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Observation of Bloch oscillations with a threshold

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We demonstrate experimentally Bloch oscillations, which occur above a certain threshold value of the effective potential gradient in lattices with specially modulated coupling between the neighboring sites. We formulate the general conditions for this phenomenon, arising due to the competition between the tilting and broadening of the transmission band, and explain why no threshold was present in any previous observations. Our experiments are performed in inhomogeneous photonic lattices, which represent the process of quantum two-mode squeezing in Fock space, underpinning a fundamental quantum-classical correspondence. [http://dx.doi.org/10.1063/1.4982879]

In 1971, Leo Esaki suggested a phenomenon that is today known as "Bloch oscillations" (BOs): the periodic motion of electrons in an atomic lattice under the action of an external electric field.[1,2] In other words, the electron in a periodic lattice undergoes an oscillatory motion instead of uniform spreading when a voltage is applied. Strikingly, this transition occurs for any voltage, as small as it may be. Being a wave phenomenon, BOs are found in any lattice system that is based on wave physics and where a linear transverse potential gradient can be applied, such as electrons in semiconductor lattices,[3] ultracold atoms,[4,5] Bose-Einstein condensates,[6] and photonic waveguide arrays.[7–9] In all implementations of BO, no threshold in the required potential gradient was observed. Recent studies provide a paradigm shift by formulating general conditions for the existence of Bloch oscillations in inhomogeneous lattices, identifying a regime of BO with a threshold.[10–12] With our work, we provide an experimental proof of this new kind of BO by utilizing photonic lattices as a platform.

The very nature of BO is identical in all lattice systems, and mathematically the key dynamical features can be captured by a set of discrete coupled mode equations,

$$i\frac{\mathrm{d}a_n}{\mathrm{d}z} = C_{n,n+1}a_{n+1} + C_{n,n-1}a_{n-1} + 2\beta n a_n\,. \qquad (1)$$

To be specific, we discuss BO implementation in photonic lattices in the form of optical waveguide arrays, since we use this platform for the experimental demonstration of our general theoretical results. Then, $a_n(z)$ is the complex optical mode amplitude in the nth waveguide, z is the evolution coordinate, $C_{n,n-1} \equiv C_{n-1,n}$ is the coupling or hopping rate between the waveguides n and $(n-1)$. The value of β determines the local propagation constant shift between the successive waveguides, which acts as the external linear potential gradient.[13] A characteristic lattice structure is shown in Fig. 1(a), top.

In homogeneous lattices ($C_{n,n-1}$ = const), BOs occur for any potential ramp $\beta \neq 0$,[14] and the oscillation frequency is proportional to the gradient. In our work, we generalize the understanding of BO for inhomogeneous lattices with variable coupling ($C_{n,n-1} \neq$ const). Our theory explains the absence of a threshold in any experimental manifestations of BO to date, including recent studies of several inhomogeneous lattice types where also BOs occur without any evidence of a threshold.[15–17] Importantly, our general analysis exactly predicts in which cases BO can occur at all

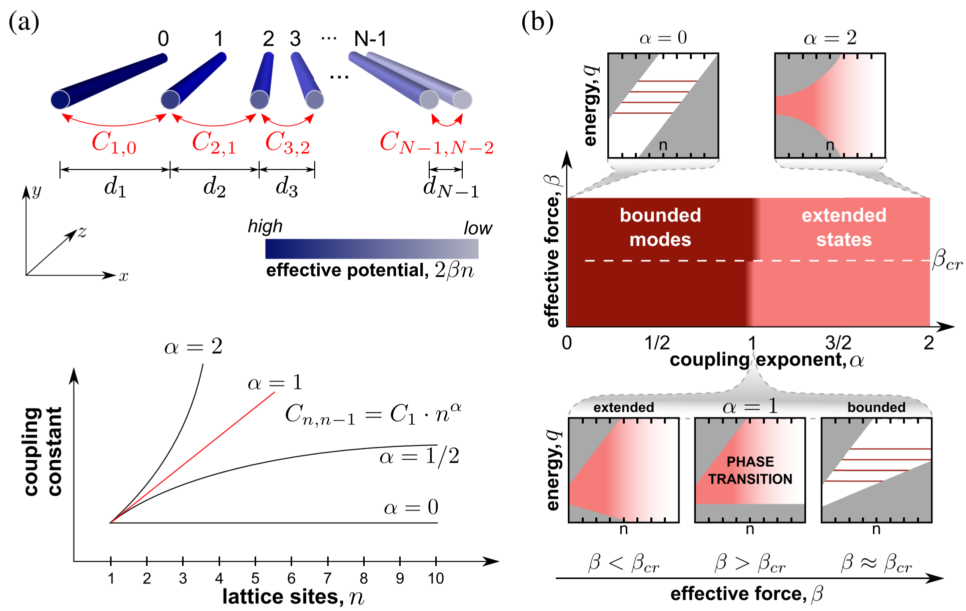

FIG. 1. *Bloch oscillations and threshold effects in inhomogeneous lattices*: (a) Top—design of a photonic lattice of N waveguides with a linear gradient of propagation constants, β, representing an effective applied potential. Bottom—characteristic polynomial dependencies of the coupling coefficient on the lattice position with the exponent α. (b) Centre—diagram indicating two regimes of bound eigenmodes or extended states depending on the coupling exponent α and the effective force β. Top—characteristic eigenmode spectra with discrete Wannier-Stark ladder of bound modes in a homogeneous lattice ($\alpha = 0$) and continuum spectrum of extended states for $\alpha = 2$. Bottom—eigenmode spectra for a linear coupling modulation ($\alpha = 1$) showing transition from continuous to discrete spectrum above the threshold effective force $\beta > \beta_{\text{cr}}$.

and pinpoints a regime when BO exhibits a threshold, providing a unifying view on several theoretical predictions.[10–12]

The appearance of BO is associated with the periodic revivals of any input state.[7–9] This is intrinsically connected to a particular structure of the eigenmode spectrum, where eigenmodes are found as solutions $a_n(z) = A_n \exp(iqz)$ with the propagation constants q defining the effective energy levels. In the BO regime, the eigenlevels q form an equidistant Wannier-Stark ladder[18] and each eigenmode is bounded. In contrast, continuum spectrum of extended modes corresponds to wavepacket broadening and absence of revivals. Then, we first perform a qualitative analysis to identify the type of spectra in lattices with different shapes of coupling modulations. Let us assume that coupling $C_{n,n-1}$ gradually varies along the lattice, although conclusions will generally apply even when this condition is not strictly satisfied. Then we consider an average coupling from site n to its left and right neighbours as $\overline{C}_n = (C_{n+1,n} + C_{n,n-1})/2$ and approximately determine transmission band around lattice site n using expressions for a locally homogeneous lattice[13] as

$$q_-(n) < q < q_+(n),\ q_\pm \approx \pm 2\overline{C}_n - 2\beta n. \tag{2}$$

If the value of q falls inside the band around lattice site n, then the wave can propagate further through the lattice, whereas outside the band it will exhibit reflection.

We now consider a broad class of inhomogeneous coupling modulations of a polynomial type, $C_{n,n-1} = C_1 \cdot n^\alpha$, as shown in Fig. 1(a), bottom. Here $\alpha \geq 0$ is a characteristic constant. We then calculate the band boundaries with Eq. (2) and plot these for different values of α in Fig. 1(b). We observe that for $\alpha < 1$ and $\beta \neq 0$, the spectrum is bounded since waves with any q will exhibit back-reflection at large n. It happens because the linear potential $(2\beta n)$ overcomes the sub-linear coupling modulation. This includes the traditional case of a homogeneous lattice ($\alpha = 0$) as well as recently considered inhomogeneous lattices where modulation shape corresponds to $\alpha = 1/2$.[15–17] On the other hand, for $\alpha > 1$, the super-linear growth in coupling exceeds the linear potential increase, and as a result, waves with any q can propagate away to arbitrarily large n, meaning that the spectrum is continuous and BO cannot happen.

Remarkably, in the intermediate case of $\alpha = 1$, a system can exhibit BO with a threshold, in agreement with previous theoretical studies.[10–12] Specifically, for $\beta < \beta_{cr}$, the spectrum is continuous and wave packets can propagate through the entire lattice. However, for $\beta > \beta_{cr}$, the modes become localized according to the band-gap structure, and accordingly the spectrum forms a discrete ladder. Here the critical value is found from the asymptotics of Eq. (2) at large n,

$$\beta_{cr} = C_1. \tag{3}$$

Such a spectral change from extended to localized states is a kind of metal-insulator phase transition, and it can occur precisely at the BO threshold.

Formation of the discrete spectrum is a necessary, but not sufficient condition for BO. It is also essential that the spectrum forms an equidistant Wannier-Stark ladder. To achieve this, all the lattice couplings need to be properly designed. To determine a class of such lattice configurations, we draw on a quantum-classical analogy to the process of two-mode squeezing in quadratically nonlinear media.[11,12] It was shown that Eq. (1) can represent the evolution of a squeezed state in number (Fock) basis, such that the wavefunction amplitudes are mapped by the respective waveguides as $\psi_{n_s,n_i} = a_n$, where $n_s = n$ and $n_i = n + \Delta$ are the photon numbers of the signal and idler modes, respectively. Here $\Delta = 0, 1, 2, \ldots$ is a free parameter, defining the difference between the idler and signal photon numbers. Such quantum-classical correspondence holds when the lattice has an inter-site hopping rate of

$$C_{n,n-1} = C_1\sqrt{n(n+\Delta)}, \tag{4}$$

and effective force β represents the phase mismatch. We note that the coupling asymptotically approaches linear dependence at large n, $C_{n,n-1} \sim C_1 n$, and it is in this regime with $\alpha = 1$ that our general analysis predicts a possibility of BO with a threshold.

Remarkably, it was indeed found theoretically that *squeezed light Bloch oscillations* occur above a threshold,[11,12] precisely as defined by Eq. (3). Any input state will then evolve in an oscillating manner with the period,

$$z_P = \frac{\pi}{b} = \frac{\pi}{\sqrt{\beta^2 - \beta_{\rm cr}^2}}\,. \tag{5}$$

For example, consider an input state with a fixed number of signal and idler photons, $n_s^{(0)} = n_i^{(0)} - \Delta$. Then the average photon number evolves according to[11]

$$\langle n_s(z)\rangle = n_s^{(0)} + \frac{\beta_{cr}^2}{|b|^2}|\sin(bz)|^2[1 + \Delta + 2n_s^{(0)}]. \tag{6}$$

This solution is also valid for the effective force below the BO threshold, $\beta < \beta_{\rm cr}$. In the latter case, b becomes imaginary and the photon number defined by Eq. (6) grows exponentially.

For our experimental studies to observe the BO with a threshold, we fabricated various waveguide lattices with different β's, but identical $C_1 \approx 0.022\,\mathrm{mm}^{-1}$. For details concerning the fabrication and characterisation of the waveguide lattices, we refer to the methods section. First, we characterize a lattice with zero effective force ($\beta = 0\,\mathrm{mm}^{-1}$) and $\Delta = 0$. In terms of squeezing analogy, this represents the vacuum state when the input light is launched into the first waveguide $n = 0$,

$$a_0(0) = 1\,, \quad a_{n>0}(0) = 0\,. \tag{7}$$

The resulting light intensity propagation is shown in Fig. 2(a). After entering the lattice, the light gradually couples to the neighbouring waveguides with a rate that is exponentially growing with z, simulating the rapid increase of the average photon number. To illustrate this, we plot the extracted average center of mass of the intensity distribution as a white line, which corresponds to the experimental implementation of the average photon number. In Fig. 2(b), we plot the theoretical intensity distribution, which is obtained by numerically integrating Eq. (1), together with the average photon number $\langle n_s(z)\rangle$ given by Eq. (6). Indeed, in this case, the photon number quickly increases, as predicted.

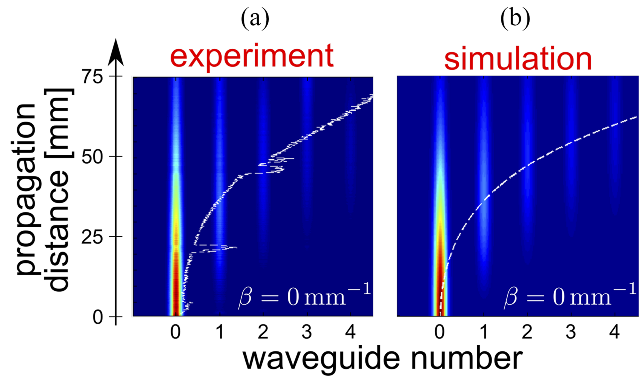

FIG. 2. *Light evolution without external force*: (a) the experimental observation and (b) numerical simulations. The lattice simulates the signal/idler photon number distribution in squeezed vacuum states in Fock space. The white lines show the center of mass of the intensity, corresponding to the average photon number $\langle n_s \rangle = \langle n_i \rangle$.

In the second part of our experiments, we gradually increase the external force in order to observe transition from the exponential growth to Bloch oscillations. The results are summarized in Fig. 3. For small values of β, the light spreads with essentially the same rate as without a potential ramp (see Figs. 3(a) and 3(e) for the experimental and theoretical results, respectively). Even when the effective force reaches the critical value ($\beta = \beta_{cr} \approx 0.022\,\mathrm{mm}^{-1}$), the experimental and numerical data indicate a monotonous light spreading with a quadratic dependence on distance, as shown in Figs. 3(b) and 3(f). However, above the critical value ($\beta > 0.022\,\mathrm{mm}^{-1}$), the light field reaches a maximum width [Figs. 3(c) and 3(g)] and eventually returns to the initial waveguide [Figs. 3(d) and 3(h)]. The latter situation is realized by implementing a detuning of $\beta = 0.047\,\mathrm{mm}^{-1}$, resulting in a Bloch

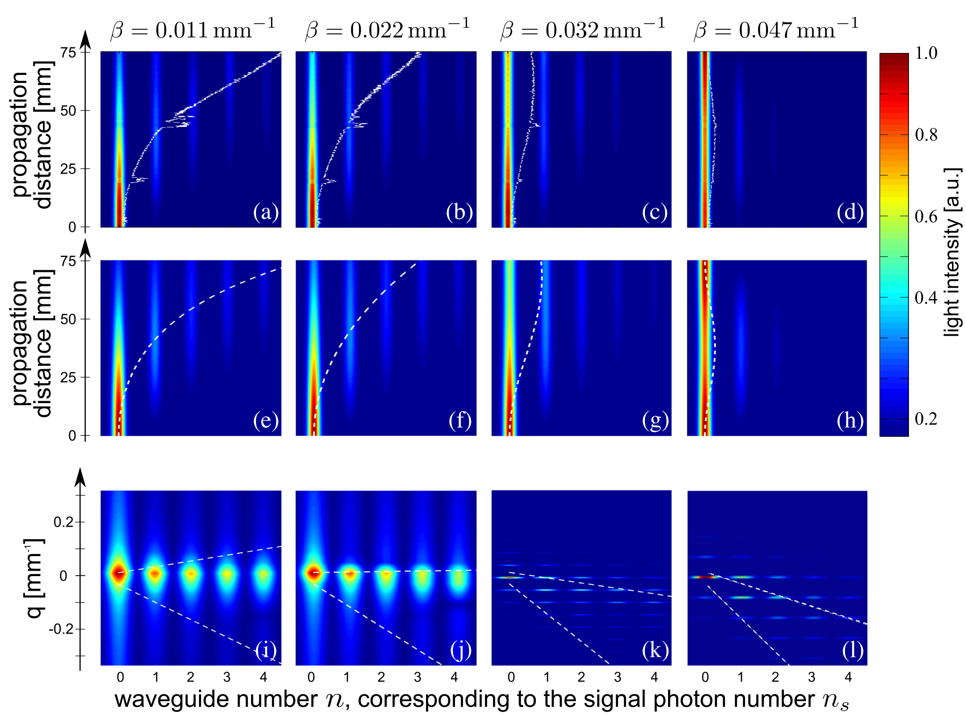

FIG. 3. *Light evolution with linear gradient*: Upper row: Experimental results: (a) $\beta = 0.011\,\mathrm{mm}^{-1}$, (b) $\beta = 0.022\,\mathrm{mm}^{-1}$, (c) $\beta = 0.032\,\mathrm{mm}^{-1}$, (d) $\beta = 0.047\,\mathrm{mm}^{-1}$. Middle row: Numerical integration of Eq. (1) with coupling constants corresponding to the experiments. The white lines represent the average signal photon number $\langle n_s \rangle$ from Eq. (6). Bottom row: The Fourier transform of the calculated complex field along the propagation direction. The dashed lines indicate the edges of the transmission band according to Eq. (2). A transition from continuous spectra to discrete ones with equidistant states when passing the critical gradient is clearly visible. In all cases, we have $\Delta = 0$.

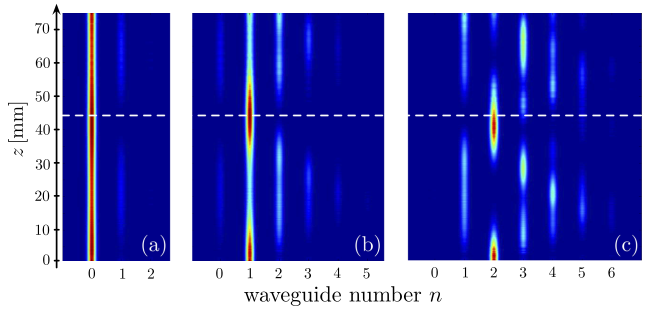

FIG. 4. *Synchronous Bloch oscillations for various* $n_s^{(0)}$: Light evolution for different input conditions. (a) $n_s^{(0)} = 0$, (b) $n_s^{(0)} = 1$, and (c) $n_s^{(0)} = 2$ in a lattice with specially modulated coupling and a detuning of $\beta = 0.072\,\text{mm}^{-1}$ and $\Delta = 0$. Although the individual light distribution strongly depends on the input waveguide, the Bloch period is constant $z_P = 45$ mm (see the dashed line).

oscillation period of $z_P = 75$ mm. Note that such threshold dynamics significantly differs from conventional Bloch oscillations with zero threshold in homogeneous lattices.[7–9] To emphasize the phase transition from extended states to Bloch oscillations, in the bottom row of Fig. 3, we show numerical plots of the Fourier transform of the field as a function of z. The dashed lines in the Fourier plot indicate the edges of transmission band, calculated according to Eq. (2). It is evident that for $\beta < \beta_{cr}$, we have a continuous spectrum in the band, and no localization. However, above the threshold the spectrum becomes discrete and equidistant (forming a Wannier-Stark ladder), as required for Bloch oscillations.

Remarkably, the period of Bloch oscillations is independent of the initial conditions. In order to experimentally verify this prediction, we implement a waveguide lattice with a strong detuning of $\beta = 0.072\,\text{mm}^{-1}$, to ensure the appearance of the Bloch oscillations. Launching the light in the first, the second, or third waveguides corresponds to the situation of having either zero ($n_s^{(0)} = 0$), one ($n_s^{(0)} = 1$), or two signal photons ($n_s^{(0)} = 2$) as the initial state. The measured intensity distribution in the lattice is shown in Figs. 4(a)–4(c). It is clearly seen that, although the individual evolution of the photon number depends on the initial condition, the period of the Bloch oscillations is in all cases identical, i.e., $z_P = 45$ mm.

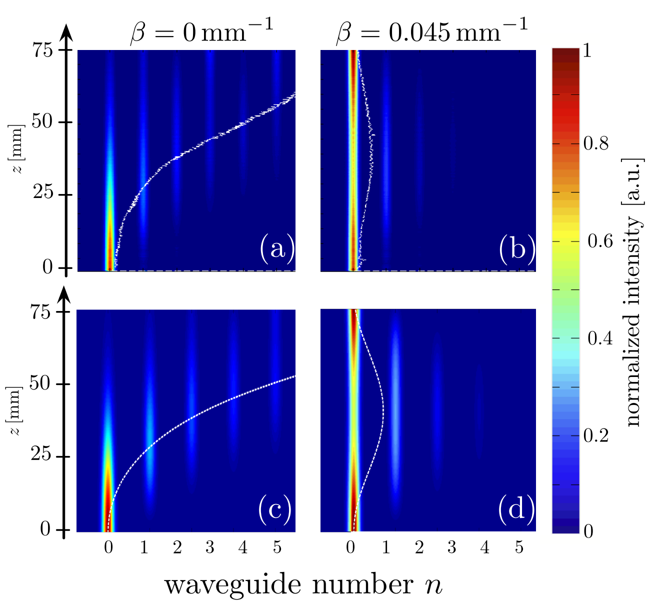

FIG. 5. *Synchronous Bloch oscillations for* $\Delta = 1$: For vanishing detuning ((a) experiment, (c) theory), the photon number grows rapidly and slightly faster than for the case $\Delta = 0$ (Fig. 2(a)). For a detuning of $\beta = 0.047\,\text{mm}^{-1}$ ((b) experiment, (d) theory), the average photon number shows squeezed light Bloch oscillations with exactly the same period of $z_p = 75$ mm as in the case $\Delta = 0$ (Fig. 3(d)). The white lines represent the average signal photon number $\langle n_s \rangle$. In all cases, we have $n_s^{(0)} = 0$.

Importantly, the Bloch oscillations are also synchronous for different values of Δ. Hence, we have fabricated two waveguide lattices with a hopping distributions corresponding to $\Delta = 1$: one with no detuning ($\beta = 0$) and one with strong detuning $\beta = 0.047\,\text{mm}^{-1}$. The experimental results are shown in Figs. 5(a) and 5(b). For comparison, numerically solutions of Eq. (1) are shown in Figs. 5(c) and 5(d). For vanishing phase-mismatch, the photon number growth is in principle similar to the case $\Delta = 0$ [cf. Fig. 2(a)], only somewhat faster. For the detuned regime above the threshold, we observe the Bloch oscillations with exactly the same period of $z_P = 75$ mm as in the case with $\Delta = 0$ [cf. Fig. 2(d)].

In summary, we have demonstrated a new regime of Bloch oscillations in inhomogeneous lattices, characterized by a phase transition from extended states to localized modes when increasing the effective force and exceeding a critical value. In analogy to the dynamics of two-mode squeezed states in Fock space, this new regime is called *squeezed light Bloch oscillations*, where the average photon number periodically returns to its initial state if the phase-mismatch between signal and idler exceeds some critical value. Our results imply various further directions of research: What would be the impact of nonlinearity? How true quantum states (such as N00N states) would evolve in our lattice? What happens in slightly disordered lattices, where only the average hopping follows a trend? These and other intriguing questions are now in reach.

See supplementary material for a detailed description of our methods and parameters.

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[1] F. Bloch, "Über die Quantenmechanik der Elektronen in Kristallgittern," Z. Phys. **52**, 555–600 (1929).
[2] L. Esaki and R. Tsu, "Superlattice and negative differential conductivity in semiconductors," IBM J. Res. Dev. **14**, 61 (1970).
[3] C. Waschke, H. G. Roskos, R. Schwedler, K. Leo, H. Kurz, and K. Kohler, "Coherent submillimeter-wave emission from Bloch oscillations in a semiconductor superlattice," Phys. Rev. Lett. **70**, 3319–3322 (1993).
[4] M. B. Dahan, E. Peik, J. Reichel, Y. Castin, and C. Salomon, "Bloch oscillations of atoms in an optical potential," Phys. Rev. Lett. **76**, 4508–4511 (1996).
[5] S. R. Wilkinson, C. F. Bharucha, K. W. Madison, Q. Niu, and M. G. Raizen, "Observation of atomic Wannier-Stark ladders in an accelerating optical potential," Phys. Rev. Lett. **76**, 4512–4515 (1996).
[6] B. P. Anderson and M. A. Kasevich, "Macroscopic quantum interference from atomic tunnel arrays," Science **282**, 1686–1689 (1998).
[7] T. Pertsch, P. Dannberg, W. Elflein, A. Brauer, and F. Lederer, "Optical Bloch oscillations in temperature tuned waveguide arrays," Phys. Rev. Lett. **83**, 4752–4755 (1999).
[8] R. Morandotti, U. Peschel, J. S. Aitchison, H. S. Eisenberg, and K. Silberberg, "Experimental observation of linear and nonlinear optical Bloch oscillations," Phys. Rev. Lett. **83**, 4756–4759 (1999).
[9] G. Lenz, I. Talanina, and C. M. de Sterke, "Bloch oscillations in an array of curved optical waveguides," Phys. Rev. Lett. **83**, 963–966 (1999).
[10] S. Longhi, "Bloch oscillations and Wannier-Stark localization in a tight-binding lattice with increasing intersite coupling," Phys. Rev. B **80**, 033106 (2009).
[11] A. A. Sukhorukov, A. S. Solntsev, and J. E. Sipe, "Classical simulation of squeezed light in optical waveguide arrays," Phys. Rev. A **87**, 053823 (2013).
[12] M. K. Nezhad, A. R. Bahrampour, M. Golshani, S. M. Mahdavi, and A. Langari, "Phase transition to spatial Bloch-like oscillation in squeezed photonic lattices," Phys. Rev. A **88**, 023801 (2013).
[13] F. Lederer, G. I. Stegeman, D. N. Christodoulides, G. Assanto, M. Segev, and Y. Silberberg, "Discrete solitons in optics," Phys. Rep. **463**, 1–126 (2008).
[14] U. Peschel, T. Pertsch, and F. Lederer, "Optical Bloch oscillations in waveguide arrays," Opt. Lett. **23**, 1701–1703 (1998).
[15] S. Longhi, "Jaynes-cummings photonic superlattices," Opt. Lett. **36**, 3407–3409 (2011).
[16] A. Perez-Leija, R. Keil, A. Szameit, A. F. Abouraddy, H. Moya-Cessa, and D. N. Christodoulides, "Tailoring the correlation and anticorrelation behavior of path-entangled photons in Glauber-Fock oscillator lattices," Phys. Rev. A **85**, 013848 (2012).
[17] R. Keil, A. Perez-Leija, P. Aleahmad, H. Moya-Cessa, S. Nolte, D. N. Christodoulides, and A. Szameit, "Observation of Bloch-like revivals in semi-infinite Glauber-Fock photonic lattices," Opt. Lett. **37**, 3801–3803 (2012).
[18] G. H. Wannier, *Elements of Solid State Theory* (University Press, 1960).