AN ACOUSTIC MODELLING BASED REMOTE ERROR SENSING APPROACH FOR QUIET ZONE GENERATION IN A NOISY ENVIRONMENT

Qiaoxi Zhu, Xiaojun Qiu and Ian Burnett

Centre for Audio, Acoustics and Vibration, Faculty of Engineering and IT, University of Technology Sydney, 15 Broadway, Ultimo, NSW, Australia

Qiaoxi.Zhu, Xiaojun.Qiu, Ian.Burnett@uts.edu.au

ABSTRACT

Remote error sensing is required in active noise control systems when they are used to create a quiet zone in a noisy environment with the constraint that the error microphones cannot be inside the zone. The challenge in remote error sensing is to estimate the sound pressure in the target zone with a small number of physical microphones outside it. The spatial harmonic decomposition method uses wave domain sound field parameterisation to reduce the required number of the error microphones but can only provide accurate estimation below a certain frequency. This paper presents an improved approach to increase the effective frequency range based on the acoustic modelling. The simulation results demonstrate the proposed method can provide more than 20 dB noise reduction up to 1650 Hz for a quiet zone with a radius of 0.1 m by using only three microphones under the studied situations.

***Index Terms*—** Active noise control, remote sensing, sound field representation, spatial harmonic decomposition, singular value decomposition

1. INTRODUCTION

It is sometimes difficult to put microphones near a listener's ears to monitor the received sound directly. The sound pressure at these positions can be estimated using remote sensing methods with an array of remote microphones [1, 2]. These techniques require a preliminary training stage [3] or an acoustic model [4] to determine the transfer functions between the sound pressures at the measured points and those at the control points. However, the noise reduction performance is quite sensitive to the accuracy of the preliminary measurement or the acoustic model estimation.

An alternative approach is to parameterise the primary and secondary sound fields over a region by superposition of basis functions [5, 6]. The basis functions can be plane wave, cylindrical wave, or spherical wave functions. They are the solutions of the homogeneous wave equation in different coordinate systems and contain wave propagation properties [5]. This parameterisation transfers the estimation of continuous sound field distribution to the estimation of coefficients of the associated basis functions.

The spatial harmonic decomposition (SHD) uses spatial harmonics (cylindrical/spherical wave functions) to describe two/three-dimensional sound fields over a region. The spatial harmonics provide the basis set which is mostly concentrated within a given finite circular/sphere region. Therefore, the SHD method allows the minimal number of basis functions (and also boundary samples) to describe a regional sound field with random sound directions, where the number is proportional to the product of the wavenumber and radius of the region [7]. This method can provide efficient active noise control performance over a space and has received significant attention in recent years [6, 8, 9].

In this paper, an acoustic modelling based singular value decomposition (SVD) method is proposed to estimate the primary and secondary sound field over a region with a few microphones. This method constructs the acoustic model based on the primary sound source direction and the secondary sound source locations. Then, the basis functions are derived through SVD to estimate the responses between the sound pressures at the boundary samples and those throughout the region of interest. The simulation shows that the proposed method performs similar to the SHD method when the sound direction of the primary sound sources is unknown. However, the proposed method can facilitate effective local active noise control over a broader frequency band than the SHD method when the primary sound sources are from a known range of directions.

This research is supported under the Australian Research Council's Linkage Project funding scheme (LP160100616), and UTS Tech Lab Blue Sky Research Grant.

2. BACKGROUND

This section will formulate the remote error sensing technique for active noise control over a region first and then introduce the spatial harmonic decomposition method.

2.1. Active noise control

The energy of the error signal $\boldsymbol{e}$ over a region is minimised to create a quiet zone

$$\text{minimise } \|\boldsymbol{e}\|^2, \tag{1}$$

where $\boldsymbol{e}$ is a V-element column vector and V is the number of control points over the region of interest. The error signal can be written as

$$\boldsymbol{e} = \boldsymbol{P}_{\mathrm{PE}} + \boldsymbol{P}_{\mathrm{SE}} = \boldsymbol{P}_{\mathrm{PE}} + \mathbf{H}_{\mathrm{SE}} \cdot \boldsymbol{w}, \tag{2}$$

where the V-element column vectors $\boldsymbol{P}_{\mathrm{PE}}$ and $\boldsymbol{P}_{\mathrm{SE}}$ represent the primary sound pressures and the secondary sound pressures at the control points, respectively, the $V \times L$ matrix $\mathbf{H}_{\mathrm{SE}}$ represents the transfer function matrix between the secondary sources and the control points, and the L-element column vector $\boldsymbol{w}$ contains the weights for the secondary sources. Substituting (2) into (1), the weights of the secondary sources are derived as

$$\boldsymbol{w} = -\mathbf{H}_{\mathrm{SE}}^{+} \cdot \boldsymbol{P}_{\mathrm{PE}}, \tag{3}$$

where the superscript "+" denotes the Moore-Penrose inverse. The weights depend on both the sound pressure distribution information of the primary sound field and the spatial transfer function information of the secondary sound sources in the control region.

To achieve noise reduction over the entire spatial region, the control points should be spatially distributed and sufficiently sampled over the region to be controlled. The conventional method [10] requires the spacing between control points to be less than half a wavelength. For a 2D circular region with a radius of R, the number of control points $V \geq \pi R^2/(\lambda/2)^2 = (kR)^2/\pi$, where λ denotes the wavelength, $k = 2\pi f/c$ denotes the wavenumber, f denotes the frequency and c is the speed of sound in air. With increased frequency or size of the region, the required number of control points increases dramatically. Unfortunately, it is hard in practice to do measurements at a large number of control points due to hardware cost and inconvenience.

2.2. Remote error sensing

The remote sensing technique uses a small number of microphones on the boundary of the controlled region to estimate the sound pressure distribution over the region $\boldsymbol{P}_{\mathrm{E}}$ from the measurement values $\boldsymbol{P}_{\mathrm{M}}$, so it has

$$\widetilde{\boldsymbol{P}}_{\mathrm{E}} = \mathbf{T}\boldsymbol{P}_{\mathrm{M}}. \tag{4}$$

where the matrix $\mathbf{T}$ contains the relationship between $\boldsymbol{P}_{\mathrm{M}}$ and $\boldsymbol{P}_{\mathrm{E}}$. The M-element column vector $\boldsymbol{P}_{\mathrm{M}}$ represents the measured sound pressures at M measured points on the boundary of the controlled region, and the V-element column vector $\widetilde{\boldsymbol{P}}_{\mathrm{E}}$ represents the estimated sound pressures at V control points over the controlled region. The number of measured points is much smaller than that of the control points ($M \ll V$) for broadband applications.

In the active noise control, the remote error sensing is to estimate the error signals over a region using a small number of microphones on its boundary. The estimated error signal is

$$\tilde{\boldsymbol{e}} = \widetilde{\boldsymbol{P}}_{\mathrm{PE}} + \widetilde{\mathbf{H}}_{\mathrm{SE}} \cdot \boldsymbol{w}. \tag{5}$$

Specifically, for the primary sound field, the estimated primary sound pressures

$$\widetilde{\boldsymbol{P}}_{\mathrm{PE}} = \mathbf{T}_{\mathrm{P}} \cdot \boldsymbol{P}_{\mathrm{PM}}, \tag{6}$$

where $\mathbf{T}_{\mathrm{P}}$ is the sensing matrix and $\boldsymbol{P}_{\mathrm{PM}}$ represents the measured sound pressures of the primary sound field. For the secondary sound field, the $M \times L$ transfer function matrix measured over the region boundary is

$$\mathbf{H}_{\mathrm{SM}} = [\boldsymbol{h}_{\mathrm{SM},1} \quad \boldsymbol{h}_{\mathrm{SM},2} \quad \cdots \quad \boldsymbol{h}_{\mathrm{SM},L}], \tag{7}$$

where $\boldsymbol{h}_{\mathrm{SM},l}$ ($l = 1, 2, \ldots, L$) represents the transfer functions between the lth secondary source and the measured points. Then the transfer functions between that source and the control points are estimated as

$$\tilde{\boldsymbol{h}}_{\mathrm{SE},l} = \mathbf{T}_{\mathrm{S},l}\boldsymbol{h}_{\mathrm{SM},l}, \tag{8}$$

where $\mathbf{T}_{\mathrm{S},l}$ is the sensing matrix for the sound field of the lth secondary source. The estimated $V \times L$ transfer function matrix is

$$\widetilde{\mathbf{H}}_{\mathrm{SE}} = [\tilde{\boldsymbol{h}}_{\mathrm{SE},1} \quad \tilde{\boldsymbol{h}}_{\mathrm{SE},2} \quad \cdots \quad \tilde{\boldsymbol{h}}_{\mathrm{SE},L}]. \tag{9}$$

So the remote error sensing is actually a problem of determining the matrices $\mathbf{T}_{\mathrm{P}}$ and $\mathbf{T}_{\mathrm{S},l}$ ($l = 1, 2, \ldots, L$) for estimating the primary sound field and the secondary sound source transfer functions over the region, respectively.

2.3. Spatial harmonic decomposition (SHD) method

In the SHD method, the sound pressure within a circular region is estimated by a series of cylindrical waves. Consider that the measured points are evenly distributed over the boundary of a circular region with a radius of R. In a polar coordinate system, the measured points are located at (R, θ_m), where $\theta_m = 2\pi m/M$, $m = 1, 2, \ldots, M$. According to the spatial Fourier transformation [5], the coefficients of the local sound field using the cylindrical waves are

$$\boldsymbol{a} = \mathbf{F}_1\boldsymbol{P}_{\mathrm{M}}, \tag{10}$$

where $\mathbf{F}_1$ is a $(2N + 1) \times M$ matrix, $N = \lceil \mathrm{e}kR/2 \rceil$ and $\lceil * \rceil$ denotes the ceiling function [7]. The nth row and mth column element of $\mathbf{F}_1$ is $\mathrm{e}^{\mathrm{j}(n-N-1)\theta_m}/[M \cdot J_{n-N-1}(kR)]$, $n = 1, 2, \ldots, 2N + 1$ and $m = 1, 2, \ldots, M$. According to the inverse transformation,

$$\boldsymbol{P}_{\mathrm{V}} = \mathbf{F}_2\boldsymbol{a}, \tag{11}$$

where $\mathbf{F}_2$ is a $V \times (2N + 1)$ matrix and its vth row and nth column element is $J_{n-N-1}(kr_v)\,\mathrm{e}^{-\mathrm{j}(n-N-1)\theta_v}$, and (r_v, θ_v) is the location of the vth control point in the polar coordinate system. Therefore,

$$\mathbf{T}_{\mathrm{SHD}} = \mathbf{F}_2\mathbf{F}_1. \tag{12}$$

In the SHD method, the primary sound field and the secondary sound source transfer function estimation use the same sensing matrix, so it has

$$\mathbf{T}_{\mathrm{P,SHD}} = \mathbf{T}_{\mathrm{S,1,SHD}} = \mathbf{T}_{\mathrm{S,2,SHD}} = \cdots = \mathbf{T}_{\mathrm{S},L,\mathrm{SHD}} = \mathbf{T}_{\mathrm{SHD}}. \tag{13}$$

The number of measured samples needs $M \geq 2N + 1$, so [11]

$$M \geq ekR + 1. \tag{14}$$

3. ACOUSTIC MODELLING BASED SINGULAR VALUE DECOMPOSITION (SVD) METHOD

In some applications, the sound propagation direction of the primary source(s) towards the controlled region is known or can be detected using other methods such as microphone array techniques [12]. Based on this prior information, an acoustic model can be constructed for the primary sound field sensing. Based on the equivalent source methods, the primary sound field can be modelled by a group of monopole point sources that generate the same sound propagation direction towards the controlled region. The transfer function matrices between the monopole point sources and the measured points or the control points, $\mathbf{H}_{\text{PM}-\text{AM}}$ and $\mathbf{H}_{\text{PE}-\text{AM}}$, can be theoretically calculated. Then, it has

$$\mathbf{T}_{\text{P,SVD}} = \mathbf{H}_{\text{PE}-\text{AM}} \cdot \mathbf{H}_{\text{PM}-\text{AM}}^{+}. \tag{15}$$

Using SVD, the transfer function matrices $\mathbf{H}_{\text{PM}-\text{AM}}$ and $\mathbf{H}_{\text{PE}-\text{AM}}$ can be decomposed as

$$\mathbf{H}_{\text{PE}-\text{AM}} = \mathbf{U}_{\text{PE}-\text{AM}} \mathbf{\Sigma}_{\text{PE}-\text{AM}} \mathbf{V}_{\text{PE}-\text{AM}}^{\text{H}}, \tag{16}$$

$$\mathbf{H}_{\text{PM}-\text{AM}} = \mathbf{U}_{\text{PM}-\text{AM}} \mathbf{\Sigma}_{\text{PM}-\text{AM}} \mathbf{V}_{\text{PM}-\text{AM}}^{\text{H}}. \tag{17}$$

Substituting (16) and (17) into (15), there is

$$\mathbf{T}_{\text{P,SVD}} = \mathbf{U}_{\text{PE}-\text{AM}} \cdot \mathbf{O}, \tag{18}$$

where $\mathbf{O} = \mathbf{\Sigma}_{\text{PE}-\text{AM}} \mathbf{V}_{\text{PE}-\text{AM}}^{\text{H}} \mathbf{V}_{\text{PM}-\text{AM}} \mathbf{\Sigma}_{\text{PM}-\text{AM}}^{+} \mathbf{U}_{\text{PM}-\text{AM}}^{\text{H}}$. Due to the feature of SVD, $\mathbf{U}_{\text{PE}-\text{AM}}$ is a unitary matrix and its columns represent orthogonal components (basis functions) of the local sound field sampled at the control points, when the sound is generated from a known direction. The matrix $\mathbf{O}$ is the operator to calculate the coefficients of these basis functions for the primary sound field from the measured primary sound pressures $\mathbf{P}_{\text{PM}}$.

The basis functions in the SVD method are similar to those of the SHD method when the acoustic model is assumed as monopole point sources evenly distributed over a circular boundary surrounding the controlled region. However, the basis functions in the SVD method are inherently more flexible and readily adapted to acoustic model of sources from known sound propagation direction [13]. Thus, the SVD method can provide more efficient basis functions to represent a local sound field, with prior information incorporated.

The matrices $\mathbf{T}_{\text{S},l,\text{SVD}}$, $l = 1, 2, \ldots, L$, for the secondary sound field sensing can be derived in the similar way. Because the location of a secondary source can be pre-defined, the acoustic model can be a set of randomly distributed monopole point sources around the known location of the lth loudspeaker to estimate the transfer function $\mathbf{h}_{\text{SE},l}$ using (8). This transfer function sensing method has been reported in the authors' previous work [14].

Different from the SHD method, the proposed acoustic-modelling-based SVD method enables the remote sensing to be geometrically adaptive to the locations of the sound source(s) and the region of interest. With prior geometric information, the SVD method can provide more efficient basis functions to represent a local sound field and result in improved broadband error sensing and active noise control over a region.

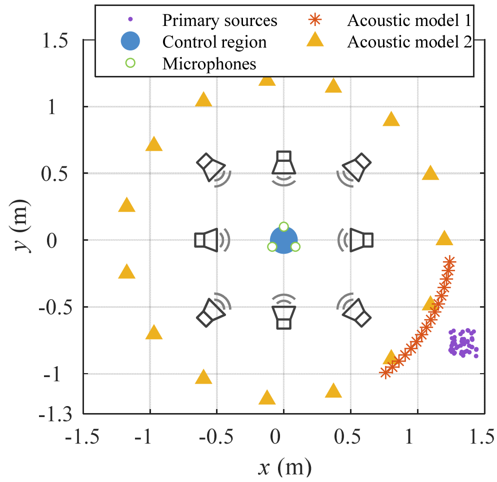

Fig. 1. System setup for active noise control of a circular region with a radius of 0.1 m. Three microphones are evenly distributed over the boundary of the region for remote error sensing. The primary sources are 48 monopole point sources located randomly within a square region $x \in [1.24, 1.44]$ m and $y \in [-0.88, -0.68]$ m. The secondary sources are eight monopole point sources surrounding the controlled region. Two acoustic models are applied in the SVD method separately. One model constitutes 16 monopole point sources over an arc to cover the sound direction of the primary sources. The other constitutes 16 monopole point sources over a circle to cover the sound waves from an unknown direction.

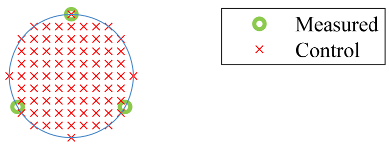

Fig. 2. Three measured points over the boundary of the region and 1257 control points evenly distributed over the region. The spacing between the control points is 0.005 m.

4. SIMULATIONS

The system setup in the simulations is illustrated in Fig. 1. Three microphones are used to sense the error signals over a circular region with a radius of 0.1 m. Eight secondary monopole point sources surround the region of interest. The primary sources are assumed to come from the direction around −30°. In the simulations under free field environment, the SVD method applies the acoustic models of monopole point sources either over an arc or over a circle. The former

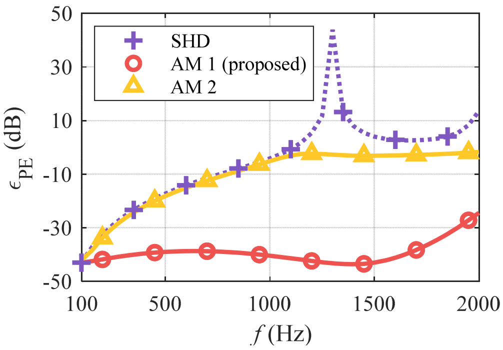

Fig. 3. Error of the primary sound field sensing over the control region, using the spatial harmonic decomposition method (SHD), the singular value decomposition method with acoustic model of sound sources over the arc (AM 1) and the singular value decomposition method with acoustic model of sound sources over the circle (AM 2), respectively.

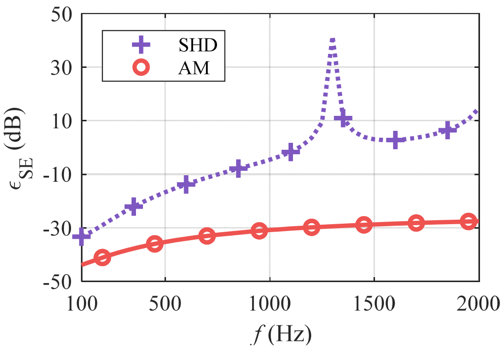

Fig. 4. Error of the secondary sound sources' transfer function sensing over the control region, using the spatial harmonic decomposition method (SHD) or the singular value decomposition method with acoustic model of sound sources around the known location of each secondary source (AM) [14], respectively.

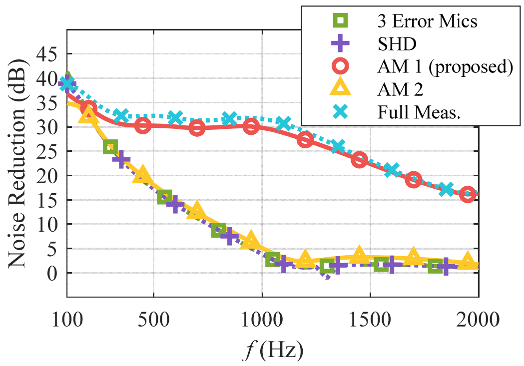

Fig. 5. Noise reduction over the control region, using the three measured sound pressures as error signals (3 Error Mics), the spatial harmonic decomposition method (SHD), the singular value decomposition method with acoustic model of sound sources over the arc (AM 1), the singular value decomposition method with acoustic model of sound sources over the circle (AM 2), and full measurement over the control points (Full Meas.), respectively.

one represents the situation where the sound propagation direction of the primary sources is available and incorporated into the remote error sensing. The latter one represents the situation where the sound propagation direction information of the primary sources is unavailable. The control points over the circular region is illustrated in Fig. 2.

The estimation errors of the primary sound field and the secondary source transfer function are evaluated in Figs. 3~4. They are defined as following:

$$\epsilon_{\mathrm{PE}} = 10 \cdot \log_{10}(\|\widetilde{\boldsymbol{P}}_{\mathrm{PE}} - \boldsymbol{P}_{\mathrm{PE}}\|^2 / \|\boldsymbol{P}_{\mathrm{PE}}\|^2), \quad (19)$$

$$\epsilon_{\mathrm{SE}} = 10 \cdot \log_{10}(\|\widetilde{\mathbf{H}}_{\mathrm{SE}} - \mathbf{H}_{\mathrm{SE}}\|^2 / \|\mathbf{H}_{\mathrm{SE}}\|^2), \quad (20)$$

Based on the error estimation in (5), the secondary source weights $\boldsymbol{w} = \widetilde{\mathbf{H}}_{\mathrm{SE}}^{+} \cdot \widetilde{\boldsymbol{P}}_{\mathrm{SE}}$ and the error signal is

$$\bar{\boldsymbol{e}} = \boldsymbol{P}_{\mathrm{PE}} - \mathbf{H}_{\mathrm{SE}} \cdot \widetilde{\mathbf{H}}_{\mathrm{SE}}^{+} \cdot \widetilde{\boldsymbol{P}}_{\mathrm{PE}}. \quad (21)$$

The noise reduction level is shown in Fig. 5, which is defined as

$$\mathrm{NR} = -10 \cdot \log_{10}(\|\bar{\boldsymbol{e}}\|^2 / \|\boldsymbol{P}_{\mathrm{PE}}\|^2). \quad (22)$$

With the geometric information of the primary/secondary sound sources being incorporated, better primary/secondary sound field sensing can be achieved. The estimation error using the proposed method ("AM 1" in Fig. 3 and "AM" in Fig. 4) is less than −27 dB from 100 Hz to 2000 Hz for both the primary and secondary sound fields. It results in improved remote error sensing than the spatial harmonic method. Finally, as shown in Fig. 5, the proposed method achieves the performance of noise reduction over the region close to that using full measurement over the control points. Though only three microphones are applied, the performance degradation is less than 2 dB, compared to the full measurement approach.

Without using the geometric information, the SVD method performs similarly to the SHD method on primary sound field sensing, which leads to similar noise reduction performance. Since only 3 microphones are used and no geometric information is applied for the sound field sensing, they perform similarly to the active noise control that use the three measured point directly (without sensing).

5. CONCLUSION

This paper proposes an acoustic-modelling-based remote error sensing method for active noise control over a region. The proposed method incorporates the geometric information of the sound sources and the controlled region into the remote error sensing. Compared with the state-of-the-art spatial harmonic decomposition method, the proposed method provides a more efficient sound field representation and results in better broadband local active noise control. Future work includes investigating applications on primary sources from multiple directions and in normal rooms.

6. REFERENCES

[1] W. Jung, S. J. Elliott, and J. Cheer, "Estimation of the Pressure at a Listener's Ears in an Active Headrest System Using the Remote Microphone Technique," *The Journal of the*

Acoustical Society of America, vol. 143, no. 5, pp. 2858-2869, 2018.

[2] X. Qiu, “A Review of Near Field Acoustic Error Sensing Strategies for Active Sound Radiation Control,” ICSV 25, 2018.

[3] W. Jung, S. J. Elliott, and J. Cheer, “Local Active Control of Road Noise inside a Vehicle,” *Mechanical Systems and Signal Processing*, vol. 121, pp. 144-157, 2019.

[4] S. J. Elliott and J. Cheer, “Modeling Local Active Sound Control with Remote Sensors in Spatially Random Pressure Fields,” *The Journal of the Acoustical Society of America*, vol. 137, no. 4, pp. 1936-1946, 2015.

[5] E. G. Williams, *Fourier Acoustics: Sound Radiation and Nearfield Acoustical Holography*. Elsevier, 1999.

[6] J. Zhang, T. D. Abhayapala, W. Zhang, P. N. Samarasinghe, and S. Jiang, “Active Noise Control over Space: A Wave Domain Approach,” *IEEE/ACM Transactions on Audio, Speech and Language Processing*, vol. 26, no. 4, pp. 774-786, 2018.

[7] R. A. Kennedy, P. Sadeghi, T. D. Abhayapala, and H. M. Jones, “Intrinsic Limits of Dimensionality and Richness in Random Multipath Fields,” *IEEE Transactions on Signal processing*, vol. 55, no. 6, pp. 2542-2556, 2007.

[8] W. Zhang, C. Hofmann, M. Buerger, T. D. Abhayapala, and W. Kellermann, “Spatial Noise-Field Control with Online Secondary Path Modeling: A Wave-Domain Approach,” *IEEE/ACM Transactions on Audio, Speech and Language Processing*, vol. 26, no. 12, pp. 2355-2370, 2018.

[9] N. Murata, J. Zhang, Y. Maeno, and Y. Mitsufuji, “Global and Local Mode-Domain Adaptive Algorithms for Spatial Active Noise Control Using Higher-Order Sources,” in ICASSP 2019-2019 IEEE International Conference on Acoustics, Speech and Signal Processing, IEEE, pp. 526-530, 2019.

[10] M. Poletti, “An Investigation of 2-D Multizone Surround Sound Systems,” in Audio Engineering Society Convention 125, Audio Engineering Society, 2008.

[11] T. Betlehem and T. D. Abhayapala, “Theory and Design of Sound Field Reproduction in Reverberant Rooms,” *The Journal of the Acoustical Society of America*, vol. 117, no. 4, pp. 2100-2111, 2005.

[12] S. Argentieri, P. Danès, and P. Souères, “A Survey on Sound Source Localization in Robotics: From Binaural to Array Processing Methods,” *Computer Speech & Language*, vol. 34, no. 1, pp. 87-112, 2015.

[13] Q. Zhu, X. Qiu, P. Coleman, and I. Burnett, “A Comparison between Two Modal Domain Methods for Personal Audio Reproduction,” *The Journal of the Acoustical Society of America*, vol. 147, no. 1, pp. 161-173, 2020.

[14] Q. Zhu, X. Qiu, P. Coleman, and I. Burnett, “Reducing Number of Transfer Function Measurement in Local Sound Field Reproduction using Acoustic Modeling,” the 23rd International Congress on Acoustics, Aachen, Germany, 2019.