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Application of Mutation Operators to Salp Swarm Algorithm

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Abstract

Salp swarm algorithm (SSA) based on the swarming behaviour of salps found in ocean, is a very competitive algorithm and has proved its worth as an excellent problem optimizer. Though SSA is a very challenging algorithm but it suffers from the problem of poor exploitation, local optima stagnation and unbalanced exploration and exploitation operations. Thus in order to mitigate these problems and improve the working properties, seven new versions of SSA are proposed in present work. All the new versions employ new set of mutation properties along with some common properties. The common properties of all the algorithms include division of generations, adaptive switching and adaptive population strategy. Overall, the proposed algorithms are self-adaptive in nature along with some added mutation properties. For performance evaluation, the proposed algorithms are subjected to variable initial population and dimension sizes. The best among the proposed is then tested on CEC 2005, CEC 2015 benchmark problems and real world problems from CEC 2011 benchmarks. Experimental and statistical results show that the proposed mutation clock SSA (MSSA) is best among all the algorithms under comparison.

Keywords: Salp swarm algorithm, mutation operators, adaptive properties, benchmark problems, nature inspired algorithms.

1. Introduction

Nature has always served as motivation for problem solving and has been continuously doing so since ages. In present age, various natural phenomena known as algorithms have been integrated to design nature based algorithms to solve various real world optimization problems at hand (Gandomi et al., 2013). These algorithms have been divided into two types namely evolutionary algorithms and swarm intelligent algorithms. Evolutionary algorithms were first formulated in the early 1970's and genetic algorithm (GA) was the first algorithm under this category (Goldberg & Holland, 1988). Other algorithms under this category include biogeography based algorithm (BBO) (Simon, 2008), differential evolution (DE) (Storn & Price, 1997), ant lion algorithm (ALO) (Mirjalili, 2015), dragonfly algorithm (DA) (Mirjalili, 2016a) and others. Swarm intelligent algorithms on the other hand are based on the swarming behavior of different species (Yang et al., 2013) and major algorithms under this category include bat algorithm (BA) (Yang, 2010), flower pollination algorithm (FPA) (Yang, 2012), particle swarm optimization (PSO) (Kennedy, 2010), ant colony

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optimization (ACO) (Dorigo & Birattari, 2010), artificial bee colony (ABC) (Karaboga & Basturk, 2007), firefly algorithm (FA) (Yang, 2009), grey wolf optimization (GWO) (Mirjalili et al., 2014), krill-herd algorithm (KHA) (Gandomi & Alavi, 2012), sine-cosine crow search algorithm (SCCSA) (Khalilpourazari & Pasandideh, 2019), hybrid particle swarm optimization and gravitational search algorithm (PSOGSA) (Mirjalili & Hashim, 2010), naked mole-rat algorithm (NMRA) (Salgotra & Singh, Salgotra & Singh) and others.

Salp swarm algorithm (SSA) (Mirjalili et al., 2017) is one among the recently introduced swarm intelligent algorithm. The algorithm is highly competitive and is based on the swarming behavior of salp species found in oceans. It consists of several stochastic operators which make it feasible to use for various optimization domains. Salps have the same structure as those of jelly fishes for food foraging, in moving from one position to another and also to pump water through their body. In SSA, the salps form salp chains which has a leader and is followed by the follower salps. As far as the parameters are concerned, the algorithm consists of only one controlling parameter which helps in maintaining a balance between the exploration and exploitation. Due to the limited use of training parameters, the SSA is found to be highly effective and has been applied to a large number of real world engineering design problems.

As far as, the recent literature is concerned, SSA has been applied to various different optimization problems. These include image segmentation of different models of fishes (Ibrahim et al., 2018), feature selection problem (Ibrahim et al., 2017), optimization of tariff plans in electrical power systems (Khalid et al., 2018), training of feed forward neural networks (Abusnaina et al., 2018), optimization of cost values in thermal, wind and hydro power plants (Das et al., 2018), parametric optimization of fuel cells (El-Fergany, 2018) and others. Apart from the application portion, various modified versions have been implemented including memetic theory based SSA (Yang et al., 2019), mathematical simplex method inspired SSA (Wang et al., 2018), multiple leader selection based SSA (Bairathi & Gopalani, 2019), binary based SSA for application to discrete optimization problems (Mirjalili et al., 2017) and others. Note that a detailed discussion on the literature has been presented in subsequent section.

Though SSA is a highly competitive algorithm and has been found to be effective for various real world applications, it suffers from the problem of local optima stagnation and hence poor exploration operation (Chen et al., 2018). The algorithm also suffers from unbalanced exploration and exploitation operation and hence has a slow convergence (Qais et al., 2019). On exploiting the basics of SSA, it can be seen that it has only one parameter to be optimized and any small change in the basic parameter makes the algorithm switch from exploration to exploitation and vice verse. Thus making it vulnerable to various problems including poor convergence and local optima stagnation. The general equation of SSA is also based on the lower and upper bounds of the problem only and doesn't include any neighbouring solutions. The use of neighbouring solutions make the algorithm more exploratory and hence helps the algorithm in moving towards particular direction. Another disadvantage of using lower and upper bound based solution update is that no information about the previous solutions is kept in the next generation. Thus providing weak exploration operation. Overall, it can be concluded that SSA suffers from various problems and more work is required to be done in order to improve its working capability.

In present work, different aspects of SSA are exploited by using seven new mutation operators to improve its overall performance. Seven different mutation operators have been identified and implemented to make the algorithm more capable in exploring and exploiting the search space. The following modifications have been added to basic SSA:

- The concept of division of iterations is followed to perform two operations. Firstly, derive new exploration equations so as to reduce the effect of random upper and lower bound and improve the exploration capabilities. Secondly, addition of a new phase based on the concepts of GWO and CS algorithms to improve the exploitation operation and add a balance between the exploration and exploitation operation.
- In order to reduce the required number of function evaluations, a linear population size reduction scheme is followed.
- Seven different mutation operators have been identified and linked to basic SSA to analyze the effect of these operators to SSA and improve overall performance. Based on these, seven new versions of SSA are proposed.
- Apart from all the above said modifications, exponential decreasing SSA parameter c_1, is also used to balance the exploration and exploitation operations.

Based on the above discussed points, SSA is modified and seven new versions are proposed. The newly proposed algorithms for performance evaluation are tested on CEC 2005 (Suganthan et al., 2005), CEC 2015 (Liang et al., 2014) benchmark problems and some real world optimization problems (Das & Suganthan, 2010). The algorithms have been subjected to multiple population and dimensional analysis and best among them is compared with the standard state-of-the-art algorithms. Apart from this, statistical tests and convergence profiles are also drawn to prove the significance of proposed algorithm statistically.

The article is divided into 6 sections with introduction in section 1, basics of SSA in section 2 and literature with respect to modified versions and application to real world problems is presented in section 3. Here it should be noted that section 3 specifically details about the various drawbacks of basic SSA and also defines the motivation behind present work. In section 4, new modified versions are proposed using the concepts of mutation operators. Here new mutation operators are proposed and based on them modifications are added in the basic SSA. The fifth section, presents the result and discussion. In this section, details of test functions, parametric details, result with respect to variable population and dimension size, effect of mutations and discussion based on results has been added. Finally, based on the results, conclusions are drawn in section 6 and future scope is highlighted. The outline of major works carried out in the paper is given in Figure 1.

2. Salp Swarm Algorithm

SSA is a recent introduction in the field of swarm intelligent algorithms, based on the navigation and foraging pattern of salps found in ocean. Salps have a body structure similar to jelly fishes and

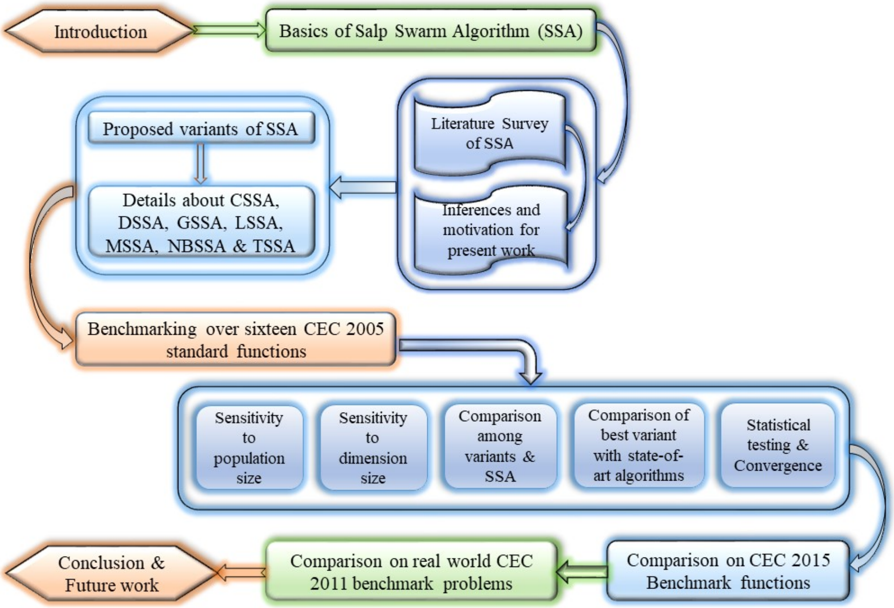

Figure 1: Outline of the paper

follow almost similar patterns for pumping water, moving forward and other tasks. These species live in groups and based on their grouping patterns, SSA has been formulated (Mirjalili et al., 2017). The algorithm uses the concept of salp chains, with leader salp as the first member of the chain and follower salps as the other members. Each salp chain has only one leader and others are the follower salps. Here it should be noted that the leader salp is meant for guiding the follower salps for searching food. The follower salps also follow each other as well in order to remain as a member of the group or salp chain.

The position update for different salps for an n-dimensional search space is represented by the amount of food source S and it is the target vector. The general equation for position update of salps with respect to food sources is given by (1)

$$Y_j^1 = \begin{cases} S_j + c_1((ub_j - lb_j)c_2 + lb_j) & C_3 \geq 0 \\ S_j - c_1((ub_j - lb_j)c_2 + lb_j) & C_3 \leq 0 \end{cases} \tag{1}$$

where Y_j^1 and S_j is the position of first salp and food source in j^{th} dimension respectively, ub_j and lb_j represents the upper and lower boundary of j^{th} dimensional problem, c_1, c_2 and C_3 are the random initialization parameters drawn from uniformly distributed parameters in the range of [0,1]. Here coefficient c_1 is the only parameter which is required to be optimized and serves as the most important factor in deciding the extent of exploration and exploitation in SSA and is given by (2):

$$c_1 = 2\, e^{-(\frac{4i}{I})^2} \tag{2}$$

where i is current iteration and I is the maximum count of iterations. The equation (1) discussed above deals with the position update of leader salps only. For position update with respect to the follower salps, the equation (3) is sued and is given by

$$Y_j^k = \frac{1}{2}\,at^2 + v_0 t \tag{3}$$

where $k \geq 2$, Y_j^k is the position of k^{th} salp with respect to j^{th} dimension, t is time per iteration, v_0 is the initial speed and $a = \frac{v_{final}}{v_0}$. Also $v_0 = 0$ is taken into consideration and hence the equation (3) becomes

$$Y_j^k = \frac{1}{2}(Y_j^k + Y_j^{k-1}) \tag{4}$$

Here it should be noted that equations (1) and (4) are used for updating the positions of leader as well as follower salps respectively. The leader salp is updated in accordance with the food source and follower salps is meant for following it until the maximum number of iterations are reached. The parameter c_1 is decreasing and helps in shifting the algorithm from exploration operation toward exploitation operation over the course of iterations. Thus, maintaining a proper balance between the global and local search and avoiding local optima stagnation problem for finding the correct estimated optimal solution.

3. Literature review

SSA because of its simple structure and limited number of parametric requirements, has been applied to a large number of optimization problems. The algorithm is very promising and fast in optimizing and hence can be considered as one among the emerging standard state-of-the-art algorithms. A lot of work has been done to improve it working capabilities and application to real world optimization problems. A binary version of SSA was proposed by using s-shaped and v-shaped transfer functions in (Faris et al., 2018), chaotic SSA using 10 chaotic maps was proposed in (Sayed et al., 2018) to improve its local optima stagnation problem, chaotic SSA based on quadratic integrate and fire neural model to optimize different functions (Majhi et al., 2019). Inertia weight controlled SSA was proposed in (Hegazy et al., 2018) to control the current best solution. This is meant for improving the reliability, local optima stagnation and switching capabilities. Another modification to convergence factor has been done in (Chen et al., 2018) to improve the switching capabilities of SSA. Gravitational search algorithm (GSA) was hybridized with SSA to avoid premature convergence in (Li et al., 2018). Here GSA was added for exploration operation whereas SSA was meant for exploitation operation. Opposition based concepts were exploited for SSA in (Bairathi & Gopalani, 2018). Here new population is initialized using the concepts of opposition based learning to improve the diversity with in the search agents. In (Wang et al., 2018), simplex method was added in SSA to diversify the population and improve the local search operation. The algorithm proposed was tested over a limited set of benchmark functions and require further investigation. Hybridized PSO-SSA was proposed by changing the position of SSA using the concepts of PSO (Ibrahim et al., 2019). SSA with sine cosine algorithm (Mirjalili, 2016b) was modified in (Singh et al., 2019) for improving its convergence rate. Different transfer function including sigmoid and arctan have also been used to implement the binary version of SSA in (Rizk-Allah et al., 2019). The reason for using these functions was to improve the mobility and multiplicity features in SSA. Controlling parameter

c_1 was remodelled for higher dimensional problems in (Qais et al., 2019). The results presented are extensive but do not pose much variations in the end results and require more investigation. Mutated SSA was proposed in (Rajalaxmi & Vidhya, 2019) to enhance diversity and avoid premature convergence. Weighted SSA was proposed in (Syed & Syed, 2019) by modifying the position using weighted average of best solutions. The modification has been added to improve the convergence properties of SSA. Chaos based SSA using five different chaotic maps was used for feature selection problem in (Hegazy et al., 2019). Dynamic adaptive weights were used by (Wu et al., 2019) to improve the equation update in SSA. The drawback with this approach is that multiplying a weight directly with the basic equation may lead to entrapment of algorithm is some local optima. Multi-leader based SSA was proposed in (Bairathi & Gopalani, 2019) to enhance the exploration and division of single group into multiple salps chains was also followed.

Apart from the various modifications, SSA has been applied to a large set of real world optimization problems. These include analysis of chemical compounds (Hussien et al., 2017), biomedical feature selection (Ibrahim et al., 2017), feature selection using SSA with chaos (Ahmed et al., 2018) and SSA with sine cosine algorithm (Neggaz et al., 2020), polymer exchange in fuel cells (El-Fergany, 2018), in power system stabilizer (Ekinci & Hekimoglu, 2018), segmentation model for fishes (Ibrahim et al., 2018), load forecasting in power systems (Wang et al., 2018), training neural networks for pattern classification (Abusnaina et al., 2018), for renewable distributor generators to minimize the power loss (Tolba et al., 2018), parameter estimation for soil water retention (Zhang et al., 2018), flow shop scheduling problem solving (Sun et al., 2018), frequency control in power systems (Kumari & Shankar, 2018), optimization of CMOS comparator and amplifier circuits (Asaithambi & Rajappa, 2018), in minimizing the cost of wind, thermal and hydro power generation (Das et al., 2018), optimizing tariff for electrical energy systems (Khalid et al., 2018), solid waste management and sewage water management (Barik & Das, 2018), minimizing power loss in distributed electric systems (Reddy & Reddy, Reddy & Reddy), optimizing values for parameter control in automatic PID controller (Mohapatra & Sahu, 2018), for solving economic load dispatch problem in power generation systems (Mallikarjuna et al., 2018), for localization problem solving to optimize time difference of arrival (TDoA) (Liu & Xu, 2018), hybridization with WOA for power generation (Alzaidi et al., 2019), in software defined networking to find the optimum number of controllers and connection between the controllers (Ateya et al., 2019), training of multilayer perceptron for sonar target classification (Khishe & Mohammadi, 2019), extracting parameters of photo voltaic cells (Abbassi et al., 2019), for solving localization problems in wireless sensor nodes (Kanoosh et al., 2019), digital infinite impulse response differentiator and integrator designing (Ali et al., 2019) and others.

3.1. Inferences and motivation behind present work

The above discussed literature poses following major drawbacks of the basic SSA algorithm:

- First of all, the algorithm consists of only one single parameter to be optimized and random values of this parameter make it more of a stagnator. That is providing one single value for the parameter to be optimized makes the algorithm perform exploration for some particular iterations and exploitation for rest of the iterations. It actually switches directly from one operation to another without any proper investigation. So more work is required to be done to improve its switching properties.

- Secondly, the SSA algorithm suffers from the problem of local optima stagnation and has poor convergence properties. It need more time to evaluate new solution and require major modifications before implementation to real world problems.
- Another major drawback is that most of the literature focused only on the basic changes in one or two properties of the basic SSA. Not much work has been done to improve the overall performance of the algorithm.
- Also, adaptive properties is a new technique to adjust the parameters of any algorithm by itself and not much work has been done to provide such operation in SSA.

Based on the points discussed above, the authors have formulated seven new version of SSA. The newly proposed algorithms aims at mitigating the problems with basic SSA. All the new version employ certain similar adaptive parameter and seven different mutation operators. Detailed discussion on the algorithmic part is given in the subsequent sections.

4. The proposed Salp Swarm Algorithms

SSA is highly competitive and has proven its worth for various real world problems. The algorithm is simple, linear and has served as a problem solver since its inception but still suffers from various problems. As pointed out in (Chen et al., 2018), the algorithm suffers from the problem of poor exploration and may lead to local optima stagnation. The algorithm also suffers form other problems such as slow convergence (Qais et al., 2019) and lack a proper balance between the diversification and intensification operation (Li et al., 2018). So in present work, seven new versions of SSA using the concepts of mutation operators are proposed. Each of the proposed algorithm follow certain common modifications and in addition to them, a new mutation operator based operation is also added to test the applicability of these operators to SSA. Here it should be noted that the proposed modifications are new and no such work has been done on improving the performance of SSA. Firstly, let us introduce the basic concepts used in the modified version and then elaborate all the concepts under different sub headings. The following points elaborate the basics of proposed modifications:

- A new concept based on division of generations is followed by using the original SSA equations for first half and new equations using the concepts of GWO and CS for second half of the iterations.
- The linear population size reduction phenomena is followed to reduce the total number of function evaluations required for solving the problem under test.
- Exponential decreasing parameter c_1, is used to balance the extent of exploration and exploitation.

- Cauchy, Gaussian, Lévy , Neighbourhood, Trigonometric and other mutation operators are exploited to improve the performance of SSA.

Based on the above said points, the proposed versions are formulated. Here it should be noted that the concept of division of generations, linear population size reduction and exponential parametric optimization is followed in all the proposed versions and has not been discussed in all the subsections.

Division of generations and exponential parametric setting: Exploration and exploitation are the two basic operations which helps in proper functioning of any optimization algorithm. An algorithm which is found to gradually decent from exploration to exploitation with balanced operations, is considered as an optimal algorithm. SSA algorithm has poor exploitation operation and requires more balanced performance. Here division of generations is followed by using two new concepts as inspired from GWO and CS to improve its overall working capability. For the first half of the generations, general equations of SSA are used whereas for the second half of generations, new equation are formulated using the concepts of GWO and CS. The reason for using equations inspired from GWO and CS is to improve the exploitation operation in SSA. As both GWO and CS algorithm are found to provide highly competitive exploitation operation (Salgotra et al., 2018), it is imperative to use them during final stages due to poor capability of SSA. Whereas in the initial stages, SSA equations are self sufficient and are better at exploration, so are used as such with mutation operators. The new solution using this equations is given by

$$\begin{cases} x_1 = x_i - A_1(C_1.x_{new} - x_i^t); \\ x_2 = x_i - A_2(C_2.x_{new} - x_i^t); \\ x_3 = x_i - A_3(C_3.x_{new} - x_i^t); \\ x_{new}^{t+1} = \frac{x_1+x_2+x_3}{3} \end{cases} \tag{5}$$

where x_{new} is the new solution and $A_1 \neq A_2 \neq A_3$ and $C_1 \neq C_2 \neq C_3$ are generated as given by $A_1 = 2a_1.r_1 - a_1$, $C_1 = 2.r_2$; $A_2 = 2a_2.r_1 - a_2$, $C_2 = 2.r_2$ and $A_3 = 2a_3.r_1 - a_3$, $C_3 = 2.r_2$. Here $a_1 \neq a_2 \neq a_3$ are linearly decreasing random numbers, $r_1 \neq r_2$ are random numbers. The above discussed equation (5) is based on the concepts of GWO. The second part is the integration of GWO with CS and use it in SSA. The equation (5) is subjected to basic global search equation of CS (Yang & Deb, 2010) and new equation is formulated as given in (6)

$$x_{new}^{t+1} = x_{new}^t + \alpha \otimes L(\lambda)(x_{best} - x_{new}^t) \tag{6}$$

where α and $L(\lambda)$ are basic and Lévy distributed random numbers. All other notations have the same significance as given by SSA.

The second step is to use a modified exponential (Chen et al., 2006) parameter c_1 instead of basic c_1. The parametric equation used is given by

$$c_1 = c_{min} + (c_{max} - c_{min}). \exp - \frac{I}{maxgen/10} \tag{7}$$

Here $c_{max} = 0.95$ and $c_{min} = 0.05$, I is the current generation and $maxgen$ is the maximum number of generations.

Linearly decreasing population adaptation: Computational complexity is a very important parameter in any optimization algorithm. With so many new optimization algorithms in use, it is the requirement of time to have minimum computational complex algorithm with maximum stable output. One parameter which makes all the algorithms equally well aligned and computationally less expensive is the population adaptation. Population size is a common factor in all the algorithms and for almost all the cases it is similar. But present case, a linearly decreasing population adaptation is followed. Thus accounting for reduction in the total number of function evaluations required for solving the problem under test. The major aim is to reduce the total computational burden of the algorithm under test. The reduction process in present case if followed in such a way that the minimum population is kept to 4. This is because only 4 minimum number of individuals are required to perform the general operation of SSA. The general equation (8) is given by

$$N(g+1) = round[(\frac{N_{min} - N_{max}}{FEs_{max}}).FEs + N_{max}] \tag{8}$$

where N_{max} and N_{min} are the minimum and maximum population sizes. The above discussed modifications are added in all the proposed version of SSA and are not discussed in all the below subsections. Note that the proposed modifications are added in the equation (6) of the algorithm and each modification is presented in the consecutive subsections.

4.1. Cauchy Salp Swarm Algorithm: CSSA

This kind of mutation operation was first implemented by Yao *et al.* (Yao et al., 1999) for conventional evolutionary algorithms. The main aim of Cauchy mutation was to produce smaller step sizes with in the bigger search space so that the algorithm is able to explore every local optimal point with in the search space. Thus overall helping the algorithm to come out of local minimal and hence reducing the problem of local optima stagnation and premature convergence. The Cauchy distribution function to generate the Cauchy random number is given by

$$y = \frac{1}{2} + \frac{1}{\pi} arctan(\frac{\delta}{g}) \tag{9}$$

The Cauchy density function is given by

$$f_{C(0,g)}(\delta) = \frac{1}{\pi} \frac{g}{g^2 + \delta^2} \tag{10}$$

Here $g = 1$ is the scale parameter, $y \in [0, 1]$ is a uniformly distributed random number. Now in order to generate the Cauchy distributed random number $C(\delta)$, equation (9) is solved and the solution is thus given by

$$\delta = tan(\pi(y - \frac{1}{2})) \tag{11}$$

This Cauchy distributed random number $C(\delta)$ is incorporated in place of c_1 parameter in the general equation (1) of the basic SSA.

4.2. Diversity Mutated Salp Swarm Algorithm: DSSA

GA is one of the oldest algorithm known till date in the field of numerical optimization. A lot of mutation strategies were exploited on GA to improve its working capabilities and one among them is diversity mutation. The concept was formulated by (Deb & Deb, 2014) and it was found that this kind of mutation improve the exploration properties of any algorithm. In present work, the same diversity mutation operator has been exploited and the algorithm thus formulated has been named as DSSA. The distribution is an adaptation of exponential distribution ($p(i) = te^{-ti}$) for $i \in [0, n-1]$. The above equation is a simple formulation and for fixed integer values n, the roots of probability distribution is given by

$$te^{-nt} - e^{-t} - t + 1 = 0 \tag{12}$$

The general equation for this kind of mutation as derived from the above equations is given by

$$c_1(t) = \frac{1}{t} log(1 - u(1 - e^{-nt})) \tag{13}$$

Here c_1 is the general parameter of SSA adapted with respect to diversity mutation, t is the iteration counter, u is a uniformly distributed random number in the range of $[0, 1]$, $p(i)$ is the probability parameter and i is any member of the population. Here it should be noted that the general equation of SSA are kept same and changes are made only in the parametric details.

4.3. Gaussian Salp Swarm Algorithm: GSSA

The density function for Gaussian mutation is given by

$$f_{G(0,\sigma^2)}(\alpha) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{\alpha^2}{2\sigma^2}} \tag{14}$$

where σ^2 is variance, α is a random number distributed in the range of [0,1]. This kind of formulation is common in evolutionary computing and aims at providing smaller step sizes to update the parent solutions. The search agents keep moving within the close proximity of the other search agents and hence explore every corner within the search space. The early formulation concepts of Gaussian distributed mutation was given by (Yao et al., 1999) for fast evolutionary programming algorithms. Salgotra *et al.* exploited the work and implemented them on FPA and it has been found that Gaussian mutation is highly effective and helps in exploring the search space in a potentially much better way (Salgotra & Singh, 2017). In present case, equation (14) is modified by using mean as zero and standard deviation as one. The general random numbers thus generated follow the patterns of Gaussian distributed random number and the equation for this case is given by

$$Y_j^1 = \begin{cases} S_j + G(\alpha)((ub_j - lb_j)c_2 + lb_j) & C_3 \geq 0 \\ S_j - G(\alpha)((ub_j - lb_j)c_2 + lb_j) & C_3 \leq 0 \end{cases} \tag{15}$$

where Y_j^1 and S_j is the position of first salp and food source in j^{th} dimension respectively, ub_j and lb_j represents the upper and lower boundary of j^{th} dimensional problem, c_1, c_2 and c_3 are the random initialization parameters drawn from uniformly distributed parameters in the range of [0,1]. Here this equation (15) is same as of the original SSA but instead of c_1, $G(\alpha)$ is used.

4.4. Lévy Flight based Salp Swarm Algorithm: LSSA

The concept of Lévy flights is not new to the field of optimization research and has been exploited on various algorithms namely CS (Yang & Deb, 2010), FPA (Yang, 2012) and others. The mechanism is based on far field randomization and uses the concepts of fatter tail optimization. Here fatter tail corresponds to more randomization, highly stochastic behavior and in turn more exploration. The general equation for this kind of mutation is given by (Yang & Deb, 2010)

$$\begin{cases} L(x) = 0.01 \times \frac{r_1 \times \sigma}{|r_2|^{1/\beta}} \quad r_1, r_2 \in [0,1]; \beta = 1.5 \\ \sigma = \left(\frac{\Gamma(1+\beta) \times sin(\frac{\pi\beta}{2})}{\Gamma(\frac{1+\beta}{2}) \times \beta \times 2(\frac{\beta-1}{2})}\right)^{1/\beta} \quad \Gamma(x) = (x-1)! \end{cases} \tag{16}$$

where $L(x)$ corresponds to Lévy distributed random number with x variable size, α, β and γ are random numbers and particular values for these functions is presented in subsequent subsections as per the problem requirement.

4.5. Mutation Clock Salp Swarm Algorithm: MSSA

In an optimization algorithm, mutation is followed using n random solutions per individual. Thus there is requirement of more individuals per random solution and hence more is the average number of functions evaluations required to perform a particular operation. So in order to overcome this problem, mutation clock based SSA is proposed and has been named as MSSA. The concept of mutation clock was formulated by (Goldberg, 1989) and aimed at enhancing the performance of GA. In mutation clock one individual is mutated, the corresponding other individual in the next iteration is generated by using exponential probability values $((p(i) = \lambda e^{-\lambda t}))$. The new mutation operator thus generated is given by $u = \int_{t=0}^{\mu} p_m e^{-p_m t} dt$. Here values of λ is the inverse of average number of mutation operations, p_m is the probability, $u \in [0,1]$ is a random variable and the final equation thus generated is given by

$$c_1 = \frac{1}{p_m} log(1-u) \tag{17}$$

Now if k^{th} variable for the i^{th} individual is mutated, the next mutation event is $((k+\mu)n) - th$ variable of $((k+\mu)/n) - th$ individual in the current population and c_1 is the general parameter of SSA. Here it should be noted that initial variable is mutated using $i = k = 1$ for every generation. Overall this strategy helps in increasing diversity among the search agents and hence improves the exploration properties of SSA algorithm.

4.6. Neighbourhood based Salp Swarm Algorithm: NBSSA

Neighbourhood based strategy has been exploited in the recent past for DE algorithm (Das et al., 2009) and has been found viable solutions for optimizing the exploration and exploitation capabilities of the algorithm. Since the neighbourhood based search improves the exploration properties of the DE algorithm, it is expected that because of the presence of two new solutions will add up in providing better results. The proposed algorithm has been named as neighbourhood based SSA

(NBSSA) and is aimed at improving the exploration properties of SSA. The algorithm consists of new random solutions to be generated while preserving the diversity among the individuals. Here X_i^t $(i = 1, 2, ...N)$ is the solution vector for N population size, of k radius neighbourhood (k is taken from 0 to $(N-1)/2$ and is a non zero vector). The major point which is to be kept in mind is that the total neighbourhood size must be less than the total population size and the solution vectors generated must be from the same neighbourhood. The general equation thus derived from whole of the solution space is given by

$$X_i^{t+1} = X_i^t + U[0,1](X_{r_1}^t - X_{r_{N-1}}^t) \tag{18}$$

Here $U[0,1]$ is a uniformly distributed random number, $X_{r_1}^t$ and $X_{r_{N-1}}^t$ are two neighbours around X_i^t and t is the generation or iteration count. Here it should be noted that the new solution generated by equation (18) are fed to the solution generated using the original equation (1) of basic SSA. The overall aim is to increase diversity and improve the exploration properties of the algorithm.

4.7. Trigonometric Salp Swarm Algorithm: TSSA

Trigonometric mutation is a new kind of mutation and is followed by using three individuals from the population. One among the members is the donor which is changed with respect to the scaled differential vector, considering two other members to be mutated. Three new individuals are generated as X_{r1}^t, X_{r2}^t and X_{r3}^t, where X_i^t is the i^{th} individual in the t^{th} generation, $r_1, r_2, r_3 = 1, ...N$ for $r_1 \neq r_2 \neq r_3 \neq i$ and N being the population size. For trigonometric mutation to take place, the members of the population need to be adapted with respect to the center of a hypothetical hyper-geometric triangle. The perturbation operation to be performed is done by using three differential vectors and their weighted sum. The general equation in this case is given by

$$V_i^{t+1} = \frac{X_{r1}^t + X_{r2}^t + X_{r3}^t}{3} + (p_2 - p_1)(X_{r1}^t - X_{r2}^t) + (p_3 - p_2)(X_{r2}^t - X_{r3}^t) + (p_1 - p_3)(X_{r3}^t - X_{r1}^t) \tag{19}$$

here p_1, p_2, p_3 are the random weights ($p_1 = 1$ and $p_2 + p_3 = 0$) that perturb and produce better individuals. In present case, the above equation (19) is adapted to formulate the new equation. After parameter adaptation, the general equation becomes

$$X_{t+1} = \frac{X_{r1}^t + X_{r2}^t + X_{r3}^t}{3} + X_{r2}^t + X_{r3}^t - 2X_{r1}^t \tag{20}$$

Here it should be noted that the three solutions used are derived from the whole population and the algorithm thus formulated is named as trigonometric SSA (TSSA). This modification is also added to increase diversity among the search agents and hence increase the exploration properties along with better convergence activities.

4.8. Computational complexity

The computational complexity in the case of SSA can be represented in the form of $O(n.d.t_{max})$ where t_{max} is the maximum number of a d dimensional problem with a population size of n. The reason for providing the complexity analysis is to find the suitable run-time for the algorithm and analyse its operation with the worst case complexities. The equation listed above is obtained from the basic population size n, dimension size d of the problem under test and the maximum number of iterations t_{max} required for finding the global optimum of the problem under test. For a single member of the population, the complexity analysis is given by the general equation O(d). This is because the dimension size of the problem remains constant. Now as the algorithm requires multiple agents or a set of population members, the general equation is enhanced to O($n.d$). Also because of the stochastic nature of the algorithms, it needs to be evaluated over multiple iterations t_{max} and hence the overall complexity becomes O($n.d.t_{max}$). This is the complexity of the original SSA. If we compare the complexity of original with respect to the proposed algorithm, there is only one adaptation and that is the linear decreasing population size. So in present case, the total computational complexity of proposed variants of SSA becomes O($n_{reduced}.d.t_{max}$). Here it should be noted that $n_{reduced}$ is the reduction per iteration for the problem under test. Also, all the proposed variants employ linear population size reduction, the overall complexity for all the variants is almost similar. Further, in comparison to basic SSA, all the proposed versions have lower computational complexity.

All the algorithms proposed above are aimed at enhancing the exploration and exploitation operation of SSA with specific focus on balancing the two phenomena. Here all the algorithms follow similar structure with changes in the type of mutation they use and hence are named differently. A generalized pseudo-code for all the algorithm is presented in Algorithm 1.

Algorithm 1: Pseudo-code of proposed MSSA algorithm
Begin
Define: population size ($\boldsymbol{N}$); (c_1)
Find initial best solution evaluate and choose the best salp
until termination criteria is met
if iterations < $\frac{maxiter}{2}$
find new solution using (1)
evaluate fitness $f(i)$
update c_1 using (7) and $\boldsymbol{N}$ using (8)
else
find new solution using (6)
evaluate fitness $f(i)$
update c_1 using (7) and $\boldsymbol{N}$ using (8)
end
find the current best
endUntil
update final best
End

5. Result and Discussion

In this section, details about the proposed variants and their applicability to various benchmark and real world problems is presented. The results are extensively discussed and performance evaluation is done using CEC 2005 (Suganthan et al., 2005), CEC 2015 (Liang et al., 2014) and other real world optimization problems. First of all, the different proposed strategies are compared with each other for variable population size and dimension sizes and then best among them is compared with standard algorithms from the literature. These algorithms are BA (Yang, 2010), FA (Yang, 2009), FPA (Yang & Deb, 2010), DE (Storn & Price, 1997), GWO (Mirjalili et al., 2014), DA (Mirjalili, 2016a), SCCSA (Khalilpourazari & Pasandideh, 2019), PSOGSA (Mirjalili & Hashim, 2010) and SSA (Mirjalili et al., 2017). The results are taken for 50 runs with best, worst, mean and standard deviation values evaluated for each of them. As far as statistical tests are concerned, Wilcoxon rank-sum test (Wilcoxon et al., 1970) and Mann-Whitney U rank tests (Ruxton, 2006) are performed using the best values of 50 runs for all the algorithms under test. More details about the same is presented in the subsequent subsections. All the algorithms are evaluated using windows 10, x64-bit system having Intel core i7 processor, 32.00 GB RAM and MATLAB version R2016a. The section also provides detailed summary of results, drawback of the proposed approach and some insightful implications. More detailed discussion about the experimental results is presented in the consecutive subsections.

5.1. Test Suite

In this paper, the performance of proposed seven mutation properties of SSA have been evaluated by taking 16 traditional numerical CEC 2005 benchmark functions listed in Table 1. The benchmark functions are classified into three categories: unimodal, multimodal and fixed dimension functions. The unimodal functions $F1 - F7$ are used to attain one global minimal solution with focusing on exploitation properties of algorithm run. The multimodal functions $F8 - F13$ have a large number of local minima and are used to test exploitation as well as exploration ability of proposed mutations. The fixed dimension functions $F13 - F15$ are used to test consistency of an algorithm to find best global minimal solution. For functions $F1 - F13$, the dimension size is taken as $D = 30$ and fitness $f_{min} = 0$. For functions $F14$, $F15$ and $F16$, dimension size is taken as $D = 2, 3, 6$ and fitness $f_{min} = 3, -3.86, -3.86$, respectively.

Table 1: CEC 2005 Benchmark functions used for comparison

Function	Dim	Range	Shift position	f_{min}
Unimodal functions				
F1(x)= $\sum_{i=1}^{n} x_i^2$	30	$[-100, 100]$	$[-30, -30, .., -30]$	0
F2(x)= $\sum_{i=1}^{n} \lvert x_i\rvert + \Pi_{i=1}^{n} \lvert x_i\rvert$	30	$[-10, 10]$	$[-3, -3, .., -3]$	0
F3(x)= $\sum_{i=1}^{n}(\sum_{j-1}^{i} x_j)^2$	30	$[-100, 100]$	$[-3, -3, .., -3]$	0
F4(x)= $max_i\{\lvert x_i\rvert, 1 \leq i \leq n\}$	30	$[-100, 100]$	$[-3, -3, .., -3]$	0
F5(x)= $\sum_{i=1}^{n-1} 100(x_{i+1} - x_i^2)^2 + (x_1 - 1)^2$	30	$[-30, 30]$	$[-3, -3, .., -3]$	0
F6(x)= $\sum_{i=1}^{n}([x_i + 0.5])^2$	30	$[-10, 10]$	$[-3, -3, .., -3]$	0
F7(x)= $\sum_{i=1}^{n} ix_i^4 = random[0, 1]$	30	$[-1.28, 1.28]$	$[-3, -3, .., -3]$	0
Multimodal functions				
F8(x)= $\sum_{i=1}^{n}[x_i sin(\sqrt{\lvert x_i\rvert})]$	30	$[-30, -30, .., -30]$		$-418.982 \times D$
F9(x)= $\sum_{i=1}^{n}[x_i^2 - 10cos(2\pi x_i) + 10]$	30	$[-5.12, 5.12]$	$[-30, -30, .., -30]$	0
F10(x)= $-20exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - exp(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)) + 20 + e$	30	$[-100, 100]$	$[-30, -30, .., -30]$	0
F11(x)= $\frac{1}{4000}\sum_{i=1}^{N} x_i^2 - \Pi_{i=1}^{N} cos(\frac{x_i}{\sqrt{i}}) + 1$	30	$[-600, 600]$	$[-30, -30, .., -30]$	0
F12(x)= $\frac{\pi}{n} 10sin(\pi y_1) + \sum_{i=1}^{n} -1(y_i - 1)^2[1 + 10sin^2(\pi y_{i+1})]$ $(y_n - 1)^2 + \sum_{i=1}^{u}(x_i, 10, 100, 4) y_i = 1 = \frac{x_i+1}{4}$	30	$[-50, 50]$	$[-30, -30, .., -30]$	0
F13(x)= $0.1(\sin^2(3\pi x_1) + \sum_{i=1}^{n}(x_i - 1)^2(1 + \sin^2(3\pi x_i + 1)))$ $+0.1((x_n - 1)^2[1 + \sin^2(2\pi x_n]) + \sum_{i=1}^{n} u(x_1, 5, 100, 4)$	30	$[-50, 50]$	$[-30, -30, .., -30]$	0
Fixed dimension functions				
F14(x)= $[1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)]*$ $[30 + (2x_1 - 3x_2)^2 * (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	$[-2, 2]$		3
F15(x)= $-\sum_{i=1}^{4} c_i \exp(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2)$	3	$[0, 1]$		-3.86
F16(x)= $-\sum_{i=1}^{4} c_i \exp(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2)$	6	$[0, 1]$		-3.86

5.2. Parameter Settings

The performance evaluation of proposed mutated salp swarm algorithm (MSSA) has been done by comparing performance of MSSA with the original SSA and another eight state-of-the-art algorithms such as BA, DA, DE, FA, SCCSA, PSOGSA, flower pollination algorithm (FPA) and grey wolf optimization (GWO). The Performance of each algorithm is again compared on the basis of 16

Table 2: Parameter Settings

Algorithm	Parameters			
	Pop Size	Dim	t_{max}	other parameters
BA	60	30	500	Loudness=0.5; Pulse rate=0.5; Stopping criteria= Max Iteration.
DA	60	30	500	w, s, a, c, f, e = *constant*
DE	60	30	500	F=0.5; CR=0.5; Stopping criteria= Max Iteration.
FA	60	30	500	α=0.5; β=0.2; $\gamma = 1$; Stopping criteria= Max Iteration.
FPA	60	30	500	P=0.8; Stopping criteria= Max Iteration.
GWO	60	30	500	α= Linearly decreased from 2 to 0; Stopping criteria= Max Iteration.
SCCSA	60	30	500	r_1, r_2, r_3, r_4
SSA	60	30	500	c_1, c_2, c_3 =*constant*
MSSA	60	30	500	c_1, c_2, c_3 =*adaptive*

different CEC 2005 numerical benchmark functions listed in Table 1. The parameter settings of these algorithms is given in Table 2 and value of these parameters has been selected from recent reported literature. in the case of BA, the loudness and pule rate are important parameters and their values are taken from (Yang, 2010). The (Mirjalili, 2016a) provided optimal parameter values in the case of DA. To improve the exploration and exploitation capability of DE, it has been found that higher value of crossover rate (CR) and lower value of scaling or control parameter (F) is an important aspect, now in present case the F and CR are taken as 0.9 and 0.5 respectively. in the case of FA, the light absorption co-efficient (γ) is an important parameter and it is selected as $\gamma = 1$ with respect to $\alpha = 0.5$ and $\beta = 0.2$ to get best results for most of the cases (Storn & Price, 1997). In FPA, only switch probability is an important parameter for analysis and it is taken as $P = 0.8$ (Yang, 2012) which fits to most of the cases. In GWO, it has been found that parameter α is acting as a decision parameter for analysing the algorithm ability of exploration and exploitation and its value is linearly decreasing number in the range of 2 to 0 (Mirjalili et al., 2014) and for SSA, parameters are generally taken as same as (Mirjalili et al., 2017). The final MSSA algorithm is proposed in this paper to show its ability to attain best optimal solution with fast convergence as compared to other algorithms. Apart from these algorithms, the common parameter values are taken as dimension size (Dim): 30 and number of runs: 50, population Size of 60 and maximum number of Iterations of 500.

5.3. Sensitivity to population Size

This subsection describes the effect of population size for the newly proposed seven different versions of SSA algorithm as discussed above by using the applications of mutation operators in accordance with conventional SSA algorithm. To analyze the effect of population, the six sets of population sizes such as $(20, 40, 60, 80, 100$ and $120)$ are used for all the algorithms and number of iterations are kept to 500.The results are measured in terms of best, worst, mean and standard deviation as given in Table 3 for population sizes of $20, 40, 60$ and in Table 4 for population sizes of $80, 100, 120$. Here, the results are obtained over 50 runs for all the algorithms under test. These results are discussed in detail as:

Population size 20: For functions $F1$, $F2$, $F4$, $F5$, $F12$ and $F13$, SSA was not able to approach the global optimum value while all other proposed variants provided competitive results and MSSA is the best among them. For function $F3$, DSSA provides the best results in comparison with other algorithms. For function $F6$, SSA provides somewhat better result among all other algorithms. For function $F7$, TSSA is found to be best. For function $F8$, all the algorithms provided the same results in terms of best values except SSA and MSSA which perform better in terms of standard deviation values. For functions $F9$ and $F11$, CSSA, DSSA, LSSA and MSSA attain a value of global optima in terms of best, worst, mean and standard deviation respectively. For function $F10$ CSSA, DSSA, LSSA and MSSA provide the same results so here it is difficult to tell which one is better. Overall for population size of 20, SSA is better for two functions, CSSA for three functions, DSSA for four functions, GSSA for no function, LSSA for three functions, MSSA for nine functions, NBSSA for no function and TSSA for one function. So, MSSA is found to be overall best among all the proposed variants for this case.

Population size 40: For functions $F1$, $F2$, $F3$, $F4$, $F7$, $F12$ and $F13$ MSSA provided the best results among all the proposed variants and approach towards near to global optimum value. For function $F5$, it is also found that MSSA provides the good results in terms of best values and a little difference exist for worst, mean and standard deviation values. For function $F6$, conventional SSA is better while the other versions provide very competitive results. For function $F8$, MSSA gives the best results in terms of standard deviation only. For functions $F9$ and $F11$ CSSA, DSSA, LSSA and MSSA attain a value near to global optimum in terms of best, worst, mean as well as standard deviation. For function $F10$, CSSA, DSSA, LSSA and MSSA also provide the same results so here it is difficult to say which one is better for these functions. In this case it is found that SSA is best for one function, CSSA and DSSA for three functions, GSSA and LSSA also for three functions, MSSA for twelve functions, NBSSA for no function, TSSA for no function. So, overall it is found that MSSA is best among all the proposed algorithms for the population size of 40.

Population size 60: For functions $F1$, $F2$, $F3$, $F4$, $F7$, $F12$ and $F13$ the proposed algorithm MSSA is found to be better among all the other algorithms. For function $F5$, GSSA gives best results in terms of best values but all the other algorithms approximately provide similar results in terms of worst, mean and standard deviation. For function $F6$, standard SSA is able to attain a value near to global optimum solution although all the proposed algorithms provided very competitive results. For function $F8$, MSSA is only algorithm to provide significantly better results in terms of standard deviation. For functions $F9$ and $F11$, proposed variants CSSA, DSSA, LSSA and MSSA attain a value of global optima while all the other algorithms suffer from local optima stagnation problem. For function $F11$, results of CSSA, DSSA, LSSA and MSSA are almost similar so here it is difficult to find the best algorithm for this function. It is found that SSA is best for only one function, CSSA and DSSA for three functions, GSSA for no function, LSSA for three functions, MSSA for eleven functions, NBSSA and TSSA for no function. Overall MSSA performs best among all the proposed variants for population size of 60.

Population size 80: In this case , for functions $F1$, $F2$, $F3$, $F4$, $F7$, $F12$ and $F13$ MSSA provides better results among all the other proposed variants. For function $F5$, the results are comparable in terms of best, worst and mean values where MSSA is found to be best. For function $F6$, SSA is again found to be best as similar to population size of 60. For function $F8$, all the algorithms provide same results in terms of best values except SSA and NBSSA which are found to provide good results in terms of standard deviation. For functions $F9$, $F10$ and $F11$ the results of CSSA, DSSA, LSSA and MSSA are almost similar . Here it is found that SSA is best for one function, CSSA and DSSA for three functions, GSSA and TSSA for no function, MSSA for eleven functions and NBSSA for one function. So this case also proves that MSSA is best for the 80 population size.

Population size 100: For functions $F1$, $F2$, $F3$, $F4$, $F7$, $F12$ and $F13$, it is found that MSSA again able to approaching towards near global optimum value. For function $F5$, MSSA provided the best results in terms of best and mean values. For function $F6$, the result of SSA is better in comparison with all other algorithms. For function $F8$, the results are comparable only in term of standard deviation where performance of MSSA is found to be best. For functions $F9$, $F10$ and $F11$, proposed variants CSSA, DSSA, LSSA and MSSA provided the almost similar results. So overall it is found that SSA is best for one function, CSSA and DSSA for three functions, GSSA and TSSA for no function, LSSA for three functions, NBSSA for no function and MSSA for twelve functions. Hence again MSSA is found to be best for this population size.

Population size 120: For functions $F1$, $F2$, $F3$, $F4$, $F5$, $F7$ and $F12$, MSSA is able to provide best results in terms of best, worst, standard deviation and even mean also. For function $F6$, SSA provided a value near to global optimum while all other variants also give competitive results. For function $F8$, results are comparable only in terms of standard deviation where MSSA is found to be best. For function $F9$, $F10$ and $F11$, results of CSSA, DSSA, LSSA and MSSA are almost similar so it is difficult to find the best algorithm for these functions. For function $F13$, MSSA attains a better results only in terms of best and mean. So in this case it is found that SSA is best for one function, CSSA and DSSA for three functions, GSSA and NBSSA for none of the function, MSSA for twelve functions and TSSA for no function. So, overall MSSA provided the best results for this population size 120.

Inferences from effect of population size: From the results, it has been analyzed that for variable population size, results become better with increase in the population size. The improvement is less for the initial population sizes of 20 and 40, but as the population size reaches 60, the overall results improve. But as population size is increased beyond 60, there is no significant improvement in the simulation results. Here, it should be noted that with increase in population size, the total number of function evaluations also increase many folds. So overall, we can say that MSSA performance is best for a population size of 60 in comparison with other algorithms.

5.4. Sensitivity to dimension Size

In this subsection, the effect of dimension size for the seven different proposed variants of SSA has been discussed. Here, the six number of dimension sets are used i.e. $(10, 30, 50, 100, 200$ and $500)$ for analyzing the results. All the results are taken for 500 iterations and rest of the parameters are same as discussed in relevant subsections. The results for dimension sizes 10, 30, 50 are given in Table 5 and for 100, 200, 500 are given in Table 6, respectively. The performance evaluation of all the algorithms are measured in terms of beat, worst, mean and standard deviation for 50 runs. The results are discussed in detail under subheadings as:

Dimension size 10: In this case, for functions $F1$, $F2$ and $F12$, MSSA is able to provide best results in terms of worst, mean and even standard deviation. For functions $F3$ and $F7$, MSSA is

Table 3: Statistical results for population size of 20, 40, 60

Function	Algorithm	Population Size 20				Population Size 40				Population Size 60			
		Best	Worst	Mean	Std	Best	Worst	Mean	Std	Best	Worst	Mean	Std
F1	SSA	2.21E-06	1.10E-03	1.19E-04	1.93E-04	1.73E-08	5.66E-08	2.94E-08	9.59E-09	8.27E-09	2.48E-08	1.66E-08	4.06E-09
	CSSA	5.07E-98	2.41E-82	7.13E-84	3.75E-83	2.82E-109	1.87E-90	3.74E-92	2.64E-91	3.71E-113	6.69E-97	1.46E-98	9.45E-98
	DSSA	2.88E-104	2.50E-84	5.01E-80	3.54E-79	3.25E-106	2.54E-88	5.25E-90	3.59E-89	1.79E-112	9.31E-94	1.94E-95	1.31E-94
	GSSA	3.04E-10	2.90E-01	1.04E-02	4.24E-02	6.12E-12	2.28E-01	9.90E-03	3.30E-02	4.72E-13	5.48E-02	2.40E-03	8.30E-03
	LSSA	9.32E-100	2.50E-84	5.84E-86	3.54E-85	5.35E-110	3.29E-90	1.13E-91	4.98E-91	1.23E-115	8.87E-96	2.41E-97	1.29E-96
	MSSA	4.93E-102	1.22E-85	3.52E-87	1.81E-86	7.37E-107	1.43E-92	3.12E-94	2.03E-93	1.41E-115	7.34E-98	2.13E-99	1.09E-98
	NBSSA	1.27E-13	6.18E+00	1.52E-01	8.75E-01	3.86E-12	2.77E-02	2.20E-03	5.50E-03	1.90E-26	2.59E-01	6.20E-03	3.67E-02
	TSSA	7.79E-14	1.02E+00	5.03E-02	1.73E-01	8.84E-11	2.84E-01	8.70E-03	4.07E-02	8.71E-10	1.62E-02	1.10E-03	3.10E-03
F2	SSA	3.45E-01	4.45E+01	5.47E+00	6.28E+00	1.08E-01	7.37E+00	1.59E+00	1.61E+00	8.70E-03	3.26E+00	9.35E-01	9.30E-01
	CSSA	1.63E-54	1.17E-44	3.06E-46	1.68E-45	2.99E-56	1.54E-48	5.09E-50	2.34E-49	1.25E-59	5.03E-51	1.65E-52	7.41E-52
	DSSA	2.94E-52	8.47E-45	4.91E-46	1.66E-45	2.89E-57	9.79E-49	6.07E-50	1.91E-49	2.51E-59	9.89E-51	4.21E-52	1.73E-51
	GSSA	1.13E-06	4.34E-01	6.15E-02	1.04E-01	1.60E-05	4.00E-01	1.91E-02	5.86E-02	9.32E-10	5.81E-02	9.80E-03	1.38E-02
	LSSA	2.32E-55	5.24E-44	1.26E-45	7.45E-45	1.18E-58	6.84E-49	2.30E-50	1.00E-49	9.61E-58	2.07E-49	4.60E-51	2.94E-50
	MSSA	3.46E-56	7.60E-46	1.83E-47	1.07E-46	1.69E-58	6.65E-49	1.38E-50	9.40E-50	1.59E-59	2.36E-53	1.60E-54	4.89E-54
	NBSSA	8.38E-06	3.43E-01	4.46E-02	6.93E-02	9.68E-09	1.48E-01	1.69E-02	2.77E-02	4.90E-08	6.65E-02	1.02E-02	1.60E-02
	TSSA	3.05E-08	5.04E-01	3.19E-02	8.16E-02	1.50E-05	1.83E-01	2.21E-02	3.28E-02	3.42E-07	1.19E-01	8.60E-02	1.90E-02
F3	SSA	9.17E+02	9.69E+03	3.38E+03	1.92E+03	2.89E+02	2.21E+03	8.92E+02	4.32E+02	4.28E+01	1.16E+03	3.43E+02	2.53E+02
	CSSA	1.88E-15	3.90E-03	1.48E-04	6.43E-04	4.76E-31	2.05E-04	4.53E-06	2.90E-05	5.47E-56	5.54E-07	1.34E-08	7.91E-08
	DSSA	1.18E-43	3.03E-02	8.68E-04	4.40E-03	4.79E-20	1.42E-04	3.12E-06	2.02E-05	1.59E-34	5.68E-06	1.17E-07	8.03E-07
	GSSA	2.40E-03	2.83E+03	2.97E+02	5.01E+02	3.04E-02	3.25E+02	4.40E+01	8.17E+01	2.22E-06	5.99E+01	4.31E+00	1.09E+01
	LSSA	5.01E-21	5.60E-03	1.39E-04	7.90E-04	6.84E-27	1.77E-05	4.06E-07	2.50E-06	8.49E-34	6.60E-09	1.36E-10	9.33E-10
	MSSA	9.43E-31	2.08E-06	4.17E-08	2.94E-07	6.24E-42	2.53E-11	5.69E-13	3.58E-12	6.61E-57	3.50E-16	1.48E-17	6.42E-17
	NBSSA	4.60E-03	2.28E+03	2.78E+02	5.02E+02	4.43E-07	5.45E+02	4.80E+01	1.01E+02	7.17E-09	1.20E+02	7.50E+00	2.33E+01
	TSSA	1.56E-02	1.94E+03	3.91E+02	5.37E+02	3.40E-03	3.63E+02	2.39E+01	5.72E+01	3.51E-06	7.09E+01	6.54E+00	1.53E+01
F4	SSA	8.81E+00	2.98E+01	1.73E+01	4.89E+00	2.87E+00	1.50E+01	8.24E+00	3.13E+00	1.06E+00	1.38E+01	5.88E+00	2.51E+00
	CSSA	1.09E-45	4.10E-34	8.60E-36	5.79E-35	2.11E-49	8.62E-40	3.19E-41	1.36E-40	5.91E-52	4.09E-42	9.82E-44	5.81E-43
	DSSA	2.58E-44	1.16E-33	4.16E-35	1.96E-34	1.02E-49	2.53E-38	7.58E-40	3.90E-39	2.30E-51	4.82E-43	2.58E-44	9.32E-44
	GSSA	1.22E-06	3.85E-01	3.62E-02	7.36E-02	5.32E-08	1.47E-01	1.50E-02	2.97E-02	4.80E-07	3.90E-02	6.00E-03	9.10E-03
	LSSA	5.16E-46	2.66E-35	2.00E-36	5.82E-36	9.39E-49	4.64E-39	1.79E-40	7.11E-40	2.15E-53	7.70E-42	2.09E-43	1.12E-42
	MSSA	4.91E-49	1.05E-34	2.17E-36	1.49E-35	1.26E-51	3.83E-41	1.80E-42	6.58E-42	2.03E-52	1.71E-43	3.98E-45	2.44E-44
	NBSSA	7.91E-06	2.24E-01	1.87E-02	3.52E-02	1.94E-09	8.60E-02	9.50E-03	1.83E-02	2.33E-06	4.69E-02	4.20E-03	7.80E-03
	TSSA	4.46E-06	6.08E-01	4.85E-02	1.12E-01	5.59E-06	1.84E-01	1.32E-02	2.87E-02	4.25E-06	7.39E-01	6.50E-03	1.30E-02
F5	SSA	3.06E+01	7.05E+03	5.94E+02	1.29E+03	1.98E+01	1.30E+03	1.47E+02	2.65E+02	1.93E+01	7.23E+02	1.08E+02	1.37E+02
	CSSA	3.86E+00	2.88E+01	2.65E+01	6.75E+00	1.39E+00	2.88E+01	2.64E+01	6.92E+00	9.07E-01	2.87E+01	2.47E+01	8.94E+00
	DSSA	2.81E+01	2.89E+01	2.87E+01	1.23E-01	2.84E+01	2.89E+01	2.87E+01	8.26E-02	2.82E+01	2.88E+01	2.86E+01	1.45E-01
	GSSA	1.47E+00	3.80E+01	1.43E+01	9.55E+00	2.79E-01	2.89E+01	9.57E+00	7.85E+00	4.70E-03	1.02E+01	2.05E+00	2.49E+00
	LSSA	9.29E+00	2.89E+01	2.80E+01	3.54E+00	2.89E-01	2.88E+01	2.68E+01	6.45E+00	5.63E-02	2.87E+01	2.47E+01	9.18E+00
	MSSA	5.13E-02	2.88E+01	2.09E+01	1.12E+01	7.05E-02	2.87E+01	1.38E+01	1.29E+01	9.20E-03	2.86E+01	2.41E+00	6.69E+00
	NBSSA	1.45E+00	3.62E+01	1.30E+01	9.47E+00	7.57E-02	2.88E+01	7.53E+00	6.80E+00	1.07E-02	2.15E+01	2.93E+00	4.48E+00
	TSSA	7.81E-02	2.91E+01	1.28E+01	8.30E+00	2.97E-01	2.50E+01	7.32E+00	6.89E+00	3.41E-02	2.69E+01	2.53E+00	4.02E+00
F6	SSA	1.12E-06	2.90E-03	1.81E-04	4.34E-04	1.56E-08	8.29E-08	3.43E-08	1.43E-08	9.49E-09	2.60E-08	1.60E-08	4.16E-09
	CSSA	2.40E-03	3.50E+00	1.14E+00	7.88E-01	2.00E-03	2.49E+00	6.75E-01	7.18E-01	4.20E-03	2.44E+00	4.97E-01	6.10E-01
	DSSA	6.70E-01	3.66E+00	2.00E+00	6.86E-01	7.44E-01	2.73E+00	1.56E+00	5.04E-01	5.65E-01	2.59E+00	1.22E-01	4.39E-01
	GSSA	4.80E-03	3.06E+00	5.20E-01	6.68E-01	4.00E-03	8.86E-01	1.36E-01	1.88E-01	3.79E-04	2.75E-01	4.99E-02	6.07E-02
	LSSA	1.75E-01	3.25E+00	1.72E+00	7.91E-01	5.26E-02	2.79E+00	1.21E+00	7.09E-01	2.24E-04	2.31E+00	8.91E-01	6.35E-01
	MSSA	6.20E-03	1.65E+00	4.02E-01	4.42E-01	3.60E-03	8.25E-01	1.05E-01	1.43E-01	3.26E-04	2.72E-01	4.30E-02	4.65E-02
	NBSSA	5.10E-03	4.88E+00	5.81E-01	9.54E-01	4.50E-03	1.45E+00	2.09E-01	3.41E-01	1.40E-03	4.01E-01	5.93E-02	7.52E-02
	TSSA	4.70E-03	1.79E+00	3.58E-01	3.77E-01	6.33E-04	1.18E+00	1.64E-01	2.40E-01	6.06E-04	5.20E-01	6.17E-02	1.00E-01
F7	SSA	7.55E-02	6.21E-01	3.11E-01	1.37E-01	2.56E-02	2.61E-01	1.20E-01	4.40E-02	2.15E-02	1.59E-01	8.57E-02	3.15E-02
	CSSA	5.37E-05	6.00E-03	1.10E-03	1.30E-03	1.09E-05	5.30E-03	7.96E-04	1.10E-03	3.95E-05	3.10E-03	5.00E-04	5.98E-04
	DSSA	1.01E-05	7.80E-03	1.20E-03	1.80E-03	5.05E-05	2.60E-03	6.67E-04	5.86E-04	7.35E-06	3.30E-03	4.56E-04	5.77E-04
	GSSA	7.75E-04	6.53E-02	1.52E-02	1.38E-02	6.17E-05	1.27E-02	3.80E-03	3.10E-03	1.66E-06	1.75E-02	2.90E-03	3.10E-03
	LSSA	1.29E-05	4.30E-03	8.09E-04	8.91E-04	1.61E-06	3.10E-03	6.60E-04	7.82E-04	8.29E-06	3.50E-03	5.41E-04	6.63E-04
	MSSA	1.04E-05	3.40E-03	7.31E-04	8.84E-04	1.11E-05	2.70E-03	5.31E-04	5.52E-04	7.96E-06	1.10E-03	3.04E-04	2.50E-04
	NBSSA	3.26E-04	9.95E-02	1.69E-02	1.62E-02	3.36E-04	2.25E-02	5.70E-03	5.30E-03	1.48E-04	1.15E-02	2.70E-03	2.60E-03
	TSSA	1.40E-03	6.83E-02	1.37E-02	1.23E-02	1.93E-04	2.38E-02	5.30E-03	5.20E-03	1.02E-04	1.11E-02	2.80E-03	3.10E-03

capable to give better results while the results of all other algorithms are still competitive. For function $F4$, the results of MSSA and CSSA are very similar for best and worst values but for mean values MSSA gives better results. For function $F5$, the results are comparable only in terms of best where NBSSA is found to be best. For functions $F6$ and $F13$, SSA is found to be best as compared to other variants of algorithms. For function $F8$, the results of MSSA found to be better only in terms of standard deviation values. For functions $F9$ and $F11$, CSSA, DSSA, LSSA, MSSA provided a results near to global optima. For function $F10$ CSSA, DSSA, LSSA and MSSA give the similar results so here it is difficult to say which one is better. So overall it is concluded that SSA is best for two functions, CSSA and DSSA for three functions, GSSA and TSSA for no function, LSSA for three functions, MSSA for ten functions and NBSSA for one function. Hence, MSSA is found to be overall best for 10 dimension size.

Dimension size 30: For functions $F1$, $F2$, $F3$, $F4$, $F7$, $F12$ and $F13$, it is found that MSSA provided the best results in terms of beat, mean, worst as well as standard deviation. For function $F5$, the results are comparable only in terms of best values where MSSA gives better results in comparison with all other variants of algorithms. For function $F6$, SSA provides the best results while results of all other algorithms are still competitive. For function $F8$, the MSSA is best in terms of standard deviation. For functions $F9$, $F10$ and $F11$, the results of CSSA, DSSA, LSSA and MSSA are almost similar so it is difficult to find the best algorithm. Hence, in this case it is

Table 3: Statistical results for population size of 20, 40, 60(Continued)

Function	Algorithm	Population Size 20				Population Size 40				Population Size 60			
		Best	Worst	Mean	Std	Best	Worst	Mean	Std	Best	Worst	Mean	Std
F8	SSA	-8.78E+03	-5.73E+03	-7.40E+03	7.74E+02	-9.04E+03	-5.98E+03	-7.36E+03	6.53E+02	-9.31E+03	-5.71E+03	-7.37E+03	7.05E+02
	CSSA	-1.25E+04	-4.86E+03	-1.05E+04	2.26E+03	-1.25E+04	-7.38E+03	-1.14E+04	1.58E+03	-1.25E+04	-5.45E+03	-1.10E+04	1.94E+03
	DSSA	-1.10E+04	-5.15E+03	-8.20E+03	1.80E+03	-1.25E+04	-5.45E+03	-8.48E+03	1.92E+03	-1.25e+04	-5.87E+03	-8.48E+03	1.90E+03
	GSSA	-1.25E+04	-1.24E+04	-1.25E+04	2.72E+01	-1.25E+04	-1.25E+04	-1.25E+04	3.99E+00	-1.25E+04	-1.25E+04	-1.25E+04	1.22E+00
	LSSA	-1.25E+04	-5.47E+03	-9.71E+03	1.99E+03	-1.25E+04	-5.87E+03	-1.01E+04	2.14E+03	-1.25E+04	-5.42E+03	-9.70E+03	2.24E+03
	MSSA	-1.25E+04	-1.25E+04	-1.25E+04	1.50E+00	-1.25E+04	-1.25E+04	-1.25E+04	1.02E+00	-1.25E+04	-1.25E+04	-1.25E+04	2.48E-01
	NBSSA	-1.25E+04	-1.24E+04	-1.25E+04	2.88E+01	-1.25E+04	-1.25E+04	-1.25E+04	4.18E+00	-1.25E+04	-1.25E+04	-1.25E+04	1.04E+00
	TSSA	-1.25E+04	-1.24E+04	-1.26E+04	2.54E+01	-1.25E+04	-1.25E+04	-1.25E+04	4.55E+00	-1.25E+04	-1.25E+04	-1.25E+04	1.55E+00
F9	SSA	3.18E+01	1.32E+02	6.93E+01	2.29E+01	1.98E+01	8.35E+01	4.81E+01	1.48E+01	1.09E+01	7.86E+01	4.49E+01	1.70E+01
	CSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	DSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	GSSA	2.50E-12	3.69E-01	1.83E-02	6.42E-02	1.29E-10	1.21E-02	4.10E-03	1.21E-02	1.71E-09	1.23E-02	5.29E-04	1.90E-03
	LSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	NBSSA	0.00E+00	6.37E-02	5.60E-03	1.21E-02	0.00E+00	3.21E-02	2.70E-03	6.30E-03	7.73E-12	9.50E-03	6.28E-04	1.70E-03
	TSSA	2.66E-11	5.31E-01	3.16E-02	1.05E-01	9.76E-11	1.84E-02	5.80E-03	1.84E-02	5.67E-11	1.00E-02	5.48E-04	1.50E-03
F10	SSA	1.50E+00	5.81E+00	3.17E+00	1.05E+00	9.31E-01	4.07E+00	2.23E+00	7.30E-01	2.55E-05	3.57E+00	1.86E+00	8.72E-01
	CSSA	8.88E-16	4.44E-15	1.45E-15	1.31E-15	8.88E-16	4.44E-15	1.52E-15	1.37E-15	8.88E-16	4.44E-15	1.52E-15	1.37E-15
	DSSA	8.88E-16	4.44E-15	1.81E-15	1.57E-15	8.88E-16	4.44E-15	1.52E-15	1.37E-15	8.88E-16	4.44E-15	1.81E-15	1.57E-15
	GSSA	5.05E-07	2.26E-01	2.23E-02	3.85E-02	5.03E-06	1.52E-01	1.53E-02	3.18E-02	1.29E-07	3.71E-02	5.00E-03	8.40E-03
	LSSA	8.88E-16	4.44E-15	1.59E-15	1.43E-15	8.88E-16	4.44E-15	1.52E-15	1.57E-15	8.88E-16	4.44E-15	1.10E-15	8.52E-16
	MSSA	8.88E-16	4.44E-15	1.81E-15	1.57E-15	8.88E-16	4.44E-15	1.52E-15	1.43E-15	8.88E-16	4.44E-15	1.31E-15	1.16E-15
	NBSSA	6.91E-11	3.50E-01	3.51E-02	7.46E-02	2.12E-09	5.37E-02	9.30E-03	1.48E-02	8.80E-07	7.72E-02	5.70E-03	1.32E-02
	TSSA	1.17E-07	8.19E-01	4.39E-02	1.43E-01	3.42E-06	1.12E-01	1.25E-02	2.14E-02	4.17E-06	3.59E-02	3.60E-03	6.30E-03
F11	SSA	1.25E-02	1.50E-01	5.45E-02	3.04E-02	2.54E-05	4.18E-02	1.00E-02	9.00E-03	9.21E-07	3.94E-02	9.60E-03	9.70E-03
	CSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	DSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	GSSA	2.38E-10	1.02E+00	5.79E-02	1.75E-01	8.77E-15	9.22E-01	5.44E-02	1.79E-01	0.00E+00	3.72E-02	2.40E-03	7.50E-03
	LSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	NBSSA	1.02E-10	4.57E-01	3.37E-02	8.00E-02	1.35E-10	2.77E-01	8.80E-03	3.99E-02	3.72E-12	2.67E-02	1.70E-03	4.30E-03
	TSSA	3.90E-11	1.03E+00	6.65E-02	1.92E-01	0.00E+00	4.66E-02	4.90E-03	1.12E-02	3.28E-11	2.71E-02	1.50E-03	4.30E-03
F12	SSA	2.10E+00	2.32E+01	9.78E+00	4.32E+00	1.54E+00	1.05E+01	5.47E+00	9.06E-02	2.62E-01	1.13E+01	4.34E+00	1.94E+00
	CSSA	3.58E-05	2.42E-01	4.84E-02	6.49E-02	1.21E-04	6.09E-01	4.24E-02	1.08E-01	2.32E-06	1.12E-01	1.66E-02	2.65E-02
	DSSA	2.91E-02	5.10E-01	1.20E-01	1.07E-01	2.06E-02	6.59E-01	1.08E-01	3.70E-03	1.30E-02	4.31E-01	8.26E-02	6.88E-02
	GSSA	6.92E-05	8.31E-02	1.10E-02	1.57E-02	4.13E-05	2.01E-02	2.50E-03	6.54E-02	4.83E-06	5.40E-03	5.67E-04	8.99E-04
	LSSA	1.19E-04	3.87E-01	8.04E-02	8.95E-02	2.71E-07	3.47E-01	4.43E-02	6.54E-02	1.65E-04	1.72E-01	3.54E-02	4.31E-02
	MSSA	8.42E-08	8.15E-02	1.13E-02	1.75E-02	4.32E-06	1.11E-02	1.50E-03	2.40E-03	9.18E-07	8.00E-03	4.39E-04	1.20E-03
	NBSSA	1.33E-04	5.10E-02	8.30E-03	1.10E-02	6.76E-05	8.40E-03	2.10E-03	2.10E-03	2.71E-06	4.70E-03	5.93E-04	8.46E-04
	TSSA	1.96E-05	3.99E-02	5.80E-03	9.10E-03	2.20E-05	1.80E-02	1.80E-03	3.20E-03	2.92E-02	3.40E-03	6.07E-04	7.62E-04
F13	SSA	6.85e-01	5.91E+01	3.15E+01	1.43E+01	5.50E-06	4.46E+01	1.06E+01	1.27E+01	7.42E-09	1.85E+01	9.91E-01	2.93E+00
	CSSA	4.20E-03	1.40E+00	3.39E-01	3.49E-01	1.80E-03	1.53E+00	1.91E-01	3.04E-01	1.10E-03	1.03E+00	1.63E-01	2.76E-01
	DSSA	4.59E-01	2.52E+00	1.08E+00	4.86E-01	2.42E-01	1.36E+00	8.38E-01	2.53E-01	2.45E-01	1.46E+00	7.66E-01	2.62E-01
	GSSA	1.70E-03	3.10E-01	7.18E-02	6.83E-02	4.72E-04	1.42E-01	3.61E-02	4.01E-02	4.37E-06	3.02E-02	7.70E-03	7.20E-03
	LSSA	6.90E-03	2.78E+00	7.25E-01	6.33E-01	1.75E-04	1.22E+00	3.33E-01	3.65E-01	4.70E-03	1.55E+00	3.39E-01	4.15E-01
	MSSA	2.50E-03	5.10E-01	7.81E-02	8.81E-02	3.45E-06	1.87E-01	2.58E-02	3.72E-02	2.33E-06	4.32E-02	7.10E-03	1.04E-02
	NBSSA	3.20E-03	4.63E-01	9.55E-02	9.78E-02	3.43E-04	1.44E-01	3.56E-02	3.40E-02	1.22E-04	6.25E-02	9.80E-03	1.64E-02
	TSSA	3.60E-03	4.11E-01	8.48E-02	8.20E-02	3.03E-04	1.70E-01	2.68E-02	3.21E-02	1.75E-05	6.10E-02	9.50E-03	1.35E-02

found that SSA is able to give best results for one function, CSSA and DSSA for three functions, GSSA and NBSSA for no function, LSSA for three functions, MSSA for twelve functions and TSSA for no function. So overall MSSA proves the best for 30 dimension size.

Dimension Size 50: In this case, for functions $F1$, $F2$, $F3$, $F7$, $F12$ and $F13$, all the versions of algorithms are able to provide competitive results where MSSA is found to be best among them. For function $F4$, CSSA is provided best results in terms of worst, mean and standard deviation. For function $F5$, results are same for worst, mean and standard deviation so here comparison is done only in terms of best where MSSA provided best results. For function $F6$, SSA results are better in comparison to all other proposed variants. For function $F8$, MSSA is able to provide comparatively better results only in terms of standard deviation. For functions $F9$, $F10$ and $F11$, results are almost similar for proposed variants such as CSSA, DSSA, LSSA and MSSA. So here it is found that SSA is best for one function, CSSA for four functions, DSSA and LSSA for three functions, GSSA and NBSSA for no function, MSSA for eleven functions and TSSA for no function. Hence again MSSA is best for this dimension size.

Dimension Size 100: For functions $F1$, $F2$, $F3$, $F7$ and $F13$, the performance of MSSA is best as compared to other variants of algorithms. For function $F4$, CSSA is able to attain a value near near to global optima. For function $F5$, all algorithms provided the almost same results in terms of mean, worst and standard deviation so here comparison is to be done only in terms of best where MSSA is found to best. For functions $F6$ and $F12$, TSSA results are better among all the other algorithms. For function $F8$, the results are comparable only in terms of standard deviation and found that MSSA is the best. For functions $F9$, $F10$ and $F11$, the performance of CSSA, DSSA,

Table 4: Statistical results for population size of 80, 100, 120

Function	Algorithm	Population Size 80				Population Size 100				Population Size 120			
		Best	Worst	Mean	Std	Best	Worst	Mean	Std	Best	Worst	Mean	Std
F1	SSA	5.49E-09	1.75E-08	1.19E-08	2.82E-09	6.45E-09	1.74E-08	1.07E-08	2.29E-09	5.77E-09	1.37E-08	9.38E-09	1.94E-09
	CSSA	1.09E-113	1.14E-97	3.19E-99	1.66E-98	3.83E-121	7.69E-104	1.54E-105	1.08E-104	1.07E-124	2.13E-107	4.68E-109	3.02E-108
	DSSA	7.00E-115	1.19E-98	2.79E-100	1.69E-99	1.64E-117	5.74E-102	1.31E-103	8.13E-103	1.20E-124	5.03E-105	1.03E-106	7.11E-106
	GSSA	3.36E-13	4.30E-03	3.64E-04	8.38E-04	2.14E-11	7.20E-03	4.11E-04	1.40E-03	1.13E-15	2.90E-03	1.66E-04	4.61E-03
	LSSA	2.28E-118	4.68E-103	1.30E-104	6.66E-104	2.50E-121	1.00E-103	3.02E-105	1.53E-104	2.20E-123	5.67E-106	1.14E-107	8.02E-107
	MSSA	1.24E-118	1.45E-103	2.92E-105	2.06E-104	1.19E-124	1.14E-107	2.31E-109	1.62E-108	3.19E-127	1.57E-111	4.51E-113	2.25E-112
	NBSSA	2.05E-15	2.73E-02	8.12E-04	3.90E-03	1.90E-18	1.20E-03	1.38E-04	2.88E-04	2.17E-13	1.17E-02	2.95E-04	1.70E-03
	TSSA	9.58E-16	6.90E-03	3.14E-04	1.00E-03	4.67E-18	9.10E-04	8.17E-05	2.07E-04	9.80E-14	1.30E-03	1.06E-04	2.56E-04
F2	SSA	1.10E-03	2.39E+00	4.46E-01	5.29E-01	1.10E-03	1.67E+00	4.04E-01	3.92E-01	1.20E-03	2.85E+00	3.84E-01	5.97E-01
	CSSA	6.54E-63	4.19E-53	1.01E-54	5.95E-54	3.14E-65	1.55E-54	3.35E-56	2.20E-55	1.01E-64	7.22E-57	1.66E-58	1.02E-57
	DSSA	5.37E-60	2.21E-53	1.17E-54	4.30E-54	6.48E-63	4.56E-54	1.04E-55	6.45E-55	1.40E-65	1.46E-55	4.24E-57	2.13E-56
	GSSA	4.05E-06	2.87E-02	5.80E-03	8.40E-03	4.50E-08	2.05E-02	2.90E-03	4.40E-03	4.14E-07	2.07E-02	2.20E-03	4.30E-03
	LSSA	1.75E-60	1.56E-53	3.84E-55	2.21E-54	4.73E-63	1.84E-53	4.08E-55	2.61E-54	7.79E-65	2.26E-57	1.68E-58	4.61E-58
	MSSA	6.25E-63	1.12E-55	4.76E-57	1.73E-56	1.82E-64	3.32E-57	8.65E-59	4.71E-58	8.36E-66	2.69E-58	1.43E-59	5.19E-59
	NBSSA	7.83E-08	2.94E-02	4.10E-03	7.10E-03	1.82E-06	3.33E-02	3.90E-03	6.80E-03	8.93E-07	2.50E-02	3.00E-03	5.00E-03
	TSSA	2.97E-09	4.10E-02	4.40E-03	8.40E-03	8.05E-08	1.71E-02	2.60E-03	4.30E-03	1.94E-07	1.92E-02	2.90E-03	4.20E-01
F3	SSA	4.02E+01	7.54E+02	1.72E+02	1.23E+02	5.56E+00	3.70E+02	1.08E+02	8.18E+01	5.20E+00	1.56E+02	5.45E+01	3.37E+01
	CSSA	1.71E-39	1.47E-11	3.11E-13	2.08E-12	1.59E-51	2.54E-13	7.06E-15	3.82E-14	1.45E-49	1.37E-17	3.19E-19	1.96E-18
	DSSA	3.89E-32	7.67E-12	1.91E-13	1.09E-12	2.94E-51	3.73E-11	7.50E-13	5.28E-12	4.56E-40	1.22E-16	4.53E-18	2.12E-17
	GSSA	3.81E-07	7.40E+01	2.96E+00	1.08E+01	2.56E-14	1.33E+01	5.49E-01	2.04E+00	2.14E-10	5.60E+00	2.08E-01	8.32E-01
	LSSA	4.70E-40	3.68E-12	1.47E-13	6.19E-13	7.73E-67	6.26E-12	1.39E-13	8.86E-13	1.25E-49	1.29E-17	3.99E-19	1.92E-18
	MSSA	3.44E-67	1.83E-16	3.67E-18	2.60E-17	5.06E-57	1.12E-17	2.24E-19	1.58E-18	1.78E-60	6.97E-21	1.39E-22	9.85E-22
	NBSSA	2.89E-10	6.30E+01	2.14E+00	9.15E+00	3.02E-11	1.00E+02	2.70E+00	1.45E+01	1.14E-06	1.03E+01	3.52E-01	1.50E+00
	TSSA	7.93E-13	2.33E+01	1.13E+00	3.52E+00	3.79E-13	1.19E+01	5.32E-01	1.92E+00	4.93E-08	4.08E+00	2.90E-01	8.72E-01
F4	SSA	3.10E-01	8.07E+00	3.63E+00	1.92E+00	3.07E-01	8.52E+00	2.51E+00	1.68E+00	2.07E-01	6.35E+00	1.89E+00	1.35E+00
	CSSA	3.45E-53	2.39E-45	7.29E-47	3.67E-46	6.55E-56	2.42E-46	5.32E-48	3.42E-47	3.89E-61	5.84E-50	2.95E-51	9.46E-51
	DSSA	1.83E-53	1.84E-43	3.94E-45	2.60E-44	4.80E-54	5.94E-47	3.08E-48	1.02E-47	9.52E-58	4.45E-48	1.57E-49	7.12E-49
	GSSA	6.11E-14	1.56E-02	2.30E-03	3.90E-03	2.25E-07	6.81E-02	3.50E-03	1.00E-02	1.83E-07	8.00E-03	1.60E-03	2.20E-03
	LSSA	2.44E-55	1.89E-45	7.75E-47	2.94E-46	4.74E-58	3.36E-47	1.46E-48	5.35E-48	1.34E-59	9.98E-50	4.57E-51	1.77E-50
	MSSA	8.96E-56	2.75E-47	1.47E-48	5.42E-48	1.31E-57	2.10E-49	1.21E-50	4.16E-50	9.19E-60	9.14E-52	2.89E-53	1.32E-52
	NBSSA	2.79E-08	2.49E-02	2.70E-03	4.50E-03	1.18E-09	9.90E-03	1.20E+03	1.90E-03	5.20E-08	2.67E-02	1.80E-03	4.50E-03
	TSSA	7.92E-07	4.19E-02	4.10E-03	7.50E-03	1.23E-07	2.35E-02	2.70E+03	4.90E-03	8.70E-07	1.13E-02	1.90E-03	2.40E-03
F5	SSA	2.23E+01	4.83E+02	8.98E+01	1.09E+02	2.14E+01	4.59E+02	6.03E+01	7.46E+01	2.26E+01	5.33E+02	7.83E+01	1.03E+02
	CSSA	7.93E-01	2.87E+01	2.33E+01	1.03E+01	6.73E-01	2.87E+01	2.27E+01	1.09E+01	2.13E-01	2.87E+01	2.37E+01	1.00E+01
	DSSA	2.80E+01	2.88E+01	2.85E+01	2.11E-01	2.78E+01	2.87E+01	2.85E+01	2.76E-01	2.79E+01	2.88E+01	2.85E+01	2.21E-01
	GSSA	2.80E-03	4.70E+00	1.05E+00	1.16E+00	1.87E-04	7.56E+00	7.07E-01	1.34E+00	1.20E-03	4.77E+00	4.18E-01	7.96E-01
	LSSA	9.37E-02	2.88E+01	2.36E+01	9.96E+00	1.85E+00	2.88E+01	2.39E+01	9.85E+00	9.99E-02	2.87E+01	2.13E+01	1.14E+01
	MSSA	6.62E-05	4.16E+00	5.49E-01	8.65E-01	1.50E-04	2.70E+00	2.40E-01	4.19E-01	2.73E-06	4.93E-01	7.84E-02	1.06E-01
	NBSSA	1.09E-05	9.11E+00	1.27E+00	1.81E+00	1.20E-03	3.99E+00	5.85E-01	8.25E-01	5.27E-04	2.64E+00	3.78E-01	5.18E-01
	TSSA	5.10E-03	3.10E+00	9.29E-01	9.17E-01	1.50E-03	2.39E+00	4.98E-01	6.29E-01	2.05E-04	1.72E+00	3.53E-01	4.56E-01
F6	SSA	7.45E-09	2.15E-08	1.26E-08	2.51E-09	6.56E-09	1.49E-08	1.07E-08	1.82E-09	4.78E-09	1.34E-08	9.43E-09	1.76E-09
	CSSA	3.80E-03	2.03E+00	4.85E-01	5.67E-01	2.69E-04	1.28E+00	2.83E-01	3.45E-01	3.00E-03	1.25E+00	2.03E-01	2.93E-01
	DSSA	3.61E-01	2.08E+00	1.14E+00	4.25E-01	2.46E-01	1.86E+00	9.31E-01	4.40E-01	1.74E-01	1.71E+00	7.38E-01	3.24E-01
	GSSA	3.56E-05	1.74E-01	2.80E-02	3.74E-02	6.93E-05	6.55E-02	1.37E-02	1.43E-02	1.56E-05	7.78E-02	1.87E-02	2.34E-02
	LSSA	2.63E-04	1.94E+00	6.06E-01	4.82E-01	4.30E-03	1.90E+00	5.03E-01	4.55E-01	2.40E-03	1.63E+00	3.91E-01	4.19E-01
	MSSA	2.28E-05	1.02E-01	1.57E-02	2.17E-02	1.16E-04	3.99E-02	6.50E-03	9.10E-03	5.39E-06	1.81E-02	2.90E-03	3.90E-03
	NBSSA	5.79E-05	2.71E-01	2.39E-02	4.08E-02	6.88E-05	8.62E-02	1.43E-02	1.65E-02	1.11E-05	5.29E-02	9.50E-03	1.11E-02
	TSSA	1.71E-05	1.12E-01	2.11E-02	2.63E-02	4.82E-05	1.01E-01	1.58E-02	1.91E-02	1.84E-05	9.01E-02	9.90E-03	1.47E-02
F7	SSA	1.20E-02	1.64E-01	5.94E-02	2.42E-02	1.47E-02	9.75E-02	4.50E-02	1.80E-02	1.12E-02	9.09E-02	3.51E-02	1.84E-02
	CSSA	1.19E-05	2.50E-03	4.08E-04	4.52E-04	1.02E-05	2.90E-03	4.01E-04	5.31E-04	1.14E-05	3.70E-03	3.03E-04	5.81E-04
	DSSA	1.17E-05	3.00E-03	4.69E-04	5.37E-04	1.92E-05	1.20E-03	4.11E-04	3.22E-04	3.38E-06	2.70E-03	3.14E-04	4.26E-04
	GSSA	5.36E-05	7.80E-03	1.70E-03	1.50E-03	5.53E-05	4.80E-03	1.40E-03	1.30E-03	6.23E-05	5.10E-03	1.20E-03	1.10E-03
	LSSA	1.92E-05	3.00E-03	4.07E-04	4.95E-04	2.36E-07	1.10E-03	2.90E-04	2.46E-04	1.95E-05	1.80E-03	3.45E-04	3.19E-04
	MSSA	8.39E-06	1.20E-03	2.65E-04	2.25E-04	4.50E-06	5.88E-04	1.46E-04	1.29E-04	6.66E-07	6.11E-04	1.42E-04	1.39E-04
	NBSSA	8.06E-05	1.01E-02	2.10E-03	2.00E-03	9.05E-05	9.40E-03	1.50E-03	1.60E-03	2.03E-04	5.70E-03	1.50E-03	1.20E-03
	TSSA	6.53E-05	1.08E-02	2.40E-03	2.20E-03	6.92E-05	8.10E-03	1.60E-03	1.50E-03	2.10E-05	4.50E-03	1.30E-03	1.20E-03

LSSA and MSSA are almost similar. So in this case SSA and GSSA is best for no function, CSSA for four functions, DSSA and LSSA for three functions, MSSA for ten functions, NBSSA for no function and TSSA for two functions. Here, Overall MSSA again provided the best results for this dimension size too.

Dimension Size 200: In this case, for functions $F1$, LSSA provided the best results in terms of mean while the results are very competitive in terms of best, worst and standard deviation. For functions $F2$, $F4$, $F7$ and $F13$, MSSA is found to be give overall best results. For function $F2$, the results are comparable only in terms of best values where the performance of CSSA is found to be best among all the variants of algorithms. For functions $F5$ and $F6$, MSSA is capable to give best results only in terms of best values while the results of all other terms are difficult to differentiate. For function $F8$, MSSA results are best in terms of standard deviation. For functions $F9$, $F10$ and $F11$ the four algorithms CSSA, DSSA, LSSA and MSSA again provided the similar results. For function $F12$, NBSSA is able to provide best results in terms of mean only. Overall it is found that SSA is best for no function, CSSA for four functions, DSSA for three functions, GSSA and TSSA for no function, lssa for four functions, MSSA for ten functions and NBSSA for one function. Hence again it is found that MSSA is best for 200 dimension size.

Dimension Size 500: For functions $F1$, $F2$, $F4$ and $F7$, SSA was not able to approach the global optimum value while all other proposed variants provided competitive results and MSSA is found to

Table 4: Statistical results for population size of 80, 100, 120(Continued)

Function	Algorithm	Population Size 80				Population Size 100				Population Size 120			
		Best	Worst	Mean	Std	Best	Worst	Mean	Std	Best	Worst	Mean	Std
F8	SSA	-9.29E+03	-5.91E+03	-7.56E+03	7.29E+02	-8.95E+03	-6.13E+03	-7.53E+03	6.79E+02	-9.03E+03	-6.18E+03	-7.62E+03	6.97E+02
	CSSA	-1.25E+04	-6.55E+03	-1.14E+04	1.78E+03	-1.25E+04	-6.03E+03	-1.12E+04	1.76E+03	-1.25E+04	-6.85E+03	-1.18E+04	1.18E+03
	DSSA	-1.12E+04	-5.93E+03	-8.68E+03	1.73E+03	-1.16E+04	-5.07E+03	-8.93E+03	1.83E+03	-1.14E+04	-4.87E+03	-8.88E+03	1.87E+03
	GSSA	-1.25E+04	-1.25E+04	-1.25E+04	5.17E-01	-1.25E+04	-1.25E+04	-1.25E+04	2.89E-01	-1.25E+04	-1.25E+04	-1.25E+04	1.88E-01
	LSSA	-1.25E+04	-5.94E+03	-1.03E+04	1.94E+03	-1.25E+04	-5.73E+03	-1.06E+04	2.08E+03	-1.25E+04	-5.82E+03	-1.08E+04	1.96E+03
	MSSA	-1.25E+04	-1.25E+04	-1.25E+04	5.80E-01	-1.25E+04	-1.25E+04	-1.25E+04	8.72E-02	-1.25E+04	-1.25E+04	-1.25E+04	3.18E-02
	NBSSA	-1.25E+04	-1.25E+04	-1.25E+04	3.70E-01	-1.25E+04	-1.25E+04	-1.25E+04	1.71E-01	-1.25E+04	-1.25E+04	-1.25E+04	1.85E-01
	TSSA	-1.25E+04	-1.25E+04	-1.25E+04	1.11E+00	-1.25E+04	-1.25E+04	-1.25E+04	5.76E-01	-1.25E+04	-1.25E+04	-1.25E+04	4.22E-01
F9	SSA	1.79E+01	1.00E+02	3.85E+01	1.62E+01	1.69E+01	6.76E+01	3.69E+01	1.22E+01	8.95E+00	6.96E+01	3.34E+01	1.42E+01
	CSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	DSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	GSSA	0.00E+00	2.60E-03	1.83E-04	5.70E-04	0.00E+00	5.70E-03	2.34E-04	9.53E-04	4.54E-11	4.65E-04	3.41E-05	9.70E-05
	LSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	NBSSA	0.00E+00	5.20E-03	2.78E-04	8.51E-04	9.66E-13	2.00E-03	1.15E-04	3.96E-04	0.00E+00	3.89E-04	2.25E-05	7.13E-05
	TSSA	3.80E-12	1.50E-03	9.27E-05	2.53E-04	0.00E+00	2.10E-03	9.16E-05	3.28E-04	6.25E-13	6.84E-04	6.12E-05	1.30E-04
F10	SSA	2.40E-05	3.62E+00	1.49E+00	8.49E-01	2.18E-05	3.02E+00	1.28E+00	8.08E-01	1.76E-05	3.51E+00	1.49E+00	9.26E-01
	CSSA	8.88E-16	4.44E-15	1.38E-15	1.24E-15	8.88E-16	4.44E-15	1.31E-15	1.16E-15	8.88E-16	4.44E-15	1.38E-15	1.24E-15
	DSSA	8.88E-16	4.44E-15	1.45E-15	1.31E-15	8.88E-16	4.44E-15	1.38E-15	1.24E-15	8.88E-16	4.44E-15	1.17E-15	9.73E-16
	GSSA	4.61E-10	2.29E-02	2.50E-03	4.70E-03	4.15E-08	2.83E-02	2.40E-03	5.60E-03	2.18E-06	7.30E-03	1.30E-03	1.90E-03
	LSSA	8.88E-16	4.44E-15	1.81E-15	1.57E-15	8.88E-16	4.44E-15	1.38E-15	1.24E-15	8.88E-16	4.44E-15	1.45E-15	1.31E-15
	MSSA	8.88E-16	4.44E-15	1.31E-15	1.16E-15	8.88E-16	4.44E-15	1.31E-15	1.16E-15	8.88E-16	4.44E-15	9.59E-16	5.02E-16
	NBSSA	2.41E-08	2.51E-02	2.40E-03	4.70E-03	1.94E-08	1.61E-02	2.00E-03	3.80E-03	1.67E-08	6.60E-03	7.39E-04	1.40E-03
	TSSA	3.53E-09	8.33E-02	3.10E-03	1.19E-02	1.10E-08	2.33E-02	1.50E-03	3.70E-03	4.34E-07	8.60E-03	1.70E-03	2.10E-03
F11	SSA	8.64E-08	3.94E-02	8.90E-03	1.16E-02	5.47E-08	4.43E-02	1.09E-02	1.23E-02	2.55E-08	3.20E-02	7.00E-03	9.20E-03
	CSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	DSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.10E-02	7.81E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	GSSA	1.11E-16	2.24E-02	1.40E-03	4.60E-03	8.49E-13	6.31E-02	1.60E-03	8.90E-03	6.66E-16	2.00E-03	1.38E-04	4.03E-04
	LSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	NBSSA	0.00E+00	2.17E-02	5.85E-04	3.10E-03	5.59E-12	9.10E-03	3.34E-04	1.40E-03	0.00E+00	2.00E-03	1.21E-04	3.70E-04
	TSSA	0.00E+00	4.59E-02	1.90E-03	7.00E-03	2.88E-15	2.79E-02	6.50E-04	4.00E-03	0.00E+00	3.25E-04	4.28E-05	7.92E-05
F12	SSA	2.69E-01	1.12E+01	3.96E+00	2.43E+00	1.32E-02	8.31E+00	2.93E+00	1.78E+00	1.55E-02	6.32E+00	2.40E+00	1.41E+00
	CSSA	2.97E-08	8.32E-02	8.20E-03	1.82E-02	1.04E-04	1.07E-01	8.30E-03	1.83E-02	1.37E-06	1.16E-01	1.13E-02	2.10E-02
	DSSA	2.29E-02	4.40E-01	7.91E-02	6.59E-02	1.81E-02	1.59E-01	5.98E-02	2.75E-02	1.52E-02	1.58E-01	5.66E-02	2.97E-02
	GSSA	3.65E-06	2.10E-03	3.20E-04	4.38E-04	1.05E-07	2.00E-03	2.14E-04	3.73E-04	1.48E-07	1.60E-03	1.32E-04	2.59E-04
	LSSA	3.57E-06	2.03E-01	3.53E-02	3.83E-02	1.05E-04	8.94E-02	2.13E-02	2.63E-02	3.43E-05	1.81E-01	1.95E-02	3.02E-02
	MSSA	1.02E-08	3.10E-03	1.99E-04	4.63E-04	1.57E-07	1.80E-03	1.19E-04	2.87E-04	2.32E-08	1.10E-03	6.88E-05	1.87E-04
	NBSSA	1.41E-06	5.30E-03	4.02E-04	8.30E-04	1.17E-08	1.20E-03	1.42E-04	2.11E-04	2.08E-07	9.77E-04	1.41E-04	2.15E-04
	TSSA	2.18E-07	2.90E-03	4.20E-04	7.45E-04	2.64E-06	1.00E-03	1.77E-04	2.31E-04	9.64E-08	4.70E-04	1.07E-04	1.20E-04
F13	SSA	8.77E-10	4.09E+00	1.65E-01	7.16E-01	7.68E-10	7.16E+00	1.49E-01	1.01E+00	6.51E-10	1.48E-01	8.00E-03	2.10E-02
	CSSA	1.80E-03	8.46E-01	1.16E-01	2.00E-01	2.90E-03	3.50E-01	5.33E-02	7.55E-02	1.00E-03	5.51E-01	9.45E-02	1.46E-01
	DSSA	3.06E-01	2.28E+00	7.02E-01	3.02E-01	1.88E-01	9.92E-01	6.09E-01	2.15E-01	1.95E-01	1.00E+00	5.72E-01	1.82E-01
	GSSA	2.55E-05	3.94E-02	6.30E-03	9.40E-03	2.18E-06	2.99E-02	3.70E-03	5.30E-03	2.03E-06	9.10E-03	1.30E-03	1.90E-03
	LSSA	1.10E-03	1.19E+00	2.07E-01	2.85E-01	1.00E-03	1.02E+00	1.61E-01	2.45E-01	5.56E-04	8.26E-01	1.67E-01	2.14E-01
	MSSA	6.21E-07	1.15E-02	2.60E-03	3.30E-03	3.02E-07	3.90E-03	6.77E-04	1.00E-03	4.25E-07	1.58E-02	9.59E-04	2.60E-03
	NBSSA	4.76E-05	7.47E-02	6.30E-03	1.12E-02	1.27E-07	1.85E-02	2.20E-03	3.70E-03	4.27E-07	1.32E-02	1.90E-03	2.50E-03
	TSSA	7.45E-06	3.16E-02	5.10E-03	6.50E-03	4.94E-06	3.02E-02	4.00E-03	6.70E-03	1.82E-05	1.59E-02	1.50E-03	2.90E-03

be best among them. For function $F3$, the results are almost same in terms of worst, mean as well as standard deviation so comparison is to be done only in terms of best values where performance of CSSA is found to best. For function $F5$, $F6$ NBSSA and TSSA provided the best results in terms of best values respectively. For function $F8$, the results are distinguishable in standard deviation values where MSSA is found to be best. For functions $F9$, $F10$ and $F11$, the results of four algorithms CSSA, DSSA, LSSA and MSSA again attains a value near to global optimum as similar to dimension size of 10. For functions $F12$ and $F13$, NBSSA and GSSA are able to provide better results respectively. Overall it is concluded that SSA is best for no function, CSSA for four functions, DSSA and LSSA for three functions, GSSA for one function, MSSA for eight functions, NBSSA and TSSA for one function. So MSSA is again found to be overall best for this case.

Inferences from effect of dimension size: From the results it has been analyzed that SSA is only best for one function $F6$ beyond dimension size 50. After this SSA is not able to provide best results for higher dimensions. So all the proposed algorithms with the help of different mutation operators provide better results for lower dimension sizes such as 10 and 30. Although the performance of proposed algorithms degrades up to certain extent at higher dimension sizes but deviation in results are under marginal limits. Here, it can be seen that proposed algorithms especially MSSA is found to be best for higher dimension sizes.

5.5. Comparison of best proposed with respect to other algorithms

5.5.1. Experimental Testing

Comparison with SSA Variants: This section details about the results for various SSA variants with respect to the proposed MSSA algorithm. The results are presented in terms of mean

Table 5: Statistical results for Dimension size of $10, 30, 50$

Function	Algorithm	Dimension Size 10				Dimension Size 30				Dimension Size 50			
		Best	Worst	Mean	Std	Best	Worst	Mean	Std	Best	Worst	Mean	Std
F1	SSA	3.70E-10	1.76E-09	8.23E-10	2.76E-10	1.73E-08	5.66E-08	2.94E-08	9.59E-09	1.50E-03	1.15E-01	2.96E-02	2.92E-02
	CSSA	7.02E-114	3.13E-96	8.95E-98	4.66E-97	2.82E-109	1.87E-90	3.74E-92	2.64E-91	5.34E-105	2.10E-90	9.36E-92	3.31E-91
	DSSA	5.06E-112	6.62E-97	1.48E-98	9.38E-98	3.25E-106	2.54E-88	5.25E-90	3.59E-89	5.20E-103	2.52E-89	1.01E-90	4.06E-90
	GSSA	8.48E-13	5.07E-02	2.10E-03	7.50E-03	6.12E-12	2.28E-01	9.90E-03	3.30E-02	1.67E-13	4.49E-01	1.11E-02	6.35E-02
	LSSA	1.63E-112	1.21E-95	2.63E-97	1.71E-96	5.33E-110	3.29E-90	1.13E-91	4.98E-91	3.66E-108	2.26E-88	4.89E-90	3.19E-89
	MSSA	5.67E-112	1.27E-97	2.61E-99	1.80E-98	7.37E-107	1.43E-92	3.12E-94	2.03E-93	7.39E-108	1.14E-89	2.31E-91	1.61E-90
	NBSSA	2.06E-15	3.67E-02	2.10E-03	6.80E-03	3.86E-12	2.77E-02	2.20E-03	5.50E-03	6.12E-14	3.75E-01	1.32E-02	5.46E-02
	TSSA	1.99E-12	3.15E-01	1.02E-02	4.61E-02	8.84E-11	2.84E-01	8.70E-03	4.07E-02	3.20E-10	5.05E-01	2.63E-02	8.26E-02
F2	SSA	3.89E-06	2.70E-01	5.40E-03	3.83E-02	1.08E-01	7.37E+00	1.59E+00	1.61E+00	2.08E+00	1.31E+01	6.38E+00	2.48E+00
	CSSA	1.05E-59	1.08E-51	6.20E-53	2.21E-52	2.99E-56	1.54E-48	5.09E-50	2.33E-49	9.89E-56	7.55E-47	1.53E-48	1.06E-47
	DSSA	4.08E-57	1.29E-50	4.52E-52	1.99E-51	2.89E-57	9.79E-49	6.07E-50	1.91E-49	3.06E-55	7.90E-47	2.86E-48	1.32E-47
	GSSA	4.13E-09	5.99E-02	6.60E-03	1.29E-02	1.60E-05	4.00E-01	1.91E-02	5.86E-02	2.53E-06	4.11E-01	3.56E-02	6.97E-02
	LSSA	1.02E-61	8.54E-50	1.93E-51	1.21E-50	1.18E-58	6.84E-49	2.30E-50	1.00E-49	6.15E-57	4.92E-47	1.29E-48	7.09E-48
	MSSA	2.45E-60	6.13E-52	1.92E-53	8.87E-53	1.69E-58	6.65E-49	1.38E-50	9.40E-50	1.42E-57	2.80E-48	5.88E-50	3.95E-49
	NBSSA	2.08E-06	8.19E-02	6.60E-03	1.51E-02	9.68E-09	1.48E-01	1.69E-02	2.77E-02	1.21E-05	2.69E-01	3.50E-02	6.21E-02
	TSSA	3.27E-07	8.14E-02	5.40E-03	1.35E-02	1.50E-05	1.83E-01	2.21E-02	3.28E-02	8.99E-06	1.73E-01	3.21E-02	4.52E-02
F3	SSA	9.59E-10	6.26E-08	6.48E-09	9.86E-09	2.89E+02	2.21E+03	8.92E+02	4.32E+02	1.96E+03	1.60E+04	6.90E+03	3.25E+03
	CSSA	2.12E-56	2.25E-20	4.51E-22	3.19E-21	4.76E-31	2.05E-04	4.53E-06	2.90E-05	5.57E-21	2.66E-01	9.00E-03	4.01E-02
	DSSA	1.86E-63	2.67E-19	6.34E-21	3.83E-20	4.79E-20	1.42E-04	3.12E-06	2.02E-05	4.67E-17	1.80E-01	5.00E-03	2.67E-02
	GSSA	4.06E-05	4.69E+01	1.33E+00	6.62E+00	3.04E-02	3.25E+02	4.40E+01	8.17E+01	4.70E-03	1.37E+03	1.48E+02	3.11E+02
	LSSA	2.17E-59	1.57E-21	5.05E-23	2.34E-22	6.84E-27	1.77E-05	4.06E-07	2.50E-06	1.49E-15	6.16E-01	1.34E-02	8.72E-02
	MSSA	1.31E-64	6.99E-27	2.84E-28	1.21E-27	6.24E-42	2.53E-11	5.69E-13	3.58E-12	1.76E-36	2.18E-07	6.41E-09	3.29E-08
	NBSSA	4.14E-07	1.92E+01	1.18E+00	3.24E+00	4.43E-07	5.45E+02	4.80E+01	1.01E+02	4.25E-08	1.58E+03	1.10E+02	2.46E+02
	TSSA	3.32E-15	9.99E+00	6.38E-01	1.65E+00	3.40E-03	3.63E+02	2.39E+01	5.72E+01	1.80E-02	1.42E+03	2.05E+02	3.08E+02
F4	SSA	1.17E-05	3.05E-05	1.80E-05	4.62E-06	2.87E+00	1.50E+01	8.24E+00	3.13E+00	1.04E+01	2.32E+01	1.63E+01	2.85E+00
	CSSA	1.44E-52	7.31E-43	5.46E-44	1.48E-43	2.11E-49	8.65E-40	3.19E-41	1.36E-40	4.73E-48	2.05E-39	8.35E-41	3.33E-40
	DSSA	7.59E-54	9.73E-40	2.13E-41	1.37E-40	1.02E-49	2.53E-38	7.58E-40	3.90E-39	1.79E-45	5.86E-37	2.38E-38	1.07E-37
	GSSA	1.28E-05	1.54E-01	1.32E-02	2.74E-02	5.32E-08	1.47E-01	1.50E-02	2.97E-02	1.07E-05	8.67E-02	1.53E-02	2.08E-02
	LSSA	1.55E-51	1.07E-39	3.71E-41	1.80E-40	9.39E-49	4.64E-39	1.79E-40	7.11E-40	1.28E-50	3.77E-38	1.25E-39	5.75E-39
	MSSA	9.26E-52	1.43E-42	3.42E-44	2.03E-43	1.26E-51	3.83E-41	1.80E-42	6.58E-42	1.36E-49	1.77E-38	4.17E-40	2.51E-39
	NBSSA	5.43E-06	4.56E-02	4.20E-03	9.10E-03	1.94E-09	8.60E-02	9.50E-03	1.83E-02	5.83E-12	8.89E-02	1.15E-02	1.82E-02
	TSSA	1.11E-06	1.36E-01	1.07E-02	2.48E-02	5.59E-06	1.84E-01	1.32E-02	2.87E-02	1.18E-05	1.83E-01	1.98E-02	3.60E-02
F5	SSA	4.52E+00	2.12E+03	1.17E+02	3.37E+02	1.98E+01	1.30E+03	1.47E+02	2.65E+02	9.84E+01	9.53E+03	7.08E+02	1.36E+03
	CSSA	2.68E-02	8.94E+00	7.09E+00	3.25E+00	1.39E+00	2.88E+01	2.64E+01	6.92E+00	3.47E+00	4.86E+01	4.35E+01	1.26E+01
	DSSA	7.95E+00	8.94E+00	8.64E+00	2.71E-01	2.84E+01	2.88E+01	2.87E+01	8.26E-02	4.83E+01	4.87E+01	4.85E+01	9.04E-02
	GSSA	3.92E-02	6.33E+00	1.50E+00	1.51E+00	2.79E-01	2.89E+01	9.57E+00	7.85E+00	1.32E-01	4.80E+01	1.34E+01	1.17E+01
	LSSA	1.32E-01	8.93E+00	7.33E+00	3.11E+00	2.89E-01	2.88E+01	2.68E+01	6.45E+00	1.99E+00	4.86E+01	4.38E+01	1.31E+01
	MSSA	1.07E-02	8.88E+00	2.55E+00	3.78E+00	7.05E-02	2.87E+01	1.38E+01	1.29E+01	4.04E-02	4.86E+01	2.63E+01	2.21E+01
	NBSSA	2.50E-03	8.95E+00	2.33E+00	2.38E+00	7.57E-02	2.87E+01	7.53E+00	6.80E+00	7.44E-02	4.87E+01	1.67E+01	1.50E+01
	TSSA	4.53E-02	7.81E+00	2.25E+00	2.13E+00	2.97E-01	2.50E+01	7.32E+00	6.89E+00	3.34E-01	4.85E+01	1.61E+01	1.32E+01
F6	SSA	1.67E-10	1.73E-09	7.51E-10	3.22E-10	1.56E-08	8.29E-08	3.43E-08	1.43E-08	5.13E-04	3.08E-01	4.38E-02	7.12E-02
	CSSA	1.02E-05	6.20E-01	1.25E-01	1.68E-01	2.00E-03	2.49E+00	6.75E-01	7.18E-01	3.20E-03	3.41E+00	1.40E+00	1.09E+00
	DSSA	8.50E-03	8.16E-01	2.34E-01	2.01E-01	7.44E-01	2.73E+00	1.56E+00	5.04E-01	1.65E+00	5.97E+00	3.35E+00	8.55E-01
	GSSA	8.14E-04	2.15E-01	3.57E-02	3.81E-02	4.00E-03	8.86E-01	1.36E-01	1.88E-01	2.30E-03	1.09E+00	2.00E-01	2.53E-01
	LSSA	6.27E-04	8.57E-01	1.93E-01	1.61E-01	5.26E-02	2.79E+00	1.21E+00	7.09E-01	5.90E-03	5.93E+00	2.10E+00	5.23E-01
	MSSA	1.71E-04	6.70E-02	1.11E-02	1.59E-02	3.60E-03	8.25E-01	1.05E-01	1.43E-01	4.30E-03	1.12E+00	1.93E-01	2.29E-01
	NBSSA	8.90E-04	2.76E-01	4.08E-02	5.60E-02	4.50E-03	1.45E+00	2.09E-01	3.41E-01	5.30E-03	1.72E+00	2.28E-01	2.72E-01
	TSSA	6.40E-04	3.81E-01	4.30E-02	6.77E-02	6.33E-04	1.18E+00	1.64E-01	2.40E-01	3.50E-03	1.70E+00	2.27E-01	3.23E-01
F7	SSA	2.10E-03	4.03E-02	1.27E-02	9.90E-03	2.56E-02	2.61E-01	1.20E-01	4.40E-02	1.44E-01	7.42E-01	3.94E-01	1.32E-01
	CSSA	5.25E-05	5.40E-03	1.00E-03	1.00E-03	1.09E-05	5.30E-03	7.96E-04	1.10E-03	5.27E-05	3.40E-03	5.78E-04	5.96E-04
	DSSA	1.55E-05	2.70E-03	8.96E-04	7.41E-04	5.05E-05	2.60E-03	6.67E-04	5.86E-04	3.68E-05	9.20E-03	1.00E-03	1.60E-03
	GSSA	1.06E-04	9.60E-03	2.60E-03	2.40E-03	6.17E-05	1.27E-02	3.80E-03	3.10E-03	4.16E-04	3.35E-02	8.40E-03	7.40E-03
	LSSA	1.75E-05	4.90E-03	9.00E-04	9.73E-04	1.61E-06	3.10E-03	6.60E-04	7.82E-04	1.06E-05	4.80E-03	7.53E-04	1.10E-03
	MSSA	3.72E-06	1.90E-03	4.04E-04	4.06E-04	1.11E-05	2.70E-03	5.31E-04	5.52E-04	7.28E-07	4.80E-03	5.31E-04	8.34E-04
	NBSSA	5.51E-05	1.25E-02	2.30E-03	2.20E-03	3.36E-04	2.25E-02	5.70E-03	5.30E-03	2.19E-04	3.98E-02	8.30E-03	9.20E-03
	TSSA	1.57E-05	2.11E-02	3.70E-03	3.60E-03	1.93E-04	2.38E-02	5.30E-03	5.20E-03	1.87E-04	4.43E-02	6.30E-03	8.20E-03

and standard deviation values in Table 7 and the dimension size for this set is 10 (as used by the other algorithms under comparison). Three versions of SSA namely salp swarm algorithm based on particle swarm optimization (SSAPSO) (Ibrahim et al., 2019), chaotic salp swarm algorithm (CSSA) (Majhi et al., 2019) and chaotic salp swarm algorithm with logistic map (CSSA5) (Sayed et al., 2018) have been employed to test the proposed MSSA. The results of mean and standard deviation are taken from their respective papers and analysis is performed on CEC 2005 unimodal and multi-modal benchmark problems. From the results, it can be seen that for $F1$, $F2$, $F4$, $F5$, $F7$, $F8$, $F9$, $F10$, $F11$ and $F12$, MSSA is found to be the best whereas for $F3$, SSAPSO was found as the best. Apart from that CSSA was found best on $F6$ and $F13$. Thus overall we can say that, CSSA5 did not performed well on any of the functions, SSAPSO for one function, CSSA for two functions and for rest of the ten functions, MSSA performed exceptionally well. From the statistical Friedman rank (f-rank) (Tejani et al., 2018) and Wilcoxon rank-sum test further prove the significance of the proposed algorithm.

Comparison with other State-of-the-art algorithms: In this section, the simulation results of different algorithms as given in Table 8 have been discussed. The results are presented in terms of best, worst, mean and standard deviation values for 30 Dimension problems. It has been analyzed from the table that for functions $F1$, $F2$, $F3$, $F4$ and $F5$, the performance of proposed MSSA is found to be best and none of the algorithm is capable to reach the performance of MSSA. For

Table 5: Statistical results for dimension size of $10, 30, 50$(Continued)

Function	Algorithm	Dimension Size 10				Dimension Size 30				Dimension Size 50			
		Best	Worst	Mean	Std	Best	Worst	Mean	Std	Best	Worst	Mean	Std
F8	SSA	-3.61E+03	-2.18E+03	-2.83E+03	3.62E+02	-9.04E+03	-5.98E+03	-7.36E+03	6.53E+02	-1.44E+04	-9.59E+03	-1.20E+04	1.04E+03
	CSSA	-4.18E+03	-2.79E+03	-4.05E+03	3.57E+02	-1.25E+04	+7.36E+03	-1.14E+04	1.58E+03	-2.09E+04	-1.22E+04	-1.78E+04	2.99E+03
	DSSA	-3.94E+03	-4.17E+03	-3.16E+03	4.43E+02	-1.25E+04	-5.45E+03	-8.48E+03	1.92E+03	-1.76E+04	-7.43E+03	-1.18E+04	2.76E+03
	GSSA	-4.18E+03	-4.17E+03	-4.18E+03	2.07E+00	-1.25E+04	-1.25E+04	-1.25E+04	3.99E+00	-2.09E+04	-2.07E+04	-2.09E+04	2.25E+01
	LSSA	-4.18E+03	-2.46E+03	-3.60E+03	6.53E+02	-1.25E+04	-5.87E+03	-1.01E+04	2.14E+03	-2.09E+04	-6.88E+03	-1.53E+04	4.07E+03
	MSSA	-4.18E+03	-4.18E+03	-4.18E+03	1.30E-01	-1.25E+04	-1.25E+04	-1.25E+04	1.01E+00	-2.09E+04	-2.09E+04	-2.09E+04	1.50E+00
	NBSSA	-4.18E+03	-4.18E+03	-4.18E+03	1.17E+00	-1.25E+04	-1.25E+04	-1.25E+04	4.18E+00	-2.09E+04	-2.09E+04	-2.09E+04	6.93E+00
	TSSA	-4.18E+03	-4.17E+03	-4.18E+03	1.84E+00	-1.25E+04	-1.25E+04	-1.25E+04	4.55E+00	-2.09E+04	-2.08E+04	-2.09E+04	1.32E+01
F9	SSA	5.96E+00	4.37E+01	1.82E+01	8.23E+00	1.98E+01	8.35E+01	4.81E+01	1.48E+01	3.58E+01	1.35E+02	7.84E+01	2.33E+01
	CSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	DSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	GSSA	1.20E-11	5.91E-02	2.00E-03	8.60E-03	1.29E-10	7.22E-02	4.10E-03	1.21E-02	4.54E-13	3.70E-01	1.19E-02	5.28E-02
	LSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	NBSSA	9.11E-11	1.29E-02	6.87E-04	2.20E-03	0.00E+00	3.21E-02	2.70E-03	6.30E-03	0.00E+00	3.80E-02	2.80E-03	7.10E-03
	TSSA	1.47E-12	3.65E-02	1.20E-03	5.30E-03	9.76E-11	9.69E-02	5.80E-03	1.84E-02	1.69E-10	8.02E-02	6.10E-03	1.65E-02
F10	SSA	6.19E-06	3.73E+00	7.32E-01	9.70E-01	9.31E-01	4.07E+00	2.23E+00	7.30E-01	2.01E+00	5.97E+00	3.81E+00	9.51E-01
	CSSA	8.88E-16	4.44E-15	1.74E-15	1.53E-15	8.88E-16	4.44E-15	1.52E-15	1.37E-15	8.88E-16	4.44E-15	1.45E-15	1.31E-15
	DSSA	8.88E-16	4.44E-15	1.52E-15	1.37E-15	8.88E-16	4.44E-15	1.52E-15	1.37E-15	8.88E-16	4.44E-15	1.52E-15	1.37E-15
	GSSA	1.92E-06	9.89E-02	1.22E-02	2.15E-02	5.03E-06	1.52E-01	1.53E-02	3.18E-02	2.45E-07	1.17E-01	1.34E-02	2.51E-02
	LSSA	8.88E-16	4.44E-15	1.45E-15	1.31E-15	8.88E-16	4.44E-15	1.81E-15	1.57E-15	8.88E-16	4.44E-15	1.52E-15	1.37E-15
	MSSA	8.88E-16	4.44E-15	1.74E-15	1.53E-15	8.88E-16	4.44E-15	1.59E-15	1.43E-15	8.88E-16	4.44E-15	1.66E-15	1.48E-15
	NBSSA	4.44E-06	2.35E-01	1.15E-02	3.53E-02	2.12E-09	5.37E-02	9.30E-03	1.48E-02	1.92E-12	5.01E-02	8.70E-03	1.22E-02
	TSSA	6.86E-07	2.07E-01	1.62E-02	3.78E-02	3.42E-06	1.12E-01	1.25E-02	2.14E-02	2.03E-08	8.42E-02	9.80E-03	1.59E-02
F11	SSA	5.91E-02	7.62E-01	2.65E-01	1.55E-01	2.54E-05	4.18E-02	1.00E-02	9.00E-03	4.51E-02	3.67E-01	1.53E-01	6.23E-02
	CSSA	0.00E+00	9.62E-01	5.60E-02	1.97E-01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	DSSA	0.00E+00	7.26E-01	1.45E-02	1.02E-01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	GSSA	9.54E-11	2.63E-01	1.29E-02	4.50E-02	8.77E-15	9.22E-01	5.44E-02	1.79E-01	5.17E-11	3.08E-01	1.97E-02	5.67E-02
	LSSA	0.00E+00	4.61E-01	9.20E-03	6.53E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	NBSSA	3.03E-09	9.92E-01	6.28E-02	2.19E-01	1.35E-10	2.77E-01	8.80E-03	3.99E-02	1.56E-14	3.50E-01	2.65E-02	7.25E-02
	TSSA	5.32E-15	8.93E-01	1.12E-01	2.45E-01	0.00E+00	4.66E-02	4.90E-03	1.12E-02	4.72E-09	2.15E-01	1.21E-02	3.37E-02
F12	SSA	4.54E-12	3.52E+00	5.21E-01	8.56E-01	1.54E+00	1.05E+01	5.47E+00	2.16E+00	3.44E+00	2.04E+01	9.50E+00	4.00E+00
	CSSA	1.10E-04	3.83E-01	4.99E-02	6.91E-02	1.21E-04	6.09E-01	4.24E-02	9.06E-02	1.34E-05	1.86E-01	2.55E-02	4.24E-02
	DSSA	1.90E-03	1.57E+00	9.42E-02	2.20E-01	2.06E-02	6.59E-01	1.08E-01	1.08E-01	3.64E-02	4.37E-01	1.09E-01	7.71E-02
	GSSA	2.69E-05	9.60E-02	4.70E-03	1.36E-02	4.13E-05	2.01E-02	2.50E-03	3.70E-03	1.77E-05	7.30E-03	1.60E-03	1.70E-03
	LSSA	4.46E-05	1.55E-01	5.69E-02	4.63E-02	2.71E-07	3.47E-01	4.43E-02	6.54E-02	6.59E-04	1.97E-01	4.08E-02	4.53E-02
	MSSA	1.30E-06	5.60E-03	9.77E-04	1.40E-03	4.32E-06	1.11E-02	1.50E-03	2.40E-03	2.24E-05	1.02E-02	1.40E-03	1.90E-03
	NBSSA	5.80E-05	3.77E-02	3.40E-03	6.00E-03	6.76E-05	8.40E-02	2.10E-03	2.10E-03	2.15E-06	8.30E-03	1.70E-03	1.90E-03
	TSSA	1.92E-05	8.59E-02	6.00E-03	1.39E-02	2.20E-05	1.80E-02	1.80E-03	3.20E-03	7.71E-05	1.29E-02	1.80E-03	2.40E-03
F13	SSA	3.15E-11	2.10E-02	2.80E-03	5.70E-03	5.50E-06	4.46E+01	1.06E+01	1.27E+01	3.47E+01	9.58E+01	6.48E+01	1.48E+01
	CSSA	2.42E-06	3.71E-01	9.97E-02	1.07E-01	1.80E-03	1.53E+00	1.91E-01	3.04E-01	2.34E-04	1.44E+00	2.81E-01	3.32E-01
	DSSA	4.10E-02	6.94E-01	2.10E-01	1.38E-01	2.42E-01	1.36E+00	8.38E-01	2.52E-01	6.38E-01	2.80E+00	1.60E+00	4.33E-01
	GSSA	5.78E-05	7.94E-02	1.59E-02	1.87E-02	4.72E-04	1.42E-01	3.61E-02	4.01E-02	1.57E-04	1.99E-01	3.71E-02	4.81E-02
	LSSA	2.34E-04	5.08E-01	1.42E-01	1.28E-01	1.75E-04	1.22E+00	3.33E-01	3.65E-01	4.27E-04	2.89E+00	6.25E-01	6.99E-01
	MSSA	1.42E-06	7.49E-02	9.40E-03	1.68E-02	3.45E-06	1.87E-01	2.58E-02	3.72E-02	3.28E-05	2.29E-01	3.49E-02	4.82E-02
	NBSSA	3.54E-04	6.12E-02	1.21E-02	1.25E-02	3.43E-04	1.44E-01	3.56E-02	3.40E-02	6.15E-04	2.25E-01	4.60E-02	4.73E-02
	TSSA	2.47E-04	7.07E-02	1.40E-02	1.43E-02	3.03E-04	1.70E-01	2.68E-02	3.21E-02	2.18E-04	1.78E-01	3.87E-02	4.53E-02

functions $F6$ and $F13$, proposed MSSA results are competitive but PSOGSA provided the best results. For functions $F7$ and $F8$, MSSA is the best algorithm as compared to other algorithms. For functions $F9$ and $F11$, the proposed MSSA attains a value of global optimum solution. For functions $F10$ and $F12$, MSSA, SCCSA, GWO and FA are capable to approach near global optima while rest of the algorithms suffer from local optima stagnation problem, but MSSA is found to be best. For the fixed dimension functions such as $F14$ and $F15$ the results are comparable only in terms of standard deviation where DE and SSA provided the best results respectively. In the same way, for function $F16$, the results are differentiated only in terms of mean values and FPA performance is found to be better among all the other algorithms under consideration. So overall, proposed MSSA is found to be best for 11 benchmark functions out of the total 16 functions, SSA for 1 functions, DE for 1 function, PSOGSA for 2 function, FPA for 1 function and BA, DA, FA, SCCSA and GWO for none of the function.

5.5.2. Statistical Testing

For statistical performance evaluation of proposed MSSA, two statistically significance tests namely Wilcoxon rank-sum test (Wilcoxon et al., 1970) and Mann-Whitney U rank sum test (Tersoff, 1988) are performed. In Wilcoxon rank-sum test, two different samples of best values are used and correspondingly p-value is generated. The two algorithms are compared at 5% significance level to show the statistical relevance between them. The results for algorithms under comparison are shown in Table 8. Here the proposed MSSA is compared with other algorithms and comparison is performed as MSSA with BA, MSSA/DA, MSSA/DE, MSSA/FA, MSSA/FPA, MSSA/GWO, MSSA/SCCSA, MSSA/PSOGSA and MSSA/SSA to prove the efficiency of MSSA statistically. The

Table 6: Statistical results for Dimension size of $100, 200, 500$

Function	Algorithm	Dimension Size 100				Dimension Size 200				Dimension Size 500			
		Best	Worst	Mean	Std	Best	Worst	Mean	Std	Best	Worst	Mean	Std
F1	SSA	3.04E+02	9.56E+02	5.98E+02	1.75E+02	9.51E+03	1.53E+04	1.19E+04	1.43E+03	6.24E+04	8.15E+04	7.12E+04	4.78E+03
	CSSA	4.58E-106	9.32E-85	1.91E-86	1.31E-85	7.18E-108	6.42E-83	1.28E-84	9.08E-84	9.04E-100	1.49E-86	8.68E-88	2.98E-87
	DSSA	2.02E-103	2.11E-85	5.09E-87	3.00E-86	8.96E-101	6.12E-86	2.07E-87	9.96E-87	2.60E-98	4.45E-86	2.02E-87	7.59E-87
	GSSA	9.87E-13	2.37E+00	7.56E-02	3.38E-01	3.09E-07	1.51E+00	6.84E-02	2.21E-01	1.50E-09	2.18E+01	4.80E-01	3.08E+00
	LSSA	3.03E-102	1.10E-86	2.21E-88	1.56E-87	3.31E-100	1.62E-88	9.69E-90	3.13E-89	1.79E-100	3.43E-83	6.89E-85	4.85E-84
	MSSA	3.07E-107	3.32E-92	1.91E-93	7.14E-93	5.49E-105	2.29E-86	4.59E-88	3.23E-87	7.63E-102	9.26E-87	4.31E-88	1.76E-87
	NBSSA	2.01E-10	3.39E-01	3.00E-02	6.37E-02	8.21E-11	3.50E-01	1.81E-02	5.88E-02	6.66E-10	1.36E+00	7.97E-02	2.50E-01
	TSSA	1.22E-09	7.15E-01	2.81E-02	1.06E-01	2.04E-08	3.31E-01	1.85E-02	5.16E-02	1.79E-07	1.36E+00	7.67E-02	2.18E-01
F2	SSA	2.28E+01	5.45E+01	3.73E+01	6.10E+00	1.10E+02	1.57E+02	1.30E+02	9.84E+00	4.31E+02	5.11E+02	4.68E+00	1.89E+01
	CSSA	4.30E-54	2.61E-46	5.50E-48	3.70E-47	2.38E-54	4.24E-47	2.74E-48	7.81E-48	1.91E-54	7.69E-46	3.51E-47	1.45E-46
	DSSA	1.24E-53	3.59E-47	3.25E-48	8.28E-48	2.38E-54	1.50E-46	8.27E-48	2.69E-47	3.74E-54	3.65E-46	1.96E-47	6.84E-47
	GSSA	5.61E-06	5.47E-01	9.11E-02	1.30E-01	2.35E-08	5.90E-01	6.73E-02	1.04E-01	1.54E-05	2.41E+00	3.01E-01	4.78E-01
	LSSA	3.60E-54	2.14E-47	8.36E-49	3.36E-48	9.27E-55	1.02E-46	6.51E-48	2.13E-47	3.85E-53	6.13E-46	1.80E-47	8.79E-47
	MSSA	3.40E-57	6.89E-49	3.88E-50	1.23E-49	1.40E-58	3.22E-48	1.90E-49	6.20E-49	2.10E-55	1.66E-46	4.60E-48	2.41E-47
	NBSSA	3.46E-05	1.36E+00	7.53E-02	2.04E-01	1.03E-05	1.11E+00	1.08E-01	2.16E-01	4.45E-05	1.70E+00	2.20E-01	3.25E-01
	TSSA	7.11E-06	2.14E-01	4.18E-02	5.26E-02	2.10E-03	1.14E+00	1.37E-01	1.96E-01	8.46E-05	1.94E+00	2.41E-01	4.60E-01
F3	SSA	1.11E+04	9.57E+04	3.74E+04	2.04E+04	4.80E+04	4.06E+05	1.55E+05	7.74E+04	2.76E+05	2.12E+06	9.98E+05	4.57E+05
	CSSA	3.59E-13	2.02E+01	1.10E+00	3.06E+00	2.89E-22	2.31E+04	7.15E+02	3.30E+03	3.89E-27	7.36E+05	5.57e+04	1.26E+05
	DSSA	2.99E-08	2.44E+02	9.89E+00	3.75E+01	2.53E-04	9.11E+03	7.44E+02	1.68E+03	1.54E-01	2.86E+05	2.40E+04	4.92E+04
	GSSA	4.10E-03	1.16E+04	2.03E+03	2.89E+03	1.50E-03	3.77E+04	9.93E+03	1.12E+04	1.45E+02	1.15E+06	1.67E+05	2.33E+05
	LSSA	1.56E-07	1.19E+03	4.03E+01	1.78E+02	1.32E-02	1.02E+04	7.21E+02	1.91E+03	1.27E+00	7.40E+05	4.86E+04	1.41E+05
	MSSA	1.79E-19	4.15E-02	8.77E-04	5.90E-03	7.39E-18	2.20E+02	4.62E+00	3.11E+01	3.79E-09	1.81E+02	1.34E+01	3.94E+01
	NBSSA	1.40E-01	2.81E+04	2.71E+03	5.20E+03	9.67E-01	6.82E+04	1.23E+04	1.53E+04	1.18E+03	1.34E+06	1.52E+05	2.28E+05
	TSSA	1.57E+00	1.45E+04	2.08E+03	3.21E+03	1.89E+00	1.10E+05	1.20E+04	2.02E+04	7.27E-02	1.44E+06	1.50E+05	2.33E+05
F4	SSA	1.90E+01	3.21E+01	2.41E+01	2.80E+00	2.54E+01	3.62E+01	3.03E+01	2.59E+00	3.17E+01	4.30E+01	3.60E+01	2.32E+00
	CSSA	3.45E-48	2.68E-38	1.02E-39	4.15E-39	9.95E-46	5.76E-38	1.88E-39	8.42E-39	1.57E-46	6.08E-36	2.08E-37	9.26E-37
	DSSA	1.13E-45	4.00E-38	1.49E-39	5.92E-39	4.18E-46	4.18E-36	8.49E-38	5.92E-37	3.15E-43	4.29E-35	1.02E-36	6.10E-36
	GSSA	1.91E-06	9.74E-02	1.36E-02	1.87E-02	2.23E-06	3.87E-01	3.50E-02	6.51E-02	5.67E-06	3.79E-01	3.46E-02	6.40E-02
	LSSA	3.56E-48	4.31E-36	8.94E-38	6.09E-37	1.97E-45	4.51E-37	1.35E-38	6.54E-38	2.03E-44	2.72E-36	8.89E-38	4.04E-37
	MSSA	1.62E-47	4.64E-37	1.02E-38	6.56E-38	3.52E-48	6.29E-38	1.34E-39	8.94E-39	4.31E-46	2.57E-37	7.90E-39	3.75E-38
	NBSSA	3.41E-06	2.21E-01	2.33E-02	3.73E-02	2.32E-06	2.02E-01	3.45E-02	5.05E-02	1.32E-04	2.12E-01	3.00E-02	3.85E-02
	TSSA	1.31E-06	1.12E-01	1.83E-02	2.65E-02	4.10E-05	1.15E-01	2.00E-02	2.53E-02	8.54E-04	3.31E-01	4.39E-02	5.94E-02
F5	SSA	1.10E+04	1.63E+05	5.37E+04	3.03E+04	6.62E+05	3.86E+06	2.00E+06	6.16E+05	1.60E+07	3.07E+07	2.22E+07	3.05E+06
	CSSA	4.15E-01	9.83E+01	8.69E+01	2.61E+01	1.19E-01	1.97E+02	1.79E+02	5.08E+01	1.53E+01	4.96E+02	4.83E+02	6.86E+01
	DSSA	9.80E+01	9.84E+01	9.82E+01	7.55E-02	1.97E+02	1.98E+02	1.97E+02	1.24E-01	4.95E+02	4.96E+02	4.96E+02	3.32E-01
	GSSA	1.65E-01	8.83E+01	2.49E+01	2.43E+01	3.22E+00	2.04E+02	7.94E+01	6.25E+01	4.35E-01	5.17E+02	1.84E+02	1.65E+02
	LSSA	1.64E+01	9.84E+01	9.65E+01	1.15E+01	9.21E+01	1.98E+02	1.93E+02	1.76E+01	9.41E+01	4.96E+02	4.75E+02	8.06E+01
	MSSA	1.85E-02	9.81E+01	4.84E+01	4.39E+01	1.65E-01	1.97E+02	6.62E+01	8.35E+01	4.35E-01	4.96E+02	2.61E+02	2.18E+02
	NBSSA	4.36E-01	9.90E+01	3.08E+01	2.72E+01	2.48E+00	1.97E+02	6.72E+01	5.99E+01	4.61E-02	5.02E+02	2.17E+02	1.72E+02
	TSSA	1.31E-01	1.02E+02	2.79E+01	2.61E+01	1.10E+00	1.97E+02	7.40E+01	5.43E+01	2.94E+00	5.17E+02	1.82E+02	1.52E+02
F6	SSA	4.12E+02	1.08E+03	6.87E+02	1.85E+02	8.27E+03	1.45E+04	1.15E+04	1.51E+03	6.43E+04	8.56E+04	7.19E+04	4.38E+03
	CSSA	3.16E-01	1.04E+01	4.16E+00	2.80E+00	4.90E-02	2.04E+01	8.55E+00	5.73E+00	2.36E-01	5.96E+01	1.66E+01	1.56E+01
	DSSA	5.20E+00	1.15E+01	7.90E+00	1.66E+00	9.23E+00	2.51E+01	1.71E+01	3.49E+00	3.60E+01	6.41E+01	4.66E+01	6.10E+00
	GSSA	1.58E-02	1.95E+00	4.78E-01	4.38E-01	2.63E-02	3.44E+00	9.43E-01	9.70E-01	1.18E-01	1.31E+01	2.66E+00	2.79E+00
	LSSA	1.35E-01	1.29E+01	6.03E+00	2.98E+00	9.39E-02	2.35E+01	1.15E+01	6.98E+00	1.17E+00	7.08E+01	3.17E+01	1.99E+01
	MSSA	1.02E-02	4.33E+00	5.60E-01	7.16E-01	3.70E-03	4.10E+00	1.12E+00	1.01E+00	5.24E-02	2.23E+01	3.49E+00	3.85E+00
	NBSSA	1.58E-02	3.53E+00	5.84E-01	6.70E-01	2.71E-02	4.67E+00	1.25E+00	1.19E+00	5.52E-02	1.05E+01	2.53E+00	2.44E+00
	TSSA	6.40E-03	2.52E+00	5.01E-01	5.08E-01	8.05E-02	5.72E+00	8.64E-01	1.18E+00	5.11E-02	1.85E+01	3.53E+00	3.25E+00
F7	SSA	7.05E-01	2.60E+00	1.85E+00	4.18E-01	7.30E+00	1.89E+01	1.17E+01	2.48E+00	1.26E+02	2.24E+02	1.68E+02	2.14E+01
	CSSA	1.24E-06	3.70E-03	6.65E-04	8.56E-04	1.92E-05	4.10E-03	5.11E-04	6.27E-04	2.72E-05	7.20E-03	7.47E-04	1.30E-03
	DSSA	1.95E-05	6.10E-03	1.20E-03	1.50E-03	2.58E-05	3.20E-03	6.72E-04	7.28E-04	1.69E-05	5.20E-03	1.30E-03	1.40E-03
	GSSA	4.24E-04	5.09E-02	9.20E-03	9.40E-03	3.18E-04	6.31E-02	1.57E-02	1.37E-02	9.94E-05	6.38E-02	1.37E-02	1.34E-02
	LSSA	9.21E-06	4.10E-03	6.81E-04	7.31E-04	2.65E-06	8.90E-03	8.01E-04	1.40E-03	9.50E-06	8.60E-03	9.81E-04	1.40E-03
	MSSA	1.61E-05	4.10E-03	4.86E-04	6.59E-04	1.53E-05	3.00E-03	4.67E-04	5.94E-04	1.62E-05	5.10E-03	9.19E-04	1.10E-03
	NBSSA	4.05E-04	6.08E-02	1.12E-02	1.32E-02	7.52E-04	6.42E-02	1.68E-02	1.45E-02	1.77E-04	1.53E-01	1.92E-02	2.52E-02
	TSSA	7.78E-04	6.95E-02	1.13E-02	1.31E-02	1.75E-04	5.66E-02	1.32E-02	1.39E-02	5.32E-04	1.15E-01	2.13E-02	2.26E-02

best algorithm cannot be compared with itself so NA (Not Applicable) is used in the Table. If the two algorithms under comparison provided the same results statistically or if there is no statistical relevance between them then '$\sim$' sign is inserted. From the results in Table 8, it has been found that performance of MSSA is significantly better for maximum number of cases and is statistically best among the algorithms under comparison.

The second statistical test used here is Mann- Whitney U rank sum test. This test is also a non-parametric test and is executed at 5% significance level. Here the proposed MSSA is compared with other existing algorithm as given in Table 8. If significant difference is not observed between two samples of best values then '=' sign is inserted in its place. In the same way if significant difference is observed between the algorithms then '+' sign is used and '$-$' sign is used for worst performing algorithm. As from the Table 8 it can be seen that it consist of '+' sign for MSSA in maximum number of cases, it is proved that performance of proposed MSSA is statistically better among all the other algorithms.

5.6. Convergence profiles

In this section convergence profiles of different algorithms has been compared. Here, the proposed MSSA is compared over different benchmark functions as shown in Figure 2. From the convergence plots it has been analyzed that convergence of proposed MSSA converges gradually during the initial iterations and faster towards the later iterations. This is due to fact that a good algorithm

Table 6: Statistical results for Dimension size of $100, 200, 500$(Conitnued)

Function	Algorithm	Dimension Size 100				Dimension Size 200				Dimension Size 500			
		Best	Worst	Mean	Std	Best	Worst	Mean	Std	Best	Worst	Mean	Std
F8	SSA	-2.51E+04	-1.86E+04	-2.21E+04	1.74E+03	-4.12E+04	-3.02E+04	-3.66E+04	2.68E+03	-7.36E+04	-5.31E+04	-6.39E+04	4.63E+03
	CSSA	-4.18E+04	-1.54E+04	-3.20E+04	7.03E+03	-8.37E+04	-2.90E+04	-6.69E+04	1.43E+04	-2.09E+05	-1.42E+05	-1.75E+05	2.95E+04
	DSSA	-3.37E+04	-1.41E+04	-2.42E+04	5.54E+03	-6.53E+04	-2.09E+04	-4.51E+04	1.06E+04	-1.81E+05	-6.86E+04	-1.16E+05	2.46E+04
	GSSA	-4.18E+04	-4.17E+04	-4.18E+04	1.87E+01	-8.37E+04	-8.36E+04	-8.37E+04	2.10E+01	-2.09E+05	-2.09E+05	-2.09E+05	4.67E+01
	LSSA	-4.18E+04	-1.43E+04	-2.65E+04	7.23E+03	-8.37E+04	-2.57E+04	-6.37E+04	1.73E+04	-2.09E+05	-8.02E+04	-1.52E+05	3.58E+04
	MSSA	-4.18E+04	-4.18E+04	-4.18E+04	2.17E+00	-8.37E+04	-8.37E+04	-8.37E+04	4.06E+00	-2.09E+05	-2.09E+05	-2.09E+05	1.68E+01
	NBSSA	-4.18E+04	-4.18E+04	-4.18E+04	1.39E+01	-8.37E+04	-8.36E+04	-8.37E+04	4.01E+01	-2.09E+05	-2.09E+05	-2.09E+05	7.60E+01
	TSSA	-4.18E+04	-4.18E+04	-4.18E+04	9.19E+00	-8.37E+04	-8.37E+04	-8.37E+04	2.97E+01	-2.09E+05	-2.09E+05	-2.09E+05	7.50E+01
F9	SSA	1.10E+02	2.83E+02	1.91E+02	4.62E+01	5.56E+02	8.99E+02	7.16E+02	6.41E+01	2.64E+03	3.16E+03	2.91E+03	1.07E+02
	CSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	DSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	GSSA	6.70E-12	1.35E-01	7.70E-03	2.09E-02	0.00E+00	6.73E-01	3.98E-02	1.15E-01	1.06E-07	1.17E+00	6.85E-02	1.90E-01
	LSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	NBSSA	3.53E-07	5.28E-02	5.20E-03	1.22E-02	8.45E-11	4.98E-01	4.35E-02	1.01E-01	5.55E-09	2.25E+00	1.28E-01	3.95E-01
	TSSA	1.90E-08	1.90E-01	1.31E-02	3.65E-02	9.81E-08	1.29E+00	4.01E-02	1.88E-01	9.86E-08	4.87E+00	2.44E-01	8.77E-01
F10	SSA	6.47E+00	1.14E+01	8.82E+00	9.77E-01	1.08E+01	1.33E+01	1.20E+01	6.02E-01	1.28E+01	1.39E+01	1.33E+01	2.50E-01
	CSSA	8.88E-16	4.44E-15	1.81E-15	1.57E-15	8.88E-16	4.44E-15	1.95E-15	1.64E-15	8.88E-16	4.44E-15	1.52E-15	1.37E-15
	DSSA	8.88E-16	4.44E-15	1.45E-15	1.57E-15	8.88E-16	4.44E-15	1.66E-15	1.48E-15	8.88E-16	4.44E-15	1.45E-15	1.31E-15
	GSSA	1.33E-06	7.79E-02	1.00E-02	1.39E-02	5.90E-10	2.39E-01	1.73E-02	4.08E-02	5.81E-06	5.72E-02	1.19E-02	1.54E-02
	LSSA	8.88E-16	4.44E-15	1.66E-15	1.48E-15	8.88E-16	4.44E-15	1.52E-15	1.37E-15	8.88E-16	4.44E-15	1.66E-15	1.48E-15
	MSSA	8.88E-16	4.44E-15	1.66E-15	1.48E-15	8.88E-16	4.44E-15	1.45E-15	1.31E-15	8.88E-16	4.44E-15	1.59E-15	1.43E-15
	NBSSA	4.21E-08	7.75E-02	1.22E-02	1.46E-02	1.58E-06	1.78E-01	1.49E-02	3.13E-02	1.85E-06	7.64E-02	1.04E-02	1.69E-02
	TSSA	7.00E-05	9.52E-02	1.08E-02	1.69E-02	2.00E-05	2.07E-01	1.65E-02	3.50E-02	2.71E-05	1.51E-01	1.80E-02	3.08E-02
F11	SSA	3.97E+00	1.22E+01	7.13E+00	1.96E+00	7.64E+01	1.45E+02	1.04E+02	1.32E+01	5.40E+02	7.10E+02	6.35E+02	4.25E+01
	CSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	DSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	GSSA	2.31E-13	1.19E-01	1.44E-02	2.67E-02	4.88E-09	1.96E-01	1.72E-02	4.29E-02	3.75E-10	3.52E-01	3.20E-02	7.77E-02
	LSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	MSSA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	NBSSA	2.13E-10	5.83E-01	3.35E-02	1.04E-01	1.40E-11	1.29E-01	1.79E-02	3.04E-02	4.66E-08	1.06E+00	4.78E-02	1.60E-01
	TSSA	4.84E-12	7.82E-01	2.98E-02	1.22E-01	1.05E-13	4.39E-01	3.19E-02	8.06E-02	9.79E-08	1.10E+00	5.63E-02	1.82E-01
F12	SSA	9.52E+00	2.69E+01	1.87E+01	4.77E+00	2.56E+01	2.75E+03	1.21E+02	3.84E+02	1.31E+04	1.09E+06	2.12E+05	1.86E+05
	CSSA	1.99E-04	1.97E-01	3.34E-02	4.54E-02	4.62E-07	1.91E-01	3.45E-02	4.71E-02	4.21E-05	1.97E-01	1.88E-02	3.79E-02
	DSSA	4.46E-02	2.60E-01	1.30E-01	5.00E-02	1.14E-01	2.34E-01	1.58E-01	2.80E-02	1.26E-01	3.47E-01	1.93E-01	3.94E-02
	GSSA	4.00E-06	8.10E-03	1.40E-03	1.80E-03	4.79E-06	6.00E-03	9.90E-04	1.10E-03	1.23E-05	1.50E-02	1.40E-03	2.40E-03
	LSSA	2.02E-06	1.95E-01	3.43E-02	5.37E-02	1.89E-05	2.05E-01	6.28E-02	6.77E-02	2.03E-04	3.61E-01	7.32E-02	9.10E-02
	MSSA	1.08E-05	1.18E-02	1.80E-03	2.30E-03	1.42E-06	6.30E-03	1.00E-03	1.40E-03	6.97E-06	2.03E-02	1.90E-03	3.50E-03
	NBSSA	5.83E-05	8.50E-03	1.70E-03	2.00E-03	2.46E-07	4.00E-03	9.67E-04	1.00E-03	2.79E-05	1.02E-02	8.96E-04	1.60E-03
	TSSA	6.78E-05	8.50E-03	1.60E-03	2.00E-03	5.52E-06	4.40E-03	1.00E-03	9.14E-04	3.88E-05	4.20E-03	1.00E-03	1.10E-03
F13	SSA	1.49E+02	3.85E+03	3.86E+02	5.63E+02	7.09E+04	1.44E+06	3.64E+05	2.96E+05	7.41E+06	2.48E+07	1.45E+07	3.97E+06
	CSSA	2.00E-03	4.42E+00	9.77E-01	1.22E+00	1.18E-01	1.03E+01	2.56E+00	2.97E+00	6.87E-02	2.86E+01	4.61E+00	7.08E+00
	DSSA	2.31E+00	5.59E+00	3.74E+00	7.40E-01	6.84E+00	1.38E+01	8.77E+00	1.25E+00	1.94E+01	3.95E+01	2.48E+01	3.88E+00
	GSSA	7.54E-04	3.69E-01	6.71E-02	8.41E-02	3.90E-03	4.39E-01	1.14E-01	1.26E-01	8.07E-04	1.74E+00	3.16E-01	4.11E-01
	LSSA	7.90E-02	5.67E+00	1.99E+00	1.71E+00	4.44E-02	9.98E+00	3.17E+00	3.52E+00	7.99E-02	2.63E+01	7.92E+00	9.49E+00
	MSSA	3.53E-04	3.46E-01	8.53E-02	1.00E-01	6.96E-05	7.43E-01	1.40E-01	1.84E-01	4.50E-03	2.93E+00	5.57E-01	6.93E-01
	NBSSA	3.92E-04	2.89E-01	7.14E-02	7.38E-02	2.49E-04	8.46E-01	1.17E-01	1.74E-01	8.10E-03	2.65E+00	4.40E-01	5.25E-01
	TSSA	4.80E-04	3.60E-01	6.21E-02	6.16E-02	1.50E-03	4.73E-01	1.12E-01	1.32E-01	3.00E-03	1.98E+00	2.87E-01	3.82E-01

requires large exploration steps during initial stages but towards the later stages it must move from exploration to exploitation phase and converge faster. Here, the algorithm has to maintain a balance between exploration and exploitation so that algorithm converge to optimal solution. Hence, from the convergence profiles it can be seen that proposed MSSA is able to converge at the end of the iterations and attaining a value near to optimal solution. So, overall we can say that MSSA is better in terms of fitness values.

5.7. Comparison on CEC2015 benchmark problems

This section describes the performance evaluation of proposed MSSA with respect to BA, DA, FA, DE, FPA, GWO and SSA on CEC2015 benchmark functions. The benchmark functions used here are 2 unimodal functions ($F_1 - F_2$), 3 multimodal functions ($F_3 - F_5$), 3 hybrid functions ($F6 - F8$) and 7 composition functions ($F_9 - F_{15}$) as given in Table 9. In this case, the common parameters such as population size, dimension size, number of runs and maximum number of iterations are selected as 50, 30, 50 and 25000, respectively. The results are compared in terms of best, worst, mean and standard deviation values for all the algorithms and are listed in Table 10.

It has been analyzed from the Table 10 that for functions F_1, F_6, F_8 and F_{10}, proposed MSSA provides best results as compared to other algorithms. For function F_2, the results of MSSA is highly competitive but FA is found to be best. For function $F3$, the results are comparable only in terms of standard deviation where BA performance is significantly better among the other algorithms. For functions $F4$, $F7$ and $F9$, all the algorithms have same performance in terms of best, worst and mean values while difference exist for standard deviation values so here MSSA provides lower standard deviation in comparison with other algorithms. For function $F5$, the performance of MSSA is little

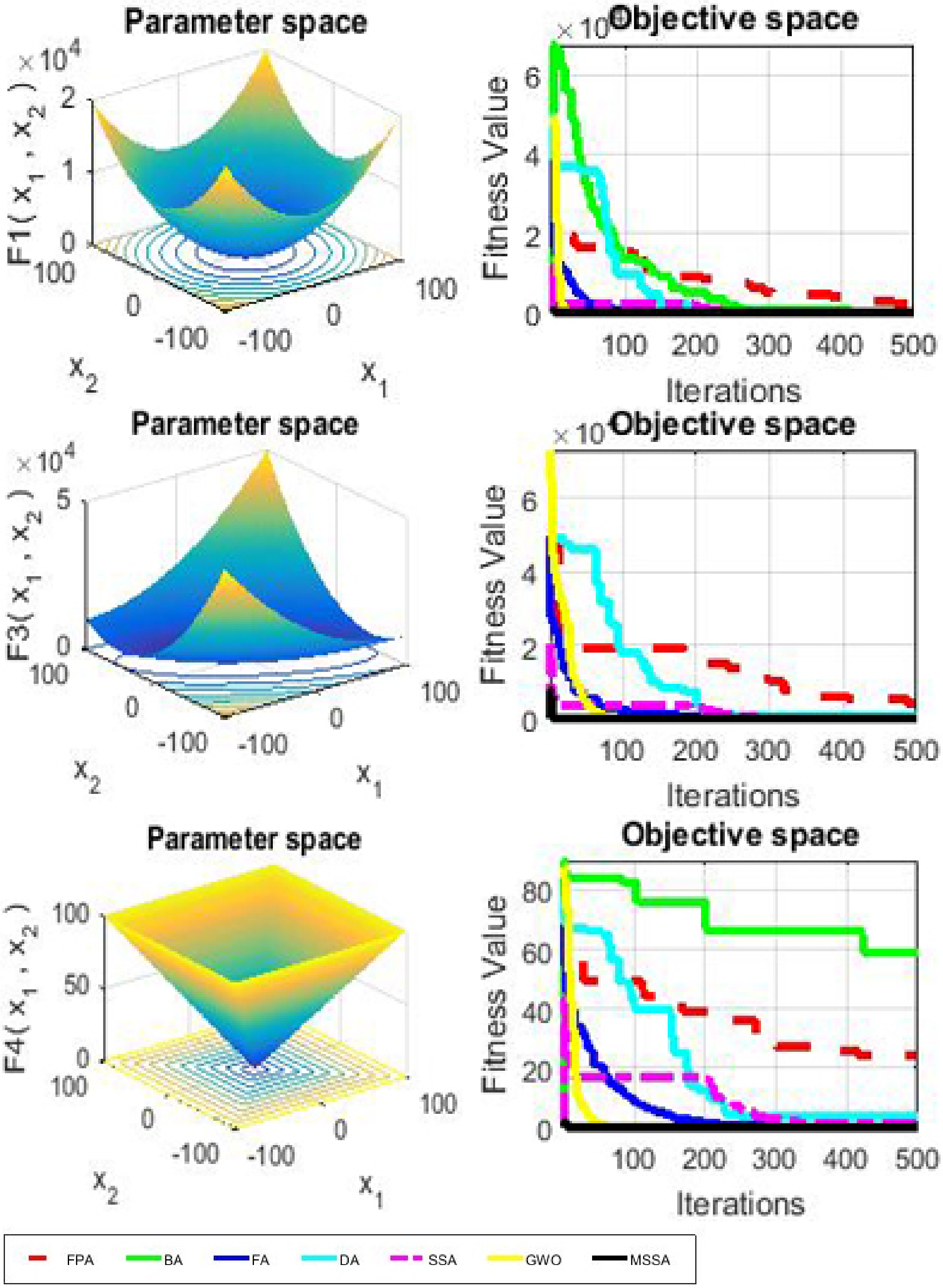

Figure 2: Convergence profiles of MSSA versus others algorithms

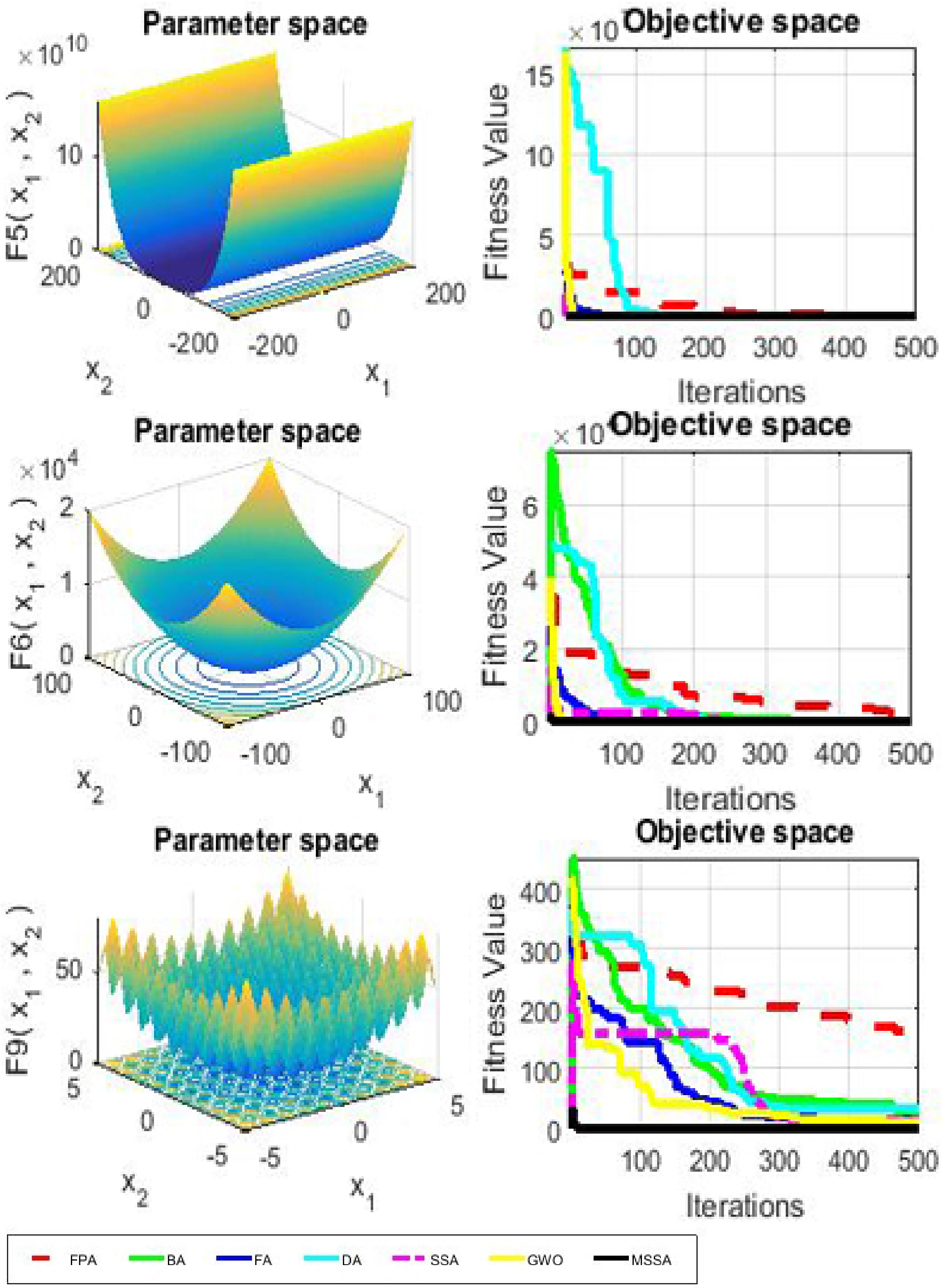

Figure 2: (Contd.) Convergence profiles of MSSA versus others algorithms

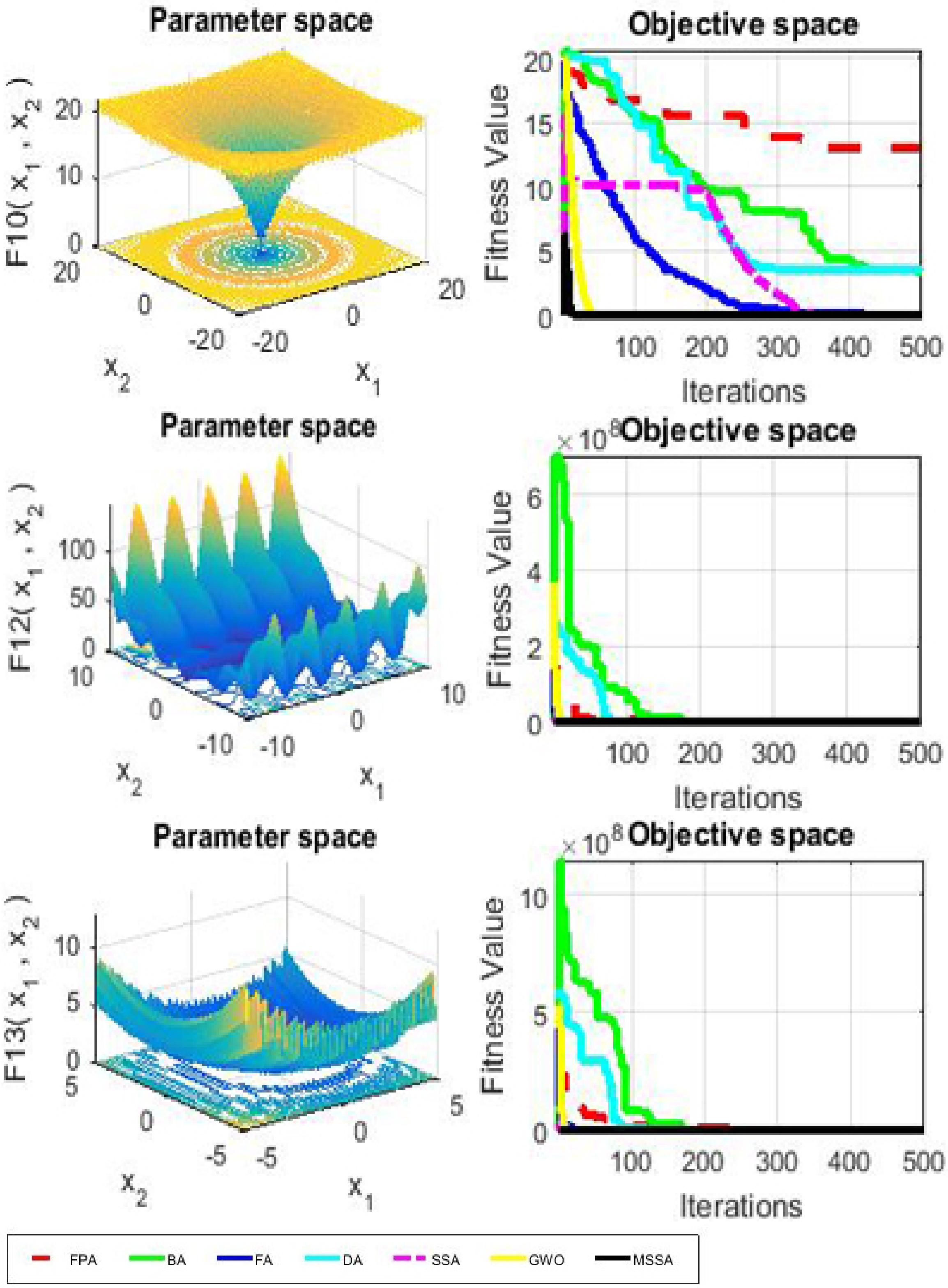

Figure 2: (Contd.) Convergence profiles of MSSA versus others algorithms

Table 7: Statistical results of comparison with SSA variants ($D = 10$)

Function		SSA	SSAPSO (Ibrahim et al., 2019)	CSSA (Majhi et al., 2019)	CSSA5 (Sayed et al., 2018)	MSSA
$F1$	mean	8.23E-10	8.17E-10	1.74E-50	1.40E-09	**2.61E-99**
	std	2.76E-10	6.82E-10	1.34E-50	3.31E-10	**1.80E-98**
	p-rank	−	−	−	−	
	f-rank	4	3	2	5	1
$F2$	mean	5.40E-03	8.21E-23	1.77E-16	4.14E-06	**1.92E-53**
	std	3.83E-02	2.80E-22	4.49E-16	1.68E-06	**8.87E-53**
	p-rank	−	−	−	−	
	f-rank	5	2	3	4	1
$F3$	mean	6.48E-09	**1.09E-34**	1.54E-10	9.01E-10	2.84E-28
	std	9.86E-09	**4.09E-34**	2.56E-10	4.87E-10	1.21E-27
	p-rank	−	+	−	−	
	f-rank	5	1	3	4	2
$F4$	mean	1.80E-05	8.77E-21	1.40E-07	1.08E-05	**3.42E-44**
	std	4.62E-06	3.16E-20	3.11E-07	1.55E-06	**2.03E-43**
	p-rank	−	−	−	−	
	f-rank	5	2	3	4	1
$F5$	mean	1.17E+02	7.87E+00	1.95E+01	8.94E+00	**2.55E+00**
	std	3.37E+02	2.62E-01	3.40E+01	**1.92E-02**	3.78E+00
	p-rank	−	−	−	−	
	f-rank	5	2	4	3	1
$F6$	mean	7.51E-10	6.40E-10	**6.74E-32**	1.83E-10	1.11E-02
	std	3.22E-10	3.76E-10	**1.41E-31**	5.02E-11	1.59E-02
	p-rank	+	+	+	+	
	f-rank	4	3	1	2	5
$F7$	mean	1.27E-02	9.12E-04	3.40E-03	9.30E-04	**4.04E-04**
	std	9.90E-03	8.48E-04	1.90E-03	1.94E-04	**4.06E-04**
	p-rank	−	−	−	−	
	f-rank	5	2	4	3	1
$F8$	mean	-2.83E+03	-2.68E+03	-2.80E+03	-2.51E+03	-4.18E+03
	std	3.26E+02	1.74E+02	2.56E+02	3.35E+02	**1.30E-01**
	p-rank	−	−	−	−	
	f-rank	4	2	3	5	1
$F9$	mean	1.82E+01	1.47E-14	1.74E-08	1.30E-10	**0.00E+00**
	std	8.23E+00	3.43E-14	3.40E-09	2.26E-10	**0.00E+00**
	p-rank	−	−	−	−	
	f-rank	5	2	4	3	1
$F10$	mean	7.32E-01	1.82E-15	5.50E-15	7.03E-06	**1.74E-15**
	std	9.70E-01	**1.46E-15**	1.71E-15	2.41E-06	1.53E-15
	p-rank	−	−	−	−	
	f-rank	5	2	3	4	1
$F11$	mean	2.65E-01	3.30E-13	**0.00E+00**	2.84E-09	**0.00E+00**
	std	1.55E-01	9.35E-13	1.64E-01	1.48E-09	**0.00E+00**
	p-rank	−	−	−	−	
	f-rank	5	3	2	4	1
$F12$	mean	5.21E-01	1.50E-01	1.49E-01	1.46E-01	**9.77E-04**
	std	8.56E-01	4.26E-02	8.09E-02	6.27E-02	**1.40E-03**
	p-rank	−	−	−	−	
	f-rank	5	4	3	2	1
$F13$	mean	2.80E-03	1.76E-05	**8.06E-32**	1.50E-03	9.40E-03
	std	5.70E-03	1.50E-05	**3.50E-32**	1.10E-03	1.68E-02
	p-rank	+	+	+	+	
	f-rank	4	2	1	3	5
w/l/t		2/11/0	3/10/0	2/11/0	2/11/0	NA
Overall f-rank value		61	30	37	46	22
Overall f-rank		5	2	3	4	1

bit better in terms of best values only. For function $F11$, the results of all the algorithms are almost similar, so here it is difficult to comment which algorithm is best for this function. For functions $F12$ and $F13$, FA is best in terms of standard deviation value. For function $F14$, performance of MSSA is best in terms of mean values. For function F_{15}, SSA provides the best results for standard deviation values. So, from the 15 functions used here for comparison, MSSA is found to be best for 10 functions, FA for three functions, SSA and BA for one function whereas DA, DE, FPA and GWO for none of the function.

5.8. Comparison on real World CEC2011 benchmarks problems

In this section, the performance of proposed algorithm has been tested to solve real world problems. The detailed descriptions of some selected applications is as follows:

5.8.1. Parameter Estimation for Frequency-wave (FM) sound waves

The frequency modulated (FM) signal analysis is improtant in wireless communication in terms of modern music system sound wave applications. In this case, FM multi-model function is a six dimensional optimization problem in which the optimization vector of sound wave X= $\{\alpha_1, \omega_1, \alpha_2, \omega_2, \alpha_3, \omega_3\}$ and is given in equation (21) and the parametric detail is listed in Table 11. The aim of this work is to define a fitness function so as to reduce the error between estimated sound and the actual sound. This problem has been tackled by GA in (Horner & Haken, 1993), (Herrera & Lozano, 2000). The

Table 8: Statistical results of comparison with other algorithms

Function	Algorithm	Best	Worst	Mean	Std	p-value	U-rank	Function	Algorithm	Best	Worst	Mean	Std	p-value	U-rank
F1	BA	4.68E+03	2.46E+04	1.26E+04	4.54E+03	7.06E-18	+	F5	BA	3.92E+05	2.39E+07	6.07E+06	4.83E+06	7.06E-18	+
	DA	3.27E+01	2.12E+03	5.69E+02	5.60E+02	7.06E-18	+		DA	8.11E+02	7.49E+05	8.36E+04	1.67E+05	7.06E-18	+
	DE	4.44E+04	7.61E+04	6.36E+04	6.39E+03	7.06E-18	+		DE	1.17E+08	2.93E+08	2.29E+08	4.17E+07	7.06E-18	+
	FA	6.10E-03	3.50E-02	1.49E-02	6.20E-03	7.06E-18	+		FA	2.76E+01	1.62E+03	1.04E+02	2.29E+02	8.99E-17	+
	FPA	2.09E+03	6.83E+03	4.59E+03	1.01E+03	7.06E-18	+		FPA	5.12E+05	3.69E+06	1.96E+06	8.27E+05	7.06E-18	+
	GWO	5.51E-40	1.05E-37	2.04E-38	2.46E-38	7.06E-18	+		GWO	2.53E+01	2.85E+01	2.66E+01	**7.24E-01**	4.65E-13	+
	SCCSA	2.77E-79	2.27E-67	9.22E-69	3.81E-68	7.06E-18	+		SCCSA	3.39E+00	8.71E+00	5.90E+00	9.13E-01	7.06E-18	+
	PSOGSA	2.75E-21	1.00E+04	2.00E+02	1.41E+03	7.06E-18	+		PSOGSA	6.74E-01	9.00E+04	7.21E+03	2.46E+04	7.06E-18	+
	SSA	9.59E-09	2.82E-08	1.75E-08	5.00E-09	7.06E-18	+		SSA	2.60E+01	1.14E+03	1.24E+02	1.99E+02	2.39E-16	+
	MSSA	**4.10E-117**	**1.07E-99**	**3.83E-101**	**1.74E-100**	NA			MSSA	**8.90E-03**	**2.87E+01**	**3.17E+00**	7.78E+00	NA	
F2	BA	8.79E+00	6.05E+05	2.31E+04	1.07E+05	7.06E-18	+	F6	BA	5.69E+03	2.41E+04	1.19E+04	4.24E+03	7.06E-18	+
	DA	1.57E+00	2.36E+01	1.24E+01	5.96E+00	7.06E-18	+		DA	3.59E+01	2.93E+03	6.31E+02	7.13E+02	7.06E-18	+
	DE	6.13E+01	5.07E+08	1.02E+07	7.18E+07	7.06E-18	+		DE	3.47E+04	7.32E+04	6.20E+04	7.22E+03	7.06E-18	+
	FA	2.69E-01	7.46E-01	4.70E-01	1.23E-01	7.06E-18	+		FA	4.60E-03	3.76E-02	1.36E-02	6.90E-03	7.06E-18	−
	FPA	3.37E+01	9.86E+01	6.12E+01	1.36E+01	7.06E-18	+		FPA	2.65E+03	7.54E+03	4.87E+03	9.78E+02	7.06E-18	+
	GWO	2.65E-23	4.26E-22	1.22E-22	8.75E-23	7.06E-18	+		GWO	8.90E-06	1.00E+00	2.94E-01	2.61E-01	7.06E-18	+
	SCCSA	1.03E-45	2.93E-39	8.25E-41	4.19E-40	7.06E-18	+		SCCSA	1.24E-09	2.72E-07	4.14E-08	5.22E-08	7.06E-18	−
	PSOGSA	1.58E-10	1.20E+01	1.03E+00	3.15E+00	7.06E-18	+		PSOGSA	**3.94E-21**	**2.20E-20**	**1.13E-20**	**3.94E-21**	NA	−
	SSA	1.45E-02	3.57E+00	8.95E-01	9.02E-01	7.06E-18	+		SSA	7.31E-09	2.78E-08	1.64E-08	4.65E-09	7.06E-18	−
	MSSA	**9.46E-62**	**1.45E-52**	**3.32E-54**	**2.05E-53**	NA			MSSA	7.50E-05	1.07E-01	2.63E-02	2.48E-02	7.06E-18	
F3	BA	8.87E+03	7.99E+04	3.39E+04	1.49E+04	7.06E-18	+	F7	BA	4.51E-01	7.58E+00	3.47E+00	1.52E+00	7.06E-18	+
	DA	3.80E+02	2.70E+04	7.38E+03	6.42E+03	7.06E-18	+		DA	1.02E-02	2.83E+00	2.41E-01	4.07E-01	7.06E-18	+
	DE	4.50E+04	1.58E+05	9.17E+04	2.75E+04	7.06E-18	+		DE	4.66E+01	1.59E+02	1.08E+02	2.76E+01	7.06E-18	+
	FA	3.92E+02	2.74E+03	1.44E+03	5.72E+02	7.06E-18	+		FA	7.60E-03	1.71E-01	2.95E-02	2.50E-02	7.06E-18	+
	FPA	3.97E+03	2.08E+04	1.03E+04	3.56E+03	7.06E-18	+		FPA	2.81E-01	2.59E+00	1.33E+00	5.88E-01	7.06E-18	+
	GWO	1.67E-12	1.83E-08	1.67E-09	3.84E-09	7.06E-18	+		GWO	6.48E-05	1.40E-03	4.62E-04	2.99E-04	4.50E-03	+
	SCCSA	3.63E-44	**2.00E-29**	**4.31E-31**	**2.83E-30**	7.06E-18	+		SCCSA	6.04E-06	1.05E-02	1.33E-03	1.72E-03	7.06E-18	+
	PSOGSA	1.43E-20	1.00E+04	1.50E+03	2.71E+03	7.06E-18	+		PSOGSA	6.88E-04	1.94E-02	6.87E-03	4.24E-03	7.06E-18	+
	SSA	6.32E+01	1.61E+03	3.91E+02	3.57E+02	7.06E-18	+		SSA	3.49E-02	1.29E-01	7.80E-02	2.49E-02	7.06E-18	+
	MSSA	**1.96E-64**	8.63E-16	4.38E-17	1.63E-16	NA			MSSA	**1.33E-05**	**1.30E-03**	**3.19E-04**	**2.92E-04**	NA	
F4	BA	2.44E+01	6.48E+01	4.27E+01	7.94E+00	7.06E-18	+	F8	BA	-Inf	-Inf	-Inf	NaN	3.31E-20	+
	DA	3.38E+00	4.00E+01	1.92E+01	9.41E+00	7.06E-18	+		DA	-7.86E+03	-4.49E+03	-5.84E+03	7.89E+02	7.06E-18	+
	DE	7.73E+01	9.13E+01	8.58E+01	2.90E+00	7.06E-18	+		DE	-8.85E+04	-7.56E+04	-9.52E+04	6.52E+01	7.06E-18	+
	FA	7.93E-02	2.41E+00	4.00E-01	4.40E-01	7.06E-18	+		FA	-7.75E+03	-4.02E+03	-5.55E+03	8.47E+02	7.06E-18	+
	FPA	2.42E+01	4.32E+01	3.45E+01	4.67E+00	7.06E-18	+		FPA	-5.21E+42	-1.33E+36	-1.43E+41	7.61E+41	7.06E-18	+
	GWO	7.35E-11	7.47E-09	1.83E-09	1.66E-09	7.06E-18	+		GWO	-1.92E+54	-1.97E+47	-1.56E+53	4.11E+53	7.06E-18	+
	SCCSA	8.31E-26	6.11E-16	2.15E-17	1.06E-16	7.06E-18	+		SCCSA	-3.72E+03	-2.43E+03	-3.07E+03	3.19E+02	7.06E-18	+
	PSOGSA	3.54E-11	1.35E+01	1.04E+00	2.30E+00	7.06E-18	+		PSOGSA	-3.72E+03	-2.12E+03	-3.08E+03	3.55E+02	7.06E-18	+
	SSA	1.51E+00	1.11E+01	5.31E+00	2.38E+00	7.06E-18	+		SSA	-9.45E+03	-6.36E+03	-7.55E+03	7.07E+02	7.06E-18	+
	MSSA	**1.20E-53**	**6.26E-43**	**1.26E-44**	**8.86E-44**	NA			MSSA	-1.25E+04	-1.25E+04	-1.25E+04	**2.15E-01**	NA	

expressions for proposed estimation and targeted sound waves are given as:

$$y(t) = \alpha_1 . \sin\{\omega_1 .t.\theta + \alpha_2 . \sin\{\omega_2 .t.\theta + \alpha_3 . \sin\{\omega_3 .t.\theta\}\}\} \tag{21}$$

$$y_0(t) = (1.0) \sin\{(5.0).t.\theta + (1.5). \sin\{(4.8).t.\theta + (2.0). \sin\{(4.9).t.\theta\}\}\} \tag{22}$$

where $\theta = 2\pi/100$ and proposed parameters are defined in the range [-6.4 6.35]. The fitness value is calculated by summation of square errors between the estimated sound (21) and targeted sound (22) as:

$$f(\overrightarrow{X}) = \sum_{t=0}^{100} \{y(t) - y_0(t)\}^2 \tag{23}$$

5.8.2. Lennard-Jones Potential Problem

It is a potential energy minimization problem (Hoare, 1979), (Moloi & Ali, 2005), in which involvement of pure Lennard-Jone (LJ) clusters has been considered to reduce the molecular potential energy level. It is a multi-model optimization problem, having exponential number of local minima and the lattice structure of LJ cluster. To attain the global minima, structures have used on the basis of Mackay icosahedrons which are listed in the cambridge cluster Database (http://www-wales.ch.cam.ac.uk/CCD.html). In this case, the aim of our proposed algorithm is to organise atoms in such a way that the molecule has minimum energy. The optimal atom organization define its fitness and capability to reduce its potential. The potential for N number of atoms is calculated as (25).

$$\overrightarrow{S_i} = [\overrightarrow{x_j}, \overrightarrow{y_j}, \overrightarrow{z_j}], \qquad j = 1, 2, 3, ..., N \tag{24}$$

Table 8: Statistical results of comparison with other algorithms (continued)

Function	Algorithm	Best	Worst	Mean	Std	p-value	U-rank	Function	Algorithm	Best	Worst	Mean	Std	p-value	U-rank
F9	BA	2.08E+01	1.33E+02	7.19E+01	2.34E+01	3.31E-20	+	F13	BA	1.29E+05	9.13E+07	1.89E+07	1.94E+07	7.06E-18	+
	DA	3.49E+01	2.64E+02	1.12E+02	4.64E+01	3.31E-20	+		DA	5.69E+00	3.11E+05	2.22E+04	6.70E+04	2.62E-17	+
	DE	3.48E+02	4.65E+02	4.19E+02	2.63E+01	3.31E-20	+		DE	5.82E+08	1.41E+09	9.78E+08	1.94E+08	7.06E-18	+
	FA	1.52E+01	4.36E+01	2.58E+01	7.74E+00	3.31E-20	+		FA	7.69E-04	**1.45E-02**	**3.60E-03**	**2.10E-03**	2.70E-03	+
	FPA	1.64E+02	2.35E+02	2.04E+02	1.45E+01	3.31E-20	+		FPA	2.69E+05	6.62E+06	1.92E+06	1.36E+06	7.06E-18	+
	GWO	0.00E+00	1.22E+01	2.21E+00	3.38E+00	5.89E-15	+		GWO	6.16E-05	5.84E-01	2.42E-01	1.37E-01	1.41E-06	+
	SCCSA	0.00E+00	1.90E+01	5.46E+00	5.62E+00	3.31E-20	+		SCCSA	7.81E-06	2.99E-01	2.01E-02	7.23E-02	7.06E-18	+
	PSOGSA	7.96E+00	7.86E+01	3.10E+01	1.53E+01	3.31E-20	+		PSOGSA	**9.81E-22**	2.51E+00	5.42E-02	3.55E-01	NA	+
	SSA	1.49E+01	7.36E+01	3.94E+01	1.44E+01	3.31E-20	+		SSA	6.39E-09	2.24E+01	1.44E+00	4.72E+00	7.06E-18	+
	MSSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	NA			MSSA	1.20E-05	5.39E-02	5.60E-03	9.90E-03	1.96E-02	
F10	BA	1.16E+01	1.63E+01	1.46E+01	9.61E-01	8.69E-19	+	F14	BA	3.00E+00	3.00E+01	4.08E+00	5.34E+00	5.62E-18	+
	DA	3.61E+00	9.89E+00	7.03E+00	1.51E+00	8.69E-19	+		DA	3.00E+00	3.00E+00	3.00E+00	1.19E-05	1.42e-09	+
	DE	1.95E+01	2.08E+01	2.04E+01	2.75E-01	8.69E-19	+		DE	3.00E+00	3.00E+00	3.00E+00	**3.35E-15**	NA	+
	FA	3.17E-02	1.23E-01	6.82E-02	2.33E-02	8.69E-19	+		FA	3.00E+00	3.00E+00	3.00E+00	3.59E-08	5.62E-18	+
	FPA	1.29E+01	1.69E+01	1.47E+01	9.70E-01	8.69E-19	+		FPA	3.00E+00	3.00E+00	3.00E+00	6.50E-09	5.62E-18	+
	GWO	2.22E-14	3.99E-14	3.17E-14	3.46E-15	3.91E-19	+		GWO	3.00E+00	3.00E+00	3.00E+00	8.46E-06	5.62E-18	+
	SCCSA	8.88E-16	**8.88E-16**	**8.88E-16**	**9.96E-32**	8.69E-19	+		SCCSA	3.00E+00	3.00E+00	3.00E+00	8.93E-09	5.62E-18	+
	PSOGSA	4.24E-13	1.52E-11	6.69E-12	3.43E-12	8.69E-19	+		PSOGSA	3.00E+00	8.40E+01	9.48E+00	2.21E+01	5.62E-18	+
	SSA	2.65E-05	3.46E+00	1.75E+00	8.11E-01	8.69E-19	+		SSA	3.00E+00	3.00E+00	3.00E+00	1.52E-13	5.97E-18	+
	MSSA	**8.88E-16**	4.44E-15	1.81E-15	1.57E-15	NA			MSSA	3.00E+00	3.00E+00	3.01E+00	3.64E-02	5.62E-18	
F11	BA	7.76E-11	1.88E+01	2.08E+00	3.39E+00	3.31E-20	+	F15	BA	-3.86E+00	-5.31E-01	-3.42E+00	9.03E-01	7.06E-18	+
	DA	0.00E+00	1.18E-02	2.37E-04	1.70E-03	4.33E-02	+		DA	-3.86E+00	-2.39E+00	-3.72E+00	3.41E-01	7.06E-18	+
	DE	0.00E+00	1.45E-01	5.80E-03	2.88E-02	6.50E-03	+		DE	-3.86E+00	-3.85E+00	-3.86E+00	5.40E-04	1.90E-11	+
	FA	1.58E-08	8.55E-06	1.28E-06	1.72E-06	3.31E-20	+		FA	-3.86E+00	-3.28E+00	-3.80E+00	1.38E-01	7.06E-18	+
	FPA	2.37E-08	4.11E-05	8.29E-06	1.03E-05	3.31E-20	+		FPA	-3.86E+00	-3.77E+00	-3.85E+00	1.82E-02	7.06E-18	+
	GWO	0.00E+00	1.45E-01	2.90E-03	2.06E-02	3.27E-01	−		GWO	-3.86E+00	-3.85E+00	-3.85E+00	4.10E-03	7.06E-18	+
	SCCSA	0.00E+00	1.75E-01	3.33E-02	4.56E-02	3.31E-20	+		SCCSA	-3.86E+00	-3.20E+00	-3.26E+00	6.00E-02	7.06E-18	+
	PSOGSA	3.93E-02	9.45E-01	2.28E-01	1.85E-01	3.31E-20	+		PSOGSA	-3.86E+00	-3.86E+00	-3.86E+00	0.00E+00	7.06E-18	+
	SSA	8.65E-15	1.18E-10	1.41E-11	2.12E-11	3.31E-20	+		SSA	-3.86E+00	-3.86E+00	-3.86E+00	**6.61E-12**	NA	+
	MSSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	NA			MSSA	-3.86E+00	-3.66E+00	-3.80E+00	5.17E-02	7.06E-18	
F12	BA	1.32E+04	3.38E+07	3.70E+06	5.99E+06	7.06E-18	+	F16	BA	-3.32E+00	-6.60E-03	-1.47E+00	1.32E+00	7.14E-08	+
	DA	5.72E-01	1.04E+04	3.54E+02	1.72E+03	7.06E-18	+		DA	-3.32E+00	-5.39E-05	-1.96E+00	1.18E+00	1.04E-12	+
	DE	2.00E+08	7.21E+08	5.17E+08	1.28E+08	7.06E-18	+		DE	-3.32E+00	-6.50E-01	-2.91E+00	6.70E-01	1.53E-17	−
	FA	1.49E-04	2.80E-03	7.20E-04	6.85E-04	9.34E-07	+		FA	-3.32E+00	-5.54E-01	-2.48E+00	1.00E+00	8.99E-17	+
	FPA	3.48E+01	1.35E+06	1.21E+05	2.32E+05	7.06E-18	+		FPA	-1.56E+00	**-1.27E-05**	**-2.75E-01**	3.48E-01	NA	+
	GWO	1.65E-06	7.26E-02	2.26E-02	1.69E-02	1.94E-13	+		GWO	-3.32E+00	-3.07E+00	-3.24E+00	6.53E-02	7.06E-18	+
	SCCSA	1.94E-07	6.03E-02	1.34E-02	1.60E-02	7.06E-18	+		SCCSA	-3.32E+00	-3.20E+00	-3.26E+00	6.00E-02	7.06E-18	+
	PSOGSA	1.98E-22	1.12E+01	1.82E+00	2.50E+00	7.06E-18	+		PSOGSA	-3.32E+00	-2.87E+00	-3.24E+00	9.63E-02	7.06E-18	+
	SSA	4.14E-01	1.08E+01	3.95E+00	2.53E+00	7.06E-18	+		SSA	-3.32E+00	-2.97E+00	-3.20E+00	8.21E-02	7.06E-18	−
	MSSA	**2.16E-07**	**2.30E-03**	**3.27E-04**	**5.08E-04**	NA			MSSA	-3.31E+00	-2.86E+00	-3.19E+00	1.00E-01	7.06E-18	

Table 9: CEC 2015 Real parameter benchmark optimization functions

	No.	Functions	$F_i^* = F_i(x^*)$
Unimodal Functions	F_1	Rotated High Conditional Elliptic Function	100
	F_2	Rotated Cigar Function*	200
Multimodal Functions	F_3	Shifted and Rotated Ackley's Function	300
	F_4	Shifted and Rotated Rastrigin's Function	400
	F_5	Shifted and Rotated Schwefel's Function	500
Hybrid Functions	F_6	Hybrid Function 1($N = 3$)	600
	F_7	Hybrid Function 2($N = 3$)	700
	F_8	Hybrid Function 3($N = 3$)	800
Composition Functions	F_9	Composition Function 1($N = 3$)	900
	F_{10}	Composition Function 5($N = 3$)	1000
	F_{11}	Composition Function 6($N = 5$)	1100
	F_{12}	Composition Function 7($N = 5$)	1200
	F_{13}	Composition Function 8($N = 5$)	1300
	F_{14}	Composition Function 9($N = 7$)	1400
	F_{15}	Composition Function 10($N = 10$)	1500
		Search Range: $[-100, 100]^D$	

where S is the LJ potential vector represented in Cartesian coordinates.

$$P_N(S) = \sum_{j=1}^{N-1} \sum_{k=1}^{N} (r_{jk}^{-12} - 2.r_{jk}^{-6}) \tag{25}$$

where $r_i = \|(\overrightarrow{s_k} - \overrightarrow{s_j})\|_2$ with gradient

$$\nabla_k P_N(S) = -12 \sum_{j=1, j \neq k}^{N} (r_{jk}^{-14} - r_{jk}^{-8})(\overrightarrow{s_k} - \overrightarrow{s_j}), \qquad k = 1, 2, 3, ..., N \tag{26}$$

The variation of molecule potential $P(r) = (r^{-12} - 2.r^{-6})$ with respect to pair distance r is shown in Fig 3. It clearly illustrated that potential goes to -1 for a radius of unity (r=1). when r varies from -1 towards 0, potential increases abruptly and its value approaches to infinity near r=0. This tendency of pair potential curve increases complexity to optimize this multi-model problem.

5.8.3. Tersoff Potential Function Minimization Problem

The inter atomic potential evaluation of silicon for covalent systems is currently an interesting area for new researchers (Tersoff, 1988). One important is Tersoff potential, which governs the interaction of silicon atoms with a strong covalent bonding. It has two optimization parameters known

Table 10: Statistical results of comparison with other algorithms for CEC 2015 ($D = 30$)

Function	Algorithm	Best	Worst	Mean	Std	Function	Algorithm	Best	Worst	Mean	Std
F_1	BA	3.89E+08	4.14E+09	1.86E+09	8.39E+08	F_9	BA	1.18E+03	1.72E+03	1.40E+03	1.25E+02
	DA	4.44E+07	3.70E+08	1.60E+08	7.54E+07		DA	1.00E+03	1.10E+03	1.04E+03	1.97E+01
	DE	4.16E+07	4.59E+09	5.60E+08	6.91E+08		DE	1.03E+03	1.53E+03	1.14E+03	1.02E+02
	FA	1.28E+06	1.12E+07	5.52E+06	2.60E+06		FA	1.00E+03	1.14E+03	1.00E+03	1.94E+01
	FPA	1.72E+09	8.79E+09	4.66E+09	1.38E+09		FPA	1.33E+03	2.02E+03	1.72E+03	1.53E+02
	GWO	1.20E+07	9.22E+07	4.17E+07	2.14E+07		GWO	1.00E+03	1.20E+03	1.03E+03	4.55E+01
	SSA	3.72E+05	3.79E+08	4.32E+07	9.54E+07		SSA	1.00E+03	1.52E+03	1.06E+03	1.06E+02
	MSSA	**3.03E+05**	**7.90E+06**	**4.17E+06**	**1.90E+06**		MSSA	**1.00E+03**	**1.00E+03**	**1.00E+03**	**1.01E-01**
F_2	BA	2.78E+10	1.61E+11	8.55E+10	2.40E+10	F_{10}	BA	6.23E+05	3.65E+08	7.68E+07	8.10E+10
	DA	1.14E+09	1.33E+10	5.70E+09	2.62E+09		DA	4.49E+05	2.43E+07	6.92E+06	4.90E+06
	DE	5.13E+09	8.11E+10	2.73E+10	1.68E+10		DE	1.90E+06	3.12E+08	7.10E+07	6.07E+07
	FA	1.63E+03	**2.71E+04**	**6.42E+03**	**5.65E+03**		FA	7.18E+04	3.27E+06	5.87E+05	5.29E+05
	FPA	9.02E+10	2.18E+11	1.53E+11	2.64E+10		FPA	6.97E+07	1.01E+09	3.66E+08	1.97E+08
	GWO	1.13E+08	9.50E+09	3.65E+09	2.44E+09		GWO	1.97E+05	7.78E+06	1.79E+06	1.87E+06
	SSA	**3.80E+02**	2.74E+10	4.28E+09	5.53E+09		SSA	1.33E+04	1.33E+07	1.12E+06	2.42E+06
	MSSA	9.28E+05	4.71E+07	1.02E+07	9.36E+06		MSSA	**2.46E+03**	**5.57E+04**	**1.50E+04**	**1.22E+04**
F_3	BA	3.20E+02	3.20E+02	3.20E+02	**4.62E-06**	F_{11}	BA	1.92E+03	2.92E+03	2.73E+03	1.34E+02
	DA	3.20E+02	3.21E+02	3.20E+02	1.44E-01		DA	1.49E+03	2.80E+03	2.25E+03	4.97E+02
	DE	3.20E+02	3.21E+02	3.20E+02	4.83E-02		DE	2.28E+03	2.65E+03	2.46E+03	1.10E+02
	FA	3.20E+02	3.20E+02	3.20E+02	1.40E-03		FA	1.41E+03	1.63E+03	1.53E+03	**5.53E+01**
	FPA	3.21E+02	3.21E+02	3.21E+02	6.78E-02		FPA	2.80E+03	4.89E+03	3.16E+03	3.50E+02
	GWO	3.20E+02	3.21E+02	3.21E+02	3.59E-02		GWO	1.86E+03	2.14E+03	1.97E+03	6.70E+01
	SSA	3.20E+02	3.20E+02	3.20E+02	1.51E-01		SSA	1.40E+03	2.50E+03	2.12E+03	3.47E+02
	MSSA	3.20E+02	3.20E+02	3.20E+02	5.69E-02		MSSA	**1.10E+03**	**1.40E+03**	**1.30E+03**	1.41E+02
F_4	BA	5.72E+02	8.30E+02	7.02E+02	5.70E+01	F_{12}	BA	1.38E+03	1.44E+03	1.40E+03	8.94E+00
	DA	6.00E+02	8.24E+02	7.04E+02	5.65E+01		DA	1.30E+03	1.40E+03	1.34E+03	3.33E+01
	DE	5.65E+02	8.58E+02	6.74E+02	6.84E+01		DE	1.40E+03	1.40E+03	1.40E+03	**2.00E-04**
	FA	4.19E+02	4.78E+02	4.42E+02	1.23E+01		FA	1.30E+03	1.30E+03	1.30E+03	7.47E-01
	FPA	9.06E+02	1.21E+03	1.07E+03	5.94E+01		FPA	1.40E+03	1.44E+03	1.40E+03	9.79E+00
	GWO	4.49E+02	6.55E+02	5.12E+02	4.07E+01		GWO	1.30E+03	1.40E+03	1.34E+03	4.49E+01
	SSA	5.48E+02	7.75E+02	6.44E+02	5.39E+01		SSA	1.30E+03	1.40E+03	1.35E+03	2.91E+01
	MSSA	**4.09E+02**	**4.38E+02**	**4.27E+02**	**6.89E+00**		MSSA	1.30E+03	1.30E+03	1.30E+03	1.14E+00
F_5	BA	4.15E+03	7.21E+03	5.43E+03	7.03E+02	F_{13}	BA	1.30E+03	1.30E+03	1.30E+03	5.50E-03
	DA	4.00E+03	6.63E+03	5.42E+03	6.38E+02		DA	1.30E+03	1.30E+03	1.30E+03	1.18E+00
	DE	7.13E+03	1.02E+04	9.17E+03	7.40E+02		DE	1.30E+03	1.30E+03	1.30E+03	2.02E-04
	FA	1.86E+03	4.50E+03	3.06E+03	5.71E+02		FA	1.30E+03	1.30E+03	1.30E+03	**1.71E-04**
	FPA	7.53E+03	1.08E+04	9.89E+03	5.60E+02		FPA	1.30E+03	2.44E+03	1.37E+03	1.94E+02
	GWO	2.02E+03	8.66E+03	3.92E+03	1.49E+03		GWO	1.30E+03	1.30E+03	1.30E+03	2.59E-04
	SSA	3.02E+03	6.10E+03	4.72E+03	6.84E+02		SSA	1.30E+03	1.30E+03	1.30E+03	1.47E-01
	MSSA	**7.78E+02**	**1.75E+03**	**1.23E+03**	**2.39E+02**		MSSA	1.30E+03	1.30E+03	1.30E+03	1.30E-03
F_6	BA	2.25E+06	3.61E+08	7.55E+07	8.30E+07	F_{14}	BA	6.08E+04	1.98E+05	1.15E+05	3.13E+04
	DA	6.53E+05	3.54E+07	7.73E+06	7.34E+06		DA	4.03E+04	6.26E+04	4.85E+04	5.51E+03
	DE	3.82E+06	2.44E+08	5.47E+07	5.03E+07		DE	6.10E+03	6.06E+04	1.75E+04	1.22E+04
	FA	1.52E+04	1.12E+06	3.80E+05	2.90E+05		FA	5.92E+03	2.44E+04	1.87E+04	2.44E+03
	FPA	9.81E+07	1.08E+09	5.14E+08	2.45E+08		FPA	9.38E+04	1.71E+05	1.33E+05	1.86E+04
	GWO	3.60E+05	1.39E+07	2.06E+06	2.51E+06		GWO	**2.86E+03**	4.08E+04	2.52E+04	6.81E+03
	SSA	1.21E+04	2.69E+07	9.93E+05	3.88E+06		SSA	1.30E+03	6.31E+04	4.20E+04	5.95E+03
	MSSA	**2.40E+03**	**1.31E+05**	**1.90E+04**	**2.53E+04**		MSSA	4.33E+03	**8.41E+03**	**6.73E+03**	**1.87E+03**
F_7	BA	8.65E+02	1.75E+03	1.15E+03	2.02E+02	F_{15}	BA	9.00E+03	2.25E+05	7.81E+04	5.31E+04
	DA	7.25E+02	8.22E+02	7.52E+02	2.85E+01		DA	1.63E+03	3.93E+03	2.01E+03	4.62E+02
	DE	7.20E+02	1.13E+03	8.15E+02	9.88E+01		DE	1.74E+03	2.40E+05	1.50E+04	3.48E+04
	FA	7.11E+02	7.19E+02	7.14E+02	1.82E+00		FA	1.60E+03	1.60E+03	1.60E+03	1.70E-03
	FPA	1.34E+03	3.67E+03	2.23E+03	5.83E+02		FPA	1.07E+05	1.35E+06	6.37E+05	3.31E+05
	GWO	7.15E+02	8.22E+02	7.32E+02	2.37E+01		GWO	1.61E+03	2.57E+03	1.71E+03	2.16E+02
	SSA	7.11E+02	8.38E+02	7.36E+02	3.12E+01		SSA	1.60E+03	1.60E+03	1.60E+03	**3.38E-06**
	MSSA	**7.01E+02**	**7.06E+02**	**7.04E+02**	**1.32E+00**		MSSA	1.61E+03	1.663+03	1.63E+03	1.17E+01
F_8	BA	9.96E+05	2.27E+08	3.02E+07	4.09E+07						
	DA	1.55E+05	9.75E+06	1.97E+06	1.59E+06						
	DE	1.60E+05	5.12E+07	1.21E+07	1.08E+07						
	FA	1.41E+04	5.68E+05	1.30E+05	1.26E+05						
	FPA	1.16E+07	4.15E+08	1.68E+08	1.08E+08						
	GWO	2.53E+04	2.67E+06	4.87E+05	5.17E+05						
	SSA	4.18E+03	5.19E+06	4.19E+05	9.11E+05						
	MSSA	**1.24E+03**	**8.23E+03**	**3.23E+03**	**2.02E+03**						

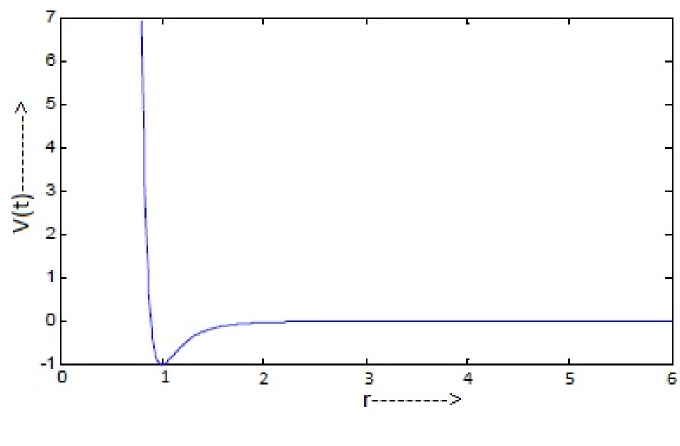

Figure 3: Molecule potential variation with respect to r

as Sc(B) and Sc(C). The molecular cluster position for N atoms is noted in Cartesian coordinates as:

$$\overrightarrow{s} = [\overrightarrow{s_1}, \overrightarrow{s_2}, ..., \overrightarrow{s_N}], \tag{27}$$

where $\overrightarrow{S_i}, i = 1, 2, 3, ..., N$ is a 3D vector presents the coordinates of the i^{th} atom, then total Tersoff potential function is defined as:

$$f(\overrightarrow{s_1}, .., \overrightarrow{s_3}) = E_1(\overrightarrow{s_1}, .., \overrightarrow{s_3}) + E_2(\overrightarrow{s_1}, .., \overrightarrow{s_3}) + ... + E_N(\overrightarrow{s_1}, .., \overrightarrow{s_3}) \tag{28}$$

Now to calculate total potential, we need to add individual potentials of each atom, which is further explained in equation (29). The individual potential is written as:

$$E_j = (1/2) \sum_{k \neq j} f_c(r_{jk}) \{P_R(r_{jk}) - B_{jk} P_R(r_{jk})\} \tag{29}$$

Here P_R is a repulsive potential; P_A is a attractive potential; r_{jk} is the distance between atoms j and k; $f_c(r_{jk})$ is probability switching function and B_{jk} is the constant that depends upon j and k atoms position and neighours of atom j and it is written as:

$$B_{jk} = (1 + \gamma^{n_j} \;\; \zeta_{jk}^{n_j})^{-1/2n_j} \tag{30}$$

$$where, \;\; \zeta_{jk} = \sum_{m \neq j} f_c(r_{jk}) \;\; g(\theta_{jkm}) \;\; exp\{\lambda_3^3 (r_{jk} - r_{jm})^3\} \tag{31}$$

where ζ_{jk} is the contribution of the neighbors of the atom j. It increases as the number m increases but the term B_{jk} decreases as ζ_{jk} increases. The exponential term is used to reduce the effect of bonds whose length is greater than r_{jk}. The term θ_{jkm} is the bond angle between bonds jk and jm, and the function g is written as:

$$g(\theta_{jkm}) = 1 + c^2/d^2 - c^2/(d^2 + [h - \cos(\theta_{jkm})]^2) \tag{32}$$

$$f_c(r_{jk}) = \begin{cases} 1, & if r_{jk} <= R - D \\ 1/2 - 1/2 \sin[\pi(r_{jk} - R)/D], & if R - D < r_{jk} < R + D \\ 0, & if r_{jk} >= R + D \end{cases} \tag{33}$$

$$P_R(r_{jk}) = Ae^{-\lambda_1 \;\; r_{jk}} \tag{34}$$

$$P_A(r_{jk}) = Be^{-\lambda_2 \;\; r_{jk}} \tag{35}$$

Here, A, B, C, D, λ_1 and λ_2 are fitted parameters. The switching function $f_c(r_{jk})$ is the boundary define parameter to calculate atom energy potential with in the boundaries (Ali & Törn, 2000). After all these analysis the fitness function is redefined to:

$$f(s) = E_1(s) + E_2(s) + ... + E_N(s), \qquad s \in \Omega \tag{36}$$

Table 11: Summary of the real world problems presented

Sr.No.	Problem	No. of Dimensions	Constraints	Bounds
1.	Parameter Estimation for FM sound waves	6	Bound Constrained	All Dimension bound between [-6.4, 6.35]
2.	Lennard-Jones Potential Problem	$3X10 = 30$ (10 atom problem)	Bound Constrained	Let $\overrightarrow{x}$ be the variable of the problem, which has three components for three atoms, six components for 4 atoms and so on. The first variable due to the second atom i.e. $x_1 \epsilon [0, 4]$, then the second and third variables are such that $x_2 \epsilon [0, 4]$ and $x_3 \epsilon [0, \pi]$. The coordinates x for any other atom is taken to be bound in the range: $[-4 - (1/4)\{(\iota - 4)/3\}, \quad 4 + (1/4)\{(\iota - 4)/3\}]$ where $[r]$ is the nearest least integer w.r.t. $r \epsilon 1$
3.	Tersoff Potential Function Minimization Problem	$3X10 = 30$ (10 atom problem)	Bound Constrained	Let $\overrightarrow{x}$ be the variable of the problem, which has three components for three atoms, six components for 4 atoms and so on. The first variable due to the second atom i.e. $x_1 \epsilon [0, 4]$, then the second and third variables are such that $x_2 \epsilon [0, 4]$ and $x_3 \epsilon [0, \pi]$. The coordinates x for any other atom is taken to be bound in the range: $[-4 - (1/4)\{(\iota - 4)/3\}, \quad 4 + (1/4)\{(\iota - 4)/3\}]$ where $[r]$ is the nearest least integer w.r.t. $r \epsilon 1$

Table 12: Comparison on Real World CEC2011 benchmarks problems

Applications	Algorithm	Best	Worst	Mean	Std
Parameter Estimation for FM sound waves	BA	1.15E+01	2.96E+01	2.49E+01	3.27E+00
	DA	9.19E+00	2.78E+01	2.26E+01	3.74E+00
	DE	1.63E+01	2.75E+01	2.59E+01	2.05E+00
	FA	1.49E+01	2.68E+01	2.32E+01	2.18E+00
	FPA	1.46E+01	2.74E+01	2.28E+01	3.08E+00
	GWO	5.36E+00	2.50E+01	1.48E+01	4.67E+00
	SSA	**3.77E-02**	2.54E+01	2.11E+01	4.54E+00
	MSSA	1.73E-01	**2.97E-01**	**2.57E-01**	**3.12E-01**
Lennard-Jones Potential Problem	BA	-2.84E+01	-1.36E+01	-2.34E+01	3.91E+00
	DA	-1.86E+01	-8.86E+00	-1.28E+01	2.25E+00
	DE	-1.18E+01	-3.79E+00	-7.75E+00	1.60E+00
	FA	-2.57E+01	-1.88E+01	-2.25E+01	1.58E+00
	FPA	-1.12E+01	-8.21E+00	-9.53E+00	7.73E-01
	GWO	-2.67E+01	-1.21E+01	-2.22E+01	2.80E+00
	SSA	-2.64E+01	-1.24E+01	-2.11E+01	3.35E+00
	MSSA	-2.17E+01	-1.14E+01	-1.60E+01	**2.61E-01**
Tersoff Potential Function Minimization Problem	BA	-3.47E+01	-8.45E+00	-2.11E+01	6.09E+00
	DA	-2.75E+01	-1.62E+01	-2.18E+01	2.55E+00
	DE	-1.81E+01	-1.04E+01	-1.40E+01	1.81E+00
	FA	-3.55E+01	-2.32E+01	-2.97E+01	2.68E+00
	FPA	-2.57E+01	-2.09E+01	-2.28E+01	1.15E+00
	GWO	-3.42E+01	-1.75E+01	-3.02E+01	3.48E+00
	SSA	-3.17E+01	-2.33E+01	-2.77E+01	2.19E+00
	MSSA	-2.66E+01	-1.49E+01	-2.02E+01	**2.38E-02**

$$where, \Omega = \{\{X_1, X_2, ..., X_n\} \mid -4.25 \leq x_1, \;\; x_i \leq 4.25, i = 4, ..., \;\; n_0 \leq x_2 \leq 4, \;\; 0 \leq x_3 \leq \pi\} \quad (37)$$

Comparison results and Inferences: To evaluate performance of proposed mutated salp swarm algorithm (MSSA) on CEC2011 real world problems, three multidimensional problems are selected in this paper. The parameters setting of FM sound wave, Lennard-Jones potential problem and Tersoff potential function minimization problem is listed in Table 11. For these applications, the proposed algorithm performance is compared with BA, DA, DE, FA, FPA, GWO and SSA in terms of best, worst,mean and standard deviation as shown in Table 12. For application of parameter estimation for FM sound wave, the results are competitive in terms of worst and mean. The SSA provides good result in terms of best value. However, MSSA is found to be best in terms of standard deviation. For Lennard-Jones potential problem, to find atom structure which requires minimum energy, the results are only comparable in terms of standard deviation where MSSA is found to be the best. It is difficult to say, which one is the good algorithm from best, worst and mean values. In third minimization problem of Tersoff potential function, it has similar results as in above second application. The results are only comparable in terms of standard deviation where only MSSA is able to attain global optima and found to be best algorithm as compared to other variants.

5.9. Analysis of Core Parameters of SSA, CSSA, SSAPSO and MSSA

Five core operations are used in SSA, CSSA, SSAPSO and MSSA and are given in Table 13. These include population size, exploration operation, exploitation operation, controlling parameters for adjusting the extent of exploration and exploitation, and the scaling factor, required to adjust the step size of global exploration operation. The simple SSA consists of basic parametric settings for all these parameters and have no adaptive properties. Instead of that, in the proposed MSSA, all of these parameters are adapted to make the algorithm self-resilient and remove the requirement of user based modifications. Here a linearly decreasing adaptive population size has been used to reduce the number of function evaluations in consecutive generations. The exploitation operation is also changed and a new phase based on the combined advantages of CS and GWO has been incorporated in the proposed version. It has already been known that the exploration operation of SSA is efficient and hence no modification has been added in this phase. The controlling parameters used in SSA are random numbers where as in present work, exponentially decreasing parametric adaptation has been followed. Thus helping the algorithm to perform extensive exploration during the initial stages and intensive exploitation towards the end. Apart from these parameters, the scaling factor plays a significant role in the performance of the algorithm and helps in proper exploration operation. MSSA employs seven new mutation operators and hence helps in overcoming the stagnation and local optima problems. If we compare the proposed MSSA with respect to other in literature like CSSA and SSAPSO, it can be said that these algorithms do not have adaptive properties and hence for those algorithms user based adaptation is very much necessary. But in MSSA, because of the presence of better adaptive strategy, end users can apply the algorithm directly without the requirement of parametric adjustments.

Table 13: Core Operations in SSA, CSSA, SSAPSO and MSSA

Core Operations	SSA	CSSA	SSAPSO	MSSA
Population size	Random number	Random number	Random number	Linearly decreasing adaptive population
Basic exploitation search	Basic	Basic	Based on either SSA or PSO	Based on combined effect of cuckoo search & GWO
Global exploration	Basic	Basic	Basic	Basic
Controlling parameters	Random number	Chaotic sequence	Random number	Exponentially decreasing with respect to iterations
Scaling Factor	Random number	Random number	Random number	Mutation Operators

5.10. Summary of results

SSA as explained above is a new algorithm and has been found to provide viable solutions for various problems. The algorithm though is competitive but is limited in scope and needs further investigation before application to various domains of research. In this section, the major modifications proposed and the summary of extensive results is presented as:

- The concept of linearly decreasing population size, adaptive parameters and division of generation is added to the basic SSA to improve its working capability.
- Apart from the above added parameters, seven new mutation operators have been exploited to the best of their potential and based on them seven new versions of SSA have been proposed.
- All the proposed versions have been tested on CEC 2005, CEC 2015 and some real world optimization problems for performance evaluation and compared with respect to existing algorithms.
- The experimental results are presented in the form of statistical and convergence profiles and it has been found that MSSA is the best among all the proposed algorithms.
- Though MSSA is best among all the algorithms under test but it has been found that it still suffers from poor convergence and more work is required to be done to improve this property.

- As the algorithm is highly competitive and hence in future can be applied to various fields of research including, workforce scheduling, unmanned aerial vehicles, medical imaging, protein protein interaction, wireless communication and others.

6. Conclusion

This paper presents new enhancements to SSA to improve its performance. Four major modifications have been added in SSA to make it highly efficient with respect to other algorithms. Adaptive parameters are added to enhance the basic parameter sets, division of iterations has been added to gradually shift from exploration to exploitation phase, linearly decreasing population adaptation is followed to reduce the total number of function evaluations and finally a new exploitation phase has been added to improve the overall working properties of SSA. Apart from these modifications, the concept of mutations has been exploited and based on that seven new mutated SSA algorithms have been proposed. These include Cauchy mutation, Gaussian mutation, Lévy mutation, neighbourhood based mutation, trigonometric mutation, mutation clock and diversity mutation. Among all these proposed mutations, experimentally for variable population size and dimension size it was found that MSSA is the best algorithm. Experimentally for CEC 2005, CEC 2015 and CEC 2011 benchmark problems also, MSSA is found to be the best. Further statistical results and convergence profiles prove the significance of MSSA over other algorithms.

As a future direction, all the proposed algorithms can be exploited and applied to problems from various domains of research including antenna design, economic load dispatch problems, medical imaging, image segmentation, signal processing, DNA sequencing, feature selection, clustering and other problems. Extended versions of MSSA can be proposed by using various other mutation strategies and adaptive properties. Dynamic switching parameters along with inertia weights can be added to improve the overall performance of the algorithm. Further properties can be improved by employing new equations instead of general equations of SSA. Apart from this, new parameter settings can be initialized for analysing the adaptive properties of new SSA variants.

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