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Research Paper

The value-at-risk of time-series momentum and contrarian trading strategies

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ABSTRACT

This paper not only provides a theoretical model for the value-at-risk of active and passive trading strategies but also discusses the substantial implications relevant to risk management. Our results suggest that, first, passive strategies are riskier than active trading strategies based on historical returns, such as momentum and contrarian strategies. Second, momentum (contrarian) trading is riskier in a bull (bear) market. Third, the value-at-risk of momentum (contrarian) strategies has a positive relation to the absolute value of the return autocorrelation, as well as a positive (negative) relation with the state of the market. Further, momentum trading strategies give a superior risk-adjusted performance compared with other strategies in international stock markets.

Keywords: value-at-risk; momentum; contrarian; risk management; return autocorrelation.

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1 INTRODUCTION AND RELATED LITERATURE

The importance of historical-return-based trading strategies, such as momentum and contrarian strategies, cannot be overestimated (see Grinblatt and Moskowitz 2004; Chordia and Shivakumar 2006; Sadka 2006; Zhu and Zhou 2009; Hou *et al* 2011; Novy-Marx 2012; Fama and French 2012; Moskowitz *et al* 2012; Bajgrowicz and Scaillet 2012; Menkhoff *et al* 2012). These trading strategies can be understood as two sides of the same coin, in that the momentum and contrarian strategies occupy opposite positions depending on the predetermined signals from historical data. However, even though numerous studies have provided theoretical and empirical results for a better understanding of these trading strategies, this research has focused on profit, neglecting the risk of the strategies. Among the few exceptions are Griffin *et al* (2003), who examine the business-cycle risk of momentum and contrarian strategies, while Hong and Satchell (2015) dissect the autocorrelation structure of momentum strategies (ie, moving-average trading rules). Nevertheless, the risk of historical-return-based trading strategies appears to be much understudied, making them a fertile area of research.

A key part of investment strategies is understanding the risk, allowing practitioners in investment companies to manage the risk of their strategies via the investment policy statement (IPS), which is drafted by an investment manager for a client. In particular, for professional investors in capital markets it is critical to have a better understanding of the nature of trading strategies, in order to avoid their positions leading them to experience serious losses. For example, following the eruption of the global financial crisis (GFC) in 2007, momentum strategies experienced huge losses, as the loser decile earned 163% while the winner decile earned 8% (Daniel and Moskowitz 2016). Our research provides researchers and practitioners with a theoretical tool to understand and manage this kind of economic phenomenon that affects market participants implementing trading strategies based on historical returns.

This paper primarily focuses on value-at-risk (VaR) as a risk measure because it is the most common and pervasive measure among financial institutions such as insurance and investment firms for understanding and managing the risk inherent in their portfolios and estimating the amount of assets that could be lost. Under the Basel III Accord, financial institutions are required to use conditional VaR (CVaR) as a risk metric; most financial institutions still use a VaR modeling approach for risk management and informed decision making because it is simple and straightforward.

One important element of this paper is its focus on the time-series momentum (TSM) rather than the cross-sectional momentum (CSM). The former is based on the time series of asset returns, while the latter is based on the cross section of asset returns. Traditional momentum studies have focused on the CSM; nonetheless, some previous studies (see, for example, Barberis *et al* 1998; Daniel *et al* 1998; Hong and Stein 1999) have examined a single risky asset, and therefore their implications are directly related to the time-series predictability, rather than the cross-sectional predictability. Likewise, rational theories of momentum (see, for example, Berk *et al* 1999; Ahn *et al* 2003; Sagi and Seasholes 2007; Liu and Zhang 2008) have also provided results pertaining to a single risky asset. In a more recent empirical study, Moskowitz *et al* (2012) investigate 58 liquid instruments (made up of indexes for stock markets, currencies, commodities and bond futures) and show that the TSM exists across all these asset classes.

This paper constructs a theoretical model to investigate the VaR of the following well-known trading strategies: buy and hold (BAH), long-only momentum (LOM), long-only contrarian (LOC), long–short momentum (LSM) and long–short contrarian (LSC). Under the assumption that returns on the underlying asset follow the first-order autoregressive (AR(1)) process, VaR equations are derived for the trading strategies considered. The respective VaR equations are standardized using a weighting variable that indicates the position of the LOM strategy. Further, this paper assesses the relation between the risk of the strategies and both the autocorrelation and the position variable.

Our theoretical results are as follows.

(1) The BAH strategy is riskier than the other trading strategies.

(2) The LOM (LOC) strategy is riskier than the LOC (LOM) strategy when the likelihood of asset returns being positive is high (low).

(3) The respective VaRs have a positive (negative) relation with return autocorrelation when the return autocorrelation is positive (negative).

Our theoretical findings provide an explanation for observations from real-world financial markets, as mentioned above. Our evidence implies that momentum strategies can be riskier either when investors overreact to the information reaching the market or when the likelihood of asset returns being positive increases. This explanation accords with the empirical results in Daniel and Moskowitz (2016), which show that crashes in momentum profits emerge when assets recover from crashes in panic states.

Based on the theoretical results, we propose hypotheses and test them using returns on indexes for 18 developed and 13 emerging stock markets. This investigation finds that some of the hypotheses are supported. Further, we extend our research to the examination of the risk-adjusted performance of trading strategies based on past returns. The most impressive empirical finding is that the momentum-based active strategies achieve both higher expected return and lower VaR, ie, higher mean-to-VaR ratio, than other trading strategies. This indicates that momentum trading can enhance the risk-adjusted performance of investment strategies.

This paper presents important findings on an aspect of the comparison of passive and active investment strategies. Numerous previous studies have contributed to the interesting debate over active versus passive investing.

On the one hand, there is a stream of empirical studies supporting the dominance of passive investing over active investing. Using the four-factor model (consisting of market, size, value and momentum factors), Carhart (1997) examines persistence in mutual fund performance and shows that expenses, turnover and load fees have a negative effect on mutual fund performance. It is well known that beating a market index such as the Standard & Poor's 500 (S&P 500), which is the most used benchmark for passive strategies, has been challenging. Previous studies show that only a small number of mutual funds succeed in outperforming the S&P 500; for example, Sorensen *et al* (1998) show that 11% of mutual funds succeeded in outperforming the S&P 500 in 1997. Further, Gruber (1996), examining the period from 1985 to 1994, shows that mutual funds earned returns that were 65 basis points lower than market indexes on average. Recent literature provides similar findings; for example, Rompotis (2009) uses metrics such as Sharpe and Treynor ratios to show that exchange-traded funds (ETFs) with active strategies earn inferior returns compared with those with passive strategies.

On the other hand, another strand of previous studies propose that active investing delivers superior returns compared with passive investing. Wermers (2000) examines the determinants of mutual fund performance to see whether active investing adds value and concludes that turnover has a positive effect on performance, and a fraction of this effect is due to managers' stock-selection skills. Similarly, Grinblatt and Titman (1989, 1993) and Wermers (1997) show that managers of mutual funds are equipped with skills to outperform their benchmark. Pástor and Stambaugh (2002) show that their sample of 503 mutual funds fails to offer a Sharpe ratio close to that of the Fama–French benchmark; nevertheless, active mutual funds perform better than passive mutual funds in replicating the performance of the Fama–French benchmark. However, the focus of the previous studies has been limited to the return or the risk-adjusted return, which has led to a lack of understanding of the risk.

This paper aims to satisfy the need for a proper theoretical tool to assess the risk of both active and passive trading strategies based on the general risk metric. The contributions of this paper are twofold. First, we build a theoretical model for the VaR of the popular trading strategies. Second, we discuss the implications for professionals evaluating the risk of their trading strategies. Our model standardizes VaR equations using a common basis that indicates the position of the LOM strategy. From the standardized VaR equations, this paper derives practical and testable implications for risk management, eg, the relation between VaR and parameters such as the return autocorrelation and the likelihood of the past return being positive. Our findings suggest that these parameters should play an important role in planning risk management strategies.

This paper is organized as follows. Section 2 describes the theoretical model regarding the underlying asset and trading strategies. Section 3 compares the VaRs for passive and active trading strategies, and assesses this relation with the return autocorrelation and with the likelihood of the past return being positive. Section 4 tests the theoretical findings presented in Section 3 using data from 18 developed and 13 emerging stock markets. Section 5 investigates the risk-adjusted performance of the trading strategies. Section 6 summarizes this paper and states our conclusions.

2 THEORETICAL MODEL

2.1 Underlying asset

A common observation of trading strategies relates to the unpredictability of asset returns (see, for example, Fama 1965; Samuelson 1965). An alternate observation (ie, asset returns are predictable using past returns) can also emerge and may suggest profitable trading strategies (see, for example, De Bondt and Thaler 1985, 1987; Jegadeesh 1990; Lehmann 1990; Jegadeesh and Titman 1993; Moskowitz *et al* 2012). Since the focus of this paper lies in examining VaR for trading strategies that are derived from the fact that we can predict asset returns using historical returns, we assume that the return on the underlying asset follows an AR(1) process:

$$r_t = \rho r_{t-1} + \varepsilon_t, \tag{2.1}$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. The autocorrelation coefficient ρ in (2.1) indicates the correlation of the current period's return with the past period's return. The autocorrelation in (2.1) does not need to be limited to the first order, as shown by previous studies (see, for example, De Bondt and Thaler 1985; Lehmann 1990; Lo and MacKinlay 1990; Jegadeesh 1990; Jegadeesh and Titman 1993; Moskowitz *et al* 2012). However, this paper limits its scope to the first-order return autocorrelation to take advantage of a simple theoretical set-up that could provide straightforward intuitions. Examining the effect of higher-order autocorrelation could be an interesting expansion of our work.

One interesting modification that can be made in (2.1) could be to allow the variance of the error term to change over time, as in well-known stochastic variance models (see, for example, Engle 1982; Bollerslev 1986). However, the aim of this paper is to compare the risks of different trading strategies in the cross section using the VaR as a risk measure, and thus this modification is not expected to deliver further meaningful implications. Therefore, this paper assumes that the variance of the error term in (2.1) is constant.

TABLE 1 Positions of the trading strategies considered in this paper.

Strategy	Position +1	Position 0	Position −1
Buy and hold	Always	N/A	N/A
Long-only momentum	$r_{t-1} > 0$	$r_{t-1} < 0$	N/A
Long-only contrarian	$r_{t-1} < 0$	$r_{t-1} > 0$	N/A
Long–short momentum	$r_{t-1} > 0$	N/A	$r_{t-1} < 0$
Long–short contrarian	$r_{t-1} < 0$	N/A	$r_{t-1} > 0$

N/A, not applicable.

2.2 Trading strategies

This paper considers trading strategies that are popular among institutional investors, ie, BAH, LOM, LOC, LSM and LSC.

Table 1 summarizes implementation details of the abovementioned trading strategies. The BAH strategy longs the underlying asset and holds it without conducting timing strategies. Hence, BAH is a passive strategy. Other strategies conduct timing strategies based on the past return. The LSM (LSC) strategy longs (shorts) the underlying asset if the past return is positive and shorts (longs) the underlying asset if the past return is negative. The LOM (LOC) strategy is similar to the LSM (LSC) strategy except for not taking a short position.

DEFINITION 2.1 The positions of the trading strategies, α_t^{LOM}, α_t^{LOC}, α_t^{LSM} and α_t^{LSC}, are defined as follows:

$$\alpha_t^{\text{LOM}} = \begin{cases} 1 & \text{if } r_{t-1} > 0, \\ 0 & \text{if } r_{t-1} < 0, \end{cases} \qquad \alpha_t^{\text{LOC}} = \begin{cases} 1 & \text{if } r_{t-1} < 0, \\ 0 & \text{if } r_{t-1} > 0, \end{cases}$$

$$\alpha_t^{\text{LSM}} = \begin{cases} 1 & \text{if } r_{t-1} > 0, \\ -1 & \text{if } r_{t-1} < 0, \end{cases} \qquad \alpha_t^{\text{LSC}} = \begin{cases} 1 & \text{if } r_{t-1} < 0, \\ -1 & \text{if } r_{t-1} > 0. \end{cases}$$

The expected value of α_t^{LOM} can be interpreted as the likelihood of the past return being positive. For example, $E(\alpha_t^{\text{LOM}}) = 0.3$ implies that the likelihood of a positive past return is 30%. Therefore, we can employ $E(\alpha_t^{\text{LOM}})$ as a measure for market states, eg, when the market is bullish, $E(\alpha_t^{\text{LOM}})$ might be greater than 0.5, whereas when the market is bearish, $E(\alpha_t^{\text{LOM}})$ might be less than 0.5. This interpretation is more plausible when the return autocorrelation in (2.1) is positive.

TABLE 2 Expected values of the positions with respect to varying levels of $E(\alpha_t^{\text{LOM}})$.

	0.00	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**1.00**
$E(\alpha_t^{\text{LOC}})$	1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.00
$E(\alpha_t^{\text{LSM}})$	–1.00	–0.80	–0.60	–0.40	–0.20	0.00	0.20	0.40	0.60	0.80	1.00
$E(\alpha_t^{\text{LSC}})$	1.00	0.80	0.60	0.40	0.20	0.00	–0.20	–0.40	–0.60	–0.80	–1.00

The table shows the relation between the expected values of the positions for the LOC, LSM and LSC trading strategies when the position of the LOM strategy increases from 0 to 1 in increments of 0.10. The results presented in the table are based on the closed-form solution.

TABLE 3 Expected values of the positions with respect to varying levels of the return autocorrelation, ρ.

	–0.99	**–0.50**	**0**	**0.50**	**0.99**
$E(\alpha_t^{\text{LOM}})$	0.4990	0.4998	0.4981	0.4989	0.4996
$E(\alpha_t^{\text{LOC}})$	–0.4990	–0.4998	–0.4981	–0.4989	–0.4996
$E(\alpha_t^{\text{LSM}})$	0.0004	0.0076	–0.0088	–0.0012	0.0059
$E(\alpha_t^{\text{LSC}})$	–0.0004	–0.0076	0.0088	0.0012	–0.0059

The results are based on 500 sets of simulated samples from 500 time periods.

REMARK 2.2 The position α_t^{LOM} defines the other positions, α_t^{LOC}, α_t^{LSM} and α_t^{LSC}, as

$$\left.\begin{aligned} \alpha_t^{\text{LOC}} &= 1 - \alpha_t^{\text{LOM}}, \\ \alpha_t^{\text{LSM}} &= 2\alpha_t^{\text{LOM}} - 1, \\ \alpha_t^{\text{LSC}} &= 1 - 2\alpha_t^{\text{LOM}}. \end{aligned}\right\} \tag{2.2}$$

PROOF The proof of Remark 2.2 is given in Appendix A online. □

Remark 2.2 demonstrates the relation between the four position variables. Table 2 presents the expected values of α_t^{LOC}, α_t^{LSM} and α_t^{LSC} given α_t^{LOM} from 0 to 1 in increments of 0.10.

2.3 $E(\alpha_t)$ and ρ

REMARK 2.3 $E(\alpha_t)$ is insensitive to the first-order return autocorrelation, ρ, where $\alpha_t = (\alpha_t^{\text{LOM}}, \alpha_t^{\text{LOC}}, \alpha_t^{\text{LSM}}, \alpha_t^{\text{LSC}})$.

As noted in Hong and Satchell (2015), it is not practical to derive the closed-form solution of the position variables. Therefore, this paper shows that $E(\alpha_t)$ is insensitive to ρ with simulated results in Table 3. 500 sets of AR(1) processes are generated in which each set has 500 time periods. For each value of ρ, the results of the 500 sets of simulations are cumulated to examine the relation between $E(\alpha_t)$ and ρ.

The positions are independent of the level of the return autocorrelation of the underlying asset, ρ, as shown in Remark 2.3 and Table 3. The results indicate that the distributional form of the error term in (2.1), rather than the value of ρ, determines the value of $E(\alpha_t)$. This has an important implication for examining VaRs of trading strategies: when examining the effect of ρ and $E(\alpha_t)$ on VaR, the interrelation between ρ and $E(\alpha_t)$ can be ignored. This result keeps our research more straightforward.

3 THEORETICAL RESULTS

3.1 VaR

This subsection shows the closed-form solutions of the VaRs for the trading strategies described in Section 2 under the assumption that returns on the underlying asset follow the AR(1) process. VaRs of the trading strategies are expressed with the likelihood of the past return being positive to standardize VaR equations. Such standardization is crucial in our research in order to compare VaRs of the various trading strategies and, in turn, understand the risk of the strategies.

REMARK 3.1 Assuming that $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, the VaR of the return on the trading strategy, r^i, can be written as

$$\mathrm{VaR}^i = -E(r^i) + q\sigma^i, \tag{3.1}$$

where i denotes the different trading strategies and q is the number of standard deviations away from the mean for the given confidence level. VaR is generally positive, although it is indeed a potential loss, and this paper follows that convention.

PROPOSITION 3.2 *The VaRs of the trading strategies can be written in terms of* $E(\alpha_t^{\mathrm{LOM}})$ *as*

$$\mathrm{VaR}^{\mathrm{BAH}} = q\sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}}, \tag{3.2}$$

$$\mathrm{VaR}^{\mathrm{LOM}} = q|E(\alpha_t^{\mathrm{LOM}})|\sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}}, \tag{3.3}$$

$$\mathrm{VaR}^{\mathrm{LOC}} = q|1 - E(\alpha_t^{\mathrm{LOM}})|\sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}}, \tag{3.4}$$

$$\mathrm{VaR}^{\mathrm{LSM}} = \mathrm{VaR}^{\mathrm{LSC}} = q|2E(\alpha_t^{\mathrm{LOM}}) - 1|\sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}}. \tag{3.5}$$

PROOF The proof of Proposition 3.2 is given in Appendix B online. □

It is important to note that this paper postulates that the position at time t is equal to its ex ante expected position, assuming that the return distribution is stable over time. It is reasonable to impose this restriction as the purpose of our model is to compare the risks of active and passive trading strategies. Further, this set-up enables our model to deliver more straightforward and intuitive suggestions while ensuring it does not violate the statistical properties of AR(1) processes; however, this approach overlooks the stochastic dependence between α_t and r_t.

Because the LSM and LSC strategies take exactly the opposite positions and the distribution of the error term is symmetric, the VaRs of these trading strategies are expected to be equal, as shown in (3.5). This is a model-specific result, which is less meaningful, and therefore should not receive too much attention. On the other hand, because the LOM and LOC strategies have asymmetric positions, VaRs of these trading strategies are expected to not be equal, as shown in (3.3) and (3.4). For this reason, this paper focuses on results for the LOM and LOC strategies as well as the BAH strategy. Hence, our results have implications that are more related to mutual funds that are prevented from taking short positions in general.

PROPOSITION 3.3 *The relative magnitudes of the VaRs of the trading strategies are as follows:*

$$\mathrm{VaR}^{\mathrm{BAH}} \geqslant \mathrm{VaR}^{\mathrm{LOC}} > \mathrm{VaR}^{\mathrm{LSM}} = \mathrm{VaR}^{\mathrm{LSC}} > \mathrm{VaR}^{\mathrm{LOM}}, \quad 0 \leqslant E(\alpha_t^{\mathrm{LOM}}) < \tfrac{1}{3}, \tag{3.6}$$
$$\mathrm{VaR}^{\mathrm{BAH}} > \mathrm{VaR}^{\mathrm{LOC}} > \mathrm{VaR}^{\mathrm{LOM}} \geqslant \mathrm{VaR}^{\mathrm{LSM}} = \mathrm{VaR}^{\mathrm{LSC}}, \quad \tfrac{1}{3} \leqslant E(\alpha_t^{\mathrm{LOM}}) < \tfrac{1}{2}, \tag{3.7}$$
$$\mathrm{VaR}^{\mathrm{BAH}} > \mathrm{VaR}^{\mathrm{LOM}} \geqslant \mathrm{VaR}^{\mathrm{LOC}} > \mathrm{VaR}^{\mathrm{LSM}} = \mathrm{VaR}^{\mathrm{LSC}}, \quad \tfrac{1}{2} \leqslant E(\alpha_t^{\mathrm{LOM}}) < \tfrac{2}{3}, \tag{3.8}$$
$$\mathrm{VaR}^{\mathrm{BAH}} \geqslant \mathrm{VaR}^{\mathrm{LOM}} > \mathrm{VaR}^{\mathrm{LSM}} = \mathrm{VaR}^{\mathrm{LSC}} \geqslant \mathrm{VaR}^{\mathrm{LOC}}, \quad \tfrac{2}{3} \leqslant E(\alpha_t^{\mathrm{LOM}}) \leqslant 1. \tag{3.9}$$

PROOF The proof of Proposition 3.3 is given in Appendix C online. □

Proposition 3.3 has two important implications for practitioners in risk management. First, the BAH strategy has the highest VaR of all the strategies, which means that timing the market with contrarian or momentum strategies could lower the risk compared with passive investing. This is a rather surprising result, as there are numerous investors who have preferred passive strategies to active strategies since ETFs became available.

Second, when the likelihood of the past return being positive is less than 0.5 (ie, in a bear market) the LOC strategy is riskier than the LOM strategy, whereas when the likelihood of the past return being positive is greater than 0.5 (ie, in a bull market) the LOM strategy is riskier than the LOC strategy. It is common for traders to use momentum-related strategies in a bull market and contrarian-related strategies in a bear market to take advantage of expected movements in capital markets stemming from investor sentiment and thus achieve superior returns. However, our results could be an indication that such enhanced performance is indeed a consequence of increased risk rather than of sentiment-based irrational behavior. This indicates that risk managers should be cautious in implementing such trading strategies, as the superior return and increased risk come hand in hand. This implication is consistent with the risk–return trade-off in classic finance theories.

3.2 VaR and ρ

The VaRs from (3.2)–(3.5) are sensitive to two parameters: the return autocorrelation, ρ, and the likelihood of the past return being positive, $E(\alpha_t^{\mathrm{LOM}})$. Since the parameters are independent of each other, as shown in Section 2.3, the relations between VaR and the two parameters are examined in this subsection and the next without considering the effect of the dependence between the parameters.

This subsection examines the relation between VaR and ρ.

PROPOSITION 3.4 *The sensitivities of the VaRs of the trading strategies with respect to ρ can be written as*

$$\frac{\partial \mathrm{VaR}^{\mathrm{BAH}}}{\partial \rho} = q\rho\sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}}\frac{1}{1-\rho^2}, \tag{3.10}$$

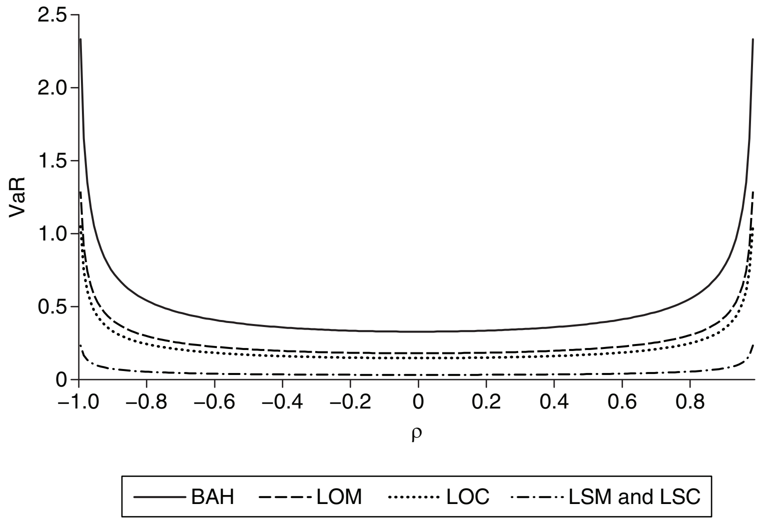
$$\frac{\partial \mathrm{VaR}^{\mathrm{LOM}}}{\partial \rho} = q\rho|E(\alpha_t^{\mathrm{LOM}})|\sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}}\frac{1}{1-\rho^2}, \tag{3.11}$$

$$\frac{\partial \mathrm{VaR}^{\mathrm{LOC}}}{\partial \rho} = q\rho|1-E(\alpha_t^{\mathrm{LOM}})|\sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}}\frac{1}{1-\rho^2}, \tag{3.12}$$

$$\frac{\partial \mathrm{VaR}^{\mathrm{LSM}}}{\partial \rho} = \frac{\partial \mathrm{VaR}^{\mathrm{LSC}}}{\partial \rho} = q\rho|2E(\alpha_t^{\mathrm{LOM}})-1|\sqrt{\frac{\sigma_\varepsilon^2}{1-\rho^2}}\frac{1}{1-\rho^2}. \tag{3.13}$$

Proposition 3.4 indicates that the VaRs have a positive relation to ρ when $0 < \rho < 1$ (indicating initial underreaction) and a negative relation to ρ when $-1 < \rho < 0$ (indicating initial overreaction). This finding accords with Hong and Satchell (2015) and has a significant consequence for risk management. The association revealed between VaR and ρ implies that VaR can be inflated (deflated) as the current return becomes more (less) correlated to the past return. In the light of behavioral studies, our result indicates that irrational reaction to arriving news can influence both the return and the risk. Therefore, Proposition 3.4 suggests that VaR estimates should be interpreted with care if statistically significant return autocorrelation exists, and that strategies for risk management should be sensitive to the level of the return autocorrelation.

FIGURE 1 The sensitivity of VaR with respect to ρ.



The graph shows the VaRs of the trading strategies given the values of the return autocorrelation, ρ. We assume that $q = 1.65$, $\sigma_{\varepsilon}^2 = 0.04$ and $E(\alpha_t^{\mathrm{LOM}}) = 0.55$.

Figure 1 visualizes the levels of the VaRs for the trading strategies against the level of the return autocorrelation. The figure implies that VaRs of the trading strategies are negatively related to ρ when $-1 < \rho < 0$, and positively related to ρ when $0 < \rho < 1$ (ie, the risk of the trading strategies increases as the absolute value of the return autocorrelation increases).

3.3 VaR and $E(\alpha_t^{\mathrm{LOM}})$

This subsection investigates the relation between VaR and the other parameter, $E(\alpha_t^{\mathrm{LOM}})$.

PROPOSITION 3.5 *The sensitivities of the VaRs of the trading strategies with respect to* $E(\alpha_t^{\mathrm{LOM}})$ *can be written as*

$$\frac{\partial \mathrm{VaR}^{\mathrm{BAH}}}{\partial E(\alpha_t^{\mathrm{LOM}})} = 0, \tag{3.14}$$

$$\frac{\partial \mathrm{VaR}^{\mathrm{LOM}}}{\partial E(\alpha_t^{\mathrm{LOM}})} = q\frac{E(\alpha_t^{\mathrm{LOM}})}{|E(\alpha_t^{\mathrm{LOM}})|}\sqrt{\frac{\sigma_{\varepsilon}^2}{1-\rho^2}}, \tag{3.15}$$

$$\frac{\partial \text{VaR}^{\text{LOC}}}{\partial E(\alpha_t^{\text{LOM}})} = -q \frac{1 - E(\alpha_t^{\text{LOM}})}{|1 - E(\alpha_t^{\text{LOM}})|} \sqrt{\frac{\sigma_\varepsilon^2}{1 - \rho^2}}, \tag{3.16}$$

$$\frac{\partial \text{VaR}^{\text{LSM}}}{\partial E(\alpha_t^{\text{LOM}})} = \frac{\partial \text{VaR}^{\text{LSC}}}{\partial E(\alpha_t^{\text{LOM}})} = q2 \frac{2E(\alpha_t^{\text{LOM}}) - 1}{|2E(\alpha_t^{\text{LOM}}) - 1|} \sqrt{\frac{\sigma_\varepsilon^2}{1 - \rho^2}}. \tag{3.17}$$

Proposition 3.5 has interesting implications for the risk of trading strategies based on historical returns. First, (3.14) indicates that VaR of the BAH strategy is not associated with $E(\alpha_t^{\text{LOM}})$. This is unsurprising because, by definition, the BAH strategy holds the underlying asset regardless of the sign of the past return.

Second, assuming that $E(\alpha_t^{\text{LOM}}) \neq 0$, (3.15) shows that VaR of the LOM strategy has a positive relation with $E(\alpha_t^{\text{LOM}})$. In contrast, assuming that $E(\alpha_t^{\text{LOM}}) \neq 1$, (3.16) implies that VaR of the LOC strategy has a negative relation with $E(\alpha_t^{\text{LOM}})$. This implication provides a further explanation for the second implication from Proposition 3.3: the risk of the LOM (LOC) strategy increases as $E(\alpha_t^{\text{LOM}})$ increases and thus the LOM (LOC) is riskier than the LOC (LOM) when $E(\alpha_t^{\text{LOM}})$ is greater (less) than 0.5. Third, the association of the VaRs of the LSM and LSC strategies with $E(\alpha_t^{\text{LOM}})$ is negative when $E(\alpha_t^{\text{LOM}})$ is less than 0.5 and positive when $E(\alpha_t^{\text{LOM}})$ is greater than 0.5. This finding suggests that the LSM and LSC strategies become riskier as $E(\alpha_t^{\text{LOM}})$ increases when $E(\alpha_t^{\text{LOM}})$ is greater than 0.5 (ie, in a bull market) and as $E(\alpha_t^{\text{LOM}})$ decreases when $E(\alpha_t^{\text{LOM}})$ is less than 0.5 (ie, in a bear market). This also confirms the second implication from Proposition 3.3.

Figure 2 summarizes the implications from Proposition 3.5. The figure shows that

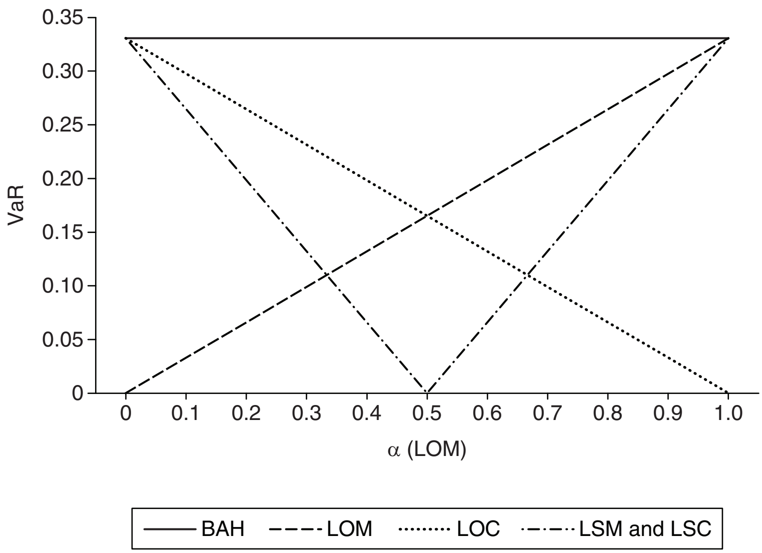
(1) the VaR of the BAH strategy is insensitive to $E(\alpha_t^{\text{LOM}})$,

(2) the LOM (LOC) strategy becomes riskier as $E(\alpha_t^{\text{LOM}})$ increases (decreases) and

(3) the LSM and LSC strategies become riskier as $E(\alpha_t^{\text{LOM}})$ increases (decreases) when $E(\alpha_t^{\text{LOM}})$ is greater (less) than 0.5.

Overall, our theoretical results from Propositions 3.3–3.5 provide interesting explanations for momentum crashes. Daniel and Moskowitz (2016) showed that momentum strategies experienced catastrophic failure in 2009, as the loser decile earned 163% and the winner decile earned 8%, and that momentum crashes occur when the market rebounds from sharp decreases. In this paper we describe this phenomenon using two parameters, ρ and $E(\alpha_t^{\text{LOM}})$: market rebounds can be characterized as the negative serial correlation, ie, $\rho < 0$, and, following the collapse in markets, the likelihood of the past return being positive increases as the market recovers, eg, $E(\alpha_t^{\text{LOM}}) > 0.5$. Our theoretical results in this section imply that trading strategies become riskier as ρ decreases from 0 and that momentum trading

FIGURE 2 The sensitivity of VaR with respect to $E(\alpha_t^{\text{LOM}})$.



The graph plots the VaRs of the trading strategies given the values of the likelihood of the past return being positive, $E(\alpha_t^{\text{LOM}})$. We assume that $q = 1.65$, $\sigma_\varepsilon^2 = 0.04$ and $\rho = 0.10$.

strategies in particular become riskier than other strategies when $E(\alpha_t^{\text{LOM}})$ is higher than, for instance, 0.7. In other words, our model suggests that the capital market circumstances around the GFC predict the momentum crashes described in Daniel and Moskowitz (2016), assuming that a fraction of the CSM is attributed to the TSM. The limitation of our model is that the same explanation can be applied to contrarian strategies when considering strategies with short selling, and therefore differentiating between momentum and contrarian long–short strategies will be an interesting development of our model in future research.

4 EMPIRICAL TESTS

4.1 Hypothesis development

This section tests the theoretical findings from Propositions 3.3–3.5 that have a practical importance to risk management. Several testable and practical hypotheses are established based on the implications from the theoretical results.

Table 4 lists the established hypotheses. The first set of hypotheses (H1A, H1B and H1C) concerns the relative magnitude of the VaRs of the trading strategies. H1A tests whether timing the market reduces the risk of the BAH strategy. H1B (H1C)

TABLE 4 Established hypotheses.

H1A	The BAH strategy has the highest level of VaR
H1B	The LOC strategy has a higher level of VaR than the LOM strategy if $E(\alpha_t^{\mathrm{LOM}})$ is less than 0.5
H1C	The LOM strategy has a higher level of VaR than the LOC strategy if $E(\alpha_t^{\mathrm{LOM}})$ is greater than 0.5
H2A	VaRs of the trading strategies have a positive relation with ρ if ρ is positive
H2B	VaRs of the trading strategies have a negative relation with ρ if ρ is negative
H3A	VaR of the BAH strategy is not related to $E(\alpha_t^{\mathrm{LOM}})$
H3B	VaR of the LOM strategy has a positive relation with $E(\alpha_t^{\mathrm{LOM}})$
H3C	VaR of the LOC strategy has a negative relation with $E(\alpha_t^{\mathrm{LOM}})$

The table presents established sets of hypotheses based on the theoretical findings from Propositions 3.3–3.5. The first set of hypotheses (ie, H1A, H1B and H1C) are derived from Proposition 3.3. The second set of hypotheses (ie, H2A and H2B) are derived from Proposition 3.4. The third set of hypotheses (ie, H3A, H3B and H3C) are derived from Proposition 3.5.

examines whether the LOM (LOC) strategy becomes riskier when $E(\alpha_t^{\mathrm{LOM}})$ is less (greater) than 0.5.

The second set of hypotheses concerns the relation between the VaRs of the trading strategies and ρ. H2A and H2B test whether the trading strategies become riskier as the absolute value of ρ approaches 1.

The third set of hypotheses concerns the relation between the VaRs of the trading strategies and $E(\alpha_t^{\mathrm{LOM}})$. H3A tests whether the BAH strategy is insensitive to $E(\alpha_t^{\mathrm{LOM}})$. H3B and H3C test whether the risk of the LOM (LOC) strategy has a positive (negative) relation to $E(\alpha_t^{\mathrm{LOM}})$.

4.2 Data

This paper examines the theoretical findings using data on international stock markets in order to provide evidence not restricted to a particular market and to retain a sufficient number of observations, and thus chooses 18 developed markets (Australia (AUS), Austria (AUT), Belgium (BEL), Switzerland (CHE), Germany (DEU), Denmark (DNK), Spain (ESP), Finland (FIN), France (FRA), the United Kingdom (GBR), Israel (ISR), Italy (ITA), Japan (JPN), the Netherlands (NLD), New Zealand (NZL), Portugal (PRT), Sweden (SWE) and the United States (USA)) and 13 emerging markets (Brazil (BRA), China (CHN), Czechia (CZE), Greece (GRC), Hungary (HUN), Indonesia (IDN), Ireland (IRL), South Korea (KOR), Mexico (MEX), Poland (POL), Russia (RUS), Turkey (TUR) and South Africa (ZAF)).

For the 31 selected markets, we obtain Morgan Stanley Capital International (MSCI) daily price indexes from Datastream and compute the log return for each market index. It is important to note that this paper treats the indexes as US dollar denominated in order to consider the perspective of US investors.

Table 5 reports descriptive statistics for the markets in the sample. The beginning date of the sample varies across the markets and the sample ends in July 2019 for all the markets. First, the table shows that developed markets earn higher expected returns with lower standard deviations than emerging markets on average. This contradicts the principle of the traditional risk–return trade-off. Second, the table shows that the return autocorrelation associated with the weak form of the efficient market hypothesis tends to be positive, except for the case of the Netherlands, and the level of the return autocorrelation appears to be higher in emerging markets than developed markets in general.

For conducting empirical tests, it is necessary to calculate the VaR as well as the parameters in our theoretical model. The most efficient method to test the theoretical results is perhaps to use the historical VaR, which is simple and accurate. To determine the VaR for each trading plan, we carry out the following steps:

(1) Compute returns based on the implementation details illustrated in Table 1.

(2) Reorganize actual historical returns in order from the worst to the best to construct the return distribution.

(3) Take the percentile value from the distribution as the historical VaR.

$E(\alpha_t^{\mathrm{LOM}})$ is calculated based on Definition 2.1, ie, we calculate the average of α_t^{LOM}, which is designed to take a value of 1 if r_{t-1} is greater than 0 and to take 0 otherwise. ρ is the regression coefficient obtained from the regression of r_t over r_{t-1}.

4.3 First set of hypotheses

Because Proposition 3.3 indicates that the relative magnitude of the VaRs of the trading strategies depends on the value of $E(\alpha_t^{\mathrm{LOM}})$, which can be interpreted as the state of the market, this paper divides the whole sample into two subsamples according to the value of $\bar{\alpha}_t^{\mathrm{LOM}}$. $\bar{\alpha}_t^{\mathrm{LOM}}$ is estimated at the end of each December with a 12-month look-back period. Then, we calculate the historical VaR using actual historical returns. This section uses 0.5 as a breakpoint, as considering other breakpoints in Proposition 3.3 is not realistic.

Table 6 presents the 1% historical VaRs of the trading strategies for the 31 developed and emerging markets included in our sample. First, the table shows that, even though the LSC strategy is riskiest, the BAH strategy appears to be riskier than the LOM and LOC strategies. This indicates that timing the market based on historical

TABLE 5 Descriptive statistics. [Table continues on next page.]

(a) Developed markets

Market	Start date	Return Mean (%)	Return SD (%)	ρ
AUS	Jan 5, 1970	4.17	21.34	0.07
AUT	Jan 2, 1970	4.76	21.42	0.08
BEL	Jan 2, 1970	5.45	19.22	0.06
CHE	Jan 2, 1970	8.16	17.98	0.03
DEU	Jan 2, 1970	5.90	21.55	0.01
DNK	Jan 2, 1970	9.15	19.07	0.05
ESP	Jan 2, 1970	2.76	22.38	0.07
FIN	Jan 4, 1982	9.54	29.33	0.02
FRA	Jan 2, 1970	5.84	21.43	0.05
GBR	Jan 2, 1970	4.72	20.69	0.05
ISR	Jan 4, 1993	2.23	22.02	0.03
ITA	Jan 2, 1970	1.88	24.26	0.07
JPN	Jan 2, 1970	6.89	20.68	0.00
NLD	Jan 2, 1970	6.95	20.21	−0.01
NZL	Jan 4, 1982	4.19	23.55	0.02
PRT	Jan 4, 1988	−1.39	21.08	0.11
SWE	Jan 2, 1970	8.41	23.83	0.05
USA	Jan 2, 1970	6.68	16.87	0.00
Mean	—	5.35	21.49	0.04
SD	—	2.81	2.74	0.03

returns reduces the risk of trading strategies and thus suggests that investors adhering to passive investment strategies should time the market based on historical returns. In practice, the active trading strategies can be implemented using ETFs. This paper concludes that this observation supports H1A in part. Further, the table shows that the LOC strategy is riskier than the LOM strategy regardless of the value of $\bar{\alpha}_t^{\mathrm{LOM}}$. This is consistent with H1B but not with H1C.

4.4 Second and third sets of hypotheses

Figure 1 shows that the relation between VaR and ρ is almost linear when $-0.2 \leqslant \rho \leqslant 0.2$, which is consistent with the normal range for ρ in market index returns. Similarly, Figure 2 shows that the relation between VaR and $E(\alpha_t^{\mathrm{LOM}})$ is linear (depending on the range of values of $E(\alpha_t^{\mathrm{LOM}})$ in the LSM and LSC strategies).

TABLE 5 Continued

(b) Emerging markets

Market	Start date	Return: Mean (%)	Return: SD (%)	ρ
BRA	Jan 4, 1988	9.51	40.79	0.11
CHN	Jan 4, 1993	−1.10	29.38	0.11
CZE	Jan 3, 1995	4.01	25.86	0.08
GRC	Jan 4, 1988	−4.71	35.74	0.09
HUN	Jan 3, 1995	7.77	33.76	0.07
IDN	Jan 4, 1988	6.68	38.01	0.16
IRL	Jan 4, 1988	1.73	24.79	0.04
KOR	Jan 4, 1988	4.32	33.28	0.06
MEX	Jan 4, 1988	11.91	28.56	0.11
POL	Jan 4, 1993	6.69	33.89	0.10
RUS	Jan 3, 1995	7.77	44.75	0.09
TUR	Jan 4, 1988	2.63	46.40	0.09
ZAF	Jan 4, 1993	5.47	27.91	0.06
Mean	—	4.82	34.09	0.09
SD	—	4.46	6.92	0.03

The table presents descriptive statistics for returns on indexes for 31 markets consisting of 18 developed and 13 emerging markets. US-dollar-denominated MSCI daily price indexes are used to compute log returns. Means and standard deviations (SD) are annualized by multiplying by 260 and the square root of 260, respectively. MSCI daily price indexes are obtained from Datastream. The sample period ends in July 2019 for all the markets in the sample. ρ is the first-order autocorrelation coefficient for returns obtained from the estimation of the autoregressive model.

Therefore, assuming that each relation is linear, to test the second and third sets of hypotheses the following regression equation is run:

$$\begin{aligned}\mathrm{VaR}_t^i &= a^i + b^i \rho_t D_t^{\rho_t>0} + c^i \rho_t D_t^{\rho_t<0} \\ &\quad + d^i \bar{\alpha}_t^{\mathrm{LOM}} D_t^{\bar{\alpha}_t^{\mathrm{LOM}}>0.5} + e^i \bar{\alpha}_t^{\mathrm{LOM}} D_t^{\bar{\alpha}_t^{\mathrm{LOM}}<0.5} + u_t^i, \end{aligned} \tag{4.1}$$

where t indexes years, i indexes trading strategies, VaR_t^i is the 1% historical VaR, $\bar{\alpha}_t^{\mathrm{LOM}}$ is the likelihood of the past return being positive and ρ_t is the return autocorrelation. The regression equation includes dummy variables because Propositions 3.4 and 3.5 suggest that the direction of the relation between VaR and each parameter depends on the value of the parameter. $D_t^{\rho_t>0}$ and $D_t^{\rho_t<0}$ indicate that ρ_t is greater or less than 0, respectively, and

$$D_t^{\bar{\alpha}_t^{\mathrm{LOM}}>0.5} \quad \text{and} \quad D_t^{\bar{\alpha}_t^{\mathrm{LOM}}<0.5}$$

TABLE 6 Test of the first set of hypotheses. [Table continues on next page.]

(a) Developed markets

	$\bar{\alpha}_t^{\mathrm{LOM}} < 0.5$					$\bar{\alpha}_t^{\mathrm{LOM}} > 0.5$				
Market	**BAH**	**LOM**	**LOC**	**LSM**	**LSC**	**BAH**	**LOM**	**LOC**	**LSM**	**LSC**
AUS	3.81	2.50	2.97	3.63	4.05	3.45	2.46	3.04	3.09	3.39
AUT	4.06	2.50	3.45	3.54	4.21	4.03	2.71	3.45	3.54	4.03
BEL	3.78	2.47	3.24	3.37	3.93	3.06	2.22	2.64	2.90	3.24
CHE	3.35	2.33	2.71	3.19	3.56	2.92	2.22	2.43	2.64	2.93
DEU	4.66	3.17	3.67	4.27	4.49	3.31	2.56	2.76	3.17	3.30
DNK	4.17	2.80	3.27	3.63	3.92	3.07	2.35	2.59	2.92	3.28
ESP	4.31	2.95	3.47	4.20	4.26	3.32	2.47	2.88	3.12	3.49
FIN	6.03	4.05	5.13	6.48	6.04	3.87	2.76	3.36	3.73	4.25
FRA	4.43	2.88	3.82	3.91	4.41	3.37	2.56	2.83	3.19	3.33
GBR	3.91	2.58	3.19	3.87	3.90	3.33	2.46	2.83	3.16	3.42
ISR	4.25	3.33	3.74	3.71	4.52	4.02	2.74	3.04	3.49	3.95
ITA	4.67	3.10	3.98	4.09	4.55	3.92	2.86	3.26	3.41	4.01
JPN	3.79	2.90	3.24	3.58	4.03	3.03	2.43	2.48	3.11	3.14
NLD	4.61	3.00	3.74	4.03	4.21	2.99	2.31	2.45	2.90	3.03
NZL	3.90	2.82	3.22	4.68	4.73	3.33	2.43	2.99	3.05	3.53
PRT	3.90	2.79	3.76	3.43	4.11	3.26	2.28	2.88	3.01	3.61
SWE	5.18	3.49	4.41	4.82	5.34	3.73	2.66	3.11	3.43	3.86
USA	3.16	2.20	2.64	3.13	3.32	2.59	1.99	2.05	2.48	2.59
Mean	4.22	2.88	3.54	3.98	4.31	3.37	2.47	2.84	3.13	3.46
SD	0.67	0.45	0.59	0.78	0.62	0.41	0.22	0.36	0.31	0.43

TABLE 6 Continued.

(b) Emerging markets

	$\bar{\alpha}_t^{\text{LOM}} < 0.5$					$\bar{\alpha}_t^{\text{LOM}} > 0.5$				
Market	**BAH**	**LOM**	**LOC**	**LSM**	**LSC**	**BAH**	**LOM**	**LOC**	**LSM**	**LSC**
BRA	7.81	4.39	6.72	6.31	8.03	6.44	4.61	5.65	6.06	6.83
CHN	5.20	3.35	4.65	4.98	6.12	4.68	3.48	4.10	4.59	5.20
CZE	5.14	3.55	4.35	4.78	5.02	4.16	2.88	3.62	3.36	4.20
GRC	6.60	5.10	5.40	6.48	6.59	5.89	3.82	5.47	5.23	6.22
HUN	5.89	3.83	4.80	5.73	5.90	5.82	3.81	5.15	5.10	6.49
IDN	7.65	4.40	6.65	6.06	8.60	4.94	3.15	4.45	4.47	6.42
IRL	5.33	3.60	4.56	4.58	5.13	3.78	2.56	2.99	3.56	3.78
KOR	6.57	4.45	5.29	5.84	7.28	4.83	3.72	4.17	4.53	5.29
MEX	6.04	3.80	5.44	5.22	6.58	4.08	3.06	3.53	3.81	4.54
POL	7.02	4.83	5.27	6.14	6.34	5.49	3.65	4.94	5.32	6.07
RUS	10.56	6.26	9.59	8.43	12.52	6.48	4.86	5.92	6.04	7.52
TUR	8.86	6.41	7.72	8.14	9.02	7.57	5.32	6.41	6.75	7.60
ZAF	6.04	3.93	5.39	5.16	6.04	4.37	3.35	3.84	4.11	4.31
Mean	6.82	4.45	5.83	5.99	7.17	5.27	3.71	4.63	4.84	5.73
SD	1.57	0.98	1.49	1.18	2.02	1.13	0.80	1.04	1.03	1.27

The table presents the 1% historical VaRs of the trading strategies. We split the whole sample into the two subsamples based on the likelihood of the past return being positive, $\bar{\alpha}_t^{\text{LOM}}$, which is estimated at the end of each December with a 12-month look-back period. Then, to determine the percent return distribution for each sample, we put actual historical returns in order from the worst to the best. The given percentile value is defined as the historical VaR.

TABLE 7 Test of the second and third sets of hypotheses.

(a) Developed markets

	a	*b*	*c*	*d*	*e*
VaR^{BAH}	3.11 (0.00)	**−1.47** **(0.04)**	1.88 (0.17)	0.12 (0.84)	1.67 (0.02)
VaR^{LOM}	−0.29 (0.00)	**−2.98** **(0.00)**	0.56 (0.61)	**5.26** **(0.00)**	**7.04** **(0.00)**
VaR^{LOC}	−0.24 (0.06)	−0.55 (0.42)	**3.27** **(0.01)**	**5.52** **(0.00)**	**7.87** **(0.00)**
VaR^{LSM}	4.49 (0.00)	**−4.09** **(0.00)**	0.57 (0.66)	−2.57 (0.01)	−1.46 (0.19)
VaR^{LSC}	4.40 (0.00)	−0.17 (0.82)	**3.29** **(0.01)**	−2.21 (0.02)	−1.08 (0.35)

(b) Emerging markets

	a	*b*	*c*	*d*	*e*
VaR^{BAH}	17.42 (0.00)	2.93 (0.15)	2.44 (0.70)	−23.53 (0.00)	−25.25 (0.00)
VaR^{LOM}	10.95 (0.00)	−1.42 (0.24)	2.38 (0.55)	**−13.40** **(0.00)**	**−14.28** **(0.00)**
VaR^{LOC}	16.35 (0.00)	**3.25** **(0.07)**	3.73 (0.46)	**−22.57** **(0.00)**	**−24.31** **(0.00)**
VaR^{LSM}	15.35 (0.00)	−0.72 (0.65)	1.16 (0.83)	−19.87 (0.00)	−21.48 (0.00)
VaR^{LSC}	16.82 (0.00)	**6.50** **(0.01)**	2.56 (0.69)	−22.68 (0.00)	−24.24 (0.00)

The table reports parameter estimates obtained from regressions of the VaRs of the trading strategies on the return autocorrelation, ρ_t, and the likelihood of the past return being positive, $\bar{\alpha}_t^{\text{LOM}}$ given by (4.1), where t indexes years, i indexes trading strategies, VaR_t^i is the 1% historical VaR, ρ_t is the return autocorrelation and $\bar{\alpha}_t^{\text{LOM}}$ is the likelihood of the past return being positive. VaR_t^i, ρ_t and $\bar{\alpha}_t^{\text{LOM}}$ are estimated at the end of each December with a 12-month look-back period. Dummy variables are incorporated into the regression as Propositions 3.4 and 3.5 indicate that the direction of the relation between the VaRs of the trading strategies and the parameters, ρ and $E(\alpha_t^{\text{LOM}})$, depends on the value of each parameter.

indicate that $\bar{\alpha}_t^{\text{LOM}}$ is greater or less than 0.5, respectively. VaR_t^i, ρ_t and $\bar{\alpha}_t^{\text{LOM}}$ are estimated at the end of each December with a 12-month look-back period.

Table 7 presents the regressions of the VaRs of the trading strategies. Part (b) shows that the coefficient estimates $\hat{b}^{\text{LOC}}$ and $\hat{b}^{\text{LSC}}$ are positive and significant at the 10% significance level. This suggests that VaRs of the LOC and LSC strategies have a positive relation with the return autocorrelation in emerging markets, which is consistent with H2A.

In contrast, part (a) shows that coefficient estimates $\hat{b}^{\mathrm{BAH}}$, $\hat{b}^{\mathrm{LOM}}$ and $\hat{b}^{\mathrm{LSM}}$ are negative and significant at the 5% significance level, indicating that VaRs of the BAH, LOM and LSM strategies have a negative association with the return autocorrelation. This result contradicts H2A. The difference in the revealed empirical findings between developed and emerging markets might be because emerging markets are more inefficient, as indicated by the higher return autocorrelation.

H2B appears to be rejected as the coefficient estimates, $\hat{c}^{i}$, are positive for VaRs of all the trading strategies in both developed and emerging markets. Overall, H2A is partially supported in emerging markets but not in developed markets, and H2B is rejected in both developed and emerging markets.

Second, VaR of the BAH strategy appears to be related to the likelihood of the past return being positive in both developed and emerging markets, and therefore H3A is rejected. Except for the case of $\hat{d}^{\mathrm{BAH}}$ in developed markets, the coefficient estimates $\hat{d}^{\mathrm{BAH}}$ and $\hat{e}^{\mathrm{BAH}}$ are significant at the 5% significance level. Further, it turns out that VaRs of the LOM and LOC strategies are positively (negatively) related to the likelihood of the past return being positive in developed (emerging) markets. The table shows that the coefficient estimates $\hat{d}^{\mathrm{BAH}}$ and $\hat{e}^{\mathrm{BAH}}$ are positive (negative) in developed (emerging) markets and both are statistically significant at the 1% significance level. Therefore, H3B is supported in developed markets and H3C is supported in emerging markets.

5 RISK-ADJUSTED PERFORMANCE

This paper has investigated passive and active trading strategies from a risk manager's perspective. Now, this section examines the risk-adjusted performance of the trading strategies considered in this research to provide researchers and practitioners with more comprehensive implications for investment strategies. To measure the risk-adjusted performance of the trading strategies, we calculate the ratio of the mean of returns to the 1% historical VaR for each trading plan.

Table 8 reports the mean-to-VaR ratios as well as means and 1% historical VaRs for the trading strategies. First, we observe that LOM and LSM strategies outperform the BAH strategies on average across markets. This is rather surprising because our findings indicate that momentum-based active trading strategies achieve superior performance. If the scope is narrowed to momentum strategies, our results indicate that timing the market using historical returns enables traders to enhance their profit and reduce their risk at the same time. Regarding contrarian-based trading strategies, their mean-to-VaR ratios tend to be negative or close to zero, resulting from the negative autocorrelation for market indexes.

Second, the mean-to-VaR ratios for the LOM strategy are analogous to those for the LSM strategy in developed markets on average, whereas the mean-to-VaR

TABLE 8 Risk-adjusted performance of the trading strategies. [Table continues on next page.]

(a) Developed markets

Market	Mean (%)					1% VaR (%)					Mean-to-VaR ratio				
	BAH	LOM	LOC	LSM	LSC	BAH	LOM	LOC	LSM	LSC	BAH	LOM	LOC	LSM	LSC
AUS	4.17	16.19	−11.74	28.22	−27.64	57.27	39.76	48.76	52.91	57.02	0.07	0.41	−0.24	0.53	−0.48
AUT	4.76	11.92	−7.79	19.08	−20.35	65.45	42.48	55.64	57.13	65.64	0.07	0.28	−0.14	0.33	−0.31
BEL	5.45	10.38	−5.89	15.31	−17.23	54.99	37.27	46.31	48.53	55.67	0.10	0.28	−0.13	0.32	−0.31
CHE	8.16	8.56	−0.78	8.97	−9.73	49.25	36.62	40.94	46.79	50.87	0.17	0.23	−0.02	0.19	−0.19
DEU	5.90	7.24	−1.31	8.59	−8.52	61.98	44.63	50.67	56.64	60.52	0.10	0.16	−0.03	0.15	−0.14
DNK	9.15	12.62	−4.35	16.08	−17.85	53.73	40.36	45.28	51.30	54.88	0.17	0.31	−0.10	0.31	−0.33
ESP	2.76	10.98	−9.00	19.20	−20.77	62.45	45.03	51.23	60.09	63.50	0.04	0.24	−0.18	0.32	−0.33
FIN	9.54	10.46	−5.81	11.38	−21.17	84.84	56.96	71.80	81.15	87.81	0.11	0.18	−0.08	0.14	−0.24
FRA	5.84	13.08	−7.01	20.32	−19.85	61.12	43.30	50.22	56.61	60.42	0.10	0.30	−0.14	0.36	−0.33
GBR	4.72	8.02	−4.18	11.31	−13.08	56.02	40.04	47.04	54.34	58.35	0.08	0.20	−0.09	0.21	−0.22
ISR	2.23	11.34	−9.23	20.45	−20.69	66.14	50.02	55.14	57.90	65.51	0.03	0.23	−0.17	0.35	−0.32
ITA	1.88	12.17	−10.54	22.45	−22.97	70.65	48.21	58.89	60.99	70.01	0.03	0.25	−0.18	0.37	−0.33
JPN	6.89	6.34	−0.60	5.79	−8.09	55.93	42.03	46.34	53.90	58.42	0.12	0.15	−0.01	0.11	−0.14
NLD	6.95	5.24	1.32	3.52	−4.31	59.43	41.26	47.29	54.46	57.57	0.12	0.13	0.03	0.06	−0.07
NZL	4.19	9.29	−9.53	14.40	−23.25	60.17	41.47	49.39	57.89	61.79	0.07	0.22	−0.19	0.25	−0.38
PRT	−1.39	14.37	−16.30	30.14	−31.21	59.84	41.21	53.99	52.12	61.37	−0.02	0.35	−0.30	0.58	−0.51
SWE	8.41	17.17	−9.75	25.94	−27.91	69.71	48.25	58.75	63.70	71.76	0.12	0.36	−0.17	0.41	−0.39
USA	6.68	6.06	0.19	5.44	−6.29	45.96	33.56	37.67	45.48	45.36	0.15	0.18	0.01	0.12	−0.14
Mean	5.35	10.64	−6.24	15.92	−17.83	60.83	42.91	50.85	56.22	61.47	0.09	0.25	−0.12	0.28	−0.29
SD	2.81	3.39	4.76	7.93	7.88	8.77	5.45	7.65	7.86	9.13	0.05	0.08	0.09	0.14	0.12

TABLE 8 Continued.

(b) Emerging markets

Market	Mean (%)					1% VaR (%)					Mean-to-VaR ratio				
	BAH	LOM	LOC	LSM	LSC	BAH	LOM	LOC	LSM	LSC	BAH	LOM	LOC	LSM	LSC
BRA	9.51	35.02	−26.44	60.53	−62.38	109.93	74.17	99.65	99.40	120.60	0.09	0.47	−0.27	0.61	−0.52
CHN	−1.10	24.18	−26.86	49.47	−52.62	82.07	54.92	72.79	76.09	90.84	−0.01	0.44	−0.37	0.65	−0.58
CZE	4.01	15.24	−12.27	26.47	−28.55	76.15	49.03	61.52	61.46	74.86	0.05	0.31	−0.20	0.43	−0.38
GRC	−4.71	21.55	−27.45	47.81	−50.20	100.26	71.05	88.20	93.03	104.87	−0.05	0.30	−0.31	0.51	−0.48
HUN	7.77	22.41	−15.16	37.05	−38.09	94.39	61.61	82.01	83.93	99.84	0.08	0.36	−0.18	0.44	−0.38
IDN	6.68	38.01	−33.87	69.34	−74.41	101.68	62.13	89.20	85.63	125.72	0.07	0.61	−0.38	0.81	−0.59
IRL	1.73	8.15	−7.30	14.58	−16.34	74.51	47.46	60.91	67.69	73.56	0.02	0.17	−0.12	0.22	−0.22
KOR	4.32	12.97	−9.73	21.61	−23.79	93.73	65.43	75.96	86.30	103.69	0.05	0.20	−0.13	0.25	−0.23
MEX	11.91	31.67	−20.44	51.42	−52.80	82.76	52.39	68.98	70.39	87.50	0.14	0.60	−0.30	0.73	−0.60
POL	6.69	17.29	−12.09	27.89	−30.86	98.76	67.86	83.77	92.16	102.12	0.07	0.25	−0.14	0.30	−0.30
RUS	7.77	37.38	−31.69	66.99	−71.15	136.87	82.45	109.58	109.95	146.34	0.06	0.45	−0.29	0.61	−0.49
TUR	2.63	29.56	−25.86	56.48	−54.34	135.82	95.11	115.74	117.65	139.69	0.02	0.31	−0.22	0.48	−0.39
ZAF	5.47	16.13	−10.97	26.79	−27.41	74.95	57.70	67.03	69.99	77.41	0.07	0.28	−0.16	0.38	−0.35
Mean	4.82	23.81	−20.01	42.80	−44.84	97.07	64.72	82.72	85.67	103.62	0.05	0.37	−0.24	0.49	−0.42
SD	4.46	9.79	9.14	18.16	18.67	20.75	13.65	17.50	16.79	23.74	0.05	0.14	0.09	0.18	0.13

The table reports the means, 1% historical VaRs and mean-to-VaR ratios for returns on the trading strategies. Means and VaRs are annualized by multiplying by 260 and the square root of 260, respectively. Mean-to-VaR ratio is defined as the ratio of the mean of returns on the given trading plan to the 1% historical VaR of return on the given trading plan.

ratios for the LSM strategy are higher than those for the LOM strategy in emerging markets on average. One potential explanation for this observation is that investors in emerging markets require higher expected risk-adjusted performance on trading strategies that require short selling. In other words, differences in, for example, costs for borrowing securities and supplies of securities between the two markets can generate this cross-sectional variation in expected performance. In other words, differences between the two markets, such as costs for borrowing securities and supplies of securities for lending, can generate this cross-sectional variation in expected performance.

From the seminal TSM research, Moskowitz *et al* (2012), it appears that the autocorrelation structure in market indexes makes momentum trading profitable. The evidence of Moskowitz *et al* then opens an interesting question about how autocorrelation affects the risk-adjusted performance of trading strategies. To answer this question, we investigate the relation between the return autocorrelation and the mean-to-VaR ratio.

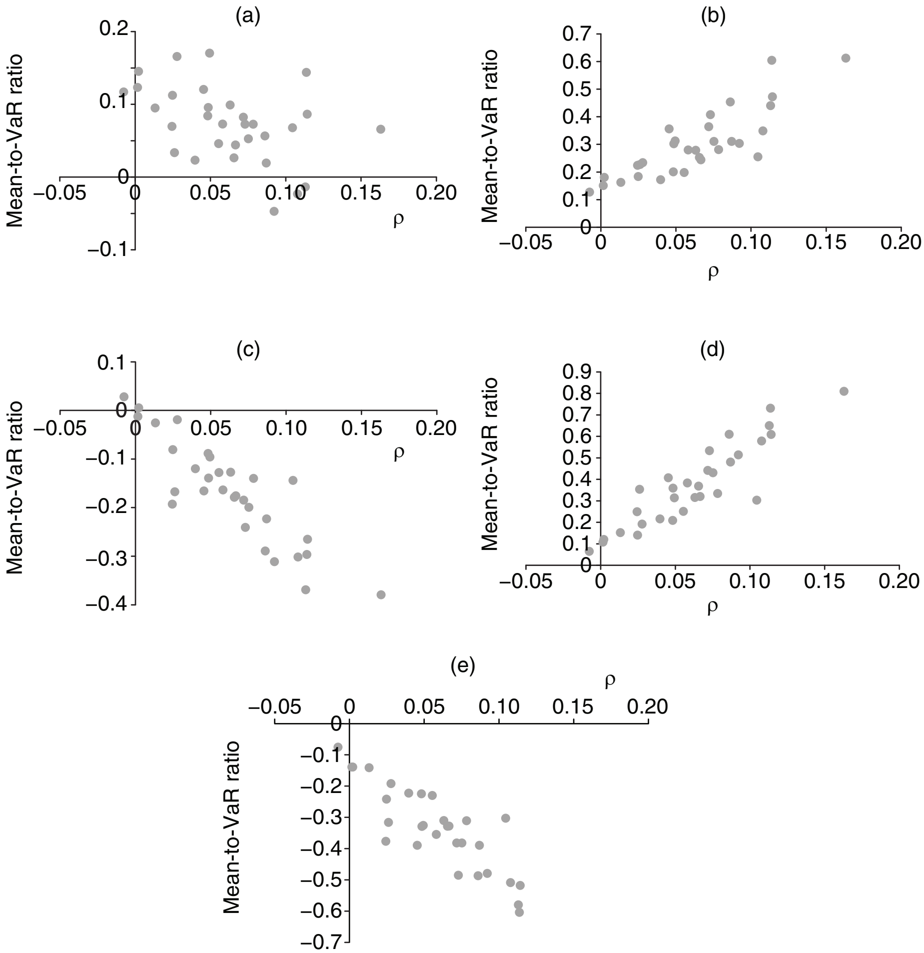
Figure 3 presents scatter plots of mean-to-VaR ratio against the return autocorrelation. The figure shows that the mean-to-VaR ratio of the BAH strategy becomes worse as the autocorrelation increases, indicating that traders in emerging markets have a greater need to time the market in order to improve their performance than traders in developed markets. Further, the figure shows that, as could be expected, the mean-to-VaR ratios of the LOM and LSM strategies (the LOC and LSC strategies) increase (decrease) as the autocorrelation increases. This empirical finding reveals a more immediate relation between the autocorrelation and risk-adjusted momentum returns that contributes to the existing TSM studies, eg, Moskowitz *et al* (2012), and that demonstrates that the return autocorrelation is associated with is associated with the mean-to-VaR ratio.

6 CONCLUSION

Our theoretical and empirical findings differ from other studies concerning the risk and return for passive and active trading strategies and provide the following insights.

First, momentum-based active trading strategies could dominate passive trading strategies, ie, it is reasonable for investors to earn higher expected return at lower risk with momentum-based trading strategies. Second, the risk of momentum trading increases in a bull market, while the risk of contrarian trading increases in a bear market. Third, VaR increases with the absolute value of the return autocorrelation. This finding makes sense intuitively but, to the best of our knowledge, has not been examined explicitly in the literature. Further, VaR is related to the likelihood of the past return being positive, which can be interpreted as the state of the

FIGURE 3 Autocorrelation versus mean-to-VaR ratio.



(a) BAH. (b) LOM. (c) LOC. (d) LSM. (e) LSC. The figures are scatter plots of Sharpe ratios against the return autocorrelations on the trading strategies.

market. Specifically, the results reveal a positive (negative) relation for momentum (contrarian) strategies. This confirms the second theoretical finding above.

The theoretical model we derive has several implications.

(1) The level of VaR of passive strategies is higher than that of active strategies. This finding implies that timing the market with momentum or contrarian strategies can provide a lower VaR level compared with passive investing.

(2) Under a bear market (in other words, when the likelihood of the past return being positive is lower than 0.5), contrarian strategies are riskier than momentum strategies. This result could be an indication that the higher expected return for momentum (contrarian) trading in a bull (bear) market is indeed at the expense of taking higher risk. This implies that risk managers should be cautious in assessing the performance of such trading results, as the higher return achieved result from taking a greater risk instead of taking advantage of irrational investors.

(3) VaR has a positive association with the first-order return autocorrelation when the return autocorrelation is positive (indicating initial underreaction), and similarly, a negative association with the first-order return autocorrelation when the return autocorrelation is negative (indicating initial overreaction). This finding implies that VaR can be inflated (deflated) as the current return is more (less) correlated to the past return, and hence VaR estimates should be interpreted with caution according to the autocorrelation structure.

Using the daily stock marker indexes for 18 developed and 13 emerging markets, we performed an empirical analysis of our theoretical findings. We established eight hypotheses from the theoretical findings and report empirical findings that support some of these hypotheses. In addition, we compared the risk-adjusted performance of passive and active trading strategies to provide researchers and practitioners with suggestions related to the return as well as the risk and found the following: first, that momentum strategies enable investors to achieve better risk-adjusted performance; and second, that the risk-adjusted performance of momentum strategies improves as the return autocorrelation increases.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

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