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Novel resonator geometry for easily manufactured tunable locally resonant metamaterial

Can Nerse* (1), Richard Schadeberg (2), Sebastian Oberst* (1)

(1) Centre for Audio, Acoustics and Vibration (CAAV), Faculty of Engineering and IT, University of Technology Sydney, Ultimo NSW 2007, Australia
(2) School of Mechatronic and Mechanical Engineering, Faculty of Engineering and IT, University of Technology Sydney, Ultimo NSW 2007, Australia
* Corresponding authors: can.nerse@uts.edu.au, sebastian.oberst@uts.edu.au

ABSTRACT
Mechanical waves and sound waves have complex propagation characteristics that are manipulated by periodic structures such as elastic metamaterials and phononic crystals for the purposes of wave guiding, vibration isolation and sound absorption. System parameters are tuned to induce auxetic physical properties such as negative effective mass density and negative Poisson's ratio. Locally resonant metamaterial (LRM) uses Fano-type interference to manipulate elastic wave propagation from the host structure by formation of a band gap due to local resonance. Not restricted by the Bragg interference limit, such sub-wavelength structures are particularly effective in attenuation of the low frequency oscillations. Tunability of the lower and upper bounds of the band gap through simple geometrical and material variations has made the LRMs a strong candidate for the noise and vibration control of automotive and industrial applications. In this study, we demonstrate a tunable LRM design that can be fabricated by injection moulding and vacuum casting. The mould for the fabrication of the resonator features a cylindrical hollow section. Pins of different diameter can be inserted into the mould to vary the material distribution in the cavity, thereby changing the resonance. A numerical model using COMSOL Multiphysics has been developed to investigate the dispersion mechanism. A parametric study of the pin diameter with respect to target band gap frequency demonstrates the capability of broadband vibration attenuation while keeping the overall size of the resonator small and constant. These results are promising for practical implementation of LRMs.

1 INTRODUCTION

In recent years, there has been a substantial interest in the development of metamaterials using novel design and methodologies for acoustic wave manipulation (Ma and Sheng, 2016), cloaking (Fan et al., 2020), vibration isolation (Zhang et al., 2018, Xu et al., 2019), energy harvesting (Tan et al., 2019), etc. The activity in this field ranges from purely academic research to practical case studies for noise and vibration control. Among such efforts, locally resonant mechanisms are of particular interest owing to their virtually limitless potential in design and implementation. In such systems, the attenuation of elastic and acoustic waves of sub-wavelength scale is commonly realized through attachment of locally resonant units and by combining different materials and geometrical features (Chang et al., 2018, El-Borgi et al., 2020). Studies by de Melo Filho et al. (2019), Van Belle et al. (2019), Jung et al. (2019) illustrate the wide range of applicability of locally resonant metamaterial (LRM) for automotive and industrial applications. Rapid prototyping via additive manufacturing is the go-to option for validation of LRM design. Sensible design choices with respect to the target host structure can be realized by quick one-off tests with experimental modal analysis. This makes 3D-printing very effective and robust for academic as well as for industrial engineering developments. However, this practice is not feasible to repeat for any range of host structures in terms of material, surface treatment, modal characteristics, etc., i.e., any design choice of LRM is prone to limitations induced by the host structure in design. Although the LRM research has advanced to the point that for any target frequency feasible designs can be produced, there are still challenges in the practical implementation with respect to manufacturing and deployment at large scale due to lack of standardised tools and assembly lines, thus requiring manual labour to a great extent. Injection moulding and vacuum casting were an ideal way to manufacture the LRM, suitable for mass-production, which if there were not the immense cost producing a wide range of molds due to various frequencies being involved.

In this paper, we demonstrate a novel resonator geometry that can be incorporated into the manufacturing processes such as injection moulding and vacuum casting. The resonator structure features a concave cylindrical section that is shaped by pin inserts in the mould (Figure 1). By inserting pins of different diamaters, material injected into the mould can be distributed differently, thereby changing the mass and stiffness distribution. This allows tuning the resonant frequency to target different band gaps for wave attenuation without requiring to adjust the mould opening, generating tunable locally resonant metamaterials (tLRM).

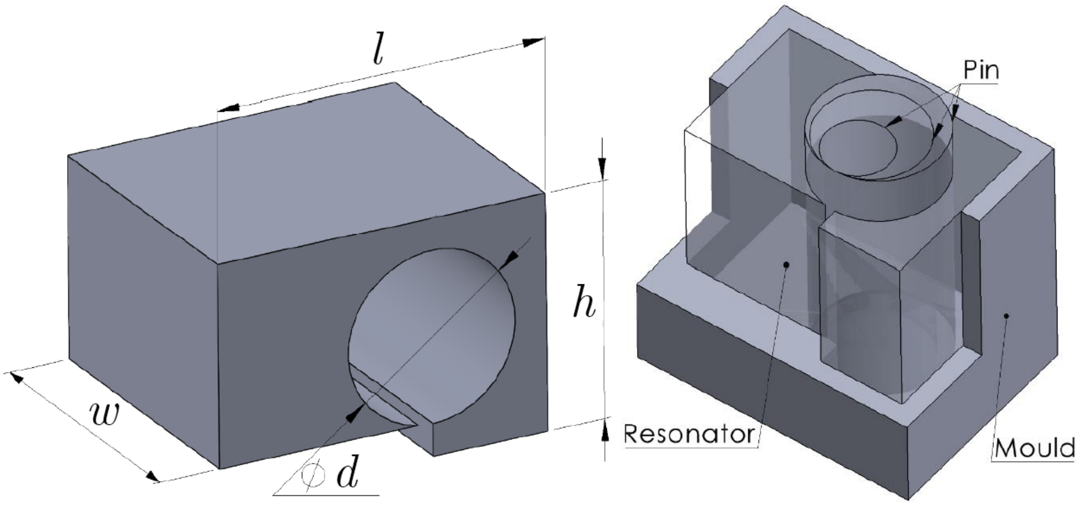

Figure 1: Proof of concept of a tLRM design. Resonator design (left), and mould for manufacture where variable pin diameter is shown (right, cutaway).

2 DESIGN AND METHODOLOGY

A cantilever beam design has been chosen to tune the first out-of-plane bending mode frequency (Figure 1). This design was shown to be effective in broadening the band gap while keeping the mass ratio low (Van Belle et al. 2019, Jung et al., 2020, Sangiuliano et al., 2020, Yu et al. 2021). By changing the diameter of the inserted pin, the inner dimensions of the resonator can be changed, thereby distributing the material over so-called spring and mass sections of the resonator. The overall package dimensions of the resonator remains constant, thus allowing to reuse of the same mould for any selection of pin diameters within geometrical constraints. Figure 1 demonstrates the sizing parameters of the resonator: width, depth, height of the resonator; diameter of the pin.

2.1 Unit Cell Band Gap Analysis

The band gap behaviour of the LRM was investigated by implementing Floquet-Bloch periodic boundary conditions on a square lattice. Figure 2 shows the Irreducible Brillouin Zone (IBZ) of the square unit cell with lattice constant s. In this study, $\mathrm{s} = 30\ \mathrm{mm}$ was considered for periodicity. The wave propagation in the coupled resonator-host system was analysed by dispersion curve analysis by solving eigenvalue problem at discrete structural wavenumbers, k_x and k_y, in the symmetry plane of $[0, \pi/s]$ The unit cell was modelled using the finite element method in COMSOL Multiphysics. The resonator and the host structure were discretised by quadratic solid and shell elements, respectively, with an element size of 1 mm to resolve the convergence of the high frequencies.

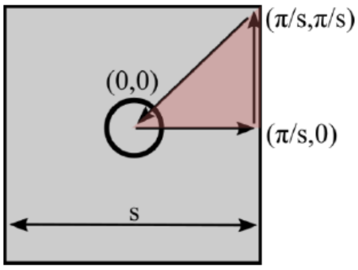

Figure 2: Irreducible Brillouin Zone in x-y plane for a square unit cell.

2.2 Parameter Tuning

The host structure material is mild steel with thickness of 2.4 mm. A few design decisions were considered in tuning of the sizing parameters to make the resonator compact such that the mass of the resonator is less than 10 grams and the height of the resonator is below 25 mm. A parameter optimisation was conducted such that $l = 25\ (\mathrm{mm})$, $10\ (\mathrm{mm}) \leq \mathrm{h} \leq 25\ (\mathrm{mm})$, $10\ (\mathrm{mm}) \leq \mathrm{w} \leq 25\ (\mathrm{mm})$ and $0 \leq \mathrm{d} \leq \mathrm{h}$. For the convenience of prototyping, the material of the resonator was chosen as photopolymer, which also has similar properties as ABS plastic – a common material in injection moulding processes. Table 1 shows the material properties of the mild steel and photopolymer, which are identified by modal testing. By parametric dispersion curve analysis, the performance of the LRM was assessed by a derivative-free optimisation that maximises the band gap width from 200 Hz to 1200 Hz. The low frequency end, which is difficult to target by conventional NVH techniques, is shown to be feasible by such novel resonator geometry as Table 2 shows the optimised sizing parameters for the target band gap frequencies of 350–390 Hz. It is noticed that even for such low resonator mass, a considerable band gap width can be obtained.

Table 1: Host (mild steel) and resonator (photopolymer) material properties

Material	Density, ρ (kg/m^3)	Elastic modulus, E (N/mm^3)	Poisson's ratio, ν	Isotropic loss factor
Mild Steel	7721	185	0.3	0.007
Photopolymer	1162	2.6	0.4	0.3

Table 2: Optimised LRM parameters for band gap frequency of 350–390 Hz.

Parameter	Value	Unit
Resonator height, h	17.5	mm
Resonator width, w	21.5	mm
Pin diameter, d	12.7	mm
Mass	6.75	g

The LRM design can be tuned with respect to target frequency band by selection of a suitable pin. Figure 3 illustrates a parametric analysis of the band gap with respect to the pin diameter. The highlighted horizontal bands are frequency intervals in which low-, mid- and high-frequency vibration attenuation are targeted. The vertical lines represent the band gap that opens up at respective pin diameter. The length of the line corresponds to band gap width, while the colour denotes whether the band gap lies within one of three target band gaps, i.e., green for inside the target band gap frequency, red for outside. At large values of pin diameter, the spring and mass sections (Figure 1) gets narrower with the reduction in stiffness outpacing the mass, thus also decreasing the first bending mode frequency. A narrow band gap at as low as 50 Hz can be obtained; however, further decrease is limited by the resonator height. At the other end of the frequency spectrum, it is shown that wide attenuation bands can be obtained by selecting an appropriately sized pin. Such band gap catalogue can be useful for practical implementation as the end-user can select a pin size for the target application, e.g., M6 pin for target frequencies of 1,129–1,250 Hz.

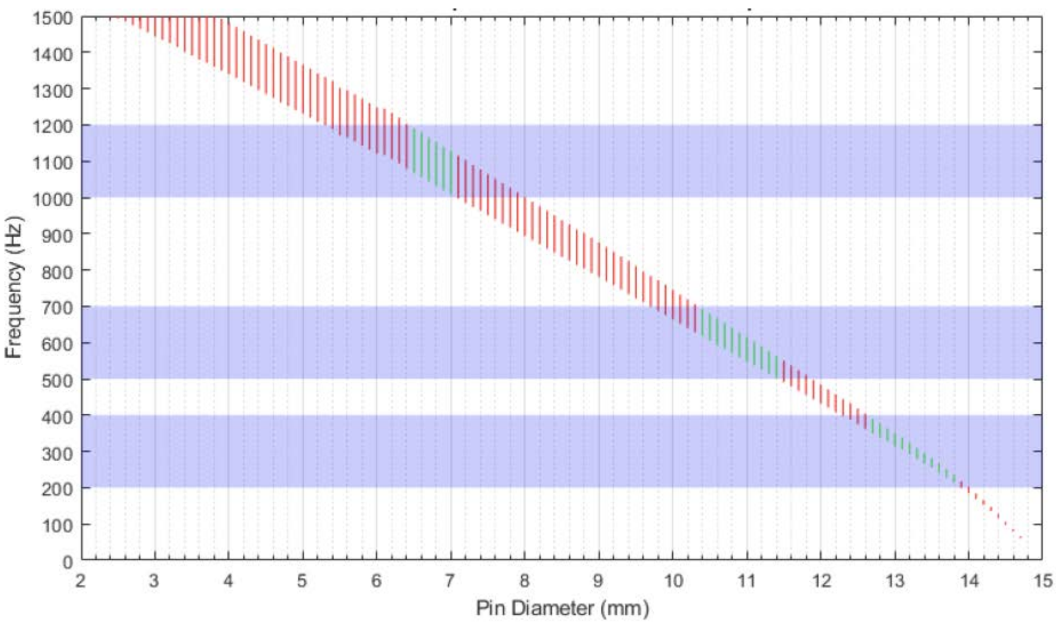

Figure 3: Band gap frequencies as a function of pin diameter. Highlighted regions correspond to the target band gap frequencies, 200–400 Hz, 500–700 Hz, 1,000–1,200 Hz. Each green or red vertical line represents a range of frequencies that are attenuated by an LRM variant, inside and outside target frequencies, respectively.

3 RESULTS AND DISCUSSION

In this section, performance of the tLRM is shown by frequency response characteristics of the tLRM treated host structure. Figure 4(a) demonstrates the dimensions and boundary conditions of the host structure, which are fixed on the short ends. Untreated host structure has only out-of-plane bending modes in the frequency interval up to 1.4 kHz, at 124 Hz, 340 Hz, 668 Hz and 1,105 Hz, three of which lie within the target band gaps in Figure 3. To effectively attenuate the host vibration at these resonant frequencies, pins of $\mathrm{d} = 13\ \mathrm{mm}$, $\mathrm{d} = 10.3\ \mathrm{mm}$ and $\mathrm{d} = 6.7\ \mathrm{mm}$ have been selected, respectively, while keeping the h and w same as the values in Table 2. A tLRM attached host structure is shown in Figure 4(b) for the resonator array with $\mathrm{d} = 10.3\ \mathrm{mm}$. To demonstrate the effective band gaps, a point harmonic load of 1 N was applied at one end, while measuring the vibration response at the opposite end. Figure 5 shows the transfer FRF of the host structure for treated and untreated cases. At the target band frequencies, tLRM treatment attenuate the vibration transmission across the host structure by at least two orders of magnitude, providing a significant reduction with only 12 unit cells corresponding to 20% mass increase. Thus, the LRM is shown to be superior to mass law.

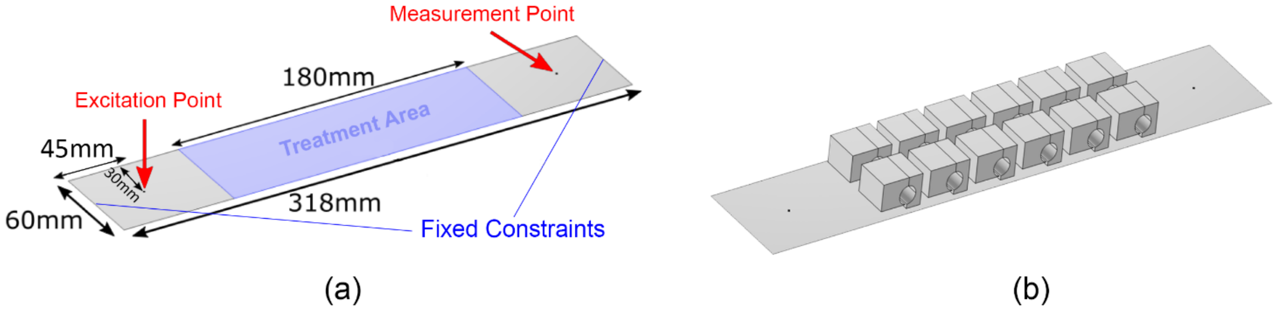

Figure 4: (a) Dimensions and boundary conditions of the host structure; (b) with tLRM treatment.

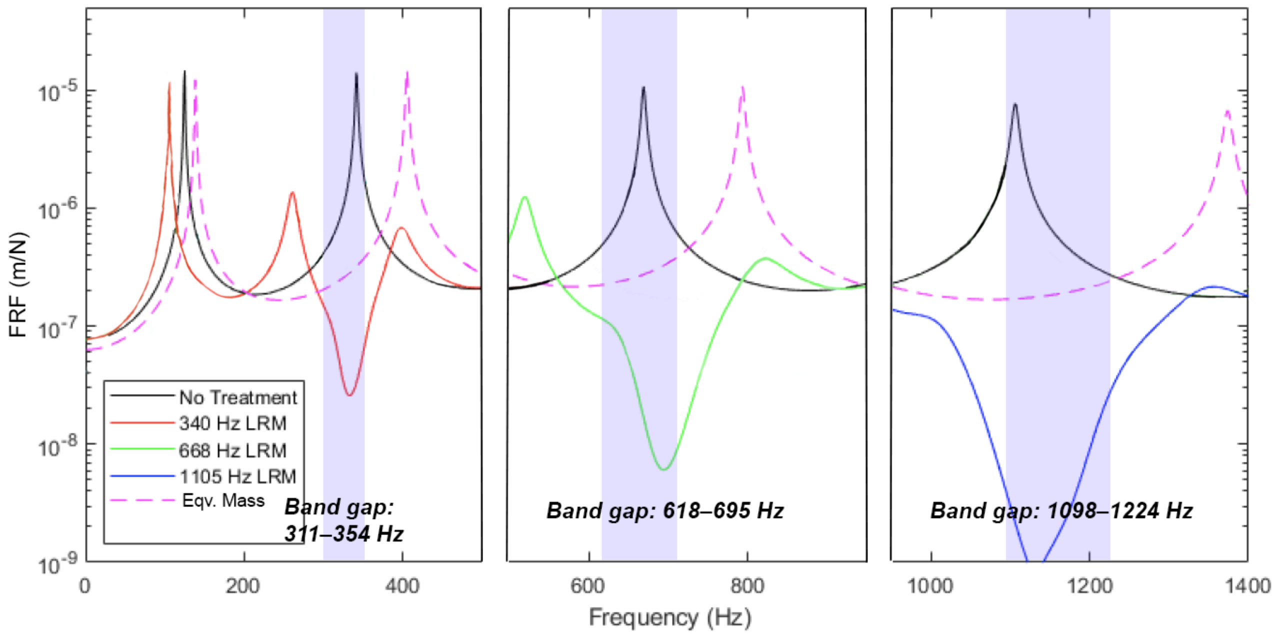

Figure 5: Low (311–354 Hz), mid (618–695 Hz) and high-frequency (1,098–1,224 Hz) performance of LRM designs are shown with respect to the untreated host structure and equivalent mass of the 1,105 Hz case.

4 CONCLUSIONS

In this paper, we have demonstrated a novel resonator geometry of tunable locally resonant metamaterials (tLRM) for the attenuation of elastic waves. The resonator features a concave cylindrical section that can be tuned by selecting appropriately sized pins inserted into an injection mould. The convenient one-parameter tuning allows *in-situ* adjustments to resonator geometry, thereby tuning the band gap frequencies, which is considered to bring a much-needed alternative for mass-manufacturing. The proof-of-concept of this study was verified by a finite element analysis. A parameter optimisation was conducted to determine the band gap frequency by a dispersion curve analysis. It is shown that the pin diameter can be selected such that the band gap can be adjusted from 50 Hz until 1,200 Hz. Elastic wave attenuation by tLRM is shown to be superior to mass law of attenuation, thereby obtaining an exceptional performance of vibration attenuation in a small lattice. Future studies will consider the periodic arrangement of LRM geometries with multiple band gaps to target broadband vibration attenuation. The presented results are very promising for combining novel noise and vibration control methodologies with conventional manufacturing processes with experimental validation being underway.

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