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BRIDGE OPERATIONAL MODAL IDENTIFICATION USING SPARSE BLIND SOURCE SEPARATION

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Abstract. *The bridge infrastructures are subjecting to continuous degradation due to ageing, environmental and excess loading. Monitoring of these structures is a key part of any maintenance strategy as it can give early warning if a bridge is becoming unsafe. Most of the current approaches are using direct measurements that installs the sensors at different specific locations on the bridge to capture the dynamic characteristics of the structure under random input, such as wind loads, traffic loads and ground motions. Based on the assumption on the white noise characteristics of the random input, structural properties of the bridge could be extracted from the vibration responses only. However, the bridge is subjected to non-stationary traffic loads, and the frequency characteristics of vibrations are varied. Especially for short-span bridges, the non-stationary traffic load excitation is significant and most of the existing output-only structural identification methods could not be used to assess the bridge condition. This study proposes a blind source separation (BSS) method using short time Fourier transform (STFT) for the analysis of non-stationary measurements in time frequency (TF) domain. The proposed method is capable of source component separation from response measurement for underdetermined problems when the number of independent measurements (sensors) is less than that of source component. The proposed method is applied to a cable-stayed bridge in the field for the operational modal identification under different traffic conditions.*

Keywords: vehicle-bridge interaction; BSS; time frequency analysis; underdetermined

1 INTRODUCTION

Vehicle-bridge interaction has a profound impact on the technologies of bridge structural health monitoring (SHM) (Sun, 2013). It is very difficult to measure the operational excitation to the bridge (such as the wind load and traffic), thus output only analysis methods are used widely for bridge SHM. Based on the assumption on the white noise characteristics of the random input, the structural properties of the bridge can be extracted from the bridge vibration responses only. However, most bridges are generally subjected to nonstationary excitations while in service (Kim and Kim, 2017). The non-stationary properties of bridge vibration under a passing vehicle is an important topic in bridge SHM (Xiao et al., 2017). Blind source separation (BSS) techniques have been widely used for structural modal identification and health monitoring (Sadhu et al., 2017) which used to recover special source components from the measured data only. Second-order blind identification and independent component analysis are two of the most used BSS methods that provide promising results when the number of sensors are greater than that of source components.

Moreover, due to the large amount of the existing bridges that have no sensor instrumentation, it would be time-saving and cost-effective if only less sensors are used. However, under the circumstance where the measurement from sensors are less than the number of active vibration modes of the system, which is referred as underdetermined problem, the traditional BSS methods do not work, because the mixing matrix is not invertible. To solve the underdetermined blind source separation (UBSS) problems, sparse representation of the sources in some domain is seek and the separation is carried out in that domain to exploit sparseness. Time-frequency (TF) techniques, such as Hilbert-Huang transform, wavelet, and short time Fourier transform (STFT) etc., have been used to analyse non-stationary signals (Zhu et al., 2012). They have also been widely used to solve UBSS problem for output only modal identification and structural damage detection (Nagarajaiah and Basu, 2010; Nagarajaiah and Yang, 2014). The advantage of TF techniques is that they are signal based methods and can be used for output only modal identification. The simplest feature based signal processing procedures in TF is via energy concentration. The idea is to analyse the behaviour of the energy distribution, i.e., the concentration of energy at certain time instant or certain frequency band or more generally, in some particular time and frequency region. Signal processing using energy concentration as a feature in the TF domain essentially consists of evaluating a TFR of the given signal (Sejdić et al., 2009). One of the well-known time-frequency representation and most used in practice is the short-time Fourier transform (STFT) (Aissa-El-Bey et al., 2007). In this study, a novel output only technique based on the spectrogram is proposed to perform the modal analysis for the bridge from the structural responses under traffic load. The proposed method is used for the condition assessment of a cable-stay field bridge.

2 VEHICLE-BRIDGE INTERACTION MODEL

Considering a vehicle-bridge interaction system as shown in **Error! Reference source not found.**, the vehicle is moving over the simply supported bridge at a constant velocity *v*. A half-vehicle model consisting of 4 degree of freedom (DOF) is used in this study. The equation of motion for the vehicle is obtained as

$$\begin{bmatrix} I_0 & 0 & 0 & 0 \\ 0 & m_0 & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{d}_1 \\ \ddot{d}_2 \\ \ddot{d}_3 \\ \ddot{d}_4 \end{Bmatrix} + \begin{bmatrix} b_1^2 c_1 + b_2^2 c_2 & b_1 c_1 - b_2 c_2 & -b_1 c_1 & b_2 c_2 \\ b_1 c_1 - b_2 c_2 & c_1 + c_2 & -c_1 & -c_2 \\ -b_1 c_1 & -c_1 & c_1 + c_3 & 0 \\ b_2 c_2 & -c_2 & 0 & c_2 + c_4 \end{bmatrix} \begin{Bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \\ \dot{d}_4 \end{Bmatrix} + \begin{bmatrix} b_1^2 k_1 + b_2^2 k_2 & b_1 k_1 - b_2 k_2 & -b_1 k_1 & b_2 k_2 \\ b_1 k_1 - b_2 k_2 & k_1 + k_2 & -k_1 & -k_2 \\ -b_1 k_1 & -k_1 & k_1 + k_3 & 0 \\ b_2 k_2 & -k_2 & 0 & k_2 + k_4 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ k_3 u_1 + c_3 \dot{u}_1 \\ k_4 u_2 + c_4 \dot{u}_2 \end{Bmatrix} \quad (1)$$

where m_0 m_1 m_2, k_1 k_2 k_3 k_4, c_1 c_2 c_3 c_4 are vehicle parameters relating to the mass, stiffness and damping of each part of the vehicle (as shown in Figure 1), respectively. I_0 is the rotational stiffness and b_1, b_2 are the distance between the axles and the gravity centre of the vehicle body. d_1, d_2, d_3, d_4 denote the vehicle displacements at each degree of freedom. u_1, u_2 are the displacements of the contact points.

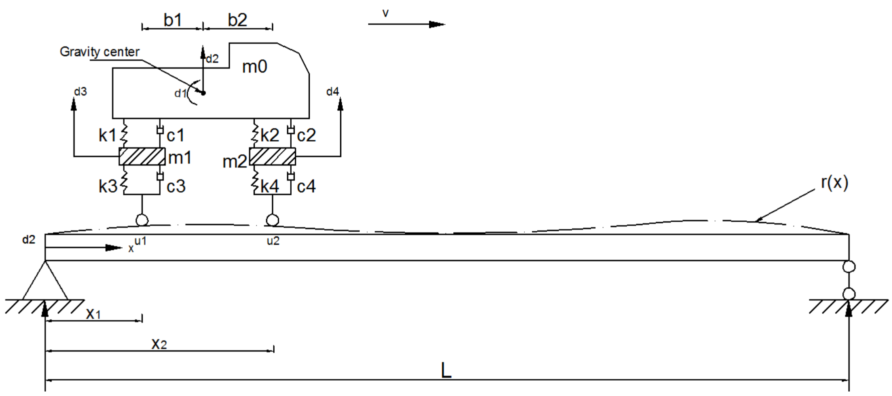


Figure 1 Vehicle-bridge interaction model

The equation of motion of the bridge is given as

$$\mathbf{M}_b\ddot{\mathbf{d}}_b + \mathbf{C}_b\dot{\mathbf{d}}_b + \mathbf{K}_b\mathbf{d}_b = \mathbf{F}_b^{\text{int}} \tag{2}$$

where $\mathbf{M}_b$, $\mathbf{C}_b$, $\mathbf{K}_b$ are the mass, damping and stiffness matrices of the bridge, respectively; $\mathbf{F}_b^{\text{int}}$ is the vector of interaction forces acting on the bridge. $\mathbf{d}_b, \dot{\mathbf{d}}_b, \ddot{\mathbf{d}}_b$ are the vectors of displacement, velocity and acceleration responses of the bridge, respectively. The displacement of the bridge can be expressed as follows with the modal superposition method

$$d_b(x,t) = \sum_{i=1}^{N} \phi_i(x)\, Y_i(t) \tag{3}$$

where N is the number of modes considered. $\phi_i(x), Y_i(t)$ are the mode shape and modal response of the ith mode, respectively.

Substituting Eq. (3) into Eq. (2) and applying the orthogonality of vibration modes, Eq. (2) becomes:

$$\ddot{Y}_i + 2\xi_i\omega_i\dot{Y}_i + \omega_i^2 Y_i = F_2(t)\phi_i|_{x=vt} + F_1(t)\phi_i|_{x=vt-b} \tag{4}$$

where ω_i, ξ_i are the natural frequency and the damping ratio of the ith mode of the bridge and $b = b1 + b2$; The interaction force acting on the bridge at the contact points is given as (Nguyen, 2015):

$$\begin{cases} F_1 = -m_1 g - m_1\ddot{d}_3 - \frac{m_0 g b_2}{b_1+b_2} - \frac{I_0\ddot{d}_1 + m_0 b_2 \ddot{d}_2}{b_1+b_2} \\ F_2 = -m_2 g - m_2\ddot{d}_4 - \frac{m_0 g b_1}{b_1+b_2} - \frac{I_0\ddot{d}_1 + m_0 b_1 \ddot{d}_2}{b_1+b_2} \end{cases} \tag{5}$$

3 SIGNAL MODEL AND ASSUMPTIONS

Let $s_i(t) \in \mathbb{C}^{(n\times1)}, i = 1, \dots n$, be n underlying source signals and denote $\mathbf{s}(t) = [\mathbf{s}_1(t), \mathbf{s}_2(t), \dots, \mathbf{s}_n(t)]^T$. At the output of the sensor array are m observed mixture signals $x_j(t)$, where $j = 1 \dots, m$, that are represented by $\mathbf{x}(t) = [\boldsymbol{x}_1(t), \boldsymbol{x}_2(t) \dots, \boldsymbol{x}_m(t)]^T$. Under the instantaneous linear mixture model, the mixture signals can be modelled as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \boldsymbol{\eta}(t) \tag{6}$$

where the mixing matrix $\mathbf{A} = [\boldsymbol{a_1}, \boldsymbol{a_2}, \dots \boldsymbol{a_N}]$ represents the transfer between the source and the mixture, and $\boldsymbol{\eta}(t)$ is additive noise vector. When n>m, it is said to be in the underdetermined case. Solving UBSS problem is to develop proper method to recover the sources and estimate the mixing matrix in Eq. (6), using only the information of the observed signals. Considering Eq. (3), it is to recover the mode shape matrix and the single-mode modal responses. Structural dynamic parameters, i.e. modal frequencies and damping ratios, can then be extracted from the modal responses.

Two following assumptions are made for the signal model: the column vectors of matric $\mathbf{A}$ are assumed to be pairwise linearly independent; that is for all $i \neq j$, $\mathbf{a}_i$ and $\mathbf{a}_j$ are linearly independent. The second assumption is the sources are assumed to have different structures and localization properties in the TF domain. More precisely, it is assumed the sources to be disjoint in the TF domain.

One of the well-known time-frequency representation (TFR) and most used in practice is the short-time Fourier transform. Let $x(t)$ denotes a complex signal and $h(t)$ a complex window function, both functions of time t. the short-time Fourier transform can be presented as

$$X_w(t, f) = \int_{-\infty}^{\infty} x(\tau)h(\tau - t)e^{-j2\pi f\tau}\, d\tau \tag{7}$$

The power spectral density (PSD) of the original signal $x(t)$ windowed by $h(t)$, called the spectrogram, is given by

$$S_w(t, f) = |X_w(t, f)|^2 \quad -\infty < t, f < \infty \tag{8}$$

The low cost of implementation for the STFT, hence for the spectrogram, together with the advantage of being free of cross terms, justifies the fact that the STFT is most used in practice (Aissa-El-Bey et al., 2007).

4 COMPONENT SEPARATION ALGORITHM FOR THE UBSS

A method for solving the UBSS problem using spectrogram (as illustrated in Figure 2) is introduced here consisting of the following four steps:

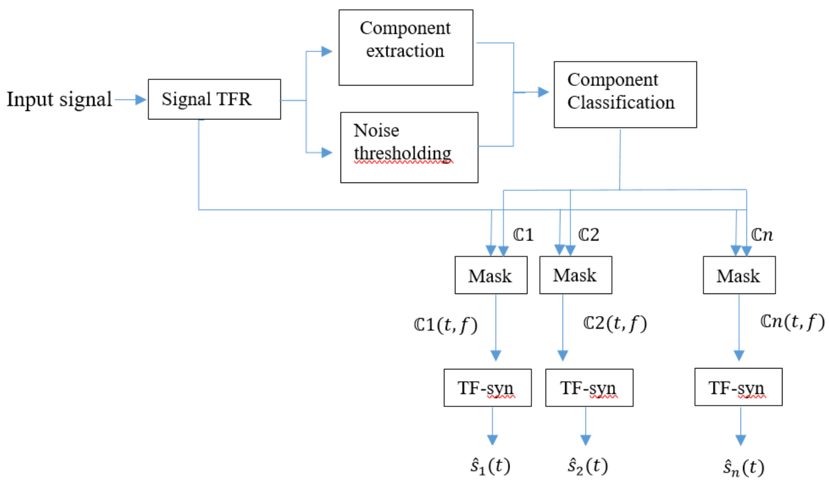


Figure 2 Flowchart of the proposed algorithm

1. Compute and spatial average the spectrogram of the signals with Eqs. (7) and (8);
2. Monocomponent extraction using “peak detection and tracking” approach
   For the component extraction, a technique similar to one of the proposed algorithms in (Barkat and Abed-Meraim, 2004) is proposed. The component separation algorithm

assumes all components of the signal exist at almost all time instants. Various components are extracted based on their peaks in the time-frequency plane. The key stages of the technique are given as following:

(1) The first step consists in noise thresholding to remove the undesired "low" energy peaks in the time-frequency domain. For each time-slice (t_p, f) of the TFR, apply a criterion for all the frequency points f_k belonging to this time-slice with a threshold ϵ_1

$$S_w(t_p, f_k) = \begin{cases} S_w(t_p, f_k) & if\ S_w(t_p, f_k) > \epsilon_1 \\ 0 & otherwise \end{cases} \tag{9}$$

Typically, $\epsilon_1 = 5\%$ of the point with maximum energy ($\max\left(S_w(t_p, f)\right)$) is selected.

(2) Peak detection to estimate the number of components.
For a noiseless and cross-terms free TFR, the number of components at a given time instant t_0 can be estimated as the number of peaks of the TFD slice $S_w(t_0, f)$. By searching and counting the peaks of TFR, it ends up with a number *d* that yields an estimate of the number of components in the signal.

(3) Component separation
   a. Assign an index to each of the *d* components in an orderly manner
   b. For each time instant *t* (starting from *t*=1), find the component frequencies as the peak positions of the TFR slice $S_w(t_p, f)$.

3. Components clustering
A classification procedure was proposed as the third stage. This component classification procedure groups the components from the second stage of the algorithm based on the minimum distance between any pair of components. If two components belong to the same actual component, their distance in the time-frequency plane is going to be smaller than the distance between the considered component and any other component. Mathematically, it is decided $S_w(t_i, f)$ and $S_w(t_j, f)$ to belong to the same class if

$$d(S_w(t_i, f), S_w(t_j, f)) < \epsilon_2 \tag{10}$$

where ϵ_2 is a properly chosen positive scalar and *d* is a distance measure. By applying the classification procedure, one can group a certain number of components from the second phase. This last number corresponds to the actual number of components in the original signal. Based on clustering information, one can define a time-frequency binary mask to separate the (t,f) region where each source is present alone (Boashash and Aïssa-El-Bey, 2018). The TF binary masking operation is defined as:

$$\hat{S}_{s,i}(t, f) = S_w(t, f)\Omega_i(t, f) \tag{11}$$

where $\hat{S}_{s,i}(t, f)$ is the estimated TFR of the *i*th source, and where

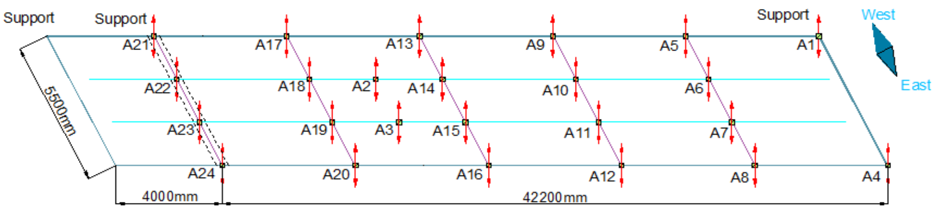
$$\Omega_i(t, f) = \begin{cases} 1, & if\ (t, f) \in \mathbb{C}_i \\ 0, & otherwise. \end{cases} \tag{12}$$

4. Signal synthesis using inverse Fourier transform is carried out to recover the original source waveforms from the separated (t, f) components in last steps.

5 MODAL FREQUENCY ASSESSMENT OF A CABLE-STAY BRIDGE

A long-term monitoring system has been installed on a cable-stayed bridge (as shown in Figure 3(a)). It is a single lane highway bridge with a span 46m and a width 5m. There are 24 accelerometers on bridge deck and Figure 3(b) shows the sensor locations. A data acquisition system continuously records the data from sensors with the sampling rate 600Hz. In this study, the vehicle-induced bridge responses are used to verify the proposed method. Three traffic cases have been considered using the full-scale field bridge monitoring system, i.e. Case 1: there is no vehicle passing over the bridge; Case 2: there is a light vehicle passing over the bridge and Case 3: there is a heavy vehicle passing the bridge. The time of record is 12 seconds for each case.

(a) The cable-stayed bridge



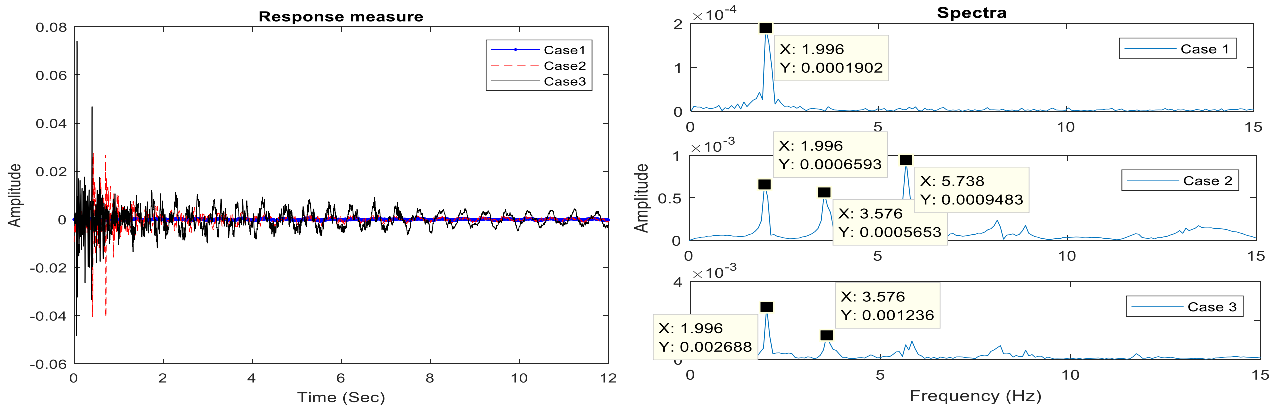
(b) Sensor location

Figure 3 Long-term monitoring of a cable-stayed bridge

Figure 4 shows the acceleration response for these three cases and the corresponding Fourier spectra from sensor A7. From Figure 4(b), only the first bridge frequency at 2.00Hz can be identified for Case 1 and the first three modes can be clearly identified for Cases 2 and 3 as 2.00Hz, 3.57Hz and 5.74Hz. The passing vehicle is an effective tool to excite the bridge for extracting the bridge frequencies. Response measurements under different cases from sensor A7, A11, A14 and A18 are used for analysis using proposed method. The response source components and their spectra for Case 1 and 2 are given in Figures 5 and 6, respectively. From Figure 5, it can be seen that three bridge modal response components are extracted from bridge responses. For Case 2, six bridge modal response components are extracted from the measurements of four sensors which demonstrate the effectiveness of proposed method for solving UBSS problems. The results for Case 3 are very similar to that of Case 2, thus only the identify frequencies are provided and summarized in Table 1 together with other two cases.

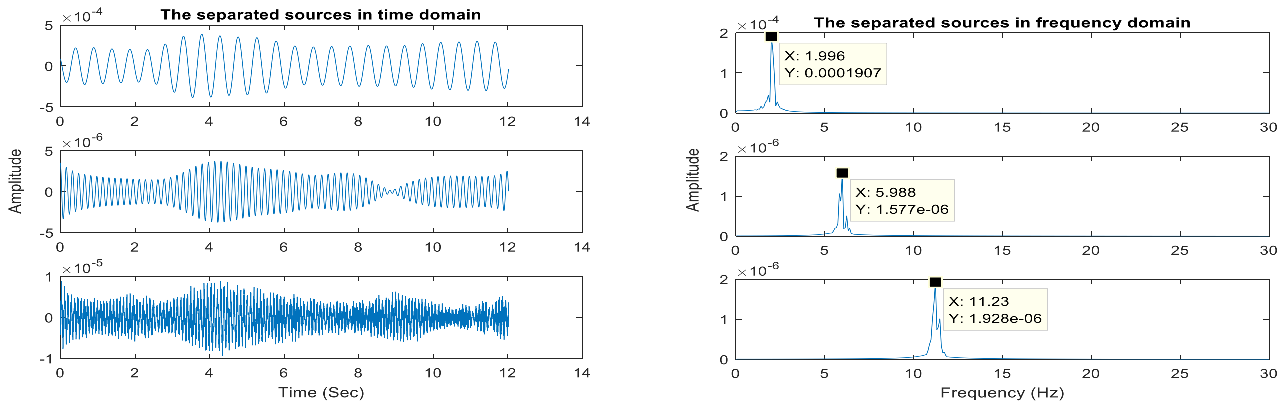
Table 1 identified frequencies of response components for different cases

	Frequency (Hz)					
	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
Case 1	2.04	--	5.95	--	--	11.23
Case 2	2.03	3.63	5.74	8.14	8.89	11.76
Case 3	2.01	3.61	5.79	8.11	8.84	11.74



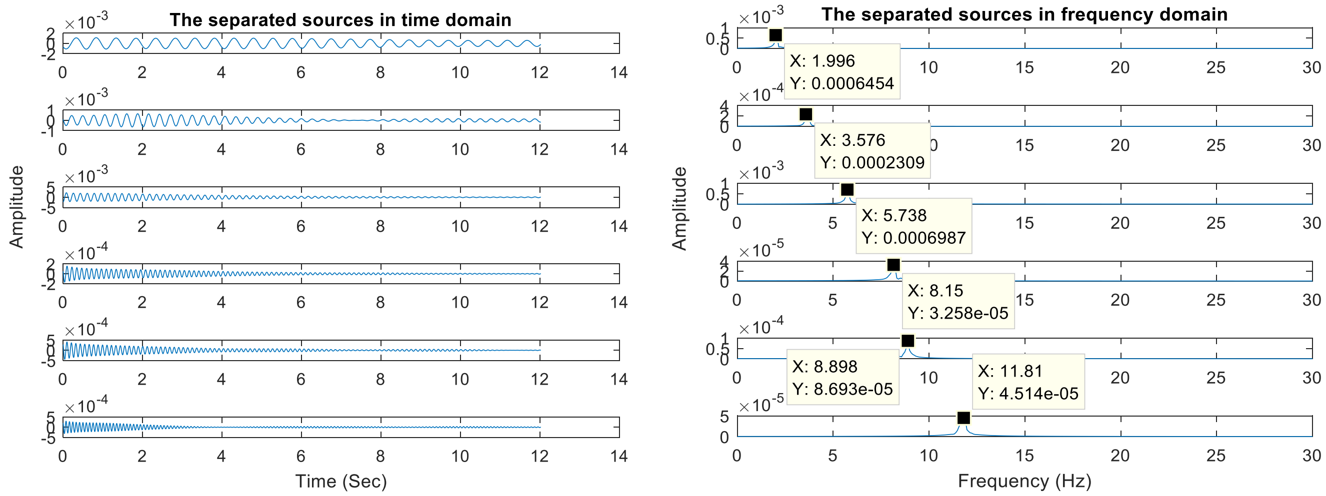
(a) Dynamic responses (b) Spectra

Figure 4 Dynamic responses and spectra from sensor A7 under different traffic cases



(a) Separated components (b) Spectra

Figure 5 Separated response components and their spectra for Case 1



(a) Separated components (b) Spectra

Figure 6 Separated response components and their spectra for Case 2

6 CONCLUSION

This study has presented a new method for the analysis of nonstationary signals in time frequency domain using spectrogram. Bridge modal responses and frequencies under traffic excitation are extracted from bridge measurements. The proposed method is capable of solving the underdetermined problems. Operational modal identification using actual field measurements of a filed bridge illustrates the effective of the algorithm.

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