

Coordinated operation of coupled transportation and power distribution systems considering stochastic routing behaviour of electric vehicles and prediction error of travel demand

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Funding information

National Natural Science Foundation of China, Grant/Award Number: 61873225; Natural Science Foundation of Hebei Province, Grant/Award Number: E2020203205

Abstract

The popularisation of electric vehicles (EVs) and the development of dynamic wireless charging technology have created an emerging trend of transportation electrification, which strengthens the coupling between electrified transportation network (ETN) and power distribution network (PDN). Meanwhile, the stochastic routing behaviour of EVs and prediction error of traffic demand pose a severe challenge to the coordinated ETN-PDN operation problem. This paper proposes a new hybrid optimisation method using stochastic user equilibrium (SUE)/information gap decision theory (IGDT) to study the impact of the unavoidable uncertainties on the coordinated ETN-PDN operation, which consists of the following two stages. In the first stage, a collaborative optimisation model based on SUE and Dist-Flow equations is established to deal with the stochastic EV routing behaviour. Built upon this model, the second stage continues to consider the traffic demand prediction error and establish a risk decision model using IGDT. The proposed model can provide proper road congestion tolls and local generator production schedules to lead to a minimum expected socio-economic cost. Also, two different coordinated operation strategies, that is, risk-seeker and risk-averse strategies, are provided to deal with the uncertainties. Case studies are carried out to demonstrate the effectiveness of the hybrid SUE/IGDT optimisation method.

1 | INTRODUCTION

With the continuous development of battery, fast charging, and dynamic wireless charging (DWC) technologies [1], electric vehicles (EVs) are gradually replacing traditional vehicles powered by fossil fuels owing to their eco-friendly and cost-efficient advantages, which creates an emerging trend of transportation electrification [2]. However, the large-scale application of EVs significantly increases the electricity demand of the power distribution network (PDN). Meanwhile, the stochastic and disordered charging behaviour of EVs may increase the power system network loss, worsen the power quality, cause distribution line congestion, which brings a series of negative effects on the safe and economic operation of PDNs. Moreover, because the

charging facilities are set up along the road of the electrified transportation network (ETN), the choices of charging location and driving route of EVs will influence the distribution of traffic flow, which further impact the operation status of ETN [3].

In order to minimise the negative impact of EVs on the PDN and ETN, an appropriate EV routing along the ETN and charging scheduling in PDN is needed [4, 5], but a single scheduling scheme considering only the PDN or ETN cannot maximise the social benefits. A scheduling scheme with a high degree of coordination between PDN and ETN can not only reduce the adverse impact of large-scale EV charging load on the PDN but also alleviate the potential congestion in ETN. Therefore, coordinated ETN-PDN operation has become an emerging hot research topic [6].

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Considering conventional plug-in charging [7, 8] for EVs, there are already existing studies on the coordinated transportation and power network operation. In [9, 10], electricity prices and road tolls are optimised simultaneously to regulate EV charging demands and driving path towards optimum costs of the coupled networks. Meanwhile, the operational data privacy of each system is considered in [10]. A comprehensive modelling framework is proposed in [11], which demonstrates the existence of equilibrium states in coupled networks. In [12], a bi-directional information exchange mechanism and a routing strategy are developed to obtain an optimum operating point of the two networks. In order to shave bi-peak and smooth bi-ramp in coupled networks, a hierarchical optimisation approach is investigated in [13]. In [14], considering the elastic response of EVs to electricity charging price, a smart charging management system and a distributed coordination pricing method are presented for the coupled network. In [15], an EV charging schedule in coupled constrained networks is studied, which analyses the impact of EV departure time on mitigating the congestion of the two networks.

Compared with conventional plug-in charging mode, the DWC mode increases the spatial-temporal nature of EV-charging operations, leading to a tighter coupling between the transportation network and power network [6]. The coupling points of the two include not only fast-charging stations with fixed geographical location but also dynamically changed locations along wireless charging lanes, which improves the enthusiasm and timeliness of interaction between users and the power grid. Therefore, the coordinated operation of the two networks in wireless charging mode has caused more attention recently. For example, reference [16] investigates the impact of mobile EV charging load on the direct current location marginal price under DWC. Reference [17] analyses the influence of limited road capacity on the coupling network operation. Considering the transportation and power systems being managed separately or simultaneously, the integrated pricing models of roads and electricity are established in [18]. In [19], an economic dispatch model for power transmission networks and traffic networks is proposed based on an alternating direction method of multipliers. Aiming at minimising the total operation cost of the two systems, an optimal traffic-power flow model is presented in [20]. On the basis of [20], a multi-period optimal traffic-power flow model is further established in [21] using a semi-dynamic traffic flow model. Note that the above studies adopt the direct current optimal power flow model or alternating current optimal power flow to describe power flow distribution, and queuing network model, Wardrop social optimal or user equilibrium (UE) [22] principles to characterise traffic flow distribution.

Based on the above discussions, it is found that the current literature suffers from the following inadequacies. On the one hand, most existing studies mainly focus on the deterministic traffic demand between origin-destination (OD) without considering the uncertainty. But from an engineering viewpoint, the time-varying traffic demand is difficult to be predicted with high accuracy. Especially for EVs, the prediction error of traffic demand cannot be ignored because the battery power consump-

tion and charging demand of EVs are significantly affected by driving speed and habits amongst other factors. On the other hand, the widely used UE principle has a strong assumption that each road traveller in ETN can accurately receive the operation state of the whole ETN, charging electricity price, congestion tolls and other information, and they are perfectly rational when choosing the path to drive. Obviously, in real life, the charging and driving route choices of road travellers are highly stochastic, for example, different drivers have different estimates and conclusions for the same path due to different subjective understanding and perception. Therefore, these uncertainties caused by the prediction error of traffic demand and stochastic routing behaviour of EVs cannot be ignored, which may have a significant impact on the coordinated operation of PDN and ETN.

In the modelling of coordinated ETN-PDN operation problem, a major challenge arises in addressing the above unavoidable uncertainties. Motivated by the aforementioned facts, this paper endeavours to present an optimisation and modelling framework to investigate the impact of the unavoidable uncertainties on the coordinated ETN-PDN operation, that is, a new hybrid optimisation method using stochastic UE and information gap decision theory (SUE/IGDT) is proposed. The major contributions of this paper are summarised as follows.

1. Based on the hybrid SUE/IGDT optimisation method, a collaborative optimisation model and a risk decision model are proposed for a coupled transportation and power distribution systems (CTPDS) considering stochastic EV routing behaviours and travel demand prediction error. The proposed hybrid model can provide proper road congestion tolls and local generator production schedules for CTPDS to lead to a minimum expected socio-economic cost while addressing the unavoidable uncertainties.
2. The SUE assignment model based on the logit function is adopted to describe EV stochastic routing behaviour and traffic flow distribution in ETN, and a comprehensive comparison between the traffic flow assignment results of SUE and UE is presented. Additionally, a sensitivity analysis is carried out to investigate the effect of the SUE stochastic parameter on the collaborative optimisation model.
3. Two different coordinated operation strategies, that is, risk-seeker strategy (RSS) and risk-averse strategy (RAS), are provided by the risk decision model for the CTPDS to deal with uncertainties. Meanwhile, impact analysis of different objective deviation factor and risk strategy selection is presented.

The remainder of this paper is organised as follows. In Section 2, the system descriptions of the CTPDS are presented, and the modelling of ETN and PDN are given. In Section 3, the hybrid SUE/IGDT optimisation method is presented. In Section 4, the solution method is developed. Various simulation results are discussed in Section 5. Finally, the main conclusions are drawn in Section 6.

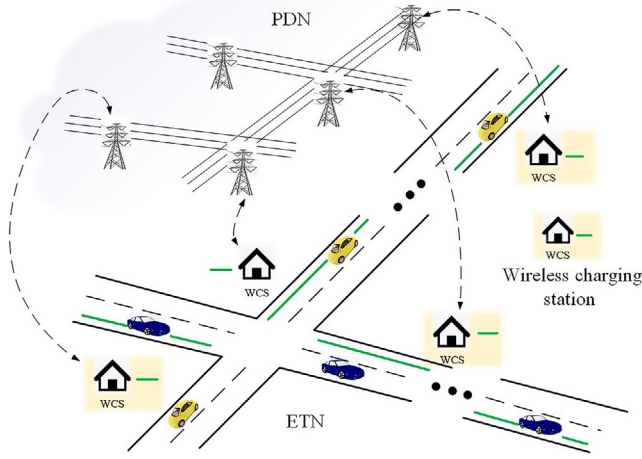


FIGURE 1 Framework of coupled transportation and power distribution systems

2 | System description and mathematical modelling

2.1 | Description of CTPDS

The system sketch of CTPDS is illustrated in Figure 1. The CTPDS is composed of ETN and PDN, which are coupled by wireless charging station and mobile EVs. When EVs need to be charged, DWC systems can continuously charge EVs through specified charging lanes installed on roadways [23]. Each charging lane in the ETN can be powered by the corresponding electrical power node/bus/substation from the PDN alone or by one electrical power node/bus/substation to multiple sections of the electrified lanes in a certain area. To simplify the analysis, we assume that each lane of ETN is equipped with wireless charging facilities and serviced by one electrical bus from the PDN. Meanwhile, similar to many existing studies [20, 21], we also assume that the presented CTPDS is managed by a non-profit entity, called the independent system operator (ISO).

2.2 | The ETN model

A simplified ETN consists of origins, destinations, intersections, lanes or roadway segments, which can be represented by a connected graph $G_T(N_T, A_T)$, where N_T and A_T denote the sets of nodes and links (or roads), respectively. A link (road) is denoted by an element $a \in A_T$, and ψ_a represents the corresponding traffic flow on the link (i.e. the number of vehicles on that link). In the ETN, every vehicle starts and completes its travel plan from an origin $r \in R$ to its destination $s \in S$, which is called travel OD pair $w \in W_{OD}$. Each OD pair $w \in W_{OD}$ is connected by a set of effective paths K_w , and the corresponding traffic demand (i.e. number of vehicles to travel in each OD pair) q_w will be assigned to these paths. Every path consists of a set of links, and this relationship can be

expressed with the help of an indicator variable $x_{a,k,w}$. If the path k includes a link a , then let $x_{a,k,w}=1$; otherwise, $x_{a,k,w}=0$. Notice that this variable is known and given in advanced according to the effective paths K_w . Let $H_{k,w}$ be the traffic flow on the path k between OD pair w , then the link flow ψ_a is given as follows:

$$\psi_a = \sum_{w=1}^{NW_{OD}} \sum_{k=1}^{NK_w} H_{k,w} x_{a,k,w}, \quad \forall a \in A_T \quad (1)$$

Equation (1) represents that the traffic flow on each link is equal to the sum of the flows on all effective paths passing through the link.

Total path travelling time is a major consideration for road travellers to choose a route, and it depends on the congestion level of each road on the route. The road congestion level is usually quantified by a latency function $\chi_a(\psi_a)$ related to link flow ψ_a , which represents travelling time needed on this link a . In the actual operation of transportation system, the more vehicles a link carries, the longer it will take to travel through that road. Therefore, the latency function $\chi_a(\psi_a)$ is an increasing function of link flow ψ_a . It is often represented by the following Bureau of Public Roads function [24]:

$$\chi_a(\psi_a) = \chi_a^0 \left[1 + 0.15 \left(\frac{\psi_a}{C_a} \right)^4 \right], \quad \forall a \in A_T \quad (2)$$

where χ_a^0 equals the length of link a divided by speed limit, called the free flow travelling time. Notation C_a denotes the link capacity.

Based on the link travelling time $\chi_a(\psi_a)$, the travelling time of each path can be calculated by $\tau_{k,w} = \sum_{a=1}^{NA_T} \chi_a(\psi_a) x_{a,k,w}$. To alleviate traffic congestion, a congestion toll T_a is charged in each link; then, drivers will choose the route according to the total travel expense. The travel expense of a driver experienced on the path k becomes the summation of travelling time cost and congestion tolls, which is given by

$$\zeta_{k,w} = \sum_{a=1}^{NA_T} [\eta \chi_a(\psi_a) + T_a] x_{a,k,w}, \quad \forall k \in K_w, w \in W_{OD} \quad (3)$$

where the weight parameter η is the monetary value of travelling time.

Assuming that all road travellers in ETN have stochastic routing behaviour, they always try to find the path with the lowest travel expense to drive, but usually they cannot correctly and accurately estimate the travel expense of each path. Thus, a consequence of all drivers' decisions will lead to an SUE [25], which can be captured by a logit model as follows:

$$\sigma_{k,w} = \frac{\exp(-\theta \zeta_{k,w})}{\sum_{k=1}^{NK_w} \exp(-\theta \zeta_{k,w})}, \quad \forall k \in K_w, w \in W_{OD} \quad (4)$$

$$H_{k,w} = q_w \sigma_{k,w}, \quad \forall k \in K_w, w \in W_{OD} \quad (5)$$

Equation (4) represents the traffic distribution proportion (or selection probability) of each effective path between each OD pair. θ is a non-negative stochastic coefficient, which represents the cognitive level (or familiarity level) of drivers to ETN. With the growth of the stochastic coefficient θ , drivers' cognitive levels become higher, bringing in a smaller variance of estimating the minimum travel expenses. Equation (5) indicates that the allocated traffic flow of each effective path is equal to the traffic demand of the corresponding OD pair multiplied by the selection probability. For simplicity, Equations (4) and (5) are called SUE conditions.

Thus, Equations (1)–(5) denote the traffic flow model of ETN, which are the ETN constraints of the proposed hybrid SUE/IGDT optimisation method to describe EV stochastic routing behaviour and traffic flow distribution in ETN. Following the paradigm in [26], it turns out that Equations (1)–(5) can be used to constitute the Karush–Kuhn–Tucker (KKT) conditions of the following optimisation problem, which is called a traffic assignment problem based on SUE (TAP-SUE):

$$\begin{aligned} \min F_{\text{TAP-SUE}} &= \eta \sum_{a=1}^{N_{A_T}} \int_0^{\psi_a} \chi_a(\varphi) d\varphi + \sum_{a=1}^{N_{A_T}} T_a \psi_a \\ &\quad + \frac{1}{\theta} \sum_{w=1}^{N_{W_{OD}}} \sum_{k=1}^{N_{K_w}} H_{k,w} (\ln H_{k,w} - 1) \\ \text{s.t. } \psi_a &= \sum_{w=1}^{N_{W_{OD}}} \sum_{p=1}^{N_{K_w}} H_{k,w} x_{a,k,w}, \quad \forall a \in A_T \\ q_w &= \sum_{k=1}^{N_{K_w}} H_{k,w}, \quad \forall w \in W_{OD} \\ H_{k,w} &\geq 0, \quad \forall k \in K_w, w \in W_{OD} \end{aligned} \quad (6)$$

Because the latency function $\chi_a(\psi_a)$ is differentiable and strictly increasing, the integral term contained in the objective function can be calculated as

$$\eta \sum_{a=1}^{N_{A_T}} \int_0^{\psi_a} \chi_a(\varphi) d\varphi = \eta \sum_{a=1}^{N_{A_T}} \chi_a^0 \left[\psi_a + 0.03 \frac{(\psi_a)^5}{(C_a)^4} \right]$$

It is easy to find that problem (6) is a convex programming problem with linear constraints and can be solved by traditional non-linear solvers. For a more detailed explanation, the readers are referred to [22, 25]. In view of the equivalence between the optimal solution of a convex program and the stationary point of its KKT conditions [27], the traffic flow model in Equations (1)–(5) can be transformed by Equation (6) to solve, which will be applied in the uncoordinated operating mode (case 1) in Section 5.

2.3 | The ETN model

A typical PDN is generally a radial network with tree topology, which can be described as a graph $GE(N_E, L_E)$, where N_E and L_E represent the sets of buses and distribution lines, respectively. In order to represent power flows of PDN, the following second-order cone (SOC)-based Dist-Flow model proposed in

[28] is employed:

$$P_{i,j} + P_{G,j} - \sum_{b=1}^{N_j} P_{j,b} - I_{i,j} R_{i,j} - P_{L,j} = 0, \quad \forall (i,j) \in L_E \quad (7)$$

$$Q_{i,j} + Q_{G,j} - \sum_{b=1}^{N_j} Q_{j,b} - I_{i,j} X_{i,j} - Q_{L,j} = 0, \quad \forall (i,j) \in L_E \quad (8)$$

$$U_i - U_j = 2 \left(R_{i,j} P_{i,j} + X_{i,j} Q_{i,j} \right) - \left[\left(R_{i,j} \right)^2 + \left(X_{i,j} \right)^2 \right] I_{i,j}, \quad \forall (i,j) \in L_E \quad (9)$$

$$I_{i,j} U_i \geq \left(P_{i,j} \right)^2 + \left(Q_{i,j} \right)^2, \quad \forall (i,j) \in L_E \quad (10)$$

Furthermore, the following security constraints of PDN are also considered:

$$U_j^{\min} \leq U_j \leq U_j^{\max}, \quad \forall j \in N_E \quad (11)$$

$$I_{i,j}^{\min} \leq I_{i,j} \leq I_{i,j}^{\max}, \quad \forall (i,j) \in L_E \quad (12)$$

$$P_{G,m}^{\min} \leq P_{G,m} \leq P_{G,m}^{\max}, \quad \forall m \in M_G \quad (13)$$

$$Q_{G,m}^{\min} \leq Q_{G,m} \leq Q_{G,m}^{\max}, \quad \forall m \in M_G \quad (14)$$

The above equations are the PDN constraints of the proposed hybrid SUE/IGDT optimisation method to describe the power flow in each distribution line and the voltage at each bus of the PDN.

2.4 | Coupling relationship between ETN and PDN

The charging demand of the wireless charging station in each link is the key coupling point of ETN and PDN. Referring to existing research in [17], the charging demand on each wireless charging station is proportional to the corresponding link traffic flow. Thus, the power demand of each bus in PDN is able to be calculated by Equation (15), which consists of two parts: One part is the regular power demand $P_{Lc,j}$, and the other part is related to the charging demand and is equal to the total traffic flow of all links served by bus j multiplied by parameter γ :

$$P_{L,j} = P_{Lc,j} + \gamma \sum_{a=1}^{N_a \in j} \psi_a, \quad \forall j \in N_E \quad (15)$$

where parameter γ denotes the charging rate of unit traffic flow and can be determined from operating experiences [20]. The equations in Equation (15) are the coupling constraints of the proposed hybrid SUE/IGDT optimisation method.

3 | COORDINATED OPERATION OF THE CTPDS WITH UNCERTAIN FACTORS BASED ON HYBRID SUE/IGDT METHOD

Considering the impact of stochastic routing behaviour of EVs and prediction error of travel demand on the CTPDS, a new hybrid SUE/IGDT optimisation method is proposed to coordinate the operation of ETN and PDN. This method is divided into two stages: In the first stage, a collaborative optimisation model considering EV stochastic routing behaviour is established. From this model, the second stage continues to consider the traffic demand prediction error and establish a risk decision model based on IGDT.

3.1 | Collaborative optimisation model of the CTPDS considering stochastic EV routing behaviour

This subsection presents a collaborative optimisation model for CTPDS to coordinate the intertwined traffic and power flows in the coupled networks. The ISO can apply the collaborative optimisation model to determine proper road congestion tolls and optimal local generator production schedules. In the proposed approach, the uncertainties related to stochastic EV routing behaviour are characterised via the SUE logit model.

Subjected to traffic flow, power flow and coupling constraints, the objective of the collaborative optimisation model is to minimise the socio-economic cost F of CTPDS based on forecasted traffic demand q_w . The corresponding optimisation model can be formulated as follows:

$$\begin{aligned} \min &= F_E + F_T \\ \text{s.t.} & \text{ Electrified transportation network constraints (1) – (5)} \\ & \text{Power distribution network constraints (7) – (14)} \\ & \text{Coupling constraint (15)} \end{aligned} \quad (16)$$

The objective function consists of two parts. The first term F_E represents the operational cost of PDN presented in Equation (17), including the power generation cost of local generators as well as the cost of purchasing power from the main grid:

$$F_E = \sum_{m=1}^{N_G} \left[a_m (P_{G,m})^2 + b_m P_{G,m} \right] + \sum_{j=1}^{N_0} \rho_{\text{sub}} P_{\text{sub},j} \quad (17)$$

The second term F_T is the total operational cost of ETN presented in Equation (18), which consists of the travelling time cost and the congestion tolls:

$$\begin{aligned} F_T &= \sum_{a=1}^{N_{AT}} \eta \chi_a (\psi_a) \psi_a + \sum_{a=1}^{N_{AT}} T_a \psi_a \\ &= \sum_{a=1}^{N_{AT}} \eta \chi_a^0 \left[1 + 0.15 \left(\frac{\psi_a}{C_a} \right)^4 \right] \psi_a + \sum_{a=1}^{N_{AT}} T_a \psi_a \end{aligned} \quad (18)$$

3.2 | Risk decision model of the CTPDS considering the prediction error of travel demand

In the collaborative optimisation model constructed in the previous subsection, it is assumed that the prediction of traffic demand is accurate, and the ETN and PDN are coordinated and optimised according to the predicted value of traffic demand q_w . However, in a practice transportation system, the traffic demand has large uncertainty. IGDT is a non-probabilistic interval optimisation method, which is very effective to handle uncertainties with large prediction error [29]. Therefore, this subsection uses the IGDT from [30] to further describe the prediction error of traffic demand and constructs the corresponding risk decision model based on the proposed collaborative optimisation model. By the proposed risk decision model, the ISO can pursue two different strategies to deal with traffic demand uncertainty: RAS and RSS.

3.2.1 | Traffic demand uncertainty model

In the existing research of IGDT, the related models describing uncertain input parameters are energy bound, slope bound, envelope bound models, and so forth. Because the fluctuation range of traffic demand has a great influence on the CTPDS, it is necessary to limit the overall uncertainty fluctuation to ensure the safety and reliability of the coupled system. Therefore, the envelope bound model [31] is used to describe the uncertainty of traffic demand, and the corresponding uncertainty set is formulated as follows:

$$\begin{cases} q_{\text{real},w} \in U(\alpha, q_w) \\ U(\alpha, q_w) = \left\{ q_{\text{real},w} : \left| \frac{q_{\text{real},w} - q_w}{q_w} \right| \leq \alpha \right\}, \quad \forall w \in W_{OD} \end{cases} \quad (19)$$

where α is the uncertainty horizon parameter, q_w and $q_{\text{real},w}$ are the forecasted traffic demand and actual traffic demand, respectively. $U(\alpha, q_w)$ indicates that the uncertainty horizon parameter α deviates from the predicted value q_w within the range of $\alpha|q_w|$.

Note that the collaborative optimisation model proposed in the first stage assumes that the prediction of traffic demand is accurate, and the ETN and PDN are coordinated and optimised according to the predicted value of traffic demand q_w . Therefore, considering the uncertainty of traffic demand, the risk decision model needs to modify the constraints related to the forecasted traffic demand. The forecasted traffic demand q_w will be replaced by the actual traffic demand $q_{\text{real},w}$. Thus, Equation (5) can be revised as

$$H_{k,w} = q_{\text{real},w} \sigma_{k,w}, \quad \forall k \in K_w, w \in W_{OD} \quad (20)$$

3.2.2 | RAS model

The RAS desires to find robust road congestion tolls and optimal production schedules of local generators to coordinate the operation of ETN and PDN in a way to be immune against higher socio-economic cost due to unfavourable traffic demand deviations from the forecasted values. This can be represented as follows:

$$\left\{ \begin{array}{l} \max \alpha_{ra} \\ \text{s.t.} \quad \max F \leq (1 + \beta_{ra}) F_0 \\ (1 - \alpha_{ra}) q_w \leq q_{\text{real},w} \leq (1 + \alpha_{ra}) q_w, \quad \forall w \in W_{OD} \\ \alpha_{ra} \geq 0, \quad \forall w \in W_{OD} \\ q_{\text{real},w} = \sum_{k=1}^{N_{K_w}} H_{k,w}, \quad \forall w \in W_{OD} \\ H_{k,w} = q_{\text{real},w} \sigma_{p,w}, \quad \forall k \in K_w, w \in W_{OD} \\ \text{ETNconstraints : (1)-(4)} \\ \text{PDNconstraints : (7) - (14)} \\ \text{Couplingconstraint(15)} \end{array} \right. \quad (21)$$

The model presented by Equation (21) indicates that RAS transforms the collaborative optimisation model into another problem to seek the maximum uncertainty of traffic demand that can be tolerated, where socio-economic cost is not exceeding $(1 + \beta_{ra})F_0$. In other words, when the traffic demand changes within the range $[(1 - \alpha_{ra})q_w, (1 + \alpha_{ra})q_w]$, the coordinated operation scheme under RAS can ensure that the total socio-economic cost is less than or equal to the expected value $(1 + \beta_{ra})F_0$. Boundary value F_0 is a socio-economic cost for the collaborative optimisation model. α_{ra} and β_{ra} denote traffic demand uncertainty and socio-economic cost deviation factor of RAS, respectively.

3.2.3 | RSS model

Contrary to the RAS, RSS is optimistically treating the uncertainty of traffic demand that may have positive effects on the objective function. Therefore, the RSS wishes to coordinate the operation of ETN and PDN at low socio-economic cost by using the positive effects, which can be represented as follows.

$$\left\{ \begin{array}{l} \min \alpha_{rs} \\ \text{s.t.} \quad \min F \leq (1 - \beta_{rs}) F_0 \\ (1 - \alpha_{rs}) q_w \leq q_{\text{real},w} \leq (1 + \alpha_{rs}) q_w, \quad \forall w \in W_{OD} \\ \alpha_{rs} \geq 0, \quad \forall w \in W_{OD} \\ q_{\text{real},w} = \sum_{k=1}^{N_{K_w}} H_{p,w}, \quad \forall w \in W_{OD} \\ H_{k,w} = q_{\text{real},w} \sigma_{k,w}, \quad \forall k \in K_w, w \in W_{OD} \\ \text{ETNconstraints : (1)-(4)} \\ \text{PDNconstraints : (7) - (14)} \\ \text{Couplingconstraint(15)} \end{array} \right. \quad (22)$$

The model presented by Equation (22) indicates that RSS transforms the collaborative optimisation model into a new problem to seek the minimum uncertainty of traffic demand that can be used, where a socio-economic cost is limited

by $(1 - \beta_{rs})F_0$. In other words, when the traffic demand changes within the range $[(1 - \alpha_{rs})q_w, (1 + \alpha_{rs})q_w]$, the coordinated operation scheme under RSS has the opportunity to make socio-economic cost less than or equal to the expected value $(1 - \beta_{rs})F_0$. Here, α_{rs} and β_{rs} denote traffic demand uncertainty and socio-economic cost deviation factor of RSS, respectively.

4 | SOLUTION METHOD

The risk decision models established in this paper are bi-level programming problems, which can be solved by the typical solution methods for bi-level models directly [32], but it may take a long computing time. To solve this issue, it is helpful to note that the proposed risk decision models can be transformed into equivalent single-level optimisation problems according to [33]. Because the road latency in Equation (2) is a high-order power function and the power demand of each bus in PDN is calculated by Equation (15), the socio-economic cost of the CTPDS is directly related to traffic demand.

For the RAS, both the operational costs in PDN and ETN in the lower-level programming model grow significantly when the traffic demand increases. Therefore, when the actual value of traffic demand is far greater than the predicted value, that is, $q_{\text{real},w} = (1 + \alpha_{ra})q_w$, the corresponding operational cost is the largest. In this case, the ‘max F ’ in Equation (21) can be reduced to ‘ F ’ equivalently, and the corresponding RAS needs the following additional Constraints (23) and (24) to replace bi-level model constraints $\min F \leq (1 - \beta_{rs})F_0$ and $(1 - \alpha_{rs})q_w \leq q_{\text{real},w} \leq (1 + \alpha_{rs})q_w$ in Equation (21):

$$F \leq (1 + \beta_{ra}) F_0 \quad (23)$$

$$q_{\text{real},w} = (1 + \alpha_{ra}) q_w, \quad \forall w \in W_{OD} \quad (24)$$

Similarly, for the RSS, both the operation costs in PDN and ETN in the lower-level programming model drop significantly when the traffic demand decreases. Therefore, when the actual value of traffic demand is far less than the predicted value, that is, $q_{\text{real},w} = (1 - \alpha_{rs})q_w$, the corresponding operational cost is the least. In this case, the ‘min F ’ in Equation (22) can be reduced to ‘ F ’ equivalently, and the corresponding RSS needs the following additional Constraints (25) and (26) to replace the bi-level model constraints $\min F \leq (1 - \beta_{rs})F_0$ and $(1 - \alpha_{rs})q_w \leq q_{\text{real},w} \leq (1 + \alpha_{rs})q_w$ in Equation (22):

$$F \leq (1 - \beta_{rs}) F \quad (25)$$

$$q_{\text{real},w} \leq (1 - \alpha_{rs}) q_w, \quad \forall w \in W_{OD} \quad (26)$$

Based on the above models, the flowchart of the proposed hybrid SUE/IGDT optimisation method is depicted in Figure 2. First, based on the forecasted value of traffic demand, the collaborative optimisation model in the first stage is solved

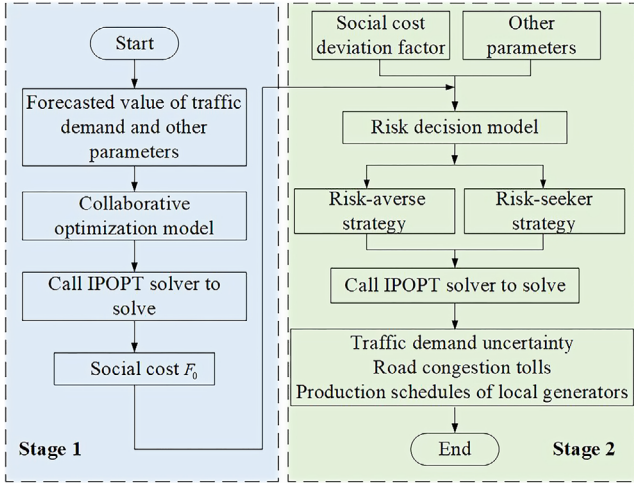


FIGURE 2 Flowchart of the proposed hybrid stochastic user equilibrium and information gap decision theory optimisation method

by IPOPT solver to obtain socio-economic cost F_0 . Then, using the acceptable socio-economic cost deviation factor, the previously obtained F_0 , and the risk decision model, the ISO selects the RAS and RSS according to their decision intention. Finally, the corresponding traffic demand uncertainty, road congestion tolls and production schedules of local generators are obtained.

5 | CASE STUDY

In this section, two test systems are adopted to demonstrate the effectiveness of the proposed hybrid SUE/IGDT method. The presented model is coded in MATLAB aided by YALMIP with IPOPT, and all case studies are implemented on a PC with Intel (R) Core (TM) i5-10210U CPU (2.11 GHz) and 16GB RAM.

In order to demonstrate the effectiveness of the proposed hybrid SUE/IGDT method for mitigating negative effects caused by uncertain factors in wireless charging mode, the following four cases are presented:

Case 1: Uncoordinated operation mode of CTPDS considering stochastic routing behaviour of EVs. There is no congestion toll to mitigate negative effects caused by traffic overload, and all EV users choose routes according to travelling time cost and self-perception. The uncoordinated operating mode is divided into two steps: In the first step, the TAP-SUE model in Equation (6) with the given link toll parameter $T_a = 0$ is solved to capture the spatial distribution of traffic flow; then, the electrified transportation authority calculates the total operation cost F_T . In the second step, the power distribution system operator will minimise the PDN operation cost F_E by solving the optimal power flow model in Equations (17), (7) to (14) with the link charging demand corresponding to the obtained traffic TAP-SUE pattern.

Case 2: Coordinated operation mode of CTPDS considering stochastic routing behaviour of EVs. This is indeed the first stage of the proposed hybrid SUE/IGDT method.

Case 3: On the basis of cases 2 and 3 further considers the prediction error of traffic demand. This is the proposed hybrid SUE/IGDT method, which contains two strategies: RAS and RSS.

Case 4: Coordinated operation mode of CTPDS considering optimal route selections of EVs (Wardrop UE). This is the modelling method in [20], which does not take into account the stochastic routing behaviour of EVs and the prediction error of traffic demand.

To ensure a feasible solution for case 1 under high traffic density, it is assumed that the lower bound of voltage can be violated at the cost of a penalty. The operation cost of PDN F_E in the four cases is modified as follows:

$$F_E = \sum_{m=1}^{N_G} \left[a_m (P_{G,m})^2 + b_m P_{G,m} \right] + \sum_{j=1}^{N_0} \rho_{sub} P_{sub,j} + \kappa \sum_{j=1}^{N_E} \Delta U_j$$

where the penalty coefficient is chosen as $\kappa=50000$ \$/p.u. [20]; meanwhile, the voltage boundary constraints in Equation (11) become the following constraints:

$$U_j^{\min} - \Delta U_j \leq U_j \leq U_j^{\max}, \Delta U_j \geq 0, \forall j \in N_E$$

It is worth emphasising that the main intention of this paper is to mitigate the impact of stochastic EV routing behaviour and the traffic demand prediction error on the coordinated operation of CTPDS. Thus, case 1 can be treated as a base case to validate the effectiveness of the presented collaborative optimisation model. In addition, compared with references [17–21], case 3 addresses the unavoidable uncertainties caused by the prediction error and stochastic routing behaviour, which are ignored in these existing studies. To demonstrate the necessity to consider these uncertainties, a similar case in [20] (called case 4) is considered and compared. The analysis and corresponding results are given as follows.

5.1 | Test system 1: Single OD pair electrified transportation system and five nodes power distribution system

The test system 1 is a simple CTPDS. Its topology is shown in Figure 3. The ETN includes one pair of origin and destination, three nodes, five links, and six paths. The PDN consists of five buses, five distribution lines and one generation unit G . The generation unit G is connected to bus 2. The characteristics of links and physical connections are summarised in Table 1. The traffic demand is 30 p.u. from T1 to T3. The base values of traffic flow and power demand are 100 vehicles/h and 10 MVA, respectively. The stochastic coefficient and objective deviation factor are set as $\theta = 1.5$ and $\beta_{ra}/\beta_{rs} = 0.1$.

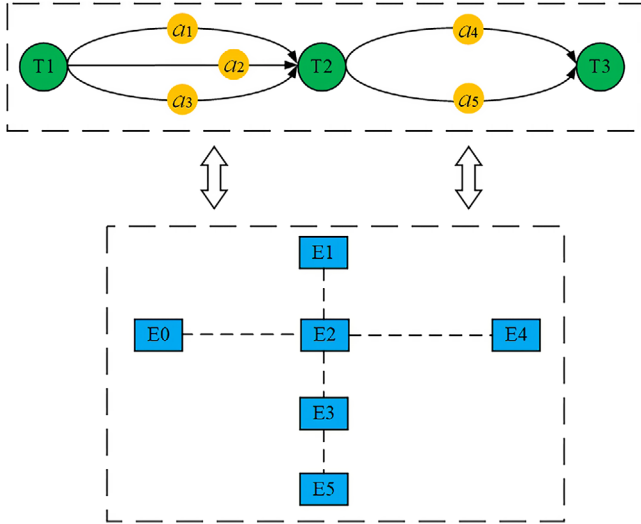


FIGURE 3 Topology of test system 1

TABLE 1 The physical connections and characteristics of electrified transportation network (ETN)

Link	C_a (p.u.)	χ_a^0 (min)	Connected power distribution network (PDN) node
a_1	6	6	E2
a_2	8	10	E4
a_3	5	6.5	E5
a_4	9.8	5	E3
a_5	7	5.5	E1

5.1.1 | Results and analysis of cases 1 to 3

To analyse the effectiveness and advantages of the proposed hybrid SUE/IGDT method, the cost, traffic flow and power flow of case 1 to 3 are compared.

The comparison results of various costs, traffic flow and power flow in cases 1 to 3 are given in Tables 2 and 3 and Fig-

TABLE 2 Costs in cases 1 to 3

Scenario	Total cost (\$)	PDN cost F_E (\$)	ETN cost F_T (\$)	Penalty cost of voltage (\$)	Uncertain parameter (%)
Case 1	1272.77	1139.41	133.36	304.43	/
Case 2	968.42	822.65	145.77	0	/
Case 3 risk-averse strategy (RAS)	1065.26	880.67	184.59	0	6.31
Case 3 risk-seeker strategy (RSS)	871.58	758.78	112.80	0	7.24

TABLE 3 Voltage magnitude and active power

Node	Voltage magnitude (p.u.)				
	E1	E2	E3	E4	E5
Case 1	0.9845	1.0199	0.9394	0.9827	0.9077
Case 2	0.9784	1.0141	0.9381	0.9736	0.9110
Case 3 RAS	0.9779	1.0154	0.9374	0.9712	0.9110
Case 3 RSS	0.9786	1.0122	0.9388	0.9760	0.9110

Node	Active power (p.u.)				
	E2-E1	E0-E2	E2-E3	E2-E4	E3-E5
Case 1	0.33	0.02	0.66	0.27	0.23
Case 2	0.33	0.12	0.62	0.29	0.19
Case 3 RAS	0.34	0.09	0.64	0.32	0.19
Case 3 RSS	0.31	0.15	0.60	0.26	0.20

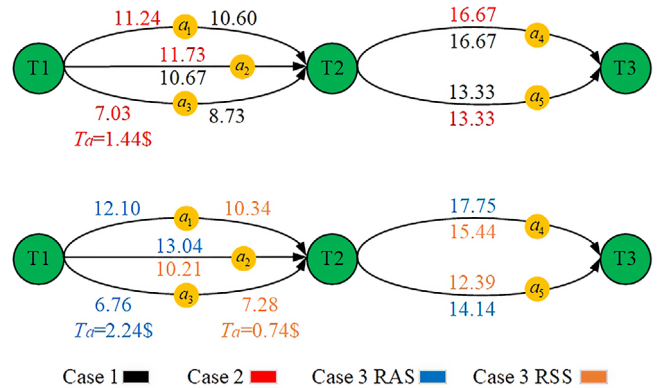


FIGURE 4 Distribution of traffic flow in electrified transportation network

ure 4, respectively. It can be seen that case 1 has the maximum total operation cost \$1272.77, which includes maximum PDN cost \$1139.41 and minimum ETN cost \$133.36. Because in the uncoordinated operation mode, most travellers choose the path with the minimum travel time, which results in heavy traffic flows of link 3. Also, link 3 is connected to the terminal buses E5 of PDN, which causes the voltage magnitude at bus E5 to drop to 0.9077 p.u., which violates the lower bound. It poses a threat to the voltage instability of PDN, and then a penalty cost of \$304.43 for voltage violation is increased in case1. In cases 2 and 3, the link T3 is charged for the toll that makes some travellers choose other routes, which helps to alleviate traffic overload on link T3. In this way, the voltage magnitudes at bus E5 reach their lower bounds but never exceed the bounds. The above indicates that the stochastic routing behaviours of EVs might pose a threat to the security of PDN in the uncoordinated operation mode, and the proposed hybrid SUE/IGDT method can reduce the total cost and realise the coordinated operation of the CTPDS.

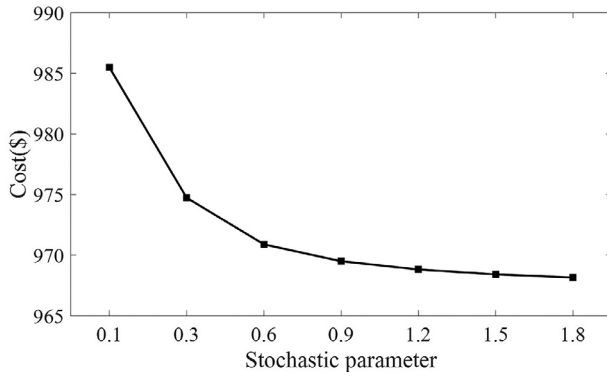


FIGURE 5 Socio-economic cost under different stochastic parameter

Compared to case 2, the impacts of uncertainty caused by travel demand prediction error is considered in case 3. In an uncertain environment, the RAS obtains the coordinated operation strategy with underestimated traffic demand by sacrificing part of the objective function values, and the RSS gives a coordinated operation strategy with overestimated traffic demand considering the positive effect of uncertainty on the objective function. For the RAS, when the upper limit of the total cost is increased to $F_{ra} = (1 + 0.1) \times \$968.42 = \$1065.26$, the maximum value of the uncertain parameter α_{ra} is 0.0631, and the toll charged in link 3 is \$2.24 more than \$1.44 in case 2. It indicates that when the total expected cost of sacrifice does not exceed 10%, the maximum travel demand prediction error that can be borne by CTPDS is 6.31%. At this point, a bigger toll is needed to compensate for the impact of this underestimated uncertainty. Conversely, for the RSS, when the upper limit of the total cost is decreased to $F_{rs} = (1 - 0.1) \times \$968.42 = \$871.58$, the minimum value of the uncertain parameter α_{rs} is 0.0724, and the toll charged in link 3 is \$0.74 less than \$1.44 in case 2. It indicates that the RSS is able to reduce 10% of the expected total cost only when the travel demand is at least 7.24% lower than the predicted value. Now, due to the decrease of congestion level, only a small toll is needed in RSS.

5.1.2 | Sensitivity analysis of stochastic parameter

The stochastic parameter θ is an important influencing factor in the SUE assignment model, and the change of this parameter will determine whether the presented collaborative optimisation model (case 2) has a promising application in the real world. Therefore, the sensitivity of the stochastic parameter θ is analysed, and the corresponding simulation results are provided in Figures 5–7.

Figure 5 shows the changes in socio-economic cost F against stochastic parameter θ . It can be seen that the socio-economic cost F decreases gradually when the stochastic parameter increases, and this is because the increase of stochastic parameter improves the cognitive level of travellers, which makes more travellers choose better routes that have a lower travel expense. However, when the stochastic parameter reaches a certain value, the socio-economic cost is no longer reduced by the accurate

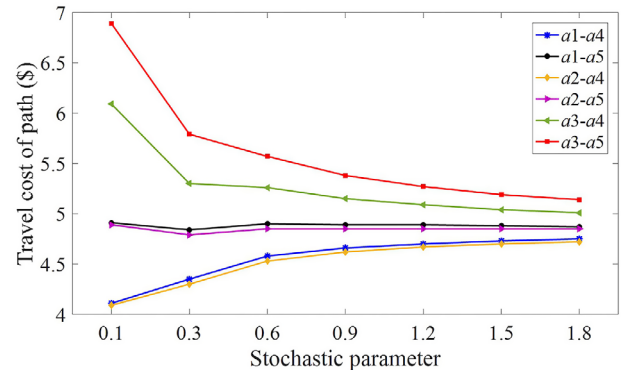


FIGURE 6 Travel cost of each path under different stochastic parameter

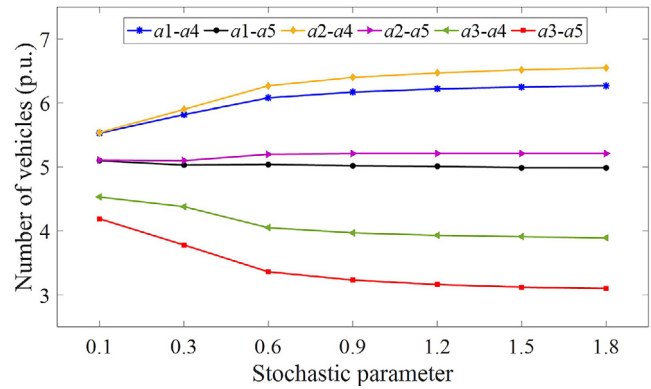


FIGURE 7 Traffic flow of each path under different stochastic parameter

estimation of travellers. This means that the individual travellers have a higher cognition level at this moment, and most of them have found out the best way to the destination according to the latest information on the congestion level and toll charge of each road.

The travel expense and traffic flow of each path under different stochastic parameter are shown in Figures 6 and 7, respectively. As can be seen, the travel expenses of path $a_3 - a_4$ and $a_3 - a_5$ decline remarkably, while that of path $a_1 - a_4$ and $a_2 - a_4$ grow when the stochastic parameter becomes bigger. Meanwhile, the gap between travel expense for each path is narrowed with the increase of the stochastic parameter, while the gap between traffic flows is widened. It further illustrates that the improvement of cognitive level enables more travellers to estimate the travel expense accurately, which makes them shift from expensive to cheaper paths. Also, the travel expense of all these paths is close to the same optimal value for a big stochastic parameter θ .

5.1.3 | Impact analysis of objective deviation factor and risk strategy selection in the risk decision model

When β_{ra} and β_{rs} change in the range of 0%–15%, the trajectories of traffic demand uncertainty α_{ra} and α_{rs} , as well as total

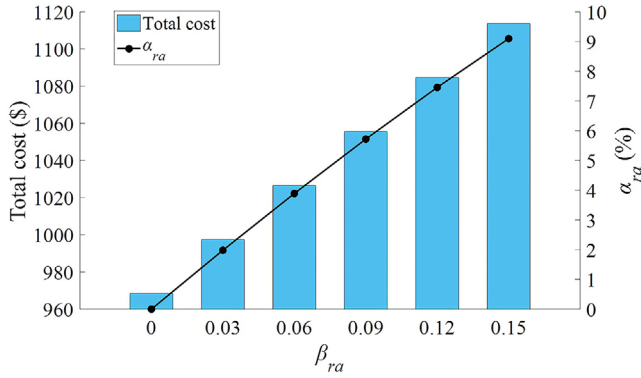


FIGURE 8 Maximum deviations of traffic demand and total socio-economic costs in risk-averse strategy with different objective deviation factors

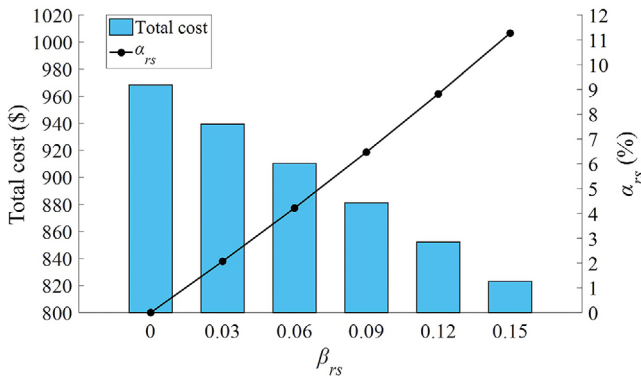


FIGURE 9 Minimum deviations of traffic demand and total socio-economic costs in risk-seeker strategy with different objective deviation factors

socio-economic costs F_{ra} and F_{rs} , are obtained by solving the risk decision model (case 3), and they are shown in Figures 8 and 9.

As presented in Figure 8, the maximum deviation of traffic demand α_{ra} increases gradually with the increase of objective deviation factor β_{ra} in the RAS; meanwhile, the total socio-economic cost F_{ra} also increases gradually. Because the RAS considers the negative impact of traffic demand prediction error, to obtain a conservative robust decision-making, more expected objective values must be sacrificed. Moreover, a higher value of α_{ra} implies a higher risk of traffic demand uncertainty.

In the RSS, as presented in Figure 9, the minimum deviations of traffic demand α_{rs} increase similarly with the increase of objective deviation factor β_{rs} ; however, the total operation cost F_{rs} decreases gradually. Because the RSS considers the positive impact of traffic demand prediction error, the uncertainty of traffic demand will bring additional revenue. A higher value of α_{rs} will give a bigger benefit brought by the uncertainty of traffic demand, which results in a smaller socio-economic cost.

In order to further analyse the impact of different risk decision models and different objective deviation factor val-

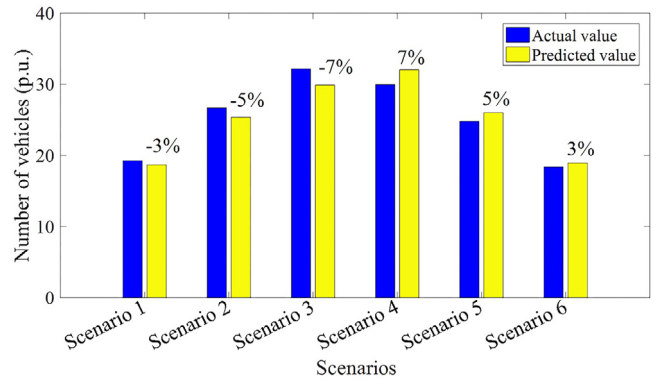


FIGURE 10 Actual and forecasted traffic demands under six scenarios

ues on the coordinated operation of the CTPDS, we specially design six scenarios with different forecasted error of traffic demand. As presented in Figure 10, in scenarios 1 to 3, the traffic demand is underestimated, whereas in scenarios 4 to 6, the traffic demand is mostly overestimated. The corresponding forecasted errors are -3%, -5%, -7%, 7%, 5% and 3%, respectively.

Table 4 displays the total socio-economic costs for coordinated operation of CTPDS under different traffic demands. It is found that in scenarios 1 to 3, the results of RAS are closer to the socio-economic cost of actual traffic demand, but in scenarios 4 to 6, the results of RSS are closer to the socio-economic cost of actual traffic demand. It confirms that for an underestimated circumstance, the RAS is more suitable, whereas for an overestimating circumstance, the RSS is more appropriate. However, the RAS (RSS) has different results for different objective deviation factors, which means that the objective deviation factor set by the system manager will have a non-negligible impact on the coordinated operation results of CTPDS.

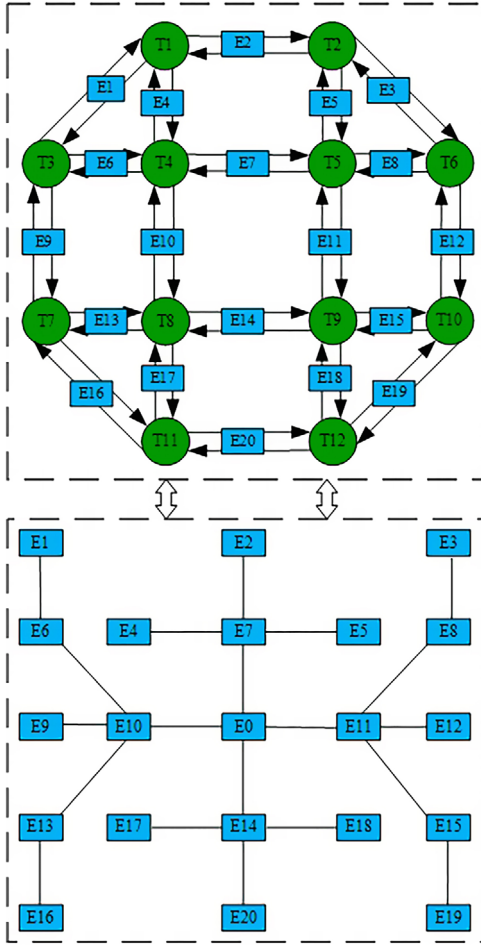
5.2 | Test system 2: 11 OD pairs electrified transportation system and 20 buses power distribution system

To further discuss the characteristics of proposed models for larger systems, the 12 nodes ETN and 20 buses PDN are integrated and employed as test system 2. Its topology is shown in Figure 11. The traffic demand and the other parameter settings have been reported elsewhere [20]. The stochastic coefficient and objective deviation factor are set as $\theta = 1.5$, and $\beta_{ra}/\beta_{rs} = 0.05$, respectively.

In test system 2, we further analyse the effect of the proposed hybrid SUE/IGDT method under different traffic density by multiplying the traffic demand with a factor ρ from 0.85 to 1.1. In addition, this coupling network is adopted to clarify the benefits of employing the SUE model rather than the Wardrop UE model and the necessity of considering traffic demand prediction error.

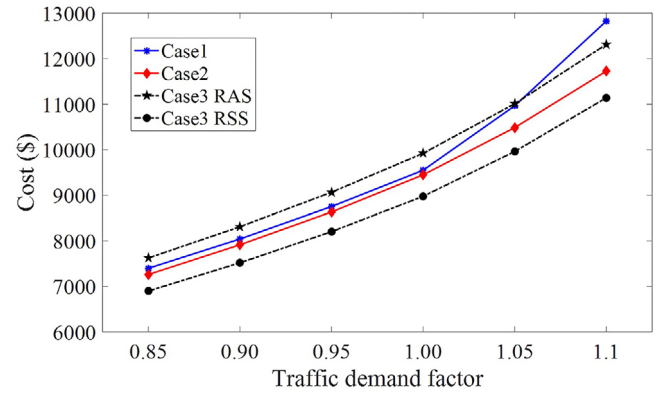
TABLE 4 Total cost for coordinated operation of coupled transportation and power distribution systems under different travel demands

Scenario	Total cost under actual traffic demand(\$)	Total cost under forecasted traffic demand(\$)	Total cost of RAS (\$)		Total cost of RSS (\$)	
			$\beta_{ra} = 5\%$	$\beta_{ra} = 10\%$	$\beta_{rs} = 5\%$	$\beta_{rs} = 10\%$
1	578.48	561.02	589.07	617.12	532.97	504.92
2	825.90	776.20	815.01	853.82	737.39	698.58
3	1079.64	963.65	1011.83	1060.02	915.47	867.29
4	966.51	1074.55	1128.28	1182.00	1020.82	967.09
5	755.83	800.39	840.41	880.43	760.37	720.35
6	551.76	568.23	596.64	625.05	539.81	511.40

**FIGURE 11** Topology of test system 2

5.2.1 | Results and analysis of cases 1 to 3 under different traffic density

Figure 12 suggests that the traffic conditions have a great influence on the total operational cost of CTPDS. For example, total socio-economic costs in cases 1 to 3 grow remarkably when the traffic demand increases, and this is because that the road latency in Equation (2) is a high-order power function, which causes higher transportation operational cost F_T , and more electricity is needed to meet the charging service, which

**FIGURE 12** Socio-economic costs under different traffic demand factors

increases the PDN operational cost F_E . Additionally, there are three other findings: (1) When the load factor $\rho \leq 1$, the total socio-economic cost of case 1 is slightly higher than that of case 2, and due to light traffic conditions, the security of PDN may not be compromised and the stochastic routing behaviour of travellers is almost close to the optimal operation state of the system; (2) when the load factor satisfies $1 \leq \rho \leq 1.05$, the PDN constraints of case 1 can still be maintained, but the cost gap with case 2 increases because a medium traffic density in case 1 leads to a higher load level of terminal buses, resulting in higher power loss through the distribution line, whereas case 2 reduces the power loss by sacrificing a small amount of the travelling time and toll cost; (3) as the traffic demand continues to increase, voltage boundary constraints cannot be maintained as shown in Figure 13 ($\rho = 1.1$); case 1 has no choice but to pay for the voltage violation, incurring a higher socio-economic cost, while cases 2 and 3 are able to prevent voltage violation by charging tolls on the roads with minimal socio-economic costs (see Table 5). The above three findings indicate that the proposed hybrid SUE/IGDT method is effective under different traffic densities.

Furthermore, different traffic densities will affect the prediction error of traffic demand. From Figure 14, it is noted that the traffic demand uncertainties α_{ra} and α_{rs} decrease gradually with the growth of traffic density level. It means that for a high traffic density, the RAS needs to sacrifice more socio-economic cost to bear the uncertainty caused by the traffic demand

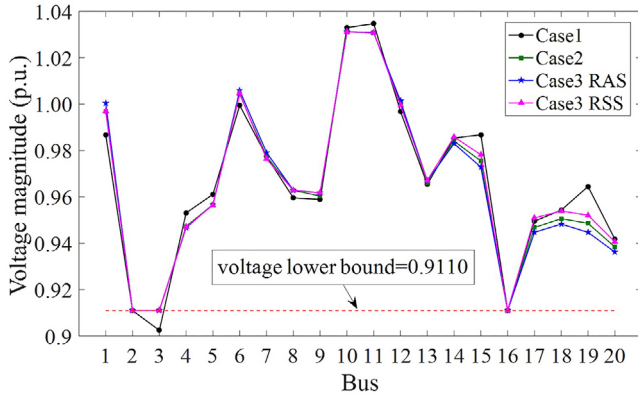


FIGURE 13 Voltage magnitude with traffic demand factor ($\rho = 1.1$)

TABLE 5 Costs in cases 1 to 3 ($\rho = 1.1$)

Case	Total cost (\$)	PDN cost F_E (\$)	ETN cost F_T (\$)	Penalty cost of voltage (\$)	Uncertain parameter (%)
Case 1	12,827.14	9520.39	3306.75	775.46	/
Case 2	11,728.28	8121.87	3606.41	0	/
Case 3 RAS	12,314.71	8277.73	4036.98	0	1.82
Case 3 RSS	11,141.88	7901.39	3240.49	0	2.03

prediction error, whereas the RSS can achieve the expected socio-economic cost with less traffic demand prediction error. It is also shown in Figure 15 that cases 2 and 3 can be solved in a reasonable time (within 140 s) in all circumstances.

To further illustrate the accuracy of SOC relaxation, the error index of each distribution line is calculated as follows:

$$\Delta_{i,j} = \left| (P_{i,j})^2 + (Q_{i,j})^2 - I_{i,j} U_i \right|, \quad \forall (i,j) \in L_E$$

Here, the traffic demand factor $\rho = 1$ is taken as an example to make a simulation analysis. The error results are shown in Table 6. It can be seen that the error results of 96.25% are less

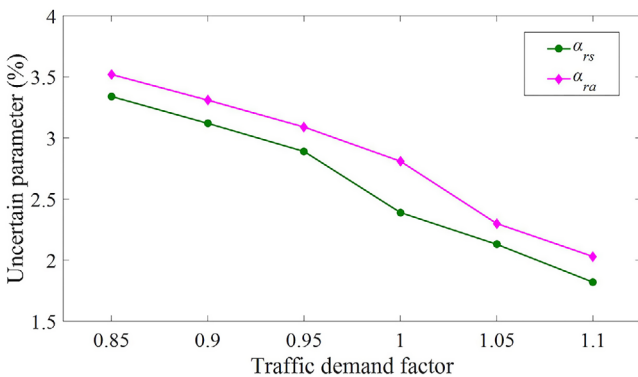


FIGURE 14 Uncertainty of traffic demand under different traffic demand factors

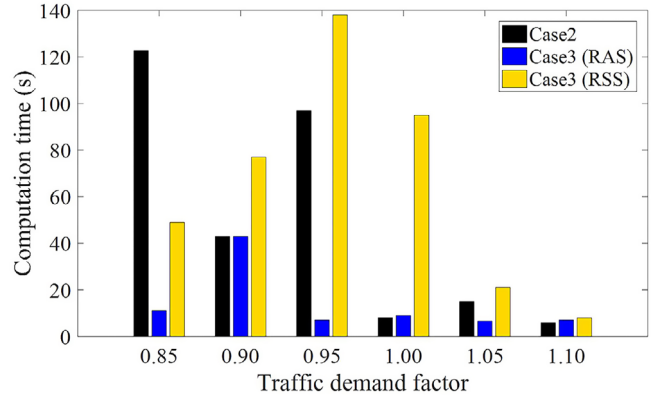


FIGURE 15 Computation times under different traffic demand factors

TABLE 6 Error index of each distribution line in cases 1 to 3

Line	Case 1	Case 2	Case 3 RAS	Case 3 RSS
E6-E1	9.98×10^{-9}	9.98×10^{-9}	9.11×10^{-9}	9.27×10^{-9}
E7-E2	9.98×10^{-9}	9.98×10^{-9}	9.10×10^{-9}	9.29×10^{-9}
E8-E3	9.98×10^{-9}	9.99×10^{-9}	9.82×10^{-9}	9.54×10^{-9}
E7-E4	9.98×10^{-9}	9.98×10^{-9}	9.15×10^{-9}	9.33×10^{-9}
E7-E5	9.98×10^{-9}	9.98×10^{-9}	9.07×10^{-9}	9.27×10^{-9}
E10-E6	9.97×10^{-9}	9.97×10^{-9}	8.87×10^{-9}	9.08×10^{-9}
E0-E7	9.96×10^{-9}	1.04×10^{-8}	8.39×10^{-9}	8.75×10^{-9}
E11-E8	9.97×10^{-9}	9.98×10^{-9}	9.52×10^{-9}	9.19×10^{-9}
E10-E9	9.98×10^{-9}	9.98×10^{-9}	9.18×10^{-9}	9.33×10^{-9}
E0-E10	9.95×10^{-9}	1.09×10^{-8}	7.96×10^{-9}	8.38×10^{-9}
E0-E11	9.94×10^{-9}	9.94×10^{-9}	6.97×10^{-9}	8.13×10^{-9}
E11-E12	9.98×10^{-9}	9.98×10^{-9}	9.25×10^{-9}	9.23×10^{-9}
E10-E13	9.98×10^{-9}	9.98×10^{-9}	9.09×10^{-9}	9.20×10^{-9}
E0-E14	9.96×10^{-9}	1.18×10^{-8}	8.30×10^{-9}	8.68×10^{-9}
E11-E15	9.97×10^{-9}	9.98×10^{-9}	9.21×10^{-9}	9.13×10^{-9}
E13-E16	9.99×10^{-9}	9.98×10^{-9}	9.43×10^{-9}	9.42×10^{-9}
E14-E17	9.98×10^{-9}	9.98×10^{-9}	9.12×10^{-9}	9.31×10^{-9}
E14-E18	9.98×10^{-9}	9.98×10^{-9}	9.09×10^{-9}	9.29×10^{-9}
E15-E19	9.98×10^{-9}	9.98×10^{-9}	9.33×10^{-9}	9.26×10^{-9}
E14-E20	9.98×10^{-9}	9.98×10^{-9}	9.03×10^{-9}	9.23×10^{-9}

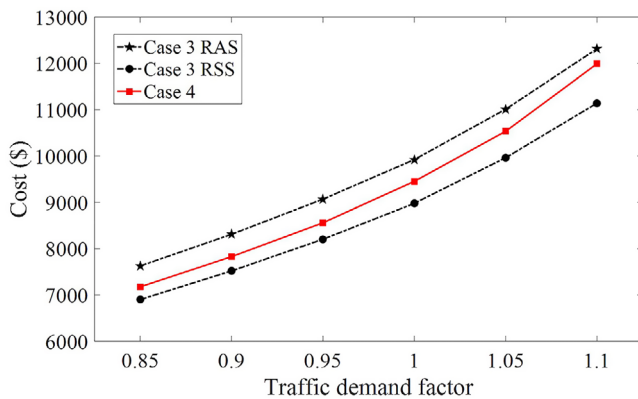
than 1×10^{-8} , and the whole error results almost close to zero, so the SOC relaxation is strictly accurate.

5.2.2 | Comparison of cases 3 and 4

First, we note that the SUE model in case 3 can simulate the practical traffic flow in a better way than case 4. This is because the Wardrop UE model in case 4 relies on a stronger assumption that each road traveller in ETN can accurately find optimal travel path, that is, the travelling costs on all used paths are equal and minimal, while cases 3 can calculate the

TABLE 7 Path selection results of cases 3 and 4 in origin-destination pair T4–T9 ($\rho = 1$)

Path		Case 3						Case 4	
		Travel cost (\$)		Path flow (p.u.)		Probability (%)		Travel cost (\$)	Path flow (p.u.)
		RAS	RSS	RAS	RSS	RAS	RSS		
1	T4-T8-T9-T10	16.84	15.29	4.25	3.61	41.55	37.12	16.36	5.88
2	T4-T5-T6-T10	17.71	15.73	1.16	1.87	11.32	19.21	16.36	0
3	T4-T5-T9-T10	16.83	15.22	4.28	4.03	41.81	41.42	16.36	4.12
4	T4-T8-T11-T12-T10	18.88	17.80	0.20	0.08	1.97	0.86	16.54	0
5	T4-T8-T9-T12-T10	18.99	17.98	0.17	0.06	1.67	0.66	16.54	0
6	T4-T5-T9-T12-T10	18.98	17.91	0.17	0.07	1.68	0.73	16.54	0

**FIGURE 16** Socio-economic costs under different traffic demand factors

selected probability and traffic flow of each effective path using the probability selection Formulas (4) and (5) to describe the stochastic routing behaviour of travellers. Here, the OD pair T1–T9 with six efficient paths is taken as an example to make a detailed analysis. The corresponding path selection results of cases 3 and 4 are listed in Table 7. It can be seen that in case 4, all travellers are assigned on paths 1 and 3 with the same minimum travelling cost, while in case 3, nearly 80% of the travellers prefer to use paths 1 and 3 with lower travelling cost, 11.32% (RAS) and 19.21% (RSS) of travellers choose the path 2, and only 5.32% (RAS) and 2.24% (RSS) of travellers choose the paths 4 to 6 with higher travelling cost. This result indicates that the SUE model in case 3 provides a solution close to the true circumstance that most travellers always try to find the path with the least travel expense, but it is impossible for everyone to identify accurately the optimal path due to the various reasons such as behaviour and cognitions. Thus, a small portion of travellers eventually chooses the paths with high travel expenses.

Second, the model formulation in case 4 has not considered the travel demand prediction error. Figure 16 shows the socio-economic costs of cases 3 and 4 under different traffic demand factors. We can see that there is a big difference in the socio-economic costs obtained by cases 3 and 4. This indicates that the travel demand prediction error has a

significant impact on the coordinated operation of PDN and ETN, and the prediction error of traffic demand cannot be ignored.

The above analysis demonstrates the benefits of employing the SUE model rather than the Wardrop UE model and the necessity of considering traffic demand prediction error. The proposed hybrid SUE/IGDT method considers the impact of stochastic routing behaviour of EVs and travel demand prediction error, and the model formulation in case 3 can provide more reasonable dispatching and helpful decisions for scheduling operators to deal with the risks brought by the uncertainties in ETN.

6 | CONCLUSION

Considering the impact of stochastic routing behaviour of travellers and travel demand prediction error, this paper proposes an optimisation modelling approach based on the hybrid SUE/IGDT method for coordinated operation of CTPDS, which is composed of the ETN and PDN, along with wireless charging stations and mobile EVs. The main conclusions drawn from simulation results are as follows:

1. The proposed hybrid SUE/IGDT method can quantify the impact of stochastic routing behaviour of travellers and traffic demand prediction error on the coordinated operation of CTPDS, and it is very effective to reduce total socio-economic cost and mitigate potential adverse effects on voltage violations of the PDN.
2. In light of the stochastic routing behaviour of travellers in a practical traffic system, the necessity of using the SUE model in the coordinated ETN-PDN is demonstrated to achieve more accurate results. Additionally, according to the sensitivity analysis of the stochastic parameter θ , the total socio-economic cost of CTPDS decreases with the improvement of road travellers' cognitive level.
3. By the impact analysis of different objective deviation factors and risk strategy selections in the risk decision model, the corresponding results can provide helpful decisions to deal with the risks brought by the uncertainties in ETN.

The effectiveness of the proposed hybrid SUE/IGDT optimisation method has been demonstrated through case studies and comparisons; however, the random and uncertain power output of renewable distributed generations (DGs) is not considered. As an immediate future step, renewable generations will be incorporated within the CTPDS, and the hybrid SUE/IGDT optimisation method will be expanded for the coordinated operation of ETN and PDN.

ACKNOWLEDGEMENTS

This work is financially supported by the [National Natural Science Foundation of China](#) (no. 61873225) and the [Natural Science Foundation of Hebei Province](#) (no. E2020203205).

NOMENCLATURE

Sets

A_T	set of links in the ETN
K_w	set of available paths between OD pair $w \in W_{OD}$
L_E	set of distribution lines in the PDN
M_G	set of local generators
N_E	set of buses in the PDN
N_T	set of nodes in the ETN
W_{OD}	set of OD pairs

Constants

χ_a^0	free flow travelling time for link $a \in A_T$
C_a	vehicular flow capacity of link $a \in A_T$
q_w	forecasted traffic demand between OD pair w
$x_{a,k,w}$	if path k passes link a , $x_{a,k,w} = 1$; otherwise, $x_{a,k,w} = 0$
NW_{OD}	numbers of OD pairs
NK_w	numbers of available paths
NA_T	numbers of links (or roads)
a_m, b_m	electricity energy production cost coefficients of generator m
$R_{i,j}, X_{i,j}$	resistance and reactance of distribution line connecting buses i and j , $(i, j) \in L_E$
$P_{G,m}^{\min}, P_{G,m}^{\max}$	active generation limits of generator m
$Q_{G,m}^{\min}, Q_{G,m}^{\max}$	reactive generation limits of generator m
U_j^{\min}, U_j^{\max}	bounds of square voltage magnitude at bus j
U_0	square voltage magnitude at the slack bus
$I_{i,j}^{\max}$	square of current flow capacity of line $(i, j) \in L_E$
$P_{Lc,j}$	regular power demand at bus j
$Q_{Lr,j}$	reactive power demand at bus j
N_G	number of generators in the PDN
N_j	number of child distribution lines connecting bus j
N_0	numbers of child distribution lines connecting the main grid
$N_{a \in j}$	numbers of links connected in bus j
ρ_{sub}	electricity price of the main grid
η	monetary value of travelling time
θ	non-negative stochastic coefficient

γ	charging rate of unit traffic flow
β_{ra}, β_{rs}	deviation factor of socio-economic cost in RAS/RSS
κ	penalty coefficient of voltage exceeding limit

Variables

ψ_a	traffic flow on link $a \in A$
$H_{k,w}$	traffic flow on path k between OD pair w
$q_{\text{real},w}$	actual traffic demand considering prediction error between OD pair w
$\tau_{k,w}$	travelling time of path k between OD pair w
$\zeta_{k,w}$	travel expense of path k between OD pair w
$\sigma_{k,w}$	traffic distribution proportion of path k between OD pair w
T_a	congestion toll of link a
$P_{i,j}, Q_{i,j}$	active/reactive power flow in line (i, j)
$P_{G,j}, Q_{G,j}$	active/reactive power generation of generator at bus j
$I_{i,j}$	square of current magnitude in line (i, j)
U_j	square of voltage magnitude at bus j
$P_{L,j}$	active power demand at bus j
α_{ra}, α_{rs}	traffic demand uncertainty in RAS/RSS

REFERENCES

- Machura, P., Li, Q.: A critical review on wireless charging for electric vehicles. *Renewable Sustainable Energy Rev.* 104, 209–234 (2019)
- Bilgin, B., et al.: Making the case for electrified transportation. *IEEE Trans. Transp. Electrif.* 1(1), 4–17 (2015)
- Xiong, J., et al.: Investigate the impacts of PEV charging facilities on integrated electric distribution system and electrified transportation system. *IEEE Trans on Transp. Electrif.* 1(2), 178–187 (2015)
- Ding, T., et al.: Optimal electric vehicle charging strategy with markov decision process and reinforcement learning technique. *IEEE Trans. Ind. Appl.* 56(5), 5811–5823 (2020)
- Ding, T., et al.: Rectangle packing problem for battery charging dispatch considering uninterrupted discrete charging rate. *IEEE Trans. Power Syst.* 34(3), 2472–2475 (2019)
- Lv, S., Wei, Z.: Coupling electricity and transportation networks to achieve dynamic wireless charging: Review and prospects. *J. Global Energy Interconnect.* 2(5), 484–491 (2019)
- Lu, Z., et al.: Modeling dynamic demand response for plugin hybrid electric vehicles based on real-time charging pricin. *IET Gener. Transm. Distrib.* 11(1), 228–235 (2017)
- Li, C., et al.: An electric vehicle routing optimization model with hybrid plug-in and wireless charging systems. *IEEE Access* 6, 27569–27578 (2018)
- He, F., et al.: Sustainability SI: Optimal prices of electricity at public charging stations for plug-in electric vehicles. *Networks Spatial Econ.* 16(1), 131–154 (2013)
- Alizadeh, M., et al.: Optimal pricing to manage electric vehicles in coupled power and transportation networks. *IEEE Trans. Control Network Syst.* 4(4), 863–875 (2017)
- Wei, W., et al.: Network equilibrium of coupled transportation and power distribution systems. *IEEE Trans. Smart Grid* 9(6), 6764–6779 (2018)
- Amini, M., Karabasoglu, O.: Optimal operation of interdependent power systems and electrified transportation Networks. *Energies* 11(1), 196 (2018)
- Jiang, H., et al.: Power-traffic coordinated operation for bi-peak shaving and bi-ramp smoothing—A hierarchical data-driven approach. *Appl. Energy*, 229, 756–766 (2018)
- Geng, L., et al.: Smart charging management system for electric vehicles in coupled transportation and power distribution systems. *Energy* 189, 116275(2019)

15. Sun, Y., et al.: EV charging schedule in coupled constrained networks of transportation and power system. *IEEE Trans. Smart Grid* 10(5), 4706–4716 (2019)
16. Ou, C., Hao, L., Zhuang, W.: Investigating wireless charging and mobility of electric vehicles on electricity market. *IEEE Trans. Ind. Electron.* 62(5), 3123–3133 (2015)
17. Wei, W., Wang, J., Lei, W.: Quantifying the impact of road capacity loss on urban electrified transportation networks: An optimization based approach. *Int. J. Transp. Sci. Technol.* 5(4), 268–288 (2016)
18. He, F., Yin, Y., Zhou, J.: Integrated pricing of roads and electricity enabled by wireless power transfer. *Transp. Res. Part C Emerging Technol.* 34(9), 1–15 (2013)
19. Manshadi, S., et al.: Wireless charging of electric vehicles in electricity and transportation network. *IEEE Trans. Smart Grid* 9(5), 4503–4512 (2018)
20. Wei, W., et al.: Optimal traffic-power flow in urban electrified transportation networks. *IEEE Trans. Smart Grid* 8(1), 84–95 (2017)
21. Lv, S., et al.: Optimal power and semi-dynamic traffic flow in urban electrified transportation network. *IEEE Trans. Smart Grid* 11, 1854–1865 (2020)
22. Wardrop, J.: Some theoretical aspects of road traffic research. *Proc. Inst. Civ. Eng.* 1(3), 325–362 (1952)
23. Ahmad, A., Alam, M., Chabaan, R.: A comprehensive review of wireless charging technologies for electric vehicles. *IEEE Trans. Transp. Electrification* 4(1), 38–63 (2018)
24. Bureau of Public Roads: *Traffic Assignment Manual*. U.S. Department of Commerce, Washington, DC (1964)
25. Daganzo, F., Sheffi, Y.: Stochastic models of traffic assignment. *Transp. Sci.* 11(3), 253–274 (1977)
26. Fisk, C.: Some developments in equilibrium traffic assignment. *Transport. Res. Part B Methodol.* 14(3), 243–255 (1980)
27. Bazaraa, M., Sherali, H., Shetty, C.: *Nonlinear Programming: Theory and Algorithms* (3rd ed.). John Wiley & Sons, Hoboken, NJ (2013)
28. Gan, G., et al.: Exact convex relaxation of optimal power flow in radial networks. *IEEE Trans. Autom. Control* 60(1), 72–87 (2015)
29. Ben-Haim, Y.: *Info-Gap Decision Theory*, pp. 267–296. Academic, San Diego, CA (2006)
30. Amirhossein, D., Mohammad, J., Behnam, M.: Short-term scheduling strategy for wind-based energy hub: A hybrid stochastic/IGDT approach. *IEEE Trans. Sustainable Energy* 10(1), 438–447 (2019)
31. Soroudi, A., Rabiee, A., Keane, A.: Information gap decision theory approach to deal with wind power uncertainty in unit commitment. *Electr. Power Syst. Res.* 145, 137–148 (2017)
32. Shafiee, S., et al.: Risk-constrained bidding and offering strategy for a merchant compressed air energy storage plant. *IEEE Trans. Power Syst.* 32(2), 946–957 (2017)
33. Chen, K., et al.: Robust restoration decision making model for distribution networks based on information gap decision theory. *IEEE Trans. Smart Grid* 6(2), 587–597 (2015)

How to cite this article: Geng L., Lu Z., Guo X., Zhang J., Li X., He L.: Coordinated operation of coupled transportation and power distribution systems considering stochastic routing behaviour of electric vehicles and prediction error of travel demand. *IET Gener. Transm. Distrib.* 15, 2112–2126 (2021).
<https://doi.org/10.1049/gtd2.12161>