bandwidth is directly affected by the dispersion properties of the reactive power:

$$BW^{\mathrm{1st}} = \frac{\Gamma_0 + \Gamma(\omega_r)}{1-\partial_\omega\triangle\omega(\omega_r)} \quad (6)$$

As illustrated in Fig. 1(b), it is clear from (6) that the emission bandwidth will be compressed (expanded) by the impact of the reactive power if the sign of the frequency derivative $\partial_\omega\triangle\omega(\omega_r)$ is negative(positive). As discussed in [10], both possibilities $\partial_\omega\triangle\omega(\omega_r) \gtrless 0$ are actually possible in dissipative or radiating systems.

In conclusion, as soon as the coupling strength goes beyond the weak-coupling regime, reactive interactions must be taken into account in order to obtain the correct bandwidth. This is true even if the spectrum continues to be characterized by a single resonant line; i.e., before the more extensively studied strong-coupling regime comes into play. For the same reason, reactive interactions can be understood as an additional degree of freedom in designing the bandwidth of quantum emitters. Naturally, having two degrees of freedom instead of one should be expected to provide new opportunities in the design of single-photon quantum sources.

III. Coupled Resonant Cavities and Fano Resonances

Coupling quantum emitters to photonic nanostructures, like resonant cavities and waveguides, provides new opportunities in tailoring their light-matter interactions [11]. Within the present context, photonic nanostructures should enable control over the dispersion properties of the reactive interaction parameters $\triangle\omega(\omega)$ and $\Gamma(\omega)$. Moving beyond conventional resonant cavities and waveguides, we illustrate this possibility by investigating the coupling of a quantum emitter to two coupled resonant cavities. In particular, as sketched in Fig. 2(a), we consider a two-level system with transition frequency ω_0, and intrinsic decay rate Γ_0, coupled to a resonant cavity that is characterized by its resonance frequency ω_1, decay rate Γ_1 and coupling strength Ω_1. This cavity is itself optically coupled to a second resonant cavity, also characterized by its resonance frequency ω_2, decay rate Γ_2 and coupling strength Ω_2. The interaction terms for this two coupled-cavity system are

$$\triangle\omega(\omega) - i\,\frac{\Gamma(\omega)}{2} = \frac{\Omega_1^2/4}{\omega-\omega_1+i\,\frac{\Gamma_1}{2} - \frac{\Omega_2^2}{4}\,\frac{1}{\omega-\omega_2+i\,\frac{\Gamma_2}{2}}} \quad (7)$$

Different responses can be obtained as functions of the quantitative relation between the cavity and emitter parameters. For example, a Fano-like resonance can be obtained by

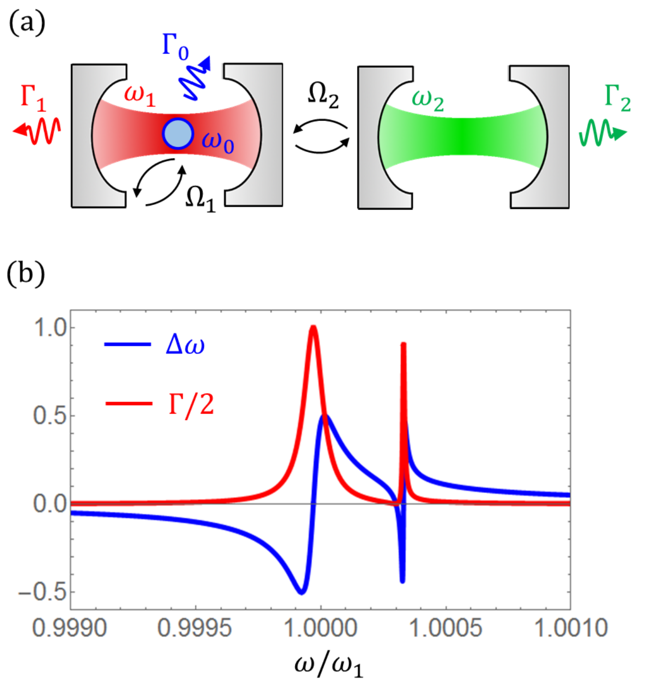

Fig. 2. Sketch of the two coupled-cavitysystem. (a) A two-level system with transition frequency ω_0 and intrinsic decay rate Γ_0 is coupled to a resonant cavity with resonance frequency ω_1, decay rate $\Gamma_1 = 10^{-4}\omega_1$ and coupling strength Ω_1. This cavity is also coupled to a second cavity with resonance frequency $\omega_2 = \omega_1 + 3\Gamma 1$, decay rate $\Gamma_2 = 10^{-6}\omega_1$ and coupling strength $\Omega_2 = 2\Gamma_1$. (b) Dispersive frequency shift and $\Delta\omega(\omega)$ and decay rate $\Gamma(\omega)$ normalized to its maximum value Ω_1^2/Γ_1.

coupling a relatively broadband cavity to a much narrower cavity. This effect is illustrated in Fig. 2(b), which depicts the frequency dispersion of the interaction terms for the parameter values $\Gamma_1 = 10^{-4}\omega_1$, $\Gamma_2 = 10^{-6}\omega_1$, $\omega_2 = \omega_1 + 3\Gamma_1$, and $\Omega_1 = 2\Gamma_1$. On the one hand, the broadband cavity at ω_1 retains its natural resonant behavior. Consequently, it could be used as a conventional cavity QED setup. On the other hand, the narrowband resonance at ω_2 becomes an asymmetric line with a minimum-maximum sequence. Furthermore, the strong dispersion around ω_2 suggests that this configuration could offer the possibility of compressing the bandwidth. In contrast with electrically small antennas where it is typically sought to maximize the bandwidth in view of their communication applications, single-photon sources benefit from narrow bandwidths. This narrow bandwidth feature helps maximize the indistinguisability of the emitted photons [12]. It would also increase the spectral resolution for metrology and spectroscopy applications with nonclassical light.

For $\Gamma_2 \ll \Gamma_1$, we can approximate $\triangle\omega(\omega_2) \simeq 0$, $\Gamma(\omega_2) = \zeta\Gamma_2$ and $\partial_\omega \triangle\omega(\omega_2) = -\zeta$, where we have defined the coupling parameter $\zeta = \Omega_1^2/\Omega_2^2$. Therefore, it is found that by increasing the coupling strength, it is possible simultaneously to increase the efficiency by enhancing the decay rate $\eta = \Gamma(\omega_2)/(\Gamma(\omega_2) + \Gamma_0)$ and to compress the bandwidth via reactive interactions. Therefore, this configuration can obtain a narrower bandwidth than that predicted in the weak-

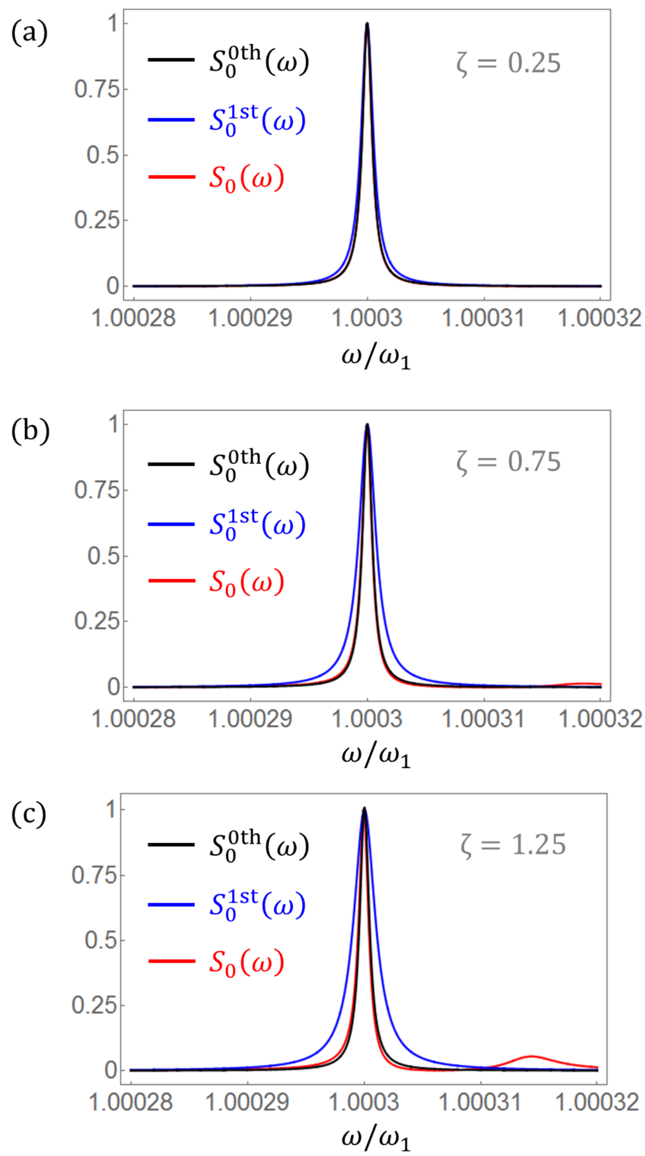

Fig. 3. Comparison between the emission spectrum $S_0(\omega)$ (red), and its zeroth $S_0^{0\text{th}}(\omega)$ (blue) and first $S_0^{1\text{st}}(\omega)$ (black) order approximations, for the coupling parameters (a) $\zeta = 0.25$, (b) $\zeta = 0.75$ and (c) $\zeta = 1.25$.

coupling regime for the same level of efficiency. This effect is illustrated in Fig. 3. A comparison between the emission spectrum $S_0(\omega)$, and its zeroth $S_0^{0\text{th}}(\omega)$ and first $S_0^{1\text{st}}(\omega)$ order approximations is given as the coupling parameter ζ increases. As expected, the bandwidth is compressed with respect to the prediction of the weak-coupling regime as ζ increases. This feature is evidenced by the narrower bandwidth for the same level of efficiency. Finally, secondary peaks start to appear in the emission spectrum for relatively large parameter values $\zeta = 1.25$. They denote the emergence of the strong coupling regime and mark the end of the regime of validity of our first-order corrections.

IV. Conclusion

In conclusion, we have demonstrated that as the response of quantum emitters deviate from the weak-coupling regime, reactive interactions must be taken into account to correctly predict the bandwidth of their emission spectra. In conjunction, we have further shown that reactive interactions must be understood as additional degrees of freedom in controlling the bandwidths of the emitted photons. These additional degrees of freedom may offer new opportunities in the design of nonclassical single-photon sources. A two coupled-cavity example was presented in which the reactive interactions between a quantum emitter and the photonic environment surrounding it break the classical direct relationship between efficiency and bandwidth and produce a narrowing of the emission bandwidth. Consequently, it has been demonstrated how one could use reactive interactions to facilitate the design of efficient and narrowband single-photon sources.

Acknowledgment

I.L. acknowledges support from Juan de la Cierva Incorporation fellowship.

References

[1] D. D. Awschalom, R. Hanson, J. Wrachtrup, and B. B. Zhou, "Quantum technologies with optically interfaced solid-state spins," *Nature Photonics*, vol. 12, no. 9, p. 516, 2018.

[2] I. Aharonovich, D. Englund, and M. Toth, "Solid-state single-photon emitters," *Nature Photonics*, vol. 10, no. 10, p. 631, 2016.

[3] P. Senellart, G. Solomon, and A. White, "High-performance semiconductor quantum-dot single-photon sources," *Nature Nanotechnology*, vol. 12, no. 11, p. 1026, 2017.

[4] M. Scully, A. Sokolov, and A. Svidzinsky, "Virtual photons: From the Lamb shift to black holes," *Opt. & Photon. News*, vol. 29, no. 2, pp. 34–40, 2018.

[5] K. Schab, L. Jelinek, M. Capek, C. Ehrenborg, D. Tayli, G. A. Vandenbosch, and M. Gustafsson, "Energy stored by radiating systems," *IEEE Access*, vol. 6, pp. 10 553–10 568, 2018.

[6] A. D. Yaghjian and S. R. Best, "Impedance, bandwidth, and q of antennas," *IEEE Trans. Antennas Propag.*, vol. 53, no. 4, pp. 1298–1324, 2005.

[7] J. L. Volakis, C.-C. Chen, and K. Fujimoto, *Small Antennas: Miniaturization Techniques & Applications*. McGraw-Hill New York, 2010.

[8] I. Liberal, I. Ederra, and R. W. Ziolkowski, "Quantum antenna arrays: The role of quantum interference on direction-dependent photon statistics," *Physical Review A*, vol. 97, no. 5, p. 053847, 2018.

[9] I. Liberal, I. Ederra, and Y. Li, "Isotropic single-photon sources," *Optics Letters*, vol. 43, no. 12, pp. 2736–2739, 2018.

[10] I. Liberal, I. Ederra, and R. W. Ziolkowski, "Control of a quantum emitters bandwidth by managing its reactive power," *Preprint available at arXiv:1808.08072*, 2018.

[11] P. Lodahl, S. Mahmoodian, and S. Stobbe, "Interfacing single photons and single quantum dots with photonic nanostructures," *Rev. Modern Phys.*, vol. 87, no. 2, p. 347, 2015.

[12] M. Karpiński, M. Jachura, L. J. Wright, and B. J. Smith, "Bandwidth manipulation of quantum light by an electro-optic time lens," *Nature Photonics*, vol. 11, no. 1, p. 53, 2017.