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Call Auctions with Contingent Orders

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Abstract: We introduce a new mechanism for call auctions which are widely used in stock exchanges. Our unique design incorporates contingent claims (buy stock A, if selling stock B) into the price discovery process. With our proposed mechanism, we show that higher liquidity during the call auctions is achieved, as well as lower volatility after the call auctions. Moreover, we show that current call auctions and the proposed mechanism have similar incentive properties. Hence, we argue that the proposed mechanism would be an improvement over the existing opening auction rules at stock exchanges.

Keywords: call auctions; stock exchanges; volatility

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1. Introduction

Intraday price volatility of stocks generally tends to be higher at the opening interval than in other trading periods during the day. Among others, Ref. [1] supports this claim using the data of Dow Jones Industrial Average between 1964–1989. Most global stock exchanges implement call auctions at the opening and closing of trading sessions to make an effective system for price discovery, reduce the opening volatility, and avoid closing price manipulations. Ref. [2] shows that the opening auction contributes to the efficiency of the opening prices, especially for the highly liquid stocks.

This paper introduces a new call auction mechanism with an innovative algorithm. This unique design lets prospective investors place contingent buy orders and determines the auction price by using these contingent orders, which will help to solve the problems mentioned above. Our mechanism is also theoretically shown to increase the liquidity level in these auctions. We also consider the incentive properties of current call auctions and the proposed new mechanism and find that they are similar.

While participating in the stock exchanges, the investors regularly update their portfolio decisions with incoming news received overnight. They may need to change their holdings on the coming trading day, but if they want to realise this change at the opening session to benefit from the concentration of liquidity at the call auction, in some cases, these traders may not have sufficient sources to execute their buy orders before completing their sell orders. One can argue that the actual delivery and payment (settlement) procedure is not instantaneous in many stock exchanges; therefore, traders can complete this exchange without using contingent orders. However, financial intermediaries are exposed to settlement risk, and the risk management considerations increase the transaction costs of these traders. On the other hand, contingent orders reduce transaction costs and provide the opportunity to simultaneously buy and sell different stocks at the opening or closing call auction.

Other potential groups of investors who may use contingent orders are "pairs traders". Pairs trading is a well-known and commonly used trading strategy. It is formed by, at first, finding two highly correlated stocks depending on historical records, then buying one and selling the other whenever the spread between them increases significantly. Ref. [3] show empirically that this simple strategy brings up to 11% average annual excess return. They also note that the profitability of this strategy, primarily used by institutional investors, depends on the traders' transaction costs. It has also been shown that call auctions are preferable to continuous auctions for these investors, as call auctions exhibit lower transaction costs and less market impact (see Snell and Tonks [4] and Economides and Schwartz [5]). Additionally, the reference price property of closing auction prices attracts institutional investors to be more active and aggressive at the closing auction [6]. An arbitrageur, with a pairs trading strategy, can avoid the risk of "buying a lower priced stock without selling a higher priced one" by using contingent orders in our proposed auction market.

Our new design for a call auction—"Call Auctions with Contingent Orders"—determines the auction price by "contingent orders". This, in turn, helps to solve the problem of "price volatility" (Remark 1) after the opening interval and achieves better "price discovery". Our new design also increases the liquidity level in the auction (Proposition 1). Since the incentive properties of present call auctions and the proposed new mechanism are similar (Propositions 2 and 3), we can theoretically argue that our new design is better compared to the current call auctions used in practice.

We acknowledge that our methodology—which is a theoretical (or "proof of concept") approach—has limitations in the sense that more research needs to be performed to establish practical advantages of our design compared to the current mechanisms in use. We discuss several possible avenues for future research in the Conclusion and Future Research section (Section 4).

The paper is structured as follows. We conclude Section 1 with a discussion of the related literature. In Section 2, we introduce the current call auction mechanism and the proposed call auction mechanism with contingent orders. In Section 3, we study the incentive properties of both types of call auctions. Section 4 concludes and flags future research.

Related Literature

We refer the reader to an extensive survey of [7] on theoretical, experimental, and empirical work on "double auctions" which subsumes the call auctions.

There are several papers (mostly empirical) that focus on call auctions. Ref. [8] examines the effect of (closing) call auctions on liquidity by exploiting the natural experiment offered by the introduction of the call auction on the Australian Stock Exchange in 1997. Many studies have performed empirical analysis of opening and/or closing call auctions. For instance, Ref. [9] studies call auctions in Singapore, Ref. [10] in Tel Aviv, Ref. [11] in Taiwan, and [12] in Hong Kong. All these papers study the standard call auctions, and there are (naturally) no studies on our proposed design for call auctions.

There is a significant body of work on price volatility in various stock exchanges. Research regarding the intraday price volatility of the stock market has documented a U-shaped pattern throughout the trading day. Refs. [13,14], by using US stock market data, are among the first to provide evidence of this pattern. Ref. [15] extends this result for the Japanese stock market via using the lunch break of the Japanese market. Moreover, they show that both the morning and afternoon sessions of the market exhibit a similar U-shaped intraday volatility pattern. Ref. [16] present comparable results for the London Stock Exchange. To explain this observation regarding intraday volatility of stocks, Ref. [17] develop a model of trading patterns in financial markets and justify the U-shaped price volatility by the interaction between strategic decisions of liquidity and informed traders. Ref. [18] claim that aggregate order imbalances contribute to the market volatility. Following this line of research, Ref. [19] conducted an empirical analysis of the role of the leveraged exchange-traded funds (ETFs) on the end-of-day volatility, and they identify a significant relationship between the leveraged ETFs rebalancing activity and the market volatility.

There are a few experimental studies of call auctions, and we refer to some of them in the Conclusion and Future Research section (Section 4).

2. Call Auctions

Different stock exchanges use very similar types of the uniform price auction mechanism for their call auction sessions. We demonstrate the general idea of these auctions below.

2.1. Current Call Auction Mechanism

Investors can only enter the call auction with limit orders, which consist of the code of the individual stock, price, and quantity for the order. In the *call auction (CA)*, buyers and sellers announce their values and quantities (v, q). Then, the *call auction price (CAP)* is chosen to be one of the prices that maximise the total sales (total sale with a price p is defined as the minimum of "total quantities demanded by buyers with values $v \geq p$" and "total quantities supplied by sellers with values $v \leq p$"). In the case that there are multiple maximisers of the total sale, the price closest to the last closing/sale price is chosen. For CAP equal to p, there could be excess demand or supply. In the case of excess demand (supply), then all matching sell (buy) orders will be executed, and the lowest buy (highest sell) offers may be partially executed. In case of multiplicity of lowest buy (highest sell) offers, these offers will be executed pro rata.

2.2. Call Auction Mechanism with Contingent Orders

Consider an investor who holds stock A and would like to buy stock B only if she sells stock A. Now, suppose we allow investors to announce their orders as "I would like to buy stock B (at most price x), if I can sell stock A (q units of the stock for a price at least y)".

After all regular limit orders and contingent orders are collected until the call auction, the *"Call Auction with Contingent Orders" (CACO)* mechanism is run by the following algorithm in order to determine the auction prices (CACOP) of different stocks:

Stage 1: All regular limit orders and selling limit orders of the contingent orders will be entered, and the price vector p_1 will be determined according to the CA mechanism described above. If any of the contingent selling limit orders is executed with p_1, corresponding buying limit orders of these contingent orders[1] are entered into the auction book, and the algorithm moves to Stage 2.

Stage k: CA mechanism determines the new price vector p_k with the updated orders in the auction book. If any of the contingent selling limit orders are executed with p_k, corresponding buying limit orders of these contingent orders are entered into the auction book, and the algorithm moves to Stage $k + 1$. Otherwise, the algorithm ends with the resulting CACOP vector p_k.

One important feature of this mechanism is that at each stage k, only new buy orders can be added to the auction book. Hence, matched sellers of stages $1, \ldots, k-1$ will also be matched in stage k and p_k is nondecreasing in k. This feature guarantees that there will be no loops in the algorithm, and it will end in finitely many steps. We now illustrate our mechanism with an example.

2.3. Example

Consider the following opening order books with two stocks.

In Table 1, C indicates that the corresponding sell order is contingent, and the sender of this contingent order wants to buy the other stock at the price in the parenthesis if her contingent claim is successfully executed. Assume that every limit order at each price was given by different investors. Below, we first consider the determination of price and volume under standard call auctions, then under call auctions with contingent orders.

Table 1. Opening Order Books.

Stock A's Limit Order Book (Last Closing Price is 1.00)				
Buy Orders		Sell Orders		
Price	Quantity	Quantity	Price	
1.01	10,000	7000	0.98	
1.00	5000	12,000	0.99	C (0.52)
0.99	15,000	4000	1.00	
0.98	7000	6000	1.01	
0.97	8000	8000	1.02	
		4000	1.03	
Stock B's Limit Order Book (Last Closing Price is 0.52)				
Buy Orders		Sell Orders		
Price	Quantity	Quantity	Price	
0.51	8000	12,000	0.50	
0.50	6000	9000	0.51	C (1.01)
0.49	12,000	8000	0.52	
0.48	8000	7000	0.53	
0.47	6000			

2.3.1. Call Auctions

By this algorithm, price and volume of the opening auction could be confirmed to be:[2]

Stock A: Price: 0.99; Volume: 19,000.
Stock B: Price: 0.50; Volume: 12,000.

After the opening auction, the first order books for the continuous auction market are given in Table 2 below.

Table 2. Order Books at the Completion of CA.

Stock A's Limit Order Book after CA			
Buy Orders		Sell Orders	
Price	Quantity	Quantity	Price
0.99	11,000	4000	1.00
0.98	7000	6000	1.01
0.97	8000	8000	1.02
		4000	1.03
Stock B's Limit Order Book after CA			
Buy Orders		Sell Orders	
Price	Quantity	Quantity	Price
0.50	6000	9000	0.51
0.49	12,000	8000	0.52
0.48	8000	7000	0.53
0.47	6000		

At the call auction, the investor willing to buy stock B if she can sell stock A turns out to be a successful seller. Therefore, after completing CA, she will be a buyer for stock B. One can confirm that in the continuous auction trading following the opening auction, she will probably buy from the investor with a contingent interest in stock A, and the latter investor will be buying stock A. These cross interactions will add to the volatility of the early periods of the continuous trading sessions.

2.3.2. Call Auctions with Contingent Orders

At the first stage of the CACO algorithm, p_1 will be identical to the resulting CA price.

Stock A: Price: 0.99; Volume: 19,000.
Stock B: Price: 0.50; Volume: 12,000.

After the first stage, the algorithm determines the already executed contingent order: the contingent order at stock A. Hence, the corresponding buy order is entered into the book of A (an order for 22,846 units of stock B with price 0.52). In the second stage, the new price vector becomes $p_2 = (0.99, 0.52)$. With this new price vector, the algorithm determines a new executed contingent order: contingent order at stock B. Hence, the corresponding buy order is entered into the book of B (an order for 4544 units of stock A with a price of 1.01). At the third stage, the new price vector becomes $p_3 = (1.00, 0.52)$. With the completion of the third stage, there are no other contingent orders, so the auction algorithm ends with CACOP equal to p_3. Resulting limitorder books with the completion of CACO are given below.

In Table 3, the italic orders are entered at the consequent stages after the first stage of the algorithm. With this algorithm, price and volume of the opening auction will be:

Stock A: Price: 1.00; Volume-19,544.
Stock B: Price: 0.52; Volume-22,846.

This example illustrates that with CACO, all the information prior to opening is efficiently used by the algorithm, and there will be no further trading at the current market structure. Comparing the outcomes of CA and CACO, we also see that CACO trading volume is greater.

Table 3. Order Books at the Completion of CACO.

Stock A's Limit Order Book for CACO (Last Closing Price is 1.00)			
Buy Orders		Sell Orders	
Price	Quantity	Quantity	Price
1.01	10,000	7000	0.98
1.00	5000	12,000	0.99
0.99	15,000	4000	1.00
0.98	7000	6000	1.01
0.97	8000	8000	1.02
		4000	1.03
1.01	*4544*		

Stock B's Limit Order Book for CACO (Last Closing Price is 0.52)			
Buy Orders		Sell Orders	
Price	Quantity	Quantity	Price
0.51	8000	12,000	0.50
0.50	6000	9000	0.51
0.49	12,000	8000	0.52
0.48	8000	7000	0.53
0.47	6000		
0.52	*22,846*		

Below, we establish that with CACO, there are no remaining matching buy and sell orders at the CACOP that can be processed into the continuous trading session. Since this is not the case in CA, we conclude that, in ceteris paribus, early continuous trading session volatility is lower with CACO than with CA. Moreover, we easily show that CACO's trading volume is greater than that of CA.

Proposition 1. *There are no remaining matching buy and sell orders at the CACO. Moreover, the trading volume of CACO is greater than or equal to the volume with CA.*

Proof. The first claim follows by definition. Since the algorithm moves to a new stage whenever there are matching buy and sell orders, there cannot be any matching buy and sell limit orders when the algorithm ends. The second claim follows by noting that the volume traded in the first stage of the CACO algorithm equals the volume traded in CA. In the following stages (if there are any), the volume traded has to increase. Therefore, the claim follows. □

Remark 1. *Since there are no remaining matching buy and sell orders at the CACO, ceteris paribus, the volatility after opening CACO will be lower than that of opening CA.*

3. Incentives Properties of Call Auctions

In a CA, it is not a weakly dominant strategy to announce the true types (values and quantities). Consider the following example.

Example 1. *Consider two buyers with value-quantity pairs* $(1, 2000)$, $(0.9, 1000)$ *and two sellers with value quantity pairs* $(0.6, 2000)$, $(0.5, 1000)$*, with the last closing/sale price equal to* 0.8*. Then, all prices in* $[0.9, 0.6]$ *maximise sales (*3000 *quantities) and* 0.8 *will be chosen as CAP. However, if the buyer 2 and both sellers announce their types truthfully, buyer* 1 *has a strict incentive to announce her value as* 0.7*, as with that deviation, the price would be* 0.7*, and she would be strictly better off.*

The above example shows that investors may have an incentive to misreport their type. However, the incentive to misreport her type arises only when that investor becomes a price setter after the deviation:

Proposition 2. *In a CA, given any announcement of other investors, an investor is never better off (compared to the truthful announcement) by announcing another type, unless with that announcement she becomes a price setter (that is, CAP is equal to her announced value).*

Proof. Consider a buyer b_i whose true value is v_i^b. If by a truthful announcement she is not a successful buyer, then CAP has to be greater than v_i^b. In that case, the only way she can be a successful buyer is to increase her type, which makes the CAP even greater, and she obtains a negative utility. Therefore, in this case there is no profitable deviation. If by a truthful announcement she is a successful buyer, then CAP has to be smaller than v_i^b. For a deviation to be profitable, b_i has to be a successful buyer after the deviation. If CAP after the deviation is not equal to the new value announcement of b_i, that means that CAP remained the same. This is because CAP is unaffected by the values announced by the successful buyers, who are not price setters. Therefore, the only way a lie can be beneficial is when after the lie, the investor becomes a price setter (as in the above example). Analogous arguments can be made for the sellers. □

A similar version of the above incentive result continues to hold in a CACO.

Proposition 3. *In a CACO, given any announcement of other investors, an investor is never better off (compared to the truthful announcement) by announcing another type, unless with* the best deviation *she becomes a price setter (that is, CACOP is equal to her announced value).*

Proof. For non-contingent investors, the arguments are the same as above. Since all the investors care about is the final price that the algorithm produces, for the buyers or the sellers of a stock, the only way to benefit is by becoming a price setter. Now consider a contingent investor who would like to buy stock A only if she can sell stock B. She can lie in two dimensions, value for A and value for B. If she is not a successful seller of stock B by announcing truthfully—in contrast with CA—she may still want to lie and become a successful seller. This may be the case if her value for stock A is sufficiently high to offset some loss in selling stock B by the profit in buying stock A. Nevertheless, this investor will become a price setter for stock B in the best deviation. Her incentive to announce values for A is the same as a regular buyer. Hence, the result follows. □

Propositions 2 and 3 establish that although CA and CACO are not truthful, the incentives from lying could be present only for "price setters". Therefore, one may expect that the behaviour of the traders may be similar in CA and CACO. On the other hand, more theoretical and experimental work is required to fully understand how bidders could game CACO and how this compares with the classic CA. Thus, although comparing CA and CACO for the same "inputs" (and ignoring the equilibrium behavior) is not entirely justified, there are indications that Proposition 1 and Remark 1 may potentially hold for the equilibrium behavior of the traders.

4. Conclusions and Future Research

In this paper, we introduce a new mechanism for call auctions, which are widely used in stock exchanges. We call this new mechanism "Call Auctions with Contingent Orders" (CACO). CACO incorporates contingent claims (such as buying stock A, if selling stock B) into the price discovery process and, in this way, achieves higher liquidity (as well as lower volatility afterwards) in comparison to regular call auctions (CAs). Although both CACO and CA lack strategyproofness, we establish that in both mechanisms, incentives from lying occur only when the players are price setters. Given this result, we believe that CACO would be a better mechanism to use at opening auctions at stock exchanges.

Our methodology in this paper is theoretical, and we only provide a "proof of concept". More research has to be carried out to validate the theoretical results, especially in experimental setups. There are several (laboratory or field) experimental studies of call auctions. For instance, Ref. [20] is one of the earlier and important papers that studies call auctions using the experimental laboratory as a test bed. This paper evaluates the comparative performance of various institutions where trades are executed via "uniform prices". More recently, Ref. [21] used controlled laboratory experiments to compare "mispricing" in double auctions, motivated by call auctions. In addition, Ref. [9] investigated the efficiency and price manipulation in the Singapore Exchange after the introduction of opening and closing call auctions in August 2000. Similarly to these papers, an experimental investigation of our proposed mechanism (in terms of performance, mispricing, and manipulation) would be vital, and it is left for future work.

Lastly, we only establish that both the current mechanism and the proposed mechanism are "similar" in terms of manipulation, yet this does not mean that there will not be any manipulation. Hence, a theoretical investigation of the manipulation and gaming of different call auctions—including our proposal—is also left for future work.

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Notes

1 The quantity of the contingent buy order is calculated by "current revenue of the investor at that stage" over "buying price announced in the contingent order, x".

[2] To see this, consider stock A. If $p_A \leq 0.97$, since there is no supply, volume is 0. If $p_A = 0.98$, then volume is 7000 (minimum of 7000 and 37,000). If $p_A = 0.99$, then volume is 19,000 (minimum of 19,000 and 30,000). If $p_A = 1.00$, the volume is 15,000 (minimum of 23,000 and 15,000). If $p_A = 1.01$, then volume is 10,000 (minimum of 29,000 and 10,000). If $p_A \geq 1.02$, since there is no demand, volume is 0. For Stock *B*, if $p_B \leq 0.49$ or $p_B \geq 0.52$, then volume is 0. If $p_B = 0.50$, then volume is 12,000. If $p_B = 0.51$, then volume 8000.

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