A Progressive Hedging Approach for Large-scale Pavement Maintenance Scheduling Under Uncertainty

Abstract

This study approaches a multi-stage stochastic mixed-integer programming model for the high-level complexity of large-scale pavement maintenance scheduling problems. The substance of some parameters in the mentioned problems is uncertain. Ignoring the uncertainty of these parameters in the pavement maintenance scheduling problems may lead to suboptimal solutions and unstable pavement conditions. In this study, annual budget and pavement deterioration rate are considered uncertainty parameters. On the other hand, pavement agencies generally face large-scale pavement networks. The complexity of the proposed stochastic model increases exponentially with the number of network sections and scenarios. The problem is solved using the Progressive Hedging Algorithm (PHA), which is suitable for large-scale stochastic programming problems, by achieving an effective decomposition over scenarios. A modified adaptive strategy for choosing the penalty parameter value is applied that aims to improve the solution process. A pavement network including 251 sections is considered the case study for this investigation, and the current study seeks optimal maintenance scheduling over a finite analysis period. The performance of the stochastic model is compared with that of the deterministic model. The results indicate that the introduced approach is competent to address uncertainty in maintenance and rehabilitation problems.

Keywords: Pavement management system; Uncertainty; Stochastic programming; Large-scale network maintenance planning; Optimization

1. Introduction

Road pavements are one of the fundamental components of a transportation infrastructure. Accordingly, they should be durable, and their condition ought to provide safe and secure roads for travelers over time. The pavement network condition significantly affects the economy, expansion, welfare, and the environment of a country. Correspondingly, pavement deterioration and maintenance, rehabilitation, and preservation of road networks have been an immense concern (Mathew and Isaac, 2014).

Implementing a well-functioning strategy to maintain pavement at acceptable levels is essential for sustainable economic growth. In the United States, more than 100 billion dollars are assigned to maintain the pavement networks annually, and the required cost has steadily increased. Due to the lack of financial resources and increase in the network maintenance price, agencies and decision-makers have investigated various strategies (Saha and Ksaibati, 2019). The road agencies look for ideal maintenance planning, which simultaneously minimizes expenditures and enhances the condition of roadway networks. Therefore, pavement maintenance and rehabilitation (M&R) optimization has attracted attention in this field. M&R optimization aims to determine the optimal treatments for each pavement section at each time to improve the condition of a network with a minimal budget. Hence, modeling the M&R problem leads to a discrete (integer) optimization problem (Ahmed et al., 2018).

Roadway networks usually comprise hundreds of pavement sections. Additionally, by increasing the number of decision variables in integer programming problems, problem complexity is increased exponentially. That is to say, the consideration of hundreds of sections in M&R scheduling optimization makes the problem non-deterministic polynomial-time hard (NP-hard). Therefore, finding the optimal solutions to large-scale M&R optimization problems is challenging and complicated, and valuable methods ought to be applied to solve these problems (Hafez et al., 2018).

On the other hand, mathematical models generally assume that all of the models' parameters are deterministic. Nonetheless, some of the criteria address uncertainty, and accordingly, considering these deterministic parameters may lead to suboptimal solutions and unreliable pavement conditions. The fluctuation of the network's yearly budget and pavement deterioration process is considerable. Budget fluctuation originates in resource limitations and changes in government policies. Moreover, pavement behavior is entirely complicated and stochastic. In other words, several components, including pavement structure, traffic loads, climate conditions, and material quality strongly influence pavement behavior and deterioration rate. The mentioned uncertainties may lead to providing inaccurate solutions if these parameters' uncertainty is not taken into account. Additionally, the implementation of inaccurate M&R scheduling may result in losing a significant amount of money and weakening the condition of the network (Menendez and Gharaibeh, 2017).

2. Background

Various approaches have been utilized to solve the M&R scheduling problem at the network level. Markovian models and integer programming models are the most-used techniques applied to plan M&R strategies. Integer programming models consider each section individually, and they provide discrete solutions for every single section (France-Mensah and O'Brien, 2018). On the flip side, Markovian models reduce the complexity of the problem and change its format from integer programming to linear programming. Markovian models categorize pavement sections into similar groups according to their characteristics and provide schedules for groups of pavement sections. Thus, these models are not usually capable of presenting the M&R plan for each section individually. As a result, Markovian models could not be qualified for scheduling M&R activities meticulously, but they may be valuable in order to analyze a group of pavements simultaneously. Moreover, Markovian models divided the pavement condition indexes (e.g., international roughness index (IRI) or pavement condition index (PCI)) into different levels. Thus, during this process, the continuous indexes convert to discrete indexes, which may reduce accuracy in the obtained solutions because pavement condition indicators are mostly continuous (Moreira et al., 2017).

According to the abovementioned concepts, integer programming models outperform Markovian models in terms of presenting M&R activities. Nevertheless, by expanding the problem's dimension, the complexity of integer programming problems is exponentially raised. Small integer problems can be easily solved by conventional techniques such as branch and bound algorithm. However, these techniques are not qualified to solve large-scale integer problems in logical time (Khiavi and Mohammadi, 2018).

To overcome this deficiency, investigators have employed decomposition techniques and metaheuristic algorithms (Karabakal et al., 1994). Decomposition techniques modify the structure of the problem, and some sub-problems are generated. Afterward, some strategies to handle the master problem and sub-problems and some strategies to create solutions are taken into account (Rahmaniani et al., 2017). Decomposition techniques solve the sub-problems and subsequently aggregate their optimal solutions. Benders decomposition, Lagrangian relaxation, and progressive hedging algorithm (PHA) are some of the most applicable decomposition techniques (Carøe and Schultz, 1999).

Karabakal et al. (1994) and Dahl and Minken (2008) used the Lagrangian relaxation method in order to decompose network M&R problems into separate sub-problems. Consequently, the decomposed problems were solved by the shortest method in the abovementioned studies. In those investigations, pavement condition index and the improvement of pavement treatments were classified into different ranges so as to be applied in the shortest path algorithm (Karabakal et al., 1994; Dahl and Minken, 2008). Likewise, Gao and Zhang (2012) used the Lagrangian relaxation method to decompose a network M&R problem. Then, they solved the obtained sub-problems by the branch-and-cut procedure. The pavement condition index does not need to be decomposed to discrete ranges in the branch-and-cut procedure (Gao and Zhang, 2012).

Metaheuristic algorithms were designed to overcome the other methods' deficiencies. That is to say, metaheuristic algorithms were fabricated to solve NP-hard problems. However, these methods cannot assess the quality of the presented solutions. Metaheuristic algorithms generally contain some parameters that considerably impact the algorithm process and quality of solutions. The metaheuristic parameters should be calibrated, and finding the optimal value of these parameters is a complicated task. Hence, metaheuristic algorithms may not yield optimal solutions due to a lack of reliability.

Metaheuristic algorithms are inspired by natural facts, social behaviors, swarm intelligence, and evolution competitions, and they solve optimization problems with a combination of random search methods and mathematical rules (Naseri et al., 2020). A genetic algorithm has been extensively applied to tackle the high-level complexity of large-scale network maintenance planning problems (Chan et al., 1994; Ferreira et al., 2002; Fwa et al., 1996; Mathew and Isaac, 2014; Meneses and Ferreira, 2012; Charles Pilson et al., 1998). Moreover, particle swarm optimization is the other most-used metaheuristic algorithm that has been used to find the optimal or near-optimal solutions to M&R problems (Tayebi et al., 2013).

On the other hand, the problems' deterministic parameters are generally taken into consideration. Nonetheless, uncertainty exists in some parameters, which leads to enhancing the precision of the model and providing trustworthy solutions.

As such, uncertainty is usually spotted in budget estimation due to several political and financial issues. Furthermore, there is a mismatch between the budget allocated to pavement maintenance and rehabilitation and the budget assigned to network expansion and new road construction. Hence, the predicted budget may significantly differ from the absolute budget allocated to M&R activities. Meanwhile, the pavement performance model is generally considered to be deterministic. Nevertheless, pavement deterioration is stochastic and probabilistic for several reasons such as traffic load variations, climate change, and these agents' interactions (Fani et al., 2020).

There are various approaches to the M&R optimization problem under uncertainty. Stochastic programming, probabilistic programming, and robust optimization are the most valuable techniques applied to consider uncertainty consequences in M&R optimization problems.

Gao et al. (2013) employed a multi-stage stochastic programming method to scrutinize the influences of uncertainty on an M&R optimization problem. The mentioned model was a Markovian model in which the budget uncertainty was considered annually. A case study with an approximate length of 16000 kilometers located in the United States was taken into account. The roads in this network were categorized into three different groups. The probabilistic transfer matrix was determined for each group, and two methods (Lagrangian relaxation and scenario reduction) were utilized to solve the problem (Gao et al., 2013).

Fan and Wang (2014) employed a stochastic integer linear programming model, and by virtue of that model budget uncertainty was taken into account in M&R scheduling. Three budget scenarios, including low, medium, and high budget, are considered each year during the analysis period. A small-scale network containing ten pavement sections was the case study for that investigation. The problem was solved under various scenarios, and the advantages of incorporating the uncertainty were taken into consideration (Fan and Wang, 2014).

Fani et al. (2020) investigated the budget uncertainty and pavement deterioration uncertainty among a mixed integer-linear programming model. To that end, a multi-stage stochastic method was modeled and applied to an M&R optimization problem. Four scenarios were introduced for each year using a combination of different deterioration rates and various budgets. Two case studies consist of four, and 21 pavement sections were analyzed. The results revealed that the introduced stochastic programming model is highly qualified to consider budget uncertainty and deterioration uncertainty simultaneously, and that method is capable of presenting optimal solutions for all scenarios (Fani et al., 2020).

In terms of the concept of optimization with chance constraints, Wu and Flintsch (2009) proposed a multi-objective Markovian model. In that model, budget uncertainty is considered for network M&R optimization problems. A normal distribution with an average value of 20 million dollars and a standard deviation of two million dollars was associated with the yearly budget. Given that the budget parameter was located on the right side of constraint, the chance constraint remained linear. The case study was a part of Virginia with a near length of 8000 kilometers (Wu and Flintsch, 2009).

Similarly, robust optimization has been applied to planning M&R activities in the pavement networks. Gao and Zhang (2009) introduced an integer programming model to solve an M&R optimization problem at the project level. The pavement performance model and the improvement gained by pavement treatments were predicted by the application of linear regression and historical data. The independent variables' coefficient of the model, as well as treatment costs, were considered to be uncertain parameters (Gao and Zhang, 2009). Ng et al. (2009) presented an integer programming model in which uncertainty was considered for the pavement deterioration process and the improvement obtained through pavement treatments (Ng et al., 2009). The budget uncertainty under a robust optimization approach was explored in Al-Amin's (2013) investigation. In this regard, an integer programming model is employed to solve a network comprising ten pavement sections. The results indicated that robust optimization is an appropriate technique to consider the probabilistic budget reduction in M&R optimization problems (Al-Amin, 2013).

To the best of the authors' knowledge, although pavement maintenance and rehabilitation scheduling have been investigated in large-scale networks, uncertainty has not received enough attention in large-scale networks due to the increasing complexity of the problem. That is to say, in the integer programming models, uncertainty has generally been considered in small networks, which is not practical for real networks because they typically contain hundreds of sections. To this end, uncertainty is taken into account for real large-scale pavement networks in this study in order to overcome the mentioned deficiencies. Whereas the decision-makers and agencies generally face large-scale networks comprising many pavement sections, proposing a model to tackle these networks could be a valuable achievement because expenditures can be managed through the mentioned model, and the precision of the optimal solution can be significantly increased.

3. Aims and scope

The current study aims to propose an algorithm to solve the pavement M&R scheduling problem for real large-scale networks under budget and pavement deterioration uncertainties. To this end, the problem is expressed as a multi-stage stochastic mixed-integer programming model so as to determine the uncertainty of the budget and pavement deterioration rate. The non-deterministic parameters are modeled by a number of possible realizations. A powerful algorithm called "progressive hedging algorithm" is employed to achieve the purposes of this study. PHA is a scenario-based decomposition technique that analyzes problems heuristically, and it can handle such large-scale stochastic problems. PHA has been profitably utilized to solve multi-stage stochastic programs with integer variables.

4. Problem formulation

In this section, the notations are initially presented. Afterward, the pavement condition index and performance model used in this study are introduced. Then, the multi-stage stochastic programming optimization model applied to considering budget uncertainty and pavement deterioration rate uncertainty in network M&R problems is described.

4.1. Notations

Table 1 presents the sets (indices), pavement condition parameters, general parameters, uncertainty parameters, and the decision variable used in this study. In this table, IRI is the international roughness index.

Table 1. The notation of the problem formulation

	Indices
$\boldsymbol{I}$	Set of pavement network sections, $\boldsymbol{I} = \{1,2,3, \dots, \mathrm{I}\}$
$\boldsymbol{K}$	Set of M&R treatments, $\boldsymbol{K} = \{1,2,3, \dots, \mathrm{K}\}$, treatment K is the most expensive and effective
$\boldsymbol{S}$	Set of uncertainty scenarios, $\boldsymbol{S} = \{1,2,3, \dots, \mathrm{S}\}$
$\boldsymbol{T}$	Set of time in the analysis period, $\boldsymbol{T} = \{1,2,3, \dots, \mathrm{T}\}$
	Pavement condition parameters
$\boldsymbol{IR_{i0}}$	Initial IRI of section i
$\boldsymbol{IR_{min}}$	Minimum possible condition of each of all sections during each time period
$\boldsymbol{IR_{max}}$	Maximum acceptable condition of each of all sections during each time period
$\boldsymbol{IR_{new}}$	Condition of pavement after reconstruction
$\boldsymbol{IR_i^*}$	IRI ideal level at the end of the analysis period
$\boldsymbol{e_{ik}}$	Performance (IRI) jump of section i because of applying treatment k
	General parameters
$\boldsymbol{A_i}$	Area of section i
$\boldsymbol{N_{ik}}$	Maximum allowable number of treatment k to be applied to section i during the analysis period
$\boldsymbol{C_{ikt}}$	Average maintenance unit cost of applying treatment k to section i at time period t
	Uncertainty parameters
$\boldsymbol{\beta^s}$	The deterioration rate in trend curve model for scenario s
$\boldsymbol{B_t^s}$	The available budget at time period t for scenario s
$\boldsymbol{p^s}$	The probability of occurrence of scenario s
$\boldsymbol{IR_{it}^s}$	The roughness of section i at time period t for scenario s
$\boldsymbol{IR_{iT}^s}$	The roughness of section i in the last year of analysis period for scenario s
$\boldsymbol{\xi_t^s}$	The realizations of the stochastic process until time period t for scenario s
	Decision variable
$\boldsymbol{x_{ikt}^s}$	The binary decision variable that indicates whether the treatment k at time period t applied to section i for the scenario s

4.2. Pavement performance indicators

Pavement performance indicators play a pivotal role in the pavement management system (PMS). With the aid of pavement performance indicators, pavement deterioration trends can be analyzed, and it is indicated that when the pavement arrives at a critical condition, the application of treatments is vital. The IRI is an essential performance index strongly correlated with pavement safety and driver convenience. IRI provides information about the pavement surface, and it has thus been considered one of the most important indicators. Driving on rough roadways results in drivers' inconvenience, velocity reduction, likely vehicle breakdown, travel time increase, and increased user costs. Various performance indicators assess pavement quality, and IRI is one of these fruitful indicators. Based on the aforementioned concepts, the IRI is taken into account as the pavement performance indicator in this study (Tsunokawa and Schofer, 1994).

Different performance models have been proposed for the IRI. One of the most applicable and well-recognized performance models is the trend curve model, which was introduced by Tsunokawa and Schofer (1994). The deterioration function of the trend curve model is illustrated in Figure 1. As can be seen, this method represents that the ratio of IRI-to-initial IRI follows a particular exponential trend. In other words, the IRI-to-initial IRI ratio can be fitted in an exponential curve over time. If the condition of pavement is equal to IR_{it^0}, the condition of this pavement at the time t^* in the future $(t^* > t)$ is evaluated based on Eq. (1).

$$IR_{it^*} = IR_{it^0} \exp(\beta(t^* - t^0)) \qquad (1)$$

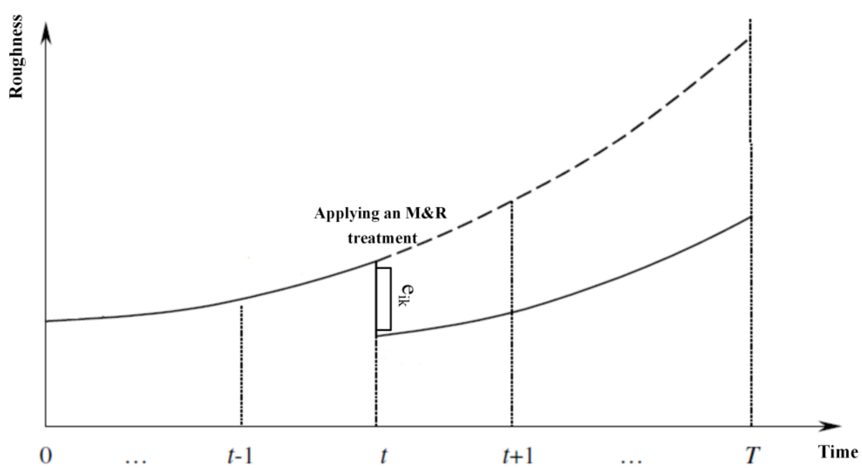

Figure 1. Roughness deterioration and the effect of an M&R treatment on roughness at the time t

The trend curve model has been utilized at the project and the network level in many studies (Gao and Zhang, 2012; Li and Madanat, 2002; Ouyang and Madanat, 2006, 2004; Seyedshohadaie et al., 2010). In the optimization model of this investigation proposed in the following section, various deterioration models can be applied based on the pavement performance indicator. In this study, the trend curve model is employed because IRI is taken into account as the pavement performance indicator. That is to say, the methodology presented in this investigation is qualified to apply to other pavement performance measurements and other deterioration functions.

4.3. Stochastic model

In this paper, the budget and deterioration rate uncertainties in M&R optimization problems are modeled through multi-stage stochastic programming. Stochastic programming is a framework for optimization problems in which some decision variables are non-deterministic. Stochastic programming aims to make a suitable decision that is feasible for all of the scenarios and optimizes an average value of stochastic variables' function and decisions (Mirhasani and Hooshmand Khaligh, 2013).

In one of the stochastic programming approaches, uncertainty is defined by a set of discrete scenarios, and the future uncertainty impact is predicted. The scenario-based stochastic programming is a valuable method in which decision-makers consider uncertainty based on determining feasible future status. For each scenario, a possibility is estimated, and it is expected that each scenario occurs with its corresponding possibility. In this situation, the purpose is to find an optimal solution that provides an appropriate level for all scenarios. For instance, if $t = 1, \dots, T$ stages exist in a problem, and ξ_t^s represents the realizations of the stochastic process until time period t for scenario s, the decision process with T stages is according to Figure 2 (a. Shapiro et al., 2009).

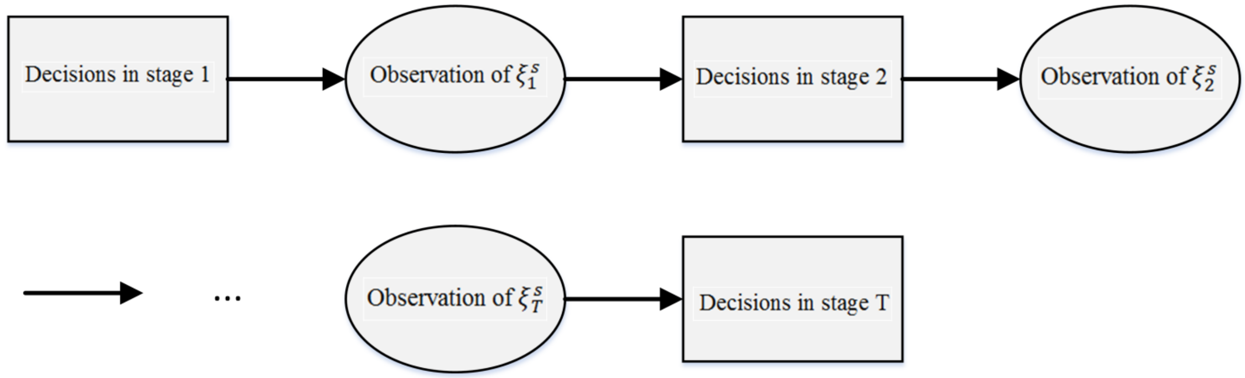

Figure 2. Decision-making steps in multi-stage stochastic programming

Regarding Figure 2, the first stage decisions are made in multi-stage stochastic programming. Afterward, a stochastic process affects the performance of first stage decisions. Subsequently, the second stage decisions are made to compensate for the likely undesirable influences of the first stage decisions. Then, the second stochastic process is realized, which impacts the first and second stages of decision performance. Consequently, the decisions are made in the third stage, and this process is continued until the final stage decisions are made (A. Shapiro et al., 2009).

Accordingly, in this investigation the uncertainty related to budget and deterioration rate is defined as a set, including the feasible combinations of budget and deterioration rate over the planning horizon. Therefore, the various amounts of budgets and different values of deterioration rates can be considered in a model. To illustrate this point, assume that a problem comprises T stages. If two feasible amounts of budget and two feasible values of deterioration rate exist in each stage (t), the number of all scenarios is equal to $\prod_{t=1}^{T} 2 \times 2 = 4^T$.

Based on the aforementioned concepts, the multi-stage stochastic mixed integer programming for M&R optimization problems is modeled according to Eqs. (2) to (10).

$$Minimize\ \sum_{i=1}^{I} \sum_{s=1}^{S} p^s |IR_{iT}^s - IR_i^*| \tag{2}$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} A_i C_{ikt} x_{ikt}^s \le B_t^s \qquad \forall t \in T, \forall s \in S \tag{3}$$

$$IR_{it}^{s} = IR_{i0}\exp(\beta^{s}t) + \sum_{j=1}^{t}\sum_{k=1}^{K-1} x_{ikt}^{s} e_{ik}\exp\big(\beta^{s}(t-j)\big) + (IR_{new} - IR_{i0}\exp(\beta^{s}t))x_{iKt}^{s} \quad \forall i \in I, \forall s \in S \tag{4}$$

$$IR_{it}^{s} \geq IR_{min} \qquad \forall t \in T, \forall s \in S \tag{5}$$

$$IR_{it}^{s} \leq IR_{max} \qquad \forall t \in T, \forall s \in S \tag{6}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{K} x_{ikt}^{s} \leq N_{ik} \qquad \forall i \in I, \forall s \in S \tag{7}$$

$$\sum_{k=1}^{K} x_{ikt}^{s} = 1 \qquad \forall i \in I, \forall t \in T, \forall s \in S \tag{8}$$

$$x_{ikt}^{s} \in \{0,1\}, IR_{it}^{s} \geq 0 \tag{9}$$

$$x_{ikt}^{m} = x_{ikt}^{n} \qquad \forall i \in I, \forall k \in K, \forall t \in T, \forall n \in S, \forall m \in S, 1 \leq m < n \leq S, \xi_{t}^{m} = \xi_{t}^{n} \tag{10}$$

Eq. (2) indicates the model objective function. According to all budget and deterioration rate scenarios, this objective function Eq. (2) minimizes the distances between pavement condition level and an ideal level in the last year of the analysis period. The pavement condition ideal level is targeted by policymakers and decision-makers. Eq. (3) displays budget constraint, in which the annual summation of M&R treatment costs ought to be lower than the yearly budget each year and for all scenarios. The pavement condition at time t is calculated based on the deterioration rate and the improvement achieved by treatments. The IRI calculation constraint is presented in Eq. (4). The last statement in Eq. (4) implies that for treatment K (reconstruction), the condition of pavement before treatment is not effective. In other words, if treatment K (reconstruction) is applied to pavement, the condition of the corresponding pavement changes to a new pavement condition (IRI_{new}), and the condition of pavement before reconstruction does not influence the condition gained after the treatment. Eq. (5) shows the minimum acceptable value of IRI, and the pavements' roughness cannot be lower than this threshold. Concerning Eq. (6), the roughness of pavements should be lower than a particular level, and this level is determined by decision-makers. The maximum allowable number of each treatment during the planning horizon is restricted by Eq. (7). The maximum allowable number of treatments is estimated based on implementation limitations, agency resources, the effective life of each treatment, and decision-makers' judgments. Eq. (8) guarantees that only one mode of M&R treatment is selected and applied to each section at each time in the analysis period. Eq. (9) reveals that x_{ikt}^{s} is a binary decision variable. That is to say, in the circumstances that a treatment is chosen for pavement at a definite time, the decision variable is one, and it otherwise equals zero. Eq. (10) presents the non-anticipativity constraints, which correlates with the decisions of similar scenarios. In each stage, two scenarios are called "indistinguishable" if their history is the same, and it cannot be predicted which of them is going to happen in the future. Otherwise, realization indicates that the history of the two scenarios are not entirely equal, and these scenarios are distinctive. The non-anticipativity constraints guarantee that a similar decision is made for two scenarios if they are indistinguishable.

5. Methodology

The model defined in the previous part is an NP-hard problem. Therefore, the PHA introduced by Rockafeller and Wets (1991) is employed to handle the high complexity of the model. The PHA is a scenario-based decomposition technique that classifies the problem into some individual sub-problems. Ultimately, single-scenario solutions are aggregated. This method utilizes a penalty parameter (quadratic) for non-anticipativity constraint violation. Rockafeller and Wets (1991) expressed that the impact of the penalty term on solution quality and PHA performance is considerable. Nevertheless, provide no procedure to optimize this parameter. In that regard, different authors have proposed various methods to update the penalty term over multiple iterations.

5.1. Progressive hedging algorithm

The M&R optimization stochastic model is presented in Eqs. (2) to (10) in section 4.3. The model can be separated into scenarios provided that non-anticipativity constraints that unite and connect all of the scenarios are overlooked. The non-anticipativity constraints can be written as follows:

$$x_{ikt}^{s} = E\left(x_{ikt}^{s'} \middle| s' \in S_t^s\right) \qquad (11)$$

Where $S_t^s = \left\{s' \middle| \xi_t^s = \xi_t^{s'}\right\}$. Eq. (11) can be changed into $x_{ikt}^{s} = \hat{x}_{ikt}^{s}$ with:

$$\hat{x}_{ikt}^{s} = \frac{\sum_{s' \in S_t^s} p_{s'} x_{ikt}^{s'}}{\sum_{s' \in S_t^s} p_{s'}} \quad (12)$$

It was recommended that augmented Lagrangian is considered in PHA modeling (Rockafeller and Wets, 1991). The augmented Lagrangian model is represented in Eqs. (13) to (20). In these equations, λ represents the Lagrange multipliers vector assigned to the non-anticipativity constraints, and $\rho > 0$ signifies a penalty parameter.

$$Minimize \ \sum_{i=1}^{I} \sum_{s=1}^{S} p^s |IR_{iT}^s - IR_i^*| + \sum_{t=1}^{T} (\lambda_t^s (x_{ikt}^s - \hat{x}_{ikt}^s) + \rho/2 \|x_{ikt}^s - \hat{x}_{ikt}^s\|^2) \quad (13)$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} A_i C_{ikt} x_{ikt}^s \leq B_t^s \qquad \forall t \in T \quad (14)$$

$$IR_{it}^s = IR_{i0} \exp(\beta^s t) + \sum_{j=1}^{t} \sum_{k=1}^{K-1} x_{ikt}^s e_{ik} \exp\big(\beta^s (t-j)\big) + (IR_{new} - IR_{i0} \exp(\beta^s t)) x_{iKt}^s \quad \forall i \in I \quad (15)$$

$$IR_{it}^s \geq IR_{min} \qquad \forall t \in T \quad (16)$$

$$IR_{it}^s \leq IR_{max} \qquad \forall t \in T \quad (17)$$

$$\sum_{t=1}^{T} \sum_{k=1}^{K} x_{ikt}^s \leq N_{ik} \qquad \forall i \in I \quad (18)$$

$$\sum_{k=1}^{K} x_{ikt}^s = 1 \qquad \forall i \in I, \forall t \in T \quad (19)$$

$$x_{ikt}^s \in \{0,1\}, IR_{it}^s \geq 0 \quad (20)$$

Additionally, Rockafeller and Wets (1991) declared that $\hat{x}_{ikt}^s$ in Eq. (13) should be fixed so as to gain full separability. Consequently, the problem should be solved repeatedly, and the amount of $\hat{x}_{ikt}^s$ and the Lagrange multipliers vector ought to be updated over consecutive resolutions. The mentioned processes are the procedure of PHA, whose steps are as follows:

First step: Set $\hat{x}^{s,0} = \left(\hat{x}_{ik1}^{s,0}, \ldots, \hat{x}_{ikT}^{s,0} \right)$ and w=0. Choose $\lambda^{s,0} = 0, \rho^0 > 0$.

Second step: Calculate $x_{ik}^{s,w+1} = \left(x_{ik1}^{s,w+1}, \ldots, x_{ikT}^{s,w+1}\right)$, s=1,…,S by solving all sub-problem scenarios regarding equations (13) to (20).

Third step: For s=1,…,S and t=1,…,T set $\hat{x}_{ikt}^{s,w+1} = \frac{\sum_{s' \in S_t^s} p_{s'} x_{ikt}^{s',w+1}}{\sum_{s' \in S_t^s} p_{s'}}$

Fourth step: Set ρ^{w+1} and $\lambda_t^{s,w+1} = \lambda_t^{s,w} + \rho^w \left(x_{ikt}^{s,w+1} - \hat{x}_{ikt}^{s,w+1}\right), t = 1, \ldots, T, s \in S$.

Fifth step: Check the termination criteria. If the criteria are met, stop the algorithm. Otherwise, update the $w \leftarrow w + 1$ and come back to the second step.

The following model needs some modifications to be completed. For instance, the fifth step requires practical termination criteria. Rockafeller and Wets (1991) claimed that Eq. (21) can be an appropriate termination criterion in order to stop the model.

$$\sqrt{\sum_{s \in S} p_s \left\| x_{ik}^{s,w+1} - \hat{x}_{ik}^{s,w} \right\|^2} \leq \varepsilon \tag{21}$$

Another modification is relevant to the primal variables' initialization. Chiche (2012) analyzed this issue and recommended setting primal variables as the sub-problem solution associated with s without the non-anticipativity constraints.

5.2. Penalty parameter update

The penalty parameter (ρ) plays a crucial role in the PHA convergence. Rockafeller and Wets (1991) applied a penalty parameter constant in order to converge the PHA. Many researchers have put this parameter into practice, and it has been revealed that the penalty parameter significantly affects the algorithm's behavior. Mulvey and Vladimirou (1991) proved that the value of ρ considerably impacts the convergence rate of the PHA. Helgason and Wallace (1991) declared that ρ ought to be as small as possible, while it should be large enough to assure the convergence.

Some studies have been conducted to examine the ideal value of the penalty parameter and its influence on affected variables. Reis et al. (2005) considered a descending factor as the penalty parameter, and this parameter was reduced gradually over multiple iterations. On the other hand, some authors have allocated ascending factors to the penalty parameter (Carpentier et al., 2012; Crainic et al., 2011). Hvattum and Løkketangen (2009) employed a restraint method to update the penalty parameter based on Eq. (21). Goncalves et al. (2012) utilized an ascending factor in proportion with non-anticipativity violation. Gul (2010) proposed a new technique to update the penalty parameter over multiple iterations. In that technique, the penalty parameter can be changed dynamically in order to enhance the PHA performance. The penalty parameter is increased when the dual variables do not progress, and it is reduced when primal variables stop progressing. Zephyr et al. (2014) applied non-anticipativity indicators and optimality-based coefficients to update the penalty parameter dynamically. Thus, the penalty parameter is allowed to be reduced or increased with that method.

Zehtabian and Bastin (2016) analyzed some of the well-recognized techniques and proposed a new adaptive method to improve the performance of the algorithm process. This novel method updated the penalty parameter efficiently and converged to optimality in all of the analyzed problems. Accordingly, the speed of the introduced method was far more than that of most of the other methods. The penalty parameter is updated intelligently according to both dual and primal spaces. In that regard, the penalty parameter is increased when non-anticipativity constraints are suitably represented. However, the penalty parameter is reduced if the approximation is not sufficient. In this investigation, the method presented by Zehtabian and Bastin (2016) is used to update the penalty parameter because of its high efficiency.

In the strategy introduced by Zehtabian and Bastin (2016), the variation of $\hat{x}^{s,w}$ is initially checked to prevent non-anticipativity constraints from being enforced when the right non-anticipativity solution is not spotted, or when the non-anticipativity approximation is not adequate. Hence, if the primal variables change considerably, the penalty parameter is not allowed to increase. Moreover, a trade-off between the quadratic penalty and the Lagrangian function is considered. To this end, if the solution is stabilized and extensive violations are observed in non-anticipativity constraints, the penalty parameter is moderately increased, and otherwise the penalty parameter is fixed. Ultimately, if none of the mentioned statuses is recognized, the primary space converges, and the penalty parameter is increased for dual space convergence. For more information about the algorithm implementation and penalty parameter update, the authors encourage the reader to peruse the details provided by Zehtabian and Bastin (2016).

6. Numerical case study

A real large-scale network is considered the case study of the proposed model. The model is solved by PHA, and the results are presented in this section. The problem modeling is coded in GAMS software. Likewise, the model is run in GAMS software. A Core i7-6700HQ computer with 2.60 GHz CPU and RAM of 16 GB is employed to run the mentioned code. The case study and its characteristics and the required parameters of optimization modeling are introduced before presenting the results.

6.1. Case study features and model parameters

A real pavement network with an approximate length of 988 kilometers is the case study of the current investigation. This case study includes 251 primary asphalt pavement sections, which are primary road, out of the cities and located in Tehran. The sections' characteristics and conditions are extracted from Iran's Road Maintenance and Transportation Organization (RMTO) (Iran's Road Maintenance and Transportation Organization, 2018).

To sum up, the average length of all sections is 3.94 kilometers. The longest and the shortest sections length in the network are equal to 16.3 and 2.4 kilometers, respectively. Moreover, the IRI is considered the performance indicator, and the average value of IRI for all network sections is 3.77 m/km in the initial year. Among all network sections, the minimum and maximum values of the IRI in the initial year are 2.24 m/km and 5.68 m/km in the order mentioned.

The pavement treatment strategies are classified into five groups, including do nothing (first strategy), preventive maintenance (second strategy), light rehabilitation (third strategy), medium rehabilitation (fourth strategy), and reconstruction (fifth strategy). Do nothing implies that no maintenance is implemented, and pavement condition does not improve. The preventive maintenance signifies chip seal, micro-surfacing, and slurry seal. Light rehabilitation includes surface milling and a thin hot asphalt overlay. Medium rehabilitation contains surface milling and more than one overlay by hot mix asphalt and cold recycling. Reconstruction implies the replacement of the entire existing pavement structure with new pavement.

The unit cost of pavement treatment is one of the vital model parameters extracted from Iran's Road Maintenance and Transportation Organization (RMTO) (Iran's Road Maintenance and Transportation Organization, 2018). The unit implementation cost of the first to the fifth treatments are zero Toman/m^2, 5000 Toman/m^2, 15000 Toman/m^2, 32000 Toman/m^2, and 65000 Toman/m^2 in the order given. The improvement reached by the implementation of the first to the fourth treatment strategies is equal to zero m/km, 0.3 m/km, 1.2 m/km, and 2 m/km, respectively. For instance, the third treatment implementation leads to a 1.2-m/km reduction in the IRI. As previously mentioned, the improvement of the fifth treatment (reconstruction) is not related to the section condition before treatment implementation, and after reconstruction, the IRI changes to a new pavement condition ($IRI_{new} = 1.5$). The mentioned improvements are extracted from Paterson (1990) and Lu and Tolliver (2012).

The RMTO allocates 66 billion Tomans for preservation, maintenance, and rehabilitation of the mentioned network, and the annual budget of the case study of this investigation is thus 66 billion Tomans (Iran's Road Maintenance and Transportation Organization, 2018). The budget reduction percentage is reckoned at 20% in order to consider uncertainty scenarios. The mentioned budget reduction is selected based on historical budget data and expert opinion. Therefore, the budget uncertainty is taken into account by considering two allocated budgets: 66 billion Tomans and a 20% budget reduction (52.8 billion Tomans).

Deterioration rate is generally determined by checking historical data. At network-level studies, a constant rate is usually assigned to the deterioration rate of all pavement sections. The deterioration rate (β) in the trend curve model (Eq. (1)) has been analyzed in several studies, and 0.05 is considered a precise and ideal deterioration rate (Li and Madanat, 2002; Ouyang and Madanat, 2004; Seyedshohadaie et al., 2010). In this investigation, the deterioration rate is detected based on the aforementioned studies and RMTO experts. In this regard, two deterioration rates, including logical rate (0.05) and pessimistic rate (0.06), are taken into consideration to address uncertainty. The pessimistic deterioration rate is considered due to the variation of pavement characteristics in the current study and the investigations, which considered a deterioration rate of 0.05. The pessimistic deterioration rate is 20% higher than the logical deterioration rate, and this incremental value is proposed by RMTO.

The minimum and maximum allowable value of IRI for each section in all years of analysis period is zero and four in the order given (Iran's Road Maintenance and Transportation Organization, 2018). Moreover, 2.2 m/km is the ideal level of IRI in the last year of the planning horizon for the noted network according to RMTO (Iran's Road Maintenance and Transportation Organization, 2018). The analysis period is considered four years, and for each year only one treatment is applied for each section.

6.2. Results of stochastic programming model

In this section, the results of the problem solved by the multi-stage stochastic programming are described, and they are compared with the outcomes presented in the deterministic model. The CPLEX solver in GAMS software is applied to solve the deterministic problems and the sub-problems of PHA.

Based on the descriptions, as mentioned earlier, there are four possible combinations of the budget and the deterioration rate in each year. Therefore, the case study of this investigation comprises 4^4=256 different scenarios for the four-year analysis period. It is assumed that the possibility of all scenarios' occurrence is equal, and each scenario happens with the possibility of $3/9 \times 10^{-3}$. The master problem contains 1285120 variables. However, the sub-problems include 5020 variables, and the decomposition technique significantly reduces the size of the problem. Thus, it can be postulated that large-scale M&R problems face high complexity and cannot be solved by conventional techniques. Nonetheless, powerful decomposition techniques such as PHA are highly qualified to address these problems.

Eq. (22) is selected as the termination criterion based on the details provided by Zehtabian and Bastin's (2016) study. Eq. (22) is the normalized version of the termination criterion suggested by Rockafeller and Wets (1991) (Eq. (21)).

$$Stopping\ Criterion = \sqrt{\frac{\sum_{s\in S} p_s \| x_{ik}^{s,w+1} - \hat{x}_{ik}^{s,w} \|^2}{max\{1, \sum_{s\in S} p_s \| \hat{x}_{ik}^{s,w} \|^2\}}} \leq \varepsilon$$

(22)

In Eq. (22), the value of ε is considered 10^{-5} based on Zehtabian and Bastin's (2016) investigation. The problem is solved for the case study, and the algorithm achieves the optimal solution after 173 iterations. The values of the stopping criterion among various iterations and their trend are illustrated in Figure 3.

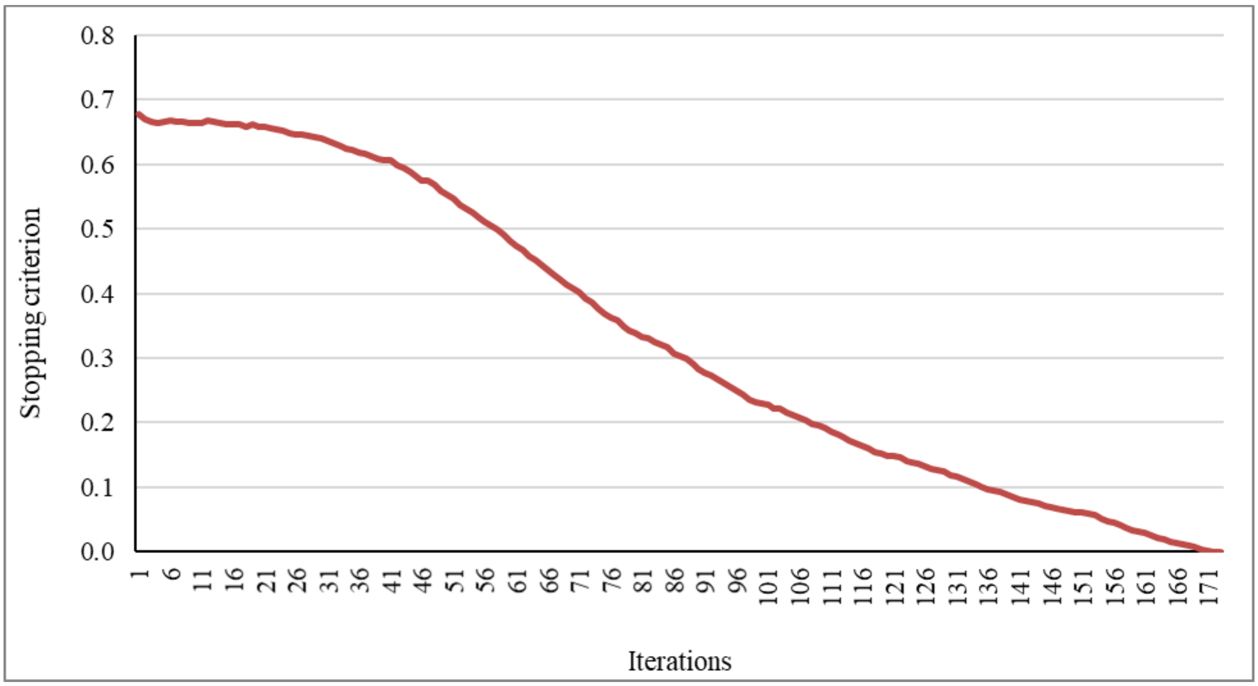

Figure 3. The values of stopping criterion over the iterations

As can be seen from the results of Figure 3, the highest amount of the stopping criterion is related to the first iteration. Furthermore, the stopping criterion value rarely increased over iterations, and it steadily reduced. Ultimately, the algorithm reaches the optimal solution in the 173rd iteration.

The objective function value obtained by the stochastic model is 126.4. The percentage of treatments selected over all scenarios in each year is presented in Table 2. That is, the values presented in Table 2 are the average value of treatment selected in 256 uncertainty scenarios. For instance, the average percentage of do nothing, preventive maintenance, light rehabilitation, medium rehabilitation, and reconstruction for all uncertainty scenarios is 68.7%, 13.2%, 12%, 5.1%, and 1%, respectively. As can be seen in the results of Table 2, the reconstruction strategy is not chosen in the second, third, or fourth years by the stochastic model. Additionally, reconstruction is applied for only one percent of sections in the firth year. The do nothing strategy is assigned to most of the sections in all years of the analysis period. Additionally, the second most-chosen strategy is preventive maintenance, and on average, it is applied for 13.2%, 22.4%, 27.5, and 26.9% of pavement sections from the first year to the fourth year, respectively. Light rehabilitation is selected more than medium rehabilitation in all of the years. Hence, it can be postulated that the stochastic model tries to enhance the condition of the network by application of preventive maintenance and light rehabilitation, which may be due to the likelihood of budget reductions in the analysis period.

Table 2. The percentage of treatments selected by stochastic model over all scenarios

Year	Do nothing	Preventive maintenance	Light rehabilitation	Medium rehabilitation	Reconstruction
1	68.7	13.2	12.0	5.1	1.0
2	57.8	22.4	19.6	0.2	0.0
3	52.1	27.5	18.0	2.5	0.0
4	55.7	26.9	14.2	3.1	0.0

The problem is solved by the deterministic expected value (EV) model to compare the results of the stochastic model with the determination model. To this end, all of the uncertainty and random parameters are considered to be their expected values, and consequently, the model is solved under the deterministic strategy known as the EV approach. For example, the budget is considered $59.4 = \frac{66 + 52.8}{2}$ in each year because each budget scenario may occur with the possibility of 50%. Subsequently, the model is solved under the EV approach, and the EV reaches an objective function of 108.9. The objective function of the optimal solution introduced by EV is 13.8% lower than that of the stochastic model, which may be due to a single scenario consideration in the EV model compared with 256 scenarios in the stochastic model.

Table 3 compares the percentage of treatment strategies introduced by the stochastic model and the expected value approach. As can be perceived, the percentage of reconstruction is exactly the same in both stochastic and deterministic models. The medium rehabilitation percent in the stochastic model is 0.3% more than that of the deterministic model. The percentage of light rehabilitation in stochastic and expected value approach is 15.9% and 16.7% in the order given. The highest level of variation is related to the percentage of preventive maintenance. The percentage of preventive maintenance selected by stochastic programming is 3% more than that of the deterministic model. Accordingly, it can be theorized that the stochastic model allocates more budget to preventive maintenance. That is, the stochastic model may try to allocate budget to a higher number of sections in order to compensate for the negative impacts of likelihood of budget reduction and possible incremental deterioration rate.

Table 3. The percentage of treatments assigned to network sections in the analysis period

	Do nothing	**Preventive maintenance**	**Light rehabilitation**	**Medium rehabilitation**	**Reconstruction**
Stochastic	58.6	22.5	15.9	2.7	0.3
Deterministic	61.1	19.5	16.7	2.4	0.3

Figure 4 illustrates the average IRI of network sections in the analysis period. A more detailed look at this figure reveals that the condition-based performances of the deterministic model and the stochastic model are approximately the same. However, the deterministic model performance is a bit better than the stochastic model performance. The slight performance-based advantage of the deterministic model over the stochastic model is due to the high number of scenarios considered by the stochastic model.

In other words, the IRI of deterministic and stochastic optimal solutions in the last year is 2.61 and 2.65, respectively. Meanwhile, the average IRI of network sections is gradually reduced, and it reaches its lowest level in the last year. The deterministic model reduces the average IRI of the network sections 9.3%, 8.8%, 8.3%, and 8.7% in the first, second, third, and fourth years of the analysis period, respectively. The network IRI is decreased 8%, 8.9%, 8.2%, and 8.6% in the first to the last year of the planning horizon, respectively.

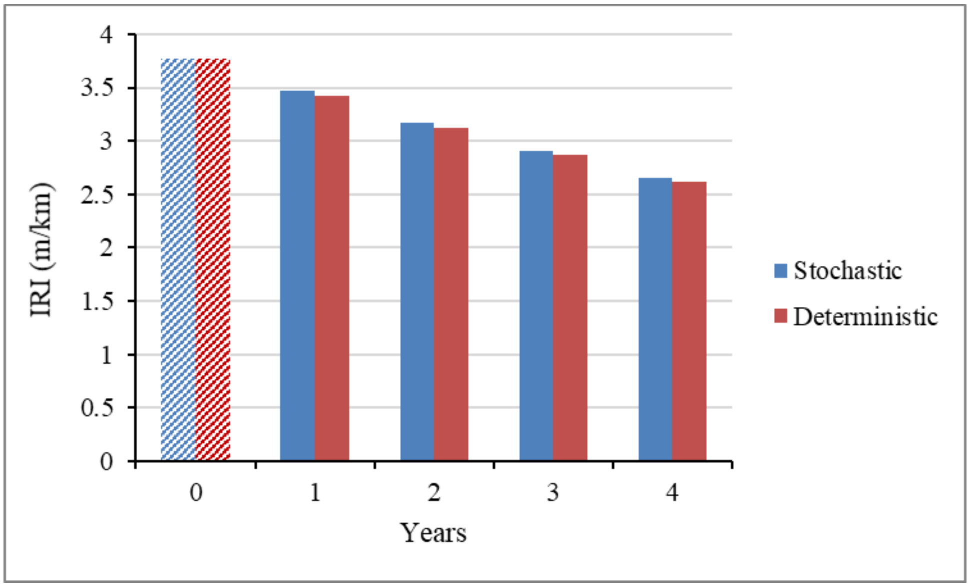

Figure 4. The condition of the network in the planning horizon

The application of stochastic models in some problems may not be valuable. Therefore, some conventional techniques are generally employed to evaluate the effectiveness of stochastic models. In the following sections, some robust yardsticks are applied to investigate the effectiveness of the stochastic model in the case study of this paper.

In the expected value (EV) approach, the average value of each uncertainty parameter is replaced with their corresponding uncertainty parameter, and this process generates a more straightforward problem, which equals the deterministic model. Assume that x_{EV} represents the optimal solution to the expected value model. In order to assess the performance of the x_{EV} solution under all scenarios, the first stage (first year) solutions of x_{EV} are considered the first stage decision variables in the stochastic model. In this mode, the stochastic problem is called the here and now problem, and the first stage decisions of this problem are fixed based on the optimal solution to the EV model. Consequently, the here and now problem is solved, and the average value of $z(x_{EV}, s)$ under all scenarios $s \in S$ is called z_{EEV} and is calculated based on Eq. (23).

$$z_{EEV} = \sum_{s \in S} p^s z(x_{EV}, s) \tag{23}$$

Therefore, z_{EEV} evaluates the next stage's variables based on x_{EV} and s. By virtue of Eq. (23), the z_{EEV} is assessed, and it equals 154.3 for the case study of this investigation. One of the vital parameters to evaluate the value of the stochastic model optimal solution is the value of the stochastic solution (VSS). The value of the stochastic solution (VSS) is calculated according to Eq. (24).

$$VSS = z_{EEV} - z_{HN} \tag{24}$$

The VSS scrutinizes the disastrous impacts of ignoring uncertainty in the model and evaluates the benefits gained by the stochastic model. The higher value of VSS represents the importance of uncertainty in the problem. The lower value of VSS signifies that the result of the expected value approach is an appropriate approximation of the optimal solution to the problem, and the application of the stochastic model is not beneficial. For the case study of this investigation, the VSS is equal to 27.9=154.3-126.4, and VSS is roughly 22% of z_{HN}, which proves that the stochastic model significantly prevails over the deterministic model.

7. Conclusions

This study focuses on pavement maintenance and rehabilitation scheduling in large-scale networks. Furthermore, uncertainty is taken into account in the aforementioned problem. To this end, the problem is formulated as a multi-stage stochastic mixed-integer programming problem. The annual budget and pavement deterioration rate are considered uncertain parameters, and two sensible states are assigned to each uncertain parameter each year. Hence, four states are generated for each stage (year) by a combination of budget uncertainty and deterioration rate uncertainty. By increasing the number of sections in the network and the number of analysis years, the complexity of the M&R problem is increased exponentially, and developing efficient optimization techniques for such a problem could be challenging. To overcome this issue, the PHA as a powerful algorithm is applied to solve the large-scale M&R problem. To this end, the master problem decomposes into scenario sub-problems. Afterward, each sub-problem is solved as a mathematical program. Consequently, scenario-dependent solutions are gradually aggregated to the ultimate optimal solution.

To analyze the performance of the introduced method, a case study including 251 primary asphalt pavement sections is taken into consideration, and the maintenance planning is investigated for a four-year period. The results indicate that the progressive hedging algorithm is highly qualified to tackle the high complexity of large-scale network M&R scheduling. The expected value approach is used to assess the effectiveness of the stochastic model. A more detailed look at the uncertainty and deterministic model's comparison reveals that both models' average condition is approximately the same. Nonetheless, the average IRI of the network sections for the deterministic model is 1.5% lower than that of the stochastic model in the last year of the analysis period because the stochastic model considers 256 scenarios simultaneously, and the deterministic model tackles only one single scenario. Moreover, the stochastic model attempts to enhance the condition of more sections at each stage. Therefore, the stochastic model allocates more funds to preventive maintenance in order to compensate for the negative influences of possible budget reduction and likelihood of incremental deterioration rate. The percentage of preventive maintenance allocated to pavement sections is 22.5% in the stochastic model, while this value is 19.5% in the deterministic model. Ultimately, some uncertainty assessment yardsticks are utilized to gauge the effectiveness of the stochastic model. These criteria indicate that the application of uncertainty in the M&R problem is vital, and the value of the stochastic solution equals 22% of z_{HN}.

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