Modeling of Complex Modes with Wave-Based Scaling

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1 Introduction

In the literature, complex modes are subject of applications that range from composites to rotating machineries [1, 2]. Complex modes have been shown to exhibit wave characteristics, and its composition with respect to traveling wave and standing wave was introduced [3]. Complex modes that result from non-uniform distribution of damping have been studied. Eigensolution and sensitivity analyses were undertaken to estimate the variation with respect to system parameters. For lightly damped systems, approximation methods were developed to obtain the complex eigenvectors from undamped modal parameters [4]. These advancements have widened the perspective in which we view the complex modes. In a more recent work, a new approach was proposed to model complex modes with a waveguide, for a simple non-proportional damping configuration [5]. It was shown that complex modes can be estimated without solving the eigenvalue problem. Complex modes can be scaled such that the imaginary part is minimized. This scaling technique was used to produce performance indices with respect to damping non-proportionality [6]. In this study, we show a wave-based scaling method on a non-proportionally damped system. It extends a previously made observation corresponding to a complex mode formation scheme [7]. It was shown that the difference between a complex mode and a real normal mode can be expressed by propagating patterns that behave like waves. Thereby, these patterns are refracted as they cross an intersection that divides damping medium. This relationship is described Snell's law. In this work we show that a beam structure that has a partially applied constrained layer damping has complex modes that behave according to the predictions. Case studies

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were conducted with respect to different damping configurations. In Sect. 2, we outline the theory of the method. Then, in Sect. 3, we show a comparison of the numerical and experimental results and give our conclusions.

2 Methods

Similar to a light beam that gets refracted at the boundary of two media, it can be argued that the complex mode featuring wave characteristics can also be represented with a similar perspective. In this regard, consider a structure with simply supported boundary conditions. Analytically, the deflection patterns for a real normal mode of beam can be expressed by a sine function, where m corresponds to the spatial wavenumber on x coordinate and L_x is the length of the beam, as shown in Eq. (1):

$$\mathrm{W}_m(x) = \sin(m\pi x/L_x) \tag{1}$$

When the system is proportionally damped, these deflection patterns retain the same shape. However, if the damping is non-uniformly distributed, as in the case of partially applied damping layer on a plate, the mode shapes are complex-valued. It is argued that the difference between a complex mode and the respective normal mode can be described propagating wave-like patterns as shown in Fig. 1. When the system is proportionally damped, the net sum of the patterns is zero, while, when the system is non-proportionally damped, we observe refraction at the individual intersections of damping. For a typical non-proportional damping configuration, the summation of patterns is a non-zero variation. Thereby, they contribute to the imaginary part of the complex mode.

As shown in Fig. 1c, the left propagating pattern is refracted at the damping intersections. This relationship is governed by Snell's law:

$$\frac{\sin(\theta_2)}{\sin(\theta_1)} = \frac{k_{m,1}}{k_{m,2}} = \frac{d_1}{d_2} \tag{2}$$

where d_1 and d_2 correspond to the refractive indices of damping regions D_1 and D_2, respectively.

3 Results and Discussion

To verify the numerical predictions, an experimental setup was prepared. A steel beam of 3 mm thickness was clamped at both ends. Shaker excitation was used at an end node to excite the beam, and uniaxial accelerometers were attached. Measurement nodes were selected so that they are equally spaced. Figure 2 shows the dimensions of the setup and locations of the transducers. Note that in the method

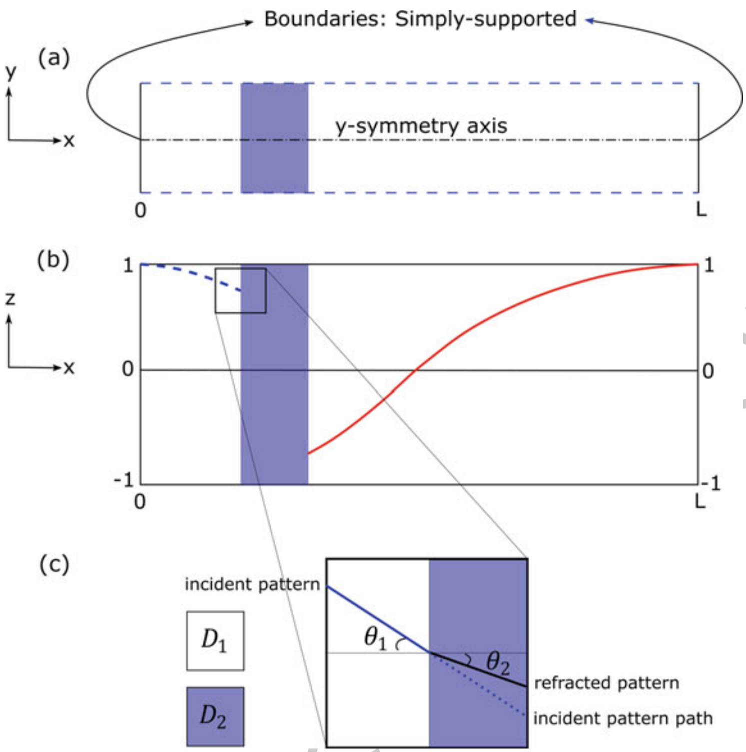

Fig. 1 Overview of the complex mode formation framework. (**a**) (x–y) section view of beam structure with a non-proportional damping patch. (**b**) Propagating patterns from the boundaries of the structure, blue dash line: left propagating pattern, red solid line: right propagating pattern. (**c**) Refraction of the propagating pattern at a damping intersection

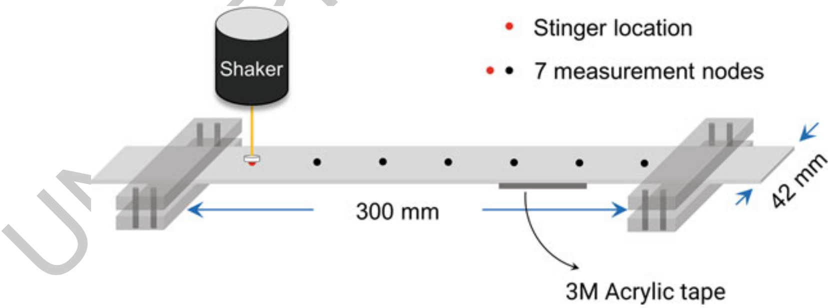

Fig. 2 Experimental setup showing the exciter and transducer locations

section we described the boundary conditions to be simply supported. Simply supported boundary conditions are the most simplistic form for analytical solution [8]. However, in an experiment setup, it is not easy to achieve it. Therefore, generally,

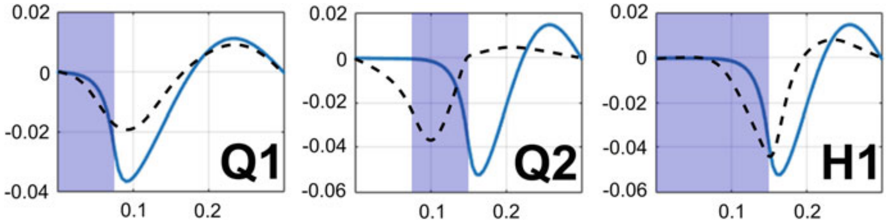

Fig. 3 Comparison of the imaginary values of the normalized first mode with respect to numerical approximation and the experiment. Blue solid line: method, black dash line: experiment

clamped boundary conditions are used. Comparing both boundary conditions, it was shown that although the modal frequencies are higher for clamped boundary condition due to increase in overall stiffness, the mode shapes are very similar to that of simply supported boundary condition [8, 9]. Hence, in the experiments we used clamped boundary conditions.

Partial damping treatment was applied by attaching three layers of 3M damping foil [10] on the bottom surface of the beam (see Fig. 2). We have tested three different cases, i.e., Q1, Q2, and H1, that refer to quarter or half size of the beam in the longitudinal direction. The complex modes that were obtained from the experiments are compared to the numerical predictions in Fig. 3.

It is observed that the overall trend is similar between numerical predictions and the experiment, especially for Q1 and H1 configurations. This has valuable implications for practical applications. One such benefit can be attained for benchmarking of the existent method, such that for cases where finite element modeling is not feasible an experimental procedure can be done for verification. In addition, in industrial applications where damping treatments are used for noise and vibration control, such approximation method can be utilized for preliminary decisions regarding the optimum location for the damping layer attachment. In the future studies, the present method can also be implemented into structural-acoustic coupling problems.

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