Genetic algorithm based filter design for quantum signal detection

Madhav Krishnan Vijayan,[1] Paraj Titum,[2] Gregory Quiroz,[2] and Christopher Ferrie,[1]

[1]Centre for Quantum Software and Information, University of Technology Sydney, Sydney, NSW 2007, Australia

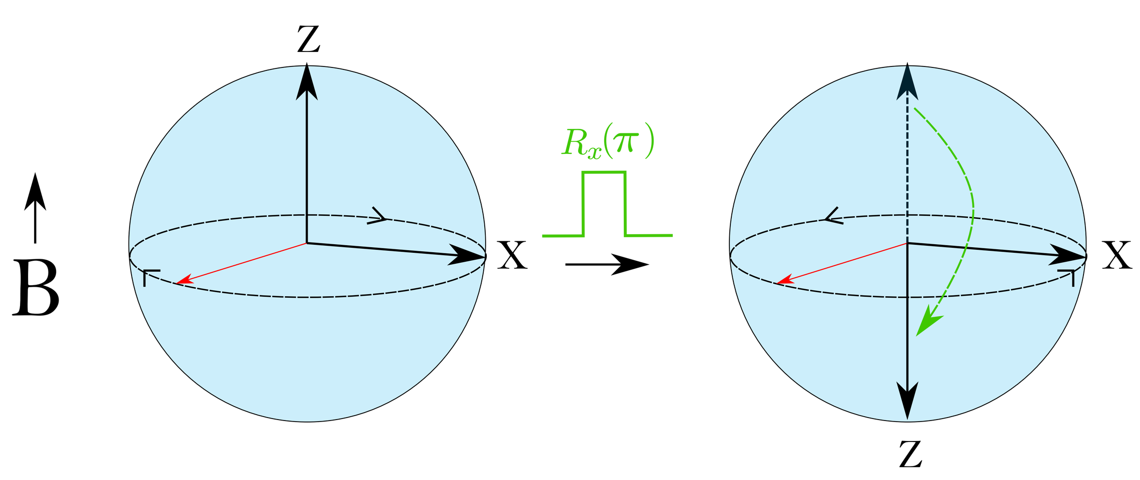
[2]The Johns Hopkins University Applied Physics Laboratory, Laurel, Maryland 20723, USA

madhavkrishnan.vijayan@uts.edu.au

1. Introduction

Genetic algorithms (GAs) are known for their ability to search complex cost landscapes overcoming local minimas; here we take advantage of this to design control pulses for quantum signal detection. Dynamical decoupling is the process of applying pulse sequences to a quantum system to mitigate the effects of environmental coupling often with the aim of prolonging its coherence lifetime. Each sequence can be understood to act as a filter which allows only certain noise frequency ranges to decohere the system. For signal detection we design filters that maximise the effect of the signal on decoherence while minimising the effect of noise. We demonstrate that a GA based control can outperform CPMG sequences for the task of quantum signal detection in the high noise regime.

2. Filter function formalism



The dynamics of a controlled qubit in a phase decohering environment is represented in a semi-classical approximation by the Hamiltonian

$$H = \left(\frac{\Omega}{2} + \beta(t)\right)\sigma_Z + \alpha(t)\sigma_X.$$

Here Ω represents the coupling with an external field , $\beta(t)$ is a random noise signal and $\alpha(t)$ is the control signal we apply along the X axis. If no control is applied, the state will accumulate an error phase $\phi = \int_0^T \beta(t)dt$. If the control signal we use is a sequence of π pulses which implements an $R_x(\pi)$ gate, this reverses the phase accumulated. If the π pulses are applied at times t_i, the total phase accumulated will be,

$$\phi = \int_{t_0}^{t_1} \beta(t)dt - \int_{t_1}^{t_2} \beta(t)dt + \ldots = \int_0^T y(t)\beta(t)dt,$$

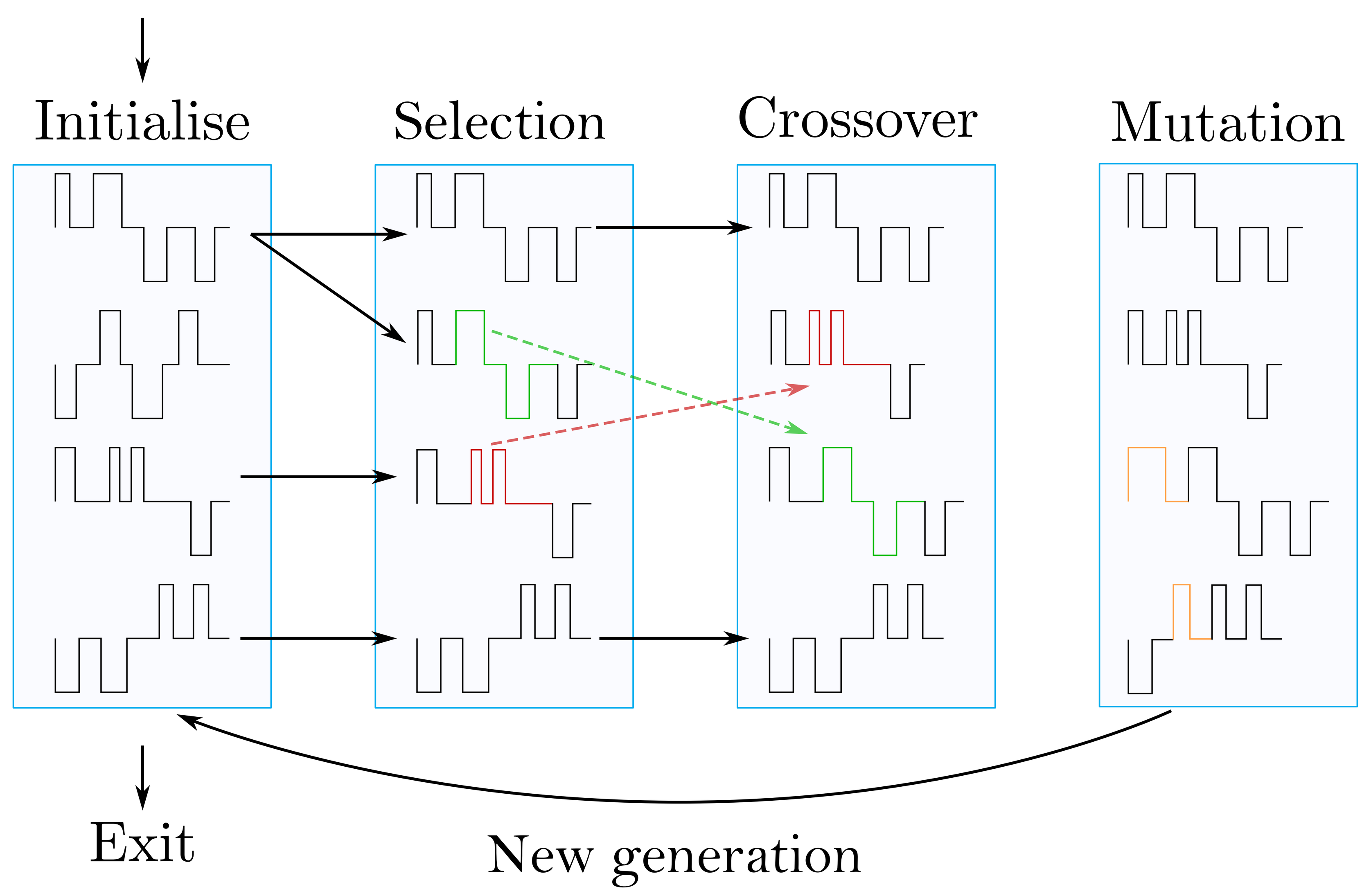
where $y(t)$ is a switching function that changes between ± 1 at times t_i. If $\beta(t)$ changes slowly with respect to $y(t)$, then most of the accumulated phase error is cancelled out. However if the variation in $\beta(t)$ is faster, then this is less effective. This dependence on frequency matching of the control and the noise is reflected in the expression for coherence of the system. Suppose we started with a state in the $+1$ eigenstate of σ_x, the coherence is given by $|\langle\sigma_x\rangle(\tau)| = e^{-\chi(\tau)}$ where,

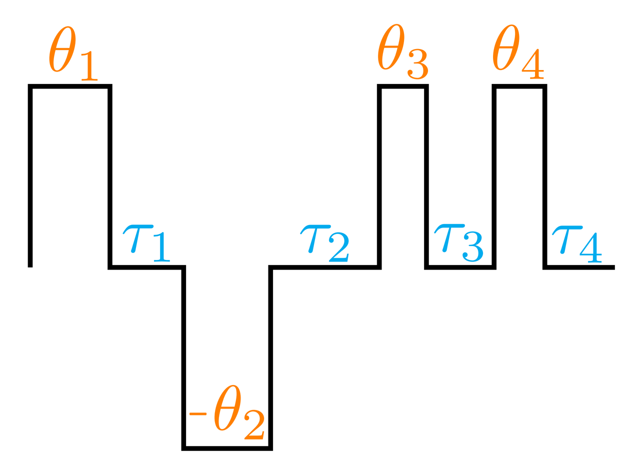
$$\chi(\tau) = \frac{2}{\pi}\int \frac{S_\beta(\omega)}{\omega^2}F(\omega\tau)d\omega,$$

here $S_\beta(\omega)$ is the power spectral density (PSD) of $\beta(t)$ and $F(\omega\tau)$ is the absolute square of the Fourier transform of $y(t)$ called the filter function. By choosing a pulse sequence which minimises this overlap, we are able to supress the effect of noise.

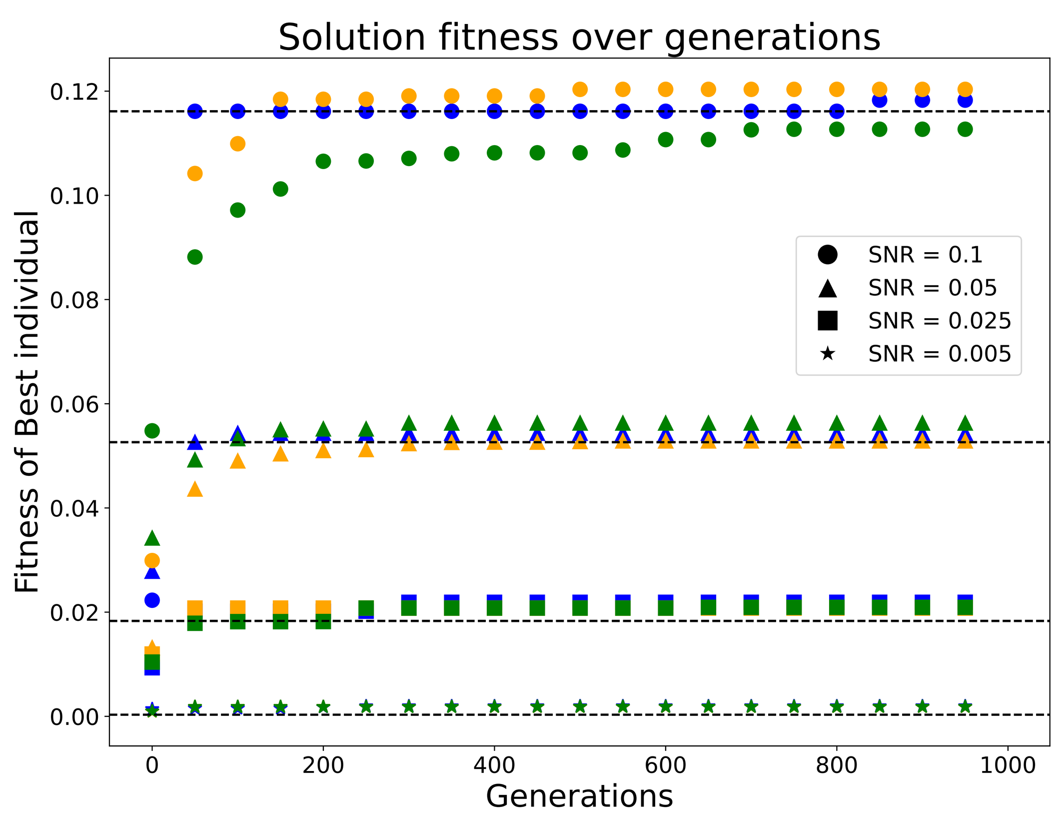
For signal detection, we use the same principles as for noise suppression. The aim is to design a pulse sequence which minimises overlap between its filter function and the noise PSD while simultaneously maximising the overlap with the signal PSD.

4. Genetic algorithm

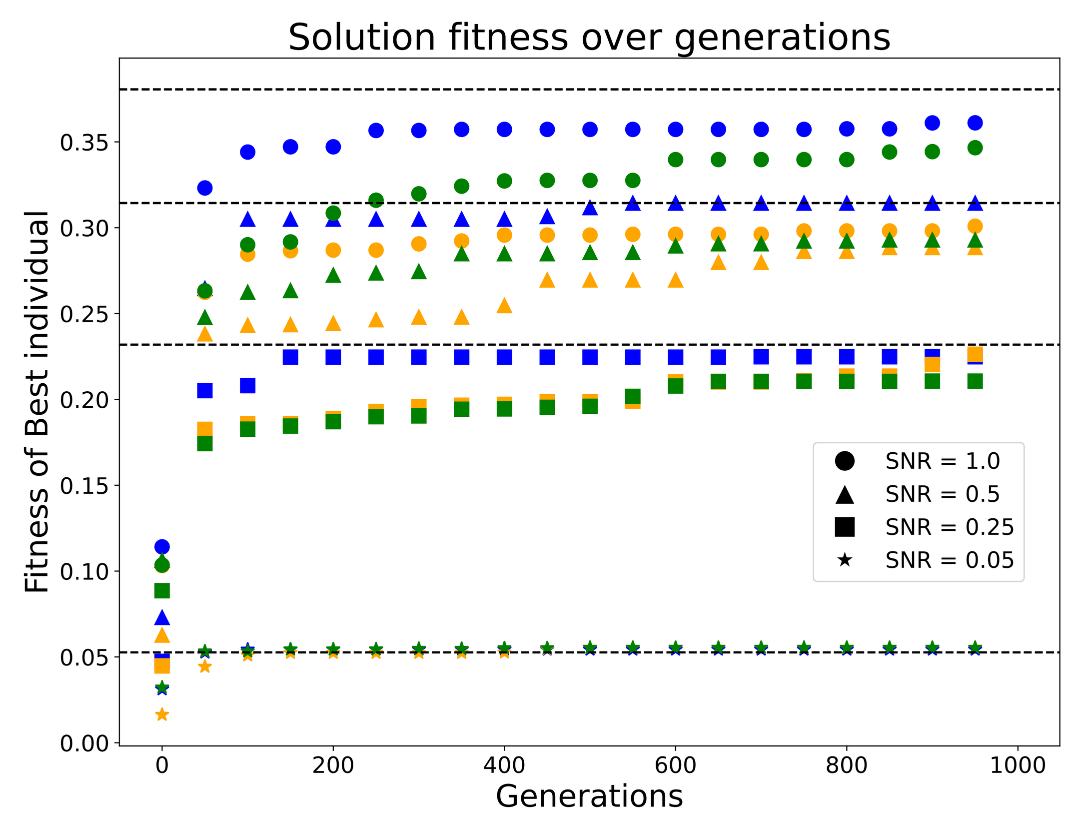




Parametric pulse sequence



High noise regime



Low noise regime

Parametric pulse sequence: More generally a control pulse sequence is parameterised by X rotation angles θ_i and the spacing between the pulses τ_i. We represent this as a rectangle pulse sequence where the width at the level position representing τ_i and the width at top and bottom position representing the X-rotation induced θ_i.

Initialisation: The population is initialised with random control pulse sequence individuals where each sequence represents a possible solution.

Selection: Individuals are selected randomly for creating the next generation with candidates with a higher fitness value having a higher chance of being selected.

Crossover: A subset of individual pairs are selected with probability P_{cx} and a random segment of parameters are interchanged between them.

Mutation: A subset of individuals are selected with probability P_m and some parameters are randomly changed.

Finally, the new generation replaces the older one and the algorithm continues until an exit condition is met such as a maximum number of generations or reaching the required fitness.