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New dynamical soliton propagation of fractional type couple modified equal-width and Boussinesq equations

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Abstract The space–time fractional coupled modified equal-width equation and the coupled Boussinesq equation are a category of fractional partial differential equations, which might be crucial mathematical feathers in nonlinear optics, solid-state physics, vibrations in the nonlinear string, ion sound waves in plasma, hydro-magnetic waves in cold plasma and many more. To assemble such new exact solutions of the mentioned equations, the Sine-Gordon expansion (SGE) technique has been proposed with inside the sense of conformable derivative and the fractional order partial differential equation that is capable to change into an ordinary differential equation by using the traveling wave transform. In this article, the SGE technique has been employed to search the higher-dimensional fractional nonlinear evolution equations and hooked up consistent soliton solutions to the faster thought fractional nonlinear evolution equations through installing use of the prolonged higher-dimensional SGE technique. The compatibility of the extended SGE technique confirms through the scoring of soliton solutions. Moreover, we explored a couple of varieties of solutions over the maple calculations, including soliton, kink types, bell types, single soliton type, dark soliton, singular kink type, and anti-bell type solutions for distinct values of constants, which have been illustrated by the usage of 3D, list-point, contour analysis, and vector plotting. It is far incredible to understand that the feature of the solutions relies upon the selection of the parameters from the figures. This takes a look at an impactful position in studying higher-dimensional fractional nonlinear evolution equations through the prolonged SGE technique.

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1. Introduction

The fractional derivatives and integrals have been discovered over three hundred years ago, but it has not always been a new discerning. In a couple of decades, large efforts were paid with inside the fields of nonlinear fractional partial differential equations (NLFPDEs) and drawn interest for their common applications in the latest medical and engineering arenas, controlled thermonuclear fusion, for instance, plasma physics, acoustics, solid-state physics, stochastic dynamical process, diffusive transport, electric network, electromagnetic theory, astrophysics, in fluid mechanics, fractional dynamics, geochemistry, manipulate theory, system identification, chemical kinematics, optical fibers, bio-genetics, solid-state physics, chemical physics, and many others fields [1-5]. The terms related to engineering standpoint including dispersion, convection, diffusion, and response are carefully related to the above cited topics, and NLFPDEs might be used to assess them correctly. Fractional calculus is the key technique of standard differentiation and integration in arbitrary order, and can be utilized to formulate and interpret the different physical nature of a non-stop transition from stationary to oscillation phenomena. For large-scale research in various sectors of engineering and physical sciences, currently, FNLEEs have turned out to be pretty famous. Some of the techniques had been evolved with the aid of using numerous researchers to perform genuine and specific stable soliton solutions of nonlinear physical models, inclusive of the modified Kudryashov method [6,7], the hyperbolic ansatz technique [8], the generalized exponential rational function method [9], the extended simple equation method [10], the sub-equation method [11], the modified Khater method [12], the Chelyshkov polynomial technique [13,14], the reproducing kernel technique [15], the two variables $(G'/G, 1/G)$-expansion technique [16-18], the generalized logistic equation method [19], the SGE technique [20-23], the F-expansion technique [24] and numerous sort of soliton [25,26] manner. The popular SGE technique became evolved primarily based on wave transformation only, and it works best for lower-dimensional FNLEEs. There are many higher-dimensional FNLEEs regarding physical mathematical problems in day-to-day life, and the understanding of them explicitly in similar soliton solutions is desired. However, new solutions of higher-dimensional FNLEEs have no longer been investigated with the aid of the SGE extending technique. Therefore, the goal of this study is to increase the computable SGE technique for higher-dimensional FNLEEs, and the implication of this approach is to set up broad-ranging stable soliton solutions to the CMEW equation [27] and the coupled Boussinesq equations [28]. These equations are converted to ODE with the help of a fractional complex transform approach to a few beneficial formulations of conformable derivatives.

The proposed fractional coupled Boussinesq equations emerge in real applications, for instance, vibrations in nonlinear string and nonlinear framework waves iron sound waves in plasma. The equation given in [29] had been advanced through Hosseini and Ansari. The authors observed its result with the use of the modified Kudryashov method [30], Hoseini *et al.* [31] defined it with the help of exp (-ϕ(ε))-expansion technique. Yaslan and Girgin identified the solution through the use of the first integral method [30]. Additionally, the CMEW equations can be emerged inside the research of drinks flow in depicting the engendering of shallow-water waves in a dynamic framework.

The present study determines the modern solutions to the above mentioned equations with the SGE method. The space–time fractional coupled Boussinesq equation and space–time fractional CMEW equations are yet to be investigated in the use of SGE technique [32,33]. This strategy has the advantage of allowing us to gather extra arbitrary constants and additional types of solutions than the above mentioned technique. It will additionally help numerical solvers in checking the correctness of the outcomes, and it can also explain the instability analysis.

The article has been organized in the following ways: In Section 2, the primary definitions and homes for conformable derivatives have been described. Section 3 illustrates the principle steps of the SGE technique. In Section 4, the precise solutions of fractional Boussinesq equation and fractional CMEW equations are given in detailed mathematical forms as they can be implemented in different commercially available mathematical software tools to see the solutions. In Section 5, the graphical illustration and discussion in different plotting tools are presented; Section 6 has been showing the comparisons of results. Finally, the conclusion of this paper has been described in Section 7.

2. Meaning and foreword

Recently, Khalil et al. [34] in 2014 introduced a new easy obedient definition of the fractional derivative known as conformable derivative. Let us consider, f: [0, ∞) → R, is a function. The order "conformable derivative" of f is defined as [35]:

$$K_\alpha(f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon} \quad (2.1)$$

supposed for all $t > 0, \alpha \in (0, 1)$. If $\lim_{t \to 0^+} f^{(\alpha)}(t)$ be real and f be α-differentiable in the domain nearly$(0, a), a > 0$, then express$f^{(\alpha)}(0) = \lim_{t \to 0^+} f^{(\alpha)}(t)$..

Theorem 1. *Assume* $\alpha \in (0, 1]$ *and at a point* $t > 0$ *as well supposed* f, g *be α-differentiable. Hence.*

- $K_\alpha(xf + yg) = xK_\alpha(f) + yK_\alpha(g)$, for all$x, y \in \mathbb{R}$.
  $K_\alpha(t^z) = ht^{K-\alpha}$, for all$z \in \mathbb{R}$.
  $K_\alpha(u) = 0$, for all constant function$f(t) = u$.
  $K_\alpha(fg) = fK_\alpha(g) + gK_\alpha(f)$.
  $K_\alpha\left(\frac{f}{g}\right) = \frac{gK_\alpha(f) - fK_\alpha(g)}{g^2}$.
  Additionally, if f is differentiable, then$KT_\alpha(f)(t) = t^{1-\alpha}\frac{df}{dt}$.

Some diverse properties just like the chain law, Gronwall's inequality, integration procedures, the Laplace transform the exponential function, and Tailor series expansion in terms of the conformable derivative [35].

Theorem 2. Assume f is an α-differentiable feature in conformable differentiable further presume that g is likewise differentiable and demarcated in variety of f, so that.

$$M_\alpha(f\circ g)(t) = t^{1-\alpha} g'(t) f_g(t). \tag{2.2}$$

3. The Sine-Gordon expansion method

Recently, many analytical and numerical approaches are being advanced with the aid of using many researchers. One of those crucial and powerful methods that can offer precise solutions with significant physical conduct is the SGE technique. The SGE technique is a really useful and crucial device for figuring out closed-shape soliton solutions to FNLEEs. In the study, the SGE technique can be defined to set up standard and wide-spectral soliton solutions to FNLEEs with admire to space and time.

The fractional Sine-Gordon equation of one dimension of the shape is reduced from the conformable shape $u(x,t) = U(\xi)$ with $\xi = a(x - vt^\alpha/\alpha)$ of the classical wave transform [36,37]:.

$$\frac{\partial^2 u}{\partial x^2} - D_t^{2\alpha} u = m^2 sinu, \;\; m \text{ is constant.} \tag{1}$$

to the ODE,

$$\frac{d^2 U}{d\xi^2} = \frac{m^2}{a^2(1-v^2)} sinU, \tag{2}$$

where v denotes velocity of the traveling wave described with inside the transformation. Some generalizations lead to the following form,.

$$\left(\frac{d(U/2)}{d\xi}\right)^2 = \frac{m^2}{a^2(1-v^2)} sin^2 U/2 + C, \tag{3}$$

where C denotes the constant of integration, C is expected zero for simplicity. Let $w(\xi) = U(\xi)/2$ and $b^2 = m^2/(a^2(1-v^2))$. Then (Eq. 3) is converted to.

$$\frac{d(\omega)}{d\xi} = bsin\omega. \tag{4}$$

Set $b = 1$ in (Eq. 4). Then from (Eq. 4) we have two important relations,

$$sin\omega(\xi) = \frac{2de^\xi}{d^2e^{2\xi}+1} = sech\xi, \text{ for } d = 1 \tag{5}$$

$$cos\omega(\xi) = \frac{d^2e^{2\xi}-1}{d^2e^{2\xi}+1} = tanh\xi, \text{ for } d = 1 \tag{6}$$

where d is the constant of integration. The form of the fractional PDE.

$$P\left(u, D_t^\alpha u, u_x, D_{tt}^{2\alpha} u, u_{xx}, ..\right) = 0, \tag{7}$$

can be reduced to an ODE.

$$G = \left(U, U', U'', \cdots\right) = 0. \tag{8}$$

A compatible wave transform is used $u(x,t) = U(\xi)$ where the transform variable ξ is defined as $a(x - vt^\alpha/\alpha)$. Then the predicted solution to (Eq. 8) takes the following form.

$$U(\xi) = A_0 + \sum_{i=1}^{s} tanh^{i-1}(\xi)(B_i sech\xi + A_i tanh\xi), \tag{9}$$

can be written as.

$$U(\omega) = A_0 + \sum_{i=1}^{s} cos^{i-1}(\omega)(B_i sin\omega + A_i cos\omega). \tag{10}$$

Owing to (Eq. 5), (Eq. 6), the process starts by determining index limit s with the aid of homogenous balance of the terms in (Eq. 8). Following the substitution the coefficients of powers of $sin\omega$, $cos\omega$ of the anticipated solution (Eq. 10) into (Eq. 8) are assumed to zero. Next, the ensuing algebraic implement is attempted to be solved for the coefficients$A_0, A_1, B_1, \cdots, a, v$. Then, the solutions are raised, if exists, via way of means of using (Eq. 5)-(Eq. 6) andξ.

4. Investigation of the solutions

In this area, we talk about the solitary wave solutions to the fractional CMEW equation and the fractional coupled Boussnesq equation, which are achieved by using the SGE technique further conformable derivative.

4.1. The space–time fractional CMEW equations

The CMEW equation is primarily based on totally coupled equal width wave equation [38,39], which turned into endorsed through Morrision *et al.* [40], and it is used as a PDE for model one-dimensional wave transmission in nonlinear media through dispersion process. Solitary wave solutions have consisted of all of the modified equations, which are called wave packets or pulses, and they may be all nonlinear wave equations with cubic nonlinearities.

Now provided the space–time fractional CMEW equation, write it down as follows:

$$D_t^\alpha u(x,t) + \varepsilon D_x^\alpha u^3(x,t) - \mu D_{xxt}^{3\alpha} u(x,t) + \varepsilon D_x^\alpha w^2(x,t) = 0, D_t^\alpha w(x,t) + \varepsilon D_t^\alpha w^3(x,t) - \mu D_{xxt}^{3\alpha} u(x,t) = 0. \tag{4.1}$$

in which ε, μ are actual parameters. These equations play a crucial policy in a fluid mechanic that is used as fashions to address a bodily description of its improvement is observed through nonlinear systems.

We consider the following wave transformation for the CMEW equations (Eq. 4.1) as follows:

$$u(x,t) = u(\xi), \xi = k\frac{x^\alpha}{\alpha} - c\frac{t^\alpha}{\alpha}, \tag{4.2}$$

wherein c is the traveling wave's speed. The equation (Eq. 4.1) is decreased to the resulting integer order ordinary differential equation (ODE) through (Eq. 4.2):

$$-cu' + \varepsilon k\left(u^3\right)' + \mu ck^2 u''' + \varepsilon k(w)' = 0,$$

$$-cw' + \varepsilon k\left(w^3\right)' + \mu ck^2 w''' = 0. \tag{4.3}$$

Integrating equation (Eq. 4. 3) with respect to ξ and neglecting the critical steady for lessen the complexity of the solutions we have.

$$-cu + \varepsilon ku^3 + \mu ck^2 u'' + \varepsilon kw^3 = 0,$$

$$-cw + \varepsilon kw^3 + \mu ck^2 w'' = 0. \tag{4.4}$$

According to the precept of balancing, from Eq. 4.4 we gain the balance number 1. Therefore, the solution of (Eq. 4.4) is

$$u(\omega) = G_0 + G_1\cos(\omega) + H_1\sin(\omega),$$

$$w(\omega) = A_0 + A_1\cos(\omega) + B_1\sin(\omega). \quad (4.5)$$

The first, second, and third derivatives of equation (Eq. 4.5) can be written as follows:

$$u^{'}(\omega) = H_1\, cos\omega\, sin\omega - G_1 sin^2\omega. \quad (4.6a)$$

$$u^{''}(\omega) = H_1\,\left[cos^2\omega\, sin\omega - sin^3\omega\right] - 2G_1\, sin^2\omega\, cos\omega. \quad (4.6b)$$

$$w^{'}(\omega) = B_1\, cos\omega\, sin\omega - A_1\, sin^2\omega. \quad (4.6c)$$

$$w^{''}(\omega) = B_1\,\left[cos^2\omega\, sin\omega - sin^3\omega\right] - 2A_1\, sin^2\omega\, cos\omega. \quad (4.6d)$$

Substituting (Eq. 4.5) - (Eq. 4.6d) into (Eq. 4.4) we have.

$$\begin{aligned}&\left(2ck^2\mu H_1 + 3k\varepsilon A_1^2B_1 - k\varepsilon B_1^3 + 3k\varepsilon G_1^2H_1 - k\varepsilon H_1^3\right)\sin(v(\psi))(\cos(v(\psi)))^2\\&+ (6k\varepsilon A_0A_1B_1 + 6k\varepsilon G_0G_1H_1)\sin(v(\psi))\cos(v(\psi))\\&+ \left(-ck^2\mu H_1 + 3k\varepsilon A_0^2B_1 + k\varepsilon B_1^3 + 3k\varepsilon G_0^2H_1 + k\varepsilon H_1^3 - cH_1\right)\sin(v(\psi))\\&+ \left(2ck^2\mu G_1 + k\varepsilon A_1^3 - 3k\varepsilon A_1B_1^2 + k\varepsilon G_1^3 - 3k\varepsilon G_1H_1^2\right)(\cos(v(\psi)))^3\\&+ \left(3k\varepsilon A_0A_1^2 - 3k\varepsilon A_0B_1^2 + 3k\varepsilon G_0G_1^2 - 3k\varepsilon G_0H_1^2\right)(\cos(v(\psi)))^2\\&+ \left(-2ck^2\mu G_1 + 3k\varepsilon A_0^2A_1 + 3k\varepsilon A_1B_1^2 + 3k\varepsilon G_0^2G_1 + 3k\varepsilon G_1H_1^2 - cG_1\right)\cos(v(\psi))\\&+ \varepsilon kA_0^3 + 3\varepsilon kA_0B_1^2 + \varepsilon kG_0^3 + 3\varepsilon kG_0H_1^2 - cG_0 = 0.\end{aligned}$$

and

$$\begin{aligned}&2ck^2\mu B_1 + 3k\varepsilon A_1^2B_1 - k\varepsilon B_1^3)\sin(v(\psi))(\cos(v(\psi)))^2\\&+ 6\varepsilon kA_0A_1B_1\sin(v(\psi))\cos(v(\psi)) + (-ck^2\mu B_1 + 3k\varepsilon A_0^2B_1\\&+ k\varepsilon B_1^3 - cB_1)\sin(v(\psi)) + (2ck^2\mu A_1 + k\varepsilon A_1^3\\&- 3k\varepsilon A_1B_1^2)(\cos(v(\psi)))^3 + (3k\varepsilon A_0A_1^2\\&- 3k\varepsilon A_0B_1^2)(\cos(v(\psi)))^2 + (-2ck^2\mu A_1 + 3k\varepsilon A_0^2A_1\\&+ 3k\varepsilon A_1B_1^2 - cA_1)\cos(v(\psi)) + \varepsilon kA_0^3 + \varepsilon kA_0B_1^2 - cA_0 = 0.\end{aligned}$$

Equating all phrases with the powers of $sin\omega\, cos\omega$ to zero, the subsequent machine may be obtained:

$$2ck^2\mu B_1 + 3k\varepsilon A_1^2B_1 - k\varepsilon B_1^3 = 0.$$

$$6\varepsilon kA_0A_1B_1 = 0.$$

$$-ck^2\mu B_1 + 3k\varepsilon A_0^2B_1 + k\varepsilon B_1^3 - cB_1 = 0.$$

$$2ck^2\mu A_1 + k\varepsilon A_1^3 - 3k\varepsilon A_1B_1^2 = 0.$$

$$3k\varepsilon A_0A_1^2 - 3k\varepsilon A_0B_1^2 = 0.$$

$$-2ck^2\mu A_1 + 3k\varepsilon A_0^2A_1 + 3k\varepsilon A_1B_1^2 - cA_1 = 0.$$

$$\varepsilon kA_0^3 + \varepsilon kA_0B_1^2 - cA_0 = 0.$$

$$2ck^2\mu H_1 + 3k\varepsilon A_1^2B_1 - k\varepsilon B_1^3 + 3k\varepsilon G_1^2H_1 - k\varepsilon H_1^3 = 0.$$

$$6k\varepsilon A_0A_1B_1 + 6k\varepsilon G_0G_1H_1 = 0.$$

$$-ck^2\mu H_1 + 3k\varepsilon A_0^2B_1 + k\varepsilon B_1^3 + 3k\varepsilon G_0^2H_1 + k\varepsilon H_1^3 - cH_1 = 0.$$

$$2ck^2\mu G_1 + k\varepsilon A_1^3 - 3k\varepsilon A_1B_1^2 + k\varepsilon G_1^3 - 3k\varepsilon G_1H_1^2 = 0.$$

$$3k\varepsilon A_0A_1^2 - 3k\varepsilon A_0B_1^2 + 3k\varepsilon G_0G_1^2 - 3k\varepsilon G_0H_1^2 = 0.$$

$$-2ck^2\mu G_1 + 3k\varepsilon A_0^2A_1 + 3k\varepsilon A_1B_1^2 + 3k\varepsilon G_0^2G_1 + 3k\varepsilon G_1H_1^2 - cG_1 = 0.$$

$$\varepsilon kA_0^3 + 3\varepsilon kA_0B_1^2 + \varepsilon kG_0^3 + 3\varepsilon kG_0H_1^2 - cG_0 = 0.$$

we have got the values of the parameter $c, k, G_0, G_1, H_1, A_0, A_1$ and B_1 by solving this system of equations with the help of computational software Maple as follows:

Set 1: $c = -0.755G_1^2\varepsilon\sqrt{-\frac{1}{2\mu}}, k = \sqrt{-\frac{1}{2\mu}}, A_0 = 0,\ A_1 = -0.755, B_1 = 0,\ G_0 = 0,\ G_1 = G_1, H_1 = 0.$

Set 2: $c = -0.377H_1^2\varepsilon\sqrt{\frac{1}{\mu}}, k = \sqrt{\frac{1}{\mu}}, A_0 = 0, A_1 = 0,\ B_1 = -0.755, G_0 = 0,\ G_1 = 0, H_1 = H_1.$

Set 3: $c = 0.755H_1^2\varepsilon\sqrt{-\frac{2}{\mu}}, k = \sqrt{-\frac{2}{\mu}},\ A_0 = 0, A_1 = 0.755i, B_1 = -0.755,\ G_0 = 0, G_1 = -0.495\sqrt{H_1}, H_1 = H_1.$

Case I. *putting the values organized in set* 1 *alongside with* (*Eq.* 4.2) *into solution* (*Eq.* 4.5), *we accomplish*

$$u_1(x,t) = G_1 \times \tanh\left(kx - \frac{ct^\alpha}{\alpha}\right),\ c = -0.755G_1^2\varepsilon\sqrt{-\frac{1}{2\mu}},\ k = \sqrt{-\frac{1}{2\mu}}$$

$$w_1(x,t) = -0.755 \times \tanh\left(kx - \frac{ct^\alpha}{\alpha}\right),\quad c = -0.755G_1^2\varepsilon\sqrt{-\frac{1}{2\mu}},\ k = \sqrt{-\frac{1}{2\mu}}$$

Case II. *Inserting the values organized in set* 2 *and using* (*Eq.* 4.2) *into solution* (*Eq.* 4.5), *we attain.*

$$u_2(x,t) = H_1 \times \text{sech}\left(kx - \frac{ct^\alpha}{\alpha}\right),\ c = -0.377H_1^2\varepsilon\sqrt{\frac{1}{\mu}},\ k = \sqrt{\frac{1}{\mu}}$$

$$w_2(x,t) = -0.755 \times \text{sech}\left(kx - \frac{ct^\alpha}{\alpha}\right),\quad c = -0.377H_1^2\varepsilon\sqrt{\frac{1}{\mu}},\ k = \sqrt{\frac{1}{\mu}}$$

Case III. *Inserting the values organized in set* 3 *and using* (*Eq.* 4.2) *into solution* (*Eq.* 4.5), *we attain*

$$u_3(x,t) = -0.495\sqrt{H_1} \times \tanh\left(kx - \frac{ct^\alpha}{\alpha}\right) + H_1 \times \text{sech}\left(kx - \frac{ct^\alpha}{\alpha}\right),\quad c = 0.755H_1^2\varepsilon\sqrt{-\frac{2}{\mu}},\ k = \sqrt{-\frac{2}{\mu}}$$

$$w_3(x,t) = 0.755i \times \tanh\left(kx - \frac{ct^\alpha}{\alpha}\right) - 0.755 \times \text{sech}\left(kx - \frac{ct^\alpha}{\alpha}\right),\quad c = 0.755H_1^2\varepsilon\sqrt{-\frac{2}{\mu}},\ k = \sqrt{-\frac{2}{\mu}}$$

The solutions derived right here are critical to defining the movement of waves and the manner they travel with inside the media, and are used as a version in PDEs for the replication of one-dimensional wave transformation in nonlinear media with dispersion processes.

4.2. The space–time fractional coupled Boussinesq equations

We want to cope with re-enact the enlargement of floor water waves via a profundity some distance now no longer the even scale, that's the midway coupled Boussinesq equations [30] in presence:

$$D_t^\alpha u(x,t) + D_x^\beta v(x,t) = 0,$$

$$D_t^\alpha v(x,t) + AD_x^\beta\left(u^2(x,t)\right) - ED_{xxx}^{3\beta}u(x,t) = 0;\ 0 < \alpha,\ \beta \le 1, \tag{4.2.1}$$

in which A and E are growing constants and D_t^α is the α order fractional derivative, in which $0 < \alpha < 1$. Boussinesq kind equations may be taken into consideration because dispersive wave propagation, the first model for nonlinear and describe the surface water waves whose horizontal scale is an awful lot large than the intensity of the water.

Introduce the subsequent fractional transformation.

$$u(x,t) = u(\xi), \xi = \frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha}. \tag{4.2.2}$$

Using Eq. (4.2.2) into Eq. (4.2.1), we get.

$$-cu' + v' = 0 \tag{4.2.3a}$$

$$-cv' + A\left(u^2\right)' - Eu''' = 0, \tag{4.2.3b}$$

in which $u' = \frac{du}{d\xi}$, integrating both equation in one time regarding travelling wave variable element ξ and trusting the essential consistent to be zero, we are able to get the underlying forms of the equations.

$$-cu + v = 0 \tag{4.2.4a}$$

$$-cv' + A\left(u^2\right)' - Eu'' = 0. \tag{4.2.4b}$$

From Eq. 4.2.4a, we attain.

$$v = cu. \tag{4.2.5}$$

Using Eq. 4.5 in Eq. 4.4b.

$$-c^2u + Au^2 - Eu'' = 0. \tag{4.2.6}$$

According to the principal of balancing, from Eq. 4.2.6, we achieve the balance number 1. Therefore, the shape of the solution of Eq. 4.2.6 is as follows.

$$u(\omega) = G_0 + G_1\ \cos(\omega) + H_1\ \sin(\omega) + G_2\ cos^2(\omega) + H_2\ \sin(\omega)\ \cos(\omega), \tag{4.2.7}$$

It is easy to find out first and second derivatives from the Eq. 4.2.7, which is necessary for the Eq. 3.2.6 that is demonstrated below.

$$u'(\omega) = -G_1 sin^2(\omega) - 2G_2\cos(\omega)sin^2(\omega) + H_1\sin(\omega)\cos(\omega) + H_2(\sin(\omega) - 2sin^3(\omega))$$

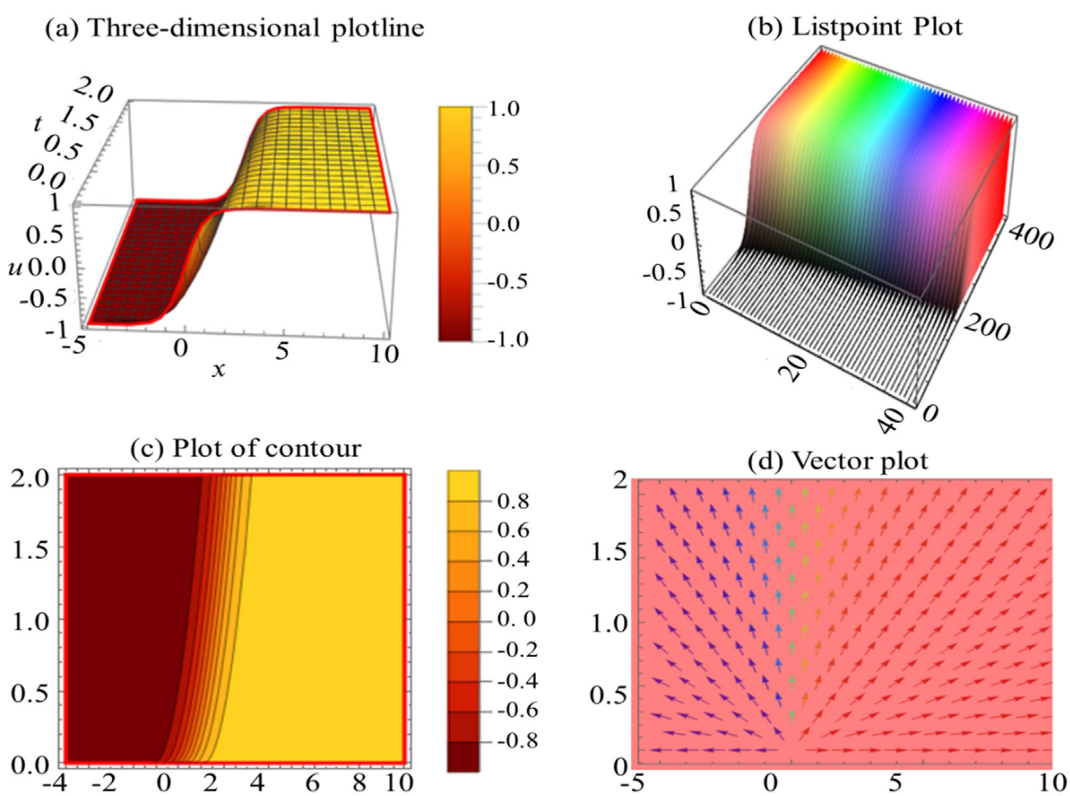

Fig. 1 Kink type wave shape of $u_1(x,t)$ when $G_1 = 1, \varepsilon = 1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ and the intervals $-5 < x < 10$ and $0 < t < 2$ for (a), $-5 < x < 10$ and $0 < t < 20$ for (b), $-5 < x < 10$ and $0 < t < 60$ for (c) and $-5 < x < 10$ and $0 < t < 2$ for (d).

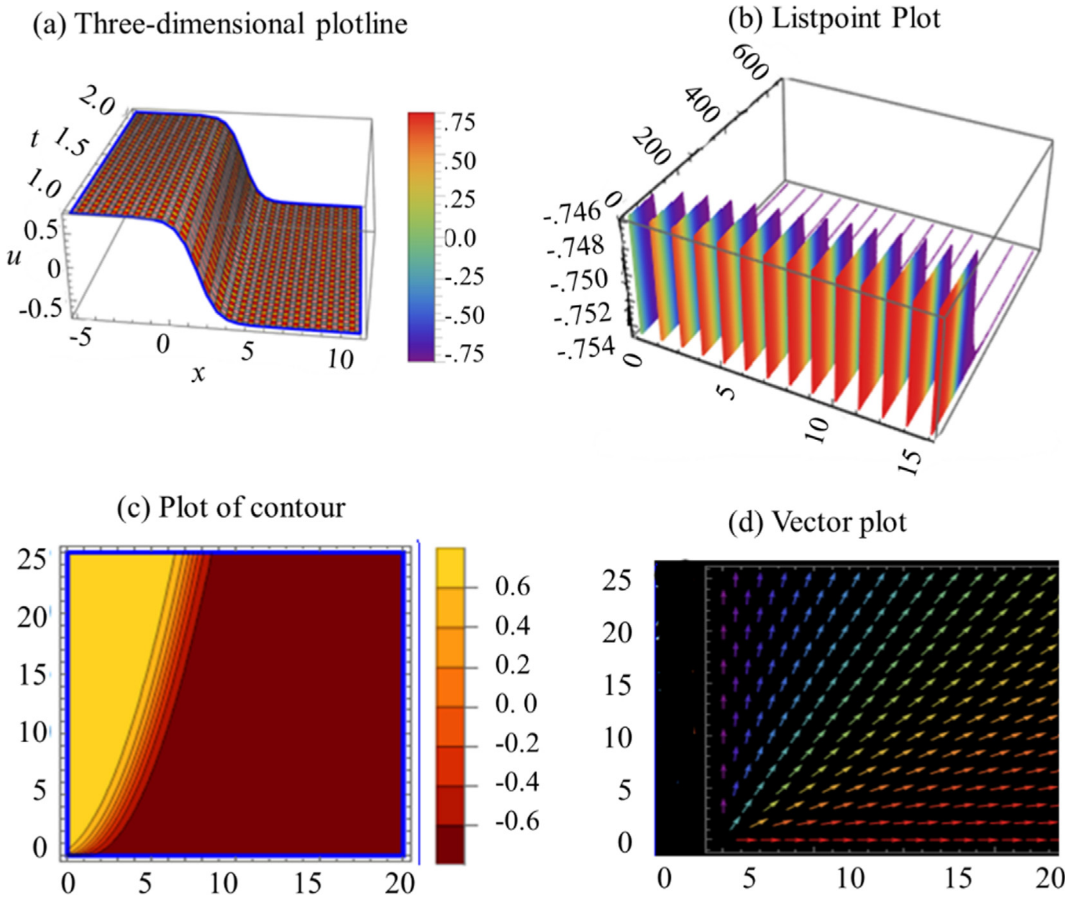

Fig. 2 Kink type wave shape of $w_1(x,t)$ when $H_1 = 1, \varepsilon = 1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ and the intervals $-5.0 < x < 10$ and $1.0 < t < 2.0$ for (a), $0.0 < x < 15$ and $0.0 < t < 5.4$ for (b), $0.0 < x < 20$ and $0.0 < t < 25$ for (c) and $0.0 < x < 20$ and $0.0 < t < 25$ for (d).

$$
\begin{aligned}
u''(\omega) = &-2G_1 sin^2(\omega)\cos(\omega) \\
&+ 2G_2 sin^2(\omega)\left(sin^2(\omega) - 2cos^2(\omega)\right) \\
&+ \left(cos^2(\omega)\sin(\omega) - sin^3(\omega)\right) + H_2\cos(\omega)(\sin(\omega) \\
&- 6sin^3(\omega)
\end{aligned}
$$

Substituting $u(\omega), u'(\omega)$ and $u''(\omega)$ in equation (4.2.6), we accomplish.

$$
\begin{aligned}
&(2AG_2H_2 - 6EH_2)(\cos(v(\sin(v(\psi))\psi)))^3 \\
&+ (2AG_1H_2 + 2AG_2H_1 - 2EH_1)\sin(v(\psi))(\cos(v(\psi)))^2 \\
&+ \left(2AG_0H_2 + 2AG_1H_1 - c^2H_2 + 5EH_2\right)\sin(v(\psi))\cos(v(\psi)) \\
&+ \left(2AG_0H_1 - c^2H_1 + EH_1\right)\sin(v(\psi)) \\
&+ \left(AG_2^2 - AH_2^2 - 6EG_2\right)(\cos(v(\psi)))^4 \\
&+ \left(2AG_0H_1 - c^2H_1 + EH_1\right)(\cos(v(\psi)))^3 \\
&+ \left(2AG_0G_2 + AG_1^2 - AH_1^2 + AH_2^2 - c^2G_2 + 8EG_2\right)(\cos(v(\psi)))^2 \\
&+ \left(2AG_0G_1 + 2AH_1H_2 - c^2G_1 + 2EG_1\right)\cos(v(\psi)) + AG_0^2 \\
&+ AH_1^2 - c^2G_0 - 2EG_2 \\
&= 0
\end{aligned}
$$

Equating all terms with the powers of $sin\omega cos\omega$ to zero, the subsequent system may be obtained.

$$2AG_2H_2 - 6EH_2 = 0$$

$$2AG_1H_2 + 2AG_2H_1 - 2EH_1 = 0$$

$$2AG_0H_2 + 2AG_1H_1 - c^2H_2 + 5EH_2 = 0$$

$$2AG_0H_1 - c^2H_1 + EH_1 = 0$$

$$AG_2^2 - AH_2^2 - 6EG_2 = 0$$

$$2AG_1G_2 - 2AH_1H_2 - 2EG_1 = 0$$

$$2AG_0G_2 + AG_1^2 - AH_1^2 + AH_2^2 - c^2G_2 + 8EG_2 = 0$$

$$2AG_0G_1 + 2AH_1H_2 - c^2G_1 + 2EG_1 = 0$$

$$AG_0^2 + AH_1^2 - c^2G_0 - 2EG_2 = 0$$

With the help of Maple, we find the values of parameter c, G_0, G_1, H_1, G_2 and H_2 by solving this system of equations as follows.

Set 1:.$c = \sqrt{E}, G_0 = -\frac{2E}{A}, G_0 = 0, G_1 = 0, G_2 = \frac{3E}{A}, H_1 = 0, H_2 = \frac{3iE}{A}$

Set 2:.$c = \sqrt{-E}, G_0 = -\frac{3E}{A}, G_0 = 0, G_1 = 0, G_2 = \frac{3E}{A}, H_1 = 0, H_2 = \frac{3iE}{A}$

Case I. *replacing the values of parameters organized in set* 1 *alongside with Eq.* 4.2.2 *into solution of Eq.* 4.2.7, *we accomplish.*

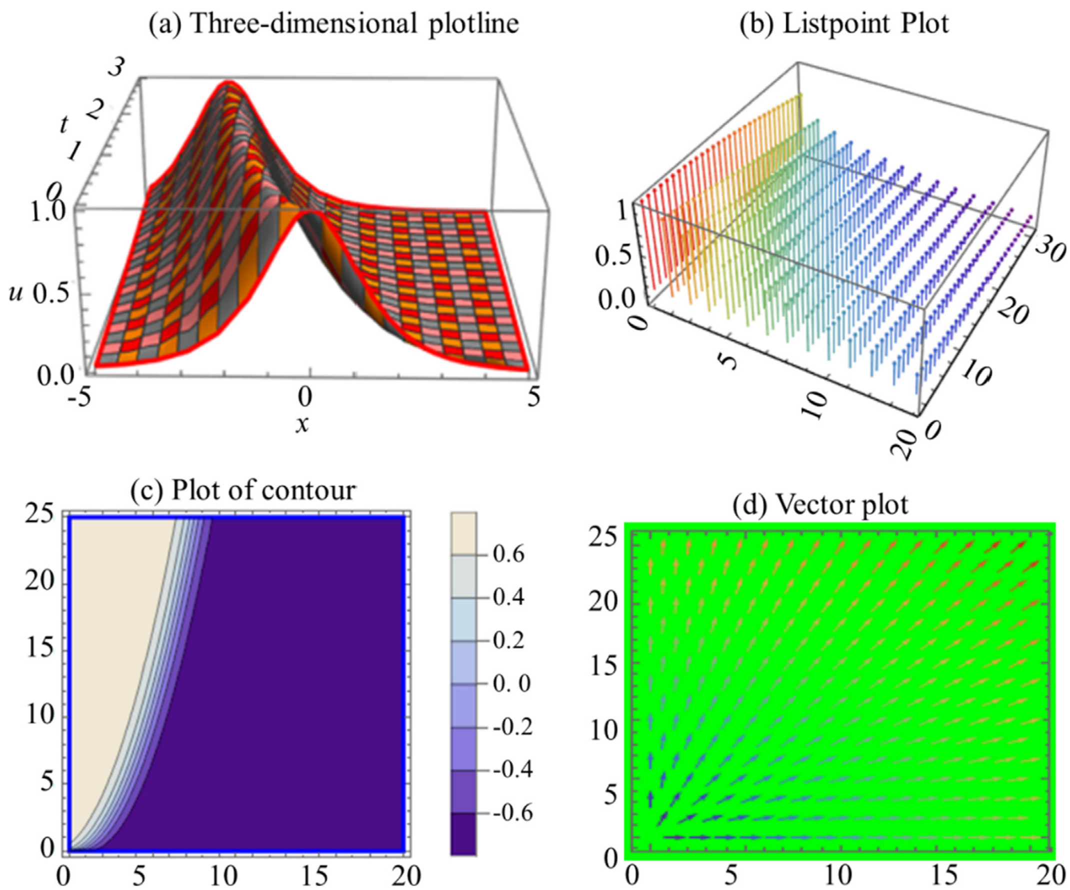

Fig. 3 Bell type wave shape of $u_2(x,t)$ when $H_1 = 1, \varepsilon = -1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ and the intervals $-5.5 < x < 5.5$ and $0.0 < t < 3.0$ for (a), $0.0 < x < 0.9$ and $0.0 < t < 5.4$ for (b) and $0.0 < x < 20$ and $0.0 < t < 25$ for both (c) and (d).

$$u_4(x,t) = -\frac{2E}{A} + \frac{3E}{A} tanh^2\left(x - \frac{ct^\alpha}{\alpha}\right) + \frac{3iE}{A}\tanh\left(x - \frac{ct^\alpha}{\alpha}\right) sech\left(x - \frac{ct^\alpha}{\alpha}\right), c = \sqrt{E}$$

$u_4(x,t)$ also can be written as the following form.

$$u_5(x,t) = -\frac{2E}{A} + \frac{3E}{A}\left(1 - sech^2\left(x - \frac{ct^\alpha}{\alpha}\right)\right) + \frac{3iE}{A}\tanh\left(x - \frac{ct^\alpha}{\alpha}\right) sech\left(x - \frac{ct^\alpha}{\alpha}\right), c = \sqrt{E}$$

Case II. *Substituting the values of parameters arranged in set* 2 *along with Eq.*4.2.2 *into solution Eq.* 4.2.7, *we accomplish.*

$$u_6(x,t) = -\frac{3E}{A} + \frac{3E}{A} tanh^2\left(x - \frac{ct^\alpha}{\alpha}\right) + \frac{3iE}{A}\tanh\left(x - \frac{ct^\alpha}{\alpha}\right) sech\left(x - \frac{ct^\alpha}{\alpha}\right), c = \sqrt{-E}$$

$u_6(x,t)$ also can be written as the following form.

$$u_7(x,t) = -\frac{3E}{A} + \frac{3E}{A}\left(1 - sech^2\left(x - \frac{ct^\alpha}{\alpha}\right)\right) + \frac{3iE}{A}\tanh\left(x - \frac{ct^\alpha}{\alpha}\right) sech\left(x - \frac{ct^\alpha}{\alpha}\right), c = \sqrt{-E}$$

The foregoing solutions could be useful for studying crucial mathematical fashions in nonlinear optics, solid-state physics, vibrations in a nonlinear string, ion sound waves in plasma, describe hydro-magnetic waves in cold plasma among other things.

5. Results and discussion

5.1. Graphical explanation

In this section, we examine the graphical depiction of the solutions derived for the time fractional CMEW problem and the space–time fractional coupled Boussinesq equation, which were generated by using Mathematica. In order to get the solution, we calculated the three-dimensional plotline, the listpoint, contour plotting, and vector plotting of the solution. All these plots depend on individual values of the indicated parameters in the obtained solution. The calculated solutions can be expressed by sketching four types of pictorial portrayals.

Solutions $u_1(x,t)$ and $w_1(x,t)$ in Fig. 1 and Fig. 2 for space–time fractional CMEW equation show kink shape wave solution within the interval $-5.0 < x < 10$ and $0.0 < t < 2.0$ with the values of $G_1 = 1, \varepsilon = 1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ for $u_1(x,t)$ and the interval $-5.0 < x < 10$ and $1.0 < t < 2.0$ with the values $H_1 = 1, \varepsilon = 1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ for $w_1(x,t)$. The most significant point of CMEW equations that were investigated here, is that

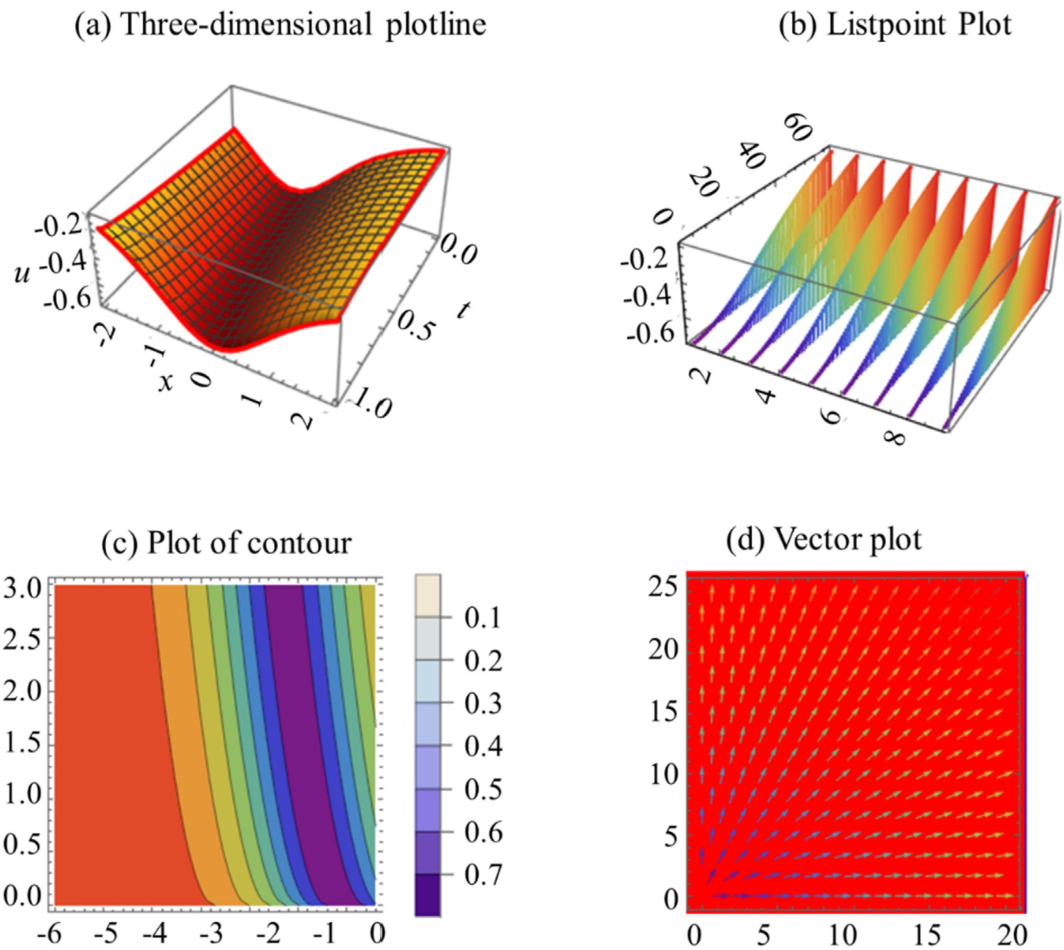

Fig. 4 Anti-bell type wave shape of $w_2(x,t)$ when $H_1 = 1, \varepsilon = 1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ and the intervals $-2.0 < x < 2.0$ and $0.0 < t < 1.0$ for (a), $0 < x < 2.0$ and $0.0 < t < 0.034$ for (b), $-6.0 < x < 0.6$ and $0.0 < t < 3.0$ for (c) and $0.0 < x < 20$ and $0.0 < t < 25$ for (d).

the kink solution, that rises or descends from the effect from one asymptotic position to another that will affect from of the effect on nonlinearity and dispersion.

Solutions $u_2(x,t)$ in Fig. 3 for space–time fractional CMEW equation represents the bell shape wave solution inside the interval $-5 < x < 5$ and $0 < t < 3$ with the values $H_1 = 1, \varepsilon = -1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$. Bell shape wave is a ceaseless flow through no breaks among standards. The top of the shape represents the most probable event in a series of data, while all other possible circumstances are symmetrically distributed around the mean, creating a downward-sloping curve on each side of the peak. The range of the bell shape is described by its standard deviation. Bell shape waves are used generally in statistics, including in analyzing economic and financial data.

The solution $u_2(x,t)$ in Fig. 4 for space–time fractional CMEW equations represents anti-kink wave solution for the values $H_1 = 1, \varepsilon = 1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ with the interval $-2.0 < x < 2.0$ and $0.0 < t < 1.0$. In anti-kink solitons, the velocity does not depend on the wave amplitude.

Fig. 5, Fig. 6 and Fig. 9 represent the single solitons shape solution of $u_3(x,t)$, $w_3(x,t)$ and $w_3(x,t)$ for the values of $H_1 = 1, \varepsilon = 1, \alpha = \frac{1}{4}, \mu = \frac{1}{2}$ with the interval $-1.0 < x < 1.0$ and $0.0 < t < 1.0$, $H_1 = 1, \varepsilon = 1, \alpha = \frac{1}{4}, \mu = \frac{1}{2}$ and the intervals $-100 < x < 100$ and $-10 < t < 10$ and $E = A = -1, \alpha = \frac{1}{2}$ with the interval $0.0 < x < 20$ and $0.0 < t < 25$. Single solitons are the type of solitary wave that has a chaotic nature, which is commonly asymptotic irregularity. Whenever the middle point of the solitary wave is fanciful, the single solitons can be bound by chaotic solitary wave solution.

For the case of space–time fractional coupled Boussinesq equations, Fig. 7 demonstrates singular kink shape wave solution of $u_4(x,t)$ considering the values of $E = A = 1, \alpha = \frac{1}{2}$ and with the interval $-0.1 < x < 20$ and $-0.1 < t < 50$ and Fig. 8 also demonstrates singular kink shape wave solution for the solution of $u_5(x,t)$ considering the values of $E = A = 1, \alpha = \frac{1}{2}$ and with the interval $-0.1 < x < 20$ and$-0.1 < t < 50$. Singular kink solution is another kind of travelling wave solution which comes from infinity as in trigonometry.

For the case of space–time fractional coupled Boussinesq equations, Fig. 10 represents dark soliton shape wave solution of $u_6(x,t)$ considering the values of $H_1 = 1, \varepsilon = -1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ and the values $0.0 < x < 20$ and$0.0 < t < 25$. The dark soliton is a localized surface "wave envelope" that causes a temporary decrease in wave amplitude. A common point of bright solitons is their robustness. This property is very important for ensuring practical applications in optical communications. Also, optical solitons emerge unchanged from collision process. Still, dissipative perturba-

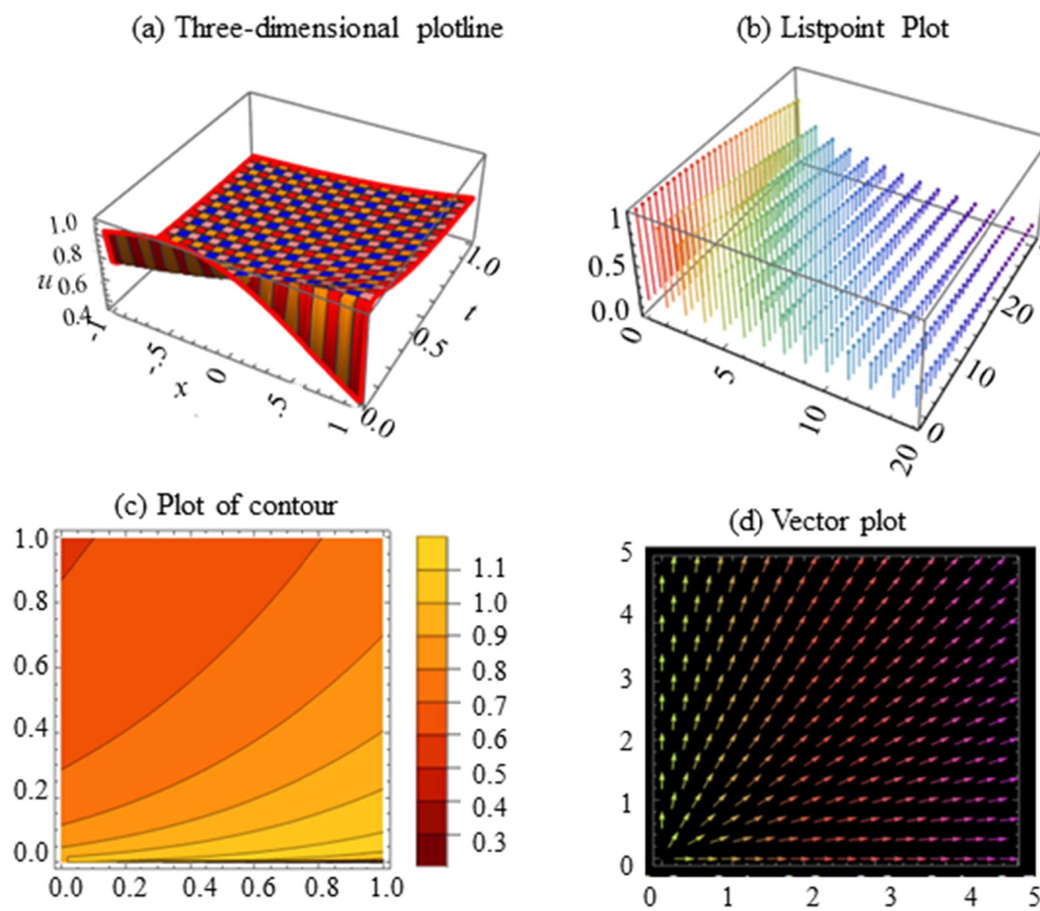

Fig. 5 Single solution type wave shape of $w_3(x,t)$ when $H_1 = 1, \varepsilon = 1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ and the intervals $-1.0 < x < 1.0$ and $0.0 < t < 1.0$ for (a), $0.0 < x < 4.5$ and $0.0 < t < 0.039$ for (b), $0.0 < x < 1.0$ and $0.0 < t < 1.0$ for (c) and $0.0 < x < 0.49$ and $0.0 < t < 5.0$ for (d).

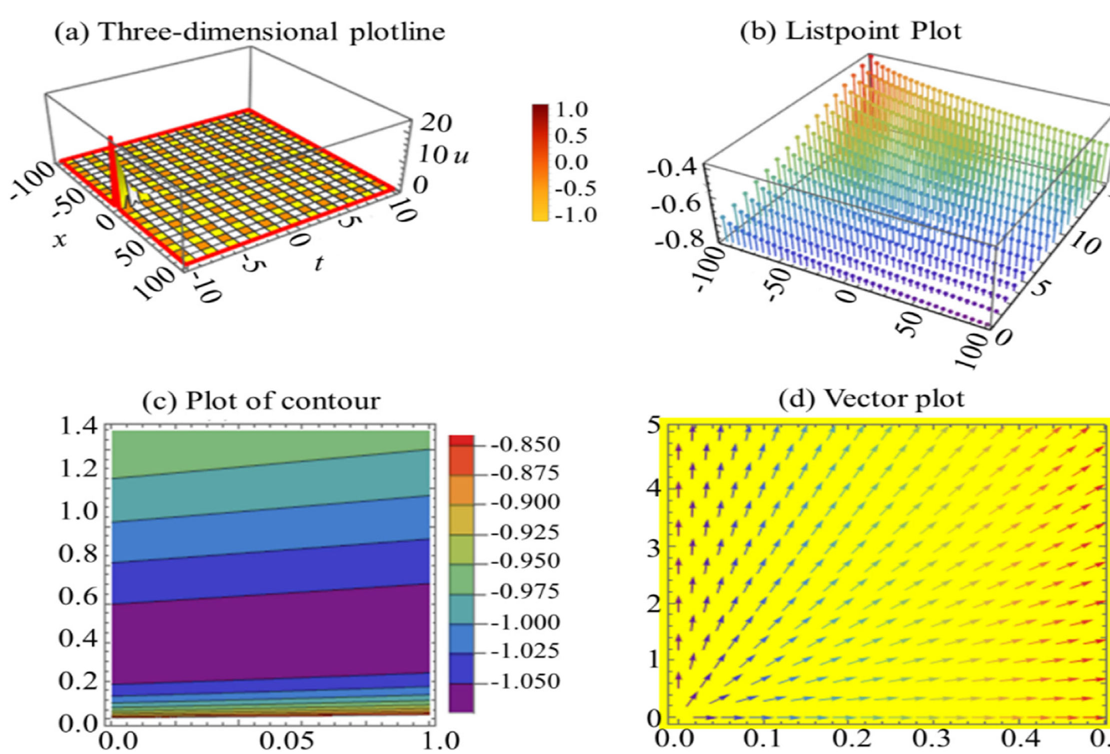

Fig. 6 Single soliton type wave shape of $u_3(x,t)$ when $H_1 = 1, \varepsilon = 1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ and the intervals $-100 < x < 100$ and $-10 < t < 10$ for (a), $0.0 < x < 0.45$ and $0.0 < t < 0.039$ for (b), $0.0 < x < 0.1$ and $0.0 < t < 1.5$ for (c) and $0.0 < x < 0.49$ and $0.0 < t < 5.0$ for (d).

tions similar as material loss or Raman scattering can destroy such solitons.

5.2. Discussion

For specific values of those parameters, we have found few acknowledged solutions such as kink, anti-kink, singular kink, single soliton, dark soliton, bell type, and bell type wave solutions with a variety of free parameters. These free parameters have significance, such as the ability to find specific solutions by changing the free parameter values of an individual solution. The impact of changing a parameter is illustrated in the diagram, and we attempted to describe the situation by providing the low and high range values. By adjusting the parameters, we can see how the wave shapes changes. Our achieved solutions have a significant impact on the field of nonlinear optics [41], solid-state physics [42], vibrations in a nonlinear string, ion sound waves in plasma [43], describe hydromagnetic waves in cold plasma [44], and so on. The activities of solitary waves have been graphically illustrated concerning space and time, revealing the higher efficiency and validity of the underline problem. We can predict space and time dependent solutions to the problem in the future by performing the necessary investigation over the parameters. Therefore, the consequences of the underlying controlling parameters on the different solutions of the problem can be used to predict the advanced method for describing the physical problems.

6. Comparison of the results

In order to verify the solutions from present model, we compare our solution with existing model [45]. The acquired results of the space–time fractional coupled Boussinesq equations by using the SGE method have compared with the results obtained by other scholars using (G'/G)-expansion method. It is fascinating to note that, some of the solutions are strikingly similar to previously developed solutions, while the others yield unique solutions. The following Table 1 provides a correlation between the results by Abazari, R. [45] solutions and the space–time fractional coupled Boussinesq equations as well as the solution of space–time fractional coupled Boussinesq equations.

The solutions in the above table are comparable. If values of arbitrary constants are given, then the solution become indistinguishable. It is critical to recognize the traveling wave solutions and solitary wave solutions of the fractional-coupled Boussinesq equations that are all new with the expection$u_4(x,t), u_6(x,t)$. These solutions are novel and highly significant, which were not previously published. The given equations have been found to be extremely important in solving the above mentioned phenomena.

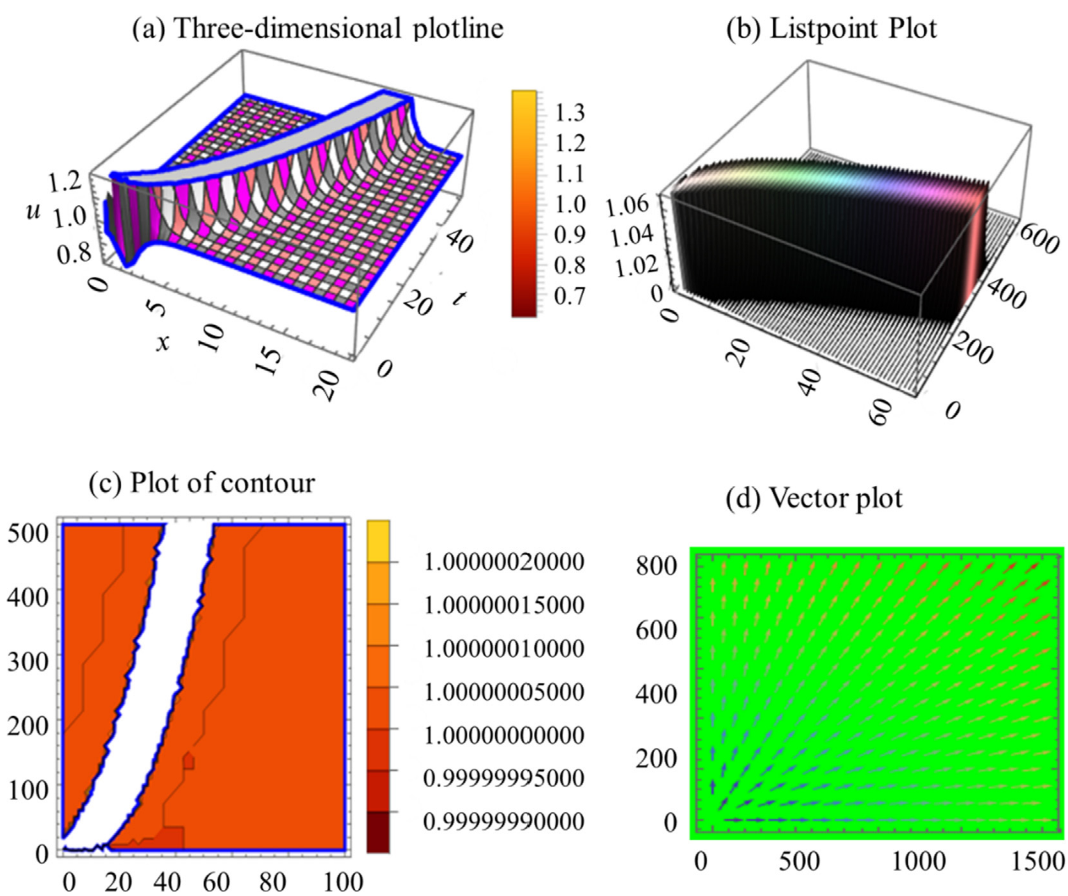

Fig. 7 Singular kink type wave shape of $u_4(x,t)$ when $E = A = 1, \alpha = \frac{1}{2}$ and the intervals $-0.1 < x < 20$ and $-0.1 < t < 50$ for (a), $0.0 < x < 20$ and $0.0 < t < 25$ for (b), $-0.1 < x < 100$ and $-0.1 < t < 50$ for (c) and $0.0 < x < 1500$ and $0.0 < t < 900$ for (d).

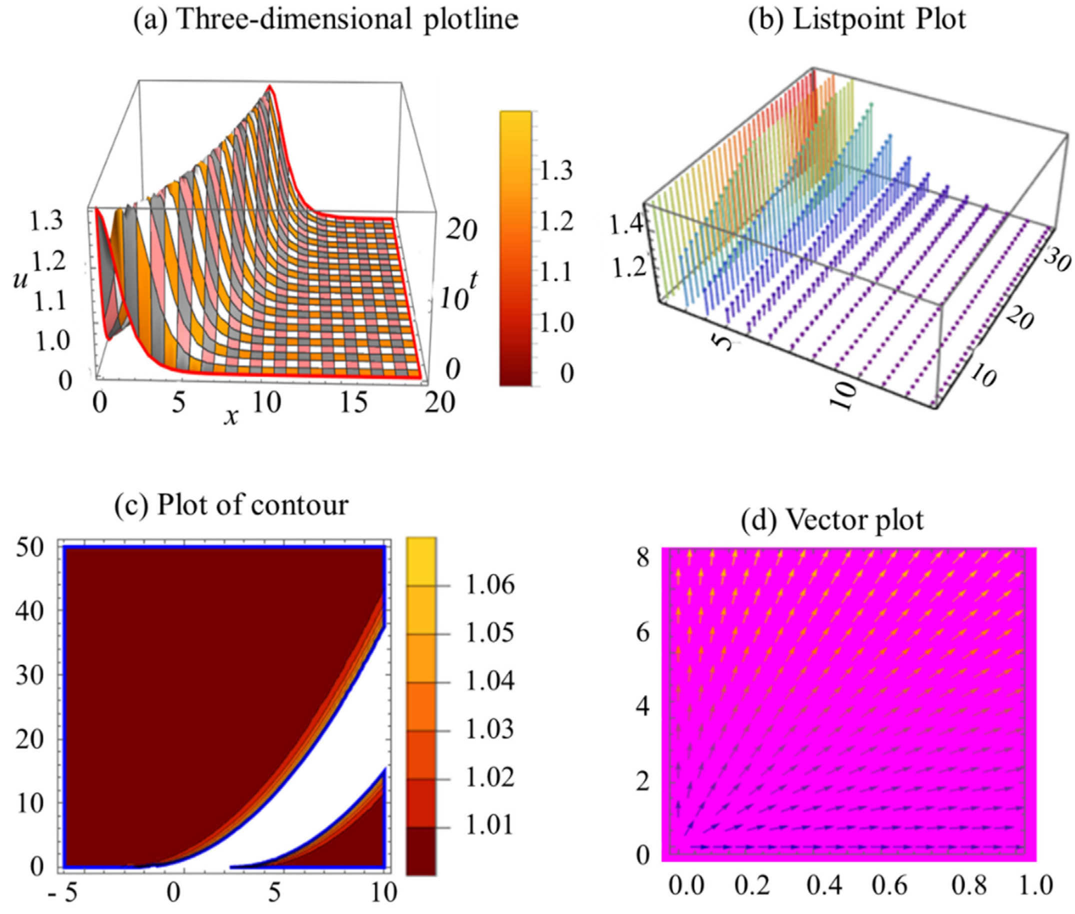

Fig. 8 Singular kink type wave shape of $u_5(x,t)$ when $E = A = 1, \alpha = \frac{1}{2}$ and the intervals $0.0 < x < 20$ and $0.0 < t < 25$ for (a), $0.5 < x < 1.5$ and $0.5 < t < 5.4$ for (b), $-6.0 < x < 10$ and $0.0 < t < 50$ for (c) and $0.0 < x < 1.0$ and $0.0 < t < 9.0$ for (d).

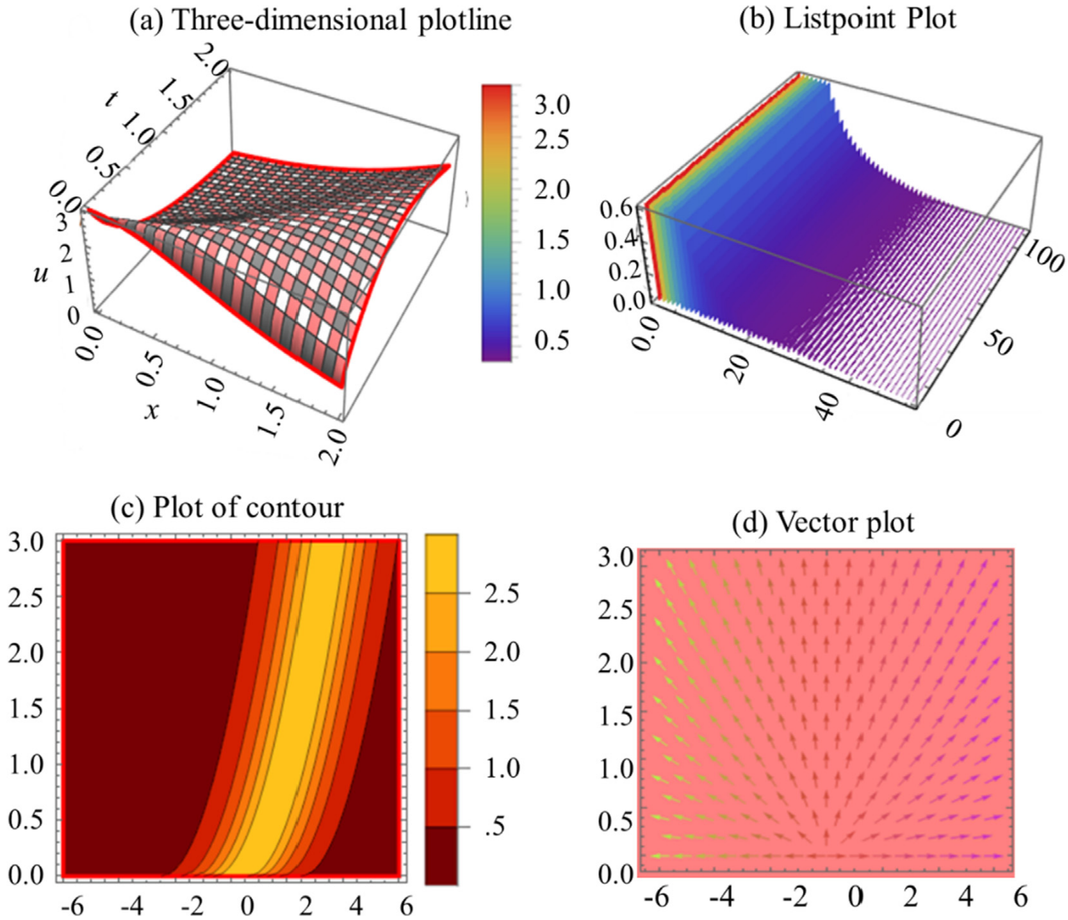

Fig. 9 Single soliton type wave shape of $u_7(x,t)$ when $E = A = -1, \alpha = \frac{1}{2}$ and the intervals $0.0 < x < 2.0$ and $0.0 < t < 2.0$ for (a), $0.0 < x < 0.1$ and $0.0 < t < 10$ for (b), $-6.0 < x < 6.0$ and $0.0 < t < 3.0$ for (c) and $-6.0 < x < 6.0$ and $0.0 < t < 3.0$ for (d).

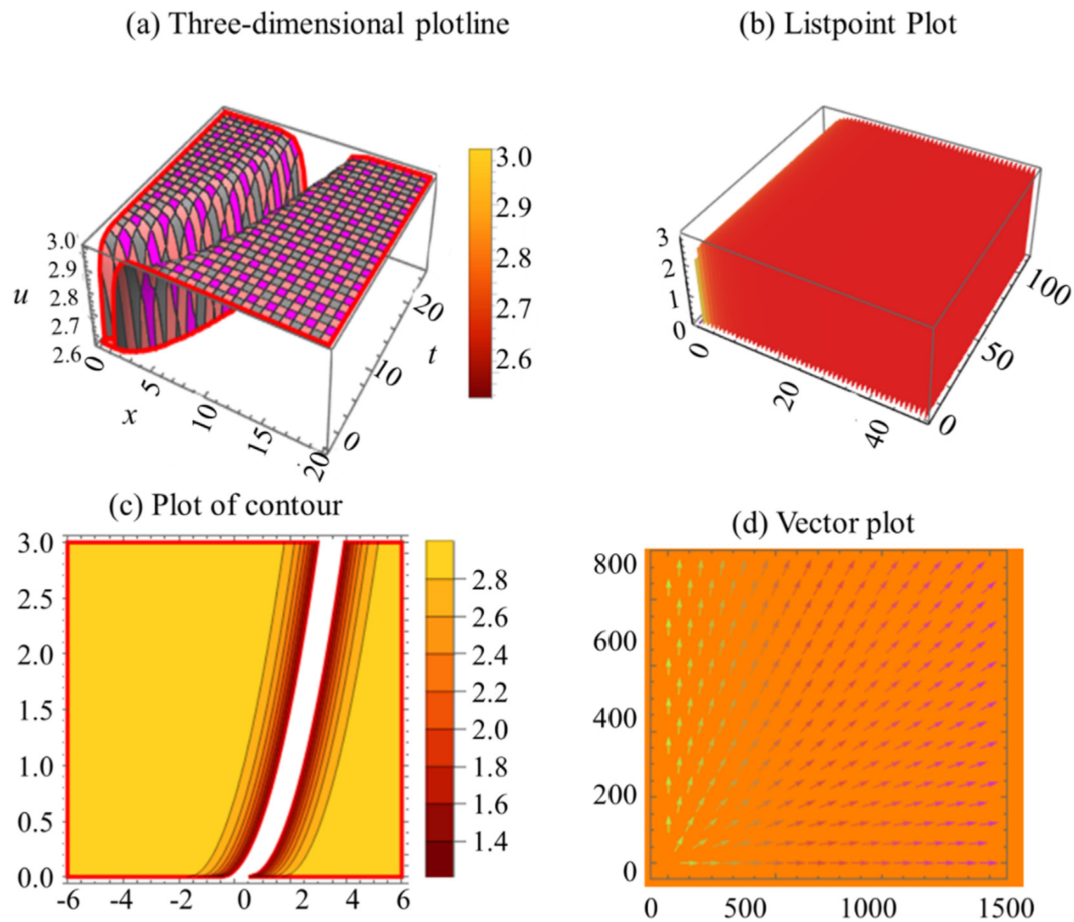

Fig. 10 Anti-bell type wave shape of $u_6(x,t)$ when $H_1 = 1, \varepsilon = -1, \alpha = \frac{1}{2}, \mu = \frac{1}{2}$ and the intervals $0.0 < x < 20$ and $0.0 < t < 25$ for (a), $0.0 < x < 0.1$ and $0.0 < t < 10$ for (b), $-6.0 < x < 6.0$ and $0.0 < t < 3.0$ for (c) and $0.0 < x < 1500$ and $0.0 < t < 900$ for (d).

Table 1 Comparison between Abazari [45] solutions and the obtained solutions of the space–time fractional coupled Boussinesq equations.

Solutions from Abazari [45]	Obtained Solutions
$u_H(x,t) = -\frac{3}{\sqrt{14}}\frac{12\tau-7}{(6\tau-7)\sqrt{\frac{1}{7-6\tau}}} \pm \frac{3}{\sqrt{14}}\frac{12\tau-7}{(6\tau-7)\sqrt{\frac{1}{7-6\tau}}} tanh^2\left(-\frac{k}{2}\sqrt{\frac{12\tau-7}{\tau k^2}}(x - 6\sqrt{\frac{2}{49-42\tau}}\tau t) - \rho_H\right).$	If $i = 0$ then $u_4(x,t) = -\frac{2E}{A} + \frac{3E}{A} tanh^2\left(x - \frac{ct^\alpha}{\alpha}\right).$
If $\tau = 1$ and$k = -2$, then $u_H(x,t) = -\frac{15}{\sqrt{14}} \pm \frac{15}{\sqrt{14}} tanh^2\left(12\sqrt{\frac{5}{4}}\left(x - 6\sqrt{\frac{2}{7}}t\right) - \rho_H\right).$	If $i = 0$ and $E = 1$ then $u_6(x,t) = -\frac{3}{A} + \frac{3}{A} tanh^2\left(x - \frac{ct^\alpha}{\alpha}\right).$
$\eta_H(x,t) = -\frac{3}{2}\frac{12\tau-7}{6\tau-7} + \frac{3}{2}\frac{12\tau-7}{6\tau-7} tanh^2\left(-\frac{k}{2}\sqrt{\frac{12\tau-7}{\tau k^2}}(x - 6\sqrt{\frac{2}{49-42\tau}}\tau t) - \rho_H\right).$	If $i = 0, A = 1$ and $E = 1$ then $u_4(x,t) = -2 + 3tanh^2\left(x - \frac{ct^\alpha}{\alpha}\right).$
If $\tau = 1$ and$k = -2$, then $\eta_H(x,t) = \frac{15}{2} - \frac{15}{2} tanh^2\left(\sqrt{\frac{5}{4}}(x - 6\sqrt{\frac{2}{7}}t) - \rho_H\right).$	$u_6(x,t) = -\frac{3E}{A} + \frac{3E}{A} tanh^2\left(x - \frac{ct^\alpha}{\alpha}\right).$

7. Conclusion

In this investigation, we have brought attention to an extension of the SGE approach to look at nonlinear fractional differential equations withinside the feel of conformable derivatives. Taking the benefit of this extension, the time-fractional CMEW equation and the distance time-fractional coupled Boussinesq equation were investigated. For specific values of those parameters, a few acknowledged kink, anti-kink, singular kink, single soliton, bell kind, and dark soliton wave solutions with a variety of free parameters. These free parameters have significant consequences, such as the ability to find specific solutions by changing the free parameter values of an individual solution. The impact of changing a parameter is illustrated in the diagram and we attempted to describe the situation by providing the low and high range values. By adjusting the parameters, we can see how the wave shape changes. Our achieved solutions have a significant impact on the field of nonlinear optics, solid-state physics, vibrations in a nonlinear string, ion sound waves in plasma, describe hydromagnetic waves in a cold plasma, and so on. The activities of solitary waves have been graphically illustrated concerning space and time, revealing the higher efficiency and validity of the claimed schemes. We can predict future work and perform the necessary tasks by recognizing how space and time changes. Therefore it is essential to know the consequences of changing parameters. Additionally, the underlying solutions could be used to study the proliferation of gravitational waves in seas, tumor growth in human body, crystals model, water waves of surface tension, blood capillaries gravitational force water waves, ion-acoustic waves in serum, etc. It is important to know that the values of unknown parameters can be calculated with the use of symbolic computational software Maple. As a result, the various inventions of exact traveling wave solutions detailed in this study may have significant importance on ocean wave motion and fluid flow research. The mentioned technique is direct, trustworthy, conformable, and effective also provides many novel physical model solutions to NLFPEEs that arise in mathematical physics, applied mathematics, and engineering.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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