RecRisk: An Enhanced Recommendation Model with Multi-facet Risk Control

Yang Xiao[a], Qingqi Pei[a,*], Lina Yao[b], Xianzhi Wang[c]

[a]State Key Laboratory of Integrated Services Networks, Xidian University, Xi'an 710071, China
[b]The School of Computer Science, the University of New South Wales, Sydney, 2032, Australia
[c]The School of Computer Science, the University of Technology Sydney, Sydney, 2007, Australia

Abstract

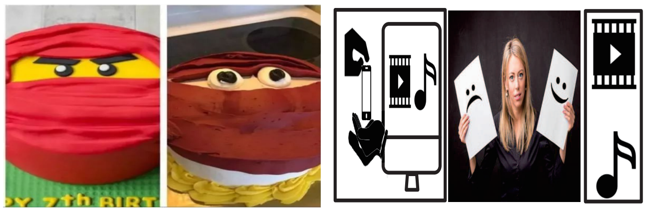
Recommender systems (RSs) play a crucial role in helping users quickly find their desired services and promoting sales of service providers in e-commerce. However, service providers intentionally upload cherry-picked service information to mislead RSs for greater profit. Such misleading recommendations pose the risk of degrading user experience and gradually undermine users' confidence in the whole service market over the long term. Worse still, most current expert recommendation methods are more susceptible to such risk because they are heavily dependent on this incomplete service information and assume it is trusted. Therefore, how to satisfy users' service requirements with risks minimized at the same time motivates our work. In this paper, we first discern two risky facets that pose significant challenges to expert recommendation: *Sense Drop* and *Blue Joy*. Then we propose a unified framework called RecRisk, which integrates trust, heat equation, and modern portfolio theory to address the above challenges. The main contributions of RecRisk are two-fold: (1) To select the services which satisfy the users' preferences, we design a trust-aware heat equation model (TAHE) that combines heat flow theory with trust elements. (2) We develop a flexible model based on modern portfolio theory to weigh users' satisfaction and services' risk facets, and finally recommend a ranked service list to users. Our experimental results demonstrate that RecRisk simultaneously achieves higher recommendation precision and decreases risks when compared to state-of-the-art approaches.

Keywords: Recommender Systems, trust, heat equation, portfolio theory

1. Introduction

The rapid development of Recommender Systems (RSs) has brought convenience for both service providers (SPs) and users in e-commerce. Through RSs, SPs can easily deliver information about the right service[1] to the right user, and therefore make increased profits in major marketplaces such as Amazon and eBay; moreover, users (or service consumers) enjoy personalized services while suffering from reduced information overload (Bobadilla et al., 2013).

However, current expert RSs merely rely on the service information, either complete or incomplete, without being able to validate its integrity (Alharthi et al., 2018). The bad news is that many SPs deliberately provide cherry-picked information about their services while neglecting negative comments or hiding negative terms such as "pay extra fees" to attract more users and gain more profits. RSs can be easily misled by such incomplete information and thereby impair users' experience in two common cases. The first case is that the recommended services may not



(a) Sense Drop (b) Blue Joy

Figure 1: Illustrative examples of two facets: (a) Sense Drop; (b) Blue Joy

meet users' preferences, indicating that the recommendation results will be of low quality. We refer to this case as *Sense Drop*. According to an e-commerce report in 2018[2], more than 32.56% complaint cases are related to Sense Drop. We show a practical example of Sense Drop in Fig.1 (a). To our surprise, in Fig.1 (a), the received cake (right) is markedly different from the recommended one (left) because the cake seller only provides the good-looking ones to RSs instead of the poor-quality cakes that the customer will actually receive. We use *Blue Joy* to name the second case that the recommended services meet user requirements but are bundled with some hidden terms and conditions. Similarly, Fig.1 (b) shows an example, where a woman is required to register via phone number in order

*Corresponding author
Email addresses: yangtomas7@gmail.com (Yang Xiao), qqpei@mail.xidian.edu.cn (Qingqi Pei), lina.yao@unsw.edu.au (Lina Yao), XIANZHI.WANG@uts.edu.au (Xianzhi Wang)

[1]we use the terms "service" and "item" interchangeably

[2]http://www.100ec.cn/zt/18yhts/

to watch movies on the recommended website despite that she does not expect to afford extra expenses such as paying extra money, or being spied by a third party when enjoying the service. Both of the above risky facets are generally reflected by users' negative feedback, such as low ratings or unfriendly text messages (Wang & Zhu, 2009; Cover & Thomas, 2012). Hence, how to efficiently guarantee accurate recommendation with risky facets lowered simultaneously is a novel challenging problem to current expert and intelligent RSs.

Although most current expert recommendation approaches (Deshpande & Karypis, 2004; Aggarwal, 2016; Jannach & Ludewig, 2017; Otebolaku & Andrade, 2017) achieve accurate recommendation via considering the influence of the user/item neighborhood, homophily effect between users including similarity, social relationships, etc. (Ardissono & Mauro, 2020; Azadjalal et al., 2017; Abbasi et al., 2014; Feng et al., 2016; Guo et al., 2016; Amal et al., 2019), they can not effectively control the risky facets of services because they assume that the service information offered by SPs is complete. Such "complete information" assumption violates the practice[3] and therefore sabotage the user experience in face of risky facets. Moreover, these studies also do not attain ideal performance in terms of accuracy, because these works neglect the transmission process of ratings. Such a process can influence users' subjective comprehension of services, which may lead to the variation of personal preference and further impact recommendation in practice (Jiang et al., 2019; Rafailidis et al., 2017).

From the perspective of risky facets, some studies (Zhu et al., 2014; Mazeh & Shmueli, 2020) associate it with the privacy problem, which is just a part of Blue Joy. Zhu et al. (2014) design a scalable and automatic approach to evaluate the privacy risks of mobile apps, which contrives app's popularity and users' security for the recommendation to assess the privacy risk. Mazeh & Shmueli (2020) propose a PDSCB recommendation algorithm, i.e., *personal data store-inspired content-based*, to protect users' privacy when enjoying services. PDSCB belongs to the category of content-based RSs, which borrows from a privacy-enhanced expert system, i.e., OpenPDS (Open Personal Data Store). Nevertheless, accounting for privacy can not fully alleviate Sense Drop due to two key limitations: a) The privacy preservation approaches can not guarantee that the recommended services will satisfy users' requirements while preventing the leakage of users' personal information during recommendation; b) Privacy preservation schemes can not fully protect users' personal preferences in the same way as they protect traditional information such as e-mail, and even if user preference is preserved, the accuracy of RSs will be sacrificed.

Motivation. To overcome the above limitations, we conjecture an ideal model called RecRisk which ensures recommendation accuracy and minimizes the risky facets simultaneously. Compared with previous expert recommendation approaches, our proposed scheme achieves two innovations: 1) For recommendation accuracy, RecRisk not only inherits the advantage of state-of-the-art methods which incorporate trust elements (Guo et al., 2012; Azadjalal et al., 2017; Gao et al., 2016; Guo et al., 2016) such as social homophily and the neighborhood effect during service selection, but also overcomes their drawbacks by modeling the rating transformation process to capture users' comprehension via heat equation. Heat equation was initially proposed to study the energy flow in a rod and has achieved success in modeling the transmission processes and precise predictions in physics and engineering (Gilkey, 2018; Ebrahimnia-Bajestan et al., 2016). Therefore, we design a trust-aware heat equation (TAHE) in a way that integrates heat equation with trust elements to guarantee the recommendation accuracy. 2) Regarding risky facets, we turn to modern portfolio theory (MPT), which was initially proposed to evaluate the risk of stocks (i.e., financial loss) and help investors gain profits in the field of finance (Markowitz, 1952). Inspired by its success of risk evaluation in finance (deLlano Paz et al., 2017; Luenberger et al., 1997; Grasse et al., 2016), we design an MPT-based model to strike an equilibrium between users' satisfaction and services' risky facets. Compared with current expert recommendation schemes which can not efficiently handle these two risky facets, our MPT-based model has been validated to cope with both *Sense Drop* and *Blue Joy* as they can be efficiently quantified by variance (Markowitz, 1952; Wang & Zhu, 2009). To this end, our RecRisk first selects services which satisfy users' requirements by TAHE, which is followed by measuring the risk facets (i.e., variance) of the selected services based on MPT, finally, ranks and recommends services based on the risky value obtained.

The main contributions of our work are summarized as follows:

- We consider the recommendation from a novel game theory's perspective and pinpoint two categories of risky facets in RSs, namely *Sense Drop* and *Blue Joy*, which have not been extensively studied in the expert and intelligent systems area.
- Compared with previous personalized recommendation schemes applied in the expert and intelligent systems area, which can not efficiently tackle risky facets, we design RecRisk, which combines heat equation, trust elements and economic theory. RecRisk promises the higher recommendation accuracy regardless of the existence of risky facets, and therefore simultaneously minimizes the value of risk facets in personalized recommendations. Moreover, we use heat equation to model the rating transmission process to further improve recommendation accuracy, which is seldom considered by previous works.
- From the perspective of theoretical contribution, our work is the first to apply heat flow theory, which is

[3] http://www.100ec.cn/zt/18yhts/

the basis of the heat equation, and prove its feasibility in expert and intelligent RSs. Our work not only explicitly analyzes and explains the limitations of heat flow theory, but also successfully extends the heat equation to service recommendation via integrating trust elements. To facilitate a better understanding of the link between RSs and Physics, we have constructed a one-one mapping between the entities in RSs (e.g., users, services, ratings) and the comparable variables in physics (e.g., energy, rod, thermal diffusivity) in Table 2.

- We have conducted extensive experiments on two real datasets to evaluate RecRisk. The evaluation results in comparison with state-of-the-art recommendation models show that RecRisk achieves better recommendation accuracy with risky facets considered.

The rest of this paper is organized as follows. Section 2 introduces the preliminaries and describes our problem. Section 3 presents RecRisk, including details of the model deduction and theoretical analysis. Section 4 reports the experiments conducted to evaluate our approach, while Section 5 presents an analysis of our experimental results. Section 6 provides an overview of trust-aware expert RSs and some risk management scheme identified by the literature. Finally, Section 7 gives some concluding remarks and outlines the future work.

2. Related Work

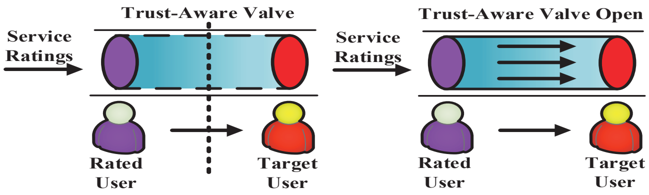
In this section, we primarily review the related work from the perspectives of trusted recommendation and service risk management.

2.1. Trusted recommendation

Many methodologies for trusted expert RSs have been designed. Some representative methods are outlined below. Yera & Martinez (2017) review the utilization of fuzzy tools in RSs. In this literature, authors analyze more than one hundred papers focused on fuzzy technique, divided into three distinct recommendation paradigms, i.e., content-based, memory-based CF and model-based CF, and extend the fuzzy tools into some new research areas such as social network information, tagging systems, etc. Pan et al. (2017) propose TDAE, a deep learning method which integrates trust information and denoising Auto-Encoders, to solve cold start and data sparsity problems. Similarly, Wu et al. (2016) propose a method based on collaborative filtering, called Collaborative Denoising Auto-Encoder (CDAE), which is similar in theory to TDAE. The difference is that CDAE aims to recommend a service list which can satisfy users' maximum preference instead of solving cold start and data sparsity problems. Pan et al. (2015) design a preference learning algorithm to learn the confidence for each uncertain examination record with the help of transaction records. Seo et al. (2017) propose an intelligent recommendation scheme, which is based on the strength of friendship between users. The proposed scheme can autonomously learn the feature of friendship from big social data to achieve service recommendation. Amal et al. (2019) describe an expert social-relational recommendation approach that computes and ranks inter-personal affinity in a social graph comprised of personal relational profiles. Parvin et al. (2019) propose a collaborative filtering method to predict the missing ratings by involving trust statements as the side information with an ant colony optimization method. Ardissono & Mauro (2020) propose a compositional trust-aware recommender system, which combines social links with global anonymous feedback about users and user contributions to enhance Top-N recommendation. Hong et al. (2019) propose a crowd-enabled framework to alleviate the limitation of *cold-start* problem. The proposed framework is based on conventional expert systems which utilize human-machine knowledge to solve the complex problem. Yuan et al. (2011) congregate two kinds of social relationship (i.e., friendship and membership) in unified matrix factorization. PMF (Mnih & Salakhutdinov, 2008) and SR2 (Ma et al., 2011), another two matrix factorization methods where the former utilizes user-rating data and the latter employs social network information to accomplish recommendation. (Ning & Karypis, 2011) design a novel Sparse Linear Method called SLIM, which is able to generate high-quality *top-N* recommendations fast. This method employs a sparse linear model in which the recommendation score for a new service can be calculated as an aggregation of other services. After SLIM's proposition, some SLIM-based methods have also been proposed such as SocSLIM with its extensions including TrusteeSLIM, TrusterSLIM, LocTrusterSLIM, LocTrusteeSLIM, LocSocSLIM (Feng et al., 2016).

2.2. Service risks managment

Another related issue is the risky facets mentioned in this work, i.e., *Sense Drop* and *Blue Joy*. few works completely solve these two issues. Previous works (Liu, 2015; Košir et al., 2014) only consider a part of *Sense Drop* from the perspective of user preference decay, but ignore other cases without users' preference decay. The work in (Zhu et al., 2014; Mazeh & Shmueli, 2020) only considers *Blue Joy* as a privacy-aware problem. Mazeh & Shmueli (2020) propose a PDSCB recommendation algorithm, i.e., *personal data store-inspired content-based*. PDSCB belongs to content-based RSs, which is based on a privacy-enhanced expert system, i.e., OpenPDS (Open Personal Data Store). The proposed approach is validated to protect users' privacy during service selection. Zhu et al. (2014) design a scalable and automatic approach to estimate the security risks of apps. This work also considers the app's popularity and users' security during recommendation and proposes an approach based on investment theory (Markowitz, 1952) to estimate the privacy risk. Ob-



(a) Trust-Aware Valve Close (b) Trust-Aware Valve Open

Figure 2: Illusion of Trust-Aware Valve

viously, current expert approaches are unable to comprehensively account for risky facets in social networks.

In summary, our scheme is unique in considering both users' preferences and risky service facets during recommendation.

3. Preliminaries

RecRisk comprises two parts: *trust-aware heat equation* (TAHE) and *MPT-based* models. Before stating explicitly, we first briefly introduce some background knowledge related to RecRisk and formulate the problem solved in this paper. The main notations used to describe the scheme are shown in Table 1.

3.1. Preliminaries

Heat equation (Haberman, 2005) is a branch of applied partial differential equation (PDE) used to describe the flow process of thermal energy in the inner part of an object. Considering a target user μ_r and rated user μ_i as two sides of a rod and assuming that μ_r agrees to accept the service provided by μ_i, this process is similar to thermal energy flowing in a one-dimensional rod. Hence, in this work, we introduce a one-dimensional heat equation and provide the explicit deduction process for estimating the value of a service.

Trust-aware valve, a value based on the trust relationship, can help people to cope with uncertainty to some extent when they make decisions on services. Apart from considering the quality of service (QoS) in RSs, μ_r often regards the trust-aware aspects between him and μ_i as another crucial element. As shown in Fig.2 (a), we regard trust-aware aspects as a valve, a service as energy, μ_r and μ_i as two sides of a rod, respectively. When μ_r trusts μ_i, i.e., the valve is open otherwise closed, he selects the service provided by this rated user, i.e., the energy flows from one side to the other side shown in Fig.2 (b). RecRisk determines several metrics: *trustworthiness, professional knowledge*,and *common-interest* to formulate a trust-aware valve when calculating estimation value.

Modern Portfolio Theory (MPT) (Markowitz, 1952) was established in 1952. The initial aim of MPT is to help investors to select appropriate stocks with a higher expected return but less risk associated with uncertainty.

Table 1: NOTATIONS

Notation	Description
μ_i	the ith rated user
μ_r	the target user
S	the list of services to be recommended
M	a set of users
S_{od}	the service ranked list determined by TAHE
$\widehat{S_{od}}$	final service ranked list derived by RecRisk
ev_i	the estimated value for s_i derived by TAHE
s_i	the ith service
r_{ij}	the rating from the ith user to jth service
r_i	the vector including the ratings of μ_i on S
t_{ab}	the trustworthiness of user a to user b
$\mathcal{I}_i$	μ_i's interest list
$s_{ci}(\mu_i, \mu_y)$	the common interest list of user μ_i and user μ_y
pc_i	μ_i's personal comprehension
μ_i^{slist}	μ_i's service list
$\mu_i^{sublist}$	the list of service in specific category of μ_i
$\lvert \cdot \rvert$	the cardinality of a set

Here if we take users' requirements and risky facets associated with recommended services as corresponding to benefit and risk in MPT, we can construct an MPT-based model to quantify the expected return and risk.

3.2. Problem Description

RecRisk aims to provide a service ranked list to the target user via weighing personal requirement and risky facets. Let S be a list of services with cardinality n, denoted by $S = \{s_1, s_2, ..., s_n\}$, and M be a set of m users who rated at least one service belonging to S, where $M = \{\mu_1, \mu_2, ..., \mu_m\}$. Each μ_i ($\mu_i \in m$) provides his rating r_{ij} on a certain service, say s_j ($s_j \in S$). We use a vector r_i to record all of the ratings offered by μ_i on S, and $r_i = \{r_{i1}, r_{i2}, ..., r_{in}\}$. All of the r_is are then combined to form a rating matrix R, where $R = \{r_{ij}\}_{m \times n}$. In addition, users in social networks own their personal relationship such as friends, relatives and colleagues. We use a directed graph $G = \{m, \xi\}$ to represent the relationship between two users, where $\xi \subseteq \{m \times m\}$ is the set of edges, representing social relationship for a pair of adjacent nodes.

Based on the statements above, our scheme RecRisk first selects services satisfying the target user's preferences by means of a *trust-aware heat equation* and thereby get a service list S_{od}, then we evaluate the satisfaction and risk facets of services in S_{od} by MPT-Based model to get the final service ranked list $\widehat{S_{od}}$ for target node.

4. Scheme Design

4.1. Overview

We first present a general introduction of RecRisk shown in Fig.3. The principal aim is to provide the target node μ_r with a service ranked list by incorporating TAHE with

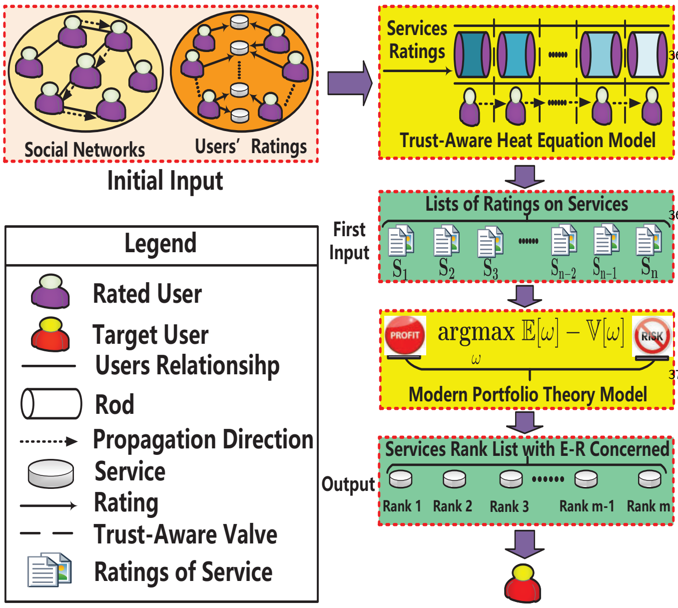


Figure 3: The Structure of RecRisk

MPT-based models. The provided services should satisfy μ_r's preferences and guarantee a minimum of risky facets. The social networks composed of the relationship among users and users' ratings are regarded as the input, colored by light red squares. Both of them are then entered into the first process, i.e., the TAHE model. This model mainly aims to calculate the final estimation value of one service by aggregating the ratings offered by all of the users who rated this service. After that, RecRisk gets the first n services (S_{od}) according to high estimation value. We extract these n services with their rating lists as the first input (colored by sea green) to enter into an MPT-based model with light green color. This model outputs a service ranked list $\widehat{S_{od}}$ to μ_r via weighing his satisfaction (i.e., expected return) and the services' risk facets (i.e., variance).

4.2. Trust-Aware Heat Equation Model

4.2.1. Metrics

In this section, we mainly introduce TAHE, the first part of RecRisk. Some metrics applied in TAHE will be firstly formulated as follows.

Metric 1 (Trustworthiness) (tv_{rb}). The trustworthiness of μ_r to μ_b, denoted by tv_{rb}, is equal to the trust that μ_r lays on μ_b according to the interaction between them. In real trust networks such as Epinions, trust opinions are expressed precisely.

Metric 2 (Professional Knowledge) (pk_{br}). This metric indicates a service recommender (SR)'s (Here is rated user) expertise to make proper recommendations to μ_r, which has two important features for SRs: 1) *Professional Knowledge* is a relatively objective metric since it considers objective information about a service such as QoS, etc. We refer to this feature as *service-related degree*, say sr_b for μ_b, which is defined as follows: $sr_b = |\mu_b^{slist} \cap \mu_r^{slist}|/|\mu_r^{slist}|$. 2) It can also influence the trust relationship between a SR and target node according to (Jiang et al., 2015). This feature is called *affiliation*, denoted by ali_{br} which can be computed according to (Jiang et al., 2015). The final pk_{br} is computed as: $pk_{br} = (\alpha_1 \cdot sr_b + \alpha_2 \cdot ali_{br})/(sr_b + ali_{br})$, where $pk_{br} \in [0, 1]$, $\alpha_1 + \alpha_2 = 1$.

Metric 3 (Common-Interest) ($s_{ci}(\mu_r, \mu_b)$). This metric, which is the last essential component of the trust-aware valve, represents the common preference among different nodes for the same service. For a target node μ_r and another node μ_b with interest lists $\mathcal{I}_r$ and $\mathcal{I}_b$ respectively, $s_{ci}(\mu_r, \mu_b)$ between μ_r and μ_b is computed as $s_{ci}(\mu_r, \mu_b) = |\mathcal{I}_r \cap \mathcal{I}_b|/|\mathcal{I}_r|$.

After determining the above three metrics between μ_r and μ_b, we get the trust-aware valve of them, i.e., T_{br} as follows,

$$T_{br} = \beta_1 \cdot tv_{rb} + \beta_2 \cdot pk_{br} + \beta_3 \cdot s_{ci}(\mu_r, \mu_b) \qquad (1)$$

where $\beta_1 + \beta_2 + \beta_3 = 1$.

Metric 4 (Personal Comprehension) (pc_r). It is known that each person has his/her unique ability to analyze, judge, study, and acknowledge information existing in the real world. We call this ability personal comprehension, denoted by pc_r. Each user has his own distinct personal comprehension of information in reality. The nature of this ability is quite similar to that of a physical concept called *Thermal Diffusivity*, which is an intrinsic characteristic of a particular substance (XU et al., 2008). In RSs, μ_r's personal comprehension pc_r is defined as follows,

$$pc_r = \frac{|\mu_r^{sublist}|}{|\mu_r^{slist}|} \qquad (2)$$

4.2.2. Heat Flow Theory-Based Principles Applied in RSs

In this section, we present the *Heat Flow Theory* (HFT)-Based principles applied in RSs, which originates from HFT principles in physics (Haberman, 2005). These principles provide solid evidence that the heat equation can be applied to RSs. The four following rules have been concluded by many physical experiments to show HFT (Haberman, 2005).

1 If the temperature remains unchangeable in a region, no thermal energy flows.
2 If there exist temperature differences, the thermal energy flows from the hotter region to the lowers.
3 The greater the temperature differences for the same substance, the greater the flow of thermal energy is.
4 The flow of thermal energy will be distinct for different materials, even with the same temperature difference. In other words, different materials have different thermal diffusivity.

Joseph Fourier[4] recognized these four rules and summarized them as *Fourier's Law* which is the general form used to construct the heat equation,

$$\phi = -K_0 \frac{\partial \omega(x, t)}{\partial x} \qquad (3)$$

[4] http://www-history.mcs.st-andrews.ac.uk/Biographies/Fourier.html

Table 2: Mapping List

Variables in RSs	Variables in Physics
target/rated user	two sides of a rod
personal comprehension	thermal diffusivity
service ratings	thermal energy
service requirement of the target user	temperature difference

where ϕ is the heat flux (the amount of thermal energy per unit time flowing from left to right per surface area). $\omega(x,t)$ is a function of temperature, x is a unit length of a rod, t is time, $\partial\omega(x,t)/\partial x$ is the derivative of the temperature, as a function of x fixed t, K_0 is thermal conductivity.

However, the heat equation can not directly be applied in RSs. This is because the target user μ_r can overlook and even reject the service from a rated user μ_i when they come into contact, i.e., there may be no rating flow between them even though a service requirement exists (i.e., temperature difference), whereas for a rod, the flow of thermal energy must occur once a temperature difference remains between the two sides. In other words, the heat equation can be tenable in RSs if and only if the trust-aware valve is considered, i.e., μ_r trusts μ_i and accept the service provided by him/her (Note: trust is one of the valves, we also consider item-neighborhood as another valve in the following experiments). Hence, to apply the heat equation, we propose another four principles according to HFT principles with the trust scenario consideration (Golbeck, 2005; Jiang et al., 2015), i.e., HFTB principles.

1 If the target user does not adopt ratings provided by his trusted rating users, no ratings will be transmitted between them.

2 If the target user adopts a user's rating, the rating will flow from this user to the target user.

3 When a user's rating on a certain service satisfies the target user's service requirement, the target user is prone to adopt this rating.

4 Different target users have different levels of personal comprehension for the same rating.

Based on our discussion, there exists a one-to-one correspondence between HFT (variables in physics) and HFTB principles with trust account (variable in RSs) shown in Table 2.

4.2.3. Detail of TAHE

In this section, we mainly offer the deduction of TAHE (see Fig.4) according to (Haberman, 2005). Here, we assume that μ_r trusts μ_i according to the trust-aware valve. Given a rod with a constant cross-sectional area $\overline{\mathcal{S}}$ and length L, the mass of this rod is uniformly distributed, we set the flow direction of energy/ratings from $x=0$ to $x=L$. Then we list some notations used in the field of physics in order to elaborate.

- $e(x,t)$: the amount of thermal energy per unit volume is called *thermal energy density*.

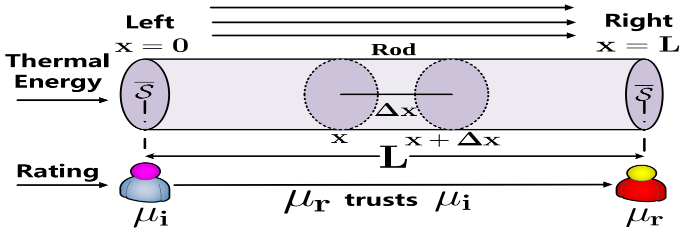


Figure 4: The Structure of TAHE

- $\phi(x,t)$: the amount of thermal energy per unit time flowing from left to right per surface area is called *heat flux*.

- $Q(x,t)$: thermal energy per unit volume generated by the inner part of substance per unit time is called *heat source*.

As shown in Fig.4, we take a thin slice between x and $x+\Delta x$, where Δx is extremely small. According to the definition of $e(x,t)$, we can obtain the thermal energy of this slide expressed as follows,

$$\text{thermal energy} = e(x,t)\overline{\mathcal{S}}\Delta x \tag{4}$$

According to conservation of heat energy (Mach, 2014), we have

$$\begin{aligned}\frac{\partial}{\partial t}[e(x,t)\overline{\mathcal{S}}\Delta x] &\approx \phi(x,t)\overline{\mathcal{S}} - \phi(x+\Delta x,t)\overline{\mathcal{S}} \\ &+ Q(x,t)\overline{\mathcal{S}}\Delta x\end{aligned} \tag{5}$$

When $\Delta x \to 0$, we get formula (6) after simplification.

$$\frac{\partial e(x,t)}{\partial t} = -\frac{\partial \phi(x,t)}{\partial x} + Q(x,t) \tag{6}$$

Now, we consider from the perspective of temperature. As shown in Fig.4, let $\omega(x,t)$ be a function of temperature, and get the heat capacity formula as follows,

$$e(x,t) = c(x)\omega(x,t)\rho(x)\overline{\mathcal{S}}\Delta x \tag{7}$$

where c(x) is the *special heat* of the rod, $\rho(x)$ is its density. Both of these variables are constant. We substitute formula (7) into (6) and get formula (8),

$$c(x)\rho(x)\frac{\partial \omega(x,t)}{\partial t} = -\frac{\partial \phi(x,t)}{\partial x} + Q(x,t) \tag{8}$$

we combine formula (3), (7) with (8) and get formula (9) after simplification.

$$\frac{\partial \omega(x,t)}{\partial t} = K\frac{\partial^2 \omega(x,t)}{\partial x^2} + Q(x,t) \tag{9}$$

where $K = K_0/c(x)\rho(x)$ is thermal diffusivity (here K can be regarded as personal comprehension), $Q(x,t)$ is an inner source which may be a constant value (zero or non-zero) or only related to x in some conditions. In RSs, $Q(x,t)$ reflects the number of services previously purchased by users.

To obtain a unique feasible solution of equation (9), one boundary condition (BC) at each side and an initial condition (IC) must be replenished and get the final formula (10).

$$\begin{cases} \text{PDE}: \dfrac{\partial \omega(x,t)}{\partial t} = k\dfrac{\partial^2 \omega(x,t)}{\partial x^2} + Q(x,t) \\ \qquad 0 \le x \le L, t \ge 0. \\ \text{BC1}: \omega(0,t) = l(t), t \ge 0. \\ \text{BC2}: \omega(L,t) = r(t), t \ge 0. \\ \text{IC}: \omega(x,0) = f(x), 0 \le x \le L. \end{cases} \tag{10}$$

4.2.4. Application of TAHE

In this section, we mainly introduce the application of the TAHE model in RSs and solve the equation defined in formula (10). Two scenarios should be taken into consideration here: one is that the target user μ_r has not previously purchased the services resembling the service to be recommended, and the other is that μ_r has previously purchased a similar service.

For the first scenario. Given a service $s_i \in S$ and its rating list r_i ($r = \{r_{1i}, r_{2i}, ..., r_{li}\}$) provided by rated user μ_i, we first compute μ_r's personal comprehension pc_r by formula (2) and select trusted rated users for him by computing the trust-aware valve T_{ir} between μ_r and μ_i according to formula (1). If T_{ir} is larger than a pre-established threshold we set, it means that μ_r trusts μ_i, then we regard μ_i and μ_r as the left and right sides of a channel and construct the TAHE model based on formula (10).

For μ_i, supposing that T_{ir} is larger than the threshold, we first establish BC and IC. Let r_{ij} be the rating provided by μ_i on s_j, the left BC can be established as: $\omega(0,t) = r_{ij}$. After r_{ij} being publicized, μ_r will produce his own opinions on s_j based on r_{ij}. Because μ_r didnot buy similar services in his historical list, his opinions on s_j depend on the trust-aware valve between μ_r and μ_i. Hence, the right BC can be determined by $\omega(L,t) = T_{ir} \cdot r_{ij}$. Observing from real life, we know that a loss of energy takes place when it is transmitted in a rod, so it is with the transmission of information. Hence, we use an exponential function to describe this process and regard it as an IC, i.e., $\omega(x,0) = e^{-x}$.

In this scenario, where μ_r didn't purchase a similar service, his personal comprehension on this service and $Q(x,t)$ are 0. However, in order to utilize the heat equation, thermal diffusivity can not be 0; thus, we use e^x to transform 0 to 1, i.e., $K = e^{pc_r}$. Finally, we get the TAHE model for the first scenario, i.e.

$$\begin{cases} \text{PDE}: \dfrac{\partial \omega(x,t)}{\partial t} = \dfrac{\partial^2 \omega(x,t)}{\partial x^2} \\ \qquad 0 \le x \le L, t \ge 0. \\ \text{BC1}: \omega(0,t) = r_{ji}, t \ge 0. \\ \text{BC2}: \omega(L,t) = T_{jr} \cdot r_{ji}, t \ge 0. \\ \text{IC}: \omega(x,0) = e^{-x}, 0 \le x \le L. \end{cases} \tag{11}$$

For the second scenario. In this scenario, the assumption that T_{ir} is larger than the threshold is established in the same way as in the first scenario. μ_r has previously purchased the similar services historically, therefore, his personal comprehension pc_r on s_j and $Q(x,t)$ is not zero. pc_r is computed by formula (2). Q(x,t) is computed as $Q(x,t) = log|\mu_r^{sublist}|$, where s_j and other services he purchased are in the same category.

Then we construct BC and IC. According to energy conservation laws, for μ_r and μ_i, we have

$$\begin{aligned} pc_i \cdot r_{ij} + pc_r \cdot \widehat{r_i} &= (pc_i + pc_r) \cdot r_i^{ave} \\ r_i^{ave} &= \frac{pc_i \cdot r_{ij} + pc_r \cdot \widehat{r_i}}{pc_i + pc_r} \\ \omega(L,t) &= r_i^{ave} \end{aligned} \tag{12}$$

where $\widehat{r_i}$ is an average value for μ_r's ratings on similar services he purchased. It is computed as follows,

$$\widehat{r_i} = \frac{\sum_{d=1}^{n1} r_{id}}{n1} \tag{13}$$

where $n1$ is the number of similar services μ_r purchased

Finally, we get the TAHE model for the second scenario, i.e.,

$$\begin{cases} \text{PDE}: \dfrac{\partial \omega(x,t)}{\partial t} = K\dfrac{\partial^2 \omega(x,t)}{\partial x^2} + Q(x,t) \\ \qquad 0 \le x \le L, t \ge 0. \\ \text{BC1}: \omega(0,t) = r_{ji}, t \ge 0. \\ \text{BC2}: \omega(L,t) = r_i^{ave}, t \ge 0. \\ \text{IC}: \omega(x,0) = e^{-x}, 0 \le x \le L. \end{cases} \tag{14}$$

where K is determined $K = e^{pc_r}$.

After constructing the TAHE model shown in formula (11) and (14), we use Crank-Nicolson method (Çelik & Duman, 2012) to solve the TAHE model. Crank-Nicolson method, an efficient finite element method to solve the partial differential equation, has been proved to be unconditionally stable and arrives at a unique convergent solution. Actually, the process of solving TAHE, defined as formula (11) and formula (14), is generally analogous. Taking the TAHE indicated as formula (11) for example, we display some details of the Crank-Nicolson method.

Crank-Nicolson Method. Firstly, we divide the time t and the rod of length L into N_t and N_L equal intervals (i.e., Δt and Δx), respectively. We get $t_0 = 0, t_1 = \Delta t, t_2 = 2\Delta t, ..., t_{N_t} = t$ and $x_0 = 0, x_1 = \Delta x, x_2 = 2\Delta x, ..., x_{N_L} = L$. Secondly, we use definite difference approximation (Haberman, 2005) to replace partial derivatives as follows.

$$\begin{aligned} \frac{\partial \omega(x,t)}{\partial t} &= \frac{\omega(x_j, t_m) - \omega(x_j, t_{m-1})}{\Delta t} \\ \frac{\partial \omega(x,t)}{\partial x^2} &= \frac{1}{2\Delta x^2}(\omega(x_{j+1}, t_m) - 2\omega(x_j, t_m) + \omega(x_{j-1}, t_m) \\ &\quad + \omega(x_{j+1}, t_{m-1}) - 2\omega(x_j, t_{m-1}) + \omega(x_{j-1}, t_{m-1})) \end{aligned} \tag{15}$$

Algorithm 1 TAHE Model for μ_r

Input: S; M; N_L; N_t; ζ;

for each $s_j \in S$ **do**
    Compute $Q(x,t)$ for μ_r
    for each rated user $\mu_i \in M$ rated s_j **do**
        Compute trust-aware valve T_{ir} by formula (1).
        if $T_{ir} \geq \zeta$ **then**
            if $Q(x,t) > 0$ **then**
                Compute k, $Q(x,t)$, $\omega(0,t)$, $\omega(L,t)$ for μ_i
                by formula (12), (13).
                for $m \in N_t$ **do**
                    Compute $\boldsymbol{\omega_L^{N_t}}$ by formula (15), (17).
                end for
            else
                Compute k, $Q(x,t)$, $\omega(0,t)$, $\omega(L,t)$ for μ_i
                by formula (11).
                for $m \in N_t$ **do**
                    Compute $\boldsymbol{\omega_L^{N_t}}$ by formula (15), (16).
                end for
            end if
        end if
    end for
    Compute ev_i for s_i.
end for

Output: S_{od}

In order to state conveniently, we use the following notation $\omega(x_j, t_m) \equiv \omega_j^m$ to indicate the accurate solution at the jth point at time t_m for formula (11) (in the same way as in (14)). By combining formula (14) with (15), we can get the following new equation after simplification,

$$\begin{cases} \mathrm{PDE}: -\sigma\omega_{j-1}^m + (2+2\sigma)\omega_j^m - \sigma\omega_{j+1}^m = \\ \qquad \sigma\omega_{j-1}^{m-1} + (2-2\sigma)\omega_j^{m-1} + \sigma\omega_{j+1}^{m-1} \\ \mathrm{BC1}: \omega_0^m = r_{ji}, t \geq 0. \\ \mathrm{BC2}: \omega_L^m = T_{jr} \cdot r_{ji}, t \geq 0. \\ \mathrm{IC}: \omega_j^0 = e^{-x_j}, 0 \leq x \leq L. \end{cases} \tag{16}$$

where $\sigma = \Delta t/(\Delta x)^2$, $j = \{1, 2, ..., N_L\}$, $m = \{1, 2, ..., N_t\}$. Let $\boldsymbol{\omega^m} = \{\omega_1^m, \omega_2^m, ..., \omega_{N_L-1}^m\}$.

After N_L iterations, vector $\boldsymbol{\omega_L^{N_t}}$ is our solution, and $\boldsymbol{\omega_L^{N_t}} = \{\omega_1^{N_t}, \omega_2^{N_t}, ..., \omega_{N_L}^{N_t}\}$. The average value of $\boldsymbol{\omega_L^{N_t}}$ is our final result, the estimated value of s_i, i.e., ev_i. Identically, we can apply Crank-Nicolson method in formula (14) and get

$$\begin{cases} \mathrm{PDE}: -\widetilde{\sigma}\omega_{j-1}^m + (2+2\widetilde{\sigma})\omega_j^m - \widetilde{\sigma}\omega_{j+1}^m = \\ \qquad \widetilde{\sigma}\omega_{j-1}^{m-1} + (2-2\widetilde{\sigma})\omega_j^{m-1} + \widetilde{\sigma}\omega_{j+1}^{m-1} + Q(x,t) \\ \mathrm{BC1}: \omega_0^m = r_{ji}, t \geq 0. \\ \mathrm{BC2}: \omega_L^m = r_i^{ave} \cdot r_{ji}, t \geq 0. \\ \mathrm{IC}: \omega_j^0 = e^{-x_j}, 0 \leq x \leq L. \end{cases} \tag{17}$$

Table 3: Variables in Algorithm 1

Variable	Description	Variable	Description
M	a set of rated users	$\omega(L,t)$	right BC2
N_L	intervals related with length L	ζ	threshold
N_t	intervals related with time t	$Q(x,t)$	$log\|\mu_r^{sublist}\|$
$\omega(0,t)$	left BC1	k	e^{pc_r}

where $\widetilde{\sigma} = k\Delta t/(\Delta x)^2$ $\boldsymbol{Q} = \{Q(x,t), Q(x,t), ..., Q(x,t)\}$.

Once each s_i's estimate value ev_i is determined, S_{od} is obtained. Algorithm 1 presents the complete process of constructing and solving the TAHE model. We summarize the main notations applied in Algorithm 1 in Table 1 and Table 3.

4.3. MPT-Based Model

The second part is the MPT-based model. After determining the recommended service list S_{od} which satisfies μ_r's requirement, the risky facets of services in S_{od} should be considered by the MPT-based model. As noted above, MPT was originally proposed to help investors select the right stocks to gain the expected return (i.e., profits) while assuming the least risk (i.e., loss of funds) in the finance market. Two essential concepts derived from MPT (Markowitz, 1952) should initially be defined before application.

Definition 1 (**Expected Return**). In portfolio theory (Markowitz, 1952; Francis & Kim, 2013), expected return is often referred to as the future benefit discounted back to the present. For example, if Alice invests one million in real estate and gains five million in five years, five million is **expected return** corresponding to one million after five years. In RSs, the expected return means that the required services satisfy the target user's personal preference and deliver a good experience.

Definition 2 (**Risk**). In portfolio theory (Markowitz, 1952; Francis & Kim, 2013), risk is often referred to as a latent and undesirable loss of benefit. Taking Alice as an example again, if Alice invests 1 million and gains nothing (0 million) in five years, 0 million is **risk** corresponding to 1 millions. In this work, risk in finance is identical to the risky facets in RSs (referred to as Sense Drop and Blue Joy) that can be quantified by variance (Markowitz, 1952).

Specifically, we can regard S_{od} as a service portfolio including n services with an accordant weight ψ_i for each service s_i, i.e.,

$$S_{od} = \{(s_i, \psi_i)\}, \quad s.t., \sum_i \psi_i = 1. \tag{18}$$

where the weight ψ_i indicates how much attention the RSs desire μ_r to lay on s_i. In other words, ψ_i can be used to decide the final ranks of services. In order to get the weight ψ_i, we first get the expected return.

Given a service $s_i \in S_{od}$, we can derive s_i's ratings provided by rated users, denoted by $r_i = \{r_{1i}, r_{2i}, ..., r_{ni}\}$.

The expected return and risk of s_i are computed as follows.

$$
\begin{aligned}
r_i^{Ave} &= \frac{\sum_j r_{ji}}{n} \\
r_i^{Var} &= \frac{\sum_j (r_{ji} - r_i^{Ave})^2}{n}
\end{aligned} \tag{19}
$$

For portfolio S_{od}, the expected return and variance can be computed according to (Wang & Zhu, 2009)

$$
\begin{aligned}
\mathbb{E}(S_{od}) &= \sum_{i}^{n} \psi_i \cdot r_i^{Ave} \\
\mathbb{V}(S_{od}) &= \sum_{i=1}^{n} \psi_i^2 r_i^{Var} + 2 \sum_{i=1}^{n} \sum_{k=i+1}^{n} \psi_i \psi_k r_i^{Var} r_k^{Var} \rho_{ik}
\end{aligned} \tag{20}
$$

where ρ_{ik} is a correlation coefficient between s_i and s_k, which is computed using formula (21),

$$
\rho_{ik} = \frac{Num_{ik}}{Num_i + Num_k - Num_{ik}} \tag{21}
$$

here Num_i is the number of users who purchased s_i, Num_{ik} is the number of common users who bought s_i and s_k.

In our problem, the final target is to get a list of weights $\boldsymbol{\psi}$ to maximize the expected return and minimize the risk (variance) of the services portfolio S_{od}. That is to say, it is necessary to solve the following optimization problem.

$$
\begin{aligned}
&arg \max_{\boldsymbol{\psi}} \ \mathbb{E}(S_{od}) - b\mathbb{V}(S_{od}) \\
&s.t. \sum_{i} \psi_i = 1
\end{aligned} \tag{22}
$$

where b is the specified risk preference parameter (Wang & Zhu, 2009). Here we compute b as $b = fix(log(|G_r| + 1))$, where G_r is a set of services with low ratings provided by μ_r, $fix(x)$ gets the integer part of x. In our experiments, the low rating is set to be smaller than 4 with the range from 1 to 5. The above optimization problem can be solved by approach mentioned in (Zhang et al., 2013). To be more specific, we can obtain the optimal weight $\boldsymbol{\psi}$ by

$$
\boldsymbol{\psi} = \frac{\begin{vmatrix} 1 & \boldsymbol{1'\Sigma^{-1}r} \\ \mathbb{E}(S_{od}) & \boldsymbol{r'\Sigma^{-1}r} \end{vmatrix} \boldsymbol{\Sigma^{-1}1} + \begin{vmatrix} \boldsymbol{1'\Sigma^{-1}1} & 1 \\ \boldsymbol{r'\Sigma^{-1}1} & \mathbb{E}(S_{od}) \end{vmatrix} \boldsymbol{\Sigma^{-1}r}}{\begin{vmatrix} \boldsymbol{1'\Sigma^{-1}1} & \boldsymbol{1'\Sigma^{-1}r} \\ \boldsymbol{r'\Sigma^{-1}1} & \boldsymbol{r'\Sigma^{-1}r} \end{vmatrix}} \tag{23}
$$

where $\boldsymbol{\Sigma}$ is a covariance matrix, $\boldsymbol{r} = \{r_1^{Ave}, r_2^{Ave}, ..., r_n^{Ave}\}'$, and $\mathbb{E}(S_{od})$ can be computed by

$$
\mathbb{E}(S_{od}) = \frac{(xz - y^2)^2 - 2b(x\boldsymbol{r} - y\boldsymbol{1})'\Sigma^{-1}(z\boldsymbol{1} - y\boldsymbol{r})}{2b(x\boldsymbol{r} - y\boldsymbol{1})'\Sigma^{-1}(x\boldsymbol{r} - y\boldsymbol{1})} \tag{24}
$$

where $x = \boldsymbol{1'\Sigma^{-1}1}$, $y = \boldsymbol{r'\Sigma^{-1}1}$, $z = \boldsymbol{r'\Sigma^{-1}r}$. The solution ψ shown in formula (23) determines the final services rank list $\widehat{S_{od}}$ for μ_r.

Table 4: Statistics of datasets

datasets	Users	Services	Ratings	Links	Den_r	Den_c
Epinions	40163	139738	664824	487183	0.01%	0.03%
Epinions_s	2000	3000	100116	243961	1.67%	2.71%
Epinions_l	10000	5000	245512	421824	0.49%	0.42%
Flixster	147612	48794	8196077	7058819	0.11%	0.03%
Flixster_s	2000	3000	2170993	188556	36.18%	4.72%
Flixster_l	10000	5000	5446772	766524	10.89%	3.07%

Note: 'Dens_r' represents the density on ratings, 'Dens_c' represents the density on trust or friendship relationship. Epinions and Flixster are the originial datasets we download, while Epinions_s, Epinions_l and Flixster_s, Flixster_l are what we apply in the experiments.

Table 5: Parameters Setting

Parameters	Value	Utilization
α_1	0.5	Compute pk_{br}
α_2	0.5	
β_1	0.35	Compute T_{ar}
β_2	0.35	
β_3	0.3	
L	1	Solve the TAHE model
t	1	
N_L	6	
N_t	6	

5. Evaluation

5.1. Datasets Description

We employ two datasets to verify the efficiency of ReCRisk: One dataset contains directed trust relationship while the other contains friendship.

The first dataset is Epinions, which is derived from a famous consumer review site and has been widely used for research into trust-based RSs. This dataset[5] includes the information of both user-service ratings and user trust relationships. It contains 40,163 users who rated 139,738 different services at least once. The total number of reviews and trust statements is 664,824 and 487,183, respectively. Another dataset is Flixster[6], which contains 147,612 users who rated 48794 services. The total number of reviews provided by users is 8,196,077, while the total number of friendship links is 7,058,819. In our experiments, we adopt the same setting as in the literature (Feng et al., 2016) to reduce the computational cost of other compared schemes, i.e., we first sort all services and users respectively by counting the number of services rated by each user and the number of users who rate each service in descending order, then select the top 2,000/10,000 users with the top 3,000/5,000 services and the relevant ratings and social links as the subset[7]. The details of datasets are shown in Table 4.

5.2. Parameters Setting

In this section, we will elaborate on the parameters utilized in the experiments. Table 5 lists the given values for some parameters.

[5] http://www.trustlet.org/downloaded_epinions.html

[6] https://drive.google.com/file/d/0ByR_snA9vkyQNExhRVRxQU1NUjA/view

[7] https://drive.google.com/file/d/0ByR_snA9vkyQNExhRVRxQU1NUjA/view

In Table 5, L and t are constant values, which are set as 1 because we aim to study energy transmission in a rod per unit time and length. α_i ($i \in \{1,2,3\}$), β_i ($i \in \{1,2\}$), N_L and N_t can be adjusted, we can change the value of these variables to validate the robustness of our scheme. The effect of different values on the scheme will be stated in the following section. Another parameter ζ mentioned in algorithm 1 is computed as follows: 1) Compute each user's trust-aware valve for the target user μ_r. 2) Get the average of these values, ζ.

5.3. Compared Schemes

The proposed scheme RecRisk will be compared with several state-of-the-art schemes as follows:

- *MF-based* schemes: We select two representative matrix factorization-based schemes, i.e., PMF (Mnih & Salakhutdinov, 2008) and SR2 (Ma et al., 2011). PMF only uses the user-service rating data while SR2 exploits the social network information by regularizing PMF in addition to consideration of user-service rating data. In our experiments, we set the regularization weight of PMF as 0.5, and those for SR2 as 0.5 and 0.25.

- *SLIM*: SLIM generated a Top-N recommendation via aggregating from user purchase/rating profile(Ning & Karypis, 2011). In our experiments, the regularization weights are set as 1 and 0.5.

- *SocSLIM*: SocSLIM (Feng et al., 2016), incorporate social links into SLIM to accomplish recommendation for the target user. It includes other extensions such as TrusteeSLIM, TrusterSLIM, LocTrusteeSLIM, LocTrusterSLIM, LocSocSLIM and LocDSLIM. These extensions are also taken into account in our comparison experiments. The parameters of these schemes can be seen in (Feng et al., 2016).

- *SPMC* (Cai et al., 2017). SPMC incorporates social links into sequential information to improve the recommendation performance. In our experiment, we set the dimension of latent vector, regularization weights and learning rate as 20, 0.1 and 0.05, respectively.

- *SREPS* (Liu et al., 2018). SREPS takes the essential preferences space into account to model the differences between user preferences in RSs and in social networks. In our experiments, we set the parameter α and β as 0.4 and 0.1 for Epinions, 0.3 and 0.2 for Flixster.

- *TCRec* (Lee et al., 2018). TCRec is a matrix factorization method which focuses on the relationship between trustor and trustee and adopt trustors' relationship as latent features. In our experiments, we set its parameters λ_μ, λ_v, λ_s as 0.1, λ_b as 0.01 and β as 100.

Table 6: Comparison on Epinions_s dataset ($N = 10$)

Schemes	HR	ARHR
PMF	0.0712 (**+179.6%**)	0.0268 (**+96.64%**)
SR2	0.0704 (**+182.8%**)	0.0269 (**+95.91%**)
SLIM	0.0987 (**+101.7%**)	0.0451 (**+16.85%**)
LocDSLIM	0.1012 (**+96.74%**)	0.0447 (**+17.90%**)
LocTrusteeSLIM	0.1097 (**+81.49%**)	0.0463 (**+13.82%**)
LocTrusterSLIM	0.1165 (**+70.90%**)	0.0503 (**+4.77%**)
TrusteeSLIM	0.1088 (**+83.00%**)	0.0452 (**+16.59%**)
TrusterSLIM	0.1122 (**+77.45%**)	0.0498 (**+5.82%**)
SPMC	0.1266 (**+57.27%**)	0.0510 (**+3.33%**)
SREPS	0.1307 (**+52.33%**)	0.0516 (**+2.13%**)
TCRec	0.1225 (**+62.53%**)	0.0507 (**+3.95%**)
RecRisk	**0.1991**	**0.0527**

Table 7: Comparison on Flixster_s dataset ($N = 10$)

Schemes	HR	ARHR
PMF	0.1644 (**+135.1%**)	0.0711 (**+64.84%**)
SR2	0.1701 (**+127.2%**)	0.0706 (**+66.01%**)
SLIM	0.1859 (**+107.9%**)	0.0858 (**+36.60%**)
LocDSLIM	0.1944 (**+98.82%**)	0.0933 (**+25.62%**)
LocSocSLIM	0.2033 (**+90.11%**)	0.0976 (**+20.08%**)
SocSLIM	0.2116 (**+82.66%**)	0.1006 (**+16.50%**)
SPMC	0.2464 (**+56.86%**)	0.1057 (**+10.88%**)
SREPS	0.2685 (**+43.95%**)	0.1106 (**+5.97%**)
TCRec	0.2379 (**+62.46%**)	0.1033 (**+13.46%**)
RecRisk	**0.3865**	**0.1172**

Note: The compared schemes in Table 7 is less than that in table 6 because LocSocSLIM includes LocTrusteeSLIM and LocTrusterSLIM, while SocSLIM includes TrusteeSLIM and TrusterSLIM. Hence, we use LocSocSLIM and SocSLIM to represent these four schemes in Table 7.

5.4. Evaluation Methods and Metrics

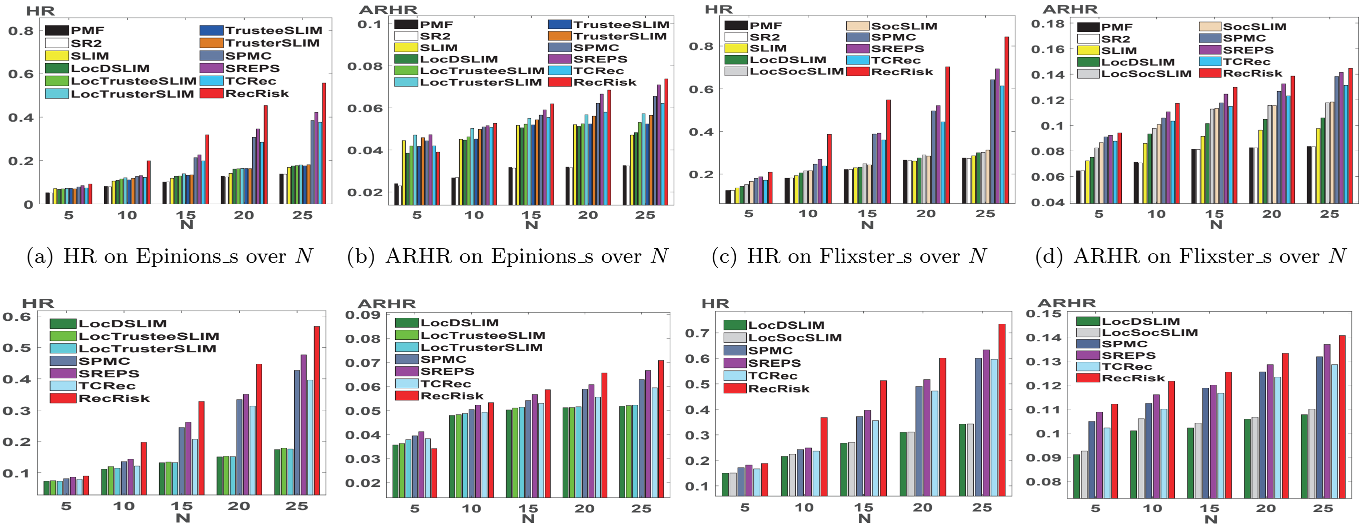
In our experiments, we apply a five-time *Leave-One-Out Cross Validation* to evaluate the performance of the various schemes. In each run, each of the dataset is split into two subsets, i.e., a training set and a testing set by randomly selecting one of the rated services for each user and putting it into the testing set. The remaining of the data is regarded as a training set, a size-N recommendation list in a descendent sequence is then generated via our scheme. We vary N as 5,10,15,20,25 to compare the difference in results.

The recommendation quality is measured via Hit Rate (HR) and Average Reciprocal Hit Rank (ARHR) (Ning & Karypis, 2011; He et al., 2017; Nikolakopoulos & Karypis, 2019; Peker & Kocyigit, 2016)[8]. HR is computed as follows,

$$HR = \frac{\#hits}{\#users} \qquad ARHR = \frac{1}{\#users}\sum_{i=1}^{\#hits}\frac{1}{p_i} \tag{25}$$

where #users denotes the total number of users (e.g., 2,000 and 10,000 in our experiments), #hits is the number of users whose service in the testing set is recommended in the *Top-N* recommendation list. Another metric is ARHR, which is defined as above, where if a service of a user is hit, p is the position of the service in the ranked recommendation list. ARHR is a weighted version of HR, and it

[8] There exist other metrics apart from HR and ARHR, such as Precision, Recall, NDCG, etc. We choose HR and ARHR because of faster computation.

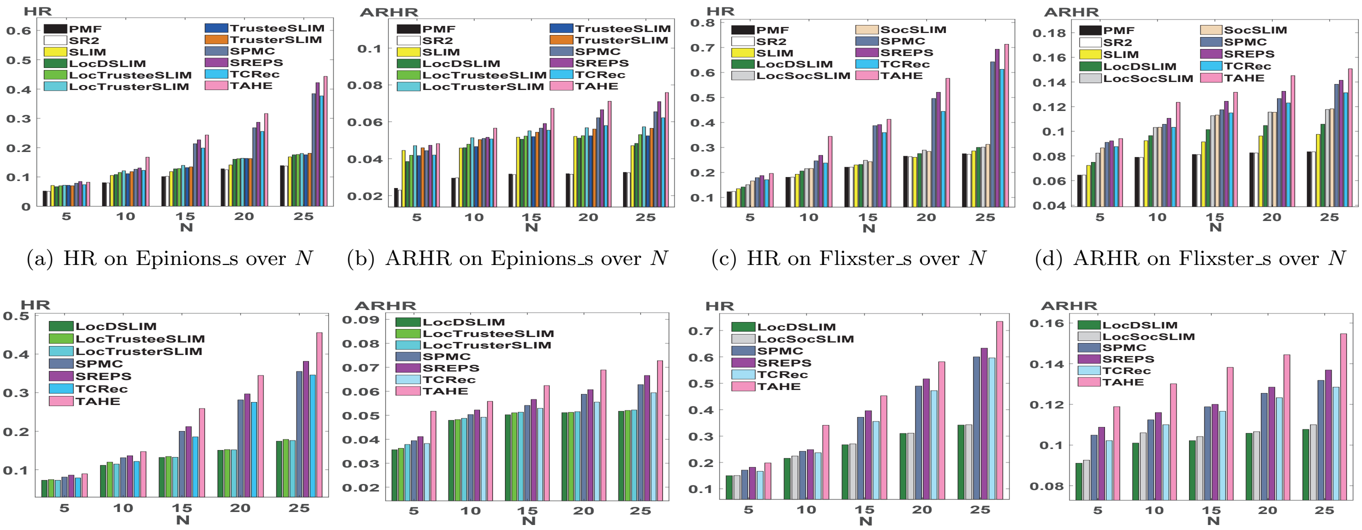


(a) HR on Epinions_s over N (b) ARHR on Epinions_s over N (c) HR on Flixster_s over N (d) ARHR on Flixster_s over N

(e) HR on Epinions_l over N (f) ARHR on Epinions_l over N (g) HR on Flixster_l over N (h) ARHR on Flixster_l over N

Note: Becase of remarkable comparison with PMF, SR2 and the original SLIM and SocSLIM shown in Table 6, Therefore, we only compare LocDSLIM, LocSocSLIM (including LocTrusteeSLIM and LocTrusterSLIM) in Fig.5 (e)-(h) on large datasets, i.e., Epinions_s, Flixster_l.

Figure 5: Recommendation with Risky Facets on datasets



(a) HR on Epinions_s over N (b) ARHR on Epinions_s over N (c) HR on Flixster_s over N (d) ARHR on Flixster_s over N

(e) HR on Epinions_l over N (f) ARHR on Epinions_l over N (g) HR on Flixster_l over N (h) ARHR on Flixster_l over N

Figure 6: Risky Facets-Free Recommendation on datasets

measures how strongly a service is recommended, in which the reciprocal of the hit position at the recommendation list.

As for risky facets evaluation, we use **standard derivation** to measure it in the final recommendation output. The standard derivation is a measure used to quantify the amount of variation or dispersion of a set of data values, and is widely used to measure the risk associated with a portfolio of assets in finance (Markowitz, 1952; Bruno & Shin, 2015). The standard derivation of each s_i in ranked list, (denoted by σ_i) is computed as $\sigma_i = \sqrt{r_i^{Var}}$, where r_i^{Var} is computed as in equation (19). The smaller the value σ_i, the less risky facets a service posesses according to the property of standard derivation (Bruno & Shin, 2015). All experiments are carried out using Matlab R2017b on a PC with Intel 4-core 2.8GHz CPU and 16GB RAM.

6. Experimental Results and Analysis

6.1. Top-N Recommendation Performance on RecRisk

We summarize the different experimental results between RecRisk and other compared schemes over the Epinions_s and Flixster_s datasets shown in Table 6 and Table 7, where N is set as 10. The symbol '+' indicates the increase in the proportion of RecRisk compared with other schemes. The results show that our scheme outperforms the best current scheme, i.e., SREPS. In terms of HR, ReCRisk is about twice as high as PMF, SR2 and SLIM. In comparison with other schemes based on SLIM, the increase in HR is more than 65%. As for ARHR, the highest values attained by current schemes are 0.0516 and 0.1106 (SREPS), respectively. However, our scheme gets a higher value than SERPS with a 2.13% and 5.97% increase, separately. Higher HR and ARHR values indicate that RecRisk can efficiently rank the services preferred by the target user in the top position.

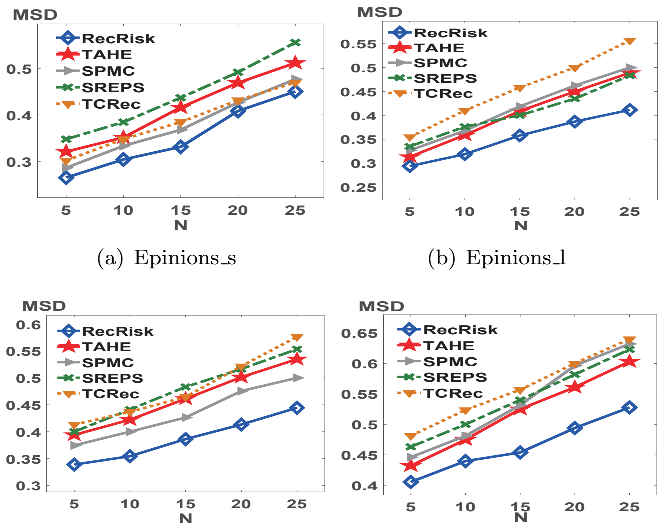
Fig.5 shows the performance of the schemes under different N over the Epinions_s, Epinions_l, Flixster_s and Flixster_l datasets, here we use parameters identical to those presented in Table 5. Some insights derived from Fig. 5 are shown as follows: 1) For both Epinions_s, Flixster_s, Epinions_l and Flixster_l datasets, the of HR and ARHR values for each scheme increase steadily as N increases. 2) Although RecRisk can not achieve the highest ARHR value ($N = 5$) over the Epinions_s dataset shown in Fig.5 (b), it also exceeds PMF and SR2. Except that, our scheme does hit the highest HR and ARHR values on the Epinions_s, Epinions_l, Flixster_s, or Flixster_l datasets. 3) RecRisk generally achieves a significant improvement in HR and ARHR except for when $N = 5, 10$, as shown in Fig.5 (f).

Moreover, the results also reveal the following: 1) The latest schemes, i.e., SPMC, SREPS and TCRec, attain significant results in HR and ARHR compared with SLIM-based schemes. 2) SLIM-based schemes, including LocDSLIM and SocSLIM attain slightly better recommendation performance than PMF and SR2. 3) The social network-based model, LocSocSLIM (including LocTrusteeSLIM and LocTrusterSLIM), makes more accurate recommendations than SLIM and LocDSLIM that do not exploit the social relationship between users. As a result, recommendation schemes with social relationship account contribute to performance improvement.

To summarize, RecRisk accomplishes more accuracy recommendations compared to current state-of-the-art schemes from the evaluation. In addition, RecRisk also considers the risky facets during recommendation while other schemes do not involve this condition.

6.2. Top-N Recommendation Performance on TAHE

The analysis of recommendation performance in Section 5.1 does not cover the risky facets-free scenarios in RSs. In fact, TAHE, the first part of RecRisk, can be applied to a risk facets-free scenario, as shown in Fig.6. Here, we adopt the same parameters setting shown in Table 7 and Fig.5, and generate two points from Fig.6: 1) For risk-free scenarios, both HR and ARHR of each scheme constantly have larger values on these four datasets as N increases; 2) TAHE has a significant improvement for these four data compared with schemes. For HR, the average increase varies from 15.32% ($N = 5$) to 94.52% ($N = 25$) compared with PMF and SR2. For ARHR, although the current best, i,e., SPMC, SREPS, and TCRec outperform other SLIM-based or matrix factorization-based schemes, on average, TAHE can attain higher ARHR value with a 13.57% increase. The above results show the proposed scheme can also be applied in risk facet-free RSs with recommendation performance guaranteed.



(a) Epinions_s (b) Epinions_l

(c) Flixster_s (d) Flixster_l

Figure 7: Risky Facets Evaluation

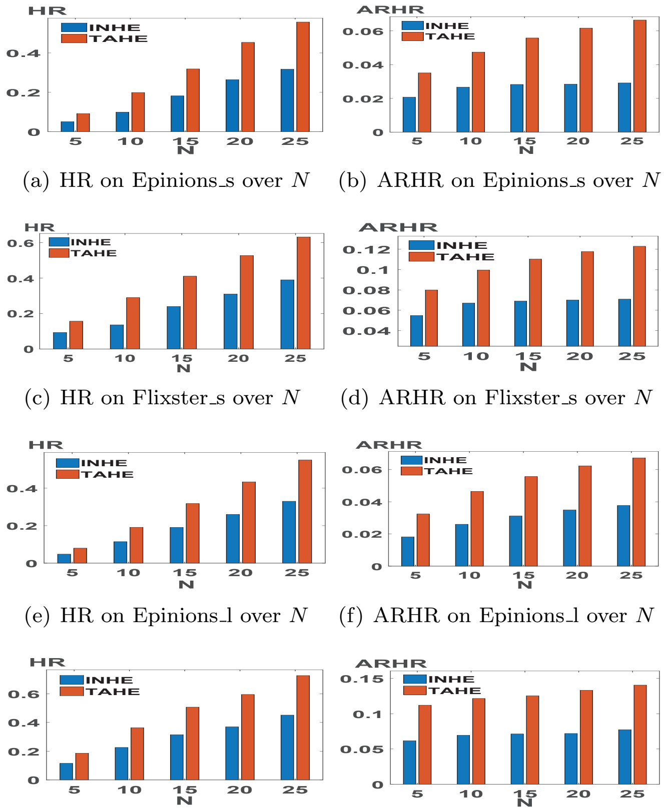
6.3. Risky Facets Evaluation

Because SPMC, SREPS, and TCRec achieve remarkable improvement in recommendation performance compared with SLIM-based and matrix factorization-based approaches, here we evaluate the risky facets of ranked service lists derived by SPMC, SREPS, TCRec, TAHE, and RecRisk. We first extract the top-N ranked services, then compare the average value of risky facets (i.e., the mean standard derivation (MSD) (Bruno & Shin, 2015)) of them.

The results of the MPT-based model are shown in Fig.7. From Fig.7, we can summarize the following viewpoints: 1) MSD becomes larger as N increases. 2) The MSD of RecRisk (colored in blue) is below those of other four schemes. 3) The difference in MSD between TAHE, SPMC, SREPS, TCRec, and RecRisk is becoming increasingly larger as N increases. To conclude, the MPT-based model is validated to efficiently control the risky facets in the final recommendation result.

6.4. Trust-Aware Effect in TAHE

In this section, we mainly discuss the effect exerted by the trust-aware valve on recommendation performance. The trust-aware valve is only related to the TAHE model rather than the MPT-based model. Hence, here we only consider the first part of RecRisk, i.e., TAHE. To be persuaded, we compare TAHE with general heat equation (HE). Based on the discussion in Section 4.2.2, we know that HE can't directly be applied in RSs because of the difference between physics and RSs. As a result, to apply HE, we use the item-neighborhood mechanism (Deshpande & Karypis, 2004) to replace the trust-aware valve as a switch



(a) HR on Epinions_s over N (b) ARHR on Epinions_s over N

(c) HR on Flixster_s over N (d) ARHR on Flixster_s over N

(e) HR on Epinions_l over N (f) ARHR on Epinions_l over N

(g) HR on Flixster_l over N (h) ARHR on Flixster_l over N

Figure 8: Trust-Aware Effect in TAHE on datasets

to control whether or not the target node adopts rated users' rating. The results between item-neighbourhood HE (INHE) and TAHE are shown in Fig.8.

In Fig.8, the comparison is significant because the height difference between INHE (in blue) and TAHE (in red) is quite evident. On average, TAHE achieves more than a 40% increase in HR and ARHR compared with INHE, which validates that trust-aware elements do improve recommendation performance in practice.

6.5. Effect of Parameters on Recommendation Performance

In this subsection, we primarily discuss the effect of the parameters recorded in Table 5 on the performance of RecRisk. We use control variable method to accomplish the experiments on parameters. Control variable method is a scientific research method, which keeps one parameter changeable while the other parameters are held unchangeable during experiments. For example, when we study the effect induced by αs, we fix the values of other parameters, βs and N_L and N_t. When considering the efficiency and computational ability of devices, we randomly select 300 users from the Epinions_s and Flixster_s datasets to estimate parameters.

Parameters α**s**. The effects of α_1 and α_2 are presented in Table 8. For ARHR@10, both Epinions and Flixster attain the highest value at $\alpha_1 = 0.2$ and $\alpha_2 = 0.8$,

Table 8: The Effect of αs

	1	2	3	4	5	MAE
α_1	0.2	0.4	0.5	0.6	0.8	
α_2	0.8	0.6	0.5	0.4	0.2	
Epinions (300 users)						
HR@10	0.2573	0.2600	0.2613	0.2667	0.2540	0.34%
ARHR@10	0.0786	0.0708	0.0768	0.0678	0.0645	0.48%
Flixster (300 users)						
HR@10	0.4339	0.4306	0.4279	0.4219	0.4153	0.59%
ARHR@10	0.1482	0.1442	0.1457	0.1456	0.1443	0.11%

Table 9: The Effect of βs (β_1=0.35)

	1	2	3	4	5	MAE
β_2	0.1	0.2	0.3	0.4	0.5	
β_3	0.55	0.45	0.35	0.25	0.15	
Epinions (300 users)						
HR@10	0.2620	0.2547	0.2713	0.2627	0.2727	0.59%
ARHR@10	0.0778	0.0669	0.0703	0.0733	0.0720	0.28%
Flixster (300 users)						
HR@10	0.4266	0.4246	0.4292	0.4339	0.4405	0.5%
ARHR@10	0.1436	0.1405	0.1453	0.1426	0.1390	0.2%

Table 10: The Effect of βs (β_2=0.35)

	1	2	3	4	5	MAE
β_1	0.1	0.2	0.3	0.4	0.5	
β_3	0.55	0.45	0.35	0.25	0.15	
Epinions (300 users)						
HR@10	0.2513	0.2613	0.2507	0.2747	0.2753	0.99%
ARHR@10	0.0739	0.0766	0.0671	0.0754	0.0688	0.35%
Flixster (300 users)						
HR@10	0.4206	0.4233	0.4286	0.4252	0.4412	0.57%
ARHR@10	0.1432	0.1436	0.1444	0.1458	0.1430	0.088%

Table 11: The Effect of βs (β_3=0.3)

	1	2	3	4	5	MAE
β_1	0.1	0.2	0.3	0.4	0.5	
β_2	0.6	0.5	0.4	0.3	0.2	
Epinions (300 users)						
HR@10	0.2720	0.2673	0.2587	0.2547	0.2707	0.64%
ARHR@10	0.0735	0.0797	0.0703	0.0753	0.0698	0.3%
Flixster (300 users)						
HR@10	0.4106	0.4279	0.4266	0.4207	0.4226	0.48%
ARHR@10	0.1393	0.1436	0.1449	0.1460	0.1456	0.19%

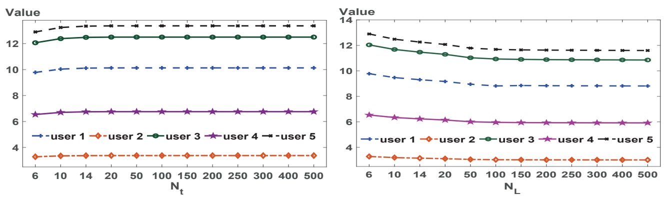
that is, 0.0786 and 0.1482 respectively. For HR@10, Epinions attains the highest at $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$, while Flixster gets it at $\alpha_1 = 0.2$ and $\alpha_2 = 0.8$. From the perspective of both HR and ARHR, Epinions gets the best result at $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, while Flixster's best result is at $\alpha_1 = 0.2$, $\alpha_2 = 0.8$.

Parameters β**s**. Table 9, 10 and 11 reflect the effects of β_1, β_2 and β_3 on recommendation performance. For HR@10, the effect induced by β_2 is more apparent than that of β_1 and β_3 because the mean absolute value (MAE) of β_2 is the highest. For ARHR, β_1, β_2 and β_3 have approximate effect on the Epinions dataset, while β_2 has an apparently slighter effect on Flixster dataset (MAE: 0.088%) compared with β_1 and β_3 (MAE: 0.2% and 0.19%). In addition, compared with αs on HR@10, the influence on recommendation performance yielded by βs is more evident. After comprehensive analysis, it can be concluded that RecRisk will attain relatively better recommendation performance at β_1=0.4, $\beta_2 = 0.35$, β_3=0.25, αs=0.5.

Parameters N_L **and** N_t. N_L and N_t are only related to solving the TAHE model, i.e., Crank-Nicolson method. Therefore, we must keep the parameters related to the

Table 12: The Effect of N_L and N_t

$N_L = 6$					
N_t	6	8	10	12	14
Value	9.7813	9.9405	10.0342	10.0869	10.1118
	3.2852	3.3279	3.3529	3.3669	3.3736
	12.0646	12.2646	12.3825	12.4488	12.4802
	6.5398	6.6408	6.7002	6.7336	6.7459
	12.8981	1.3110	13.3448	133.1131	13.2398
$N_t = 6$					
N_t	6	8	10	12	14
Value	9.7813	9.6033	9.4753	9.3809	9.3089
	32852	3.2341	3.1979	3.1713	3.1512
	12.0406	11.8419	11.6817	11.5634	11.4732
	6.5398	6.4251	6.3429	6.2823	6.2362
	12.8981	12.6592	12.4871	12.3601	12.2634



(a) $N_L = 6$ (b) $N_t = 6$

Figure 9: **Parameters** N_L and N_t

Crank-Nicolson method as constants (except for N_L or N_t when we study it). We execute Algorithm 1 according to the values of N_L or N_t shown in Table 12 and Fig.9. Due to the limited space, we only show the variation of N_L and N_t of five users. The results of other users (which are not shown) corresponds to these five users after statistic analysis. Table 12 and Fig.9 show that: 1) the value increases with N_t becomes larger (when N_L is fixed) but decreases as N_L turns larger (when we fix N_t); 2) Fig. 9 shows the value begins to converge when N_L and N_t exceed 50. This indicates the convergence ability of the Crank-Nicolson Method in solving the TAHE model. 3) The line in Fig.9 belongs to one-one mapping links, which show that the variation of N_L or N_t does not affect the sequence of the services ranked list that we derive.

To summarize, the impact of the variation of parameters on RSs is slight because the MAE is less than 1%. This validates the robustness of RecRisk.

7. Conclusion and Future Work

In this work, we have explored a novel latent problem, i.e., risky facets in expert and intelligent RSs, which have not been studied by the previous works. Here we consider two risky facets: *Sense Drop* and *Blue Joy*, and propose RecRisk, which can simultaneously guarantee high recommendation performance and minimize the associated risks. RecRisk first helps the target user to select those trusted services which satisfy her personal preference via constructing and solving a trust-aware heat equation (TAHE), which considers the variation of personal preference and trust-aware factors between users. Second, we try to strike a balance between user satisfaction and the accompanying risky facets associated with services via a scalable model based on modern portfolio theory. Finally, services are ranked via weighing risky facets and satisfaction and then recommended to target users. Our experimental results show that RecRisk outperforms state-of-the-art recommendation models in terms of HR and ARHR while also considering risky facets.

Although our experimental results have validated the superiority of our RecRisk in terms of recommendation accuracy and risky facets management, there are several limitations related to RecRisk. The first limitation is that we need higher computation complexity. RecRisk consumes more computation time in getting the final result compared with some classical approaches such as PMF or SLIM. That is why we just test our scheme on a fraction of datasets. In the future, more optimization work should be engaged in reducing computation consumption. The second limitation is that RecRisk relies on special datasets. RecRisk is just applied in those datasets which contain social relationships such as Epinions, Ciao, Douban, etc. The performance may be impaired once it is tested on the datasets without social links. The last limitation is that our scheme is just verified on the public datasets rather than in practice. More realistic results would be more persuasive. In the future, we plan to obtain more information related to social relationships among users, such as emotion, typical shopping events, etc. Such information may contribute to better recommendation via constructing a more precise trust-aware valve value.

Based on our work, there are several promising potential future directions for advanced research into expert and intelligent RSs:

- The formation of the trust-aware valve is comparatively simple, which may not be appropriate in more sophisticated scenarios. We can design an intelligent trust judgment method that can autonomously compute trust value via deep learning methodologies such as active learning or reinforcement learning.
- For intelligent RSs, users' emotions such as joy, apathy, indifference or sadness could be critical to their ratings on services. Such ratings along with subjective attitudes also pose a new challenge for accurate recommendation.
- Currently, most intelligent recommendation schemes, including ours, mainly regard social links as a supplementary tool to accomplish item recommendation. However, social scientists have validated that users' preferences and their social links are mutually influenced (Shu et al., 2018; Jiang et al., 2014). Therefore, how to explicitly extract such mutual relationships is also inspiring.

Acknowledgements

This work is supported by the National Key Research and Development Program of China under Grant 2018YFE0126000, The Key Program of NSFC-Tongyong Union

Foundation under Grant U1636209, the National Natural Science Foundation of Chine under Grant 61902292, and the Key Research and Development Programs of Shaanxi under Grant 2019ZDLGY13-07 and 2019ZDLGY13-04. The corresponding author is Qingqi Pei.

References

Abbasi, M. A., Tang, J., & Liu, H. (2014). Trust-aware recommender systems. *Machine Learning book on computational trust, Chapman & Hall/CRC Press*, .

Aggarwal, C. C. (2016). Neighborhood-based collaborative filtering. In *Recommender Systems* (pp. 29–70). Springer.

Alharthi, H., Inkpen, D., & Szpakowicz, S. (2018). A survey of book recommender systems. *Journal of Intelligent Information Systems*, *51*, 139–160.

Amal, S., Tsai, C.-H., Brusilovsky, P., Kuflik, T., & Minkov, E. (2019). Relational social recommendation: Application to the academic domain. *Expert Systems with Applications*, *124*, 182–195.

Ardissono, L., & Mauro, N. (2020). A compositional model of multifaceted trust for personalized item recommendation. *Expert Systems with Applications*, *140*, 112880.

Azadjalal, M. M., Moradi, P., Abdollahpouri, A., & Jalili, M. (2017). A trust-aware recommendation method based on pareto dominance and confidence concepts. *Knowledge-Based Systems*, *116*, 130–143.

Bobadilla, J., Ortega, F., Hernando, A., & Gutiérrez, A. (2013). Recommender systems survey. *Knowledge-based systems*, *46*, 109–132.

Bruno, V., & Shin, H. S. (2015). Capital flows and the risk-taking channel of monetary policy. *Journal of Monetary Economics*, *71*, 119–132.

Cai, C., He, R., & McAuley, J. (2017). Spmc: socially-aware personalized markov chains for sparse sequential recommendation. *arXiv preprint arXiv:1708.04497*, .

Çelik, C., & Duman, M. (2012). Crank–nicolson method for the fractional diffusion equation with the riesz fractional derivative. *Journal of Computational Physics*, *231*, 1743–1750.

Cover, T. M., & Thomas, J. A. (2012). *Elements of information theory*. John Wiley & Sons.

Deshpande, M., & Karypis, G. (2004). Item-based top-n recommendation algorithms. *ACM Transactions on Information Systems (TOIS)*, *22*, 143–177.

Ebrahimnia-Bajestan, E., Moghadam, M. C., Niazmand, H., Daungthongsuk, W., & Wongwises, S. (2016). Experimental and numerical investigation of nanofluids heat transfer characteristics for application in solar heat exchangers. *International Journal of Heat and Mass Transfer*, *92*, 1041–1052.

Feng, X., Sharma, A., Srivastava, J., Wu, S., & Tang, Z. (2016). Social network regularized sparse linear model for top-n recommendation. *Engineering applications of artificial intelligence*, *51*, 5–15.

Francis, J. C., & Kim, D. (2013). *Modern portfolio theory: Foundations, analysis, and new developments* volume 795. John Wiley & Sons.

Gao, P., Miao, H., Baras, J. S., & Golbeck, J. (2016). Star: Semiring trust inference for trust-aware social recommenders. In *Proceedings of the 10th ACM Conference on Recommender Systems* (pp. 301–308). ACM.

Gilkey, P. B. (2018). *Invariance theory: the heat equation and the Atiyah-Singer index theorem*. CRC press.

Golbeck, J. A. (2005). *Computing and applying trust in web-based social networks*. Ph.D. thesis.

Grasse, N. J., Whaley, K. M., & Ihrke, D. M. (2016). Modern portfolio theory and nonprofit arts organizations: Identifying the efficient frontier. *Nonprofit and Voluntary Sector Quarterly*, *45*, 825–843.

Guo, G., Zhang, J., & Thalmann, D. (2012). A simple but effective method to incorporate trusted neighbors in recommender systems. In *International Conference on User Modeling, Adaptation, and Personalization* (pp. 114–125). Springer.

Guo, G., Zhang, J., & Yorke-Smith, N. (2016). A novel recommendation model regularized with user trust and item ratings. *IEEE Transactions on Knowledge and Data Engineering*, *28*, 1607–1620.

Haberman, R. (2005). *Applied Partial Differential Equations: With Fourier series and boundary value problems*. (4th ed.). China Machine Press.

He, X., Liao, L., Zhang, H., Nie, L., Hu, X., & Chua, T.-S. (2017). Neural collaborative filtering. In *Proceedings of the 26th international conference on world wide web* (pp. 173–182).

Hong, D.-G., Lee, Y.-C., Lee, J., & Kim, S.-W. (2019). Crowdstart: Warming up cold-start items using crowdsourcing. *Expert Systems with Applications*, *138*, 112813.

Jannach, D., & Ludewig, M. (2017). When recurrent neural networks meet the neighborhood for session-based recommendation. In *Proceedings of the Eleventh ACM Conference on Recommender Systems* (pp. 306–310). ACM.

Jiang, L., Cheng, Y., Yang, L., Li, J., Yan, H., & Wang, X. (2019). A trust-based collaborative filtering algorithm for e-commerce recommendation system. *Journal of Ambient Intelligence and Humanized Computing*, *10*, 3023–3034.

Jiang, M., Cui, P., Wang, F., Zhu, W., & Yang, S. (2014). Scalable recommendation with social contextual information. *IEEE Transactions on Knowledge and Data Engineering*, *26*, 2789–2802.

Jiang, W., Wu, J., & Wang, G. (2015). On selecting recommenders for trust evaluation in online social networks. *ACM Trans. Internet Techn.*, *15*, 14–1.

Košir, D., Kononenko, I., & Bosnić, Z. (2014). Web user profiles with time-decay and prototyping. *Applied intelligence*, *41*, 1081–1096.

Lee, J., Noh, G., Oh, H., & Kim, C.-k. (2018). Trustor clustering with an improved recommender system based on social relationships. *Information Systems*, *77*, 118–128.

Liu, C.-Y., Zhou, C., Wu, J., Hu, Y., & Guo, L. (2018). Social recommendation with an essential preference space. In *Thirty-Second AAAI Conference on Artificial Intelligence*.

Liu, X. (2015). Modeling users' dynamic preference for personalized recommendation. In *Twenty-Fourth International Joint Conference on Artificial Intelligence*.

Luenberger, D. G. et al. (1997). Investment science. *OUP Catalogue*, .

Ma, H., Zhou, D., Liu, C., Lyu, M. R., & King, I. (2011). Recommender systems with social regularization. In *Proceedings of the Fourth ACM International Conference on Web Search and Data Mining* WSDM '11 (pp. 287–296). New York, NY, USA: ACM. doi:10.1145/1935826.1935877.

Mach, E. (2014). *History and Root of the Principle of the Conservation of Energy*. Cambridge University Press.

Markowitz, H. (1952). Portfolio selection. *The journal of finance*, *7*, 77–91.

Mazeh, I., & Shmueli, E. (2020). A personal data store approach for recommender systems: enhancing privacy without sacrificing accuracy. *Expert Systems with Applications*, *139*, 112858.

Mnih, A., & Salakhutdinov, R. R. (2008). Probabilistic matrix factorization. In *Advances in neural information processing systems* (pp. 1257–1264).

Nikolakopoulos, A. N., & Karypis, G. (2019). Boosting item-based collaborative filtering via nearly uncoupled random walks. *arXiv preprint arXiv:1909.03579*, .

Ning, X., & Karypis, G. (2011). Slim: Sparse linear methods for top-n recommender systems. In *Data Mining (ICDM), 2011 IEEE 11th International Conference on* (pp. 497–506). IEEE.

Otebolaku, A. M., & Andrade, M. T. (2017). Context-aware personalization using neighborhood-based context similarity. *Wireless Personal Communications*, *94*, 1595–1618.

Pan, W., Zhong, H., Xu, C., & Ming, Z. (2015). Adaptive bayesian personalized ranking for heterogeneous implicit feedbacks. *Knowledge-Based Systems*, *73*, 173–180.

Pan, Y., He, F., & Yu, H. (2017). Trust-aware top-n recommender systems with correlative denoising autoencoder. *arXiv preprint* *arXiv:1703.01760*, .

Parvin, H., Moradi, P., & Esmaeili, S. (2019). Tcfaco: Trust-aware collaborative filtering method based on ant colony optimization. *Expert Systems with Applications*, *118*, 152–168.

deLlano Paz, F., Calvo-Silvosa, A., Antelo, S. I., & Soares, I. (2017). Energy planning and modern portfolio theory: A review. *Renewable and Sustainable Energy Reviews*, *77*, 636–651.

Peker, S., & Kocyigit, A. (2016). mrhr: a modified reciprocal hit rank metric for ranking evaluation of multiple preferences in top-n recommender systems. In *International Conference on Artificial Intelligence: Methodology, Systems, and Applications* (pp. 320–329). Springer.

Rafailidis, D., Kefalas, P., & Manolopoulos, Y. (2017). Preference dynamics with multimodal user-item interactions in social media recommendation. *Expert Systems with Applications*, *74*, 11–18.

Seo, Y.-D., Kim, Y.-G., Lee, E., & Baik, D.-K. (2017). Personalized recommender system based on friendship strength in social network services. *Expert Systems with Applications*, *69*, 135–148.

Shu, K., Wang, S., Tang, J., Wang, Y., & Liu, H. (2018). Crossfire: Cross media joint friend and item recommendations. In *Proceedings of the Eleventh ACM International Conference on Web Search and Data Mining* (pp. 522–530).

Wang, J., & Zhu, J. (2009). Portfolio theory of information retrieval. In *Proceedings of the 32nd international ACM SIGIR conference on Research and development in information retrieval* (pp. 115–122). ACM.

Wu, Y., DuBois, C., Zheng, A. X., & Ester, M. (2016). Collaborative denoising auto-encoders for top-n recommender systems. In *Proceedings of the Ninth ACM International Conference on Web Search and Data Mining* (pp. 153–162). ACM.

XU, K.-z., CHANG, C.-r., SONG, J.-r., GAO, H.-x., MA, H.-x., ZHAO, F.-q., & HU, R.-z. (2008). Specific heat capacity, thermodynamic properties and adiabatic time-to-explosion of rdx. *Chinese Journal of Explosives & Propellants*, *4*, 010.

Yera, R., & Martinez, L. (2017). Fuzzy tools in recommender systems: A survey. *International Journal of Computational Intelligence Systems*, *10*, 776–803.

Yuan, Q., Chen, L., & Zhao, S. (2011). Factorization vs. regularization: fusing heterogeneous social relationships in top-n recommendation. In *Proceedings of the fifth ACM conference on Recommender systems* (pp. 245–252). ACM.

Zhang, W., Wang, J., Chen, B., & Zhao, X. (2013). To personalize or not: a risk management perspective. In *Proceedings of the 7th ACM Conference on Recommender Systems* (pp. 229–236). ACM.

Zhu, H., Xiong, H., Ge, Y., & Chen, E. (2014). Mobile app recommendations with security and privacy awareness. In *Proceedings of the 20th ACM SIGKDD international conference on Knowledge discovery and data mining* (pp. 951–960). ACM.