Cascaded Nonlinear Attitude Observer and Simultaneous Localisation and Mapping

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Abstract

This paper presents a novel integration of the nonlinear observer theory and simultaneous localisation and mapping for aerial navigation applications. This extends the previous work by the authors in which a nonlinear observer was applied to the attitude estimation and integrated navigation problem. The key novelty of this work is in the feedback correction mechanism from the linear SLAM estimator to the nonlinear observer, which enables the attitude correction from the feature position measurements. We utilise the relationship between the acceleration error and the attitude error, and the pseudo-inverse of a skew-symmetric matrix for the attitude feedback. Lyapunov-based stability analysis is provided for a simplified model without considering the gyroscope bias. Flight dataset is used to confirm the method. Thanks to the robustness of the nonlinear observer and the optimal linear estimator, the vehicle pose and map features are estimated effectively.

1 Introduction

Robust pose, or position and attitude (orientation), estimation is a fundamental problem for autonomous unmanned aerial vehicles (UAVs), in particular operating with high-speed and high-dynamics manoeuvres. With the growing number of availability of small-scale UAV platforms, the development of a robust yet simple pose estimation algorithm has drawn significant attention from robotics, control and navigation community. The most commonly used techniques have been based on extended Kalman filtering (EKF) or its nonlinear variants to handle the nonlinearity in the vehicle dynamics and observations [Vidal *et al.*, 2018][Li and Mourikis, 2013]. For example, the EKF has been widely applied for aerospace applications such as attitude-heading reference systems, integrated inertial navigation systems, and simultaneous localisation and mapping (SLAM) [Kim *et al.*, 2020 in press][Vidal *et al.*, 2018][Bjorne *et al.*, 2017].

Such filtering methods, however, are computationally demanding for an embedded processor in a small-scale platform. More importantly, it is difficult to guarantee the robustness of the filter due to the nonlinearity of the system. To address these issues, researchers have actively investigated a nonlinear observer (NLO) for the attitude estimation problem [Mahony *et al.*, 2011], GNSS/Inertial integrated navigation [Grip *et al.*, 2012], and visual-inertial SLAM [Wang and Tayebi, 2018]. One of the critical elements of such approach is in the use of accelerometer output, called a specific force as it contains both vehicular and gravitational accelerations, to estimate the gravitational direction (or the plumb-bob direction). If the vehicular dynamics are moderate or low, as in the most land vehicles or hovering drones, a complementary filtering (or EKF) of the accelerometer and gyroscope outputs can provide a robust attitude estimate as well as the gyro biases.

On the contrary, if the vehicular dynamics is sufficiently high, additional aiding information is required. [Euston *et al.*, 2008] utilises the air-data information to compensate for the centripetal acceleration by estimating the angle-of-attack during the coordinated turns of a UAV. [Grip *et al.*, 2012] fuses GNSS information to determine the acceleration using the nonlinear attitude observer and linear position observer, thus in an observer-observer framework, and [Bjorne *et al.*, 2017] investigates the use of visual information with fixed-gain nonlinear observers.

In this work, we propose a cascaded observer-estimator system, in which a nonlinear observer handles the attitude nonlinearity, while a linear Kalman filter handles the linear translational dynamics. This approach is particularly advantageous for SLAM applications, as the linear Kalman filter (LKF) can optimally handle the large-sized map features, whilst the observer-

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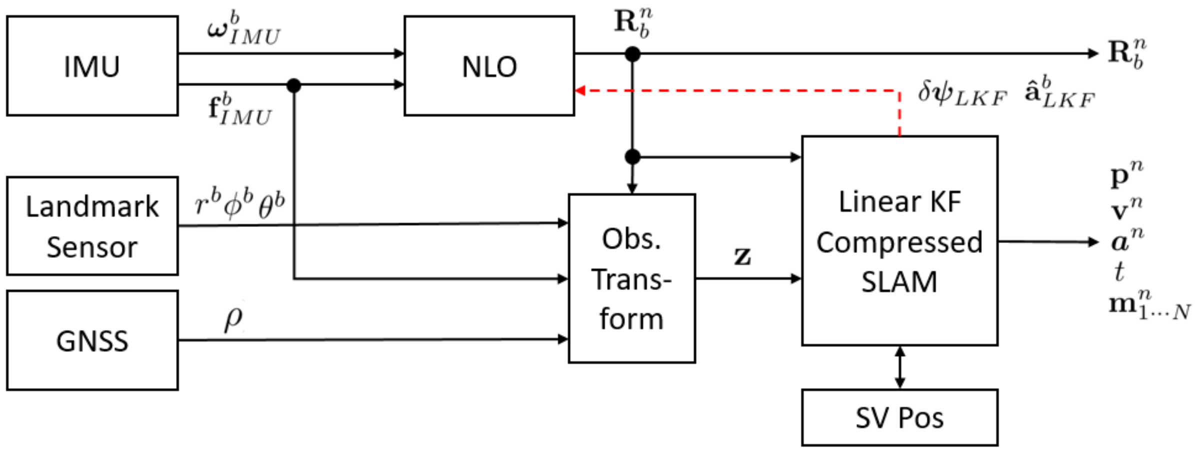

Figure 1: The cascaded observer-estimator architecture consisting of a nonlinear observer (NLO) for the attitude estimation, and a linear Kalman filter (LKF) for SLAM. The key aspect of this structure is the attitude correction term ($\delta\boldsymbol{\psi}_{LKF}$) from the LKF to enhance the NLO performance.

based method relies on the manual tuning of the gains typically from the steady-state analysis as in [Bjorne *et al.*, 2017]. The added computational complexity of LKF can be handled effectively by using the compressed estimator as in [Kim *et al.*, 2020 in press]. The main contributions of this paper are:

- An observer-estimator framework is proposed, in which a nonlinear attitude observer provides robust stability while the linear SLAM estimator offers the optimality for the linar part. The key novelty is in aiding of the observer from the acceleration error estimated from SLAM through the use of pseudo-inverse.

- An asymptotic stability proof is provided using the Lyapunov stability analysis.

- The convergence of the attitude error is verified for a high-speed UAV flight dataset.

To the best of our knowledge, integrating the nonlinear observer and linear SLAM estimator has not been addressed elsewhere, as well as its related stability analysis. Following the introduction, Section 2 provides an overview of the cascaded observer-estimator structure. Section 3 discusses the nonlinear observer for the attitude estimation, focusing on the feedback loop from the external estimator. Section 4 details the linear Kalman filter in the context of SLAM. It describes the linear dynamics and linear/nonlinear observation models. Section 5 presents an experimental study using a fixed-wing UAV flight dataset followed by conclusions.

2 Cascaded Observer-Estimator Structure

A continuous-time stochastic dynamic system with a transition model $\mathbf{f}(\cdot)$ and observation model $\mathbf{h}(\cdot)$ can be described as

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)) \tag{1}$$
$$\mathbf{z}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{v}(t)), \tag{2}$$

where $\mathbf{x}(t)$ and $\mathbf{u}(t)$ are the state and control input vector at time t, respectively, with $\mathbf{w}(t)$ and $\mathbf{v}(t)$ being the process and observation noise, respectively. The system can be further partitioned into a nonlinear (NL) and linear (L) parts as below

$$\dot{\mathbf{x}}_{NL}(t) = \mathbf{f}(\mathbf{x}_{NL}(t), \mathbf{u}_{NL}(t), \mathbf{w}_{NL}(t)) \tag{3}$$
$$\dot{\mathbf{x}}_L(t) = \mathbf{A}(t)\mathbf{x}_L(t) + \mathbf{B}\mathbf{w}_L(t), \tag{4}$$

where $\mathbf{A}$ and $\mathbf{B}$ are the linear dynamic and input matrix, respectively.

Figure 1 illustrates the cascaded observer-estimator architecture used to solve the SLAM problem in this work. The nonlinear observer utilises the IMU (inertial measurement unit) angular-velocity measurement ($\boldsymbol{\omega}^b_{IMU}$) as an input to the nonlinear attitude dynamic model. The specific force ($\mathbf{f}^b_{IMU}$) is used as a nonlinear observation by providing an *indirect* angular measurement through the estimated gravity direction. The output of the nonlinear observer is the estimated rotation matrix ($\hat{\mathbf{R}}^n_b$) which converts a vector defined in the body frame[1] to the one in the navigational frame[2].

[1]The body frame is defined with roll (x-axis), pitch (y-axis) and yaw (z-axis).

[2]The navigation frame is defined with North (x-axis), East

The observer output is used to transform the observations, that is, $3D$ feature and IMU specific force measurements are converted to the ones in the navigational frame, which enables the linear observation updates in the filter. A linear Kalman filter (LKF) accounts for the linear translational dynamics of the system, and a constant-acceleration model is utilised in this work. A highly efficient compressed fusion algorithm is incorporated by partitioning the state into a local and global state [Kim *et al.*, 2020 in press]. Compared to the other cascaded methods, our approach has a feedback correction loop from LKF to NLO to enhance performance. The underlying intuition is that any positional error observed in the LKF implies an acceleration error, which then carries information on the misalignment error. Thanks to the LKF framework, the large size of the gain matrix is not manually tuned, but optimally computed, which is the key benefits compared to the previous work as in [Grip *et al.*, 2012].

3 Nonlinear Attitude Observer

The attitude of the vehicle is estimated using a nonlinear observer. The attitude can be parameterized in several different ways, such as quaternion or Euler angles, we adopt the rotational matrix parameterization $\mathbf{R}_b^n \in \mathbb{SO}(3)$. With the augmented bias state $\mathbf{b}^b$, the nonlinear dynamic model becomes

$$\dot{\mathbf{x}}_{NL} = \mathbf{f}(\mathbf{x}_{NL}, \mathbf{u}_{NL}, \mathbf{w}_{NL}), \tag{5}$$

with,

$$\dot{\mathbf{R}}_b^n = \mathbf{R}_b^n[\boldsymbol{\omega} + \mathbf{w}_{NL}]_\times \tag{6}$$

$$\dot{\mathbf{b}}^b = \mathbf{0}, \tag{7}$$

where $[\boldsymbol{\omega} + \mathbf{w}_{NL}]_\times \in \mathbf{so}(3)$ is a skew-symmetric matrix of the input $\boldsymbol{\omega} \in \mathbb{R}^3$ and the noise $\mathbf{w}_{NL}$, with the superscript-n denoting the navigation frame and the superscript-b a body frame attached to the vehicle.

The gravity observation is a vector measurement of the plumb-bob direction from the IMU accelerometers. If the vehicle manoeuvring is not severe, the specific force measurement ($\mathbf{f}^b_{IMU}$) can be reasonably approximated to the plumb-bob direction ($\mathbf{g}^b_{IMU}$). However, this is not valid for the UAV dataset used in this work, and additional knowledge on the vehicle acceleration $\hat{\mathbf{a}}^b$ is required from the LKF,

$$\mathbf{g}^b_{IMU} = \mathbf{f}^b_{IMU} + \hat{\mathbf{a}}^b_{LKF}. \tag{8}$$

The compensated gravitational vector is then compared with the predicted gravity direction ($\hat{\mathbf{g}}^b$) from the observer, generating a multiplicative error ($\mathbf{e}$). Following the spirit in [Mahony *et al.*, 2008], a proportional and integral controller is adopted. In our observer-estimator framework, the estimator (that is SLAM) provides information on the navigational error which can be used to correct attitude estimate in the observer. To utilise this information, we implement an additional proportional feedback loop for $\delta\hat{\boldsymbol{\psi}}_{LKF}$ estimated from the LKF block (more details in Section 4.1),

$$\boldsymbol{\omega} = (\omega^b_{IMU} - \mathbf{b}^b) + \left(K_P\mathbf{e} + K_I\int \mathbf{e}dt\right) + K_A\delta\hat{\boldsymbol{\psi}}^b_{LKF},$$

where K_P, K_I, K_A are the proportional, integral and angular (misalignment) control gain, respectively.

4 Linear SLAM Estimator

Given the rotation matrix estimated from the NLO, the translational vehicle dynamics becomes linear. We employ a constant acceleration model for the vehicle augmented with a constant receiver clock-drift and a random constant landmark model.

$$\dot{\mathbf{x}}_L = \mathbf{A}\mathbf{x}_L + \mathbf{B}\mathbf{w}_L, \tag{9}$$

where $\mathbf{A}$ and $\mathbf{B}$ are defined from the linear dynamic models,

$$\dot{\mathbf{p}}^n = \mathbf{v}^n, \quad \dot{\mathbf{v}}^n = \mathbf{a}^n, \quad \dot{\mathbf{a}}^n = \mathbf{w}_1 \tag{10}$$

$$\dot{ct}_b = ct_d, \quad \dot{ct}_d = w_2 \tag{11}$$

$$\dot{\mathbf{m}}^n_{1\cdots N} = \mathbf{0}, \tag{12}$$

with the state vector $\mathbf{x}_L = [\mathbf{x}^T, \mathbf{m}^T]^T$ defined as

$$\mathbf{x} = [\mathbf{p}^{nT}, \mathbf{v}^{nT}, \mathbf{a}^{nT}, ct_b, ct_d]^T \tag{13}$$

$$\mathbf{m} = [\mathbf{m}_1^{nT}, \mathbf{m}_2^{nT}, \cdots, \mathbf{m}_N^{nT}]^T, \tag{14}$$

where, $\mathbf{p}^n$ represents the vehicle position, $\mathbf{v}^n$ the vehicle velocity, $\boldsymbol{a}^n$ the vehicle acceleration, ct_b and ct_d the receiver clock bias and drift, and $\mathbf{m}^n_i$ the i^{th}-landmark position, respectively.

The SLAM system in this work follows the all-source navigation framework, fusing three types of observations: IMU, $3D$ visual features with depth estimator, and pseudorange observations from GPS satellites. The observations and corresponding covariance are transformed into the navigation frame using the estimated attitude from the NLO, resulting in linear observation models. That is the IMU specific force measured in the body frame is transformed into that of the navigation frame, and fused in the filter. The range (r^b), bearing (ϕ^b) and elevation (θ^b) measurement from the i^{th}-landmark is also transformed to a relative $3D$ position vector in the navigation frame, enabling linear filter update. Using the linear state and observation models, and their corresponding covariance, the filter predicts and updates the state estimates and the covariance matrix.

(y-axis) and Down (z-axis). In this work, the Map Grid of Australia 1994 (MGA94) is used as a local-fixed, local-tangent frame.

4.1 Pseudo-inverse of Specific Force

The estimated acceleration error ($\tilde{\mathbf{a}}^n$) in the LKF is related to the misalignment error of the rotation matrix [Kim and Sukkarieh, 2007] as

$$\hat{\mathbf{R}}_b^n \triangleq \mathbf{R}_b^n(\mathbf{I} + [\delta\boldsymbol{\psi}]_\times) \tag{15}$$

$$\tilde{\mathbf{a}}^n = -\mathbf{f}^n \times \delta\boldsymbol{\psi} = -[\mathbf{f}^n]_\times \delta\boldsymbol{\psi}^n. \tag{16}$$

$[\mathbf{f}^n]_\times$ is a skew-symmetric matrix and thus not invertible. In this work, we utilise the SVD (singular value decomposition) of the matrix, yielding the pseudo-inverse,

$$[\mathbf{f}^n]_\times = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \tag{17}$$

$$[\mathbf{f}^n]_\times^\dagger = \mathbf{V}\boldsymbol{\Sigma}^\dagger\mathbf{U}^T, \tag{18}$$

with $\Sigma^\dagger$ being the reciprocal matrix of each non-zero element on the diagonal, leaving the zeros in place. Therefore, the feedback term to the NLO becomes

$$\delta\boldsymbol{\psi}_{LKF}^n = -[\mathbf{f}^n]_\times^\dagger \tilde{\mathbf{a}}^n. \tag{19}$$

The resulting control signal $\boldsymbol{\omega}$ becomes

$$\boldsymbol{\omega} = \underbrace{\boldsymbol{\omega}_{IMU}^b}_{\text{Feedforward}} + \underbrace{\left(K_P + \frac{K_I}{s}\right)\mathbf{e}}_{\text{Gravity Feedback}} - \underbrace{K_A\delta\boldsymbol{\psi}_{LKF}^b}_{\text{LKF Feedback}}, \tag{20}$$

in which, the first two terms are the feedforward and feedback terms from the IMU information, and the last term is for the observer-estimator coupling.

5 Stability of the Nonlinear Observer

To understand the stability of the observer-estimator system, we consider the simplified feedback terms without the bias estimation. Tthe stability analysis with the bias term was provided in [Grip *et al.*, 2012] by utilising the bias projection function in a observer-observer framework. We consider the stability of the observer-estimator system and the simplified control signal becomes

$$\boldsymbol{\omega} = K_P(\mathbf{g}^b \times \mathbf{R}\mathbf{g}^n) - K_A(\mathbf{R}_n^b[\mathbf{f}^n]_\times^\dagger \tilde{\mathbf{a}}^n), \tag{21}$$

in which, the first term accounts for the misalignment between the measured gravity and predicted one. The second term represents the misalignment due to the acceleration error provided from the LKF estimator.

Theorem 1 *The system (3) with an injection (21) becomes asymtotocailly stable for the region given a sufficiently large proportional gain K_{P1},*

$$\frac{K_{A1}\|\tilde{\mathbf{r}}_0\|}{K_{P1}\|\tilde{\mathbf{r}}\|} < \tilde{s} < 1, \tag{22}$$

in which $\tilde{s}$ and $\tilde{\mathbf{r}}$ are the scalar and vector parts of the error quaternion, and K_{A1} and K_{P1} are the feedback gains of the LKF estimator and observer, respectively.

Remark 1 *Two assumtions are used:*

- *The gyroscope bias term is assumed known and calibrated, and thus $\mathbf{K}_I = 0$*
- *The misalignment error estimated from the LKF estimator is assumed a bounded signal, that is, $\|[\mathbf{f}^n]_\times^\dagger \tilde{\mathbf{a}}^n)\| < \|\tilde{\mathbf{r}}_0\|$.*

Proof 1 *For a unit error quaternion $\tilde{q} = [\tilde{s}, \tilde{\mathbf{r}}^T]^T$, we can define the Lyapunov candidate function as*

$$V = \|\tilde{\mathbf{r}}\|^2 = 1 - \tilde{s}^2. \tag{23}$$

Along the state trajectory, $\dot{V}$ becomes

$$\begin{aligned}
\dot{V} &= -2\tilde{s}\dot{\tilde{s}} \\
&= -2\tilde{s}\left(\frac{1}{2}\mathbf{R}_b^n\tilde{\mathbf{r}}^T\boldsymbol{\sigma}^n\right) \\
&= -\tilde{s}\tilde{\mathbf{r}}^T\mathbf{R}_b^n\left(K_P(\mathbf{g}^b \times \mathbf{R}_n^b\mathbf{g}^n) - K_A(\mathbf{R}_n^b[\mathbf{f}^n]_\times^\dagger\tilde{\mathbf{a}}^n)\right) \\
&\leq -2\tilde{s}^2K_P\|\tilde{\mathbf{r}} \times \boldsymbol{\sigma}^n\|^2 + \tilde{s}K_A\tilde{\mathbf{r}}^T\tilde{\mathbf{r}}_0 \\
&\leq -2\tilde{s}^2K_P\|\tilde{\mathbf{r}}\|^2\|\boldsymbol{\sigma}^n\|^2 + \tilde{s}K_A\|\tilde{\mathbf{r}}\|\|\tilde{\mathbf{r}}_0\| \\
&\leq -2K_{P1}\|\tilde{\mathbf{r}}\|^2\tilde{s}\left(\tilde{s} - \frac{K_{A1}\|\tilde{\mathbf{r}}_0\|}{K_{P1}\|\tilde{\mathbf{r}}\|}\right),
\end{aligned}$$

where the result from [Grip et al.*, 2012] is used, and $([\mathbf{f}^n]_\times^\dagger\tilde{\mathbf{a}}^n) < \|\tilde{\mathbf{r}}_0\|$ represents the bounded misalignment angle. If $\dot{V} < 0$, then the proposed feedback system will be asymptotically stable. Thus,*

$$\dot{V} < 0 \iff \frac{K_{A1}\|\tilde{\mathbf{r}}_0\|}{K_{P1}\|\tilde{\mathbf{r}}\|} < \tilde{s} < 1, \tag{24}$$

which can be guaranteed by selecting the observer gain K_{P1} as sufficiently large compared to the estimator gain K_{A1}.

6 Experiment Results

To verify the proposed method, we utilise a flight dataset recorded from a UAV platform as detailed in [Sukkarieh *et al.*, 2003]. The dataset includes measurements from a low-grade IMU sensor, a low-cost GNSS receiver from CMC-Novatel, and a monochrome camera installed in a down-looking configuration. A total of 150 artificial visual landmarks, white plastic sheets of $1m \times 1m$ to estimate the depth, were installed on the ground, and those positions were surveyed using a real-time kinematic GNSS receiver with $\leq 20cm$ accuracy.

Figures 2(a) and 2(c) compare the Euler angles (roll and heading) with the reference-EKF outputs (dashed line in red), NLO+accelerometer (solid line in cyan), NLO+gyroscope (dashed-dot in black), and the proposed NLO+SLAM (solid blue). From the enhanced plots ((b) and (d)), it can be visibly observed that the estimated angles from NLO+SLAM converge to the EKF

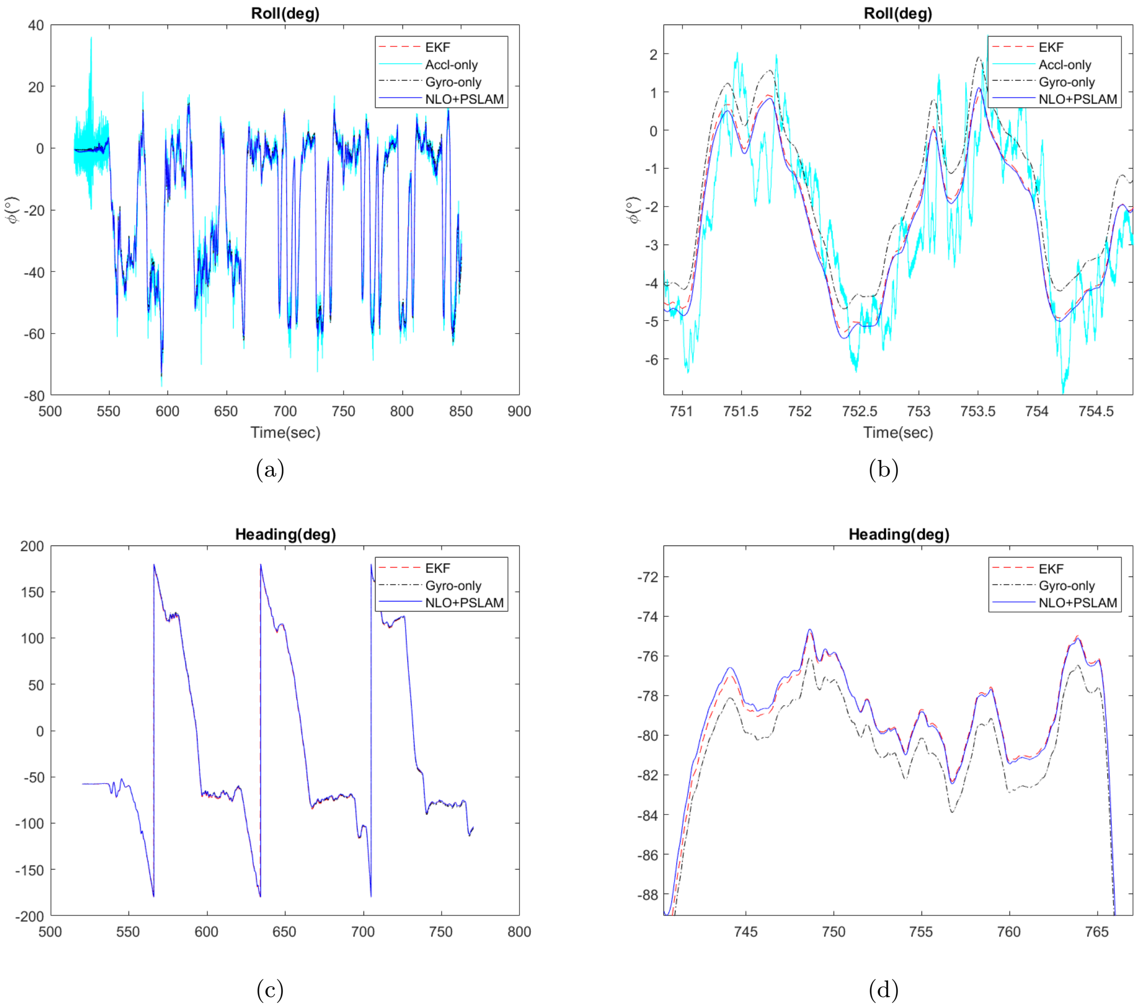

Figure 2: (a) Estimated roll from the NLO with (b) enhanced views. (c) Estimated heading from the NLO with (d) enhanced views. It can be seen that the NLO+SLAM results show convergence to the full-EKF solutions, while NLO+accelerometer (solid in cyan) or NLO+gyroscope (broken-line in black) diverge from the reference.

solutions consistently, while the NLO+gyroscope solutions diverge. The NLO+acceleration outputs show fluctuating solutions due to the low signal-to-noise ratio of the IMU acceleration. This is particularly the case for the high-speed, high-manoeuvring vehicles, in which the approximated gravity vector from the specific force contains large errors due to the dynamic manoeuvres. It can be observed that the yaw angle is indirectly observed from the LKF aiding, converging to the EKF solution. This is due to the fact that the SLAM system can estimate the acceleration error in the navigation frame, from the landmark observations, which in turn provides the information on the misalignment angles, making the full attitude state observable. This is one of the key benefits of the proposed observer-estimator method.

7 Conclusions

This article presented a novel integration of a nonlinear attitude observer and a linear SLAM estimator, in which the nonlinear observer handled the nonlinear attitude dynamics. In contrast, the linear Kalman filter estimates the linear translational dynamics of the vehicle and map. The acceleration error calculated from the SLAM filter was converted to a misalignment correction term through the pseudo-inverse of the specific force. Lyapunov-based stability analysis was provided, showing an asymptotic convergence for a region controlled by the feedback gains. For a UAV flight dataset, the proposed method confirmed the convergence of the attitude estimates to the reference solutions. Future work is on performing a more rigorous stability analysis, including the IMU biases.

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