



## Research article

## Modeling the optimal mitigation of potential impact of climate change on coastal ecosystems

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## ARTICLE INFO

## Keywords:

Modeling and simulation  
Optimal control  
Climate change  
Greenbelt  
Desulfurization  
Coastal ecosystems

## ABSTRACT

Global warming is adversely affecting the earth's climate system due to rapid emissions of greenhouse gases (GHGs). Consequently, the world's coastal ecosystems are rapidly approaching a dangerous situation. In this study, we formulate a mathematical model to assess the impact of rapid emissions of GHGs on climate change and coastal ecosystems. Furthermore, we develop a mitigation method involving two control strategies: coastal greenbelt and desulfurization. Here, greenbelt is considered in coastal areas to reduce the concentrations of GHGs by absorbing the environmental carbon dioxide (CO<sub>2</sub>), whereas desulfurization is considered in factories and industries to reduce GHG emissions by controlling the release of harmful sulfur compounds. The model and how it can control the situation are analytically verified. Numerical results of this study are confirmed by comparison with other studies that examine different scenarios. Results show that both control strategies can mitigate GHG concentrations, curtail global warming and to some extent manage climate change. The results further reveal that both control strategies are more effective than one control method. Overall, the results suggest that the concentrations of GHGs and the effects of climate change can be controlled by adopting sufficient coastal greenbelt and desulfurization techniques in various industries.

## 1. Introduction

The Earth's environment is closely interconnected with living species, big or small. Greenhouse gas (GHG) emissions have risen in the two centuries due to unchecked population growth, modernization, industrialization, and urbanization. About 83% of atmospheric GHGs is produced from man-made materials/sources and human activities [1]. Of the man-made sources, about 59% of GHGs originate from industry [2] which significantly increases the concentrations and amounts of GHGs in the atmosphere. This is coupled with the indiscriminate cutting down of the world's remaining forest areas (Figure A1, Appendix A), which are the 'lungs' of the planet's environment by absorbing approximately 32.6 gigatonnes of CO<sub>2</sub> every year [3]. As a result, the atmospheric temperature is rising as the concentrations of GHGs increase, and so is the climate changing proportional to global warming [4, 5]. For these two reasons, destructive natural phenomena such as tornados, tsunamis, cyclones, droughts, floods, rising seawater levels, and acidification are inevitable [6]. Human societies and flora and fauna suffer a variety of skin diseases due to incoming and worsening UV radiation [7]. About 137 species of temperature-sensitive plants, animals, and insects are

becoming extinct every day due to global warming [7]. If this current situation remains, the annual emissions of GHGs will reach 1.34 billion tons by 2030 and 56 billion tons by 2050 [8]. Subsequently, the atmospheric temperature will increase by 3.6 °C by 2036 [9] and 4.05 °C by 2100 [10] which is enough to wipe out most temperature-sensitive species [11].

Most harmful GHGs such as sulfur dioxide (SO<sub>2</sub>) damage the forest ecosystem by introducing acid rain. A large part of the total forest areas is being lost rapidly because of acid rain as well as climate change. Currently, Carbon Capture and Storage (CCS) techniques applied to the decarbonization of industries are becoming more widespread to control GHGs [12]. With these techniques, approximately 20% of CO<sub>2</sub> emissions can be reduced by 2050 [12]. Feedback control with various polymeric amines is a very useful strategy to reduce GHGs by absorbing the anthropogenic emissions of CO<sub>2</sub> [13,14]. According to recent research, the amine-based pilot plant technique is more effective in diminishing GHGs and especially flue gases [15]. Experimental results show that this technique can capture GHG emissions, approximately 9–11% more than the amine-based carbon capture plant [15]. On the other hand, desulfurization is a chemical process used to remove sulfur compounds from a

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molecule or mixture [16]. The main aim of desulfurization is to reduce the release of harmful sulfur compounds and especially SO<sub>2</sub> from different man-made sources. In order to reduce the release of sulfur components and local acid rain, desulfurization can be implemented in various industries.

Yet, the total forest area on the earth's surface is declining daily because of excess emissions of GHGs and indiscriminate deforestation [17]. There is no afforestation alternative that is enough to maintain the natural balance and overthrow the concentrations of GHGs [3, 6]. In this case, greenbelt is an effective technique to reduce the concentrations of GHGs by absorbing CO<sub>2</sub>. Greenbelt can be established near industrial areas, roadsides, coastal regions and urban areas. Greenbelt consisting of a heavy metal accumulated species was adapted around one industrial area (West Bengal, India) so that it could reduce the air pollution by absorbing pollutants emitted by industries [18]. To lower the concentrations of atmospheric chlorine, the greenbelt of *Thuya Orientalis* is very effective [19]. Greenbelt could be introduced by the roadsides of urban areas to control air pollution, emissions of carbon dioxide, and noise [20]. In this case, greenbelt is more effective in preserving both contained and uncontained urban areas [21].

There is much statistical and descriptive research describing various techniques that can reduce the concentrations of GHGs. For example, Samimi *et al.* (2013) claimed that GHGs are the most responsible for rapid climate change. These authors also disclosed some techniques such as coastal afforestation to minimize the concentrations of GHGs [22]. However, only a handful of analytical studies describe the best possible control methods introducing control strategies for reducing the rapid emissions of GHGs. For example, Biswas *et al.* (2016) developed an optimal control method using a mathematical model to curtail global warming and CO<sub>2</sub>, via utilizing aquatic ecosystems as the remediation strategy [23]. Jaschik *et al.* (2020) used a hybrid VSA-membrane technique to capture high emissions of CO<sub>2</sub>. Their study's results show that the hybrid VSA-technique is more flexible and effective than membrane systems or VSA (Vacuum Swing Absorption) [24]. Misra *et al.* (2015) adopted the reforestation technique employing a time delay nonlinear model to reduce atmospheric CO<sub>2</sub>. They disclosed that reforestation is very effective in reducing amounts of CO<sub>2</sub> [25]. Furthermore, Pendrill *et al.* (2019) stated that deforestation is the second-largest source of GHGs, and afforestation could serve as a control strategy to GHGs especially CO<sub>2</sub> [26]. There is still a lack of such research analyzing GHGs and climate change control strategies, especially for saving the world's coastal ecosystems.

To fill the gaps in the research on this topic, we aim to mitigate climate change by minimizing the concentrations of GHGs in an effort to save coastal ecosystems. In this case, we first formulate a new mathematical model to describe the harmful effects of the rising GHG emissions on climate change and coastal ecosystems. Then, we develop a control system that could mitigate the problem and takes the form of two control strategies, namely coastal greenbelt and desulfurization. Here, the coastal greenbelt is adopted to mitigate the concentrations of GHGs by reducing atmospheric CO<sub>2</sub>. For the second strategy, desulfurization is embraced to reduce GHGs emitted by industries and the amount of sulfur components being released. We analytically verify both models and find the necessary conditions of best management by using Pontryagin's maximum principle in terms of Hamiltonian. Furthermore, we perform numerical simulations to validate the analytical results by comparing the results of other research papers. To investigate the effectiveness of the suggested control techniques, the simulations are integrated into three different scenarios.

The remainder of the paper is organized as follows. Section 2 explicitly describes the methods and materials which are used in this research. The mathematical model and the development of the corresponding control method are presented in Section 3. Additionally, an analysis is conducted in Appendices B and C. Section 4 presents the numerical results of this study. This section includes three different scenarios based on the efficacy of the control strategies; it also presents an explicit comparison between them. The summary and limitations of this study are outlined in Section 5.

## 2. Materials and methods

### 2.1. Study area and materials

This study investigates the global coastal area to mitigate the effects of global warming and the rapidly rising concentrations of GHGs; there is no one specific region examined here. Because global warming is a worldwide and important issue, it is not possible to reduce it or climate change by adopting some control strategies that are applicable to only one nation or region. In this study, we consider two control strategies, namely coastal greenbelt and desulfurization to diminish global warming and this means shrinking the concentrations of emitted GHGs. The coastal greenbelt can be applied to any coastal region where afforestation is lacking. The strategy plays a significant role in reducing global warming and climate change by absorbing atmospheric CO<sub>2</sub>. Meanwhile, desulfurization can be adopted by many industries because it is vital to end the release of harmful sulfur components into the air.

For parametric estimates and relevant data collection, we consider the top GHGs producing countries. The considered countries are China, United States, European Union (27), India, Russia, and Japan [17] (their annual emissions of GHGs are represented in Figure A2, Appendix A). In this case, we first studied in-depth the relationships between rapid global warming, concentrations of GHGs, and types of coastal ecosystems. Following this, we gathered relevant data and/or environmental factors from research papers, articles, monographs, annual reports, etc., of governmental and non-government organizations [1, 2, 4, 5, 6, 9, 10, 13, 15, 20, 21, 22, 24, 27, 28]. Then, we did a detailed statistical analysis of the collected data after observing carefully the system, processes, and corresponding data, etc., to obtain the parametric values. All the parametric values used in this study are secondary or estimated data. These parameters are displayed in Table 1 with the corresponding descriptions and numerical values. By comparing the numerical results with others, the parametric values are verified and then described in Section 4.

### 2.2. Methods

#### 2.2.1. Lotka-Volterra model

The Lotka-Volterra model consists of a pair of ordinary differential equations (ODEs). Generally, the model is used to describe the dynamic behavior of the species living in a biological system where one species acts as prey and another as predator [29, 30]. The general form of the Lotka-Volterra model can be written in terms of a pair of autonomous differential equations:

$$\dot{x} = xf(x, y) \text{ \& \; } \dot{y} = yg(x, y) \quad (1)$$

where  $x$  and  $y$  are the prey and predator population, respectively, and the functions  $f(x, y)$  and  $g(x, y)$  are the growth rate of the corresponding species.

#### 2.2.2. Deterministic model

The deterministic model consists of differential equations (DEs) whose solutions rely on parametric values and the initial conditions of state variables [30]. This model is widely employed in environmental management, epidemiology, biomathematics, etc., to describe how dynamic systems function [30]. Basically, the model is formulated based on the competition model, epidemic model, chaos theory, Lotka-Volterra model, and Logistic growth model.

#### 2.2.3. Optimal control problem

An optimal control problem (OCP) is concerned with the state variable and control variable. Let  $x(t)$  be the state variable and  $u(t)$  be the control variable of an OCP, then  $x(t)$  satisfies the following differential equation:

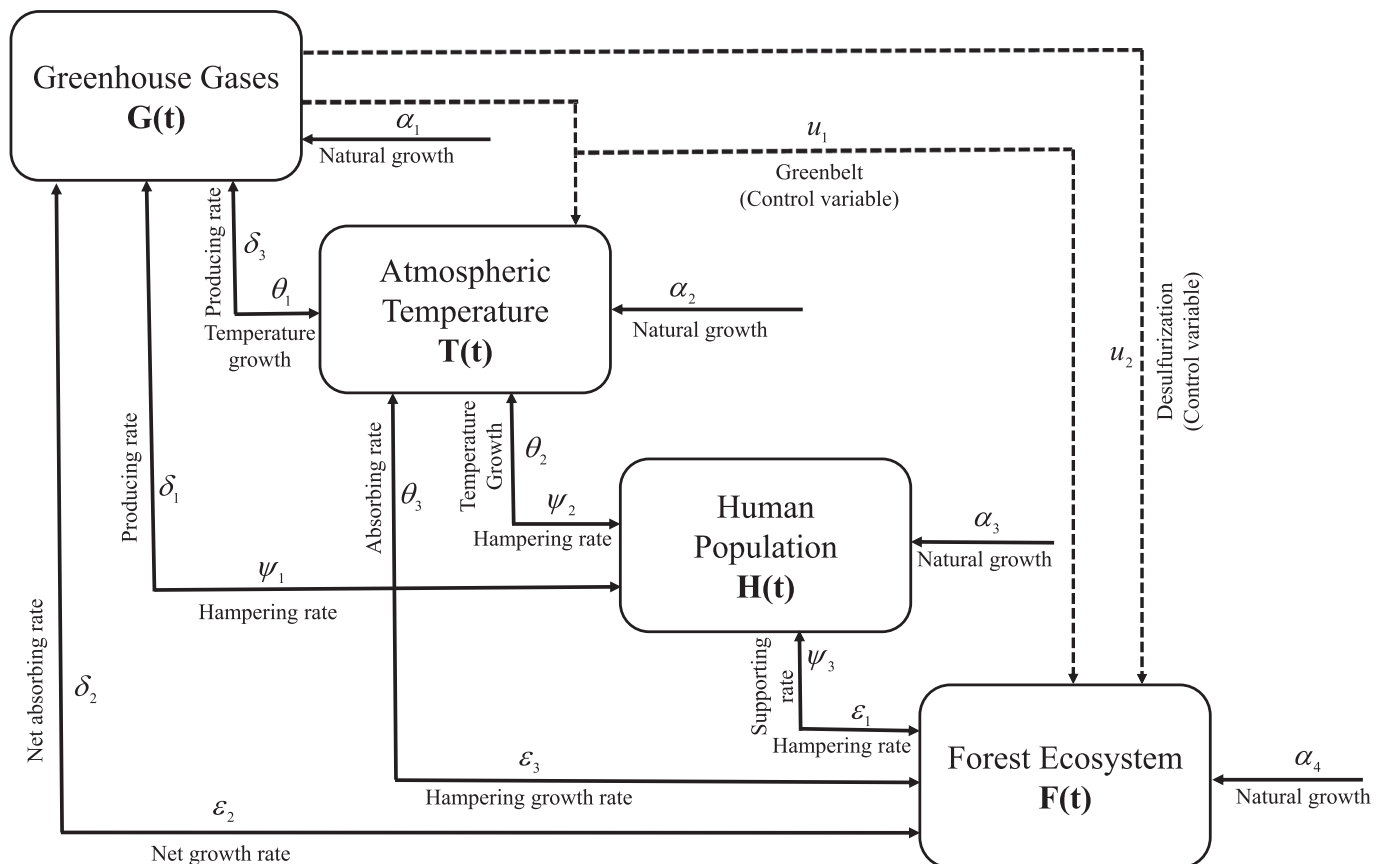
$$\dot{x}(t) = g(t, x(t), u(t)) \quad (2)$$

**Table 1.** An explicit description of the parameters used in this study including numerical values.

Parameter	Descriptions	Parametric value	Units	Source
$\alpha_1$	Normal concentrations rate of GHGs	0.00015	$\text{Kg km}^{-2}$	[4, 5, 13], calculated
$\delta_1$	Producing rate of GHGs by the human population	0.025	$\text{Kg km}^{-2}$	[4, 6, 15, 24], calculated
$\delta_2$	Absorbing rate of $\text{CO}_2$ by forest ecosystems	0.0023	$\text{Kg km}^{-2}$	[13, 20, 21], calculated
$\delta_3$	Concentration rate of GHGs after natural disasters	0.0005	$\text{Kg km}^{-2}$	[1, 6], calculated
$\alpha_2$	Natural growth rate of atmospheric temperature	0.1	$^\circ\text{C}$	[1, 9, 10]
$\theta_1$	Growth rate of atmospheric temperature due to GHGs	0.67	$^\circ\text{C}$	[1, 9, 10], calculated
$\theta_2$	Increasing rate of atmospheric temperature by the human population	0.0055	$^\circ\text{C}$	[4, 22], calculated
$\theta_3$	Absorbing rate of atmospheric temperature by forest ecosystems	0.0225	$^\circ\text{C}$	[10, 22], calculated
$\alpha_3$	Natural growth rate of the human population	0.000015	Thousand <sup>-1</sup>	[6, 28]
$\psi_1$	Decreasing rate of human population due to harmful GHGs	0.58	Thousand <sup>-1</sup>	[13, 15, 28], calculated
$\psi_2$	Hampering rate of human population due to global warming	0.29	Thousand <sup>-1</sup>	[1, 28], calculated
$\psi_3$	Increasing rate of human population with the help of forest ecosystems	0.00956	Thousand <sup>-1</sup>	[6, 28], calculated
$\alpha_4$	Natural growth rate of forest ecosystems near coastal areas	0.05	$\text{km}^{-2}$	[2, 6]
$\epsilon_1$	Deforestation rate caused by human beings	0.095	$\text{km}^{-2}$	[2, 6]
$\epsilon_2$	Net growth rate of forest ecosystems with the help of $\text{CO}_2$	0.00122	$\text{km}^{-2}$	[2, 6], calculated
$\epsilon_3$	Decreasing rate of forest cover due to global warming	0.0513	$\text{km}^{-2}$	[1, 2, 6], calculated
$a$	Saturation constant	0.01	-	[6]
$k_1$	Carrying capacity of the human population	1000	$\text{km}^{-2}$	[6, 27], calculated
$k_2$	Carrying capacity of forest ecosystems	100000	$\text{km}^{-2}$	[6, 27], calculated

here the function  $g$  is continuously differentiable. The objective of the OCP is to find a piecewise continuous control  $u(t)$  and the corresponding state variable  $x(t)$  to optimize (maximize or minimize) a specific objective functional [31]. An OCP can be described in the following way:

$$\begin{aligned}
 &\text{Maximize or minimize } J(x, u) = \int_a^b L(t, x(t), u(t)) dt \\
 &\text{Subject to } \dot{x}(t) = g(t, x(t), u(t)) \text{ a.e. } t \in [a, b] \\
 &\quad u(t) \in U \text{ a.e. } t \in [a, b] \\
 &\quad x(a) \in x_0 \text{ a.e. } t \in [a, b] \\
 &\quad \text{and } x(b) \in \mathbb{R}^+ \text{ is free}
 \end{aligned}
 \tag{3}$$



**Figure 1.** The schematic diagram of the effect of environmental GHGs on global warming and coastal ecosystems with controls.

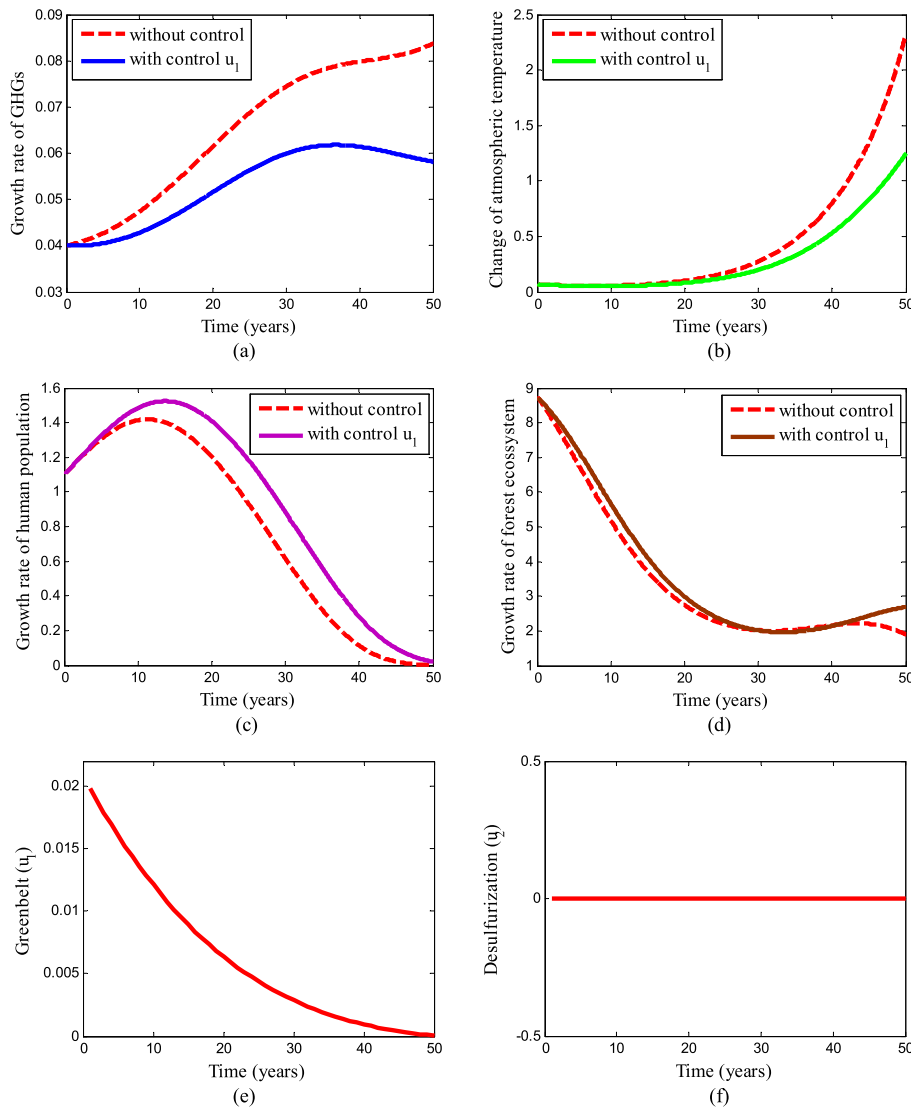


Figure 2. Numerical results of the dynamic model (4) and control model (6) for Scenario I. The figures represent the (a): growth rate of GHGs, (b): rising rate of atmospheric temperature, (c): growth rate of human population near coastal areas, (d): growth rate of forest ecosystems near coastal areas, (e): control profile of  $u_1 \in (0, 0.02]$ , (f): control profile of  $u_2 = 0$ . Here, the discrete lines and continuous lines represent the time series before and after adopting the coastal greenbelt only, respectively.

here  $u(t)$  and  $x(t)$  both are piecewise continuous differentiable and  $[a, b]$  is the time interval where  $a, b \in \mathbb{R}^+$  and  $a < b$ . In optimal control problem (3),  $u(t)$  belongs to a certain space  $U$  which may be a piecewise continuous function or a space of measurable function which satisfies all the constraints of the problem. Therefore,  $(x^*, u^*)$  is the optimal solution of the OCP if the cost can be minimized for all processes involved [32].

### 2.3. Programming language

To investigate the dynamic behavior of the suggested system under the designated control strategies, we have used solver ode45 in MATLAB programming language.

### 3. Model formulation

Mathematical modeling can help explain natural phenomena and helps in the design of management, analysis, control strategy, and better prediction [6]. Optimal control is another important mathematical tool widely used in environmental management along with disease and/or infection minimization [33]. The purpose of this study is to suggest how to save the world's coastal ecosystems by controlling climate change as much as possible, and the concentrations of GHGs. It

is why we first formulate a four compartmental mathematical model describing the impact of rapid climate change on coastal ecosystems. Then, we develop an optimal control model based on the newly formulated model. In this case, we embrace coastal greenbelt and desulfurization as two control variables so that they can mitigate climate change and GHG emissions. The four dynamic variables that we considered are: the concentrations of GHGs ( $G(t)$ ) emitted by various industrial activities and man-made sources; atmospheric temperature ( $T(t)$ ) which changes the earth's climate; the human population near coastal areas ( $H(t)$ ) which simply worsens deforestation; and the forest ecosystems near coastal areas ( $F(t)$ ) which are being damaged due to rapid climate change and concentrations of GHGs.

Here  $\alpha_1, \alpha_2, \alpha_3$  and  $\alpha_4$  are respectively the natural growth rate of  $G(t), T(t), H(t)$ , and  $F(t)$  [6, 34]. Many human societies are emitting GHGs and these are increasing in the atmosphere [1]. Here,  $\delta_1 HG$  presents the concentrations of GHGs caused by human activities. The rapid concentration of GHGs increases the atmospheric temperature [4] and threatens the way that people live [6]. However, it does promote forests' photosynthetic activity so this could in fact promote better forest density [6]. Here,  $\theta_1 GT$  is the increase in atmospheric temperature,  $\psi_1 GH$  is the decline in the human population, and  $\frac{\epsilon_2 F}{a+G}$  presents the promotion of

coastal forest ecosystems due to the rapidly rising concentrations of GHGs. People directly contribute to global warming by firing forest areas, deforestation, coal plants, burning fossil fuels, massive burning operations, etc. [6]. Here,  $\theta_2HT$  is the rise in atmospheric temperature, whereas  $\varepsilon_1HF$  is the decline in coastal ecosystems due to humans activities. Rapid global warming and climate change are reducing the viability of coastal people's lives through forest fires, cyclones, tsunami, tornados, floods, and droughts [6, 25, 35, 36]. Here,  $\psi_2TH$  presents the decrease in human population, whereas  $\varepsilon_3TF$  presents the destruction of coastal systems due to rapid global warming. Conversely, the coastal forest area absorbs GHGs (CO<sub>2</sub>), protects coastal societies from various calamities, and is a source of food and shelter [27, 28, 37, 38]. Here,  $\delta_2FG$  and  $\theta_3FT$  present the absorptions of  $G(t)$  and  $T(t)$  respectively by forest ecosystems, whereas  $\psi_3FH$  is the increase in human population made possible by the support of forest ecosystems. Here  $a(0 \leq a < 1)$  is the saturation constant, whereas  $k_1$  and  $k_2$  are respectively the carrying capacity of  $H(t)$  and  $F(t)$ . Parameters used in this study are described in Table 1.

Considering the interrelationships among the dynamic variables, we formulate a mathematical model consisting of the following nonlinear ordinary differential equations (NODEs) [6, 29, 30, 34]:

$$\begin{aligned} \frac{dG}{dt} &= \alpha_1G + \delta_1HG - \delta_2FG + \delta_3T \\ \frac{dT}{dt} &= \alpha_2T + \theta_1GT + \theta_2HT - \theta_3FT \\ \frac{dH}{dt} &= \alpha_3H \left(1 - \frac{H}{k_1}\right) - \psi_1GH - \psi_2TH + \psi_3FH \\ \frac{dF}{dt} &= \alpha_4F \left(1 - \frac{F}{k_2}\right) - \varepsilon_1HF + \frac{\varepsilon_2F}{a+G} - \varepsilon_3TF \end{aligned} \tag{4}$$

$$\text{with initial conditions } G_0 = G(0) > 0, T_0 = T(0) > 0, H_0 = H(0) \geq 0, F_0 = F(0) \geq 0 \tag{5}$$

Analytical analysis of the model (4) is documented in Appendix B.

To minimize the emissions as well as the concentrations of GHGs, we consider two control variables ( $u_1, u_2$ ): (a)  $u_1$  is adopted for coastal greenbelt that can reduce the concentration of GHGs by absorbing atmospheric CO<sub>2</sub>; and (b)  $u_2$  is adopted for desulfurization which can hinder the release of harmful sulfur components. This is done by absorbing or converting them to another component having less toxicity.

$$\mathbf{x}(t) = \begin{pmatrix} G(t) \\ T(t) \\ H(t) \\ F(t) \end{pmatrix} \mathbf{g}(t, \mathbf{x}(t), \mathbf{u}(t)) = \begin{pmatrix} \alpha_1G + \delta_1HG - \delta_2FG + \delta_3T - (u_1 + u_2)G \\ \alpha_2T + \theta_1GT + \theta_2HT - \theta_3FT - u_1GT \\ \alpha_3H \left(1 - \frac{H}{k_1}\right) - \psi_1GH - \psi_2TH + \psi_3FH \\ \alpha_4F \left(1 - \frac{F}{k_2}\right) - \varepsilon_1HF + \frac{\varepsilon_2F}{a+G} - \varepsilon_3TF + (u_1 + u_2)F \end{pmatrix}, \mathbf{u}(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$$

Thus, controls  $u_1$  and  $u_2$  can abate the concentrations and emissions of GHGs and reverse – to some extent - climate change. These control variables can also improve the world's coastal ecosystems promoting the growth of trees and reducing acid rain. Considering the control variables  $u_1$  and  $u_2$ , the dynamic model (4) can be developed via the following NODEs [31, 33, 39]:

$$\begin{aligned} \frac{dG}{dt} &= \alpha_1G + \delta_1HG - \delta_2FG + \delta_3T - (u_1 + u_2)G \\ \frac{dT}{dt} &= \alpha_2T + \theta_1GT + \theta_2HT - \theta_3FT - u_1GT \\ \frac{dH}{dt} &= \alpha_3H \left(1 - \frac{H}{k_1}\right) - \psi_1GH - \psi_2TH + \psi_3FH \\ \frac{dF}{dt} &= \alpha_4F \left(1 - \frac{F}{k_2}\right) - \varepsilon_1HF + \frac{\varepsilon_2F}{a+G} - \varepsilon_3TF + (u_1 + u_2)F \end{aligned} \tag{6}$$

with the same initial conditions (5).

Now, the dynamic model (6) is an optimal control model. The control variables can be defined in terms of the following Lebesgue measurable control set:

$$U = \{ (u_1(t), u_2(t)) : 0 \leq u_i(t) \leq 1, i=1, 2 \text{ a.e. } t \in [0, T_p] \}$$

where  $T_p$  denotes a preselected time interval when controls are applied. Here  $u_1 = 0$  represents that no greenbelt is implemented in coastal areas, and  $u_2 = 0$  shows that no desulfurization is to curtail sources that emit GHGs. When the control variables  $u_1$  and  $u_2$  are fully implemented in the system, then  $u_1 = 1$  and  $u_2 = 1$ .

The control model (6) aims to obtain an objective functional that can minimize the concentration of GHGs and improve the coastal ecosystems at minimum cost. Therefore, the objective functional of the control model (6) is given by

$$\text{Minimize } J(u_1, u_2) = \int_0^{T_p} \left( G(t) + \frac{A}{2}u_1^2 + \frac{B}{2}u_2^2 \right) dt \tag{7}$$

where  $A$  and  $B$  are the weight parameters that balance the costs of control measurements.

Control model (6) can be reformulated below using the objective functional (7) and state constraints:

$$(P) \begin{cases} \text{Minimize } J(\mathbf{x}, \mathbf{u}) = \int_0^{T_p} L(t, \mathbf{x}(t), \mathbf{u}(t)) dt \\ \text{Subject to } \dot{\mathbf{x}} = \mathbf{g}(t, \mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{u}(t) \in U, \quad \forall t \in [0, T_p] \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases} \tag{8}$$

where,

and the performance indexing integrand is denoted by:

$$L(t, \mathbf{x}(t), \mathbf{u}(t)) = G(t) + \frac{A}{2}u_1^2 + \frac{B}{2}u_2^2 \tag{9}$$

The reformulated control model (8) is an optimal control problem. We analyze the characterization of the optimal control problem (P) and

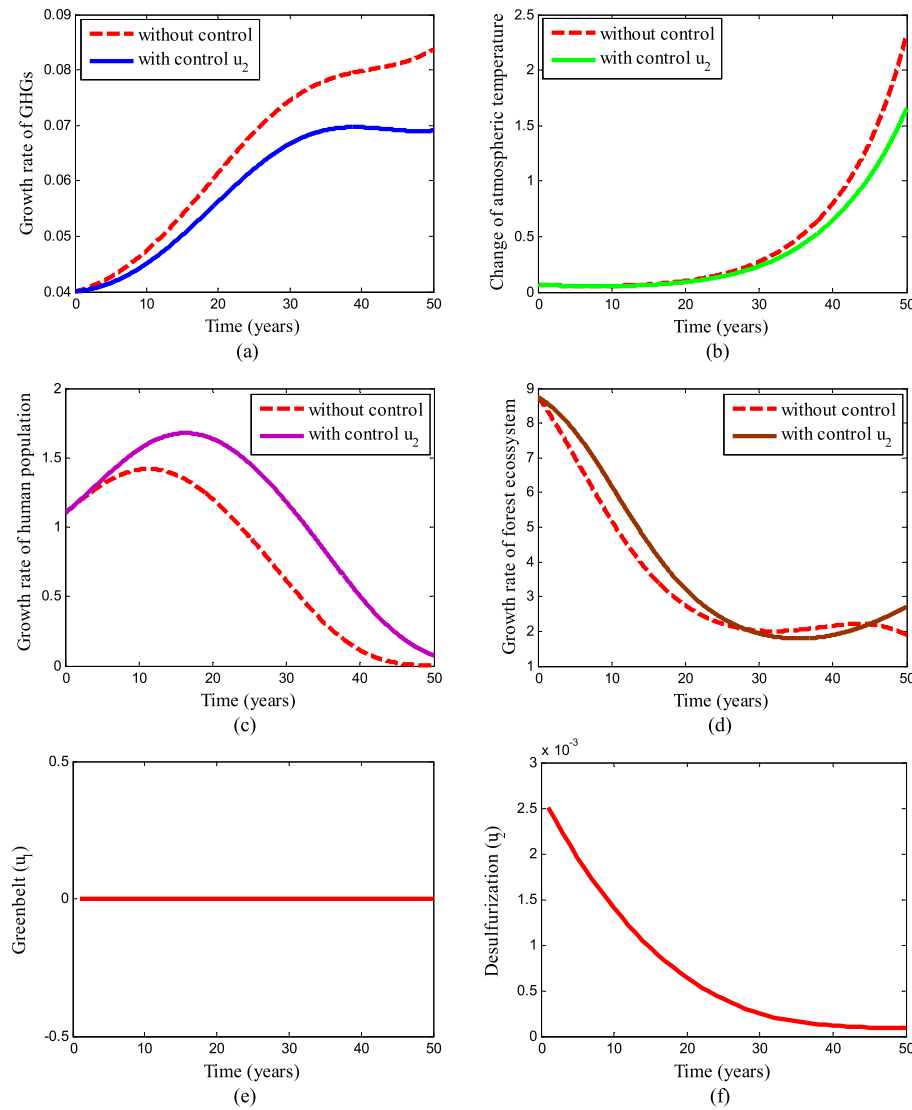


Figure 3. Numerical results of the dynamic model (4) and control model (6) for Scenario II. The figures show the (a): growth rate of GHGs, (b): rising rate of atmospheric temperature, (c): growth rate of human population near coastal areas, (d): growth rate of forest ecosystems near coastal areas, (e): control profile of  $u_1 = 0$ , (f): control profile of  $u_2 \in (0, 0.0025]$ . Here, the discrete lines and continuous lines represent the time series before and after adopting the desulfurization only, respectively.

find out the necessary conditions for optimality are documented in Appendix C. The schematic diagram of the control model (6) is presented in Figure 1.

#### 4. Results and discussion

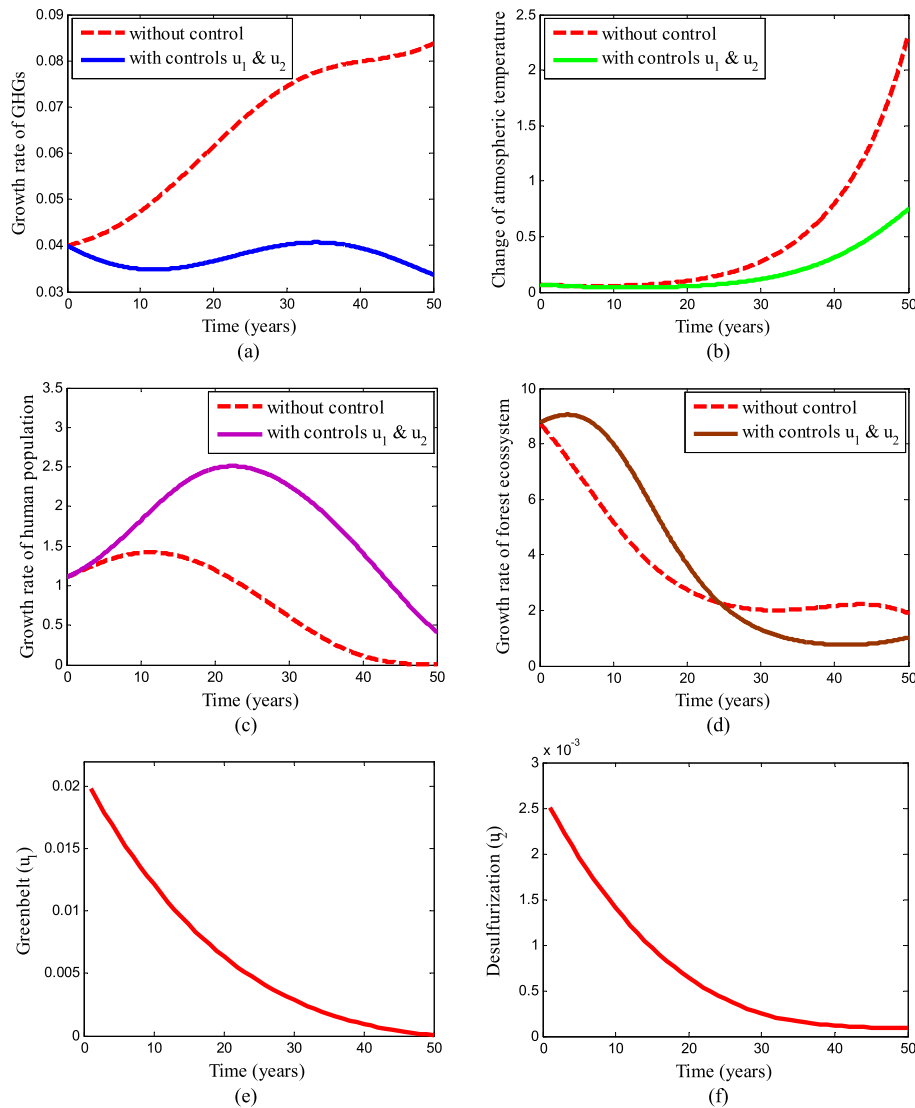
In this section, we perform numerical simulations to illustrate the numerical results of the dynamic model (4) and control model (6). The simulation of the control model (6) is carried out using the forward-backward sweep method [31, 40, 41]. A set of parametric values is employed for the detailed investigations shown in Table 1. Considering the initial values of the dynamic variables are  $G_0 = 0.04$ ,  $T_0 = 0.07$ ,  $H_0 = 1.1$ ,  $F_0 = 8.75$  and the simulations for time  $T_p = 50$  years. This is due to the long-term management along with weight parameters  $A = 800$  and  $B = 300$ . To illustrate the effectiveness of the adopted control strategies, we consider three scenarios for (i)  $u_1 \neq 0$  and  $u_2 = 0$ , (ii)  $u_1 = 0$  and  $u_2 \neq 0$ , and (iii)  $u_1 \neq 0$  and  $u_2 \neq 0$ . Here, the optimal values of the controls are  $0 \leq u_1 \leq 0.02$  and  $0 \leq u_2 \leq 0.0025$  which are similar to Joshi [39] and Biswas et al. [33]. We conducted simulations for the dynamic model (4) to compare the results obtained before and after implementing the control strategies, and got the results by referring to Figures 2, 3, and 4 as "without controls". The three scenarios are explained in more detail below.

#### Scenario I

In this scenario, we adopted the greenbelt only (i.e.  $u_1 \neq 0$ ,  $u_2 = 0$ ) for the system as a control variable to reduce GHGs, and the objective functional  $J$  is optimized with  $u_1 \in (0, 0.02]$ . The changes in dynamic variables under the control variable  $u_1$  are represented in Figure 2.

In the absence of control strategies, the concentrations of GHGs rise rapidly due to the low absorption capacity of  $CO_2$  by forests. If the current situation continues, the GHGs may increase from 0.04% to 0.084% in the next 50 years [4, 6] as displayed in Figure 2(a). When the coastal greenbelt is implemented in the system, the intensity of the concentrations of GHGs drops gradually to approximately 0.058% instead of 0.084% [18, 19, 20, 21]. Due to the reason of fall in the concentrations of GHGs under the control strategy, the atmospheric temperature can rise from 0.07 °C and reach approximately 1.75 °C, and not 0.07 °C to 2.30 °C in the next 50 years [1, 6, 9, 10]. This is represented in Figure 2(b). It seems that the control technique can greatly diminish the threat of global warming. As a consequence, the human population slightly increases because of less environmental pollution and warming [6] as shown in Figure 2(c). When the concentrations of GHGs and global warming declines under the control strategy, the forest ecosystem slowly recovers and regains some density [2, 6]. The growth rate of the forest ecosystem can approach approximately  $2.75 \text{ km}^{-2}$  instead of  $1.96 \text{ km}^{-2}$  in the next 50 years after adopting the control





**Figure 4.** Numerical results of the dynamic model (4) and control model (6) for Scenario III. The figures represent the (a): growth rate of GHGs, (b): rising rate of atmospheric temperature, (c): growth rate of human population near coastal areas, (d): growth rate of forest ecosystems near coastal areas, (e): control profile of  $u_1 \in (0, 0.02]$ , (f): control profile of  $u_2 \in (0, 0.0025]$ . Here, the discrete lines and continuous lines represent the time series before and after adopting the coastal greenbelt and desulfurization simultaneously, respectively.

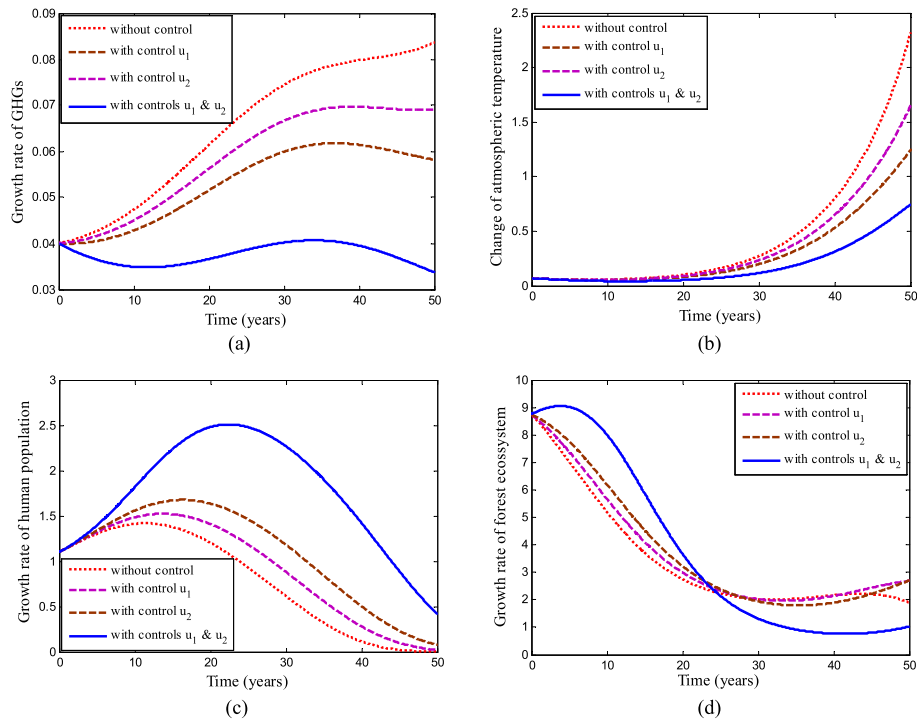
strategy  $u_1$  as represented in Figure 2(d). Figures 2(e) and 2(f) show, respectively, the control profiles of  $u_1$  and  $u_2$  for this scenario. Figure 2(e) illustrates the coastal greenbelt is maintained full effort ( $u_1^{\max} = 0.02$ ) at the beginning and slowly reaches zero at the end of the intervention. Previous studies reported that the control efforts should be adopted fully at the start [18, 19, 20, 21]. Our finding discloses that similar efforts made for the coastal greenbelt will reduce GHG concentrations. Figure 2(f) shows that  $u_2$  is not adopted in this scenario. Figure 2 reveals that if we can implement the coastal greenbelt as much as possible, it can save ecosystems by reducing GHGs as well as global warming.

## Scenario II

In this scenario, we adopted desulfurization only (i.e.  $u_1 = 0$ ,  $u_2 \neq 0$ ) in the system as a control variable to reduce the emissions of sulfur components. It did this by absorbing or converting to another component of less toxicity. In this scenario, the objective functional  $J$  is optimized with  $u_2 \in (0, 0.0025]$ . Figure 3 below shows the progressive changes in the dynamic variables under the control strategy  $u_2$ .

When desulfurization occurs in the world's industries, it reduces harmful sulfur components being released, especially  $\text{SO}_2$  [12,14,15]. As

a result, this can lead to less GHG emissions from 0.04% to 0.0685% and not 0.04%–0.084% as shown in Figure 3(a). Because of the emissions reduction under this control strategy, the atmospheric temperature rises but then drops from 2.30 °C to 1.66 °C [1, 6, 9, 10] as shown in Figure 3(b). Figure 3(c) presents that the reduction in GHGs emissions and atmospheric temperature increases the growth of the human population considerably [6]. According to the figure, the human race's growth rate may increase from 0.001% to 0.1% in the next 50 years under the control strategy. Due to the decline in the concentrations of GHGs and global warming, the rate at which the forest ecosystems can recover will increase [2, 6] as depicted in Figure 3(d). The figure indicates that the growth rate of forest ecosystems can reach approximately 2.85  $\text{km}^{-2}$  instead of 1.96  $\text{km}^{-2}$  when the strategy has been completed. The control profiles of this scenario are represented in Figures 3(e) and 3(f). Here, Figure 3(e) shows  $u_1$  is not adopted in this scenario. Figure 3(f) shows that desulfurization must be fully and consistently maintained ( $u_2^{\max} = 0.0025$ ) at the beginning and gradually falls when the intervention ends. This outcome seems to be consistent with previous studies [12, 13, 14, 15] which reported that if desulfurization is fully implemented, it can significantly reduce the emissions of industrial GHGs. So Figure 3 illustrates that desulfurization can mitigate GHGs emissions and global warming.



**Figure 5.** A numerical comparison between Scenarios I-III. (a)–(d) respectively present the time series of the growth of GHGs, atmospheric temperature, human population, and forest ecosystems near coastal areas under three different control scenarios.

### Scenario III

In this section, we adopted both coastal greenbelt and desulfurization into one system that can absorb GHGs and reduce their emissions at the same time. In this case, the objective functional  $J$  is optimized with  $u_1 \in (0, 0.02]$  and  $u_2 \in (0, 0.0025]$ . The numerical changes in the dynamic variables under both the active controls are illustrated in Figure 4.

When both greenbelt and desulfurization are implemented simultaneously, the GHGs significantly decline from the current rate, specifically from 0.04% to 0.034% in the next 50 years [12, 14, 15, 18, 19, 20, 21]. These changes are shown in Figure 4(a). In the case of having both control strategies, the rise in atmospheric temperature reaches approximately 0.75 °C instead of 2.30 °C after the time interval due to a significant decline in the concentration of GHGs [1, 6, 9, 10]. The numerical outcome is shown in Figure 4(b). The human population's growth rate rose sharply at a lower concentration of GHGs [6] as displayed in Figure 4(c). According to this figure, the growth rate of the human population can reach 0.44% after the time interval. As usual, the fall in the concentrations of harmful GHGs gives the forest ecosystem time to recover significantly when both controls are applied [2, 6]. However, if the control strategies are carried out for a long period of time, it can really cut CO<sub>2</sub> concentration to minimal levels [2]. If the concentration of CO<sub>2</sub> drops below the minimum level, it will significantly reduce the photosynthetic activity of the plant [6]. Consequently, it will reduce the density of coastal forests. The changes are shown in Figure 4(d). Figures 4(e) and 4(f) represent the profiles of  $u_1$  and  $u_2$  respectively. Figure 4(e) confirms that the maximum and minimum efforts of the greenbelt profile are the same as Scenario I, and the results accord with other research [18, 19, 20, 21]. Similarly, Figure 4(f) presents the profile of desulfurization over the time interval which is the same as Scenario II, and the results are similar to those reported elsewhere [12, 13, 14, 15]. However, Figure 4 explicitly indicates that if we can employ both coastal greenbelt and desulfurization simultaneously, GHGs and global warming will be reduced significantly.

Next, we compared and analysed the results of all scenarios. Specifically, we considered the same parametric values and initial values that were already used in Scenarios I-III. The changes in the growth of dynamic variables for three scenarios are displayed in Figure 5. From Figure 5, it is seen that the concentrations of GHGs and the increasing atmospheric temperature significantly decline under both control strategies and not just one. Conversely, the growth rate of the human population and forest ecosystem near coastal areas is dramatically evident in both control strategies. However, if the control strategies are applied over the long-term, it can further break down the minimum concentration of CO<sub>2</sub>. In turn, this means that the density of forest ecosystems will decrease the photosynthesis activities of trees.

From Scenarios I-III, it is concluded that Scenario III is the best strategy for minimizing the concentrations of GHGs and reducing the threat of global warming, as well as enriching or simply saving the coastal ecosystems.

### 5. Conclusion

In this study, we have formulated a new mathematical model (4) to assess the impact of rapid emissions of GHGs on climate change and coastal ecosystems. We have also developed a control system (6) by considering coastal greenbelt and desulfurization as the control variables. Model (4) has been verified by analysis. We have found the necessary best possible conditions for managing problem (P) by using Pontryagin's maximum principle in terms of Hamiltonian. Furthermore, we have verified the results of this study by comparing them to what other studies found, employing numerical simulations.

The analysis findings described in three different scenarios were based on an investigation of the effectiveness of chosen control strategies. From Scenarios I-III, it is seen that both control strategies can effectively reduce the emissions and concentrations of GHGs, whereas the coastal greenbelt is more effective than desulfurization. However, when both control strategies are adopted to system (6)



simultaneously instead of only one control strategy, this process significantly minimizes the GHGs. As a result, it can markedly help the coastal ecosystems to regrow, which is part of the overall strategy to combat climate change and global warming. Therefore, Scenario III is the best strategy to minimize the concentrations of GHGs and global warming. Since global warming is proportionally related to the concentrations of GHGs, it is more convenient to minimize global warming and climate change by reducing industrial emissions and have the GHGs absorbed in some way. Overall, the results show that coastal greenbelt and industrial desulfurization have good environmental remediation potential.

The parametric values of this study are mostly related to environmental factors, so the only limitation of this study is that the analytical results may change when the corresponding variables in nature also change. Since this study describes the effective strategies for better atmospheric and coastal environmental management, it is incumbent upon all governments to design programs now and in the future to save the planet's ecosystems.

## Declarations

### Author contribution statement

Sajib Mandal: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Md. Sirajul Islam & Md. Haider Ali Biswas: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Sonia Akter: Contributed reagents, materials, analysis tools or data.

### Funding statement

This work was supported by the Ministry of Science and Technology, Bangladesh, Government of the People's Republic of Bangladesh (39.00.0000.012.002.04.19–06, Session: 2019–2020).

### Data availability statement

Data included in article/supp. material/referenced in article.

### Declaration of interests statement

The authors declare no conflict of interest.

### Additional information

Supplementary content related to this article has been published online at <https://doi.org/10.1016/j.heliyon.2021.e07401>.

## Acknowledgements

The authors gratefully appreciate the thoughtful comments of the editors and reviewers that helped us to improve this paper. The authors also deeply acknowledge Mr Phillip V. Thomas, Professional language editor and proof-reader, South Australia, for his great support in improving the language of this manuscript.

## References

- National Research Council, *Advancing the Science of Climate Change*, National Academies Press, Washington, DC, 2010.
- T.A. Boden, R.J. Marland, G. Andres, Global, Regional, and National Fossil-Fuel CO<sub>2</sub> Emissions (1751–2014), 2017. United States.
- Y. Pan, R.A. Birdsey, O.L. Phillips, R.B. Jackson, The structure, distribution, and biomass of the world's forests, *Annu. Rev. Ecol. Evol. Syst.* 44 (2013) 593–662.
- T.S. Ledley, E.T. Sundquist, S.E. Schwartz, D.K. Hall, J.D. Fellows, T.L. Killeen, Climate change and greenhouse gases, *Eos* 80 (39) (1999) 453–474.
- M. Rock, M.R.M. Saade, M. Balouktsi, F.N. Rasmussen, H. Birgisdottir, R. Frischknecht, G. Habert, T. Lützkendorf, A. Passera, Embodied GHG emissions of buildings – the hidden challenge for effective climate change mitigation, *Appl. Energy* 258 (2020) 114107.
- S. Mandal, M.S. Islam, M.H.A. Biswas, Modeling the potential impact of climate change on living beings near coastal areas, *model, Earth Syst. Environ.* (2020).
- S. Sahney, M.J. Benton, H.J. Falcon-Lang, Rainforest collapse triggered Pennsylvanian tetrapod diversification in Euramerica, *Geology* 38 (12) (2010) 1079–1082.
- Centre for International Environmental Law (CIEL), 2019. <https://www.ciel.org/news/plasticandclimate/>.
- S. Solomon, D. Qin, M. Manning, Z. Chen, M. Marquis, K.B. Averyt, M. Tignor, H.L. Miller, *Climate Change 2007: the Physical Science Basis – Contribution of Working Group I (WG1) to the Fourth Assessment Report (AR4) of the Intergovernmental Panel on Climate Change (IPCC)*, Cambridge University Press, Cambridge, United Kingdom and New York, USA, 2007.
- A. Sanchez-Lugo, P. Berrisford, C. Morice, A. Argüez, State of the climate in 2018, *Spec. Suppl. Bull. Am. Meteorol. Soc.* 99 (8) (2018) S11–S12.
- C. Mora, The projected timing of climate departure from recent variability, *Nature* 502 (7470) (2013) 183–187.
- D. Leeson, N.M. Dowell, N. Shah, C. Petit, P.S. Fennell, A Techno-economic analysis and systematic review of carbon capture and storage (CCS) applied to the iron and steel, cement, oil refining and pulp and paper industries, *Int. J. Greenh. Gas Control* 61 (2017) 71–84.
- B. Chu, S. Duncan, A. Papachristodoulou, C. Hepburn, Analysis and control design of sustainable policies for greenhouse gas emissions, *Appl. Therm. Eng.* 53 (2013) 420–431.
- A.M. Varghese, G.N. Karanikolos, CO<sub>2</sub> capture adsorbents functionalized by amine-bearing polymers: a review, *Int. J. Greenh. Gas Control.* 96 (2020) 103005.
- A. Krotki, L.W. Solny, M. Stec, T. Spietz, A. Wilk, T. Chwola, K. Jastrzab, Experimental results of advanced technological modifications for a CO<sub>2</sub> capture process using amine scrubbing, *Int. J. Greenh. Gas Control.* 96 (2020) 103014.
- I. Babich, Science and technology of novel processes for deep desulfurization of oil refinery streams: a review, *Fuel* 82 (6) (2003) 607–631.
- The State of the World's Forests 2020. Forests, Biodiversity and People, FAO and UNEP, Rome, 2020.
- D. Karmakar, P.K. Padhy, Air pollution tolerance, anticipated performance and metal accumulation indices of plant species for greenbelt development in urban industrial area, *Chemosphere* 237 (2019) 124522.
- J.P. Dimbour, A. Dandrieux, G. Dusserre, Reduction of chlorine concentrations by using a greenbelt, *J. Los. Preven. Proc. Indus.* 15 (5) (2002) 329–334.
- M.N. Islam, K.S. Rahman, M.M. Bahar, M.A. Habib, N. Hattori, Pollution attenuation by roadside greenbelt in the around urban area, *Urban For. Urban Green.* 11 (4) (2012) 460–464.
- S. Siedentop, S. Fina, A. Krehl, Greenbelts in Germany's regional plans-An effective growth management policy? *Landsc. Urban Plann.* 145 (2016) 71–82.
- A. Samimi, H.R. Tajik, S. Azizkhani, S. Dokhani, E. Godini, B. Almasinia, M. Samimi, Examine the effects of greenhouse gases on climate change, *World Sci. J.* 1 (12) (2013) 130–143.
- M.H.A. Biswas, T. Rahman, N. Haque, Modeling the potential impacts of global climate change in Bangladesh: an optimal control approach, *J. Fund. Appl. Sci.* 8 (1) (2016) 1–19.
- M. Jaschik, M. Tanczyk, J. Jaschik, A. Janusz-Cygan, The performance of a hybrid VSA-membrane process for the capture of CO<sub>2</sub> from flue gas, *Int. J. Greenh. Gas Control.* 97 (2020) 103037.
- A.K. Misra, M. Verma, E. Venturino, Modeling the control of atmospheric carbon dioxide through reforestation: effect of time delay, *Model, Earth Syst. Environ.* 1 (3) (2015) 1–17.
- F. Pendrill, U.M. Perssona, J. Godar, T. Kastner, D. Moran, S. Schmidt, R. Wood, Agricultural and forestry trade drives large share of tropical deforestation emissions, *Global Environ. Change* 56 (2019) 1–10.
- S. Devi, R.P. Mishra, A mathematical model to see the effects of increasing environmental temperature on plant–pollinator interactions, *Model, Earth Syst. Environ.* 6 (2020) 1315–1329.
- S. Babbar, R. Babbar, Causal dynamics of CO<sub>2</sub> source emissions and population in India using Bayesian approach, *Model. Earth Syst. Environ.* 4 (1) (2018) 339–348.
- F. Brauer, C. Castillo-Chavez, *Mathematical Models in Population Biology and Epidemiology*, Springer, Heidelberg, 2000.
- H.I. Freedman, *Deterministic Mathematical Models in Population Ecology*, Marcel Dekker, New York, 1980.
- S. Lenhart, J.T. Workman, *Optimal Control Applied to Biological Models*, Chapman and Hall/CRC press, New York, 2007.
- G.C. Jahn, L.P. Almazan, J.B. Pacia, Effect of nitrogen fertilizer on the intrinsic rate of increase of *hysterozoea setariae* (Thomas) (*Homoptera: Aphididae*) on rice (*Oryza sativa* L.), *Environ. Entomol.* 34 (4) (2005) 938–943.
- M.H.A. Biswas, M.M. Haque, U.K. Mallick, Optimal control strategy for the immunotherapeutic treatment of HIV infection with state constraint, *Optim. Contr. Appl. Methods* (2019) 1–12.
- S. Mandal, M.S. Islam, M.H.A. Biswas, Modeling and analytical analysis of the effect of atmospheric temperature to the planktonic ecosystem in oceans, in: *Lecture Notes in Networks and Systems* 137, Springer, Singapore, 2021, pp. 131–140.
- B.M. Sanderson, B.C. O'Neill, J.T. Kiehl, G.A. Meehl, R. Knutti, W.M. Washington, The response of the climate system to very high greenhouse gas emission scenarios, *Environ. Res. Lett.* 6 (34005) (2011) 1–11.
- S. Mandal, M.S. Islam, M.H.A. Biswas, Modeling the impact of carbon dioxide on marine plankton, *Int. J. Math. Comput. Sci.* 14 (2020) 197–202.

- [37] R. Islam, M.M. Islam, M.N. Islam, M.N. Islam, S. Sen, R.K. Faisal, Climate change adaptation strategies: a prospect toward crop modelling and food security management, *Model, Earth Syst. Environ.* 6 (2) (2020) 769–777.
- [38] D.D. Barros Santiago, W.L.E. Correia Filho, J.F.D. Oliveira-Júnior, C.A.D. Silva Junior, Mathematical modeling and use of orbital products in the environmental degradation of the Araripe Forest in the Brazilian Northeast, *Model, Earth Syst. Environ.* 5 (4) (2019) 1429–1441.
- [39] H.R. Joshi, Optimal control of an HIV immunology model, *Optim. Contr. Appl. Methods* 23 (4) (2002) 199–213.
- [40] M.Z. Ndi, F.R. Berkania, D. Tambaru, M. Lobo, Ariyanto, B.S. Djahi, Optimal control strategy for the effect of hard water consumption on kidney-related diseases, *BMC Res. Notes* 13 (10) (2020) 201.
- [41] J.P. Sharp, A.P. Browning, T. Mapder, C.M. Baker, K. Burrage, M.J. Matthew, Designing combination therapies using multiple optimal controls, *J. Theor. Biol.* 497 (2020) 110277.