Pavement maintenance and rehabilitation planning optimisation under budget and pavement deterioration uncertainty

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Abstract

One of the key parts of a pavement management system is the maintenance and rehabilitation planning. In highway agencies, the plan is usually developed under the assumption that all parameters are known with certainty. In practice, there are various parameters that are afflicted with large uncertainty. Ignoring the uncertainty may lead to a suboptimal plan which can adversely affect the network conditions. The objective of this study is to develop an optimisation framework for network-level pavement maintenance and rehabilitation planning considering the uncertain nature of pavement deterioration and the budget with an applicable approach. A multistage stochastic mixed-integer programming model is proposed to find the optimal plan that is feasible for all possible scenarios of uncertainty and optimise the expectation of objective function. Two case studies of 4 and 21 pavement sections are presented to show the applicability of the proposed method. The value of stochastic solution and the expected value of perfect information which are the indices for evaluating the benefits of using the stochastic model are, respectively, 30% and 85% of the objective function of here and now model for the first case study and 26% and 42% of it regarding the second one. The indices' values are high, which indicate the effectiveness of the stochastic solution.

Keywords: Pavement management, Maintenance, Uncertainty, Integer programming, Stochastic programming

1. Introduction

The development of transport road network plays a vital role in achieving national development, economic development, and growth and brings notable social benefits. The agency and user costs tend to increase with the deterioration of road conditions significantly. Maintenance and rehabilitation (M&R) is the key to the preservation of pavements and ensure they remain productive throughout their lifespan. It is of great importance to manage the M&R strategies time and cost-effectively (Mathew and Isaac, 2014).

One of the main objectives of a pavement management system (PMS) is to select and optimise an efficient M&R plan in the network and project level. At the network level, decisions regarding the budgets and general resource allocations are made, and the overall strategy of the pavement network is determined; while at the project level, the focus is on performance and maintenance of individual and specific sections (Meneses and Ferreira, 2015).

At the network level, a number of mathematical models have been presented for pavement maintenance planning. They have mainly solved the problem under the assumption of deterministic parameters. However, in reality, randomness and uncertainties are inherent and several areas might contain uncertainty in the entire process. Two of the main parameters contributing to the uncertainty in pavement M&R planning problem are the budget and the pavement deterioration.

The budget is subject to economic fluctuations and political interventions and cannot consider as a constant parameter. In our country, Road Maintenance & Transportation Organization (RMTO) (RMTO, 2019) is responsible for budgeting and planning the maintenance strategy of the road network of the country. The budget of RMTO has fluctuated over the years and for example, it has not been increased over the last two years, unlike the predictions. The budget decreased from 1804 billion Iranian Tomans (1 US dollar = 11500 Tomans on Sep 2, 2019) in 2017 to 1680 billion Tomans in 2018, which reflects a 6/8 % decrease in budget. Besides, statistics show a 26% decrease in budget between 2017 and 2016 (RMTO, 2019). Furthermore, splitting the budget between maintenance and other activities like safety measures is another challenge. Hence, the actual allocation of funds to maintenance may deviate from the initial estimate.

In many mathematical models, the pavement deterioration is also considered as a deterministic process. Deterioration of pavement is caused by various factors, including age, environment, traffic, the strength of pavement, and material properties. Many of these factors are not predictable, and on the other hand, the interaction between these factors is not identified. Accordingly, the deterioration of pavement is a probabilistic phenomenon, and a reliable pavement performance model should be used to estimate the pavement condition. Ignoring the random nature of budget and pavement deterioration may provide an inapplicable solution, and it is reasonable to explore the effect of uncertainty on the optimal decision (Zhang and Gao, 2016).

2. Basic Concept and Literature review

Since the early 1980s, several optimisation models have been developed for network-level pavement M&R planning. Generally, there are two dominant approaches in the literature, of which one solves the problem concerning the uncertainty of the parameters, especially uncertainty in pavement deterioration, and the other is based on the assumption that all parameters are deterministically known quantities. The most popular models of the first approach are the models which are based on the theory of Markov Decision Processes (MDP). Integer programming-based models are the prominent models regarding the latter approach.

2.1. MDP-based models

The Markov chain is the most widely used probabilistic approach for modeling the deterioration of pavements. MDP-based models commonly lead to a linear programming problem. In these models, the network sections are divided by their characteristics, and the results are usually expressed as the percentage of each group that should receive each level of treatment (de la Garza *et al.*, 2011). The inability to develop a pavement M&R plan for each section is a disadvantage of these models compared to integer programming models. Additionally, the pavement must be discretised in terms of its condition indicator in the MDP framework, and this procedure could result in reduced accuracy in the solution because pavement condition indicators are mostly continuous (such as pavement condition index, international roughness index).

Golabi et al. (Golabi, Kulkarni and Way, 1982) developed pioneering work in applying MDP for the Arizona PMS. They calculated that the implementation of the M&R optimal plan saved $14 million during the first year. Mbwana et al. (Mbwana and Turnquist, 1996) proposed a large-scale linear program model that is derived from an underlying dynamic programming formulation. The MDP-based models were extended by applying the latent Markov decision process (LMDP) that relaxes the assumptions of error-free facility inspections (Guignier and Madanat, 1999) and developing randomised policies in conjunction with network-level constraints, like condition standards (Smilowitz and Madanat, 2000). Moreover, Wu and Flintsch (Wu and Flintsch, 2009) proposed a linear programming approach for pavement preservation planning that uses multi-objective optimisation.

2.2. Integer programming-based models

Several researchers used approaches based on integer programming (IP) or mixed-integer programming (MIP) for planning optimal M&R strategies. The solution of integer programming addresses the question of when, where, and what M&R treatment should be performed and identifies the best treatment for each section individually. The computational complexity (in general, NP-hard) involved in the IP problems is a challenge, which makes it difficult to solve large-scale problems efficiently (Denysiuk *et al.,* 2017).

Li et al. proposed a cost-effectiveness-based integer M&R planning on a year-by-year basis (Li, Haas and Huot, 1998). The minimum budget requirements for maintaining a prescribed level of the pavement network performance was determined. A sample network of 5 sections was used as a pilot study. Wang et al. established a multi-objective MIP model for M&R planning, including constraints of available annual budgets and minimum requirements on pavement conditions and were applied it to a case study with ten sections (Wang, Zhang and Machemehl, 2003). A MIP model for optimal highway pavement rehabilitation was proposed by Ouyang and Madanat (Ouyang and Madanat, 2004). Two solution approaches, a branch-and-bound algorithm, and a greedy heuristic were presented and evaluated using a network of 3 sections. Chakroborty et al. were presented a binary linear IP formulation for an optimum determination of maintenance strategies for a network of 42 sections (Chakroborty, Agarwal and Das, 2012). A comparative study on the budget allocation between three methods of cost-benefit analysis (CBA), integer programming (IP), and a “decision tree + needs-based” allocation was carried out by Mensah et al. (France-Mensah and O’Brien William, 2018). The findings showed that the IP approach was more effective than others. Karabakal et al., Dahl and Minken and Gao and Zhang used the Lagrangian decomposition method to decompose and solve a large-scale pavement M&R planning problem (Karabakal, Bean and Lohmann, 1994)(Dahl and Minken, 2008)(Gao and Zhang, 2012).

2.3. Stochastic approaches

In mathematical programming, most of the problems are solved in the case of deterministic conditions. In real cases, however, these parameters are subject to uncertainty and may undergo significant variations. Likewise, in the pavement M&R planning problem, the uncertainty in some parameters is considerable and should take into account.

The budget and pavement deterioration are the parameters that mostly contribute to the uncertainty of the problem. The budget allocated to the highway agencies is subject to uncertainty due to various financial and political conditions. If the budget decreases from the original estimate during one or more years of the planning period, some parts of the scheduled maintenance plan either could not be completed or maybe delayed, and rescheduling is therefore needed. It also causes potential pavement condition fluctuations compared with expectations (Al-Amin, 2013). Besides, it is rational to employ a probabilistic approach to model the pavement deterioration process because the predicted pavement performance is sensitive to many factors such as the accuracy of the performance model, traffic load, weather condition, and structural properties and the uncertainty in pavement performance is associated with the reduction in the reliability of pavement M&R plan (Chootinan *et al.*, 2006).

Several optimisation approaches have been introduced to deal with uncertainty, and its consequences include stochastic programming, probabilistic programming, robust optimisation, fuzzy programming, etc. Table 1 shows a summary of the studies on the pavement network and project-level M&R planning considering the uncertainty of important parameters. In Table 10, the parameter of uncertainty, the approach to deal with uncertainty, MDP, or IP based modeling, the level of maintenance, and the case study of the research are presented.

As shown in Table 1, the uncertainty in pavement deterioration and the budget in the network level is often investigated using MDP framework. As was mentioned before, two major drawbacks of this approach are attributed to the discretization of the pavement condition and the inability to provide an M&R plan for each section individually. Furthermore, the uncertainties of the parameters regarding the IP approach have been addressed in a few previous studies. The gaps in the related literature are, firstly, the uncertainty in pavement deterioration has not been studied using stochastic models. Secondly, the previous studies have been considered the total value of the budget as a random variable. Whereas it could be more practical to incorporate the budget uncertainty in a new different manner. In this paper, the budget is divided into two parts, a certain part which is defined as the minimum available budget that can be guaranteed to happen and an uncertain remaining part which represents the budget uncertainty. Therefore, it is intended to fill the gap and consider the uncertainty of pavement performance and the budget using a novel approach in this research.

3. Objective and Scope

The main purpose of this study is to establish a network-level M&R plan considering budget and pavement deterioration uncertainty. To achieve this goal, the problem is formulated as a multistage stochastic mixed-integer programming model. The uncertainty of the parameters is described by a finite number of possible realizations. The multistage stochastic model seeks a solution that is feasible and optimal for all possible parameter choices. In other words, the model aims to find a solution that will perform well on average.

4. Methodology

In this section, steps to establish the methodology are presented. Selecting the pavement condition indicator and identifying the required models to consider in optimisation is the first step. The next steps include developing and solving the pavement M&R mathematical programming model in case of deterministic and stochastic budget and pavement deterioration, respectively. The flowchart in Figure 5 describes the steps adopted for this research. The details of each step are explained in the following. Moreover, the sets, parameters, and variables used in the model are defined in section 3.1.

4.1. Notations

The sets, parameters, and variables used in the model are defined in Table 11.

4.2. Pavement condition indicator and the required models for optimisation

Selecting pavement condition indicator and identifying the required models such as pavement performance model, benefit effectiveness of pavement treatments and M&R treatment unit cost are considered as the prerequisites for the development of any pavement M&R optimisation model. Surface roughness is a key parameter in evaluating pavement deterioration level. Roughness is an important pavement characteristic that affects not only ride quality but also vehicle operating costs, fuel consumption, road safety, and maintenance costs. The International Roughness Index (IRI) is the most commonly employed pavement roughness index and used as the performance indicator in this research. It has been proved there is a good correlation between IRI and distresses (Sandra and Sarkar, 2013).

Several performance models have been proposed to predict IRI. A typical deterministic model for pavement M&R planning is the following model which is presented by Tsunokawa and Schofer (1994) (Tsunokawa and Schofer, 1994).

$$IR_{it^*} = IR_{it^0} \exp(\beta(t^* - t^0) \qquad (1)$$

In fact, the deterioration process is modeled as an exponential function of time. This model is widely used as the performance model in network and project level M&R optimisation (Tsunokawa and Schofer, 1994)(Li and Madanat, 2002)(Ouyang and Madanat, 2004)(Seyedshohadaie, Damnjanovic and Butenko, 2010).

The application of each treatment leads to an increase in pavement condition. Treatment effectiveness in network-level planning mainly refers to the performance jump in pavement condition just upon treatment. Determining the precise degree of effectiveness is a complicated process due to various factors that influence it, such as pretreatment condition, the quality of performing M&R activities, etc. The performance jump can be indicated in average value and estimated through historical data, expert assessment, or as a function of treatment and other variables (Labi and Sinha, 2003).

In addition, the unit cost incurred by the agency is calculated based on the sum of the operational costs of each treatment.

4.3. Pavement M&R planning: deterministic case

Consider a road network $\boldsymbol{I} = \{1,2,3,\dots,\mathrm{I}\}$ with I pavement sections. $\boldsymbol{K} = \{1,2,3,\dots,\mathrm{K}\}$ is defined as the set of M&R treatments, which treatment K is the most effective and expensive one. $\boldsymbol{T} = \{1,2,3,\dots,T\}$ is referred to as the planning horizon as a discrete set of time periods. Pavement sections deteriorate continuously in time due to traffic loading, climatic loading and aging and optimal M&R decisions are made at discrete time points.

The deterministic integer linear programming model for network-level pavement M&R planning is formulated as follows:

$$\text{Minimise } \sum_{\mathrm{i}=1}^{\mathrm{I}} |\mathrm{IR}_{\mathrm{iT}} - \mathrm{IR}_{\mathrm{i}}^{*}| \quad (2)$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} A_i C_{ikt} x_{ikt} \leq B_t \qquad \forall t \in \boldsymbol{T} \quad (3)$$

$$IR_{it} = IR_{i0} \exp(\beta_i t) + \sum_{j=1}^{t} \sum_{k=1}^{K-1} x_{ikt} e_{ik} \exp\big(\beta_i (t-j)\big) + (IR_{new} - IR_{i0} \exp(\beta_i t)) x_{iKt} \ \ \forall i \in \boldsymbol{I}, \forall t \in \boldsymbol{T} \quad (4)$$

$$IR_{it} \geq IR_{min} \qquad \forall i \in \boldsymbol{I}, \forall t \in \boldsymbol{T} \quad (5)$$

$$IR_{it} \leq IR_{max} \qquad \forall i \in \boldsymbol{I}, \forall t \in \boldsymbol{T} \quad (6)$$

$$\overline{IR}_t = \frac{\sum_{i=1}^{I} IR_{it} A_i}{\sum_{i=1}^{I} A_i} \qquad \forall t \in \boldsymbol{T} \quad (7)$$

$$\overline{IR}_t \leq IR_t^{network} \quad (8)$$

$$\forall t \in \boldsymbol{T} \sum_{k=1}^{K} X_{ikt} = 1 \qquad \forall i \in \boldsymbol{I}, \forall t \in \boldsymbol{T} \quad (9)$$

$$X_{ikt} \in \{0,1\} \qquad \forall i \in \boldsymbol{I}, \forall t \in \boldsymbol{T}, \forall k \in \boldsymbol{K} \quad (10)$$

Eq. (2) indicates the objective function of pavement M&R planning, which is to minimise the deviations of the condition of each section at the end of the planning horizon from a defined ideal condition. Eq. (3) guarantees that the total costs of M&R activities in each time period do not exceed the available budget. Eq. (4) calculates the pavement condition of each section over time according to deterioration rate β_i and performance jump e_{ik}. The last term in Eq. (4) represents that in the case of reconstruction, the pavement condition could be considered as a new pavement regardless of its previous condition. Eq. (5) presents the best possible condition of each of all sections in each time period. Eq. (6) ensures that the condition of each section in each time period satisfies a predefined maximum acceptable condition. Eq. (7) calculates the area-weighted average condition of all sections in the network and Eq. (8) states the maximum requirement on this value in each time period. Eq. (9) ensures that only one M&R treatment should be selected for each section in each time point. Equation (10) defines the binary decision variable X_{ikt} of section i at time period t regarding treatment k, which takes the value one if the treatment selected and 0 otherwise.

4.4. Pavement M&R planning: a stochastic approach

The model described in Eqs. (2)-(10) assumes that all parameters are deterministic and known, whereas, in this section, a new stochastic model is proposed to address decision-making under uncertainties. In this paper, a multistage stochastic programming model is developed to deal with the uncertainty of pavement deterioration and budget. The uncertainty is described by a finite number of possible realizations, which is expressed by a scenario tree. Decisions are made at the nodes in a scenario tree, and the branches show realizations of the uncertain variables. Each scenario is a description of a sequence of events and is assigned to a path from the root to leaf in a scenario tree (Shapiro, Dentcheva and Ruszczyski, 2009). Suppose that the uncertainty of both pavement deterioration and budget each year is considered as two possible realizations with the same probability. Thus, the two-year scenario tree of the problem with a totally of 16 scenarios is presented in Figure 6.

In the case of budget uncertainty, the total available budget has been expressed as a random variable in past studies. Practically, it is unclear that after how many months from the beginning of the year, the absolute value of the budget is known because the budget allocation process is time-consuming. For example, suppose that the budget is allocated to a highway agency after the first four months of a year. As a result, it means that decision-makers have to wait for four months until the budget is known, and the optimal M&R treatments regarding the allocated budget and its scenario should be applied. To tackle the problem through a practical approach, the annual budget is divided into two certain and uncertain parts in this research. The certain part (B_t^{min}) is considered as the minimum available budget and a decision variable determines the optimal M&R treatments based on this part of the budget. In fact, the model selects M&R activities with a higher priority. The remaining part of the budget is considered under uncertainty and represented in terms of several scenarios ($\tilde{B}_t^s$). A decision variable aims to find the optimal M&R treatments based on the realization of different scenarios of this additional part.

The uncertainty of pavement deterioration rate is characterised as a set of scenarios (β^s). Consequently, different deterioration rates can be included in the model. For example, in a problem with T stages, if the additional budget and the deterioration rate in stage t respectively associated with M_t and N_t possible states, the total number of scenarios will be equal to $\prod_{t=1}^{T} M_t \times N_t$.

The multistage stochastic mixed-integer programming model of network-level pavement M&R planning problem is formulated by equations (11)-(18).

$$Minimize \sum_{i=1}^{I} \sum_{s=1}^{S} p^s |IR_{i5}^s - IR_i^*| \tag{11}$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} A_i C_{ikt}(x_{ikt} + y_{ikt}^s) \leq B_t^{min} + \tilde{B}_t^s \qquad \forall t \in \boldsymbol{T}, \forall s \in \boldsymbol{S} \tag{12}$$

$$IR_{it}^s = IR_{i0} \exp(\beta^s t) + \sum_{j=1}^{t} \sum_{k=1}^{K-1} (x_{ikt} + y_{ikt}^s) e_{ik} \exp\big(\beta^s (t-j)\big) + (IR_{new} - IR_{i0} \exp(\beta^s t))(x_{iKt} + y_{iKt}^s) \quad \forall i \in \boldsymbol{I}, \forall t \in \boldsymbol{T}, \forall s \in \tag{13}$$

$$IR_{it}^{s} \geq IR_{min} \qquad \forall i \in \boldsymbol{I}, \forall t \in \boldsymbol{T}, \forall s \in \boldsymbol{S} \tag{14}$$

$$IR_{it}^{s} \leq IR_{max} \qquad \forall i \in \boldsymbol{I}, \forall t \in \boldsymbol{T}, \forall s \in \boldsymbol{S} \tag{15}$$

$$\overline{IR}_{t}^{s} = \frac{\sum_{i=1}^{I} IR_{it}^{s} A_i}{\sum_{i=1}^{I} A_i} \qquad \forall t \in \boldsymbol{T},, \forall s \in \boldsymbol{S} \tag{16}$$

$$\overline{IR}_{t}^{s} \leq IR_{t}^{network} \qquad \forall t \in \boldsymbol{T}, \forall s \in \boldsymbol{S} \tag{17}$$

$$\sum_{k=1}^{K}(x_{ikt} + y_{ikt}^{s}) = 1 \qquad \forall i \in \boldsymbol{I}, \forall t \in \boldsymbol{T}, \forall s \in \boldsymbol{S} \tag{18}$$

$$x_{ikt} \in \{0,1\}, y_{ikt}^{s} \in \{0,1\} \qquad \forall i \in \boldsymbol{I}, \forall k \in \boldsymbol{K}, \forall t \in \boldsymbol{T}, \forall s \in \boldsymbol{S} \tag{19}$$

$$y_{ikt}^{m} = y_{ikt}^{n} \qquad \forall i \in \boldsymbol{I}, \forall k \in \boldsymbol{K}, \forall t \in \boldsymbol{T}, \forall n \in \boldsymbol{S}, \forall m \in \boldsymbol{S}, 1 \leq m < n \leq S, \xi_{t}^{m} = \xi_{t}^{n} \tag{20}$$

The objective function of the model is defined in Eq. (12) to minimise the expected deviations of the condition of each section at the end of the planning horizon from a defined ideal condition with regard to the overall space of possible scenarios. The Equations (13) to (19) are similarly developed as the Equations (3) to (10) except that they are defined for each scenario. The binary decision variables x_{ikt} and y_{ikt}^{s} are defined regarding the deterministic and stochastic parts of the problem, respectively. Eq. (20) represent the nonanticipativity constraints, which reflect the fact that decisions made at t^{th} stage need only depend on the information of realised uncertainties up to stage t. In other words, the scenarios that share the same history in each stage cannot be distinguished and should present the same results. Decisions for different scenarios are linked by nonanticipativity constraints. For example, in the scenario tree of Figure 6, it can be found that scenarios 1 to 4 are indistinguishable in the first year because the budget and the deterioration rate of these scenarios are the same in year 1 ($\xi_1^1 = \xi_1^2 = \xi_1^3 = \xi_1^4$). Therefore, the nonanticipativity constraints will require the first four scenarios obtaining the same results in year 1.

5. Numerical Results

To illustrate the methodology proposed and assess the effectiveness of the stochastic programming approach, the results of two pavement network case studies are presented in this section. The computational tasks are performed on a laptop with Intel Core 7 CPU (2.50GHz) and 16.0 GB RAM with General Algebraic Modeling System (GAMS).

5.1. Case study 1

A pavement network of 4 sections with a total area of approximately 168000 m^2 is used as the case study. The pavement distress data collection and analysis procedure was conducted using automated equipment. The required data for the case study were obtained from RMTO database (ORM (Office of Road Maintenance), 2019). The main specifications of the example network are as follows.

5.1.1. Initial condition

As discussed earlier in section 3.2, the International Roughness Index (IRI) is used as the performance indicator in this research. The area, number of lanes, and initial condition of each section are presented in Table *12*.

5.1.2. Planning horizon

The optimisation model in this paper aims to develop pavement M&R planning over a medium-term planning horizon of 5 years. Each road section is assumed to receive only one M&R treatment in each year.

5.1.3. Budget

A budget of 2 billion Toman per year is planned to establish M&R strategies, where the mean decrease of 20% from the predicted value of budget was observed based on the historical data of RMTO database (ORM (Office of Road Maintenance), 2019).

Therefore, two possible realizations of the budget in each year are set to be 2 and 1.6 billion Toman for the stochastic approach. Moreover, the minimum budget (B_t^{min}) is assumed to be 60% of the allocated budget and equal to 1.2 billion Toman. As a result, the minimum budget is estimated to be 1.2 billion Toman, and two possible additional cases of 0.4 and 0.8 billion Toman are considered as the stochastic states of budget.

5.1.4. M&R treatments

Generally, network-level M&R planning requires less detailed distress data than the project level. M&R treatments in this study are grouped into five categories: do nothing, preventive maintenance, rehabilitation type 1, rehabilitation type 2, reconstruction. Do nothing strategy consists of no maintenance works and means maintaining the current condition. The M&R actions attributed to the remaining four categories are as follows:

- Preventive maintenance: Crack sealing, fog seal, chip seal, microsurfacing, slurry seal
- Light rehabilitation: Surface milling, thin HMA overlay, stabilise base and seal, 4 to 6 cm HMA overlay
- Medium rehabilitation: Surface milling and thick HMA overlay, Cold recycling, 8 to 12 cm HMA overlay
- Reconstruction: Replacement of the entire existing pavement structure, reconstruction of the base and surface

The average value of performance jump and the unit cost concerning the application of each M&R treatment are proposed in Table 13. The average value of the performance jump is estimated based on the works of Lu and Tulliver (Lu and Tolliver, 2012)and Paterson (Paterson, 1990). Pavement treatment effectiveness and the average reduction in IRI for preventive maintenance were investigated by Lu and Tulliver (Lu and

Tolliver, 2012). Paterson (Paterson, 1990) developed rehabilitation effectiveness under various pavement thicknesses. In addition, the unit cost of each treatment incurred by the agency was obtained from RMTO database (ORM (Office of Road Maintenance), 2019).

5.1.5. Pavement deterioration rate

Pavement deterioration rate (β) can be calculated from historical data. A constant rate has been often employed for all sections in network-level planning. β in the performance model of equation (1) has been set as 0.05 in several previous studies (see (Li and Madanat, 2002),(Ouyang and Madanat, 2004) and (Seyedshohadaie, Damnjanovic and Butenko, 2010)). In this study, after discussions with RMTO experts, it was concluded that, due to the differences in weather conditions of the sample network in our country compared to the mentioned studies, two deterioration rates of 0.05 and 20% higher than that equal to 0.06 are considered as the possible realizations in each year.

5.1.6. Requirement limits on IRI

The minimum and maximum of IRI in terms of pavement condition of each section in each year is considered 0 and 4, respectively. In addition, the ideal IRI correspond to RMTO objectives is set at 2.2 (RMTO, 2019).

5.1.7. Stochastic programming results

The total number of scenarios in a multistage stochastic programming model for four possible combinations of deterioration rate and budget in each year is equal to 4^5=1024. Assuming all scenarios with equal probability of occurrence, each scenario occurs with probability $9/77 \times 10^{-4}$. The objective function value of the optimal solution found by the stochastic model is equal to 0.365.

The pavement M&R plan in cases of the minimum budget and the uncertainty scenarios of the additional budget and deterioration rate is proposed in Table 14 and Table 15, respectively. For the sake of brevity, the M&R treatment of categories 2 to 5 are listed in the following tables and the years with do-nothing strategy are not presented in the tables.

Table 14 shows which plan is given priority concerning the minimum budget and the selected M&R plan should be applied regardless of the additional budget value. According to Table 14, preventive maintenance in the first year for sections 1 and 2, light rehabilitation in the first year for section 3, and preventive maintenance in the third year for section 4 are selected based on the minimum value of the budget.

The other M&R treatments concerning the uncertainty scenarios of the additional budget and deterioration rate are briefly presented in Table 15. Due to a large number of scenarios, the suggested M&R treatments for each section along with the percentage of occurrence among all scenarios are indicated in Table 15. That is, the values in this table represent the percentage of all scenarios that each M&R treatment is suggested. For example, preventive maintenance in 69% of the scenarios and medium rehabilitation in 31% of them are suggested for section 3 in the second year. The values in Table 15 can identify the importance and priority of the selected M&R treatments. If an M&R treatment in a year is selected regarding 100% of the scenarios, it can be considered equivalent to the selected treatments in Table 14 because it should be performed in all cases regardless of the occurrence of any scenarios. For example, Preventive maintenance for section 4 in the first year is actually required for all scenarios.

The average performance condition of each section over all 1024 scenarios during the planning period is depicted in Figure 7. For example, the average IRI of section 1 decreases from 2.92 in the first year to 2.23 m/km in the last year.

To compare the results of the stochastic with the deterministic solution, the problem is solved based on the expected value (EV) approach. All random parameters are replaced by their expected values in the EV approach, and a deterministic problem is solved. The objective function of the optimal solution of the EV model is 0.014. The solution of the EV model is better than the stochastic solution because the EV model solved the problem under a single scenario in comparison with 1024 scenarios of the stochastic model.

The percent of each treatment assigned to the network sections over the planning horizon regarding the optimal solution of the deterministic and stochastic approach is presented in Table 16. As shown in Table 16, the stochastic model suggested a higher number of preventive maintenance to optimise the network condition fluctuations compared to the deterministic one. The stochastic model provides a solution each year which aims to compensate for any bad effects that might have been experienced as a result of the previous-years decisions. Accordingly, the stochastic model tries to distribute the annual budget to more pavement sections in order to reduce the adverse effects of the changes in the pavement deterioration rate and the budget.

In the following section, the advantage of using the stochastic approach is investigated.

5.1.8. The advantage of using the stochastic approach

There are different methods for modeling multistage stochastic programming problems. The results that have so far been discussed in section 5.2. are corresponded to the so-called here and now (HN) decision. The here-and-now method reflects the fact that decisions are made before perfect information is acquired. The solution to this method

(z_{HN}) is not optimal for any outcome, but it is the best for many outcomes considered altogether. The other methods, such as EV and wait-and-see (WS), generally seek to transform the stochastic problem into a deterministic one.

It is mentioned that the EV model is expressed by replacing the random parameters by their expected values. Suppose that $\mathrm{x_{EV}}$ is an optimal solution to the EV problem. To evaluate the performance of $\mathrm{x_{EV}}$ for each scenario, it is needed to fix the values of the first stage decision variables of the EV problem in the HN problem. After solving the HN, the expected result of using the EV solution (EEV) will be estimated according to Eq. (21).

$$z_{EEV} = \sum_{s \in S} p^s z(x_{EV}, s) \quad (21)$$

z_{EEV} investigates the solution of the next stages as a function of x_{EV} and each scenario. The value of z_{EEV} for the case study is obtained 0.471. The value of a stochastic solution (VSS) is a measure that allows assessing the advantage of using a stochastic programming approach. It is calculated by Eq. (22):

$$VSS = z_{EEV} - z_{HN} \quad (22)$$

The VSS indicates the cost associated with ignoring uncertainty in the problem and measures the expected gain from solving a stochastic model. A high amount of VSS shows that uncertainty is important for the optimal solution and utilising stochastic model is necessary. VSS is obtained as $VSS = 0.471 - 0.365 = 0.106$ in the case study which is approximately 30% of z_{HN} and represents the benefit of using a stochastic model for the problem.

In the WS approach, the decision-maker is allowed to make optimal decisions after observing the realization of random parameters. It can be calculated by solving the optimisation problem for each scenario, one by one and taking the mean value of all the deterministic solutions. If the optimal solution of the model corresponding to the s^th^ scenario is z^{*s}, z_{WS} will be obtained according to Eq. (23):

$$z_{WS} = \sum_{s \in S} p^s \, z^{*s} \tag{23}$$

Another index for evaluating the advantage of stochastic modeling is the expected value of perfect information (EVPI). EVPI indicates that a decision-maker would be willing to spend in gaining full information about the future. It is calculated using Eq. (24):

$$EVPI = z_{HN} - z_{WS} \tag{24}$$

The large EVPI represents the value of the information about the future and indicates that there is a risk of variability in the expected objective value without perfect information. For the case study, z_{WS} is obtained 0.061. As a result, EVPI is equal to 0.365-0.061=0.304. It is approximately 85% of z_{HN} which demonstrates the high cost of ignoring the uncertainty.

5.2. Case study 2

To explore the applicability of the proposed methodology more, another case study with 21 pavement sections, a planning period of 4 years, and 5 M&R alternatives is adopted. It is worth mentioning that the computational complexity of the proposed stochastic MIP model increases exponentially with the number of pavement sections and scenarios. Direct methods are inadequate for large-scale networks with a large number of scenarios. Various decomposition methods have been applied in such problems to deal with this limitation, and it might be of interest in further research studies.

The data for the case study was taken from RMTO like the first case study and it was collected with automatic equipment (ORM (Office of Road Maintenance), 2019).

5.2.1. Input data

A network of 21 pavement sections of primary roads is taken into account. The minimum, maximum, and average value of the area and initial condition of the network sections are 11031, 195216 and 65485 m^2 and 2.38, 5.09, and 3.61 m/km, respectively. The planning horizon is assumed to be 4 years. The estimated budget is approximately 7 billion Toman per year, according to the RMTO database (ORM (Office of Road Maintenance), 2019). But, similar to the previous example, the budget is considered to be stochastic and will be 7 billion or 20% less than it (5.6 billion). The minimum budget (B_t^{min}) is assumed to be 60% of the total assigned budget and equal to 4.2 billion Toman. Ergo, the minimum budget is evaluated to be 4.2 billion Toman, and two possible additional cases of 1.4 and 2.8 billion Toman are the stochastic states of the budget. Other required data, including the types of M&R treatments, Pavement deterioration rate, and its possible states and the requirement limits on IRI are the same as described for the first case study.

5.2.2. Stochastic programming results and the advantage of using the proposed method

In this case study, it is assumed that there are four possible combinations of the budget and pavement deterioration each year, which yields a total of 4^4=256 scenarios with equal probabilities for this problem. The objective function value of the stochastic problem (HN) is equal to 5.561.

Table 8 indicates a brief overview of the optimum M&R plan proposed by the stochastic model. The numbers in Table 8 indicate the percent of each M&R treatment selected for the network sections over all scenarios in each year during the planning period. For example, in the third year, the model assigned 33.3, 29, 32.9, and 4.8 percent of all possible alternatives to do nothing, preventive maintenance, light rehabilitation, and medium rehabilitation, respectively.

The optimal value of the EV model for the second case study is equal to 2.75. Similar to the first case study, the objective function of the EV solution is much better than the HN one because the HN model considers all scenarios and contains the nonanticipativity constraints, whereas the EV model only considers the average values of the stochastic parameters. The percent of each M&R treatment planned for the network sections over all scenarios during the planning period concerning the deterministic (EV) and stochastic (HN) solution is shown in Table 9. The results presented in Table 9 are roughly consistent with the results in the previous case study that the stochastic model aims to distribute the annual budget to more pavement sections through selecting more preventive maintenance (41.7%) compared with the deterministic case (26.9%). The variation of average IRI of all pavement sections during the planning period for the deterministic and stochastic approach is illustrated in Figure 4. As shown in Figure 4, despite incorporating all uncertainty scenarios, the stochastic model could improve the average condition of the network efficiently toward the ideal condition.

To investigate the advantage of using the stochastic model, the EEV and WS models are solved. The optimal solutions of the EEV and WS models are equal to 6.99 and 3.21, respectively. Therefore, VSS is equal to 6.99-5.56=1.43 which is 26% of z_{HN} and denotes that uncertainty significantly affects the optimal solution. Moreover, EVPI is equal to 5.56-3.21=2.35 that is 42% of z_{HN}. The large value of EVPI indicates a high additional profit when perfect information is reached.

Conclusion

Developing an effective M&R strategy plays a significant role in modern PMS. There are various mathematical models for solving the pavement M&R planning problem. Most of the models deal with the deterministic situation. In fact, there are several uncertain parameters in the model, which could greatly affect the optimal strategy. But, few stochastic versions of the models have been proposed to match a real-world situation. A multistage stochastic mixed-integer programming model for pavement M&R planning was established in this paper to address the uncertainty of pavement deterioration and the budget.

The uncertainty of pavement deterioration was represented by a set of distinct realizations of the deterioration rate in the performance function. The budget was divided into two parts; one is associated with the minimum value of the budget, which is considered deterministic, and the M&R activities with the highest priority are selected based on this part. The other is the remaining part of the budget which is denoted as a number of possible realizations, and a decision variable finds the optimal M&R treatments according to the realization of different additional budget scenarios. The following conclusions could be drawn from the results of the present study:

- Stochastic programming approach can be used to account for the uncertainty of pavement deterioration and budget and develop optimal pavement M&R plan for each section individually which could be highly useful for decision-makers
- Two pavement network case studies with 4 and 21 sections are presented as an application of the proposed methodology. It is showed that the stochastic model selected more number of preventive maintenance, 40.55% of total M&R actions over the planning horizon compared to 30% regarding the deterministic model for the first case study and likewise, 41.71% versus 26.9% of total M&R treatments for the second case study. It seems that the stochastic model aims to divide the available budget between more pavement sections to mitigate the negative effects of the uncertainty in the parameters concerning future years.
- The benefits of using a stochastic model compared to a deterministic approach investigated using VSS and EVPI indices. The VSS values for the two case studies

are 30% and 26% of z_{HN}, respectively. Additionally, the EVPI values are 85% and 42% of z_{HN} for the first and second case studies, respectively. The relatively large values of the indices demonstrate that the solutions of the multistage stochastic programming model, instead of the underlying deterministic one, are useful.

It is important to make the point that the high computational complexity of the mixed-integer programming models hinders the practical application of the proposed stochastic pavement M&R planning model for large-scale networks. Some decomposition techniques have been proposed to cope with the computational complexity of the problems with a large-scale dimension. It would be valuable for future research to develop an algorithm to overcome this limitation so that the results would be more aligned with real networks with a large number of sections.

References

Al-Amin, M. (2013) *Impact of budget uncertainty on network-level pavement condition : a robust optimization approach*. The University of Texas at Austin.

Chakroborty, P., Agarwal, P. K. and Das, A. (2012) 'Comprehensive pavement maintenance strategies for road networks through optimal allocation of resources', *Transportation Planning and Technology*. Routledge, 35(3), pp. 317–339. doi: 10.1080/03081060.2012.671044.

Chootinan, P. *et al.* (2006) 'A multi-year pavement maintenance program using a stochastic simulation-based genetic algorithm approach', *Chootinan, P. et al. (2006) 'A multi-year pavement maintenance program using a stochastic simulation-based genetic algorithm approach', Transportation Research Part A: Policy and Practice. Pergamon, 40(9), pp. 725–743. doi: 10.1016/J.TRA.2005.12.003.Trans*. Pergamon, 40(9), pp. 725–743. doi: 10.1016/J.TRA.2005.12.003.

Dahl, G. and Minken, H. (2008) 'Methods based on discrete optimization for finding road network rehabilitation strategies', *Computers & Operations Research*. Pergamon, 35(7), pp. 2193–2208. doi: 10.1016/J.COR.2006.10.015.

Denysiuk, R. *et al.* (2017) 'Two-stage multiobjective optimization of maintenance scheduling for pavements', *Journal of Infrastructure Systems*, 23(3). doi: 10.1061/(ASCE)IS.1943-555X.0000355.

Fan, (David)Wei and Wang, F. (2014) 'Managing Pavement Maintenance and Rehabilitation Projects under Budget Uncertainties', *Journal of Transportation Systems Engineering and Information Technology*, 14(6), pp. 92–100. doi: 10.1016/S1570-6672(13)60145-2.

France-Mensah, J. and O'Brien William, J. (2018) 'Budget Allocation Models for Pavement Maintenance and Rehabilitation: Comparative Case Study', *Journal of Management in Engineering*. American Society of Civil Engineers, 34(2), p. 5018002. doi: 10.1061/(ASCE)ME.1943-5479.0000599.

Gao, L., Guo, R. and Zhang, Z. (2011) 'An augmented Lagrangian decomposition approach for infrastructure maintenance and rehabilitation decisions under budget uncertainty', *Structure and Infrastructure Engineering*. doi: 10.1080/15732479.2011.557388.

Gao, L. and Zhang, Z. (2009) 'Robust Optimization for Managing Pavement Maintenance and Rehabilitation', *Transportation Research Record: Journal of the Transportation Research Board*, 2084(1), pp. 55–61. doi: 10.3141/2084-07.

Gao, L. and Zhang, Z. (2012) 'An Approximation Approach to Large-Scale Pavement Maintenance and Rehabilitation Problem', *Transportation Research Record: Journal of the Transportation Research Board*. Transportation Research Board, 2304, pp. 1–14. doi: 10.3141/2304-13.

Golabi, K., Kulkarni, R. B. and Way, G. B. (1982) 'A Statewide Pavement Management System', *Interfaces*. INFORMS, 12(6), pp. 5–21. Available at: http://www.jstor.org/stable/25060344.
Guignier, F. and Madanat, S. (1999) 'Optimization of infrastructure systems maintenance and improvement policies', *Journal of Infrastructure Systems*, 5(4), pp. 124–134. doi: 10.1061/(ASCE)1076-0342(1999)5:4(124).
Karabakal, N., Bean, J. and Lohmann, J. (1994) *Scheduling pavement maintenance with deterministic deterioration and budget constraints*.
Kuhn, K. D. and Madanat, S. M. (2005) 'Model uncertainty and the management of a system of infrastructure facilities', *Transportation Research Part C: Emerging Technologies*, 13(5–6). doi: 10.1016/j.trc.2006.02.001.
de la Garza, J. M. *et al.* (2011) 'Network-level optimization of pavement maintenance renewal strategies', *Advanced Engineering Informatics*. Elsevier, 25(4), pp. 699–712. doi: 10.1016/J.AEI.2011.08.002.
Labi, S. and Sinha, K. C. (2003) 'Measures of short-term effectiveness of highway pavement maintenance', *Journal of Transportation Engineering*, 129(6), pp. 673–683. doi: 10.1061/(ASCE)0733-947X(2003)129:6(673).
Li, N., Haas, R. and Huot, M. (1998) 'Integer programming of maintenance and rehabilitation treatments for pavement networks', *Transportation Research Record*, (1629), pp. 242–248. doi: 10.3141/1629-27.
Li, Y. and Madanat, S. (2002) 'A steady-state solution for the optimal pavement resurfacing problem', *Transportation Research Part A: Policy and Practice*. Pergamon, 36(6), pp. 525–535. doi: 10.1016/S0965-8564(01)00020-9.
Lu, P. and Tolliver, D. (2012) 'Pavement treatment short-term effectiveness in IRI change using long-term pavement program data', *Journal of Transportation Engineering*, 138(11), pp. 1297–1302. doi: 10.1061/(ASCE)TE.1943-5436.0000446.
Mathew, B. S. and Isaac, K. P. (2014) 'Optimisation of maintenance strategy for rural road network using genetic algorithm', *International Journal of Pavement Engineering*, 15(4), pp. 352–360. doi: 10.1080/10298436.2013.806807.
Mbwana, J. R. and Turnquist, M. A. (1996) 'Optimization modeling for enhanced network-level pavement management system', *Transportation Research Record*, (1524).
Meneses, S. and Ferreira, A. (2015) 'Flexible pavement maintenance programming considering the minimisation of maintenance and rehabilitation costs and the maximisation of the residual value of pavements', *International Journal of Pavement Engineering*. Taylor & Francis, 16(7), pp. 571–586. doi: 10.1080/10298436.2014.943207.
Ng, M., Zhang, Z. and Travis Waller, S. (2011) 'The price of uncertainty in pavement infrastructure management planning: An integer programming approach', *Transportation Research Part C: Emerging Technologies*, 19(6), pp. 1326–1338. doi: 10.1016/j.trc.2011.03.003.
ORM (Office of Road Maintenance) (2019) *Roads information [Data set], Iran's Road Maintenance and Transportation Organization (RMTO), Tehran, Iran.*
Ouyang, Y. and Madanat, S. (2004) 'Optimal scheduling of rehabilitation activities for multiple pavement facilities: Exact and approximate solutions', *Transportation Research Part A: Policy and Practice*, 38(5), pp. 347–365. doi: 10.1016/j.tra.2003.10.007.
Paterson, W. D. O. (1990) 'Quantifying the effectiveness of pavement maintenance and rehabilitation', in *Proceedings at the 6th REAAA Conference*. Kuala Lumpur, Malaysia.
RMTO (2019) *Iran's Road Maintenance and Transportation Organization (RMTO), Tehran, Iran.* Available at: http://www.rmto.ir (Accessed: 20 September 2019).
Sandra, A. K. and Sarkar, A. K. (2013) 'Development of a model for estimating International Roughness Index from pavement distresses', *International Journal of Pavement Engineering*. Taylor & Francis, 14(8), pp. 715–724. doi: 10.1080/10298436.2012.703322.
Seyedshohadaie, S. R., Damnjanovic, I. and Butenko, S. (2010) 'Risk-based maintenance and rehabilitation decisions for transportation infrastructure networks', *Transportation Research Part A: Policy and Practice*, 44(4), pp. 236–248. doi: 10.1016/j.tra.2010.01.005.
Shapiro, A., Dentcheva, D. and Ruszczyski, A. (2009) *Lectures on Stochastic Programming, MOS-SIAM Series on Optimization*. Society for Industrial and Applied Mathematics. doi: doi:10.1137/1.9780898718751.
Smilowitz, K. and Madanat, S. (2000) 'Optimal inspection and maintenance policies for infrastructure networks', *Computer-Aided Civil and Infrastructure Engineering*, 15(1), pp. 5–13.
Tsunokawa, K. and Schofer, J. L. (1994) 'Trend curve optimal control model for highway pavement maintenance: Case study and evaluation', *Transportation Research Part A: Policy and Practice*. Pergamon, 28(2), pp. 151–166. doi: 10.1016/0965-8564(94)90035-3.
Wang, F., Zhang, Z. and Machemehl, R. (2003) 'Decision-Making Problem for Managing Pavement Maintenance and Rehabilitation Projects', *Transportation Research Record: Journal of the*

Transportation Research Board, 1853(1853), pp. 21–28. doi: 10.3141/1853-03.
Wu, Z. and Flintsch, G. W. (2009) 'Pavement Preservation Optimization Considering Multiple Objectives and Budget Variability', *Journal of Transportation Engineering*, 135(5), pp. 305–315. doi: 10.1061/(ASCE)TE.1943-5436.0000006.
Zhang, Z. and Gao, L. (2016) 'A nested modelling approach to infrastructure performance characterisation', *International Journal of Pavement Engineering*, 8436(April), pp. 1–7. doi: 10.1080/10298436.2016.1172712.

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Table 10. A summary of the studies on pavement network and project-level M&R planning considering the uncertainty of important parameters

Research	Parameter of uncertainty	The approach to deal with uncertainty	MDP or IP based modeling	level of maintenance	case study
Kuhn and Madanat (2005) (Kuhn and Madanat, 2005)	Transition probabilities	Robust Optimisation	MDP	Network	A network of 8360 square meters
Gao and Zhang (2008) (Gao and Zhang, 2009)	The variables in linear performance prediction and maintenance effect models	Robust Optimisation	IP	Project	-
Wu and Flintsch (2009) (Wu and Flintsch, 2009)	Budget	probabilistic programming	MDP	Network	A network of 16,000 lane km
Seyedshohadaie et al. (2010) (Seyedshohadaie, Damnjanovic and Butenko, 2010)	Pavement deterioration rate	Optimisation under risk constraints	MDP	Network	20 pavement sections
Ng et al. (2011) (Ng, Zhang and Travis Waller, 2011)	Pavement deterioration rate	Robust Optimisation	IP	Project	-
Gao et al. (2011) (Gao, Guo and Zhang, 2011)	Budget	Stochastic programming	MDP	Network	A network of 16,400 lane km
Al-Amin (2013) (Al-Amin, 2013)	Budget	Robust Optimisation	IP	Network	10 Pavement Sections
Fan and Wang (2014) (Fan and Wang, 2014)	Budget	Stochastic programming	IP	Network	10 Pavement sections

Table 11. The sets, parameters and variables

Sets/Indices	
$\boldsymbol{I}$	Set of pavement network sections, $\boldsymbol{I} = \{1,2,3, \dots, \mathrm{I}\}$
$\boldsymbol{T}$	Set of planning horizon, $\boldsymbol{T} = \{1,2,3, \dots, \mathrm{T}\}$
$\boldsymbol{K}$	Set of M&R treatments, $\boldsymbol{K} = \{1,2,3, \dots, \mathrm{K}\}$, treatment K is the most effective and expensive
$\boldsymbol{S}$	Set of scenarios, $\boldsymbol{S} = \{1,2,3, \dots, \mathrm{S}\}$
Parameters	
$\boldsymbol{IR_{i0}}$	Initial IRI of section i
$\boldsymbol{IR_{it}}$	IRI for section i at time period t
$\boldsymbol{IR_{iT}}$	IRI of section i at the end of the planning horizon (Tth ear)
$\boldsymbol{IR_{min}}$	Minimum possible condition of each of all sections during each time period
$\boldsymbol{IR_{max}}$	Maximum acceptable condition of each of all sections during each time period
$\boldsymbol{IR_{new}}$	condition of pavement after reconstruction
$\boldsymbol{IR_i^*}$	Ideal condition at the end of the planning horizon
$\boldsymbol{\overline{IR}_t}$	Average condition of the network at time period t
$\boldsymbol{IR_t^{network}}$	Maximum possible average condition of the network sections
$\boldsymbol{A_i}$	Area of section i
$\boldsymbol{\beta_i}$	Deterioration rate of section i
$\boldsymbol{\beta_i^s}$	Deterioration rate of section I for scenario s
$\boldsymbol{t^*}$	Discrete time, taken to be a year, $\boldsymbol{t^*} = 1,2,3, \dots, T$
$\boldsymbol{t^0}$	The beginning of the planning horizon
$\boldsymbol{B_t}$	Budget at time period t
$\boldsymbol{B_t^{min}}$	The minimum available budget at time period t
$\boldsymbol{B_t^s}$	The realization of the additional part of the budget at time period t for scenario s
$\boldsymbol{p^s}$	The probability of occurrence of scenario s, $\sum_{s \in S} p^s = 1$
$\boldsymbol{\xi_t^s}$	The realizations of the stochastic process until time period t for scenario s
$\boldsymbol{C_{ikt}}$	Average maintenance unit cost of applying treatment k to section i at time period t
$\boldsymbol{e_{ik}}$	Performance jump of section i because of applying treatment k
$\boldsymbol{N_{ik}}$	Maximum allowable number of treatment k can be applied to section i during the planning horizon
Variables	
$\boldsymbol{x_{ikt}}$	The binary decision variable of section i at time period t regarding treatment k, valued at 1 if selected, 0 otherwise;
$\boldsymbol{y_{ikt}^s}$	The binary decision variable of section i at time period t regarding treatment k for scenario s, valued at 1 if selected and 0 otherwise.

Table 12. Initial IRI of pavement sections

Pavement section	Number of lanes	Area (m^2)	Initial IRI (m/km)
1	3	38771	3.55
2	3	48759	2.41
3	3	75360	4.32
4	3	43305	2.81

Table 13. Performance jump and the unit cost of each M&R treatment

Maintenance and rehabilitation treatment	Cost (Toman/m^2)	Performance jump (m/km)
1	0	0
2	5000	0.3
3	15000	1.2
4	32000	2
5	65000	Restore pavement condition to its original condition, $\mathrm{IRI}_{\mathrm{new}} = 1.5$

Table 14. M&R plan in case of the minimum budget for the first case study (x_{ikt})

Section.Year	M&R treatment	
	2	3
1.1	1	0
2.1	1	0
3.1	0	1
4.3	1	0

Table 15. The percent of selected M&R treatments in case of the uncertainty scenarios for the first case study (y_{itk}^{s})

Section 1		Section 2		Section 3		Section 4	
Year.M&R	Percent of senarios	Year.M&R	Percent of senarios	Year.M&R	Percent of senarios	Year.M&R	Percent of senarios
2.3	50	2.2	63	2.2	69	1.2	100
3.2	14	3.2	31	2.4	31	2.2	75
3.3	23	4.2	61	3.2	17	4.2	26
4.2	3	5.2	39	3.3	3	5.2	6
4.3	14			3.4	39		
5.2	1			4.2	5		
5.3	13			4.3	4		
				4.4	15		
				5.2	1		
				5.3	6		
				5.4	8		

Table 16. The percent of each treatment assigned to the network sections over the planning horizon

Treatment	**Deterministic**	**Stochastic**
Do nothing	50	44.15
Preventive maintenance	30	40.55
Light rehabilitation	15	10.65
Medium rehabilitation	5	4.65
Reconstruction	0	0

Table 17. The percent of each M&R treatment selected for the network sections over all scenarios in each year

Year	Do nothing	Preventive maintenance	Light rehabilitation	Medium rehabilitation	Reconstruction
1	33.3	29.0	32.9	4.8	0
2	39.6	47.0	13.4	0	0
3	36.2	43.7	19.6	0.5	0
4	40.2	47.1	11.8	0.9	0

Table 18. The percent of each M&R treatment during the planning period concerning the deterministic and stochastic solution

Treatment	**Deterministic**	**Stochastic**
Do nothing	48.33	37.31
Preventive maintenance	26.90	41.71
Light rehabilitation	21.90	19.44
Medium rehabilitation	2.86	1.54
Reconstruction	0.00	0.00

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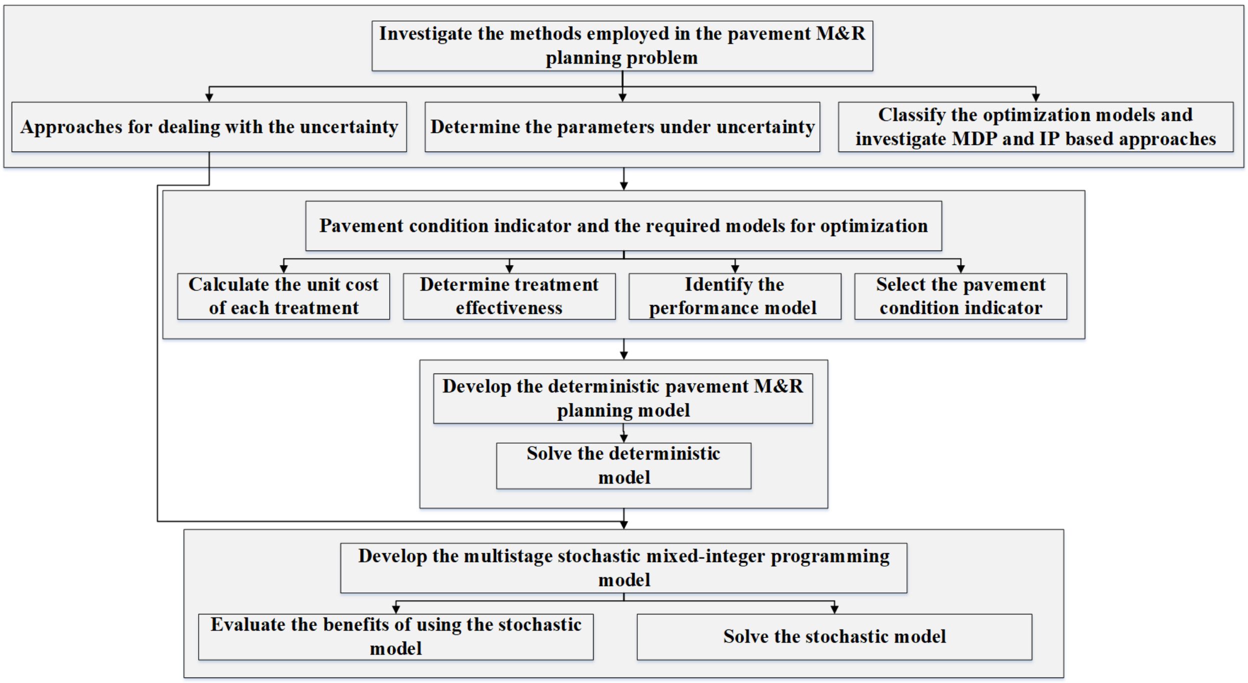

Figure 5. The steps adopted for this research

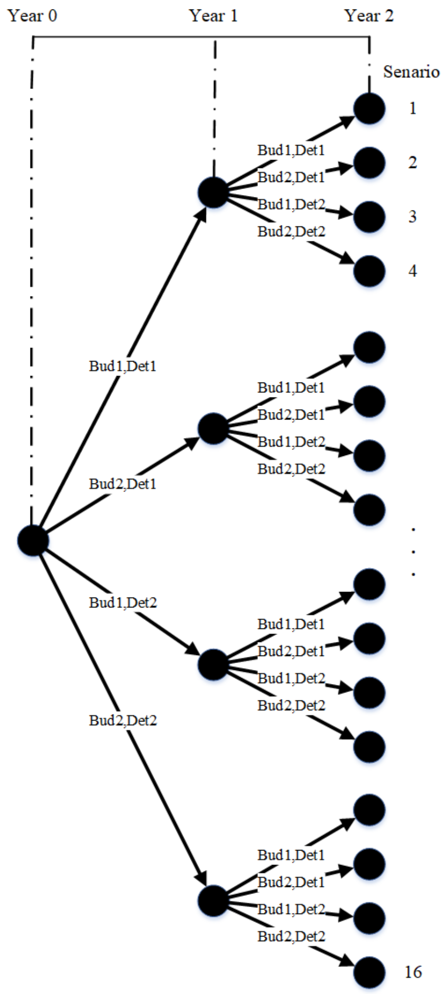

Figure 6. Two-year scenario tree under the uncertainty of pavement deterioration and budget

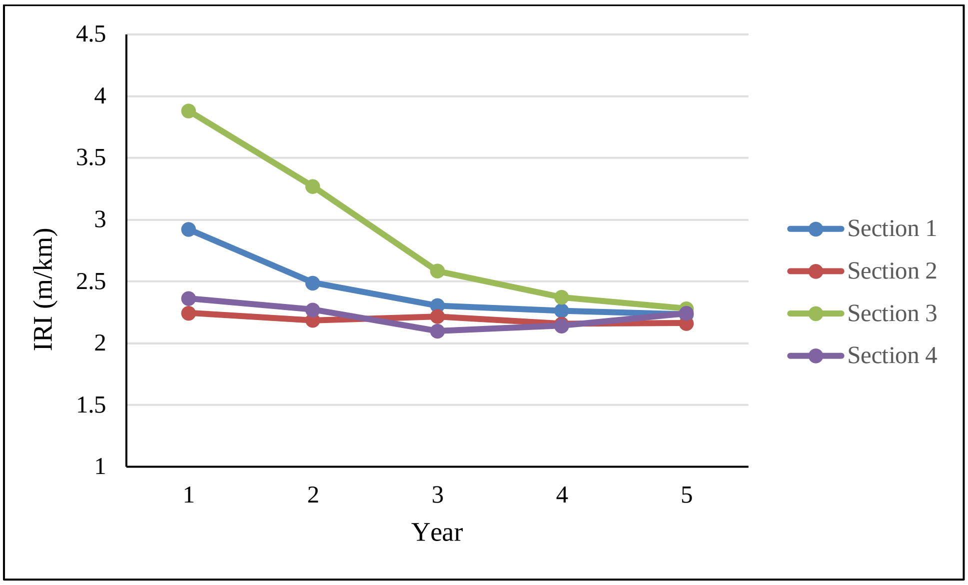

Figure 7. The average IRI of each section over all 1024 scenarios during the planning period

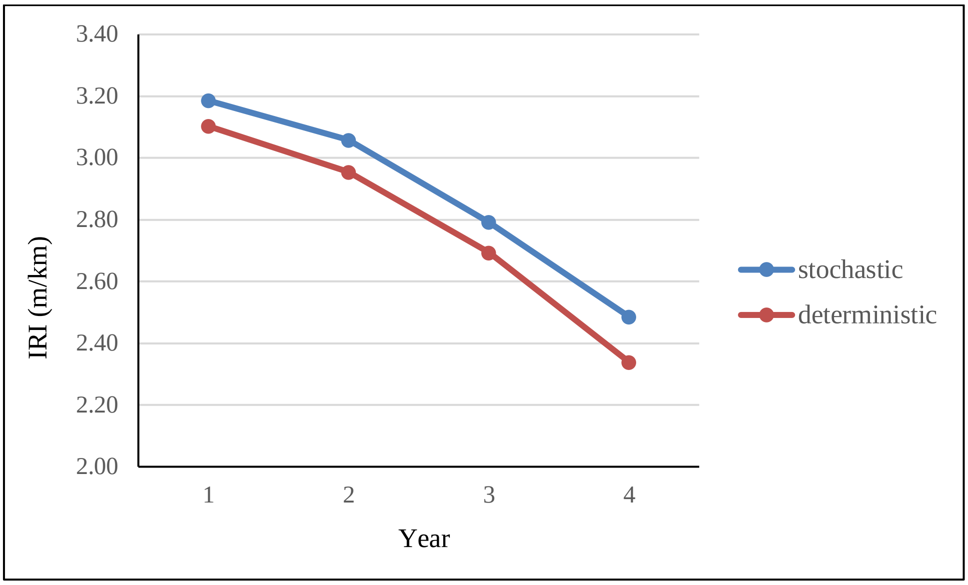

Figure 8. The average IRI of all pavement sections during the planning period for the deterministic and stochastic approach