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Modulated acoustic radiation force in a carrier standing wave

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Introduction

For many applications, such as sorting, separation, and manipulation of differently shaped particles [1-3], it is desirable to be able to control the direction of the acoustic radiation force (F_{rad}) and produce e.g., alternating pushing and pulling forces [4]. Using progressive acoustic plane waves, it has been demonstrated that pulsating spherical carriers experience a force direction reversal owing to radiation energy originating from the pulsating surface [5] with the limitation that the oscillation of the particle is limited to its boundaries. Here, and by using an acoustic standing plane wave, oscillating positive and negative radiation forces studied theoretically and numerically. By using Gor'kov's formulation, the acoustic contrast factor [6], is calculated for modulated acoustic field quantities. The ability of producing both positive and negative forces leads to the ability to control the vibration of an object. Fushimi et al. suggested an experimental model of a nonlinear stiffness model of the trapping of a solid particle in a single-axis acoustic levitator [7]. Here, by using their experimental data the presented analytical and numerical model are validated and discussed.

Nonlinear dynamic model of a spherical object inside a single-axis acoustic levitator

Let's consider a naturally buoyant spherical object (particle) with rigid body oscillations in form of $Z_p(t) = A\sin(\omega_o t)$ within an ideal fluid and subjected to an incident pressure standing wave $P_{in} = P_a \cos\left(kz(t)\right)\cos\left(\omega t\right)$, as schematised in Figure 1. The particle can be assumed to be a harmonic oscillator with mass of M_p and nonlinear stiffness k_s which oscillates about its equilibrium position e at distance z from the pressure anti-node.

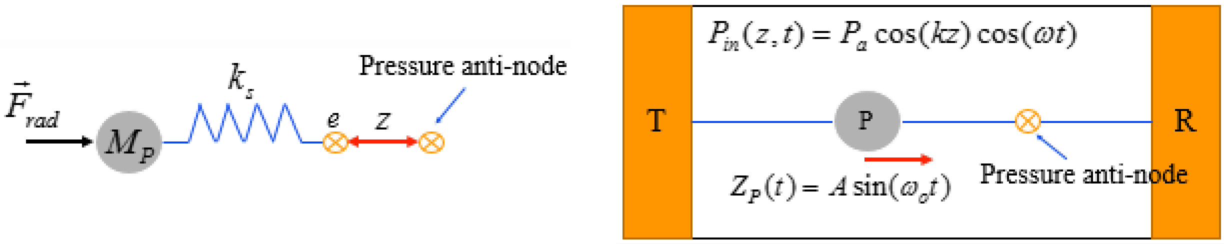

Figure 1: A schematic of an oscillatory particle suspended between a transducer and a reflector with its mechanical model.

Using this mathematical model, the incident pressure field can therefore be re-written as

$$P_{in}(z,t) = \left(P_a C_1(t)\cos(kz) + P_a C_2(t)\sin(kz)\right) \tag{1}$$

with $C_1(t) = \cos\left(\omega t\right)\cos\left(\left(Ak\right)\sin\left(\omega_o t\right)\right)$ and $C_2(t) = \cos\left(\omega t\right)\sin\left(\left(Ak\right)\sin\left(\omega_o t\right)\right)$.

According to Gor'kov's theory [7], F_{rad} is calculated as

$$F_{rad} = 4k\pi r^3 E_{ac}\Phi(\tilde{\kappa},\tilde{\rho})\sin\left(2kz\right) \tag{2}$$

where k is the wave number, r is the particle radius, E_{ac} is the acoustic energy density [7], Φ is the acoustic contrast factor, and $\tilde{\kappa}$ and $\tilde{\rho}$ represent the relative compressibility and density between, particle and host fluid, respectively. To verify the equations, we study F_{rad} as exerted by a standing wave in form of a background pressure field, Eq. (1), and its effect on a particle immersed in water using a 2D axisymmetric geometry (COMSOL Multiphysics 5.5). Radiation boundary conditions are used to mimic the particle's free space scattering by conducting a finite element simulation.

Analytical and numerical examples

We conduct a parametric analysis to calculate F_{rad} on an oscillatory particle comparing analytical and numerical simulations. The parameters used for the calculations are: $c_1 = 6559$ $\left[\mathrm{m/s}\right]$, $\rho_1 = 2000$ $\left[\mathrm{kg/m^3}\right]$,

$\lambda = 1.48$ [mm], $z = \lambda / 8$ [mm], $c_0 = 1480$ [m/s], $\rho_0 = 1000$ [kg/m^3], $\omega = 1$ [MHz], and $\omega_o = 10$ [KHz]. When, c being the speed of sound and ρ is the density with indices 0 and 1 denoting the fluid and the particle, respectively, z is the distance between the pressure anti-node and the center of the particle in the wave direction, and λ is the wavelength. Figure 2 shows the mean value of time functions of Eq. (1) and an acoustic contrast factor Φ for different normalized amplitude, Ak. F_{rad} changes its sign, indicating direction reversal.

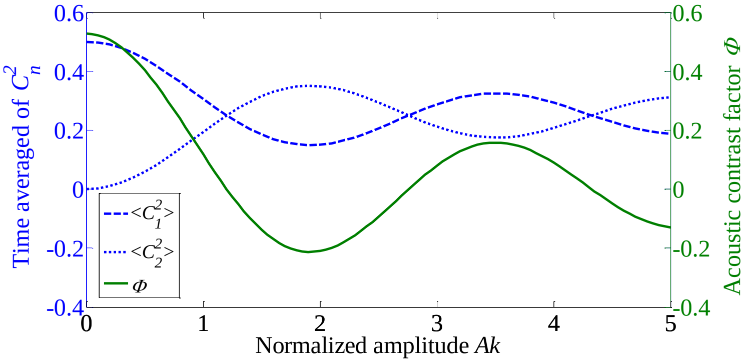

Figure 2: The acoustic contrast factor and $\langle C_n(t)^2 \rangle$, $n = 1, 2$ for different value of the Ak.

Figure 3 depicts the acoustic radiation force function (Y_{st}) [2] as dimensionless value of F_{rad}, against the dimensionless particle radius kr for two different values of Ak. Y_{st} shows that the different values of the positive and negative of F_{rad} can be achieved.

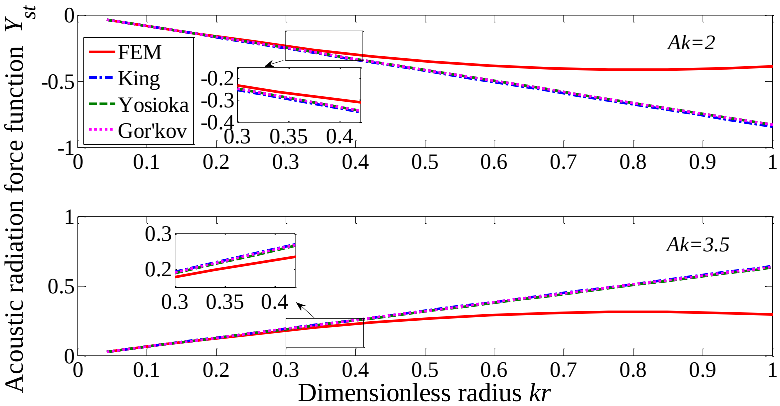

Figure 3: A comparison of the Y_{st} between numerical and analytical results for an oscillatory particle in a standing acoustic wave field over.

A single degree-of-freedom nonlinear dynamical equation of motion can be written as

$$\ddot{\theta} + \alpha |\dot{\theta}| \dot{\theta} + \beta \sin(\theta) = \gamma \cos(\theta) \sin(\omega_o t) \quad (3)$$

when $\theta(t) = \sigma z(t)$ and α, β, γ, and σ are constant coefficients.

Conclusion

Due to the vibrating particle, the modulated incident acoustic pressure produces positive, zero and negative F_{rad}. This suggests that the exposure of the particle to e.g., externally applied vibrations can provide an alternative means of controlling F_{rad}. A single degree-of-freedom nonlinear model of a solid particle trapped inside a mid-air single-axis levitator has been presented and will be compared to validate experimental findings of Fushimi [7].

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