Common Agency-Based Economic Model for Energy Contract in Electric Vehicle Networks

Yuris Mulya Saputra[1], Diep N. Nguyen[1], Dinh Thai Hoang[1], Eryk Dutkiewicz[1], and Markus Dominik Mueck[2]
[1] School of Electrical and Data Engineering, University of Technology Sydney, Australia
[2] Intel Corporation

***Abstract*—The rapid adoption of electric or hybrid vehicles (EVs) has called for wide deployment of charging stations. These stations can be launched/owned by different owners, referred to as charging station providers (CSPs), which make energy contracts with a smart grid provider (SGP). However, there exists a shortage of mutual economic strategy between the SGP and CSPs in an energy request/transfer competition due to the selfish nature among them. In this paper, we propose an economic model leveraging a multi-principal single-agent (referred to as *common agency*) contract policy, aiming at maximizing the utilities of multiple CSPs while optimizing the utility of the SGP in an EV network. In particular, we first develop the common agency-based contract problem as a non-cooperative energy contract optimization problem, in which each SP can maximize its utility given the common constraints from the SGP and the contracts of other CSPs. To deal with this problem, we develop an iterative energy contract algorithm to find an equilibrium contract solution where the contracts from the CSPs can produce maximum utilities of the CSPs and satisfy the constraints of the SGP. Through numerical results, we show that our proposed model can improve the social welfare of the EV network up to 54% and the utilities of CSPs up to 60% compared with the baseline method in which each CSP obtains the amount of energy that is proportional to its energy request.**

Keywords- Energy demand, electric vehicle, charging station, common agency, and contract theory.

I. INTRODUCTION

As one of the green solutions, electric vehicles (EVs) have revolutionized transportation systems thanks to their high energy efficiency and gas emission reduction. Referring to a global EV viewpoint from International Energy Agency, the predicted number of EVs including battery EVs and plug-in hybrid EVs worldwide will reach up to 255 millions in 2030 [1]. This outlook will trigger a plethora of electricity demand for EVs in the power market and wide deployment of charging stations (CSs) for both public and private usages.

Commonly, CSs are launched/owned by different providers, referred to as CS providers (CSPs). In this case, the CSPs control their CSs to transfer energy (from a smart grid provider, SGP, through the power grid system) to EVs once they charge their batteries at the CSs in the real-time. However, there exists a problem when the CSs receive high charging demands from the EVs simultaneously during peak hour, e.g., before and after working time. As such, the SGP and CSPs may suffer from a heavy energy transfer congestion and overload in the distribution network, and high energy transfer cost from the SGP, respectively [2]. For that, CSPs may also reserve/store the energy from the SGP at their CSs' energy storage in advance during a certain period [3], [4]. In this way, the SGP can stabilize the energy transfer in the power grid system when a large number of EVs require to charge energy at CSs simultaneously. Moreover, the CSPs can minimize the energy cost to serve the EVs as the energy stored in advance may have lower price [5]. Nonetheless, the above energy reservation is still lack of mutual economic strategy due to the selfish nature between the SGP and CSPs. Hence, it is still challenging to maximize the profits for both the SGP and CSPs concurrently.

To concurrently maximize the profits/utilities of both SGP and CSPs, it is critical to study the strategic self-interest interaction among them. In [8], the authors propose a Stackelberg game model to study the interaction between the energy provider (as a leader which sets the apropriate energy price) and the energy buyers (as followers which adjust their energy demands based on the announced energy price). Likewise, the authors in [9] propose a multi-stage Stackelberg game to optimize EV charging considering the energy trading interaction among the SGP (as a leader), CSPs (as followers), and EVs (as followers). Nonetheless, both approaches, i.e., [8] and [9], only work well if the SGP and CSPs have full information from each other. However, in practice, SGPs and CSPs usually have some private information, e.g., energy capacity and demands, which may not be available for optimization in advance, and thus other appropriate economic models should be studied.

To address the problems, the contract theory-based approach can be a promising solution. This approach focuses on the mutual agreements between the principals which offer contracts and the agents which accept/reject the contracts [10]. In this way, the principals can maximize the utility under the individual rationality (IR) and incentive compatibility (IC) constraints from the agents. The former constraint is to guarantee that the agents can obtain a non-negative utility while the latter one is to maximize the utility when the appropriate contracts from the principals are applied. For example, a contract game-based approach in which an energy consumer (as the principal) offers contracts, i.e., the amount of energy and the corresponding payment, to the multi-type energy providers (as the agents) is introduced in [11]. In [12], the authors propose a contract-based method on power trading between a mobile CSP and multiple EVs under complete and incomplete information, e.g., remaining battery life, of EVs. Alternatively, the authors in [13] introduce a contract-based charging strategy for EV clusters along with an iterative algorithm to optimize a CSP's utility. Nevertheless, the above approaches only consider a sin-

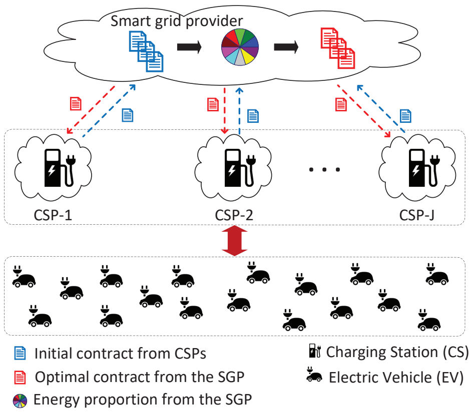


Fig. 1: System model of an EV network.

gle principal optimization where energy contracts are offered to the multiple agents. In fact, a competition among multiple principals to attract the agent and maximize their utilities is more challenging and general in practice. Furthermore, they only consider the contract interaction between a CSP and EVs (or an EV and CSPs), where the aggrement between the SGP and CSPs to stabilize energy transfer in the power grid system are usually left out.

To address the above challenges, in this paper, we introduce a new economic model for an EV network using a multi-principal single-agent (referred to as *common agency*) contract [15] where the CSPs operate as the principals which offer contracts containing energy demand and payment, while the SGP works as the agent which optimizes the offered contracts correspondingly. The aim of this model is to maximize the utilities of CSPs and improve the social welfare, i.e., overall utilities of the SGP and CSPs in the EV network, given that the SGP and CSPs have private information. Specifically, we formulate the contract model as a non-cooperative energy contract optimization problem in which each CSP maximizes its utility under common IR and IC constraints from the SGP and other CSPs' contracts. To do so, we develop an iterative algorithm at the SGP to not only achieve the optimal contracts but also find the equilibrium contract solution. This solution can achieve the social welfare less than 10% compared with that of the cooperative contract, i.e., when all the CSPs collaborate to share their energy demands and offer only one contract to the SGP. Through numerical results, we demonstrate that our proposed model can enhance the social welfare of the EV network, i.e., up to 54%, and the utilities of participating CSPs up to 60% compared with the baseline method.

II. System Model

The system model of the considered EV network is illustrated in Fig. 1. We consider J CSs belonging to J different CSPs. Each CSP can store the energy from the power sources through the power grid system at its CS's energy storage prior to EV charging. To monitor all charging transactions of EVs, each CSP uses a log file containing consumed energy of the EVs' transactions for a certain period. Based on the total energy usage transactions at particular period, each CSP can send an initial energy contract, i.e., energy demand and offered payment, to the SGP at different time before optimal contract optimization process at the SGP. For example, the SGP can collect all initial contracts from the CSPs on the weekdays and send final optimal contracts to them on the weekend.

Let $\mathcal{J} = \{1, \dots, j, \dots, J\}$ denote the set of CSPs in the considered area which request energy from the SGP. The SGP can supply an amount of energy with the capacity of A for the CSPs at a particular period. We set ζ to be the SGP's energy transfer price unit for the CSPs. Additionally, the energy charging price per MWh for EVs at CSP-j is denoted by α_j. We define ψ to be the possible type of the SGP which represents the willingness to transfer energy for the CSPs [7] based on the SGP's energy capacity, where ψ is between the minimum type $\underline{\psi}$ and the maximum type $\overline{\psi}$. In practice, the types of SGP can be expressed as its energy capacities, and thus an SGP with higher type is usually willing to transfer more energy due to its larger energy capacity [11]. The type of SGP is usually a private information which the SGP does not want to share with the CSPs due to its economic benefit. Hence, in practice, the CSPs only can observe the distribution of the SGP's type, i.e., $\rho(\psi)$ [11]–[13], e.g., through observations about public transactions of the SGP in the energy trading market.

By observing $\rho(\psi)$, each CSP may offer different energy contract to the SGP. Then, based on energy demand requests from the CSPs, the SGP can determine the optimal proportions for the CSPs according to its capacity. Specifically, we denote $\boldsymbol{\omega} = [\omega_1, \dots, \omega_j, \dots, \omega_J]$ (because the SGP exactly knows its own type) as a vector of real proportions that the SGP with type ψ will transfer the requested energy to all the CSPs, where $0 \leq \omega_j \leq 1$. In addition, we define $\boldsymbol{\delta}(\psi) = [\delta_1(\psi), \dots, \delta_j(\psi), \dots, \delta_J(\psi)]$ as a vector of energy amount requested by the CSPs, given the possible SGP's types $\psi, \forall \psi \in [\underline{\psi}, \overline{\psi}]$. Each CSP-$j$ may also offer various payments for different types of the SGP. In this case, we denote $\boldsymbol{\beta}(\psi) = [\beta_1(\psi), \dots, \beta_j(\psi), \dots, \beta_J(\psi)]$ as a vector of offered payments from the CSPs for the possible SGP's types $\psi, \forall \psi \in [\underline{\psi}, \overline{\psi}]$. To this end, when the SGP has a higher type, the CSPs can try to request more energy (and offer higher payments correspondingly) to the SGP and vice versa [7]. Otherwise, if a CSP requests more energy demand to a lower type SGP, the SGP may provide even lower energy proportion to that CSP to ensure that the SGP can still maximize the utilities of all CSPs fairly given its energy capacity.

III. Utility Functions for the SGP and CSPs

In this section, we elaborate the utility functions for the SGP and CSPs. Given the energy demand $\boldsymbol{\delta}(\psi)$ and the payment $\boldsymbol{\beta}(\psi)$, the utility function of the SGP with type ψ can be determined as follows:

$$\psi S(\boldsymbol{\omega}, \boldsymbol{\beta}(\psi)) - C(\boldsymbol{\omega}, \boldsymbol{\delta}(\psi)), \tag{1}$$

where ψ in the first component is to characterize the weight of the $S(\boldsymbol{\omega}, \boldsymbol{\beta}(\psi))$ of the SGP with type ψ. The SGP with a higher type should have a higher weight due to its willingness to transfer more energy [14]. Additionally, $S(\boldsymbol{\omega}, \boldsymbol{\beta}(\psi))$ and $C(\boldsymbol{\omega}, \boldsymbol{\delta}(\psi))$ refer to the satisfaction and cost functions of the SGP, respectively. In particular, we adopt a natural logarithm function that is widely used to quantify the satisfaction of energy sellers [11]–[13], which can be expressed by $S(\boldsymbol{\omega}, \boldsymbol{\beta}(\psi)) = \ln\bigg(1 + \sum_{j=1}^{J} \omega_j \beta_j(\psi)\bigg)$. This equation specifies that the satisfaction function follows the law of diminishing returns, i.e., the satisfaction increases as the payments from the CSPs increase. However, the SGP may have less interest to enhance the satisfaction when the current amount of energy in its energy storage becomes smaller due to energy transfer. Moreover, the cost function of SGP is imposed when the SGP acquires energy to serve the CSPs' requests [7]. Thus, it can be calculated by $C(\boldsymbol{\omega}, \boldsymbol{\delta}(\psi)) = \gamma \sum_{j=1}^{J} \omega_j \delta_j(\psi)$, where $\gamma > 0$ is the energy transfer cost per energy unit through the power grid system, i.e., the distribution network between the SGP and the CSPs including power flow and voltage usages. From (1), the SGP with type ψ needs to maximize its own utility as the optimization problem $(\mathbf{Q}_1)$ below:

$$(\mathbf{Q}_1) \quad \max_{\boldsymbol{\omega}} \ \psi S(\boldsymbol{\omega}, \boldsymbol{\beta}(\psi)) - C(\boldsymbol{\omega}, \boldsymbol{\delta}(\psi)), \tag{2}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} \omega_j \delta_j(\psi) \leq A(\psi), \tag{3}$$

$$0 \leq \omega_j \leq 1, \forall j \in \mathcal{J}, \tag{4}$$

where $A(\psi)$ indicates the energy capacity for the SGP with type ψ. From $(\mathbf{Q}_1)$, we can obtain the optimal proportion $\hat{\boldsymbol{\omega}}$, where $\hat{\boldsymbol{\omega}} = [\hat{\omega}_1, \ldots, \hat{\omega}_j, \ldots, \hat{\omega}_J]$, straightforwardly. The reason is that the objective function in (2) is convex due to the concavity of satisfaction function and the linearity of cost function in the first and second component, respectively. Additionally, the constraints (3) and (4) are linear, and thus we can obtain the optimal $\hat{\boldsymbol{\omega}}$ using some well-known tools such as CPLEX [16]. Taking into account $\hat{\omega}_j, \forall j \in \mathcal{J}$, each CSP can obtain a certain amount of energy from the SGP with the expense of energy transfer payment. Hence, the expected utility function of CSP-j can be written as follows [10]:

$$\mu_j(\boldsymbol{\beta}(\psi), \boldsymbol{\delta}(\psi)) = \int_{\underline{\psi}}^{\overline{\psi}} \Big(\alpha_j \hat{\omega}_j \delta_j(\psi) - \hat{\omega}_j \beta_j(\psi)\Big) \rho(\psi) d\psi, \tag{5}$$

where $\alpha_j > 0$ is the energy price unit per MWh for EVs at CSP-j. The (5) implies the expected profit of CSP-j obtained from its EVs given the distribution of the SGP's possible types.

In the contract design, the CSPs send contracts $\big(\beta_j(\psi), \delta_j(\psi)\big), \forall j \in \mathcal{J}, \forall \psi \in [\underline{\psi}, \overline{\psi}]$, to the SGP, to maximize their utility functions under the following two contraints from the SGP, i.e., individual rationality (IR) and incentive compatibility (IC) [10]. These constraints must be satisfied to guarantee the feasibility of the contracts.

Definition 1. *Individual Rationality (IR): The utility function of the SGP must be equal or greater than its utility level tolerance $\hat{\mu}_{SGP}$ = 0, to satisfy a non-negative utility, i.e.,*

$$\psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi)) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi)) \geq \hat{\mu}_{SGP}, \forall \psi \in [\underline{\psi}, \overline{\psi}]. \tag{6}$$

Definition 2. *Incentive Compatibility (IC): The SGP with type ψ will prefer to select a contract design for its own type ψ rather than that with another type $\hat{\psi}$, i.e.,*

$$\begin{aligned}&\psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi)) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi)) \geq \psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\hat{\psi})) - \\ &C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\hat{\psi})), \psi \neq \hat{\psi}, \forall \psi, \hat{\psi} \in [\underline{\psi}, \overline{\psi}].\end{aligned} \tag{7}$$

Based on Definition 1 and 2, the IR constraints in (6) guarantee that the SGP with a certain type will always participate in the energy transfer process. Furthermore, the IC constraints in (7) ensure that the SGP can always obtain the maximum utility if an appropriate contract, i.e., the contract designed for that type of the SGP, is employed [10].

IV. Energy Contract Formulation

From (1)-(7), we can formulate and simplify the energy contract optimization in the following sections.

A. Non-Cooperative Energy Contract

We design a non-cooperative energy contract model where multiple CSPs offer contracts independently to the SGP. Given the (5), the energy contract optimization problem $(\mathbf{Q}_2)$ to maximize the utility for each CSP-j independently at the SGP is described as follows:

$$(\mathbf{Q}_2) \quad \max_{\boldsymbol{\beta}(\psi), \boldsymbol{\delta}(\psi)} \ \mu_j(\boldsymbol{\beta}(\psi), \boldsymbol{\delta}(\psi)), \forall j \in \mathcal{J}, \tag{8}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} \hat{\omega}_j \delta_j(\psi) \leq A(\psi), \tag{9}$$

$$\psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi)) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi)) \geq 0, \forall \psi \in [\underline{\psi}, \overline{\psi}], \tag{10}$$

$$\begin{aligned}&\psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi)) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi)) \geq \psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\hat{\psi})) - \\ &C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\hat{\psi})), \psi \neq \hat{\psi}, \forall \psi, \hat{\psi} \in [\underline{\psi}, \overline{\psi}],\end{aligned} \tag{11}$$

where constraints (9) indicate that the total actual energy allocation from CSPs must not exceed the energy capacity of the SGP with type ψ. Moreover, the constraints (10) and (11) refer to the IR and IC constraints from the SGP. From $(\mathbf{Q}_2)$, the optimal contract of CSP-j is influenced by the SGP's choice of $\hat{\omega}_j$. In this case, each CSP-j will compete with other CSPs in a competition strategy to attract the SGP and find an equilibrium solution as defined in Definition 3.

Definition 3. *Equilibrium solution for non-cooperative energy contract: The optimal contracts $(\boldsymbol{\beta}^*(\psi), \boldsymbol{\delta}^*(\psi))$ or $\big(\beta_j^*(\psi), \delta_j^*(\psi)\big)$, $\forall j \in \mathcal{J}, \forall \psi \in [\underline{\psi}, \overline{\psi}]$, are called the equilibrium solutions of the $(\mathbf{Q}_2)$ if and only if the following conditions satisfy*

$$\begin{aligned}&\mu_j(\boldsymbol{\beta}^*(\psi), \boldsymbol{\delta}^*(\psi)) \geq \mu_j(\beta_j(\psi), \delta_j(\psi), \boldsymbol{\beta}^*_{-j}(\psi), \boldsymbol{\delta}^*_{-j}(\psi)), \\ &\forall j \in \mathcal{J}, \forall \psi \in [\underline{\psi}, \overline{\psi}],\end{aligned} \tag{12}$$

for all $\big(\beta_j(\psi), \delta_j(\psi)\big) \neq \big(\beta_j^*(\psi), \delta_j^*(\psi)\big)$ that meets the constraints (10) and (11).

Particularly, given the equilibrium contract selections of other CSPs, i.e., $(\boldsymbol{\beta}^*_{-j}(\psi), \boldsymbol{\delta}^*_{-j}(\psi)), \forall \psi \in [\underline{\psi}, \overline{\psi}]$, the utility function of CSP-j with the equilibrium contract choice $\big(\beta_j^*(\psi), \delta_j^*(\psi)\big), \forall \psi \in [\underline{\psi}, \overline{\psi}]$, has to be the highest utility among other utilities of the CSP-j. In other words, the conditions in (12) imply that there is no CSP-j which can improve its utility by unilaterally deviating from its optimal contracts $\left(\beta_j^*(\psi), \delta_j^*(\psi)\right), \forall \psi \in [\underline{\psi}, \overline{\psi}]$.

B. Energy Contract Simplification

From $(\mathbf{Q}_2)$, we can observe that the computational complexity of solving $(\mathbf{Q}_2)$ grows quadratically when the number of possible types increases in the iterative process. To address this problem, we can simplify the $(\mathbf{Q}_2)$ by reducing the number of IR constraints in (10) and IC constraints in (11) [10]. For that, we first state Lemma 1 as follows.

Lemma 1. *Let $(\boldsymbol{\beta}, \boldsymbol{\delta})$ denote any feasible contracts from CSPs to the SGP such that if $\psi \geq \hat{\psi}$, then $\boldsymbol{\beta}(\psi) \geq \boldsymbol{\beta}(\hat{\psi})$, where ψ and $\hat{\psi} \in [\underline{\psi}, \overline{\psi}]$.*

Proof. Due to limited space, we provide high-level ideas to prove Lemma 1 as follows. In particular, we can use IC constraints from the Definition 2. As such, we first prove that $\boldsymbol{\beta}(\psi) \geq \boldsymbol{\beta}(\hat{\psi})$ if and only if $\psi \geq \hat{\psi}$. Then, we can show that $\psi \geq \hat{\psi}$ if and only if $\boldsymbol{\beta}(\psi) \geq \boldsymbol{\beta}(\hat{\psi})$. □

Based on Lemma 1, intuitively, we can further state that the contract requires a higher energy demand if the SGP obtains a higher payment, i.e., if $\boldsymbol{\beta}(\psi) \geq \boldsymbol{\beta}(\hat{\psi})$, then $\boldsymbol{\delta}(\psi) \geq \boldsymbol{\delta}(\hat{\psi})$ [7]. As a result, the following Proposition 1 can be achieved.

Proposition 1. *For any feasible contract $(\boldsymbol{\beta}, \boldsymbol{\delta})$ with $\psi \geq \hat{\psi}$, ψ and $\hat{\psi} \in [\underline{\psi}, \overline{\psi}]$, the utility of the SGP must satisfy*

$$\begin{aligned} &\psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi)) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi)) \geq \\ &\quad \hat{\psi} S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\hat{\psi})) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\hat{\psi})). \end{aligned} \tag{13}$$

Proof. Due to limited space, we provide high-level ideas to prove. Specifically, from Lemma 1, if $\boldsymbol{\beta}(\psi) \geq \boldsymbol{\beta}(\hat{\psi})$, then $\boldsymbol{\delta}(\psi) \geq \boldsymbol{\delta}(\hat{\psi})$. Thus, if $\psi \geq \hat{\psi}$, we have $\psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi)) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi)) \geq \hat{\psi} S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\hat{\psi})) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\hat{\psi}))$. □

From Proposition 1, we can reduce the number of IR constraints using the $\underline{\psi}$. In particular, as the IC constraints hold, we have

$$\begin{aligned} &\psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi)) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi)) \geq \psi S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\underline{\psi})) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\underline{\psi})) \\ &\geq \underline{\psi} S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\underline{\psi})) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\underline{\psi})). \end{aligned} \tag{14}$$

Equation (14) specifies that the utility function of the SGP is monotonically increasing function of ψ. Hence, the IR constraints for other ψ, where $\psi \neq \underline{\psi}$, will hold if and only if the IR constraint for $\underline{\psi}$ is satisfied, i.e., the utility function of the SGP for $\underline{\psi}$ is non-negative. As such, we can replace the IR constraints in (10) into

$$\underline{\psi} S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\underline{\psi})) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\underline{\psi})) \geq 0. \tag{15}$$

In addition to the IR constraint reduction, we can further reduce the IC constraints by replacing them using the following conditions described in Lemma 2.

Lemma 2. *The IC constraints in (11) of $(\mathbf{Q}_2)$ are equivalent to the following monotonicity, i.e.,*

$$\frac{d\boldsymbol{\beta}(\psi)}{d\psi} \geq 0, \forall \psi \in [\underline{\psi}, \overline{\psi}], \tag{16}$$

and local IC conditions, i.e.,

$$\psi \frac{dS(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi))}{d\psi} - \frac{dC(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi))}{d\psi} \geq 0, \forall \psi \in [\underline{\psi}, \overline{\psi}]. \tag{17}$$

Proof. Due to limited space, we provide high-level ideas to prove Lemma 2 as follows. Particularly, the monotonicity conditions can be easily derived from Lemma 1. Then, the local IC conditions can be proved using the contradiction such that the IC constraints cannot be satisfied. □

Based on the aforementioned constraint simplification, we can rewrite the optimization problem $(\mathbf{Q}_2)$ into

$$(\mathbf{Q}_3) \quad \max_{\boldsymbol{\beta}(\psi), \boldsymbol{\delta}(\psi)} \mu_j(\boldsymbol{\beta}(\psi), \boldsymbol{\delta}(\psi)), \forall j \in \mathcal{J}, \tag{18}$$

s.t. (9) and,

$$\underline{\psi} S(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\underline{\psi})) - C(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\underline{\psi})) \geq 0, \tag{19}$$

$$\psi \frac{dS(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi))}{d\psi} - \frac{dC(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi))}{d\psi} \geq 0, \forall \psi \in [\underline{\psi}, \overline{\psi}], \tag{20}$$

$$\frac{d\boldsymbol{\beta}(\psi)}{d\psi} \geq 0, \forall \psi \in [\underline{\psi}, \overline{\psi}]. \tag{21}$$

Practically, an agent may have finite number of types [7]. Thus, we can assume that the SGP has discrete number of possible types (e.g., the SGP with low, medium, and large energy capacity correspond to three different levels of willingness to transfer energy). Subsequently, we can transform $(\mathbf{Q}_3)$ into discrete forms by first rewriting the (5) as follows:

$$\hat{\mu}_j(\boldsymbol{\beta}(\psi), \boldsymbol{\delta}(\psi)) = \sum_{\psi=\underline{\psi}}^{\overline{\psi}} \Big(\alpha_j \hat{\omega}_j \delta_j(\psi) - \hat{\omega}_j \beta_j(\psi)\Big)\rho(\psi), \tag{22}$$

where $\sum_{\psi=\underline{\psi}}^{\overline{\psi}} \rho(\psi) = 1$. We then modify the left side of the constraints (20) as follows:

$$\begin{aligned} &\psi \frac{dS(\hat{\boldsymbol{\omega}}, \boldsymbol{\beta}(\psi))}{d\psi} - \frac{dC(\hat{\boldsymbol{\omega}}, \boldsymbol{\delta}(\psi))}{d\psi} \\ &= \psi \sum_{j=1}^{J} \hat{\omega}_j \Big(\beta_j(\psi) - \beta_j(\psi - \Delta\psi)\Big) - \\ &\left(1 + \sum_{j=1}^{J} \hat{\omega}_j \beta_j(\psi)\right)\left(\gamma \sum_{j=1}^{J} \hat{\omega}_j \Big(\delta_j(\psi) - \delta_j(\psi - \Delta\psi)\Big)\right), \end{aligned} \tag{23}$$

where $\Delta\psi = \psi - \psi^*$, and ψ^* refers to the type of the SGP which is one type smaller than the type ψ. As a result, the simplified version of $(\mathbf{Q}_3)$ is

$$(\mathbf{Q}_4) \quad \max_{\boldsymbol{\beta}(\psi), \boldsymbol{\delta}(\psi)} \hat{\mu}_j(\boldsymbol{\beta}(\psi), \boldsymbol{\delta}(\psi)), \forall j \in \mathcal{J}, \tag{24}$$

s.t. (9), (19) and,

$$\begin{aligned} &\psi \sum_{j=1}^{J} \hat{\omega}_j \Big(\beta_j(\psi) - \beta_j(\psi - \Delta\psi)\Big) - \\ &\left(1 + \sum_{j=1}^{J} \hat{\omega}_j \beta_j(\psi)\right)\left(\gamma \sum_{j=1}^{J} \hat{\omega}_j \Big(\delta_j(\psi) - \delta_j(\psi - \Delta\psi)\Big)\right) \\ &\geq 0, \forall \psi \in [\underline{\psi}, \overline{\psi}], \end{aligned} \tag{25}$$

$$\beta_j(\psi) - \beta_j(\psi - \Delta\psi) \geq 0, \forall j \in \mathcal{J}, \forall \psi \in [\underline{\psi}, \overline{\psi}]. \tag{26}$$

To find the optimal contracts from $(\mathbf{Q}_4)$, we propose an iterative algorithm in Algorithm 1. In particular, we first find the optimal values of $\hat{\boldsymbol{\omega}}$ which maximize $(\mathbf{Q}_1)$. Then, we perform the iterative algorithm in which the SGP can update the possible contracts of each CSP to maximize the CSP's utility, given other CSPs' current contracts remain fixed [18]. In each iteration, the SGP tries to find the optimal contract of each CSP which maximizes the CSP's utility. The algorithm stops when the differences between the previous and current iterations' utilities of all CSPs reach the optimality tolerance η, and thus the equilibrium contract solution can be achieved.

Algorithm 1 Iterative Energy Contract Algorithm

The SGP informs the current ζ (MWh) to all CSPs
Each CSP sends $\left(\beta_j^{(0)}(\psi), \delta_j^{(0)}(\psi)\right), \forall \psi \in [\underline{\psi}, \overline{\psi}]$
Set η, $\theta = 0$, and $\hat{\mu}_j\left(\boldsymbol{\beta}^{(0)}(\psi), \boldsymbol{\delta}^{(0)}(\psi), \right), \forall j \in \mathcal{J}$
repeat
  Find $\hat{\boldsymbol{\omega}}^{(\theta)}$ which maximize $(\mathbf{Q}_1)$
  for $\forall j \in \mathcal{J}$ **do**
    Find the contracts $\left(\beta_j^{(\theta+1)}(\psi), \delta_j^{(\theta+1)}(\psi)\right), \forall \psi \in [\underline{\psi}, \overline{\psi}]$, which maximize $(\mathbf{Q}_4)$
    Compute $\hat{\mu}_j\left(\beta_j^{(\theta+1)}(\psi), \delta_j^{(\theta+1)}(\psi), \boldsymbol{\beta}_{-j}^{(\theta)}(\psi), \boldsymbol{\delta}_{-j}^{(\theta)}(\psi)\right)$
    if $\left[\hat{\mu}_j\left(\beta_j^{(\theta+1)}(\psi), \delta_j^{(\theta+1)}(\psi), \boldsymbol{\beta}_{-j}^{(\theta)}(\psi), \boldsymbol{\delta}_{-j}^{(\theta)}(\psi)\right) - \hat{\mu}_j\left(\boldsymbol{\beta}^{(\theta)}(\psi), \boldsymbol{\delta}^{(\theta)}(\psi)\right)\right] > \eta$ **then**
      Set $\beta_j^{(\theta)}(\psi) = \beta_j^{(\theta+1)}(\psi)$, $\delta_j^{(\theta)}(\psi) = \delta_j^{(\theta+1)}(\psi), \forall \psi \in [\underline{\psi}, \overline{\psi}]$
    end if
  end for
until $\hat{\mu}_j\left(\boldsymbol{\beta}^{(\theta)}(\psi), \boldsymbol{\delta}^{(\theta)}(\psi)\right), \forall j \in \mathcal{J}$ do not change
Obtain $\left(\boldsymbol{\beta}^{*}(\psi), \boldsymbol{\delta}^{*}(\psi)\right), \forall \psi \in [\underline{\psi}, \overline{\psi}]$

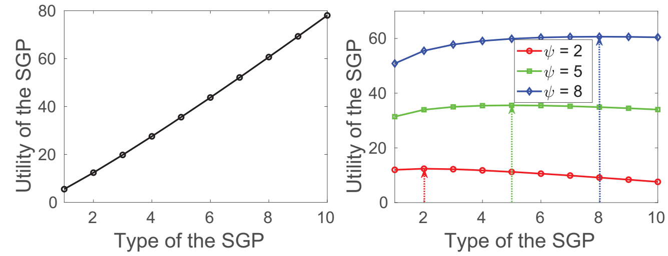
V. Performance Evaluation

A. Simulation Setup

We evaluate the proposed method using the following simulation settings. Specifically, we set the SGP's energy transfer price unit at 200AUD/MWh. We also set the energy charging price unit from the CSPs to the EVs at 220AUD/MWh [19]. Moreover, the initial energy demands for all CSPs are randomly generated using a uniform distribution between 0 and 3 MWh for a week. We consider 10 possible types of the SGP, i.e., $\underline{\psi} = 1$ and $\overline{\psi} = 10$, with the same distribution of the types, i.e., $\rho(\psi) = 0.1$. We set the energy capacity for the SGP with type ψ at 10ψ MWh. We also set $\gamma = 0.3$. Then, we compare the performance of the proposed economic model with that of the cooperative contract and the baseline method (i.e., each CSP obtains the amount of energy that is proportional to its energy request, given the SGP's energy capacity). To obtain various simulation results, we consider various number of principals, i.e., $J = 5$ to $J = 50$ CSPs.

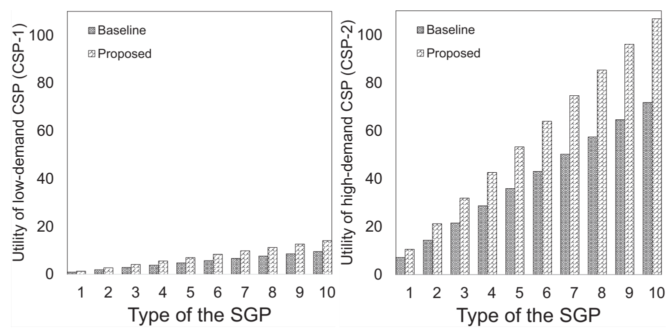
B. Simulation Results

Fig. 2 shows the validity of IR and IC constraints of the SGP in the case with 20 CSPs considered. We first can observe in Fig. 2(a) that the utility of the SGP for all possible types



(a) IR constraint (b) IC constraint

Fig. 2: The validation of IR and IC constraints of the SGP for the proposed method.



(a) CSP-1 (b) CSP-2

Fig. 3: The actual utilities of several CSPs.

achieves non-negative values, thereby satisfying IR constraints. In particular, the utility of the SGP follows non-decreasing trend with the type. The reason is that higher type of the SGP can store larger amount of energy, and thus it triggers the willingness to transfer more energy to CSPs and get higher payments from the CSPs. Then, from Fig. 2(b), we can show that IC constraints are also satisfied. Particularly, the SGP achieves the highest utility when it pretends to be its own type, instead of pretending to be the other types. For example, the SGP with the type 2, type 5, and type 8 reach the highest utility when it applies the exact contracts for the type 2, type 5, and type 8, respectively. If the SGP does not pretend to be its own type, the utility will be lower or equal to that of its true type. Because the IR and IC constraints of the SGP are guaranteed, we can find the feasible contracts for all CSPs.

We then vary the type of SGP to evaluate the actual utility of representative low- and high-demand CSPs as well as social welfare, i.e., the accumulated utilities of all the parties (including the SGP and all the CSPs) in the considered system, in Fig. 3 and Fig. 4, respectively. Specifically, the proposed method can improve the actual utility up to 48% for both low-demand CSP, i.e., CSP-1, and high-demand CSP, i.e., CSP-2, compared with that of the baseline method. In this case, we do not apply the utility of cooperative contract due to one utility optimization for all CSPs. For the social welfare, this is of importance to observe the efficiency of the whole EV

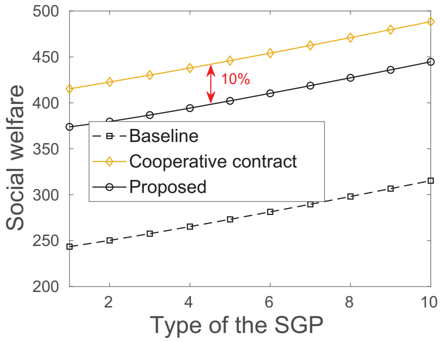


Fig. 4: Overall social welfare between the SGP and all CSPs for non-cooperative (proposed) and cooperative contracts.

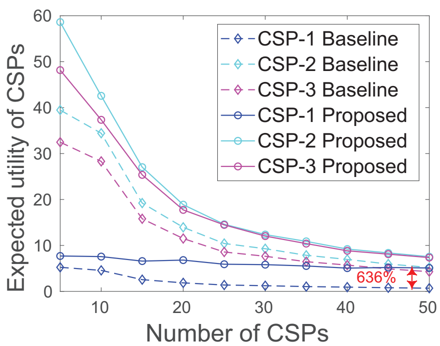


Fig. 5: The CSPs' expected utility under different methods as the number of CSPs increases.

network [10]. Align with the trend of the SGP's utility in Fig. 2(a), the social welfare can be improved when the type gets higher. As such, compared with the baseline method, the proposed method can enhance the social welfare up to 54%. In addition, compared with the scenario in which all the CSPs are cooperative, its social welfare can achieve less than 10% gap. Note that, although the cooperative contract can achieve better social welfare, it requires that all CSPs belong to the same provider or accept to share the information and join in the same contract without any competition among them.

In Fig. 5, we present the expected utilities of some CSPs, i.e., CSP-1, CSP-3, and CSP-2 (in ascending order based on their energy demands), when the number of CSPs increases. As such, we observe that the utility of each CSP gets lower as the number of CSPs increases due to the lower proportion influenced by the SGP's limited energy capacity. Compared with the baseline method, all CSPs can improve their utilities by 60% for small number of CSPs in the network. When there exists a high number of CSPs, the utility gap between the proposed and baseline methods can reach up to 636%. This implies that all CSPs can balance among their utility optimizations based on their energy request proportions.

VI. Conclusion

In this paper, we have proposed the economic model using the common agency-based contract strategy to maximize the utilities of all CSPs. First, we have formulated the common agency-based contract problem as the non-cooperative energy contract optimization problem. Then, we have developed the iterative energy contract algorithm to achieve the equilibrium contract solution for all CSPs. Through numerical results, we have shown that compared with the baseline method, our proposed method can improve the utilities of the CSPs as well as the social welfare of the EV network significantly. This is because the proposed method can help to balance the utility optimizations of all participating CSPs based on their optimal energy request proportions and contracts. In the future work, we will investigate the utility performance when communication constraints are considered.

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