A Simplified Analytical Model for Particle-Geogrid Aperture Interaction

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Abstract

In this paper, the role of aperture size on the particle-geogrid interaction is analysed based on probabilistic mechanics. Property of ballast size distribution is considered using a Weibull distribution. Particle-geogrid aperture interaction is classified into three mechanisms: interlock interaction, no interaction and sliding interaction, based on the relative particle size that is defined as the ratio of the particle size to the aperture size. Model predictions are in good agreement with a previously proposed correlation between the aperture size and the particle-geogrid interface strength from literature.

Keywords

geogrids; geosynthetics; theoretical analysis

List of notations

α	interface efficiency factor
φ	friction angle of unreinforced ballast
δ	apparent friction angle of geogrid-ballast interface
D	particle size
A	aperture size of geogrid
D/A	relative size
D_{50}	average particle size
D_{10}	the particle size that 10% ballast mass is smaller than it
D_{60}	the particle size that 60% ballast mass is smaller than it
C_u	the coefficient of uniformity
D_{min}	minimal particle size
D_{max}	maximum particle size
$\alpha(D/A)$	function of α decided by D/A
$f(D)$	probability density for a particle of size D
$F(D)$	cumulative probability function of Weibull distribution
$P(D)$	probability density function of Weibull distribution
λ	scale parameter of Weibull distribution
k	shape parameter of Weibull distribution
ξ	D/A ratio, relative size
P_i	probability of grain interlock
α_i	interface efficiency factor of interlock interaction
α_s	interface efficiency factor of sliding interaction
α_n	interface efficiency factor when particles do not interlock with a geogrid aperture
d	standard deviation of normal distribution
h	a tiny value to integrate probability density

Introduction

Geogrids have been employed to increase the stability of ballasted track in the past few decades. Several studies have investigated the interaction of the aperture size (A) of geogrids and ballast (McDowell et al., 2006; Brown et al., 2007; Indraratna et al., 2011; Sweta and Hussaini, 2018). McDowell et al. (2006) reported the optimum ratio between aperture size and nominal particle size to be in the proximity of 1.4 in relation to the peak resistance in a discrete element simulation of pull-out testing. Brown et al. (2007) concluded that the optimum aperture size for ballast with an average particle size of 50mm should be 60-80mm to achieve the lowest settlement after 3000 cycles of loading. Hussaini (2012) identified the role of aperture size on the reduction of lateral displacement of geogrid-reinforced ballast interface under cyclic loading and reported the optimum A/D_{50} ratio to be 1.21, where D_{50} is the average particle size. Indraratna et al. (2014) reported the optimum A/D_{50} ratio in the range of 1.1 for the least geogrid-reinforced ballast deformation measured in the field. Palmeira and Góngora (2016) conducted several large scale cyclic loading tests for geogrid-reinforced gravel fill and reported the effect of aperture size on the traffic benefit ratio (TBR), which is defined as the number of loading cycles of reinforced material to that of unreinforced material for the same rut depth at surface. They concluded the optimum A/D_{50} ratio to be approximately equal to 2. Indraratna et al. (2011) reported a correlation between aperture size (A) and interface efficiency factor (α) (Koerner 1998), defined as:

$$\alpha = \tan\delta \,/\, \tan\varphi \qquad (1)$$

where φ is the friction angle of unreinforced ballast and δ is the apparent friction angle at the interface of geogrid and ballast. And the optimum aperture size was reported as $1.2D_{50}$. Sweta and Hussaini (2018) identified the similar correlations between α and A/D_{50} under different shearing rates.

In terms of theoretical modelling, Jewell et al. (1984) derived a micromechanical framework for the geogrid-reinforced soil. Greenwood and Williamson (1966) developed a model for the interaction between two rough surfaces. However, the interaction between geogrid and ballast is different with above interactions, as ballast particles have various sizes resulting in different interaction mechanisms with geogrid. Therefore, this paper aims to develop an analysis to capture and quantify the relationship between the aperture size (A) and interface efficiency factor (α) using the particle size distribution.

2. Theoretical Considerations

2.1 Interface efficiency factor

For simplicity, this study assumes that particle and aperture size has major influence on the interface efficiency factor, compared to other properties of geogrids such as its thickness, tensile strength, bending stiffness of transverse members, junction strength etc. The particle-aperture interaction effect is assumed to be primarily dependent on the relative particle size, defined as the ratio of the particle size (D) to the aperture size of geogrid (A), denoted as $\mathrm{D/A}$. Thus, the interface efficiency factor (α) can be taken as the average of the interface efficiency factors of all particles, hence:

$$\alpha = \int_{D_{min}}^{D_{max}} \alpha(D/A) f(D) dD \tag{2}$$

where $\mathrm{D_{min}}$ and $\mathrm{D_{max}}$ are the minimum and maximum particle size of ballast, respectively, $\alpha(\mathrm{D/A})$ is the function of α controlled by $\mathrm{D/A}$, $\mathrm{f(D)}$ is the probability density for a particle of size D at the interface. For a given $\mathrm{f(D)}$, α is the function of A. In contrast to the empirical relationship where the interface efficiency factor is characterized using mean ballast size (D_{50}), Equation (2) considers the entire particle size distribution.

2.2 Evaluation of $\boldsymbol{f(D)}$

$\mathrm{f(D)}$ can be derived from particle size distribution (PSD) curve. In this Note, Weibull distribution (Weibull, 1951) is adopted to describe ballast PSD (Fang et al, 1993). The cumulative distribution function (CDF) of Weibull distribution is given by:

$$\mathrm{F(D)} = 1 - \mathrm{e}^{-\left(\frac{\mathrm{D}}{\lambda}\right)^{\mathrm{k}}} \tag{3}$$

The probability density function (PDF) is presented by:

$$P(D) = \frac{k}{\lambda}\left(\frac{D}{\lambda}\right)^{k-1} e^{-(D/\lambda)^{k}} \tag{4}$$

where λ is the scale parameter and k is the shape parameter. The influences of λ and k on CDF and PDF are shown in Figure 1. The PDF of Weibull Distribution can be taken as $f(D)$. The correlation between λ, k and PSD parameters can be derived as follow. Taken D as λ, Equation (3) can be derived into:

$$F(\lambda) = 1 - e^{-\left(\frac{\lambda}{\lambda}\right)^{k}} = 1 - e^{-1} \approx 0.6 \tag{5}$$

Therefore, the probability of $D \leq \lambda$ is approximately 0.6, and the physical meaning of λ is the particle size at which 60% ballast mass is smaller than, namely D_{60}. Taken D as D_{10}, where D_{10} is the particle size at which 10% ballast mass is smaller than, then Equation (3) can be re-written as:

$$F(D_{10}) = 1 - e^{-\left(\frac{D_{10}}{\lambda}\right)^{k}} \tag{6}$$

Recalling the physical meaning of λ and D_{10}, Equation (6) becomes:

$$0.1 = 1 - e^{-\left(\frac{D_{10}}{D_{60}}\right)^{k}} = 1 - e^{-\left(\frac{1}{C_u}\right)^{k}} \tag{7}$$

where, C_u is the coefficient of uniformity. Therefore, the correlation between k and C_u can be written as:

$$k = \frac{1}{\log(C_u)} \tag{8}$$

2.3 Evaluation of $\alpha(D/A)$

Following several past studies (Indraratna et al., 2011; Sweta and Hussaini, 2018 and Han et al., 2018), the role of aperture size on the shear strength of particle-geogrid interface can be commonly attributed to three distinct interaction mechanisms:

(1) Interlock Interaction: Once the interlock between geogrids and particles is formed, α is assumed to have a constant value, α_i.

(2) No Interaction: When $D < A$, unless a cluster of particles is considered to be interlocking with a given geogrid aperture, single particles are too small to interact with the geogrid. Thus, $\alpha = 1$ when $D < A$.

(3) Sliding Interaction: When $D \geq A$, if particles do not interlock geogrids with their smaller dimensions, particles are considered to be too large to form a proper interlock, thus the particle tends to slide along the grid interface. In this case, α is assumed to have the constant value, α_s. According to lab observations of Indraratna et al. (2011), Sweta and Hussaini, (2018) and Han et al. (2018), interface efficiency factor is smaller than unity when aperture size is relatively smaller than particle size. If aperture size is extremely small, particle-geogrid interaction would work like particle-geotextile interaction. Therefore, α_s should be smaller than 1 but larger than the interface efficiency factor of ballast-geotextile interaction, which is 0.8 based on the laboratory measurement for ballast-geotextile interaction as reported in an earlier study by Indraratna et al. (2011). To clarify further the evaluation of $\alpha(D/A)$, a dimensionless variable ξ is adopted to replace D/A. Therefore, $\alpha(D/A)$ can be expressed as:

$$\alpha(D/A) = \alpha(\xi) = P_i(\xi) \times \alpha_i + \left(1 - P_i(\xi)\right) \times \alpha_n \tag{9}$$

where $P_i(\xi)$ is the probability of grain interlock for a specific D/A ratio (ξ), α_n is the interface efficiency factor when particles do not interlock with a geogrid aperture , where $\alpha_n = 1$ when $\xi < 1$, and $\alpha_n = \alpha_s$ when $\xi \geq 1$. As the discontinuity of α_n at $\xi = 1$ seldom occurs in practice, the following equation is adopted to describe α_n as also plotted in Figure 2.

$$\alpha_n(\xi) = \alpha_s + (1 - \alpha_s)/\left(1 + e^{20\times(\xi-1)}\right) \tag{10}$$

The probability density of grain interlock is assumed to be a normal distribution whose mean equals to 1, and when ξ = 1, the highest probability of sustaining a particle-geogrid interlock is achieved. Therefore, $P_i(\xi)$ can be expressed as follows;

$$P_i(\xi) = \int_{\xi-h}^{\xi+h} \frac{1}{d\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\xi-1}{d}\right)^2} \tag{11}$$

where d is the standard deviation of normal distribution, h is a tiny value to integrate probability density.

To determine values of α_i, α_s, d and h discussed above, a series of direct shear tests was conducted using ballast with uniform particle size and geogrids under the same procedures suggested by Indraratna et al. (2011). Single-size particles were sieved and divided into four size intervals: 13.2-19mm, 19-26.5mm, 26.5-37.5mm, 37.5-53mm. Uniform sizes of ballast were established by averaging two sizes of intermittent sieves as 16mm, 23mm, 32mm and 45mm. Physical characteristics of geogrids used in testing are shown in Table 1. The equivalent aperture sizes (21-64mm) are derived according to the approach explained by Indraratna et al. (2011), which is the square root of the opening area that is taken as the equivalent aperture size of rectangular apertures. The diameter of the largest inscribed circle is taken as the equivalent aperture size of triangular apertures. The interface efficiency factors (α) for various A/D ratios (ξ) are shown in Table 2. Experimental results are plotted in Figure 3.

When α_i = 2.9, α_s = 0.83, d = 0.17 and h = 0.05, Equation (9) has a reasonable fit with experimental data. With obtained values of d and h, Equation (11) is plotted in Figure 4. As shown in Figure 4, the probability of grain interlock reaches the maximum of 23.1% when ξ = 1, and it decreases to zero when ξ = 0.5 and 1.5. This means that small particles whose size is smaller than 0.5A and large particles whose size is larger than 1.5A have extremely low probability of interlocking with the geogrid apertures.

3. Model Validation

To capture the effect of entire PSD, a set of large-scale direct shear test data from Indraratna et al. (2011) and from laboratory investigations using similar procedures by the Authors are used to validate the proposed model. The fresh latite ballast PSDs used in the current study and by Indraratna et al. (2011) are shown in Figure 5. Additional direct shear test on well-graded ballast with the coefficient of uniformity of 2.3 was conducted in this study. It is noted that Indraratna et al. (2011) adopted a more uniform ballast gradation with a coefficient of uniformity of 1.87. Physical characteristics of geogrids used in testing are shown in Table 1. Corresponding interface efficiency factors are shown in Table 3. The parameters used in model validation are listed in Table 4.

The comparison between model predictions and the experimental data from Indraratna et al. (2011) and current study is shown in Figure 6. It shows a good agreement between the theoretical predictions and the experimental data. It can be found that the variation of the interface friction of more uniform ballast conducted by Indraratna et al. (2011) has a slightly higher peak of α with a narrower range of aperture sizes having α > 1 when compared to that of less uniform ballast used in the current study. The reinforcing effect is provided purely by the interlock between particle and geogrids. The probability of grain interlock for specific PSD can be evaluated as follow

$$\mathrm{P_i} = \int_{D_{min}}^{D_{max}} \mathrm{P_i}(\xi) f(D) dD \tag{12}$$

When A/D_{50} = 1.35, the probabilities of grain interlock are 11.2% for more uniform PSD, and 9.3% for more well-graded PSD. The higher probability of grain interlock for more uniform PSD makes α of Indraratna et al. (2011) higher than that of the current study. However, when A/D_{50} = 2.2, the probabilities of grain interlock are 1.6% and 1.7% for more uniform and more well-graded PSD, respectively , resulting in a lower value of α for more uniform ballast. This model shows that different PSD gradations have different probabilities of grain interlock, leading to different interface behaviours. It can be seen that the model can reasonably capture the aperture size effect on different ballast gradations.

4. Model Limitations

This model considers the interaction between particles and apertures of a geogrid, hence it has limitations as explained below. Apart from the relative size of particle in relation to grid aperture size, other properties of geogrids such as its thickness, tensile strength, bending stiffness of transverse members, junction strength etc., can influence the grid-ballast interaction (Brown et al., 2007; Cuelho et al., 2014; Palmeira and Góngora, 2016). However, these parameters of geogrids have been excluded from the current mathematical model for simplicity.

5. Conclusions

In this study, the use of probabilistic analysis in predicting the correlation between aperture size (A) and the interface efficiency factor (α) by exploiting three distinct interaction mechanisms is demonstrated. The proposed three particle-geogrid interaction mechanisms, namely: interlock interaction, no interaction and sliding interaction, are consistent with the findings of laboratory investigations from other researchers. The developed analytical model considers the ballast size gradation using Weibull distribution.

The developed theoretical model is particularly advantageous when ballast size gradations have different coefficients of uniformity (C_u), which are not reflected by the D_{50}. The resulting model predictions are in good agreement with the interface efficiency factors (α) based on two sets of laboratory studies conducted by the authors and through past literature. When the particle size distribution becomes more uniform, the interface efficiency factor increases at the optimum aperture size but decreases at non-optimum aperture sizes. The model can represent well the variation of the interface behaviour between ballast and geogrids considering the geogrid's aperture size and particle size distribution, which previous models failed to capture.

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Table 1 Physical characteristics of the geogrids used in the current study

Geogrids	Type	Aperture Shape	Aperture Size, A (mm)	Rib Width (mm)	Ultimate Tensile Strength (kN/m)
G1	Extruded	Triangle	21*	1	19
G2	Woven	Square	25	6	43
G3	Welded	Square	32	7	30
G4	Woven	Rectangle	43*	8	43
G5	Extruded	Rectangle	64*	5	30

Note: * Equivalent aperture size; All geogrids are biaxial products

Table 2 Interface efficiency factors for various D/A ratios (ξ)

Geogrids Type	Uniform Ballast Size, D (mm)	D/A ratios, ξ	Interface Efficiency Factor, α
G1	23	1.09	1.15
	32	1.52	0.88
G2	23	0.92	1.26
	32	1.28	1.05
G3	16	0.5	1.00
	23	0.72	1.07
	32	1	1.33
	45	1.41	0.89
G4	23	0.53	1.01
	32	0.74	1.04
G5	23	0.36	1.00
	32	0.5	1.00

Table 3 Interface efficiency factors for well-graded PSD ($C_u = 2.3$) measured in current study

Geogrids	Interface Efficiency Factor, α
G1	0.93
G2	0.94
G3	1.11
G4	1.16
G5	1.05

Table 4 Model Parameters

Data Source	α_i	α_s	d	h	D_{min}	D_{max}	λ	k
Current Study	2.9	0.83	0.17	0.05	5	60	38	2.75
Indraratna et al. (2011)					10	60	38	3.68

Figure captions

Figure 1. (a) Weibull cumulative distribution function, Equation (3); (b) Weibull probability density function, Equation (4)

Figure 2. Correlation between α_n and ξ according to Equation (10)

Figure 3. Comparisons between Equation (9) and lab data when α_i = 2.9, α_s = 0.83, d = 0.17 and h = 0.05

Figure 4. Plot of Equation (11) when d = 0.17 and h = 0.05

Figure 5. Particle size distributions used in current study and Indraratna et al. (2011)

Figure 6. Comparisons between model predictions and experimental results