Addressing student misconceptions of phasors and AC resonance

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Abstract Summary (35 words)

Interactive Lecture Demonstrations, using a 'Predict, Observe, Discuss, Synthesise' learning cycle and audience response devices (i.e. clickers), have been used to improve students' conceptual understanding of phasors and AC resonance in an introductory electronics course.

Keywords- misconceptions, Interactive Lecture Demonstrations, clickers, electronics

I. BACKGROUND

A. *Transmissionist Teaching and Misconceptions*

Traditional transmissionist modes of teaching are largely ineffective in improving students' conceptual understanding of physics [1]. Students are not 'empty vessels', but instead hold their own conceptions about different phenomena that they have developed through their experience of the world [2]. Often at odds with accepted scientific explanations, these misconceptions are very resistant to change and are typically not overcome by transmissionist teaching [3-5].

B. *Interactive Lecture Demonstrations*

One strategy developed to directly challenge students' misconceptions is to use Interactive Lecture Demonstrations (ILDs) [6]. An ILD activity typically involves a planned sequence of iterations of the following learning cycle:

- Students make predictions about some experimental manipulation
- Students discuss their predictions with their neighbours, then reassess their predictions
- Students observe the outcome
- The lecturer facilitates a discussion with students to reconcile any differences between the observed outcome and their predictions, and help synthesise the observations into their frameworks of understanding.

ILDs have been shown to be more successful than transmissionist teaching in improving student conceptual understanding [7]. An important factor in these learning gains is the peer discussion about the experimental predictions [8, 9].

C. *Average normalised gain*

Pre- and post-testing before and after an educational intervention is an established evaluation technique [10]. The average normalised gain (g) is a measure of improvement on pre- and post-tests [1]. It is defined as:

$$g = \frac{Observed\ improvement}{Maximum\ possible\ improvement} \quad (1)$$

Although its use and interpretation are sometimes contested [11, 12], it is a well-established measure. Typical values for traditional instruction are $g \approx 0.2$, and for interactive-engagement teaching methods $g \approx 0.5$ [1]. In a previous study at Swinburne University of Technology (SUT) a value of $g = 0.22$ was reported [13], although it should be noted that the pre-test was administered after traditional lectures and so these learning gains are in addition to any that would have resulted from the traditional instruction.

II. OBJECTIVE

Lecturers in a large-enrolment first-year introductory electronics course at SUT have noted students' ongoing difficulties with understanding phasors, and the associated confusion around leading versus lagging of sinusoidal waveforms, and the concepts of instantaneous versus rms voltages and currents.

The objective of this study was to develop and evaluate the effectiveness of a new set of ILDs in improving students' conceptual understanding of phasors and AC resonance.

III. ILDS AT SWINBURNE UNIVERSITY OF TECHNOLOGY

ILDs were first introduced into first-year electronics at SUT in 2006, initially only for Operational Amplifiers [13, 14] and then in 2010 for phasors and AC resonance [15-17].

A. *Resonance ILDs in 2010*

In 2010, a multiple-choice pre-test of seven conceptual questions was given to the students prior to the ILD intervention (but after traditional instruction). Performance was poor, with an average score of only 38% (N = 86). Four of the questions were answered correctly by fewer than 30% of respondents, comparable to random chance. Such poor performance on a conceptual test, even after traditional instruction, is unfortunately all too typical [1].

However, for the subset of students (N = 21) who attended <u>all</u> the ILD sessions, there was a marked improvement in performance on the post-test. Their average scores increased significantly from 33% on the pre-test to 54% on the post-test ($p < 0.01$).

For these 21 students, $g = 0.32$. Recall that the pre-test was administered after traditional lectures and so these learning gains are in addition to any that would have resulted from the traditional instruction.

Despite these gains, two misconceptions remained prominent.

IV. Prominent Misconceptions in AC Resonance

A. *Graphical representations of leading and lagging*

In a series RLC circuit (Fig. 1), the voltages across the different components are not in phase.

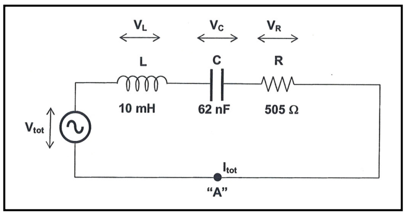

Figure 1. A typical AC series RLC circuit.

The voltage V_L across the inductor leads the voltage across the resistor, V_R, by 90°. The voltage V_R in turn leads the voltage across the capacitor, V_C, by another 90°. A 90° phase difference, as exists for example between V_R and V_C, can be represented graphically as follows:

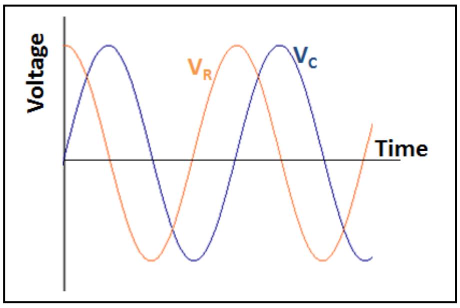

Figure 2. V_R leads V_C by 90°.

Two of the ILD activities conducted in 2010 involved written and graphical descriptions of leading and lagging. Although 90% of students could identify the correct written description of the phase difference, only 70% could graph it correctly, with **the most common error being misrepresenting leading as lagging**, or vice versa. The most related question on the pre- and post-test showed the least improvement of all the questions.

It is conjectured that this misconception confusing leading and lagging is perhaps because students see a graph like Fig. 2 above as a snapshot, in which V_C is 'winning' (i.e. leading) because it is more to the right. Such an inappropriate projection of a mechanical understanding into an electrical context has been seen elsewhere [18].

B. *Adding voltages that are out of phase*

Because of the phase differences between the different components' voltages, **the rms voltage for the entire circuit is not simply the arithmetic sum of the rms voltages** but must be calculated as follows:

$$V_{tot} = \sqrt{(V_R)^2 + (V_L - V_C)^2}\ . \tag{2}$$

In the relevant ILD activity in which students were given the voltages across the various components and had to predict the voltage across the whole circuit, 85% simply, but incorrectly, gave the arithmetic sum. For those students who attended all ILDs there was an average normalised gain of 0.38 for the relevant question on the pre- and post-test. The post-test score on this question for this group was only 43%, whereas the pre-test score was only 14%, comparable to chance (20%).

V. Phasors and the Phasor Demonstrator

Phasors are a representation of sinusoidal functions of constant amplitude, frequency, and phase, and can be visualised as a vector rotating counter-clockwise in the complex plane. The projection of the vector onto the real axis represents the measurable value of the function at a particular point in time. In RLC circuits the voltages across the different components vary with the same frequency, and so phasors are particularly useful in describing behavior in such circuits. For example in Fig. 3, it is straightforward to recognise that the magnitudes of V_R and V_C are identical and that V_R leads V_C by 90°:

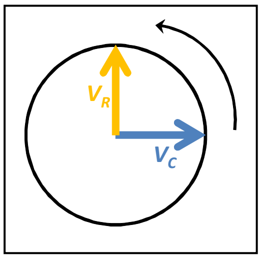

Figure 3. Phasor representation of V_R leading V_C by 90°.

Some simple geometry (Fig. 4) shows that the voltage measured across the two components lags V_R by 45° and has a magnitude given by:

$$V_{RC} = \sqrt{(V_R)^2 + (V_C)^2}. \tag{3}$$

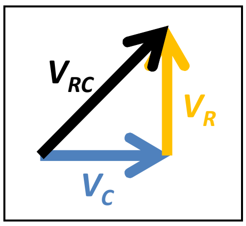

Figure 4. Adding phasors

To address the misconceptions of confusing leading with lagging, and that AC series component voltages (or phasor magnitudes) do not sum arithmetically, a physical model of phasors was developed. A white metal disc driven by a small electric motor was mounted on a motion trolley (see Fig. 5).

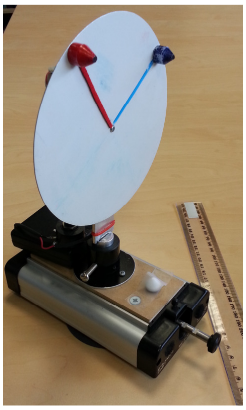

Figure 5. The phasor demonstrator

Coloured magnets are attached to the white disc to represent the actual phasors. In the ILD activities, students were presented with the apparatus and asked to make predictions about the vertical component of the displacement of the different coloured dots over time. The vertical component of the displacement of the phasor represents the sinusoidal variation with time of the voltage. Using the vertical component facilitated measurement and direct generation of a graph. Observations were made using video tracking software and footage of the phasor demonstrator rolling through the frame from left to right (see the screen shot in Fig. 6 below). The approximately constant velocity of the device meant that horizontal displacement was a proxy measure for time, and so the 'graph' could be observed empirically, direct from the footage.

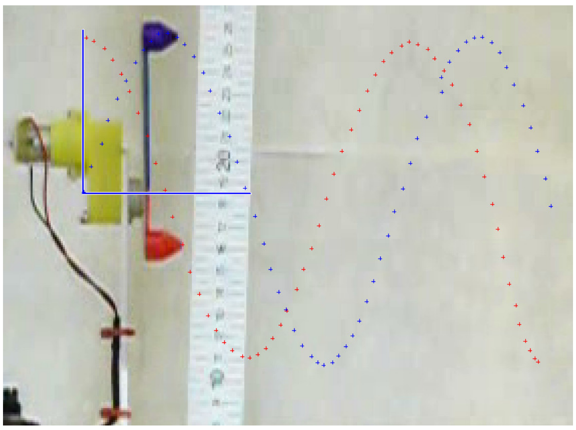

Figure 6. Observation of vertical displacement of the coloured magnets

In total, five new ILD activities were introduced in 2011 using the phasor demonstrator, in which student were asked to make predictions and observations about graphical representations, leading versus lagging, and adding phasors.

VI. Evaluation of Revised 2011 Resonance ILDs

A. *Were the two prominent misconceptions addressed?*

Different ILDs addressed the two misconceptions (see Table I).

TABLE I. CORRECT STUDENT PREDICTIONS AFTER DISCUSSION

Concept:	**ILD**	**Students identify the:**	**Correct (%)**
Describing graphs of leading and lagging correctly	1.1*	Graph of blue dot's vertical displacement	89
	1.2*	Graph of red dot's vertical displacement	90†
	1.3*	Correct description of the relative phase of the dots' motion, given the graphs	94†
Adding voltages correctly, with regard to phase	1.4*	Graph representing the sum of the two dots' vertical displacement	93
	1.5*	Algebraic expression for the magnitude of the sum of two phasors	23
	1.6	Algebraic expression for the magnitude of the sum of three phasors	46

* New ILDs in 2011

† When more than 80% of students made a correct initial prediction, we skipped the discussion phase and went straight to the observation.

After observing the graphs of the blue and red dots' vertical displacement, and their relative motion on the rotating white disc, almost all students were able to correctly describe their phase relationship (i.e. ILD1.3: Red leads Blue by 90°). They could then recognise the graph representing the sum of these two functions (ILD1.4), but struggled in recognizing the correct expression for its amplitude (ILD1.5). This improved somewhat after discussion, with twice the number of students making the correct prediction for the ostensibly more difficult question with three phasors in ILD1.6.

Two particular questions on the pre- and post-test measured students' understanding of these two concepts. Students' performance is shown below in Table II. Of the 57 students that attended the pre-test and above ILDs in 2011, only 20 subsequently also attended the post-test. The scores for this subset are given in parentheses.

TABLE II. PERFORMANCE ON PRE- TO POST-TEST

	Most relevant concept question	
	Stating that instantaneous voltages sum arithmetically, and that rms voltages do not	***Identifying the phase difference from a graph of two sinusoidal functions***
Pre-test (N = 57)	*16% (20%)*	*65% (75%)*
Post-test (N = 43)	*20% (15%)*	*85% (85%)*
g	*0.05 (-0.06)*	*0.57 (0.4)*

Although the ILDs seemed to help students understand how leading and lagging are represented graphically, misconceptions around how phasors add were persistent.

This may be because of the nature of the ILDs. The observations for ILDs 1.1-1.4 were all made empirically. Objective measurements were made of the position of the red and blue dots in each frame. Collectively these were used to construct graphs that clearly matched only one of the response options, without having to invoke the authority of the lecturer. Likewise, for ILD1.3, the counterclockwise motion of the disc and the dots' relative position allows for a simple observation of which one is leading.

However, for ILDs 1.5 and 1.6, the 'observation' of the correct algebraic expression was instead the lecturer *asserting* that the vectors could be added head-to-tail using Pythagoras' Theorem. Moving the arrows around on the board and invoking Pythagoras still somewhat relies on the authority of the lecturer to convince the students of the correct answer. The challenge therefore is to design ILD activities wherein all the predictions are validated through direct, unambiguous, empirical observation.

B. Overall

A physical model to represent phasors was developed and used in the ILDs. Other ILDs in the sequence focused on predictions and measurements of an actual RLC circuit.

A baseline test was administered to students prior to traditional instruction. Pre- and post-testing was used to infer student conceptual change. Student responses were tracked anonymously using codes.

Traditional instruction had no effect on the distribution of student responses to the three questions given before and after the traditional lectures (N = 80/57). The average normalised gain (g) was 0.01. However after the ILDs, there was a marked improvement on the 8-question test ($g = 0.26$) for the 12 students who attended both ILD sessions.

One confound however was attendance [19], with only 5 students (out of an enrolment of 148 students) attending all four sessions (baseline / pre-test & ILD 1 / ILD 2 / post-test).

VII. CONCLUSIONS

Preliminary analysis suggests that misconceptions about how leading and lagging are depicted graphically, and about how voltages with different phase add, are persistent. Overall, however, it is clear that this revised set of ILDs is much more effective than traditional instruction in improving student conceptual understanding of phasors and AC resonance.

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