Efficient Market Design with Distributional Objectives

Isa E. Hafalir
University of Technology Sydney, Australia

Fuhito Kojima
University of Tokyo, Japan

M. Bumin Yenmez
Boston College, United States

Abstract

In a matching market with distributional objectives, we study the existence of mechanisms that weakly improve the distributional objective (compared to a status-quo matching), and satisfy constrained efficiency, individual rationality, and strategy-proofness.

Our model is grounded in real-world applications including school choice, civil servant matching, teacher assignment, medical residency matching, daycare assignment, and labor markets. In these settings, agent types might be defined by characteristics such as ethnicity, gender, race, and socioeconomic status. We define the distribution of a matching as a matrix that indicates the number of each agent type at every institution and the distributional objective as a function on distributions that indicates the desirability of distributions in terms of a policy.

We first observe that some distributional objectives might not allow for a mechanism that improves the distributional objective while also satisfying constrained efficiency, individual rationality, and strategy-proofness. We then introduce a mechanism based on the top-trading cycles algorithm (Shapley and Scarf, 1974) that satisfies these properties if the distributional objective is pseudo M$^\natural$-concave, a notion of concavity for discrete functions that we introduce.

Intuitively, pseudo M$^\natural$-concavity requires that when we move from two distributions towards each other, the minimum value of the distributional objective does not decrease. We show that a distributional objective is pseudo M$^\natural$-concave if and only if its upper contour sets satisfy a notion of discrete convexity called M$^\natural$-convexity. We explore conditions under which a distributional objective is pseudo M$^\natural$-concave, including scenarios where the objective is the negative distance to a set of ideal distributions using either the Chebyshev distance or the discrete metric.

Lastly, we offer several important examples of pseudo M$^\natural$-concave distributional objectives, such as quota policies in worker-exchange market, type-specific floors and ceilings to promote diversity in school choice problems, and districts requiring the number of students assigned to them not to decrease in the interdistrict school choice problem.