Teachers' learning through an online lesson study: An analysis from the expansive learning perspective

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Abstract

Purpose-This study aimed to explore how a group of Chinese primary mathematics teachers learned through conducting an online cross-cultural lesson study between China and Australia.

Design/methodology/approach-An expansive learning theory was adopted to examine teachers' learning in a complex activity system. Multiple data including videos of research lessons, debriefings and audios of interviews were collected. From the expansive learning perspective, various contradictions (as driving forces of learning) were identified and the ways of resolving the contradictions (as enactment of learning) were located to feature teacher learning throughout the online lesson study process.

Findings-Teachers' expansive learning includes enhancing teachers' MKT and Mathematics TPACK, developing instructional design skills and capability in addressing challenges occurring in the virtual environment.

Limitations/implications-Theoretically, the study illustrated how expansive learning theory could be utilized to examine teacher collaborative learning in the online cross-cultural lesson study. Practically, this study showed reiterative cycles and experts' facilitation are crucial to expansive learning for linking research to classroom practice. However, this study did not focus on student learning in the virtual environment. Australian teachers' reciprocal learning through the online lesson study also requires further exploration.

Originality-Both online lesson study and cross-cultural collaboration are innovative. The expansive learning lens is creatively used to examine the complexity of teacher learning in such a novel environment.

Keywords Lesson study, Cross-cultural lesson study, Virtual professional learning community, Online teaching, Expansive learning.

Paper type: Research paper

1. Introduction

Much effort has been devoted to investigating different teacher professional development (PD) programs for improving students' learning in the past decades (e.g., Darling-Hammond et al., 2017). With the high reputation in practical features such as job-embedded and teacher-oriented but student learning-focused (Lewis, 2016), Lesson Study (LS, hereafter) has been well recognized as one of the most effective PD (Author et al, 2016). However, what mechanism that makes LS works effectively is still a mystery even though its affirmative impact has been empirically evident (Author et al., 2019). Furthermore, not much attention has been paid to explore what educational practitioners in

China and Japan where LS originated, may learn from online and cross-cultural LS. The rapid development of digital technology and its practical features have created opportunities to stretch the current LS to online mode for cross-cultural LS research (Isoda et al., 2017). Building on the experience of cross-cultural teacher exchange program between the UK and Shanghai (Author et al, submitted) and the online teaching experience during the time of Covid-19 pandemic in Shanghai (from Feb to June 2020), Shanghai Normal University and Association of Australian Mathematics Teachers (AAMT) collaboratively launched an online LS to promote teacher collaborative learning in June 2020. This cross-cultural online LS project focuses on exploring effective ways for teaching mathematical rich topics for primary schools in both countries. The current study aims to examine the affordance and constraints of Shanghai teachers' learning from this cross-cultural online LS during the first three teaching cycles.

2. Conceptual framework

2.1 Lesson study and teacher collaborative learning

A typical Japanese LS usually includes four stages: Study, Plan, Teach and Reflect (Lewis, 2016). One LS cycle is usually concluded at writing up a report to share the learning experiences among the LS communities. Research has provided evidence about the benefits of LS which include improving teaching and learning, remodeling teaching practice, implementing new curriculum, and linking theories to practice (e.g., Author et al, 2016).

The Chinese LS refers to a teacher PD approach that involves collaborative lesson planning, cycles of teaching with classroom observation, post-lesson debriefing and reflection, and lesson revision; these components have been evolved for over a century in China (Li, 2019). Structurally similar to the Japanese LS, the Chinese LS has its unique features such as reiterative teaching of the research lesson to different groups of students and involvement of the knowledgeable other throughout the LS process. The Chinese LS usually invites experts who are officially responsible for organizing school-based teaching activities and evaluating teachers' teaching and students' learning (Gu and Gu, 2016) to provide comments and feedback from the lens of educational researcher and leader. The Chinese LS also emphasizes both the product (lesson exemplars) and process (teacher learning).

The practice of the Chinese LS reveals that a culture of "teaching as a public activity" (Stigler et al., 2013, p.227) has been well established in China; observing and learning from each other's teaching and opening up the classroom to the public have become a norm for Chinese teachers (Li, 2019). Moreover, the school-based teaching research groups (TRG) have been in place as local professional learning communities (PLC) (DuFour, 2004) for over a half century (Chen, 2020). Some key foci and practices such as analyzing mathematics content (e.g., the difficulty, importance, and key points), teaching strategies (critical incidents), and student learning effects have been gradually developed within the PLC (Yang and Ricks, 2011). Importantly, the nature of the Chinese LS has been endorsed as a type of improvement science (Bryk et al., 2016) contributing to the improvement of mathematics and science education system wide (Author et al, 2017). Since this LS took place in a blended approach (a combination of online and onsite), relevant literature on teachers' knowledge for teaching mathematics online and virtual PLCs is briefly summarized in the following section.

2.2. Mathematical Knowledge for Teaching, Technology, Pedagogy and Content Knowledge and virtual professional learning community

Building upon Shulman's seminar work (1986) on teacher knowledge, Ball et al. (2008) extended to Mathematical Knowledge for Teaching (MKT) which comprises two key categories: subject matter knowledge and pedagogical content knowledge. Subject matter knowledge includes Common Content Knowledge (CCK), Specialized Content Knowledge (SCK) and Horizon Content Knowledge (HCK). CCK refers to the common knowledge and skills not unique to teachers. SCK denotes the knowledge unique for teaching. HCK is about the mathematical landscape of how the topics are connected between grade levels within a curriculum. Pedagogical content knowledge includes Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). (See Ball et al., (2008) for detailed interpretations).

Mishra and Koehler (2006) developed the framework of *Technology, Pedagogy and Content Knowledge* (TPACK) based on Shulman's work (1986). In general, TPACK refers to the knowledge for teaching using appropriate digital technologies. Given the context of online LS on teaching mathematics presented in this paper, teachers' knowledge about the use of digital resources and tools for teaching mathematics, and teachers' capability of using of technology to support mathematics learning are particularly relevant. (The details of the definitions of TPACK framework components refers to Mishra & Koehler, 2006).

Virtual PLCs can provide some potential benefits such as no geographic boundary access of professional development activities and easy communication between teachers. All these provide a means for developing supportive and collegial professional practice (Kelly & Antonio, 2016). However, researchers found that the knowledge exchange and sharing in virtual PLCs are often too superficial and non-critical and further identified impacting factors for productive online PLCs (e.g., Lantz-Andersson et al., 2018). These include trustful relationships among online PLC members, high quality mediators for knowledge generation, and rich content closely related to teaching practice (Lantz-Andersson et al., 2018). The results of Author et al.'s study (2020) also revealed that online LS debriefing sessions could be productive when an onsite PLC has already been established with trustful environment and an experienced mediator with knowledgeable MKT in play.

2.3 Theory of expansive learning and lesson study

The Cultural-History Activity Theory (CHAT) was originated from Vygotsky's understanding of learning; learning was social and situated activities, which were mediated by cultural tools including language and signs (Vygotsky, 1978). CHAT was then interpreted as an activity system involving three key components: a *subject* (i.e., the people) using *tools* (e.g., skills, knowledge, language) to act on an *object* (e.g., lesson) (Engeström, 1987). Engeström then further extended CHAT by identifying the social elements that play significant roles in driving the process of the activity. The social elements include the *rules* which guide subjects' actions, the *community* which influences people's actions and the *division of labor* which is referred to a group of people playing different roles for reaching the object. Overall, all different activity components are interplayed to act on the objects (Engeström, 1987). From the perspective of CHAT, LS is a typical activity system because it involves collaboration and division of labor among teachers/researchers in the community and using teaching materials and lesson plans (i.e., tools) to enact the research lesson (i.e., objects) for promoting teachers' professional development.

Engeström (2001) attempted to portray a collective and dialectical perspective of learning in a complex activity system and thus developed a new generation of CHAT (i.e., the third generation) that embedded multiple activity systems. The third generation CHAT is particularly concerned about explaining the learning that occurs when different activity systems work together on a problem (Engeström, 2001). This shared problem becomes the object of this complex system. The current study adopted the third generation in the form of a complex system comprising two interacting activity systems (see Figure 1). The two activity systems are the Chinese LS and Australian LS. When the two systems interact, the object 1 the lessons based on local practice transforms to an object 2 the lessons inspired by cross-cultural exchanges. Thus, the research lesson serves as the boundary object which is jointly constructed through the interaction of the systems and ultimately, learning across the systems will occur (Engeström, 2001; Engeström and Sannino, 2010).

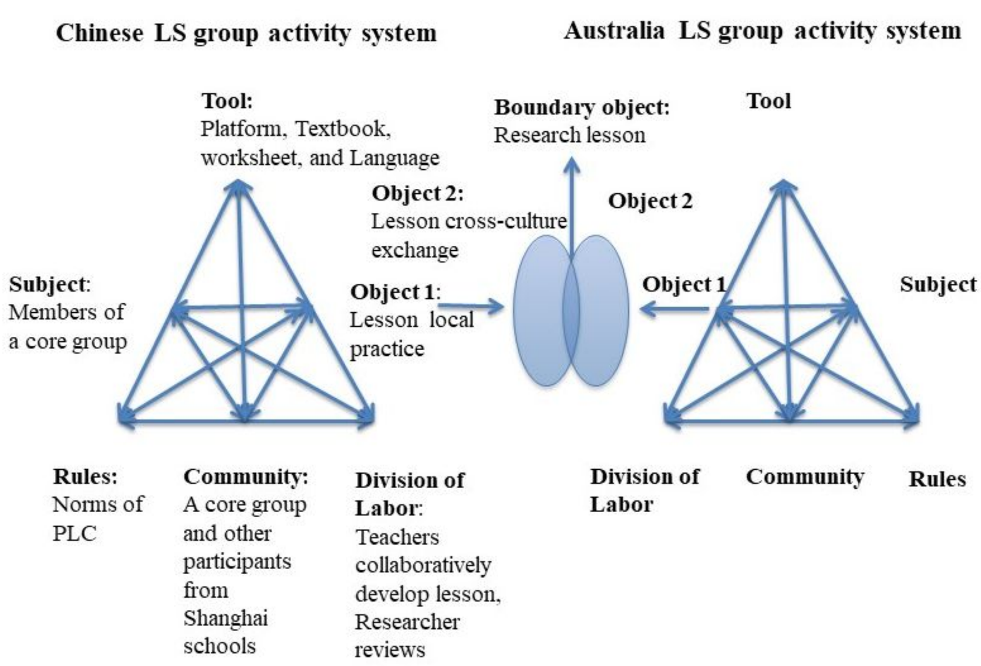

Figure 1. A complex system of a cross-culture lesson study

An important principle of CHAT is *contradiction* which refers to the 'accumulating structural tensions within and between activity systems" (Engeström, 2001, p. 137). The power of contradictions as Engeström (2016) assumed is its capacity of becoming a driving force for addressing the problems within and between system(s). Different types of contradictions may arise from (1) each of the constituent components of the central activity

(Primary Contradiction), (2) the interplay between the constituent components of the central activity (Secondary Contradiction), (3) the differences of goals/motive between the dominant form of the central activity and the culturally more advanced form of the central activity (Tertiary Contradiction) and (4) the differences between the central activity and its neighbor activities of a complex system (Quaternary Contradiction) (Engeström, 2016).

To analyze the learning within the complex activity system, Engestrom (2001) proposes the idea of expansive learning, another key principle of CHAT. *Expansive learning* is denoted as "learning in which the learners are involved in constructing and implementing a radically new, wider and more complex object and concept for their activity" (Engeström & Sannino, 2010, p. 2). Expansive learning "puts the primacy on communities as learners, … and on the formation of theoretical concepts" (p. 2). CHAT views contradictions as driving forces of expansive learning when new objects, concepts and motives emerge while people are dealing with the contradictions (Engeström & Sannino, 2010). Expansive learning among the people of the activity system(s) entails a sequence of learning actions (i.e., expansive learning cycle) that can resolve the contradictions. The expansive learning cycle includes questioning, analysis, modeling, testing and implementing the new model, and reflecting the process and consolidating the new practice (Engeström, 2001).

The nature of learning in activity systems aligns the aims and structures of LS. LS brings together teachers and researchers (i.e., activity systems) around a collective focus (a shared research lesson) in order to enhance teachers' knowledge and their teaching practice through iterative cycles of planning, implementation and reflection (i.e., expansive learning) (Wake et al., 2016). Building on this conception, Wake et al., (2016) used CHAT to examine how the process of LS promoted teacher learning though analyzing lesson plans which was taken as the boundary objects. Furthermore, the findings of Wake et al.'s study (2020) has provided evidence to how the artifacts in LS could promote teachers' teaching knowledge and thus proposed LS as a specific case of expansive learning. They believe that CHAT could be used to identify the contradictions emerged in LS, the driving forces for promoting expansive learning among teachers.

Contradictions as a driving force for expansive learning have been explored widely (e.g., Potari et al., 2018). Yet, it is largely unknown how the contradictions could be resolved to promote expansive learning in LS. This study made an explicit effort to not only identify various contradictions and but to also locate the relevant learning cycle for 'addressing these contractions' (enactment of expansive learning) in an online LS context. Specially, the study sought to address the following research questions:

1: What were the major contradictions revealed during the online LS process in the Shanghai learning community (if any)?

2: How the contradictions were resolved over the online LS process to promote expansive learning?

3. Method

3.1 The LS team

The whole LS team included a core group of three participants and 10 observers. The core group consisted of the enacting teachers (Ms. Rong), head teacher (Ms. Wong) and a researcher from a local university (the first author). Both teachers are from a private primary school in Shanghai. Both completed a bachelor's degree in primary education and have more than 10 years of teaching primary mathematics. Ms Rong also had experiences in conducting research lessons in the UK. The university researcher has been engaging in mathematics teacher education research for years. The other participants included mathematics teachers from the exchange program. The researcher and members of the Shanghai LS team have developed close collaborative working relationship from multiple teacher professional development programs. The Australian LS team included three school teachers and two university educators.

3.2 Mathematics curriculum

The school system and mathematics curriculum in Shanghai and South Australia are quite different. The elementary school is from Grades 1 to 5 and Grades 1 to 7 in Shanghai and South Australia respectively. For the content of probability, the Shanghai elementary mathematics curriculum requires students to develop some basic concepts such as certain and uncertain events, using simple words to represent the likelihood of uncertain events, frequency of outcomes in an experiment, equiprobable and sample space of simple events. Using fractions to represent the probability of events is not within the Shanghai elementary curriculum (SMEC, 2004). In South Australia, in addition to the basic concepts of probability, using fractions (percentage and decimals) to represent probability is a key learning outcome in the Australian mathematics curriculum (ACARA, 2020).

3.3 Process of the online Lesson Study

This LS was a teacher exchange project between the AAMT and Shanghai Normal University based on the Memorandum of Understanding on mathematics teacher education in 2019. Due to the Covid-19 pandemic, the LS team agreed on using Zoom to conduct the cross-cultural LS on teaching probability in Shanghai and South Australia. Three lessons including one onsite and two online lessons were conducted. For Rong's onsite Shanghai lesson, around 45 students sat in rows in a classroom. For the Australian online class, around 20 students sat in groups of 4 attending Rong's teaching. The class was assisted by 2 teachers. For the online Shanghai class, around 20 students sat in a classroom where two teachers assisted Rong to organize classroom activities. The very draft of the lesson plan was created by Rong. The process of the LS is displayed in Figure 2.

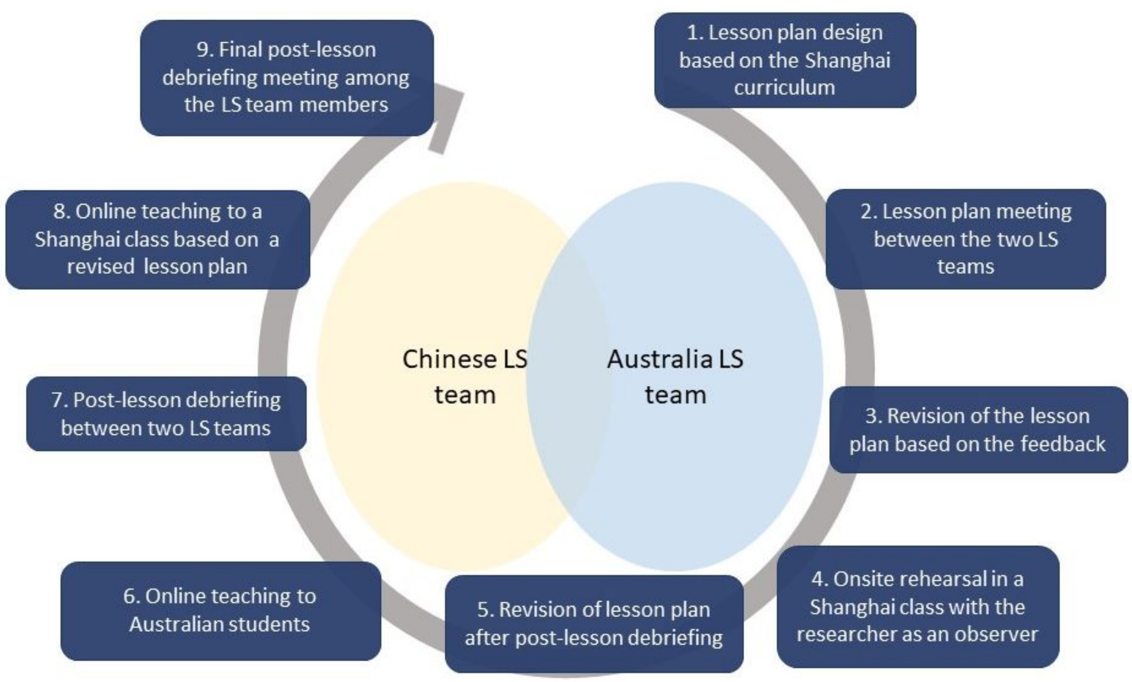

Figure 2. The process of the online LS in a cross-culture activity system

3.4 Data collection and analysis

In the LS, the data collected include: (1) three versions of lesson plan (D1); (2) videos of three lessons (D2); (3) videos of three post-lesson meetings (D3); and (4) audio recording of interviews with the Ms. Rong and Ms. Wong (D4). All the teaching and debriefing videos/audios were transcribed verbatim by research assistants. The accuracy of the transcription was checked by the researcher. The analysis was started with reading through the data thoroughly for comparing the similarities and differences between the three lessons. An online meeting was followed for identifying differences across the lessons and exploring the reasons. Then, regular meetings for discussing the contradictions were held to minimize the risk of misinterpreting the data. Based on extensive discussions over multiple meetings, the three authors reached consents on the contradictions at three levels: Primary, Secondary and Tertiary. Although various contradictions have been identified, this research paper focused on discussing the contradictions emerged from the Chinese teachers' understanding of the concept of probability and use of online resources (i.e., MKT and TPACK). Then, each author focused on one level of contradictions, and identified the expansive learning cycle for resolving the contradictions. It was then followed by further discussing the accuracy and coherence of findings and cross check for reliability.

4. Results

The results presented in the following section were attempted to provide answers to the research questions.

4.1 The primary contradictions and strategic actions

One primary contradiction was identified. *Contradiction 1* was about the disparity between teachers' and researchers' understanding of basic concept of probability: making sense of experimental probability and presenting the outcomes in fractions versus transforming from experimental probability to theoretical probability through using experiment and computer simulation. With Rong's teaching experiences, the learning goals were set to "develop students' understanding of basic concept of probability and fairness of game, and to use fractions to represent the probabilities through exploratory activities, such as tossing a coin and drawing balls from a box" (D1-1). In the first teaching, the lesson began with an activity focused on tossing a coin moving to a demo animation to make sense of fairness (i.e., equiprobable). This was attempted to show the occurrence of heads or tails approaching half-half and the half-half occurrence maintained as the number of toss increased towards infinite. This tossing activity of itself was a good investigation. However, the contradiction arose when the teacher used fraction to describe the probability too soon and suggested it was half because "there are two equal faces" (D2-1). In the post-lesson debriefing, the researcher pointed this out and explicitly explained the relationship between occurrence (i.e., experimental probability) and theoretical probability and emphasized the goal of this lesson should focus on developing students' understanding of this relationship as from which the concept of equiprobable and quantification of probability would be developed naturally.

> Researcher: *What is occurrence and frequency ratio? […] I think 500 times is just an occurrence, but what is the frequency ratio? Let's say there are 50 times head up in 500 times of tossing. Divide 50 by 500, and this is called a frequency ratio [for head]. Well, when the number of tosses increases infinitely, the frequency ratio will stabilize at a value, then we define this value as a probability, probability is defined by the frequency ratio. Through this experiment, we define the notion of experimental probability. […] What does theoretical probability mean? You don't need to do experiments. Let's say I have a coin with head and tail. The chance of getting either one theoretically is 1/2, because there are two outcomes of the event. Theoretical probability involves sample space and event outcomes. Let's say there are 6 numbers [N=6] in the sample space [for a dice], and the probability of getting each number [M=1]is the same, so I use M/N to represent it, right? (D3-1)*

The teachers of LS team held the same idea as Rong. It was evident in their discussion of how to implement the tasks. The teachers' major discussion was around how many times tossing a coin would be more appropriate to naturally emerge the half-half occurrence so that the chance of 1/2 could be verified. The researcher found the teachers' direction pedagogically inappropriate and drew teachers' attention to investigating the fairness of tossing a coin, which was determined by the equiprobability of outcomes. The experiment was not to verify the chance of 1/2.

> Researcher: *The student said that it was fair, and another student said the probability is equal, […] and then you keep saying that the gap (between the occurrence of head and tail) would become smaller (with increasing the number of toss), as if everyone has already known it was 1/2. […] everyone targeted ½ eventually. But what I mean is that students have to make conjecture when they talk about fairness. In fact, all experiments require testing for hypotheses before any conclusion. (D3-1)*

The researcher emphasized that the outcomes generated from tossing a coin was to inspire students to predict the equiprobability when they observed the occurrence of heads and tails came closer with increasing trials. Following this argument, a school teacher suggested that before tossing the coin, teachers should have made clear to the students that the coin was made of uniform texture so that the notion of equiprobable could be discerned easier.

> *A school teacher: For … experiments, it is implicitly assumed that the coin is of uniform texture, so it is equiprobable, […] What I want to say is even if we do not explicitly mention the notion of equiprobable, when introducing the coin and balls, we need to make this assumption clear to make the experiments more scientific (D3-3)*

Then, Rong suggested using computer simulation to generate large sample data. She believed that the experiment could induce a discussion on connecting experimental to theoretical probability so that 'equiprobable' could emerge naturally. This contradiction was eventually settled through making sense of learning aims of the research lesson.

4.2 The secondary contradictions and strategic actions

Two secondary contradictions occurred between tool and object at this secondary level. *Contradiction 2* occurred during Rong's online teaching with Australian students [D2-2]. A technical issue obstructed direct interaction between Rong and the Australian students. When the students were using the microphones on their iPad to interact with Rong, the voices from different groups interfered with each other. The Australian teachers then realized that technically, the system did not allow users to have more than one microphone working synchronously in one space. Then, they had to turn off their microphones and communicated with Rong through their Australian teachers using the main microphone. Thus, the interaction between Rong and students became "asynchronous" so that the lesson was not finished as scheduled.

Rong originally planned to monitor the investigation tasks by herself. However, in the online lessons she was not able to observe students' participations and provide support in time as in an actual classroom. She was disappointed as she believed that teacher's guidance on student participation was extremely important.

> Rong: *[…] if I wanted to use this online teaching model again, it may be more suitable for the lessons that focus on computations or conceptual knowledge. For online classroom, if the students do not manipulate [the activities] properly, or if some issues happen, it is not possible to guide them in time.* (D4-1)

This contradiction was not resolved eventually but the LS team has been inspired to explore different online teaching design, use of digital devices and its implementation to enhance the teaching efficiency.

Contradiction 3 was displayed when the teachers attempted to scaffold students' learning. The learning aim was distorted when the teacher scaffolded her students during the implementation of the task. In the first lesson (D2-1), after the students have tossed a coin for 30 times, Rong initiated conservations with the students to explain that the occurrence of head and tail would approach half-half. However, the teacher's scaffolding was found deviated.

> Rong: *Tell me about the outcomes of your trials.*
>
> Zhang: *[…] I got 19 heads and 11 tails.*

Rong: *Before tossing the coin, we guessed how likely it was to get the head and the tail. So why after doing this, we found that for some groups, there were more heads, for other groups, there were more tails. Why? Maybe this effect isn't particularly noticeable yet due to the small number of trials, right? So this way, we have five students in each group, we put all the results together, and see how many heads and tails in each group*. […]

Rong: … *how many heads and how many tails? Not much of a difference now, is it? They're close enough, aren't they? Especially Group 3, they are all the same, … there's a slight difference between the number of heads and tails in Group 5 and Group 1, right?* […]

In this conservation, apparently the teacher was using the investigation results to verify the half-half occurrence of head and tails instead of using the data to develop the concept of equiprobable and form a conjecture about the tendency of occurrence as the trials increased infinitely. In the lesson, the teacher suggested that the number of trials might *not be enough to show the effect*. Though she did not explicitly make clear to what the effect she referred to, in her later utterances where she mentioned 'not much differences between the number of heads and tails' as well as 'they were close enough' were actually pointed to half-half occurrences of heads and tails. This contradiction was brought out by one researcher. The teacher was suggested not pre-assuming the equal chance in her scaffolding and explanation. The following episode illustrated how the concept of equiprobable was naturally emerged when this pre-assumption was avoided.

Teacher: *Here, 147 times for the head and 153 times for the tail. Look at the final outcomes and compare the group results and your own. What do you think? Ai.*

Student: *I think the times of the head and tail are very close, proving that they do have the same likelihood of 1/2.*

Teacher: *The number of heads and tails is very close, […] the likelihood of the head or the tail is 1/2*. (D2-3)

This contradiction was resolved in her last lessons when emphasizing the prediction of likelihood of ½ instead of verifying.

4.3 The tertiary contradictions and intended actions

From the last debriefing meeting and interviews with two core teachers, two tertiary contradictions and proposed solutions were identified. *Contradiction 4 refers to the disparity between the teacher's existing and required MKT.* After the teaching, Rong disclosed her superficial understandings of equiprobability and sample space. For instance, Mr. Wong pointed out Rong's inappropriate task design and said that "tasks should be carefully selected to better serve the learning goals of the class. […] For example, rock, scissor, paper problem was not appropriate for the lesson." (D4-2) Another teacher also challenged Rong's explanation for the fairness of the game Rock, Paper, Scissors.

> *For Rock, it could win, lose or tight. […] So, you need to list out all the outcomes in a table to analyze. There should be 9 different outcomes and each act has 3 outcomes that win. Thus, the chance for each act to win is 1/3. But the calculation is not because there are three different acts.* (D3-3)

The other participants agreed that this task involved the concept of listing sample space and should not be discussed in this lesson. This contradiction was finally settled at removing this task from the lesson. Nevertheless, her (and the LS team's) understanding of the concepts (SCK) has been developed gradually throughout the research journey; it was beyond their current knowledge of teaching about this mathematical concept (KCC). Although quantifying probabilities using fractions is not required in Shanghai curriculum, after these lessons, she eventually realized that "[…] incorporating fractions with introduction of probability in grade 5 could consolidate fraction understanding and deepen the concept of probability at the same time, it is a nice integration that addresses students' learning" (D4-1).

She also appreciated the researcher's clarification and explanation on the concept of probability and this has inspired her to consider teaching the basics from an advanced mathematics perspective (HCK).

> Teacher: *This guides me to better design classroom teaching: […] design, […] rationale, …..this opened up my horizon, I need to have an advanced perspective so that I can have deeper insight ”*. (D4-1)

This contradiction was resolved at the teacher's better understanding of the probabilistic concept as well as the curriculum.

Contradiction 5 refers to the disparity between the teacher's existing and required Mathematics TPACK. Rong's teaching with Australian students was synchronous online and some technical issues occurred that really affected her interaction with the students in time. To mitigate this constraint, several strategies were suggested for future teaching.

A worksheet with an organized table was suggested for recording data and results. The teachers also proposed different ways such as using Excel spreadsheets so that the data could be calculated automatically. (D3-3)

Regarding carrying out the experiments, the LS team realized there were some unexpected issues arising from tossing a coin. Rong and other teachers suggested providing a demonstration of activities to students before the actual experiments. (D3-3). To maintain the consistency of operating teaching materials across countries, Ms. Wong suggested providing photos of manipulatives or even videos to detail the requirements of experiments and activities. (D4-2)

Regarding the actual implementation of experiments and activities, Rong suggested asking students to carry out the experiment and record their results before lesson. The teacher then aggregated results and took it to the class for discussion. Other teachers suggested video-taping students' experiments in advance. (D3-3)

To facilitate the students' learning, Rong adopted a short animation video to show the results of experiments conducted by different statisticians. However, the effect of the demo was not satisfactory. Several teachers suggested improving the quality of the animation video or using simulation applets to generate larger number of trials. (D33) Moreover, the experts from Australia strongly suggested using computer simulation to develop the understanding of occurrence of head and tail steadily approaching half-half with increasing trials towards infinite. (D3-2) However, Rong admitted herself not knowledgeable in computer simulation due to lack of IT knowledge (e.g., Mathematics TPACK) (Mishra and Koehler, 2006).

> Teacher: *[…] it is not enough just to show an animation video, we should let students have personal experience in how the frequency rate approaches a fixed value as the trials increase … the Australian teachers said we could program applets and let students use them, but I do not know programming, […].* (D4-1)

Ms.Wong also commented on the common weakness of Shanghai elementary mathematics teachers in integrating technologies in teaching and admitted the difficulty in employing teachers with strong IT knowledge. (D4-2) Furthermore, Ms. Wong and the researcher discussed how to provide specific training programs for school teachers. (D4-2)

After the LS, Rong eventually realized that she needed to develop stronger ability in using various educational technologies and online resources. In addition, how to interact with students, understanding student learning difficulty and providing feedback in time in a virtual environment were also challenges that required her to address in the future. (D4-1)

This contradiction was not resolved eventually but the teacher has been inspired to explore different online teaching designs, use of digital devices and its implementation that suit the needs of students

5. Discussion

This study revealed that the teachers have expanded their learning through dealing with the contradictions in the LS within the Chinese cultural activities and across cultural activities. The expansive learning was discussed below.

5.1 Expansive learning through dealing with the contradictions

In the study, five contradictions and corresponding achieved or intended expansive learning are identified and summarized in Table 1.

Table 1. The contradictions and expansive learning

Different types of contradictions in	**Contradictions in the LS** (Resolved or semi-resolved)	**Expansive learning** (Achieved or intended)

expensive learning		
Primary Contradiction	**C1**. The disparity between the teacher's and the researcher's understanding of basic concept of probability	Making sense of the learning aims of the research lesson
Secondary contradictions	**C2**. Limited functions of online platform did not meet the required communication needs	Exploring different online teaching design, use of digital devices and its implementation
	C3. Inappropriate facilitation and intended goals of learning	Predicting the likelihood of ½ instead of verifying the pre-assumed value
Tertiary contradictions	**C4**. Limited and required MKT	Deepening understanding of the topic from curriculum and advanced mathematics perspectives
	C5. Limited and required Mathematics TPACK	Exploring different online teaching design, use of digital devices and its implementation

Through the entire LS process, by dealing with the contradictions, the expansive learning is evidenced in the following dimensions of the new objects, concepts, motives, or novel patterns of the activity (Engeström, 2001).

Expansive learning as transformation of objects. Through resolving C1-C5, the object of the LS was transformed from *producing* an online lesson for different cultures to establishing collaborative efforts and intelligence among participants to resolve contradictions for *developing and conducting* online lessons. As discussed earlier, a lot of issues relating to making sense of the learning aims of the lessons (e.g., making sense of experimental probability and using fractions to represent the probability) (C1), design of instructional tasks and technical problems occurred during the online teaching with the Australian students (C2 and C5). With the experts' comments and the teachers' collective reflections, the contradictions have become a driving force to urge the LS teams to shift their focus from emphasizing specific content of activities to developing well-design instructions and materials including design of the activities, administering the experiments and using computer simulation (C5). The contradictions also empowered the teachers to develop a thorough understanding of equiprobable (C4). Although the implementation of instructional tasks was not very successful in all the three teaching cycles, collective efforts among the participating teachers has been established to collaboratively resolve the issues and achieve their intended goals. Eventually their collaboration has formed a collective effort and intelligence for *developing and conducting* online teaching in general.

Expansive learning as deepening understanding of MKT and developing TAPCK for teaching online. Through dealing with the contradictions (C1, C3, C4, C5), the participating teachers have deepened their understanding of MKT for teaching the concept of probability, and Mathematics TPACK for online teaching. They corrected their misconception (C1), learned how to appropriately scaffold student learning (C3), eliminated the inappropriate mathematics tasks (C4), provided innovative ideas for teaching online (C5). They also gained insight into teaching elementary mathematics from an advanced perspective. They were motivated to develop digital knowledge and skills for teaching mathematics in a virtual environment.

Expansive learning as boundary crossing. There were multiple activity systems interacting with each other in this LS. These activity systems included the Shanghai LS team in which the central activity was situated, the research system in which the researchers worked with the teachers, the teaching system in which the Chinese teachers worked collaboratively with the Australian team. The *research lessons* eventually served as a boundary object for *developing and conducting* online lessons that required different systems to work together and to share ideas. Through working on the research lessons, some new practices (new patterns of activity) have been brought to the existing activity systems and made the expansive learning happen (Engeström and Sannino, 2010). For example, through the interaction between the teaching system and research system, the teachers' MKT has been strengthened and the researchers have gained more understanding about the teachers' challenges such as lack of MKT and Mathematics TPACK. This enabled the researchers to step out of their "ivory tower" to experience a genuine "messy" classroom. The researchers also attempted to provide different solutions such as offering relevant teacher professional training programs. Another example is that through the interactions between the Shanghai LS team and the Australian team, the Shanghai team learned two big ideas about teaching and learning probability: quantifying probability using fractions and exploring the linking of experimental and theoretical probability using simulation. Both ideas have played crucial roles in transforming object of activity, developing teachers' knowledge, and updating teachers' teaching practices.

5.2 Expansive learning through the iterative cycles of LS within a supportive PLC.

A typical LS cycle is very similar to the cycle of expansive learning: setting goals versus questioning and analysis, planning a research lesson versus modeling, teaching the lesson versus testing and implementing; and reflecting on the lesson versus reflecting. More importantly, the unique features of the Chinese LS (discussed in the earlier section) strengthen the expansive learning process via multiple ways. For instance, the involvement of experts including university researchers and teaching research specialists (Author et al, 2017) are able to enrich the research lessons by bringing in their research findings and expertise and by providing new models and instruments for assessing teaching effectiveness. The reiterative cycle of LS provides opportunities for teachers to re-design, re-test, re-reflect and re-revise the lessons. As evident in this study, throughout the LS journey, teachers revisited their understanding of the teaching concepts, re-set the learning goals, and re-develop the practicing tasks to accommodate the teaching in a virtual environment. In an activity system, the rules, community, and division of labors are fundamentally important components (Engeström, 2001). In this study, although the PLC (i.e., The Shanghai LS team) has been shifted from face-to-face to virtual PLC, the teachers and researchers worked together to provide critical comments (e.g., criticizing inappropriateness and errors in practicing tasks) and offer pertinent suggestions (e.g., use spreadsheet, and computer simulation) for improving the online lessons. These incidences suggest that the online PLC is productive (Lantz-Andersson et al., 2018). This may be related to the well-established teaching research system (Chen, 2020) and teacher collaborative culture in China (Li, 2019). The finding echoes that moving a well-established face-to-face PLC to virtual PLC could be successful when an experienced facilitator is come into play (Author et al, 2020).

5. 3 Implications

This study displayed both theoretical and practical implications. Theoretically, the study illustrated how CHAT could be applied to examine teacher collaborative learning in LS. Thus, this current study contributes to deepening the understanding of the effect and process of LS. Practically, this study also has implications for strengthening LS. Both the reiterative cycle and experts' facilitation in LS are crucial to expansive learning. The reiterative cycle provides a mechanism for expansive learning. The experts strengthen the design, evaluation, and reflection, all of which contribute to linking research to classroom teaching (Author et al, 2016), as well as promoting the development of teachers' MKT (Ni Shuilleabhain, 2016). Another practical implication is the pre-requisite for establishing a productive and effective virtual PLC. This study has provided evidence that a well-established onsite PLC and experienced facilitator are two crucial impacting factors for virtual PLC to be successful.

5.4 limitations and further studies

There are some limitations identified in this study. First, this study did not collect student data due to the constraint of online teaching platforms. Moreover, the interactions between the teacher and students were very limited. How to observe student learning on an online platform requires further investigation. Second, this study only focused on Chinese participating teachers and their learning due to the difficulty in collecting data from the Australian counterparts. Investigating how the learning of Australian teachers and reciprocal learning occur through boundary crossing is an important research topic in the future. Third, the differences between the two countries such as culture (student learning, and teacher learning) (Stigler and Herbert, 1999), curriculum, teacher professional development system may promote or hinder the teacher learning through online LS. This area should be explored appropriately.

6. Conclusion

This study examined teachers' learning through a cross-cultural online LS which aimed to address how to conduct a lesson on an online platform. The findings showed that the teachers' expansive learning including enhancing their MKT and Mathematics TPACK, developing skills in designing instructional tasks and developing the capability in using various teaching materials and tools (physical and electronic devices, online resources) were emerged through the processes of resolving the three levels contradictions. Meanwhile the study suggests that qualified facilitators and pertinent training programs are crucial for promoting teachers' expansive learning. More importantly, this study demonstrated how the CHAT theory could be employed to investigate the complexity of LS for further developing the theory and practice of LS.

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Teachers' learning through an online lesson study: An analysis from the expansive learning perspective

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Abstract

Purpose-This study aimed to explore how a group of Chinese primary mathematics teachers learned through conducting an online cross-cultural lesson study between China and Australia.

Design/methodology/approach-An expansive learning theory was adopted to examine teachers' learning through collective activities across different activity systems. Multiple data sets including videos of research lessons, debriefings and audios of interviews were collected. From the expansive learning perspective, based on a fine-grained qualitative data analysis. various contradictions (as driving forces of learning) were identified and the ways of resolving the contradictions (as enactment of learning) were located to feature teacher learning throughout the online lesson study process.

Findings-Teachers' expansive learning includes enhancing teachers' MKT and Mathematics TPACK, developing instructional design skills and capability in addressing challenges occurring in the virtual environment were identified.

Limitations/implications-Theoretically, the study illustrated how expansive learning theory could be utilized to examine teacher collaborative learning in the online cross-cultural lesson study. Practically, this study showed reiterative cycles and experts' facilitation are crucial to expansive learning for linking research to classroom practice. However, this study did not focus on student learning in the virtual environment. Australian teachers' reciprocal learning through the online lesson study also requires further exploration.

Originality-Both online lesson study and cross-cultural collaboration are innovative. The expansive learning lens is creatively used to examine the complexity of teacher learning in such a novel environment.

Keywords Lesson study, Online lesson study, Virtual professional learning community, Online teaching, Expansive learning.

Paper type Research paper

1. Introduction

Much effort has been devoted to investigating different teacher professional development (PD) programs for improving students' learning in the past decades (e.g., Darling-Hammond et al., 2017). With the high reputation in practical features such as job-embedded, teacher-oriented but student learning-focused, and closely connected to learning outcomes (Lewis, 2016; Murata, 2011), Lesson Study (LS, hereafter) has been well recognized as one of the most effective PD (Author, 2016; Lewis and Perry, 2017). However, what the mechanism that makes LS works effectively is still a mystery even though its affirmative impact has been well documented and empirically evident (Authors, 2019). Furthermore, not much attention has been paid to explore what educational practitioners in China and Japan where LS originated, may learn from online and cross-cultural LS. The rapid development of advanced digital technology and its practical features have created opportunities to stretch the current LS to an online mode for cross-cultural LS research (Isoda et al., 2017). Building on the experience of cross-cultural teacher exchange program between the UK and Shanghai (Authors, submitted) and the online teaching experience during the time of Covid-19 pandemic in Shanghai (from Feb to June 2020), Shanghai Normal University and Association of Australian Mathematics Teachers (AAMT) collaboratively launched an online LS and webinar program to promote teacher collaborative learning in June 2020. This cross-cultural online LS project focuses on exploring effective ways for teaching mathematical rich topics for primary schools in both countries. The current study aims to examine the affordance and constraints of Shanghai teachers' learning from this cross-cultural online LS during the first three teaching cycles.

2. Conceptual framework

2.1 Lesson study and teacher collaborative learning

A typical Japanese LS usually includes four stages: Study, Plan, Teach and Reflect (Lewis, 2016). One whole phase of LS is usually concluded at writing up a report to share the learning experiences among the LS communities. Research has provided evidence about the benefits of LS which include improving teaching and learning, remodeling teaching practice, implementing new curriculum, and linking theories to practice (e.g., Author and colleagues, 2016; Lewis, 2016).

The Chinese LS refers to a teacher PD approach that involves collaborative lesson planning, cycles of teaching with classroom observation, post-lesson debriefing and reflection, and lesson revision; these components have been evolved for over a century in China (Li, 2019). Structurally similar to the Japanese LS, the Chinese LS has developed its unique features such as reiterative teaching cycles conducted by the same teacher for each research lesson and immediate feedback from experts and LS team in the post-lesson debriefing meetings. The Chinese LS usually invites experts who are officially responsible for organizing school-based teaching activities and evaluating teachers' teaching and students' learning (Gu and Gu, 2016) to provide comments and feedback from the lens of educational researcher and leader. The Chinese LS also emphasizes both the product (lesson exemplars) and process (teacher learning).

The practice of the Chinese LS reveals that a "culture of teaching as public activity" (Stigler et al., 2013) has been developed in China; observing and learning from each other's teaching and opening up the classroom to the public have become a norm for the Chinese teachers (Li, 2019). Furthermore, the nature of the Chinese LS has been endorsed as a type of improvement science (Bryk et al., 2016) contributing to the improvement of mathematics and science education system wide (author and colleagues, 2017).

2.2 Theory of expansive learning and lesson study

Cultural-History Activity Theory (CHAT) is a theory for understanding how a group of people achieve their objects (Engeström, 2001). Different tools that are mediated by the cultural and historical roots of the system such as community, rules and division of labor are interplayed to achieve the objects. The third generation of CHAT (Engeström, 2001) attempts to portray a collective and dialectical perspective of learning for extending the analysis of one single activity system to a complex activity system that involved interactions between different single systems (Engeström, 2001).

An important principle of CHAT is *contradiction* which is denoted as the 'historically accumulating structural tensions within and between activity systems' (Engeström, 2001, p. 137). The power of contradictions as Engeström (2016) assumed is its capacity of becoming a driving force for addressing the problems within and between system(s). Different types of contradictions may arise from (1) each of the constituent components of the central activity (Primary Contradiction), (2) the interplay between the constituent components of the central activity (Secondary Contradiction), (3) the differences of goals/motive between the dominant form of the central activity and the culturally more advanced form of the central activity (Tertiary Contradiction) and (4) the differences between the central activity and its neighbor activities of a complex system (Quaternary Contradiction) (Engeström, 2016).

Another key principle of CHAT is *expansive learning* which is denoted as 'learning in which the learners are involved in constructing and implementing a radically new, wider and more complex object and concept for their activity' (Engeström and Sannino, 2010, p. 2). Expansive learning "puts the primacy on communities as learners, on transformation and creation of culture, on horizontal movement and hybridization, and on the formation of theoretical concepts" (p. 2). CHAT views contradictions as driving forces of expansive learning when new goals/objects, concepts and motives emerge while the people are dealing with the contradictions within the activity system(s) (Engeström and Sannino, 2010). Expansive learning among the people of the activity system(s) entails a sequence of learning actions (i.e., expansive learning cycle) that are capable of resolving the contradictions. The expansive learning cycle includes questioning, analysis, modeling, testing and implementing the new model, and reflecting the process and consolidating the new practice (Engeström, 2001).

Limited research used CHAT as a framework to understand the nature of teacher professional learning. One example was that Potari (2013) used CHAT to explore the relationship between theory and practice in mathematics teacher PD and commented that CHAT offered researchers a powerful tool to connect individuals to different social aspects in mathematics education research. In a recent study of Potari et al., (2018), CHAT was employed to identify the contradictions between activity systems of mathematics teaching, mathematics education research, and educational policy, and to explore how these they interacted in the process of developing a mathematics curriculum. Building on the existing research, Wake et al., (2016) used CHAT to examine how LS processes promoted teacher learning in PD programs. They concluded that lesson plans as one of the boundary objects played a critical role in facilitating teachers' learning.

These studies have provided evidence to how CHAT in general, and theory of expansive learning in particular, could be used to investigate teacher learning in LS by means of identifying the contradictions. The current study adopted the third generation of CHAT in the form of two interacting activity systems (see Figure 1). In this study, the two activity systems were the Chinese LS and Australian LS. The research lesson served as the boundary object of both systems to explore and expand their learning through boundary crossing (Engeström and Sannino, 2010).

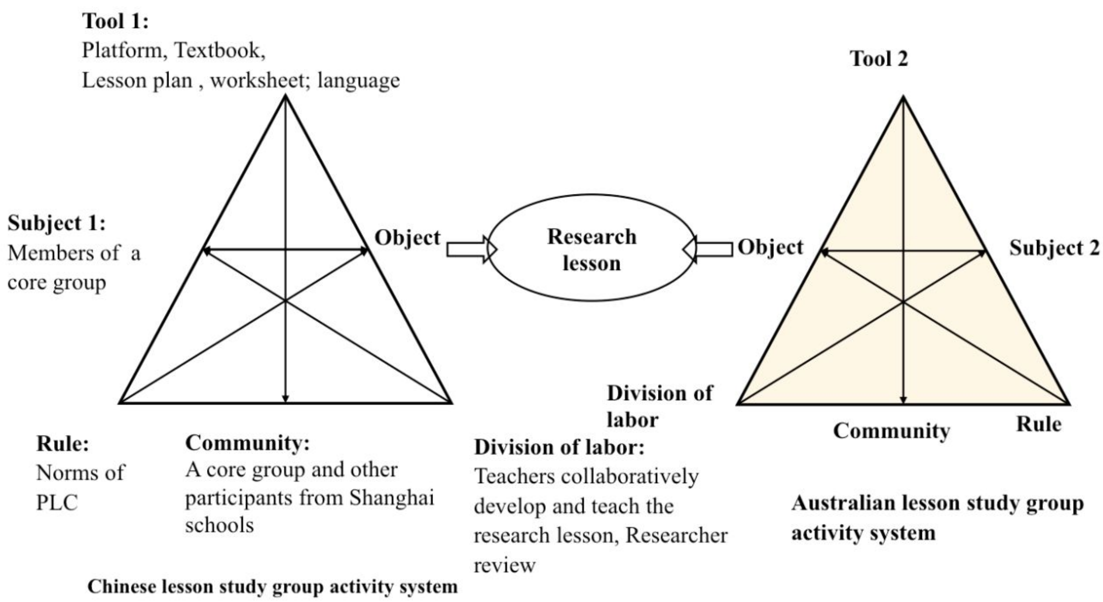

Figure 2. Systems of online lesson study

Many studies have focused on contradictions as a driving force for learning (e.g., Wake et al., 2016). Yet, it is largely unknown how the contradictions could be resolved to promote expansive learning. This study made an explicit effort to not only identify various contradictions (driving forces of expansive learning) and but to also locate the relevant learning cycle for addressing these contractions (enactment of expansive learning) in an online LS context. Specially, the study sought to address the following research questions:

1: What were the major contradictions revealed during the online LS process in the Shanghai learning community (if any)?

2: How the contradictions were resolved over the online LS process to promote expansive learning?

3. Method

3.1 The LS team

The whole LS team included a core group of three participants and 10 observers and reviewers. The core group consisted of the enacting teachers (Ms. Rong), head teacher (Ms. Wong) and a researcher from a local university (the first author). Both teachers are from a high-respected private primary school in Shanghai. Both completed a bachelor's degree in primary education and have more than 10 years of teaching primary mathematics. Ms Rong also had experiences in conducting research lessons in the UK. Ms. Wong is the head teacher of mathematics in the primary school. The university researcher has been engaging in mathematics teacher education research for years. The other participants included mathematics teachers from the exchange program. The Australian LS team included three school teachers and two university educators.

3.2 Mathematics curriculum and the teaching topic

The school system and mathematics curriculum in Shanghai and South Australian are quite different. In Shanghai, elementary school is from Grades 1 to 5 while in South Australia, it is from Grades 1 to 7. For the content of probability, the Shanghai elementary mathematics curriculum requires students to develop some basic concepts such as certain and uncertain events, using simple words to represent the likelihood of uncertain events, frequency of outcomes in an experiment, equiprobable and sample space of simple events. Using fractions to represent the probability of events is not within the Shanghai elementary curriculum (SMEC, 2004). In South Australia, in addition to the basic concepts of probability as in the Shanghai curriculum, using fractions (percentage and decimals) to represent probability is a key learning outcome in the Australian mathematics curriculum (ACARA, 2020).

The concept of fairness in probability looks simple to many teachers thus is easily oversimplified the embedded mathematical concept. One of the common misconceptions is that the chance of getting a 6 in tossing a die is the least among the other numbers (Langrall and Mooney, 2005). Different studies (e.g., Park and Lee, 2019) have provided evidence that in the initial stage, the concept of fairness and equiprobability could be constructed through using hands-on activities with small number of trials then gradually escalating to computer simulation with large sample data. In the next stage, Fielding-Wells (2014) suggested using encounter examples that involved non-equiprobable outcomes such as picking balls from a bag with an unbalanced number of colour balls. Edwards and Hensien (2000) believed that fairness, equiprobability, quantification of probability and more importantly probabilistic reasoning would naturally emerge through engaging students in these activities.

3.3 Process of the online Lesson Study

This LS was a teacher exchange project between the AAMT and Shanghai Normal University based on the Memorandum of Understanding on mathematics teacher education in 2019. Due to the Covid-19 pandemic, the onsite LS was cancelled. As teachers were getting used to using online platforms to teach, the LS team agreed on using one of the online platforms (e.g., Zoom) to conduct the cross-cultural LS on teaching probability in elementary schools in Shanghai and South Australia. Three lessons on the same lesson plan and teaching flow were conducted. An onsite lesson was conducted to a Shanghai class first then followed by an online lesson to an Australian class. The LS cycle was completed at an online lesson to another Shanghai class. For the onsite Shanghai lesson, there were around 45 students sitting in rows attending Ms. Rong's face-to-face teaching in the classroom. For the Australian online class, there were around 20 students sitting in groups of 4 learning through an online teaching by Rong. The class was assisted by a team of 2 teachers. For the online Shanghai class, there were around 20 students sitting in classroom, where two teachers assisted Ms. Rong to organize classroom activities. The LS process included: (1) Rong designed a lesson on probability for Grade 5 students based on the Shanghai curriculum; (2) the LS team and Australia team discussed the lesson plan on an online meeting; (3) Rong revised the lesson plan based on the feedback and included using fraction to represent probability ; (4) Rong did an onsite rehearsal teaching in a Shanghai elementary school, observed by the researcher; (5) a post-lesson debriefing meeting was conducted and Rong revised the lesson plan to incorporate the researcher's comments; (6) Rong conducted an online teaching to one class of Australian students observed by over 70 Shanghai and Australia teachers; (7) post-lesson debriefing meeting between LS team and the Australian teachers was conducted and observed by over 70 Shanghai and Australian teachers; (8) Rong slightly revised the lesson plan based on the feedback and self-reflection, then taught the lesson to another Shanghai class remotely with 10 Shanghai teachers as observers; and (9) a final post-lesson debriefing meeting among the LS team members was conducted.

3.4 Data collection and analysis

In the LS, the data collected include: (1) three versions of lesson plan（D1）; (2) videos of three lessons (D2); (3) videos of three post-lesson meetings (D3); and (4) audio recording of interviews with the Ms. Rong (D4-1) and Ms. Wong (D4-2) All the teaching and debriefing videos/audios were transcribed verbatim by research assistants (either in Chinese or English depending on original languages used). The accuracy of the transcription was checked by the researcher. All the data were stored in a folder on Google drive shared with the three authors. To identify the contradictions, the researchers repeatedly read through all the data to portray the overall picture of this study. The analysis was started with comparing the similarities and differences between the three lessons. Then, an online meeting of the three authors was devoted to identifying changes across the lessons and reasons for the differences. Once some plausible reasons have been identified, regular meetings for discussing the contradictions were held to minimize the risk of misinterpreting the data. In the meetings, each author in turns shared their own analysis and observed contradictions. Based on extensive discussions over multiple online meetings, the three authors reached consents on the observed contradictions at three levels: Primary, Secondary and Tertiary. Although various contradictions at the three levels have been identified, this research paper focused on discussing the contradictions emerged from the Chinese teachers' understanding of mathematics knowledge for teaching in general, and concept of probability and use of online resources in particular. Then, each author focused on one level of contradictions, and identified the expansive learning cycle for resolving the contradictions among the Chinese teachers. The authors have gone through the seven components of expansive learning cycle in the analysis process though the components were not explicitly indicated in the results and discussion sections. It was then followed by further discussion on the accuracy and coherence of findings presentation and cross check for reliability.

4. Results

The results presented in the following section were attempted to provide answers to the research questions: what contradictions emerged from this online LS and how the contradictions were resolved eventually.

4.1 The primary contradictions and strategic actions

One primary contradiction was identified. *Contradiction 1* was about the disparity between teachers' and researchers' understanding of basic concept of probability: making sense of experimental probability and presenting the outcomes in fractions versus transforming from experimental probability to theoretical probability through using experiment and computer simulation. With Rong's teaching experiences, the learning goals were set to "develop students' understanding of basic concept of probability and fairness of game rules, and to represent the probabilities using fractions through exploratory activities, such as tossing a coin and drawing balls from a box" (D1-1). In the first teaching, the lesson began with an activity focused on tossing a coin moving to a demo animation to make sense of fairness (i.e., equiprobable). This was attempted to show the occurrence of heads or tails approaching half-half and the half-half occurrence maintained as the number of toss increased towards infinite. This tossing activity of itself was a good investigation. However, the contradiction arose when the teacher used part-whole interpretation to describe the probability and suggested it was half because "there are two equal faces, one head, and one tail, so showing up heads is ½" (D2-1). In the post-lesson debriefing, the researcher pointed this out and explicitly explained the relationship between occurrence (i.e., experimental probability) and theoretical probability and emphasized the goal of this lesson should focus on developing students' understanding of this relationship as from which the concept of equiprobable and quantification of probability would be developed naturally.

> Researcher: *What are occurrence and frequency right? […] I think 500 is just an occurrence, but what is the frequency? Let's say there are 50 times head up in 500 times of tossing a coin. Divide 50 by 500, and this is called a frequency [for head]. Well, when the number of tosses increases infinitely, the frequency will stabilize at a value, then we define this value as a probability, probability is defined by the frequency. Through this experiment, we define the notion of experimental probability. […] What does theoretical probability mean? You don't need to do experiments. Let's say I have a coin with head and tail. The chance of getting either one is 1/2, because there are two outcomes of the event. So what is involved in theoretical probability? It is the sample space and the event*

outcomes. Let's say there are 6 numbers [N=6] in the sample space [for a die], and the probability of getting each of them [M=1]is the same, so I use M/N to represent it, right? (D3-1)

The teachers of LS team held the same idea as Rong. It was evident in their discussion of how to carry out the activities. The teachers' major discussion was around how many times tossing a coin would be more appropriate to naturally emerge the half-half occurrence so that the chance of 1/2 could be verified. The researcher found the teachers' direction pedagogically inappropriate and drew teachers' attention to investigating the fairness of tossing a coin, which was determined by the equiprobability of outcomes. The experiment was not to verify the chance of 1/2.

Researcher: *The student said that it was fair, and another student said the probability is equal, then you get students to toss 20 or 30 times, and then you keep saying that the gap (between the occurrence of head and tail) would become smaller (with increasing the number of toss), as if everyone has already known it was 1/2. In fact, everyone targeted ½ eventually. But what I mean is that students have to make conjecture when they talk about fairness. In fact, all experiments require testing for hypotheses before any conclusion. (D3-1)*

The researcher emphasized that the outcomes generated from tossing a coin 10 times, 30 times, 150 times and so on was to inspire students to predict the equiprobability when they observed the occurrence of heads and tails came closer with increasing trials. Quantifying the probability should not be introduced at an early stage. Following this argument, a school teacher suggested that before tossing the coin, teachers should have made clear to the students that the coin was made of uniform texture so that the notion of equiprobable could be discerned easier.

A school teacher: For … experiments, it is implicitly assumed that the coin is of uniform texture, so it is equiprobable, […] therefore it allows us to use a fraction to represent its chance. What I want to say is even if we do not explicitly mention the *notion of equiprobable, when introducing the coin and balls, we need to make clear this assumption to make the experiments more scientific (D3-3)*

Nevertheless, Rong looked like she partly understood the issue and suggested using computer simulation to generate large sample data to test for the half-half hypothesis. The teacher believed that the experiment could induce a discussion on connecting experimental to theoretical probability, which in turn would emerge the concept of equiprobable. Eventually, this contradiction was settled at this semi-resolved state.

4.2 The secondary contradictions and strategic actions

Two secondary contradictions occurred between tool and object at this secondary level. *Contradiction 2* occurred during Rong's online teaching with Australian students [D2-2]. A major technical issue prevented her from interacting directly with the Australian students. When the students were using the microphones on their iPad to interact with Rong, the voices from different groups interfered with each other. The Australian teachers then realized that technically, the system did not allow users to have more than one microphone working synchronously in one space. Then, they had to turn off their microphones and communicated with Rong through their Australian teachers using the main microphone in their classroom. Thus, the conversation between Rong and students became an "asynchronous interaction" so that the lesson was not finished as scheduled.

Rong originally planned to monitor the investigation group activities by herself. However, in the online lessons she was not able to observe students' activities and to provide necessary support in time as in an actual classroom. She was disappointed as she believed that teacher's guidance on student activities was extremely important.

Rong: *[…] After the class, I feel that if I wanted to use this teaching model again, it may be more suitable for the lessons that focus on teaching computations or conceptual knowledge. For the online classroom, if the students do not manipulate [the activities] properly, or if some issues happen, it is not convenient to guide them in time.* （D4-1）

Contradiction 3 was displayed when the teachers attempted to scaffold students' learning. The learning goal was distorted when the teacher scaffolded her students during the implementation of activities. In the first lesson (D2-1), after the students have tossed a coin for 30 times, Rong initiated conservations with the students in an attempt to explain that the occurrence of head and tail would approach half-half. However, the teacher's scaffolding was found deviated.

Rong: *Tell me about the outcomes of your trials.*

Zhang: *[…] I got 19 heads and 11 tails.*

Rong: *Before tossing the coin, we guessed how likely it was to get the head and the tail. So why after doing this, we found that for some groups, there were more heads, for other groups, there were more tails. Why? Maybe this effect isn't particularly noticeable yet due to the small number of trials, right? So this way, we have five students in each group, we put all the results together, and see how many heads and tails in each group*. […]

Rong: … *how many heads and how many tails? Not much of a difference now, is it? They're close enough, aren't they? Especially Group 3, they are all the same, … there's a slight difference between the number of heads and tails in Group 5 and Group 1, right?* […]

In this conservation, apparently the teacher was using the investigation results to verify the half-half occurrence of head and tails instead of using the data to develop the concept of equiprobable and form a conjecture about the tendency of occurrence as the trials increased infinitely. In the lesson, the teacher suggested that the number of trials might *not be enough to show the effect*. Though she did not explicitly make clear to what the effect she referred to, in her later utterances where she mentioned 'not much differences between the number of heads and tails' as well as 'they were close enough' were actually pointed to half-half occurrences of heads and tails. This contradiction was brought out by one researcher. The teacher was suggested to not pre-assume the equal chance in her scaffolding and explanation. This contradiction was resolved eventually in her later lessons. The following episode illustrated how the concept of equiprobable was naturally emerged when this pre-assumption was avoided.

Teacher: *Here, 147 times for the head and 153 times for the tail. Look at the final outcomes and compare the group results and your own. What do you think? Ai.*

Student: *I think the times of the head and tail are very close, proving that they do have the same likelihood of 1/2.*

Teacher: *The number of heads and tails is very close, […] the likelihood of the head or the tail is 1/2.* (D2-3)

4.3 The tertiary contradictions and intended actions

Based on the last debriefing meeting and interviews with two core teachers, the following the tertiary contradictions and proposed solutions were identified. *Contradiction 4 refers to the disparity between the teacher's existing and required mathematics knowledge for teaching (MKT).* After the teaching, Rong disclosed her superficial understanding of fairness and equiprobability, and understanding of representing probabilities using fractions (e.g., MKT refer to Ball et al., 2008). For instance, Mr. Wong pointed out Rong's inappropriate task design and said that "tasks should be carefully selected to better serve the learning goals of the class. […] For example, rock, scissor, paper problem was not appropriate for the lesson." (D4-2) Another teacher also challenged Rong's explanation for the fairness of the game Rock, Paper, Scissors.

> *For Rock, it could win, lose or tight. […] So, you need to list out all the outcomes in a table to analyze. There should be 9 different outcomes and each act has 3 outcomes that win. Thus, the chance for each act to win is 1/3. But the calculation is not because there are three different acts.* (D3-3)

The other participants agreed that this task involved the concept of listing sample space and should not be discussed in this lesson. This contradiction was finally settled at removing this task from the lesson. Nevertheless, her (and the LS team's) understanding of the concepts has been developed gradually throughout the research journey; it was beyond their current knowledge of teaching about the mathematical concepts. Although quantifying probabilities using fractions is not required in Shanghai curriculum, after these lessons, she eventually realized that "[…] incorporating fractions with introduction of probability in grade 5 could consolidate fraction understanding and deepen the concept of probability at the same time, it is a nice integration that addresses students' learning" (D4-1)

She also appreciated the researcher's clarification and explanation on the concept of probability and this has inspired her to consider teaching the basics from an advanced mathematics perspective.

> Teacher: *This guides me to better design classroom teaching: […] design, […] rationale, ...this opened up my horizon, I need to have an advanced perspective so that I can have deeper insight ".* (D4-1)

Contradiction 5 refers to the disparity between the teacher's existing and required Mathematics TPACK. Rong's teaching with Australian students was synchronous online and some technical issues occurred that really affected her interaction with the students in time. To mitigate this constraint, several strategies were suggested for future teaching.

A worksheet with an organized table was suggested for recording student's data and for adding up the data as group results. During the debriefing meeting, the teachers proposed different ways such as using Excel spreadsheets so that the data could be calculated automatically. (D3-3)

Regarding carrying out the experiments in the online teaching, the LS team realized there were some unexpected issues arising from tossing a coin. Rong and other teachers suggested providing a demonstration of activities to students before the actual experiments. (D3-3) To maintain the consistency of operating teaching materials across countries, Ms. Wong suggested providing photos of manipulatives or even videos to detail the requirements of experiments and activities. (D4-2)

Regarding the actual implementation of experiments and activities, Rong suggested asking students to carry out the experiment and record their results before lesson. The teacher then aggregated results and took it to the class for discussion. Other teachers suggested video-taping students' experiments in advance. (D3-3)

To facilitate the students' learning, Rong adopted a short animation video to show the results of experiments conducted by different statisticians. However, the effect of the demo was not satisfactory. Several teachers suggested improving the quality of the animation video or to even use simulation applets to generate larger trials. (D33) Moreover, the experts from Australia strongly suggested using computer simulation to develop the understanding of occurrence of head and tail steadily approaching half-half with increasing trials towards infinite. (D3-2) However, Rong admitted herself not knowledgeable in computer simulation due to lack of IT knowledge (e.g., Mathematics TPACK) (Mishra and Koehler, 2006).

> Teacher: *[…] it is not enough just to show an animation video, we should let students have personal experience in how the frequency approaches a fixed value as the trials increase … the Australian teachers said we could program applets and let students use them, but I do not know programming, […].* (D4-1)

Ms.Wong also commented on the common weakness of Shanghai elementary mathematics teachers in integrating technologies in teaching and admitted the difficulty in employing teachers with strong IT knowledge.(D4-2) Furthermore, Ms. Wong and the researcher in the interview discussed how to provide specific training programs for school teachers. (D4-2)

After the LS, Rong eventually realized that she needed to develop stronger ability in using various educational technologies and online resources; it was beyond her expectation. In addition, how to interact with students, understanding student learning difficulty and providing feedback in time in a virtual environment were also challenges that required her to address in the future. (D4-1)

5. Discussion

This study revealed that the teachers have expanded their learning through dealing with the contradictions in the LS within the Chinese cultural activities and across cultural activities. The expansive learning was evidenced in three areas in this study.

5.1 Expansive learning as transformation of objects

In this study, the object of this LS was transformed from *producing* an online lesson for different cultures to establishing collaborative efforts and intelligence among participants to resolve contradictions for *developing and conducting* online lessons. As discussed earlier, a lot of issues relating to the learning goals of activities (e.g., using fractions to represent the probability), design of activities and technical problems occurred in the online teaching with the Australian students. With the experts' comments and the teachers' collective reflections, the contradictions have become a driving force to urge the LS teams to shift their focus from emphasizing specific content of activities to developing well-design instructional materials including design of the activities, administering the experiments and using computer simulation. The contradictions also empowered the teachers to develop a thorough understanding of equiprobable. Although the implementation of activities was not very successful in all the three teaching cycles, collective efforts among the participating teachers has been established to collaboratively resolve the issues and achieve their goals. Eventually their collaboration has formed a collective effort and intelligence for *developing and conducting* online teaching. The teachers' MKT and Mathematics TPACK were improved as well.

5.2 Expansive learning as boundary crossing and networking building

There were multiple activity systems interacting with each other in this LS. These activity systems included the Shanghai LS team in which the central activity was operated, the research system in which the researchers worked with the teachers, the teaching system in which the teachers worked collaboratively and the Australian team. The research lessons eventually served as a boundary object for *developing and conducting* online lessons that required different systems to work together and to share ideas. Through working on the research lessons, some new practices have been brought to the existing activity systems and made the expansive learning happen (Engeström and Sannino, 2010). In this LS, the Shanghai LS team were made aware of two critical ideas through discussion and ideas exchange. The two ideas were how to effectively organize and administer activities with specific skills required and how to use computer simulation. The object of the LS has been transformed from teaching specific mathematical concepts to *developing and conducting* online lessons in general.

Regarding interaction between the research system and Shanghai LS team, the researchers not only widened Rong's teaching perspectives but also fostered her understanding of MKT in general and the teaching of probability in particular. More importantly, Rong was made aware that for quality teaching, it was necessary to understand elementary mathematics from an advanced perspective.

For the research system, the researchers understood better the teachers' challenges such as lack of MKT and Mathematics TPACK in their teaching. This helped the researchers step out of their "ivory tower" to experience a genuine "messy" classroom. The researchers attempted to seek different solutions such as offering relevant teacher professional training programs. For the teaching system, the head teacher was made aware of the need for teachers to participate in pertinent PD to develop their Mathematics TPACK.

5.3 Expansive learning as a cycle of learning actions within a supportive community.

A typical LS cycle is very similar to the cycle of expansive learning: setting goals versus questioning and analysis, planning a research lesson versus modeling, teaching the lesson versus testing and implementing; and reflecting on the lesson versus reflecting. More importantly, the unique features of the Chinese LS (discussed in the earlier section) strengthen the expansive learning process via multiple ways. For instance, the involvement of experts including university researchers and teaching research specialists (Author and colleagues, 2017) are able to enrich the research lessons by bringing in their research findings and expertise and by providing new models and instruments for assessing teaching effectiveness. The reiterative cycle of LS provides opportunities for teachers to re-design, re-test, re-reflect and re-revise the lessons. As evident in this study, throughout the LS journey, teachers revisited their understanding of the teaching concepts, re-set the learning goals and re-develop the practicing tasks to accommodate the teaching in a virtual environment. So that, the unexpected challenges from the online teaching as well as teachers' lack of MKT could be addressed.

In an activity system, the rules, community, and division of labors are fundamentally important components (Engeström, 2001). In this study, although the professional learning community (i.e., The LS team) has been shifted from face-to-face to online community, the teachers and researchers still made critical comments (e.g., inappropriateness and errors in practicing tasks) and offered suggestions frankly (e.g., use spreadsheet, and computer simulation) for improving the lessons in the online environment. The existing literature indicates that teachers' conversation and discussion are usually superficial and non-critical in a virtual PLC (e.g., AlBalushi and Al-Abdali, 2015), and suggests that contents relevant to teachers' practice, a trustful relationship, and effective facilitation are important for a virtual PLC to be productive (e.g., Lantz-Andersson et al., 2018). In fact, in China, a systemwide teaching research system (Chen, 2020) and teacher collaborative culture have been well established, all of which in turn develop the norm and routine in onsite LS (Li, 2019). The current LS has illustrated that through concrete facilitation from researchers and trustful relationship between teachers and researchers, the transformation of PLC from face-to-face to online could be successful.

5. 4 Implications

This study displayed both theoretical and practical implications. Theoretically, the study illustrated how CHAT could be utilized to examine teacher collaborative learning in LS. The findings showed that the participating teachers' learning could be described as expansive learning. The expansive learning provided a useful lens for investigating the complexity of LS. Thus, this current study contributes to deepening the understanding of the effect and process of LS.

Practically, this study also has implications for strengthening LS. Both the reiterative cycle and experts' facilitation in LS are crucial to expansive learning. The reiterative cycle provides a mechanism for expansive learning. The experts strengthen the design, evaluation, and reflection, all of which contribute to linking research to classroom teaching (Huang et al., 2016), as well as promoting the development of teachers' MKT (Ni Shuilleabhain, 2016).

5.5 limitations and further studies

There are some limitations identified in this study. First, this study did not collect students due to the constraint of online teaching platforms. Moreover, the interactions between the teacher and students were very limited. How to observe student learning on an online platform requires further investigation. Second, this study only focused on Chinese participating teachers and their learning due to the difficulty in collecting data from the

Australian counterparts. Investigating how the learning of Australian teachers and reciprocal learning occur through boundary crossing is an important research topic in the future. Third, the differences between the two countries such as culture (student learning, and teacher learning) (Stigler and Herbert, 1999), curriculum, teacher professional development system may promote or hinder the teacher learning through online LS. This area should be explored appropriately.

6. Conclusion

This study examined teachers' learning through a cross-cultural online LS which aimed to address how to conduct a lesson on an online platform. The findings showed that the teachers' expansive learning including enhancing their MKT and Mathematics TPACK, developing skills in designing instructional tasks (experiment activity) and developing the capability in using various teaching materials and tools (physical and electronic devices, online resources) were emerged through the processes of resolving the three levels contradictions. More importantly, this study demonstrated how the CHAT theory could be employed to investigate the complexity of LS for further developing the theory and the practice of LS.

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