3 Mortgage loans, debt default and the emergence of banking crises

3.1 Mortgage and banking crises

The current financial crisis in the USA can be characterised by a fast expansion of mortgage loans to households in order to purchase real estate. A difference to the traditional Minskyan crisis and the previous type of financial crisis is that borrowers are not firms but households. The Japanese crisis was characterised by firms and banks borrowing to invest in real estate, and the East Asian crisis was characterised by firms and banks borrowing foreign-denominated debt. In the current worldwide crisis, financial fragility that has led borrowers from hedge to speculative and even Ponzi positions involves the dynamics of household income and interest payments.

There are three main reasons explaining the large increase in mortgage credit: competition between financial institutions; the interaction between real estate prices and credit constraints; financial innovations. These three elements have induced banks to relax screening and monitoring of borrowers and to increase the quantity of credit supplied to households. First, in the aftermath of the crash of the dotcom bubble in 2000 and in an environment characterised by low interest rates, banks, under the pressure of financial intermediaries such as hedge funds, found in mortgage debt a highly profitable business. Second, increasing real estate prices contributed to the relaxation of credit rationing. The increasing value of collateral has reduced default risk and has led banks to increase credit, in line with the financial accelerator model. Third, financial engineering is central to understanding why banks have reduced the screening and monitoring of borrowers.

Securitisation has been the major financial innovation. In 2006, securitisation concerned 87 per cent of prime mortgages and 75 per cent of subprime mortgages (Ashcraft and Schuermann, 2007). Securitisation aims at transforming loans into liquid assets in order to transfer the credit risk to the market. Mortgages with different qualities are bundled together and the resulting financial products are usually sold to the market through special purpose vehicles (SPVs) in the form of collateralised debt obligations (CDOs) or residential mortgage-backed securities (RMBSs). The belief was that credit risk was transferred to the market-led banks to expand the supply of credit, leading to a worsening of the average quality of loans. In addition, securitisation reduced the

need of banks to increase their own funds following credit expansion. In a traditional banking sector, banks are exposed to credit risk and must provision reserves to cover the risk accordingly. As risks were transferred to the market, banks were not forced to provision for them. Although securitisation was supposed to reduce the exposure of banks to risk, it in fact induced banks to take on more risk.

As a result, low income households were able to access credit, the so-called subprime mortgages. While subprime borrowers accounted for 9 per cent of borrowers in 2000, this proportion had increased to 20 per cent by 2006 (Dell Ariccia *et al.* (2008)). This large increase in mortgage debt produced a deterioration of household incomes and soaring debt defaults.

However, the mortgage crisis led to a banking crisis because securitisation failed to protect banks against credit risk. SPVs turned out not to be independent of banks, as banks were funding these companies and were guaranteeing the emission of mortgage-backed securities. More importantly, banks bought asset-backed and mortgage-backed securities, so that banks were still exposed to default risk but had not built reserves accordingly.

In this chapter, we study the very first elements of the crisis: the macroeconomic effects of household debt. We leave aside for the moment the role played by real estate prices, financial innovations and debt default on credit rationing. We are mainly concerned with understanding the dynamics between household debt, interest payments, wages and aggregate demand. The starting framework is a Goodwinian cycle, in which household consumption and debt as well as independent investment behaviour is introduced in order to assess whether it adds stability or instability to the baseline dynamics.

We make use of an approach originally formulated in supply side terms only and introduce Keynesian elements into it. The central features of this approach to Keynesian macrodynamic theory, and its application to the study of the financial market and boom-bust cycles, are the mechanisms generating non-cleared markets and the phenomenon of disequilibrium that is recurrently present in certain markets such as the labour or goods markets. In contrast to the mainstream which generally stresses the clearing of all markets at each instant of time,[1] in our modelling approach, as it will be stressed on several occasions, disequilibrium situations are the main driving force of wage/price inflation dynamics. Moreover, disequilibrium effects in financial markets are often generated by overleveraging in the real sector, in the household sector as well as in the financial sector of the economy. Some of the markets may act as either stabilising or destabilising forces through a variety of different macroeconomic channels such as the real wage feedback channel, the product market, the financial market as well as debt devaluation channels. Thus we demonstrate that there are indeed different (and also quite valid) possibilities to specify and analyse the dynamics of the macroeconomy than just in the currently fashionable Dynamic Stochastic General Equilibrium (DSGE) framework.

[1] In our view an heroic assumption in a continuous-time modelling framework.

Due to the fact that in our modelling approach the stability of the analysed dynamical system is not imposed *ab initio* by assumptions of rational expectations, which require that the economy always 'jumps' to some stable manifold and therefore always converges to the steady state after any type of shock, its stability properties (and its analysis) are based on the relative strength of the interacting macroeconomic and financial feedback channels. The ongoing occurrence of 'bubbles' and 'herding' in the financial markets across the world, as well as the large macroeconomic imbalances present nowadays in the global economy through overleveraging, show that such divergent paths do indeed take place in significant and sometimes long-lasting ways.

In this chapter we add to the crisis analysis of the preceding chapter by considering a sequence of models (a sequence of parameter scenarios for a single three-dimensional dynamical model in fact) that run from situations of stable excessive overconsumption of worker households, to their weakly excessive overconsumption and a certain degree of instability, and from there to the situation of a strict credit rationing of worker households. In the fourth model type we then consider actions that rescue the economy from this last instability scenario and imply a path back to economic stability that is based on investment stimuli and monetary policy regarding the loan and the default rate on the credit market.

In the latter type of economy, the loan rate will again fall to a low value and workers' debt will reduce in time through their now positive savings. In the limit, this economy could even converge to a situation where workers are lending to asset holders, since they have a positive savings rate in the steady state. Yet, we view this phase as representing only a transient phase where the economy recovers to a certain degree from too high mortgage debt to its normal functioning, maybe subsequently followed again by the emergence of some behaviour that leads the macroeconomy back to the stable overconsumption situation from which we started. The sequence of events analysed in this chapter may play itself out in actual historical events.

In light of the quotation from Minsky at the beginning of Chapter 1 it may well be, if the transient phase is long enough, that the first type of excessive overconsumption reappears and that the whole process starts again, although possibly in a different historical garb. It is our view that certain features of Minsky's financial instability hypothesis[2] have indeed reoccurred in the present financial crisis.

3.2 A Keynes–Goodwin model with mortgage loans and debt default

We consider a Keynesian model where income distribution matters and where workers purchase goods and houses with a marginal propensity to consume significantly larger than one.[3] The worker households therefore need credit, supplied by asset holders (acting as commercial banks), and have to pay interest on their outstanding debt.

[2] See Minsky (1992).

[3] This chapter is based on the baseline model of a mortgage crises developed in Charpe *et al.* (2009). There the authors investigate overconsumption of worker households, their credit rationing and bailout monetary policy. Here we generalise this treatment of overconsumption driven mortgage crises towards an inclusion of commercial banks as lending and depositary institutions.

In this initial situation, the steady state is attracting, due to the fact that goods market equilibrium is profit-led (although aggregate demand is wage-led), since economic activity here depends, as an exception, positively on workers' debt. A marginal propensity somewhat closer to one however makes the steady state repelling, since economic activity then becomes wage-led as well. The stable excessive overconsumption case is thus a fragile one that can easily turn from a stable boom to explosiveness and from there through induced processes of credit rationing into a devastating bust. In such a situation the public authorities may prevent the worst by specifically stimulating investment, discouraging consumption of indebted workers to a certain degree and by acting as a creditor of last resort, purchasing loans where otherwise debt default (and bankruptcy regarding house ownership) would occur. This bailout policy, accompanied by supporting decreases in the loan rate on debt, re-establishes stability for the economy and reduces the loss of homes of worker households.

The model of this section consists of two household types: workers and asset holders, and firms that are owned by the asset holders (through their real investment contributions). Workers have wage income which they totally spend on goods consumption and the purchase of houses (part of which is financed through loans from the asset holders). Their combined marginal propensity to consume, $c_w = c_g^w + c_h^w$, is therefore assumed to be larger than one. Workers' real wage income, Y_w, is however reduced through the interest they have to pay on their actual real loans, Λ_w^a, and the excess of their spending over this income determines the amount of new loans $\dot{\Lambda}_w$, they need for their intended purchase of new houses. Since the model allows for debt default this rate of change is however not the rate of change of actual loans. In equations this all reads as

Workers:

$$Y_w = vY - i\Lambda_w^a, \quad v = \frac{\omega L^d}{Y} = \frac{\omega}{z} \quad \text{the wage share} \tag{3.1}$$

$$C_w = (c_g^w + c_h^w)Y_w, \quad c_w = c_g^w + c_h^w > 1 \tag{3.2}$$

$$\dot{\Lambda}_w = (c_w - 1)Y_w = -S_w \tag{3.3}$$

$$\dot{K}_w = c_h^w Y_w - \varphi_b K_w, \quad \varphi_b K_w = \varphi_d \Lambda_w^a \tag{3.4}$$

$$\hat{L} = n = const \tag{3.5}$$

In these equations we denote the real wage by ω (and the given state of labour productivity by $z = Y/L^d$). Labour supply L is subject to natural growth and employment L^d is determined in the sector of firms (see below). The symbol i denotes the loan rate on workers' real debt Λ_w^a and the demand of workers for new loans is here not subject to credit rationing (by asset holders), although there is debt default of amount $\varphi_d \Lambda_w^a$ that depends on the loan rate and that translates itself into bankruptcy (decay of the stock of houses) of the amount of houses $\varphi_b K_w$ (from which the owners are removed and which becomes useless as has been seen in the mortgage crisis in the US economy). This loss of housing capital reduces of course the rate of change of the housing stock of workers as shown in equation (3.4). Note finally that the consumption function of workers is the only behavioural assumption that is made in this module of the model.

Asset Holders:

$$Y_c = rK + i\Lambda_w^a, \quad r = \frac{Y - \delta K - vY}{K} \tag{3.6}$$

$$C_c = 0 \quad [S_c = Y_c] \tag{3.7}$$

$$\dot{\Lambda}_w^a = \dot{\Lambda}_w - \varphi_d \Lambda_w^a \tag{3.8}$$

$$W = R + K + \Lambda_w^a \tag{3.9}$$

$$\dot{R} = Y_c - \dot{\Lambda}_w - I = S_c + S_w - I = S - I \tag{3.10}$$

Asset holders, who play the role of commercial banks in this chapter, do not consume and spend their real profit income rK (with r the rate of profit of firms) and their interest income, through their savings, on new debt given to workers $\dot{\Lambda}_w$, on the new capital goods given to firms and on new reserve holdings $\dot{R}$.[4] They do not ration worker households' purchases with respect to their financing decisions and are also completely passive with respect to their demand for new reserves. In this formulation they are therefore fairly passive suppliers of funds. They do however set the loan rate for the credit market which by and large is a given magnitude in this chapter (see however Section 3.6 for an exception). Note here finally that the chapter still ignores a resale market for houses and thus cannot say anything on the occurrence of booms and busts in such a market (as they preceded the subprime crisis in the US economy).

The sector of firms is also still formulated in a very simple manner. Firms produce output according to effective demand Y (by means of a fixed proportions technology with y^p the potential output-capital ratio and $z = Y/L^d$ their employment function with a given labour productivity z) and are choosing their rate of net investment I[5] by the excess of their rate of profit over its steady state value. Moreover, the trend term in their investment behaviour is simply given by the natural rate of growth in order to avoid any discussion about how natural growth and capital stock growth adjust to each other.[6]

Firms:

$$u = Y/Y^p \quad \text{the rate of capacity utilisation} \tag{3.11}$$

$$y^p = Y^p/K = const \quad \text{the potential output capital ratio} \tag{3.12}$$

4 In the subsequent analysis we will assume goods market equilibrium ($S = S_c + S_w = I + \delta K$) and can then show that the change in R is always zero. We therefore ignore the variable R altogether by setting it equal to zero in the following.

5 Depreciation δK is retained by firms as replacement for worn-out capital goods.

6 We will ignore in this chapter all effects that can result from changes in the rate of capacity utilisation of firms. This would demand the integration of a wage-price spiral as considered in Flaschel and Krolzig (2006) from the theoretical as well as from an empirical point of view. Such a wage-price spiral is also needed when the steady state assumption $y_o = y^p$, see below, is to be derived as an implied condition.

$$z = Y/L^d = const \quad \text{labour productivity} \tag{3.13}$$

$$I/K = i_f(r - r_o) + n, \quad r = \frac{(1-v)Y}{K} - \delta. \tag{3.14}$$

The present chapter concentrates on the interaction between indebted workers and credit supplying asset holders (playing the role of commercial banks). The sector of firms is therefore considered as fairly tranquil and not subject to volatile investment behaviour. There is in particular no debt financing of investment. Instead it is assumed throughout the chapter that asset holders directly invest part of their income into real capital stock formation and this at a rate that is smaller than one as far as the comparison with excess profitability (or loss) is concerned. Note here however that this gives rise in the real growth dynamics to a Goodwin (1967) type profit squeeze mechanism that can be either wage-led or profit-led. The debt feedback chain of the economy is therefore integrated with such a growth cycle model in this chapter and investigated with respect to the consequences it has in such a framework.

The final law of motion of this model of fluctuating growth and indebtedness in the housing sector concerns the real wage dynamic that drives the real part of the economy in the form of a real wage Phillips curve (PC) (a conventional textbook PC with myopic perfect foresight regarding price inflation)

Real Wage Adjustment:

$$\hat{\omega} = \beta_{we}(e - \bar{e}). \tag{3.15}$$

The variable $e = \frac{L^d}{L}$ denotes the rate of employment, with Y the demand driven output level of firms. Real wages are driven by excess demand pressure $e - \bar{e}$ on the labour market (with $\bar{e}$ the normal rate of employment). Such real wage dynamics are obtained from a conventional wage PC if myopic perfect foresight is assumed for its accelerator term.

3.3 Excessive overconsumption and an attracting steady state

The 3D dynamics implied by the model of the preceding section to be investigated in this section in a special case, is based on a PC distributive cycle mechanism interacting with a Goodwin (1967) type growth dynamics and a law of motion for the debt to capital ratio of workers. The system, in terms of the state variables, can be written as[7]

$$\hat{v} = \beta_{we}(y/l - \bar{e}), \quad v = \frac{\omega}{z}, \tag{3.16}$$

$$\hat{l} = -i_f(r - r_o), \quad r = (1-v)y - \delta, \quad l = \frac{zL}{K}. \tag{3.17}$$

7 The differential equations for v and l follow easily from their ratio definitions. The differential equation for λ_w^a is obtained by taking the logarithm of the ratio defining it and then time-differentiating it and substituting in the extensive form equations.

$$\dot{\lambda}_w^a = (c_w - 1)(vy - i\lambda_w^a) - (i_f(r - r_o) + n + \varphi_d)\lambda_w^a, \quad \lambda_w^a = \frac{\Lambda_w^a}{K}, \tag{3.18}$$

with the goods market equilibrium or IS expression for the output capital ratio

$$y = \frac{c_w i\lambda_w^a + i_f(\delta + r_o) - n - \delta}{(c_w - i_f)v + i_f - 1}. \tag{3.19}$$

This IS curve is easily obtained from the goods market equilibrium equation

$$Y = c_w(vY - i\Lambda_w^a) + (i_f(r - r_o) + n + \delta)K. \tag{3.20}$$

by transforming it into intensive form and by solving it for the output-capital ratio y. Note that the budget equations of the two households of the model imply $\dot{R} = 0$ if IS equilibrium is assumed.[8]

We assume in this section that

$$c_w > 1 + (1 - i_f)(1 - v_o)/v_o > 1 > i_f \quad \text{iff} \quad c_w > 1 > i_f > \frac{1 - c_w v_o}{1 - v_o}$$

holds true. In this case the Goodwin subcycle is characterised by stability and the marginal propensity of workers to consume is very large, and may be empirically beyond all reasonable values. We therefore consider this starting case to be an extreme situation of excessive overconsumption backed up by unrestricted loans. We have in this case that the denominator of the IS curve in (3.19) is positive and we will show below that the numerator of equation (3.19) is also positive when evaluated at the steady state value of λ_w^a. This assumption implies that goods demand is extremely 'wage-led', which is a natural assumption in the case where the marginal propensity to consume is larger than one. However this extreme case implies that the ratio y depends negatively on the wage share v (which one would not expect it to be true in a wage-led economy), since the denominator in the IS curve in equation (3.19) is positive, and it moreover implies a positive dependence of y on the debt to capital ratio λ_w^a (around the steady state of the model, to be discussed below).

The law of motion (3.17) for the labour-capital ratio l is easily obtained by means of the standard rules for growth rate calculations. The law of motion (3.18) for the wage share v is a simple consequence of our real wage PC when labour productivity is assumed to be constant.

We also have a further law of motion, for the ratio $k_w = K_w/K$, which however does not feed back into the above dynamics and which reads

$$\hat{k}_w = c_h^w(vy - i\lambda_w^a)/k_w - [i_f(r - r_o) + n] - \varphi_d\lambda_w^a/k_w. \tag{3.21}$$

Equation (3.21) shows how bankruptcy of worker households accompanies their debt default concerning credit they have obtained from the asset holders. Of course these

[8] On this basis we have

$$\dot{R} = rK + i\Lambda_w^a - \dot{\Lambda}_w - I = rK + i\Lambda_w^a - (c_w - 1)(vY - i\Lambda_w^a) - I = Y - \delta K - C - I = 0.$$

two laws of motion should feed back into the baseline dynamics (3.16), (3.17) and (3.18) in future extensions of the model. Note also that there is no resale market for houses in the present formulation of the model so that houses can be treated as consumption goods and need not be classified as investment goods, as is done in the system of national accounts. By assumption part of the housing stock has been financed by mortgages, and may therefore be subject to bankruptcy if debt default occurs on such loans.

Proposition 3.1 (The reference balanced growth path)
Assume that the risk premium r_o is given in an appropriate way (to be determined below). There is then in general a locally uniquely determined interior steady state of the dynamics (3.16)–(3.18) which is given by[9]

$$y_o = y^p, \tag{3.22}$$

$$e_o = \bar{e}, \quad l_o = y_o/\bar{e}, \tag{3.23}$$

$$y_{wo} = \frac{y_o - \delta - n}{c_w}, \tag{3.24}$$

$$\lambda_{wo}^a = \frac{c_w - 1}{n + \varphi_d} y_{wo}, \tag{3.25}$$

$$v_o = \frac{y_{wo} + i\lambda_{wo}^a}{y_o} = \frac{y_{wo}}{y_o}\left(1 + i\frac{c_w - 1}{n + \varphi_d}\right), \tag{3.26}$$

$$r_o = (1 - v_o)y_o - \delta. \tag{3.27}$$

Proof: The first two steady state values (3.22) and (3.23) have been set from the outside (and are determined in this way if the law of motion for the inflationary climate is added to the model). The goods market equilibrium then gives the equation for workers' income (3.24), which in turn can be used to determine the steady state debt ratio (3.25) by setting $\dot{\lambda}_w^a = 0$. The steady wage share (3.26) is then obtained from the definition of workers' income. Finally. the target rate of profit r_o in (3.27) is determined in such a way that the wage share allows for $\dot{l} = 0$. □

If one assumes the special case $i = n + \varphi_d$ one gets in particular that

$$y_o = y^p, \tag{3.28}$$

$$e_o = \bar{e}, \ l_o = y_o/\bar{e}, \tag{3.29}$$

$$y_{wo} = \frac{y_o - \delta - n}{c_w}, \tag{3.30}$$

$$\lambda_{wo}^a = (c_w - 1)y_{wo}/i, \tag{3.31}$$

[9] The condition $y_o = y^p$ is an assumption that can only be proved to be valid if a more complete framework is used that allows the integration of inflation and anti-inflationary monetary policy. The condition on r_o is therefore used here as a substitute for the introduction of that broader theoretical framework.

$$v_o = \frac{y_{wo} + i\lambda^a_{wo}}{y_o} = \frac{c_w y_{wo}}{y_o} = \frac{y_o - \delta - n}{y_o} > 0, \tag{3.32}$$

$$r_o = (1 - v_o)y_o - \delta = n, \tag{3.33}$$

which ensures that $v_o \in (0, 1)$ holds true and $r_o = n$, the so-called Cambridge equation (for the case $s_c = 1$). Note that this situation implies that the steady state rate of profit r_o is then equal to the rate of return on loans $(i\lambda^a_w - \varphi_d\lambda^a_w)/\lambda^a_w$. The analysis that follows will investigate, with the exception of cases where the loan rate is assumed to be close to zero (see also Section 3.6 on monetary policy), cases that are close to this special situation.

Remark 3.1 (1) It is obvious from the above that the steady state values are of meaningful size. They are also meaningful in the border case $i = 0$ (keeping $n + \varphi_d$ fixed). Note that the value of r_o has been adjusted in such a way that the assumption $i_f(\cdot) = 0$ is fulfilled at the steady state values.

(2) Assuming as in Goodwin (1967) a parameter value $c_w = 1$ gives as a special case $\lambda^a_w = 0$, to be considered later on. Note however that the investment function (through its interest rate dependence) still preserves the Keynesian IS curve and thus preserves the situation of a Goodwin growth cycle model that is demand driven on its market for goods. Considering the IS curve in this case in more detail gives

$$y = \frac{i_f(\delta + r_o) - n - \delta}{(i_f - 1)(1 - v)},$$

which exhibits a negative denominator in the case of $i_f < 1$.

We consider in the following however the case of a strongly wage-led goods demand characterised by

$$c_w > 1 + (1 - i_f)(1 - v_o)/v_o > 1 > i_f, \tag{3.34}$$

already briefly mentioned above. In this case we have a positive denominator in the expression for the IS curve of the model given by

$$y = \frac{c_w i\lambda^a_w + i_f(\delta + r_o) - n - \delta}{(c_w - i_f)v + i_f - 1}. \tag{3.35}$$

Under condition (3.34) the numerator and denominator of (3.35) are both positive at the steady state. For the partial derivatives of the functions y, vy, r with respect to v we find that[10]

$$\frac{\partial y}{\partial v} = -y\frac{c_w - i_f}{D} < 0, \quad \frac{\partial(vy)}{\partial v} = y\frac{i_f - 1}{D} < 0, \quad \frac{\partial r}{\partial v} = -y\frac{c_w - 1}{D} < 0. \tag{3.36}$$

[10] Note that in the subsequent analysis it is convenient to introduce a specific symbol for the denominator of (3.35) at the steady state, namely

$$D = (c_w - i_f)v_o + i_f - 1.$$

For the derivative $\frac{\partial y}{\partial \lambda^a_w}$ we in addition always have

$$\frac{\partial y}{\partial \lambda^a_w} = \frac{c_w i}{D} > 0, \quad \frac{\partial r}{\partial \lambda^a_w} = \frac{c_w i}{D}(1 - v) > 0$$

which transforms itself directly into corresponding derivatives for vy, $r = (1-v)y-\delta$. The partial derivatives with respect to the state variable v in equation (3.36) show in particular that these derivatives do not depend at the steady state on the interest rate i so that the influence of i on the Jacobian of the dynamics only comes in to play when the partial derivative $\frac{\partial y}{\partial \lambda^a_w} = c_w i/D$ is involved. Note that we will use for brevity y_v in place of $\frac{\partial y}{\partial v}$, etc. in the following discussion.

The sign of the derivative $y_v < 0$ shows the somewhat striking result that the goods market dynamics appear to be profit-led, although aggregate demand per unit of capital is clearly wage-led, because of $y^d_v = (c_w - i_f)y > 0$. This indicates that empirical findings on the role of income distribution on the market for goods (where we in fact can only observe empirically the interactions of demand and supply) which state that the goods market behaviour is profit-led (decreases with increases in the wage share) can be completely in line with the unobservable fact that planned goods demand depends positively on the wage share (so that it is wage-led). We note also that in the extreme case here considered we have a positive effect of increases in the loan rate on the state of the goods market. This is a further astonishing characteristic of the very excessive consumption case being considered here.

The Jacobian of the dynamic (3.16)–(3.18) reads in the special case being considered (note that we use the abbreviation $k = \beta_{we}/l_o$)[11]

$$J = \begin{pmatrix} k y_v v_o & -k\bar{e}v_o & k y_{\lambda^a_w} v_o \\ -i_f r_v l_o & 0 & -i_f r_{\lambda^a_w} l_o \\ (c_w - 1)(vy)_v - i_f r_v \lambda^a_{wo} & 0 & (c_w - 2)i - i_f r_{\lambda^a_w}\lambda^a_{wo} \end{pmatrix}. \tag{3.37}$$

To determine the sign of the determinant of this Jacobian we therefore have to investigate the sign of the determinant

$$\begin{vmatrix} y_v & -\bar{e} & y_{\lambda^a_w} \\ -r_v & 0 & -r_{\lambda^a_w} \\ (c_w - 1)(vy)_v - i_f r_v \lambda^a_{wo} & 0 & (c_w - 2)i - i_f r_{\lambda^a_w}\lambda^a_{wo} \end{vmatrix}$$

$$= \begin{vmatrix} -y\frac{c_w - i_f}{D} & -\bar{e} & \frac{c_w i}{D} \\ y\frac{c_w - 1}{D} & 0 & -(1 - v_o)\frac{c_w i}{D} \\ (c_w - 1)y\frac{i_f - 1}{D} + i_f y\frac{c_w - 1}{D}\lambda^a_{wo} & 0 & (c_w - 2)i - i_f(1 - v_o)\frac{c_w i}{D}\lambda^a_{wo} \end{vmatrix}$$

[11] In the special case being considered here we have $i = n + \varphi_d$, and hence the $(c_w - 2)$ term in the Jacobian J due to this assumption.

$$= \frac{yi}{D}\begin{vmatrix} -(c_w - i_f) & -\bar{e} & \frac{c_w}{D} \\ c_w - 1 & 0 & -(1-v_o)\frac{c_w}{D} \\ (c_w-1)(i_f-1) + i_f(c_w-1)\lambda^a_{wo} & 0 & (c_w-2) - i_f(1-v_o)\frac{c_w}{D}\lambda^a_{wo} \end{vmatrix}.$$

The sign of this determinant is therefore equal to the sign of its sub-determinant

$$\begin{vmatrix} 1 & -(1-v_o)\frac{c_w}{D} \\ i_f - 1 + i_f\lambda^a_{wo} & (c_w-2) - i_f(1-v_o)\frac{c_w}{D}\lambda^a_{wo} \end{vmatrix}.$$

The sign of the latter determinant depends upon the expression

$$c_w - 2 - i_f(1-v_o)\frac{c_w}{D}\lambda^a_{wo} + (1-v_o)\frac{c_w}{D}(i_f - 1 + i_f\lambda^a_{wo})$$

which finally simplifies to

$$c_w - 2 + (1-v_o)\frac{c_w}{D}(i_f - 1). \tag{3.38}$$

The expression in (3.38) is negative if there holds $i_f \leq 1$ and if c_w is not too large (an assumption which can be justified from the empirical point of view). In the case $i_f > 1$ we calculate from $D = (c_w - i_f)v_o + i_f - 1 > i_f - 1 > 0$ that

$$c_w - 2 + (1-v_o)\frac{c_w}{D}(i_f - 1) < c_w - 2 + (1-v_o)c_w < 1.5c_w - 2 < 0$$

for all $i_f < c_w$ if $c_w < 4/3$, and $v_o > 0.5$ holds true. Thus we get a negative determinant $|J|(\equiv -a_3)$ for all empirically plausible parameter values $c_w > 1$ and $v_o = (y_o - \delta - n)/y_o$.

The assumption on the size of c_w implies that the trace of $J(\equiv -a_1)$ is negative as well. For the sum of the principal minors of order two we have

$$a_2 \equiv \frac{ky_ov_o(c_w-1)}{D}i_fy_o + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}.$$

From the above calculations we have

$$J_2 = \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}$$

$$= kv_o\begin{vmatrix} y_v & y_{\lambda^a_w} \\ (c_w-1)(vy)_v - i_fr_v\lambda^a_{wo} & (c_w-2)i - i_fr_{\lambda^a_w}\lambda^a_{wo} \end{vmatrix}$$

$$= kv_o\begin{vmatrix} -y_o\frac{c_w-i_f}{D} & \frac{c_wi}{D} \\ (c_w-1)y_o\frac{i_f-1}{D} + i_fy_o\frac{c_w-1}{D}\lambda^a_{wo} & (c_w-2)i - i_f(1-v_o)\frac{c_wi}{D}\lambda^a_{wo} \end{vmatrix}$$

$$= \frac{kv_oy_oi}{D^2}\begin{vmatrix} -(c_w-i_f) & c_w \\ (c_w-1)(i_f-1)+i_f(c_w-1)\lambda^a_{wo} & D(c_w-2) - i_f(1-v_o)c_w\lambda^a_{wo} \end{vmatrix}$$

$$= \frac{ky_ov_o(c_w-1)}{D}\frac{i}{D}\begin{vmatrix} -(c_w-i_f)/(c_w-1) & c_w \\ i_f(1+\lambda^a_{wo}) - 1 & D(c_w-2) - i_fc_w(1-v_o)\lambda^a_{wo} \end{vmatrix}$$

A sufficient assumption to obtain $a_2 > 0$ is to assume that the loan rate i is chosen sufficiently small, such that, should the considered determinant be negative, it is dominated by the first principal minor $\frac{ky_ov_o(1-c_w)}{D}i_fy_o$.

The remaining Routh–Hurwitz condition for local asymptotic stability is

$$a_1a_2 - a_3 > 0 \quad \text{where} \quad a_1 = -\text{ trace } J, a_3 = -|J|.$$

The determinant of J is however dominated by the remaining terms in a_1a_2, if the adjustment speed of money wages β_{we} is chosen sufficiently large, since it enters a_1, a_2, a_3 linearly with a positive slope and thus in a_1a_2 through a positively sloped quadratic term.

Therefore to summarise we have

$$a_1 > 0, a_2 > 0, a_3 > 0, a_1a_2 - a_3 > 0$$

for the coefficients of the characteristic polynomial of the matrix J (the Routh–Hurwitz conditions) and have thus shown the local asymptotic stability of the steady state of the model. This gives us the proposition:[12]

Proposition 3.2 (Stability for a 'normal' range of parameter values)
Assume that i is sufficiently small and β_{we} sufficiently large. Then the steady state of the dynamical system (3.16)–(3.18) is locally attracting for all empirically relevant parameter sizes of the model.

This result is perhaps not as surprising as one may at first think in a significantly debt driven economy. Default so far only appears as if it were a gift from asset holders to worker households, since bankruptcy (the loss of a house) does not yet feed back into the considered dynamics. Moreover, the Goodwin part of the model (the interaction of the state variables v and l) is here of a convergent type, while the only destabilising feedback loop is the one between the state variables v and λ^a_w through their positive interaction as shown through the entries J_{13} and J_{31}. Debt has a positive effect on the growth rate of the wage share and the latter may have a positive effect on the time rate of change of the debt to capital ratio. But this accelerating mechanism is not really dominant in the overall interaction of the three state variables that are considered here.

There is however an element in the considered situation that can lead the investigated dynamics into a situation where the steady state becomes unstable and where the assumed degree of wage flexibility will lead the economy towards explosive fluctuations. This case comes about when it is assumed that the considered excessive overconsumption stimulates asset holders to invest in real capital at a rate i_f that is larger than c_w. In this case we get $y_v > 0$ and thus a wage-led goods market behaviour. Since the stability proof of this section required for a degree of wage flexibility β_{we} that is sufficiently high, it is then very likely that the trace of J, given by

$$\beta_{we}y_vv_o - (c_w - 1)i - i_fr_{\lambda^a_w}\lambda^a_w - (n + \varphi_d),$$

[12] This proposition also holds true for all $c_w > 1 \geq i_f$.

becomes positive and the steady state becomes repelling. We will investigate such a situation in more detail in Section 3.5.

3.4 Weakly excessive overconsumption and a repelling steady state

We consider now the only mildly excessive wage-led case

$$i_f < 1 < c_w < 1 + (1 - i_f)(1 - v_o)/v_o. \tag{3.39}$$

In this situation the propensity to consume of workers c_w is still greater than 1 but is bounded above as shown in equation (3.39). This case we shall dub as weakly excessive consumption.

On the basis of the assumption (3.39) we now have a significantly changed situation in the expression (3.35) for the goods market equilibrium expression at the steady state since the sign of the denominator D changes sign. For the partial derivatives of the functions y, vy, r with respect to v we therefore now have

$$y_v = -y\frac{c_w - i_f}{D} > 0,\ (vy)_v = y\frac{i_f - 1}{D} > 0,\ y_v = -y\frac{c_w - 1}{D} > 0. \tag{3.40}$$

In addition for the derivative $(vy)_{\lambda_w^a}$ we continue to have

$$(vy)_{\lambda_w^a} = \frac{c_w i}{D} < 0$$

which transforms itself directly into corresponding derivatives for $vy, r = (1 - v)y - \delta$. Note that all partial derivatives now have the opposite sign as compared with the overconsumption case considered in the previous section. The sign of the derivative $y_v > 0$ now shows the normal result that the goods market dynamics appear to be wage-led when aggregate goods demand per unit of capital is wage-led, that is $y_v^d = (c_w - i_f)y > 0$. Moreover interest rate effects, via rising loan rates, are now contractionary since $y_i < 0$ (instead of increasing activity) as one would intuitively expect in a debt driven economy.

Under the above assumption we have for the determinant of the Jacobian the result

$$|J| = \begin{vmatrix} 0 & -k\frac{y_o}{l_o}v_o & 0 \\ -i_f r_v l_o & 0 & -i_f r_{\lambda_w^a} l_o \\ (c_w - 1)(vy)_v & 0 & (c_w - 1)i - (n + \varphi_d) \end{vmatrix} \approx \begin{vmatrix} 0 & - & 0 \\ - & 0 & + \\ - & 0 & -n - \varphi_d \end{vmatrix}.$$

and thus can assume that this determinant is always positive for values of $c_w - 1$ and i that are of empirical relevance (under a relatively normal working of the economy). This assumption also no longer implies that the trace of J is negative. For the sum of the principal minors of order two we have from the above considerations that

$$a_2 = \begin{vmatrix} ky_v v_o & -k\frac{y_o}{l_o}v_o \\ -i_f r_v l_o & 0 \end{vmatrix} + \begin{vmatrix} ky_v v_o & ky_{\lambda_w^a} v_o \\ (c_w - 1)(vy)_v - i_f r_v \lambda_w^a & (c_w - 1)i - i_f r_{\lambda_w^a}\lambda_w^a - (n + \varphi_d) \end{vmatrix}.$$

Using the expressions and signs for the partial derivatives in (3.40) we see that a_2 has the same sign as the expression

$$\begin{vmatrix} - & -k\frac{y_o}{l_o}v_o \\ i_f\frac{y_o(c_w - 1)}{D}l_o & 0 \end{vmatrix} + \frac{ky_o v_o c_w i}{D^2}\begin{vmatrix} -(c_w - i_f) & 1 \\ (c_w - 1)[i_f(1 + \lambda_{wo}^a) - 1] & -i_f(1 - v_o)\lambda_{wo}^a - \frac{n + \varphi_d}{c_w i D} \end{vmatrix}.$$

We conjecture that the second determinant of this expression is very likely to be negative. The validity of the conditions

$$a_1 > 0, a_2 > 0, a_3 > 0, a_1 a_2 - a_3 > 0$$

on the coefficients of the characteristic polynomial of the matrix J (the Routh–Hurwitz conditions) is therefore in general not likely to hold from various perspectives, implying the proposition:

Proposition 3.3 (Instability of the weakly excessive consumption case)
Assume $i_f < 1 < c_w < 1+(1-i_f)(1-v_o)/v_o$. The steady state of the dynamical system (3.16)–(3.18) is locally repelling for empirically plausible choices of the parameter values of the model.

The case of weakly excessive overconsumption in the sector of worker households therefore does not represent a viable situation, in particular since the previously stable dynamics of the Goodwin substructure with its state variables v, l have now become totally unstable. This is further exemplified by the following proposition which states that even very low loan rates i cannot be of help in the considered case.

Proposition 3.4 (Instability for small values of the loan rate)
Assume that $i = 0 < n + \varphi_d$. Then the steady state of the dynamical system (3.16)–(3.18) is locally repelling. This result also holds for all loan rates chosen sufficiently small.

Proof: It is easily shown that the sign structure of the Jacobian in the case $i = 0$ is of the qualitative form

$$J = \begin{pmatrix} ky_v v_o & -k\bar{e}v_o & ky_{\lambda_w^a} v_o \\ -i_f r_v l_o & 0 & -i_f r_{\lambda_w^a} l_o \\ (c_w - 1)(vy)_v - i_f r_v \lambda_{wo}^a & 0 & (c_w - 2)i - i_f r_{\lambda_w^a}\lambda_{wo}^a \end{pmatrix}$$
$$= \begin{pmatrix} + & - & 0 \\ - & 0 & 0 \\ ? & 0 & - \end{pmatrix}.$$

This immediately implies the instability of the (wage-led) Goodwin part of the model (capturing the interaction of v and l), since the subsystem corresponding to the state variables v and l does not depend on the third state variable λ_w^a. The real parts of the

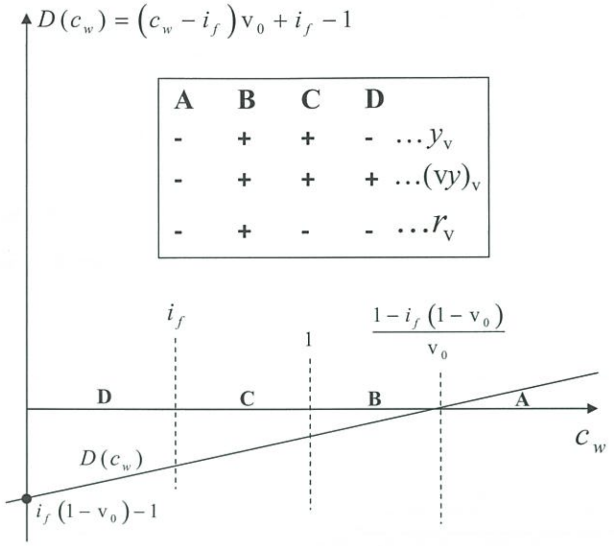


Figure 3.1 A summary of the stability scenarios for a varying parameter c_w. The table in the figure gives the signs of y_v, $(vy)_v$ and r_v in the various scenarios – A, B, C, D

eigenvalues are therefore not all negative, a fact that extends to all sufficiently small i, since the eigenvalues depend continuously on the parameters of the model. □

So far we have investigated the parameter situations denoted by A and B in Figure 3.1. We have found out that case A (see Section 3.3) represents by and large a stable, although exceptional performance of the economy (since a wage-led aggregate demand is here represented by a profit-led IS curve, due to a high debt to capital ratio), while case B (this section) does not result in a viable situation. A further conclusion here is that if case B prevails, investment should be stimulated to the extent such that $i_f > \frac{1-c_w v_o}{1-v_o}$ holds true. For example through a loan rate policy that lowers the loan rate to a sufficient degree such that this inequality can be assured (assuming that the parameter i_f depends negatively on the loan rate i). Stimulating investment in this way shifts the $D(c_w)$ line of Figure 3.1 upwards until the point is reached where the value of c_w, which currently characterises the economy, has been moved into the stable region A.

However it may not be possible to stimulate investment decisions to such a degree (such that situation A becomes established) starting from the situation B considered in this section. Credit rationing, to be considered in the next section, may then need to be enforced – leading eventually to the situation C as shown in Figure 3.1. In situation C the entry J_{31} in the matrix J has become unambiguously negative which increases the destabilising elements in the Routh–Hurwitz coefficient a_2. The overall situation is clear in this case, it represents an unstable situation, since we still have a positive entry J_{11} that will destabilise the economy if wages are sufficiently flexible with respect to demand pressure on the market for labour. Since the stability proof of Section 3.3 required a degree of wage flexibility β_{we} that is sufficiently high it is thus very likely

in this case that the trace of J, given by

$$\beta_{we} y_v v_o - (c_w - 1)i - i_f r_{\lambda_w^a} \lambda_w^a - (n + \varphi_d),$$

is then positive and the steady state becomes repelling. We will investigate this situation in the next section.

If there is no possibility of a return to the (fragile) situation A, because the debt to capital ratios involved may be considered as too exceptional, policy can therefore only attempt to establish a situation as represented by case D (to be considered in Section 3.6) where a profit-led regime is re-established on a non-exceptional basis, meaning that both aggregate demand as well as the IS curve are depending negatively on the share of wages in national income, which will be the content of Proposition 3.6.

3.5 Credit rationing, reduced consumption and the emergence of mortgage crises

We start from the case of excessive overconsumption and assume now that asset holders stop lending to worker households due to the just observed tendency of the economy to become an unstable one or simply due to a change in their risk perception concerning worker households' debt. One may for example assume that the loan rate i (and with it the default rate φ_d, see the next section for more details on this) has been increasing in a stepwise fashion until a level has been reached that persuades asset holders to stop their lending to worker households. The immediate consequence is that the propensity to consume is forced to the value 1 (or close to 1 if some lending still goes on, the case we considered in the preceding section). In the following analysis we therefore consider the stability features for the case $c_w \le 1$.

When $c_w \le 1$ (or sufficiently close to 1) as in the preceding section we have that $D < 0$, but now in addition also $\lambda_{wo}^a < 0$. The assumed shock to worker households' propensity to consume has an immediate effect on the goods market equilibrium, which we recall from equation (3.19) may be written

$$y = \frac{c_w i \lambda_w^a + i_f(\delta + r_o) - n - \delta}{(c_w - i_f)v + i_f - 1},$$

but now with $(c_w - i_f)v + i_f - 1 < 0$. The change in IS equilibrium resulting from the change in the parameter c_w is obtained by taking the total differential of the IS equation, so that

$$dc_w vy + [(c_w - i_f)v + i_f - 1]dy = dc_w i \lambda_w^a = 0$$

Rearranging this equation appropriately yields

$$\frac{dy}{dc_w} = \frac{vy - i\lambda_w^a}{(1 - i_f)(1 - v)}$$

Viewed from the situation $c_w = 1$ we thus have the result that a positive shock to c_w increases economic activity (since the steady state value of λ_w^a is zero at this position), that is, in reversed terms, the jump to a marginal propensity to consume of unity in the above considered crisis situation is contractionary and would lead the economy into the deflationary region if it were at its steady state position initially. This would induce a process of debt deflation and thus an increasing debt and interest burden for the worker households.

The steady state position is now given by

$$y_o = y^p \; l_o = y_o/\bar{e}, \; y_{wo} = y_o - \delta - n, \; \lambda^a_{wo} = 0, \; v_o = y_{wo},$$

and for the employed partial derivatives we have in this case (since the denominator in the IS curve is now negative)

$$y_v = -y\frac{1 - i_f}{D} > 0, \quad (vy)_v = y\frac{i_f - 1}{D} > 0, \quad y_v = 0.$$

For the derivative $(vy)_{\lambda^a_w}$ we find that $(vy)_{\lambda^a_w} = \frac{c_w i}{D} < 0$.

Proposition 3.5 (Full credit rationing implies a repelling steady state) *Assume $c_w = 1$. Then: The steady state of the dynamical system (3.16)–(3.18) is surrounded by explosive forces. This then also holds for all marginal propensities c_w chosen sufficiently close to* 1.

Proof: The Jacobian of the dynamics at the steady state now reads as

$$J = \begin{pmatrix} ky_v v_o & -k\bar{e}v_o & ky_{\lambda^a_w} v_o \\ -i_f r_v l_o & 0 & -i_f r_{\lambda^a_w} l_o \\ (c_w - 1)(vy)_v - i_f r_v \lambda^a_{wo} & 0 & (c_w - 2)i - i_f r_{\lambda^a_w} \lambda^a_{wo} \end{pmatrix}$$
$$= \begin{pmatrix} ky_v v_o & -k\frac{y_o}{l_o} v_o & ky_{\lambda^a_w} v_o \\ 0 & 0 & -i_f r_{\lambda^a_w} l_o \\ 0 & 0 & -(n + \varphi_d) \end{pmatrix}.$$

This gives the qualitative sign structure

$$J = \begin{pmatrix} + & - & - \\ 0 & 0 & + \\ 0 & 0 & - \end{pmatrix}.$$

This immediately implies the instability of the dynamics, since two of the principal minors of order 2 are zero while the third one is negative, since we can calculate that $a_2 < 0$. □

Note that the determinant of the Jacobian J is zero in the situation $c_w = 1$, and non-zero (positive or negative) for $c_w < 1$. There will be one eigenvector direction where the dynamics move very slowly, and there is in addition always one unstable (and one stable[13]) root, where the former drives the system away from the steady state.

[13] Since $a_2 > 0$ holds.

In the case of an arbitrary $c_w \in (i_f, 1)$ we get for the Jacobian the qualitative structure

$$J = \begin{pmatrix} + & - & - \\ + & 0 & + \\ - & 0 & - \end{pmatrix},$$

since there holds

$$J_{31} = (c_w - 1)y\frac{i_f - 1}{D} - i_f(-y)\frac{c_w - 1}{D}\lambda^a_{wo} = (c_w - 1)y\frac{1 - i_f}{|D|} + i_f y\frac{1 - c_w}{|D|}\lambda^a_{wo} < 0$$

due to $\lambda^a_{wo} < 0$. We therefore get as sufficient condition for instability:

$$a_2 < 0 \quad \text{iff} \quad -J_{12}J_{21} < -J_{11}J_{33} + J_{13}J_{31}.$$

The last condition states that the stability created by the profit-led Goodwin (1967) subcycle of the model is overcome by the feedback chain between the wage share v and the debt to capital ratio λ^a_w. Of course further instability scenarios may come about through the trace of J if wages are sufficiently flexible with respect to the labour market gap and through the determinant of J if $-J_{23}J_{31}$ becomes dominant. The case where workers save in order to reduce their prevailing level of debt therefore does not at all represent a situation where recovery and convergence towards the steady state of the economy can be expected.

3.6 Monetary policy in a mortgage crisis

In order to determine what monetary policy should do in the situation we have investigated in the preceding section we now consider the rise in the loan rate i that triggered the crisis in more detail. We consider for this purpose the situation shown in Figure 3.2.

The basic assumption in Figure 3.2 is that the default rate φ_d is an increasing convex function of the loan rate i. We add to this function the term $r_o = n$ and obtain an expression (as a function of the loan rate i) that shows the minimum loan rate needed to obtain as rate of return on loans the profit rate of the sector of firms. We next assume that the actual loan rate tends, under normal conditions, to the minimum loan rate as shown in the situation A. Such an adjustment leads in the limit to the loan rate i_o and thus to a fairly tranquil situation where the loan rate as well as the default rate is small, a situation that is attracting smaller as well as larger loan rates in an asymptotically stable way. Yet as Figure 3.2 shows, the basin of attraction of the loan rate i_o is limited to the right by the loan rate i_1, since the considered stable adjustment process becomes an unstable one to the right of i_1.

Consider now the case B in Figure 3.2. We here assume that external events have led to this increase in the loan rate (maybe in a stepwise fashion) where asset holders now insist on a loan rate that leads to a higher rate of return on their loans than the one in the sector of firms. Assume that the increase in the loan rate eventually leads to a loan rate that is higher than i_1, then the financial market of the economy will be pushed into the unstable region where the loan rate gives rise to an upward spiralling process C. We

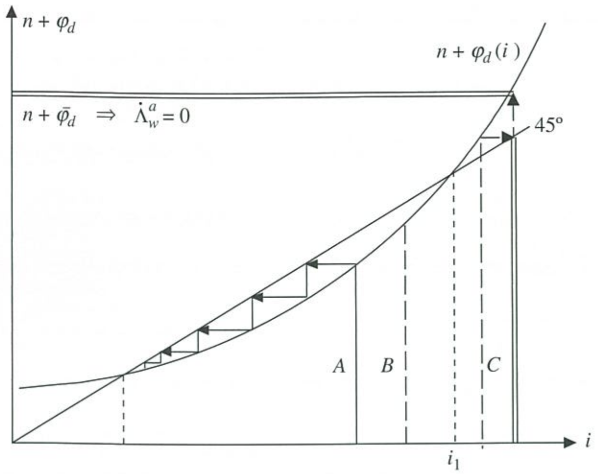


Figure 3.2 Loan rate adjustment dynamics

next assume that this process comes to an end at the value determined by the double line, since asset holders then stop giving credit to worker households as discussed in the preceding section (due to the high default rate that accompanies the then existing loan rate). The economy is then trapped into a situation where the loan rate as well as the default rate is very high and where the propensity to consume houses is zero and the overall propensity to consume therefore is less than one.

In this still very basic scenario (as far as mortgage crises are concerned) the policy to be adopted by the central bank (which is now assumed to enter the stage) is still a simple one. The first step in the solution to the credit crisis is simply to buy an amount of bad loans φ_c that allows for a loan rate i smaller than i_1 for the bad loans $\bar{\varphi}_d(i) - \varphi_c$ that then remain in the sector of asset holders. This increases the reserves of asset holders and must be conditioned on their acceptance of the minimum loan rate that is consistent with the debt default situation they are then facing. If this acceptance is followed in all subsequent steps we may then expect that the process leading back to the loan rate i_o may again become effective, now in the situation where the overall propensity of worker households to consume is less than 1 (and where debt of worker households is reduced in each point in time through their now positive savings rate). The question then becomes how the private sector of the economy (the real part of the economy and debt decumulation) behaves in such a situation.

The downward correction of the loan rate just considered is however not yet sufficient in order to stabilise the economy, since the Jacobian we considered at the end of the preceding section is plagued by a wage-led situation as far as the entry J_{11} is concerned and exhibits a destabilising feedback mechanism through the entries J_{13} and J_{31} which are both negative and thus endanger the positivity of a_2. In order to obtain a situation where the stability of the private sector can be assured we consider again the sign of the determinant of J as given by the expression (3.38). We continue to investigate the case $c_w < 1, i_f < 1$, that is the situation where $D < 0$ holds true. We assume now the situation where $1 > i_f > c_w > 0$ has been established, e.g. through forced savings of worker households, based on their debt that the central bank is now holding.

Simple empirical estimates as they were considered with regard to equation (3.38), or just the assumption that i_f is sufficiently close to 1, then ensure that the determinant of the matrix J is negative. Moreover we have in this case $y_v < 0$, $(vy)_v > 0$, $r_v < 0$, $y_{\lambda^a_w} < 0$, $r_{\lambda^a_w} < 0$ and thus get in this case the sign distribution

$$J = \begin{pmatrix} ky_v v_o & -k\bar{e}v_o & ky_{\lambda^a_w} v_o \\ -i_f r_v l_o & 0 & -i_f r_{\lambda^a_w} l_o \\ (c_w - 1)(vy)_v - i_f r_v \lambda^a_{wo} & 0 & (c_w - 2)i - i_f r_{\lambda^a_w} \lambda^a_{wo} \end{pmatrix}$$
$$= \begin{pmatrix} - & - & - \\ + & 0 & + \\ - & 0 & - \end{pmatrix}.$$

This immediately implies the stability of the Goodwin part of the model.

Proposition 3.6 (Stability for small values of i and flexible wage adjustment) *Assume $c_w < i_f$, and that the parameter k is sufficiently large and $i = n + \varphi_d(i)$ is sufficiently small. Then the steady state of the dynamical system (3.16)–(3.18) is locally attracting.*

Proof: We have already that $a_1 = -\text{trace } J > 0$ and $a_3 > 0$. Choosing k sufficiently large also gives $a_2 > 0$ if the entries in the last column of J are made sufficiently small, and this can be done by choosing a low value of the loan rate i. The latter situation also ensures $a_1 a_2 - a_3 > 0$, since a_3 can be made small relative to $a_1 a_2$ in this way. □

In view of what we have shown with respect to Figure 3.1, see also Figure 3.3, the first objective of the central bank thus should be to support the maintenance of the loan rate at sufficiently low level. In a next step consumption should be discouraged or/and investment encouraged. One may for example assume that the value of i_f depends negatively on the loan rate i. Improving the investment climate to a sufficient degree may therefore be one policy option of the central bank. Moving i_f upwards increases not only the domain A and thus makes the excessive case more robust, but it also increases the domain D and thus allows the economy to settle down in an enlarged region D (after credit rationing and the reduction of c_w to a value smaller than 1 have occurred) where the steady state is again attracting and where current mortgage debt is decreasing over time.

The central bank starts by buying a fraction α of the volume of defaults, the bad loans $\varphi_d \lambda^a_w$. This increases the disposable income of asset holders[14] and leads to an

[14] Note that we here assume that the disposable income of asset holders is given by:

$$Y_c = rK + i\Lambda^a_w + \alpha\varphi_d(i)\Lambda^a_w.$$

This however does not alter the steady state of the model, since aggregate defaults remain the same (implying no change in the law of motion for λ^a_w), although part of them is now held by the central bank.

increase of either their reserve holding R (which we have ignored so far) or their investment, leading to an adjustment of the parameter i_f that may become permanent even if the central bank stops buying bad loans. In the case it continues to do so we have the situation that bad loans no longer lead to bankruptcy (loss of homes) of the corresponding worker households. One may assume in addition that the central bank starts demanding some low interest payment from them, but we do not consider this here explicitly.

Since asset holders now face less bad loans, in fact $(1-\alpha)\varphi_d\lambda_w^a$ due to the bailout exercised by the central bank, we have moreover assumed that the loan rate falls, as was intended by the central bank. This induces a further reduction in the amount of bad loans and thus (taken together) improves not only investment in real capital formation, but also the credit situation worker households are facing. There is less default and less bankruptcy (which was accompanied by the destruction of workers' homes) and cheaper credit for worker households and, as we have argued, increased robustness in the stability scenario of the private sector that this policy implies.

We summarise the stability analysis of this chapter by Figure 3.3 which provides another representation of what we have already considered in Figure 3.1. In Figure 3.3 we show this from the perspective of the borderline case that separates a positive denominator in the IS curve from a negative one (where debt increases are contractionary). We have that all marginal propensities c_w above this borderline represent the case of excessive consumption where the debt to capital ratio (in the steady state) is so high that it implies a profit-led IS curve despite a wage-led aggregate demand function. The cases B and C are again the unstable ones and case D represents a stable situation, since

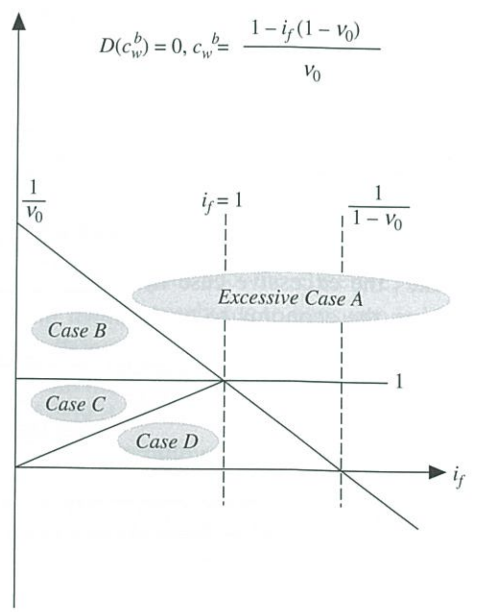


Figure 3.3 A alternative summary of the stability scenarios for a varying parameter c_w

demand as well as goods market equilibrium are profit-led so that flexible wages are then stabilising the economy.

The domains A and D are therefore the ones where the economy may be able to reproduce itself in the long run. The central bank should then only take care to keep the investment climate in an appropriate range and the loan rate on workers' debt reasonably low. Nevertheless it is still possible to conceive cases where there is a latent tendency towards instability (approached through investment weaknesses or investment booms), since in particular the regime of overconsumption may not be the best of all accumulation regimes.

3.7 Adding commercial banking

In this section we extend the model by allowing now for commercial banks as lending and depositary institutions in place of the lending activities of asset holders we have so far assumed as the basis for the mortgage loans to worker households. We thus no longer have to consider the reserve holdings R of asset holders which in fact could be, and indeed was, assumed to be zero due to the IS equilibrium assumption on the market for goods. Besides serving the investment plans of firms, asset holders now hold their remaining wealth as saving deposits at the commercial banks which now lend these saving deposits to a certain extent to the workers and in addition hold reserves in view of the deposits they have received from the asset holders. In a first step we will assume that reserves are adjusted towards a level of desired reserves at each moment in time, and that the remainder is given to worker households as a given mortgage loan rate. The worker households accept this supply of credit and invest it in residential space. This channels the savings of asset holders into consumption demand of workers and thus gives rise to the validity of Say's Law whereby the Keynesian regime is removed from the model (since reserves are also goods demand in this real model of capital and mortgage accumulation). We then investigate the ways in which Keynesian goods market rationing can again be introduced into this modelling framework and the business cycle implications to which it gives rise.

As before we have the sector of workers given by

Workers:

$$Y_w = vY - i_c\Lambda_w^a, \tag{3.41}$$

$$C_w = Y_w + \dot{\Lambda}_w, \tag{3.42}$$

$$\dot{K}_w = \dot{\Lambda}_w - \varphi_b K_w, \quad \varphi_b K_w = \varphi_d \Lambda_w^a, \tag{3.43}$$

$$\hat{L} = n = const. \tag{3.44}$$

Note however that the consumption of residential space by worker households is now dependent on the credit supply of banks and thus 'rationed' through the commercial banking sector (at the given loan rate i_c). All supplied credit (and only credit) is now assumed to be invested by worker households in residential space.

Asset holders no longer supply credit, but put all income that is not invested into their stock of real capital as saving deposits in the commercial banking sector (at a given interest rate i_d on these saving deposits):

Asset Holders:

$$Y_a = rK + i_d D, \quad r = y(1-v) - \delta, \tag{3.45}$$
$$C_a = 0, \quad [S_a = Y_a], \tag{3.46}$$
$$\dot{D} = Y_a - I, \tag{3.47}$$
$$W_a = K + D, \tag{3.48}$$
$$\dot{K} = I = i_f(r - r_o)K + nK. \tag{3.49}$$

The new sector is that of commercial banks, which use the new saving deposits $\dot{D}$ of asset holders to provide new credit $\dot{\Lambda}_w$ to worker households at a given level of actual reserve changes $\dot{R}$ at each moment of time. There is a given loan rate–deposit rate spread in this banking sector, implying that there are profits Y_c accruing to commercial banks, which are used to supply further credit to the sector of worker households. The net worth of commercial banks is given by W_c. Thus we have:

Commercial Banks (CBs):

$$Y_c = i_c \Lambda_w^a - i_d D, \tag{3.50}$$
$$C_c = 0 \quad [S_c = Y_c], \tag{3.51}$$
$$\dot{\Lambda}_w^a = \dot{D} + Y_c - \dot{R} - \varphi_d \Lambda_w^a, \tag{3.52}$$
$$\dot{\Lambda}_w = \dot{D} + Y_c - \dot{R}, \tag{3.53}$$
$$W_c = \Lambda_w^a + R - D. \tag{3.54}$$

Reserves are adjusted with some delay towards desired reserves, which provides a rule that allows us to determine residually the volume of loans supplied to workers:

$$\dot{R} = \beta_r(\psi^* - \psi)D = \beta_r(\psi^* D - R), \quad \psi = R/D, \tag{3.55}$$
$$i_c = (1+\mu)i_d. \tag{3.56}$$

We assume that the loan rate on mortgages i_c is given by a markup on the interest rate paid on the saving deposits of asset holders. For completeness we also introduce a very simple central bank sector into this extension of the baseline model of this chapter, which holds the reserves R of commercial banks and which sets the interest rate on saving deposits (by legislation), but which does not yet supply any high-powered money.

The Central Bank (CB):

$$R = R, \tag{3.57}$$
$$i_d = i. \tag{3.58}$$

The central bank therefore has just one instrument to influence the working of the economy (the rate of interest on asset holders' deposits). Otherwise it just controls the reserves held by the commercial banks, by setting the desired reserve ratio ψ^*, eventually enforcing higher minimum reserves if it decrees that the commercial banking sector is not working properly.

The next set of equations check that income matches expenditure in our set-up of a credit-financed economy:

$$\begin{aligned}
Y_w + Y_a + Y_c &= (vY - i_c\Lambda_w^a) + (Y(1-v) - \delta K + i_d D) + (i_c\Lambda_w^a - i_d D) = Y - \delta K,\\
C_w + I_f + I_c &= (vY - i_c\Lambda_w^a + \dot{\Lambda}_w) + I + \dot{R}\\
&= (vY - i_c\Lambda_w^a + \dot{\Lambda}_w) + (rK + i_d D - \dot{D}) + (i_c\Lambda_w^a - i_d D + \dot{D} - \dot{\Lambda}_w)\\
&= Y - \delta K.
\end{aligned}$$

Note that we consider only real goods (in a one-sector framework) which implies that also reserves of commercial banks represent real commodity inventories. The activities in the real sector therefore concern consumption proper and housing demand of worker households, net investment of firms and reserve holdings of commercial banks, describe a consumption function of worker households, an investment function of firms and the reserve adjustment of commercial banks. All remaining activities are residual ones.

The above extension of the baseline model of this chapter gives as new laws of motion:

$$\begin{aligned}
\hat{D} &= \frac{\dot{D}}{K}\frac{K}{D} = [r + i_d d - (i_f(r - r_o) + n)]/d, \quad r = y(1-v) - \delta,\\
\hat{R} &= \frac{\dot{R}}{K}\frac{K}{R} = \beta_r(\psi^* d - \rho)/\rho, \quad \psi^* = const, \rho = R/K,\\
\hat{\Lambda}_w^a &= \frac{\dot{\Lambda}_w^a}{K}\frac{K}{\Lambda_w^a}\\
&= [r + i_d d - (i_f(r - r_o) + n) + i_c\lambda_w^a - i_d d - \beta_r(\psi^* d - \rho) - \varphi_d\lambda_w^a]/\lambda_w^a\\
&= [r - (i_f(r - r_o) + n) + i_c\lambda_w^a - \beta_r(\psi^* d - \rho) - \varphi_d\lambda_w^a]/\lambda_w^a.
\end{aligned}$$

This in turn gives the intensive form representation of the model as

$$\begin{aligned}
\dot{d} &= y(1-v) - \delta + i_d d - (i_f(y(1-v) - \delta - r_o) + n) - (i_f(y(1-v) - \delta - r_o) + n)d,\\
\dot{\rho} &= \beta_r(\psi^* d - \rho) - (i_f(y(1-v) - \delta - r_o) + n)\rho,\\
\dot{\lambda}_w^a &= y(1-v) - \delta - (i_f(y(1-v) - \delta - r_o) + n) + i_c\lambda_w^a - \beta_r(\psi^* d - \rho)\\
&\quad - (i_f(y(1-v) - \delta - r_o) + n + \varphi_d)\lambda_w^a,
\end{aligned}$$

which is quite a different situation compared to with the original one (where asset holders supplied the credit to the worker households) and where only a single law of motion was needed to describe the dynamics of credit financing, namely (see equation (3.18))

$$\dot{\lambda}_w^a = (c_w - 1)(vy - i\lambda_w^a) - (i_f(r - r_o) + n + \varphi_d)\lambda_w^a, \quad \lambda_w^a = \frac{\Lambda_w^a}{K}.$$

The goods market equilibrium is now determined by

$$\begin{aligned} Y &= C_w + I_f + I_c + \delta K \\ &= (vY - i_c\Lambda_w^a + \dot{\Lambda}_w) + (rK + i_d D - \dot{D}) + (i_c\Lambda_w^a - i_d D + \dot{D} - \dot{\Lambda}_w) + \delta K, \end{aligned}$$

which in intensive form gives rise to

$$\begin{aligned} y &= (vy - i_c\lambda_w^a + \dot{\Lambda}_w/K) + (r + i_d d - \dot{D}/K) + (i_c\lambda_w^a - i_d d + \dot{D}/K - \dot{\Lambda}_w/K) + \delta \\ &= vy - i_c\lambda_w^a + r + i_d d + i_c\lambda_w^a - i_d d + \delta = y. \end{aligned}$$

We thus get that Say's Law is valid in such an economy, that is the goods market is in equilibrium at all levels of output Y, quite in contrast to the cases we have investigated earlier in this chapter. This holds since consumption is now endogenously determined by workers' disposable income and the credit that is supplied out of the saving deposits of asset holders by the commercial banks to them. Summing up the full dynamics of this extended case is given by the five laws of motion

$$\hat{v} = \beta_{we}(y^p/(zl) - \bar{e}), \tag{3.59}$$

$$\hat{l} = -i_f(y^p(1 - v) - r_o), \tag{3.60}$$

$$\dot{d} = y^p(1 - v) - \delta + i_d d - [i_f(y^p(1 - v) - \delta - r_o) + n](1 + d), \tag{3.61}$$

$$\dot{\rho} = \beta_r(\psi^* d - \rho) - (i_f(y^p(1 - v) - \delta - r_o) + n)\rho, \tag{3.62}$$

$$\begin{aligned} \dot{\lambda}_w^a = {} & y^p(1 - v) - \delta + i_c\lambda_w^a - \beta_r(\psi^* d - \rho) \\ & - [i_f(y^p(1 - v) - \delta - r_o) + n](1 + \lambda_w^a) - \varphi_d\lambda_w^a. \end{aligned} \tag{3.63}$$

The first two differential equations (3.59) and (3.60) provide the conventional Goodwin (1967) closed orbits growth cycle dynamics, but these are now appended with laws of motion for deposits d, actual debt λ_w^a and reserves ρ, all measured per unit of capital.

It is easily shown that the appended dynamics are asymptotically stable if the interest rates i_d, i_c are chosen sufficiently small, since the Jacobian at the steady state of the dynamical system (3.59)–(3.63) is characterised by

$$J = \begin{pmatrix} - & 0 & 0 \\ + & - & 0 \\ - & + & - \end{pmatrix}.$$

The interior steady state itself is given by

$$l_o = \frac{y^p}{z\bar{e}}, \quad v_o = \frac{y^p - r_o}{y^p}, \quad d_o = \frac{r_o - n}{n - i_d}, \quad \rho_o = \frac{\psi^* d_o}{1 + n/\beta_r}, \quad \lambda_{wo}^a = \frac{r_o - n(1 + \rho_o)}{\varphi_d + n - i_c}.$$

This implies that r_o must be sufficiently large and i_d, i_c sufficiently small in order to provide a meaningful set of steady state conditions.

The end result of the extension of the baseline model of this chapter is that we have returned to a Goodwin type supply driven model of the distributive cycle, augmented by stable dynamics of deposits of asset holders, reserves of commercial banks and mortgage debt of workers (all per unit of capital). This is a classical growth cycle model where asset holders not only accumulate real capital and where workers not only consume their real wages, but also buy residential space through the credit that is supplied to them by the commercial banks. We consider this as an interesting extension of the original Goodwin (1967) model, but one that has lost the Keynes component through the specific formulation of credit supply driven housing consumption of workers.

In order to show in a simple way how effective demand problems can be re-introduced into the extension of the baseline model of this chapter (which in fact is more a modification, since the consumption function of workers has been changed in this section) we now add the existence of high-powered money to the model, supplied by the central bank in line with the demand for this asset by asset holders in order to fix the interest rate on savings deposits, which we have already assumed above. We assume that asset holders want to add to their high-powered money holdings (or reduce it) according to a function $\dot{M} = \dot{M}(Y, r, i)$ with partial derivatives $\dot{M}_1 < 0$, $\dot{M}_2 < 0$, $\dot{M}_3 < 0$. Hence this function assumes that new money hoardings by asset holders decrease with economic activity (are negative in the boom and positive in the bust), decrease with declining profitability of firms (for a given state of economic activity) and decrease when the interest rate on the saving deposits of asset holders is increasing.

We assume moreover that the interest rate i is set by the central bank as a decreasing function of the rate of profit r with the value of i we have so far considered given by $i(r_o)$. We finally assume that there is a unique level of economic activity Y_o where $\dot{M}(Y_o, r_o, i(r_o))$ is zero.[15] This level of economic activity therefore now prevails in the steady state of the economy which is given as before, since $\dot{M}$ is zero there and the flow conditions of the model being of the same type as the previous ones. The central bank activities are now characterised by the expressions:

Central Bank (CB):

$$\begin{aligned} R + M &= R + M, \\ i_d &= i(r), \\ \dot{M} &= \dot{M}(Y, r, i). \end{aligned}$$

These equations summarise the balance sheet of the central bank, its interest rate policy and its new supply of high-powered money. Money supply is here still of a helicopter type, since there are no other financial assets in the present formulation of the model that can be subject to open market operations of the central bank. This is a provisional

15 This assumes that the condition $\dot{M}_2 + \dot{M}_3 i'(r) \neq 0$ is fulfilled.

assumption for the time being which will be removed in the more advanced model types considered in this book. It serves to illustrate in the simplest way possible how hoarding can lead to Keynesian effective demand problems in an otherwise classical framework.

For the consumption demand of workers we now have the expressions

$$\begin{aligned} C_w &= vY - i_c\Lambda_w^a + Y_a - I - \dot{M} + Y_c - \dot{R} \\ &= vY - i_c\Lambda_w^a + rK + i_d D - I - \dot{M} + i_c\Lambda_w^a - i_d D - \dot{R} \\ &= vY + rK - I - \dot{M} - \dot{R} \\ &= Y - \delta K - I - \dot{M} - \dot{R}. \end{aligned}$$

From the last expression we have for total demand $Y^d = C_w + I + \delta K + \dot{R}$ the simple representation

$$Y^d = Y - \dot{M}.$$

IS equilibrium then implies, as in the case of the baseline model, that such a change in asset holdings must be zero. It follows that $\dot{M}(Y, r, i(r)) = 0$, from which by means of the implicit function theorem we obtain $Y = Y(r)$, the derivative of which is given by

$$Y'(r) = -\frac{\dot{M}_2 + \dot{M}_3 i'(r)}{\dot{M}_1}. \tag{3.64}$$

We thus now have that economic activity is a decreasing or increasing function of the wage share v (since we have assumed that $\dot{M}_2 + \dot{M}_3 i'(r) \neq 0$) and this in a way that is completely different from the one we have used in the baseline model of this chapter. The real dynamics of the model of this section therefore now become, due to what we have here assumed for workers' consumption and the money demand of asset holders:

$$\hat{v} = \beta_{we}(y(v)/(zl) - \bar{e}), \tag{3.65}$$

$$\hat{l} = -i_f(y(v)(1-v) - r_o) = -i_f(r(v) - r_o). \tag{3.66}$$

We consider in the following analysis only situations where the rate of profit remains a negative function of the wage share, so that the effect of the wage share on Y is not that positive that it makes the overall effect of the wage share on the rate of profit a positive one. In this case the Jacobian of the real dynamics of the model is characterised at the steady state by

$$J = \begin{pmatrix} \beta_{we} y'(v_o)/(zl_o)v_o & - \\ + & 0 \end{pmatrix}. \tag{3.67}$$

In order to get the result that the steady state of the real dynamics is asymptotically stable we must therefore postulate that the monetary policy of the central bank is conducted in such a way that the condition

$$Y'(r) = -\frac{\dot{M}_2 + \dot{M}_3 i'(r)}{\dot{M}_1} > 0$$

holds true. This is exactly the case when the condition

$$\frac{dY(r(v))}{dv} = \frac{\dot{M}_2 r'(v) + \dot{M}_3 i'(r(v))r'(v)}{|\dot{M}_1|} < 0, \quad \text{that is,} \quad |i'(r)| > \frac{|\dot{M}_2|}{|\dot{M}_3|}$$

is fulfilled.

We thus get the result that the goods market appears, due to the money demand of asset holders, as if economic activity were wage-led. The interaction of the result on the market for goods with the law of motion of the wage share is thus an unstable one if an active interest rate policy of the central bank is absent, while an interest rate policy that is exercised with sufficient strength can remove this type of instability from the considered dynamics. We thus have the end result that credit rationing of a certain type can make the economy unstable and so there is a need for monetary policy in order to make it sustainable. This section therefore indicates in conjunction with the earlier sections how the baseline model of this chapter may be augmented in order to arrive at a Keynes–Goodwin growth cycle model where a variety of stability scenarios can be explored. In the next chapter we will by contrast focus on the debt financing behaviour of firms and the instability problems to which this gives rise.

3.8 Conclusions and outlook

In this chapter we have investigated a second type of crisis scenario, now within a closed economy and concerning worker households and their indebtedness. Adding to the crisis scenarios of Charpe *et al.* (2009), we have considered here a sequence of models (a sequence of parameter scenarios for a single three-dimensional dynamic model in fact) which run from situations of stable excessive overconsumption of worker households, to weakly excessive overconsumption and a certain degree of instability, and from there to the situation of a strict credit rationing of worker households. In the fourth model type, we considered actions that can rescue the economy from this last instability scenario and that imply a solution that leads the economy back to economic stability, with mortgage debt shrinking in time. The return to stability is based on investment stimuli (or alternatively, and not so attractive, further reductions in workers' marginal propensity to consume), coupled with a monetary policy that aims at a decrease in the loan rate and the default rate on the credit market.

In this latter type of economy, the loan rate will fall to a low value again and workers' debt will reduce in time through their now positive savings. In the limit, this economy would even converge to a situation where workers are lending to asset holders, since they have a positive savings rate in the steady state. Yet, in our view this would only be a transient phase in which the economy recovers to a certain degree from too high mortgage debt back to its normal functioning, perhaps subsequently followed again by the emergence of some behavioural parameter changes which lead the macroeconomy back to the stable overconsumption situation from which we started. The sequence of events analysed in this chapter thus may repeat itself in historical time.

In the next chapter we go on from workers' indebtedness to firms' debt accumulation and from there to processes of debt deflation as another important feedback channel that can endanger the viability of the entire economy in a downward direction. Compared with the considered exchange rate crises in Chapter 2 and the mortgage crises of the present chapter, the Fisher (1933) process of accelerating debt deflation in the private sector may be one that is very difficult to attenuate or even reverse by economic policy.

3.9 Appendix: some simulation studies of the baseline model

We consider in this appendix the system (3.16)–(3.18) in its various configurations from the numerical point of view.

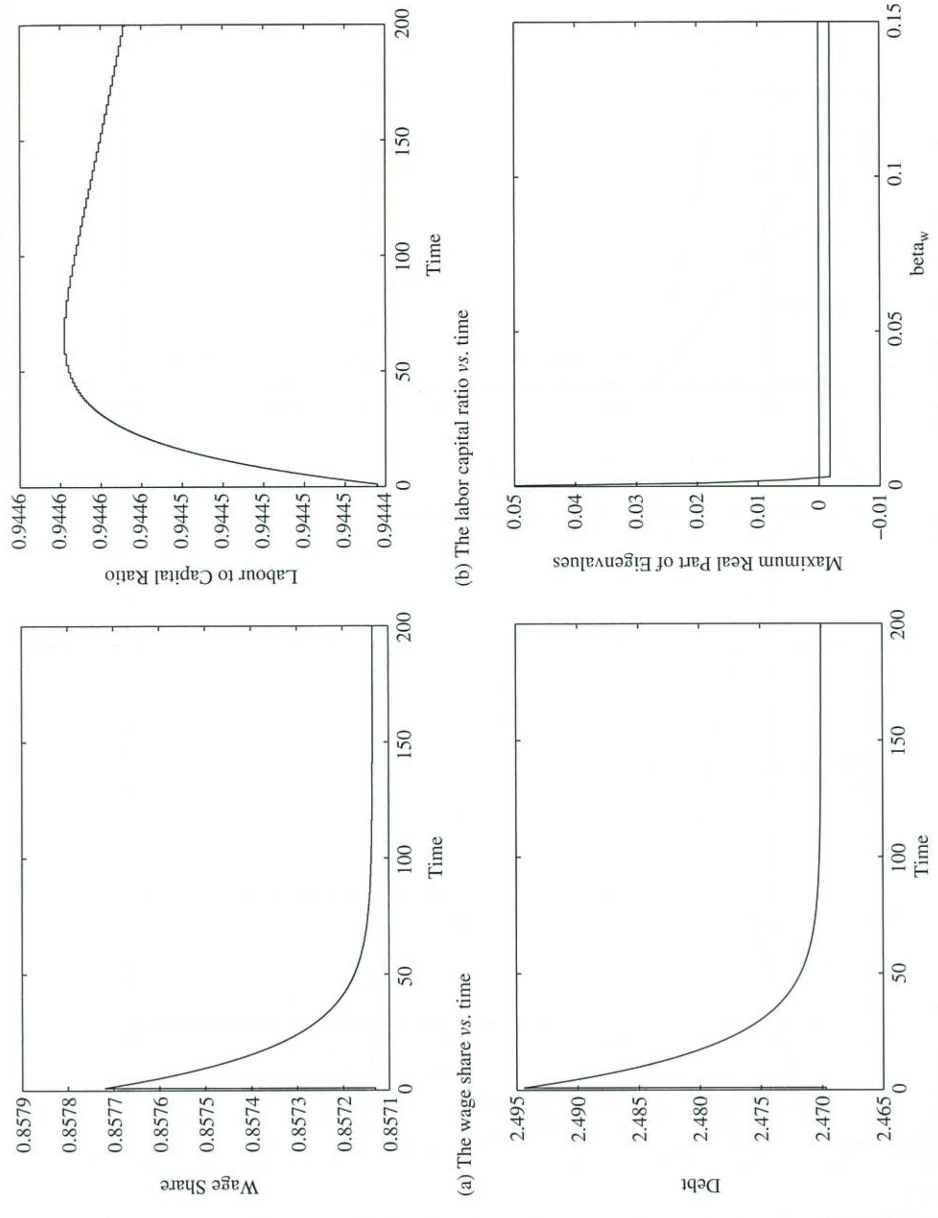
Excessive overconsumption $(c_w > 1 + (1 - i_f)(1 - v_o)/v_o > 1 > i_f)$
The first set of simulations illustrates the case of a profit-led economy. The propensity to consume is set at 1.2, which corresponds clearly to an excessive overconsumption case. In addition, $c_w < 4/3$, implies that the goods market dynamics are profit-led, even though aggregate demand is wage-led. The value of i_f is fixed at 0.02 and is smaller than c_w also implying that the impact of the wage share on output is negative.

The dynamics of the economy following a 1 per cent debt shock are displayed in Figure 3.4. The steady state is attracting even though convergence takes place at a slow pace. Convergence takes place as a result of the stable interaction between the wage share and the output to capital ratio, and despite the cumulative channel between debt and the wage share. As shown by the maximum real part of eigenvalues in Figure 3.4(d), increasing wage flexibility increases the stability of the system. Similarly, reducing β_w to 0.01 increases shock-dependence and economic fluctuations as we see from Figure 3.5.

Weakly excessive consumption $(i_f < 1 < c_w < 1 + (1 - i_f)(1 - v_o)/v_o)$
We now consider the case of weakly excessive overconsumption by way of Figure 3.6:

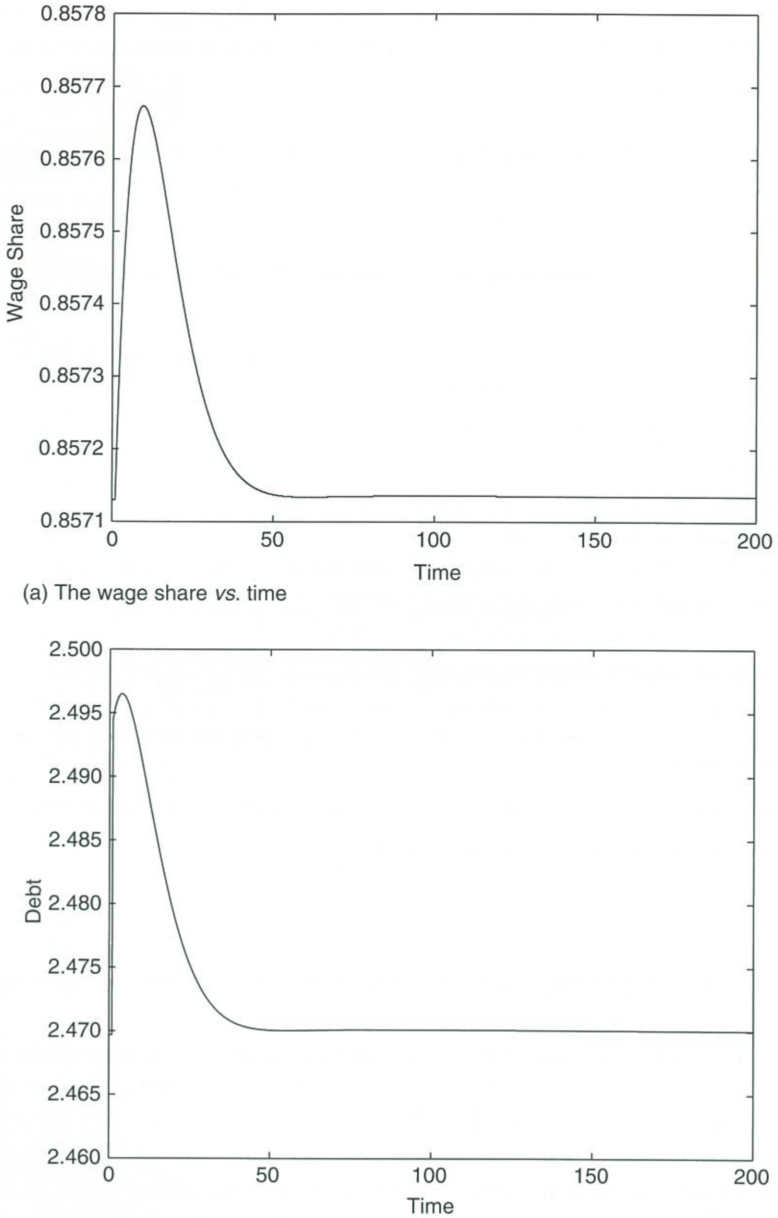
Figure 3.6(a) represents the wage-led case, where the value of c_w has been lowered from 1.2 to 1.05 such that the condition $1 < c_w < (1 + (1 - i_f)(1 - v_0)/v_0)$ holds. In the wage-led case, the unstable interaction between the wage share and the output to capital ratio is strongly unstable. The dynamics of the wage share and the output are cumulative. Any increase in the wage share fosters output, which feeds back positively on the wage share as employment increases. In the wage-led case, the Goodwin part of the model is strongly unstable. As stated in Proposition 3.4, the system loses stability even in the case $i = 0$ which turns off the impact of debt on the system. Forcing β_w to zero allows the unstable dynamic interaction between the wage share and output to be turned off. In such a case, the economy is stable as shown in Figure 3.6(b).

The overall impact of the propensity to consume on the stability of the system is summarised by the maximum real part of eigenvalues represented in Figure 3.6(c) and 3.6(d). In Figure 3.6(c), the propensity to consume lies between 1 and 1.1. At



(a) The wage share *vs.* time
(b) The labor capital ratio *vs.* time
(c) The debt level *vs.* time
(d) The max. real part of eigen-values *vs.* β_w

Figure 3.4 The dynamics of the economy following a 1 per cent debt shock – the profit-led case

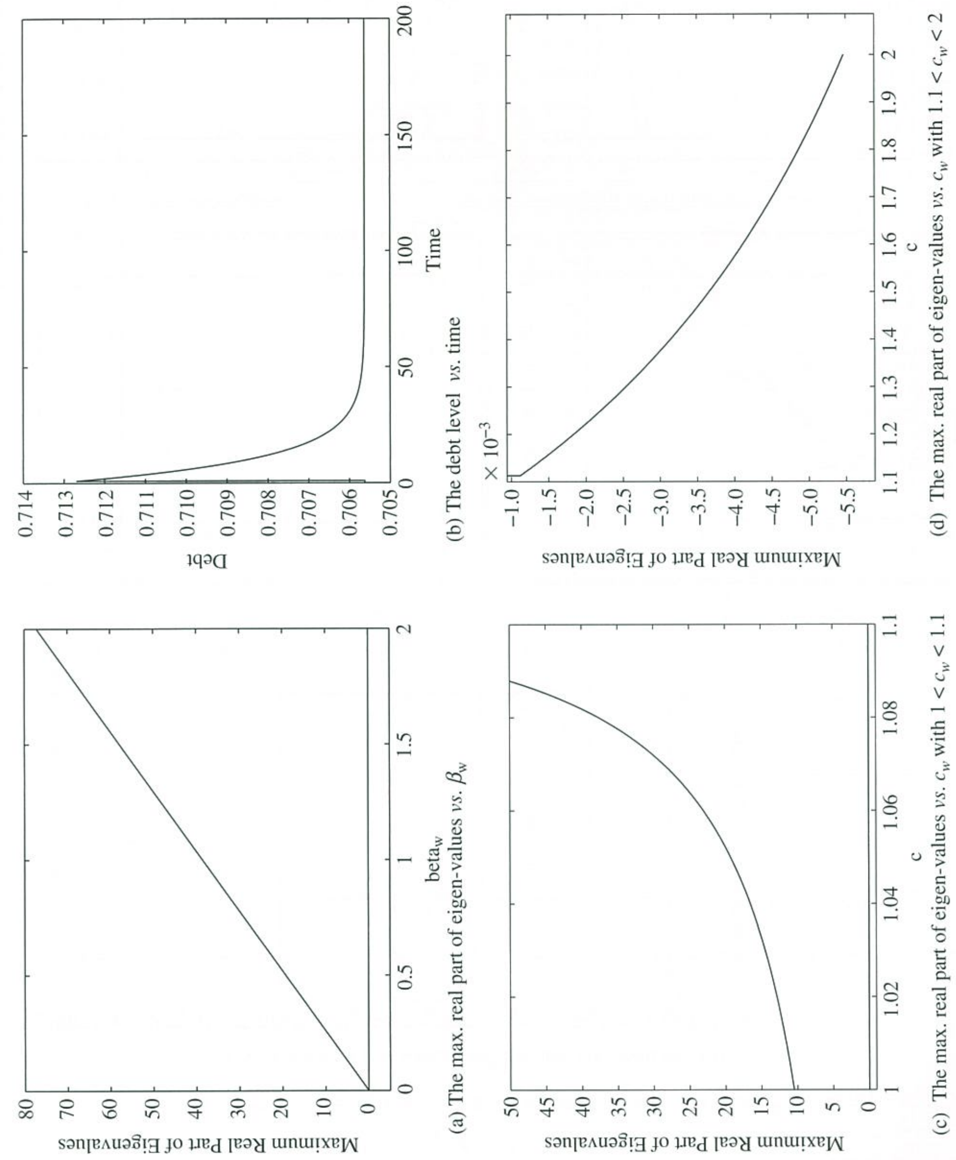


(a) The wage share *vs.* time

(b) The debt level *vs.* time

Figure 3.5 The dynamics of wage share and debt. The case of weak wage adjustment

$c_w = 1$, households can be seen as credit constrained. Households consume all their income as they do not have access to debt financed consumption. Full credit rationing implies a repelling steady state as proved in Proposition 3.5. Eigenvalues are positive and quickly increasing with respect to c_w. The economy is wage-led and unstable



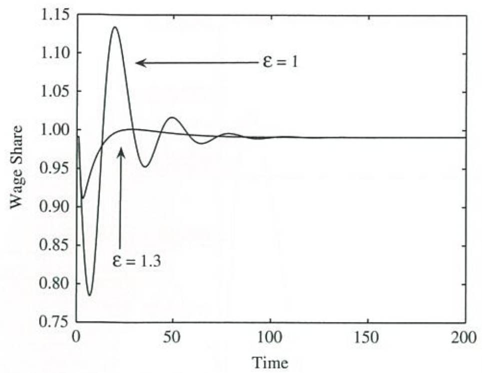
(a) The max. real part of eigen-values *vs.* β_w

(b) The debt level *vs.* time

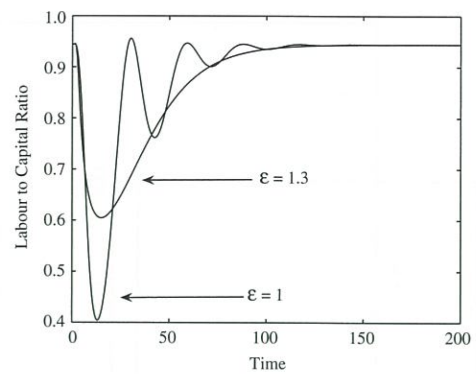
(c) The max. real part of eigen-values *vs.* c_w with $1 < c_w < 1.1$

(d) The max. real part of eigen-values *vs.* c_w with $1.1 < c_w < 2$

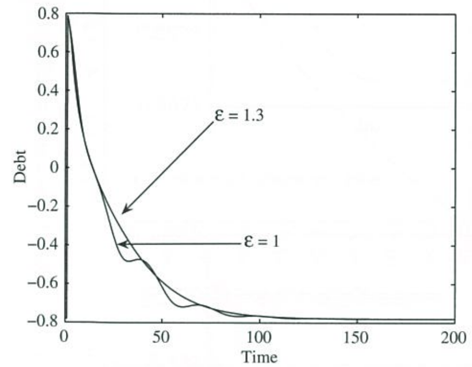
Figure 3.6 Eigenvalues and debt in the wage-led case



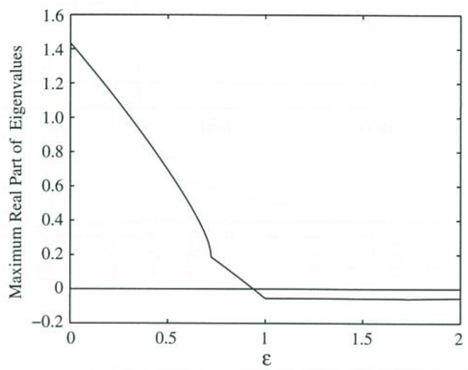
(a) The wage share *vs*. time.



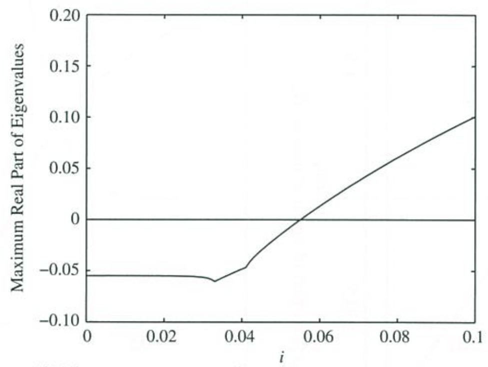
(b) The labor-capital ratio *vs*. time.



(c) The debt level *vs*. time.



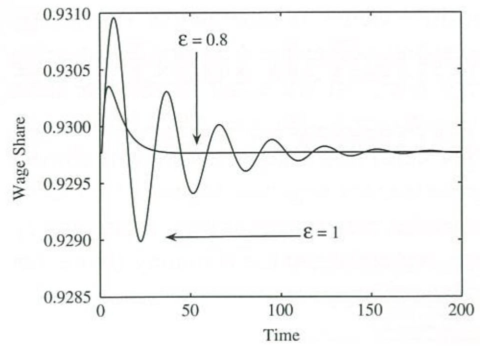
(d) The max. real part of eigen-values *vs*. 1_f.



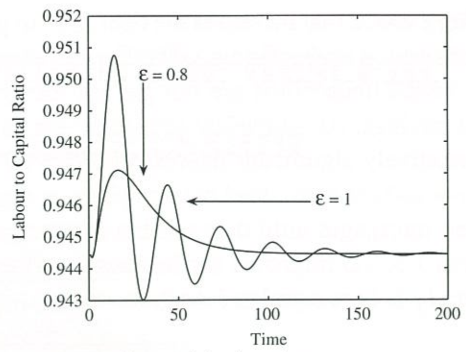
(d) The max. real part of eigen-values *vs*. i with $0 < i < 0.1$.

Figure 3.7 Stabilising the investment climate in the case when $i_f > 1$ and $c_w < 1$

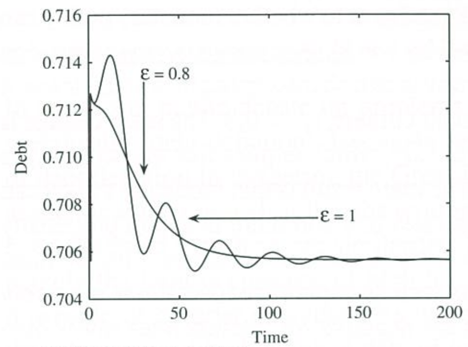
given the output, wage share interaction. Up to $c_w = 1.1$, the economy is wage-led and unstable. In Figure 3.6(d), the propensity to consume lies between 1.1 and 2, where the economy is profit-led. Now eigenvalues are negative and decreasing in c_w. The system converges given the stability of the wage, output interaction.



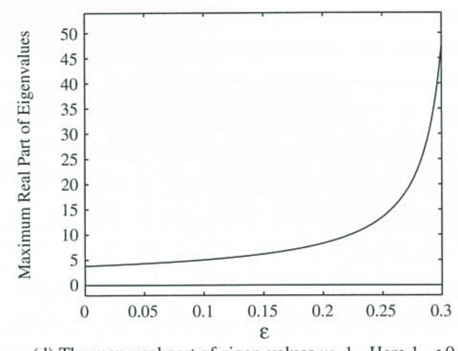
(a) The wage share *vs*. time.



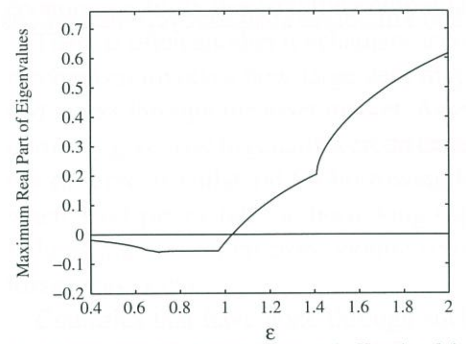
(b) The labor-capital ratio *vs*. time.



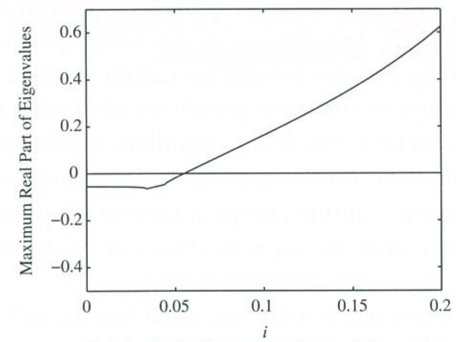
(c) The debt level *vs*. time.



(d) The max. real part of eigen-values *vs*. 1_f. Here $1_f < 0.3$.



(e) The max. real part of eigen-values *vs*. 1_f. Here $1_f < 0.4$.



(f) $0.4 < 1_f < 2$. The max. real part of eigen-values *vs*. i. Here $0 < i < 0.2$.

Figure 3.8 Stabilising the investment climate in the case $i_f > 1$ and $c_w > 1$

***Transition from crisis to 'pre-crisis' level** ($i_f \geq 1$ **and** $c_w < 1$)*

Figure 3.7 illustrates the role of monetary policy in stabilising consumer debt dynamics. The interest rate is kept low at 2 per cent, the investment climate variable i_f is larger than the propensity to consume c_w, which at $c_w = 0.95$ is lower than 1. A propensity to consume lower than 1 implies a negative debt ratio at the steady state. The aim of the simulations is to understand whether monetary policy can stabilise the economy after a shock that increases the debt ratio to positive values. In other words, the simulations aim at understanding whether monetary policy can restore a 'pre crisis' situation in which households are not getting mortgage debt, but are either rationed or have net savings. As shown by the two eigenvalues diagrams, Figures 3.7(d) and 3.7(e), a relatively significant interest rate ($i > 10$ per cent) and a good investment climate ($i_f > c_w$) are associated with stability, as eigenvalues are negative. Households reduce their mortgages until they reach a situation in which they are net savers. Increasing i_f from 1 to 1.3 improves the investment climate and stabilises the economy (Note that here i_f is here equal to $i + \epsilon$).

***Weakly excessive overconsumption and monetary policy** ($i_f > 1$ **and** $c_w > 1$)*

With respect to the previous simulations, they enquire as to whether monetary policy can stabilise the economy when it is characterised by weakly excessive overconsumption and strong instability. In Figure 3.8, the economy is still characterised by a small interest rate, $i = 2$ per cent, and a rather good investment climate, $\imath_f = 0.8$. The main change is that the propensity to consume is now equal to 1.05, which implies that the economy is in the unstable case illustrated in Figure 3.6. The main result is that monetary policy can restore stability if it manages to keep interest rates low and there is a high propensity to invest profits. However, contrary to the previous case, the investment climate must belong to a corridor. Negative expectations as well as euphoric expectations are likely to make the economy unstable. Eigenvalues are negative for interest rates lower than 6 per cent, as shown in Figure 3.8(f). In addition, eigenvalues are still negative for i_f lying between 0.4 and 1. Increasing i_f from 0.8 to 1 increases business cycle fluctuations and slows down convergence.

4 Debt deflation and the descent into economic depression

4.1 The debt deflation debate

In the recent public debate on problems of the world economy, 'deflation', or more specifically 'debt deflation', has again become an important topic. The possible role of debt deflation in triggering the Great Depression of the 1930s has come back into academic studies as well as into the writings of economic and financial journalists. It has been observed that there are similarities between recent global trends and the 1930s, namely the joint occurrence of high levels of debt and falling prices, the dangerous downside of cheaper cars and TVs when debt is high. Debt deflation thus concerns the interaction of high nominal debt of firms, households and countries and shrinking economic activity due to falling output prices and increasing real debt.

There is often another mechanism accompanying the one just mentioned. That other mechanism involves how large debt may exert an impact on macroeconomic activity and works through the asset market. Asset price inflation during economic expansions normally gives rise to generous credit expansion and lending booms. Assets with inflated prices serve as collateral for borrowing by firms, households or countries. In contrast when asset prices fall the borrowing capacity of economic agents shrinks, financial failures may set in, macroeconomic activity decreases and consequently large output losses may occur.

Countries that have gone through such booms and busts are some Asian countries (in particular Japan), Russia and Brazil in 1998 and 1999. In all of those countries asset price inflation and lending booms entailed a subsequent debt crisis and asset price deflation. Thus, the usual mechanism of debt deflation due to falling output prices has been accompanied by the asset price deflation mechanism.[1]

Some academic commentators have also criticised the single-minded preoccupation of certain central banks and the IMF with inflation, and have suggested that providing some room for inflation should be of help in preventing a global financial crisis. Moreover, global growth strategies, and the elements they could contain, continue to be

[1] For a recent detailed study employing a different approach, namely asymmetric information theory, see Mishkin (1998).

discussed in academic and policy circles. The need for a fundamental restructuring of the IMF and World Bank and a new financial architecture is continuously stressed based on the judgement that the world has, in the last decade, faced several severe financial challenges, with the latest being the biggest since the 1930s. Debt dynamics can be, as Reinhart and Rogoff (2009) have argued, a very destabilising force and therefore appear to be an important problem that the world economy may be facing.

This issue was visibly exemplified in the recent financial market meltdown in 2007–9. It began with the very large indebtedness in the US subprime (mortgage) market in 2007, evolved as a credit crisis through the US banking system in 2008/9, and subsequently spread worldwide, causing a worldwide financial panic, and staggering declines in global growth rates. This time, the usual boom-bust mechanism with the risen asset prices and a credit boom was reinforced by new financial innovations; specifically, the development of new financial intermediations through complex securities, such as mortgage-backed securities (MBS), CDO and credit default swaps (CDS). The complex securities, which were supposed to outsource and diversify idiosyncratic risk, have, jointly with the changes in the macroeconomic environment, actually accelerated risk taking and the boom, but also the bust. First through high asset prices and high leveraging and then, second, on the downside through the instability of credit via a credit crunch. These innovations provided the underlying financial intermediation mechanism through which the asset price boom and busts were fuelled. Although the actual way in which boom-bust cycles in asset prices and borrowing and lending evolve may change over time the mechanisms at work are very similar; for further details we refer the reader to Bernard and Semmler (2009).

Modern macroeconomic theory, as it has evolved since the Second World War, has paid scant attention to the above described mechanism of debt deflation. No doubt this is due to the fact that during that time the major economies in the world experienced a long period of growth followed by a long period of inflation from which we have only recently emerged. The classic study of debt deflation remains that of Fisher (1933), although Minsky (1975, 1982) in his writings on the financial instability hypothesis continued to warn of the dangers of another great depression. More recently Keen (2000) has focused attention on the Fisher debt effect and Minsky's financial instability hypothesis. There is therefore an urgent need for economists to model the process of debt deflation in its interaction with monetary and fiscal policies that may stop the process of rising debt, falling output and asset prices, and a collapse into depression.

In this chapter[2] we embed the process of debt accumulation and debt deflation via a sequence of partial models of debt accumulation and price deflation into fully integrated and consistent (with respect to budget constraints) macroeconomic models of closed and open economies. At the core of the model will be firms that finance fixed investment as well as involuntary inventory investment not from retained earnings, but by loans

[2] This chapter represents a reformulation and extension of the original contribution on debt deflation by Chiarella *et al.* (2001a), where the Fisher (1933) approach to debt deflation was embedded into a theory driven structural macroeconometric framework of the KMG variety.

from the credit market. In the current chapter we neglect equity finance. Our model will thus focus mainly on the first mechanism of the debt deflation process, the destabilising role of flexible wages and prices in economies with high nominal debt. The destabilising role of asset prices will be by and large neglected.[3]

Our macroeconomic model contains a sufficient number of agents and markets to capture the essential dynamic features of modern macroeconomies, and stresses the dynamic interaction between the main feedback loops of capital accumulation, debt accumulation, price and wage inflation (deflation), exchange rate appreciation (depreciation), inventory accumulation and government monetary and fiscal policies. Our modelling framework relies on previous work by the authors and contributions by other co-authors.[4] The essential difference is that here we focus on debt-financed investment of firms in place of pure equity financing considered in the earlier papers. We will thus add a further important feedback loop missing in our earlier approach to macro modelling, namely, from a partial point of view, the destabilising Fisher debt effect of deflationary (or inflationary) phases of capital accumulation arising from the creditor–debtor relationship between asset-owning households, banks and firms.

Keen (2000) has investigated the Fisher debt effect, between firms and financial intermediaries, in the context of an augmented classical growth cycle model of Goodwin (1967) type. He has found that it may imply local asymptotic stability for the overshooting mechanism of the growth cycle, but the overshooting can lead to instability, for high debt outside a corridor around the steady state of the model. In addition Keen provides an interesting discussion of Fisher's vision of the interaction of over-indebtedness and deflation and of Minsky's financial instability hypothesis. He extends the proposed model of the interaction of indebted firms and income distribution to also include a study of the role of government policies in such an environment. He focuses on nominal adjustment processes in place of the real ones of the classical growth cycle model.

We will start from Keen's 3D model of this process, expand it by flexible prices (to obtain a 4D model), by a Metzlerian quantity adjustment process, inflationary expectations and an interest rate policy rule and will finally provide a general 10D dynamical system exhibiting a complete representation of stock-flow interactions, adjusting prices and quantities, asset market dynamics and fiscal and monetary policy rules. While we concentrate on debt accumulation and its real implications in the lower dimensional versions we will nevertheless have a full set of stock-flow interactions,[5] but not yet alternative financing instruments of firms (equity, debt and retained profits) in the general 10D version of the dynamics that we consider.

[3] For work on credit market, economic activity and the destabilising role of asset price inflation and deflation, see Minsky (1975) and Mishkin (1998).

[4] See Chiarella and Flaschel (2000).

[5] Such interactions were still totally excluded in the basic 6D Keynesian price/quantity dynamics derived from the models of Keynesian monetary growth in Chiarella and Flaschel (2000); see the following for a brief representation of this KMG disequilibrium approach to AS-AD growth.

In the general framework we develop on the basis of the final model of this chapter in Part II and III we will discuss important further issues in the development of debt deflation, such as credit rationing, bankruptcy, bank and foreign exchange market crisis and domestic or foreign policy intervention. These issues need to be investigated, however, in much more detailed ways in future research in order to allow a full treatment of the dangers of the joint occurrence of debt and deflation in certain areas of the world economy or on a worldwide scale.

4.2 3D debt accumulation

In this chapter we develop the core Keynes—Metzler—Goodwin (KMG) dynamics with pure debt financing (and retained earnings) in place of pure equity financing; see Chiarella and Flaschel (2000) for their original formulation and Chiarella *et al.* (2005) for the further development of this type of analysis. We proceed in a stepwise fashion by starting from a simple 3D supply side dynamics as in Goodwin (1967), but now with debt in addition to pure profits as a means of financing the investment projects of firms. At the next step we introduce a law of motion for the price level and can therefore then start to consider deflationary processes in addition to the debt accumulation dynamics of the 3D case. This 4D extension also makes use, in a preliminary way, of a demand side restriction for the output decision of firms and thus departs from full capacity growth (where deflationary processes concerning the price level are hard to justify) towards fluctuating capacity utilisation of firms.

Yet, the above approach to aggregate demand and the role it plays in debt accumulation and deflationary processes still represents a simplifying approximation of a complete and consistently formulated delayed adjustment process of the output decision of firms towards fluctuating aggregate demand, as it is part of the 6D KMG approach to economic growth as developed in Chiarella and Flaschel (2000, Ch. 6).

In a third step we therefore add the delayed Metzlerian output-inventory adjustment mechanism to the considered 4D dynamics and arrive thereby at a 7D dynamical system of the KMG variety, with the law of motion for the debt to capital ratio as a new differential equation and the sole representation of the stock-flow interaction of actual economies. We furthermore also add a law of motion for the rate of interest which increases the dimension again by one. In contrast to the KMG growth model considered by Chiarella and Flaschel (2000, Ch. 6), see here Section 4.4, we now have two further laws of motion, since this earlier approach determined the rate of interest by an equilibrium condition and since the evolution of equity quantities and prices did not yet feed back into the price-quantity-growth dynamics of this basic prototype model of the dynamics of Keynesian monetary growth.

We started in Chiarella and Flaschel (1999c) and Chiarella *et al.* (2001a, 2001b) the analysis of the macrodynamics of debt deflation with a very general 20D model (of applied type) and then approached the understanding of the role of debt and deflation in such a framework from an extended 3D supply side growth cycle dynamics of Goodwin (1967) type, as formulated and investigated in Keen (2000), which included

loans to firms and thus debt financing of (part of) their investment expenditures in a very fundamental way. In the present chapter we will pursue an opposite approach by starting from the basic 3D framework of the Keen (2000) model and by developing it into 4D and 8D models which make it more and more complete and also consistent in its feedback structure between prices, quantities, expectations and rates of interest in particular.

Taken together, and based on the linear behavioural assumptions still used in our earlier approaches to debt and deflation, the equations of the Keen (2000) model can be represented as a 3D dynamical system in the state variables $v = wL^d/pY^p$ the wage share, $e = L^d/L$ the rate of employment, and $\lambda = \Lambda/pK$ the debt to capital ratio of the firms, as shown below. We note that the price level p is kept fixed in this core version of the Keen model[6] (and set equal to one for notational simplicity) and that the rate of interest $\bar{i}$ is also a given magnitude in this model, just as the reference (minimum) target rate of profit $\bar{r}$ which is compared with the actual pure rate of profit $r = y^p(1-v) - \delta - \bar{i}\lambda$ by firms in their investment decision.

In contrast to the macrodynamics developed in Chiarella and Flaschel (2000) we give budget equations a role to play in the price-quantity-growth dynamics of their KMG prototype model. However, only the budget equation of firms will really be of importance in the models of debt and deflation in this section, since the dynamics of the government budget constraint and interest payments of the government are still suppressed in the 7D model by way of appropriate assumptions and thus left for future research.

The 3D dynamics which are to be investigated in this section, and which are based on a Phillips curve (PC) mechanism, debt and profit driven investment behaviour and the budget restriction of firms, read (note that we still have $p = 1$):

$$\hat{v} = \beta_w(e - \bar{e}) - n_l, \quad e = L^d/L, \tag{4.1}$$

$$\hat{e} = g_k - (n + n_l), \quad g_k = I/K = \alpha_r^k(r - \bar{r}), \tag{4.2}$$

$$\dot{\lambda} = g_k(1 - \lambda) - r, \quad \lambda = \Lambda/K, \tag{4.3}$$

with the following definitions and supplementary algebraic equations[7]

$$\begin{aligned} e &= L^d/L, && L^d = Y/z, \hat{z} = n_l = \text{ const.}, \hat{L} = n = \text{ const.}, \\ C_w &= wL^d && \text{(budget equation: workers)}, \\ C_b &= \bar{i}\Lambda - \dot{\Lambda} && \text{(budget equation: financial capitalists)}, \\ I &= rK + \dot{\Lambda} && \text{(budget equation: industrial capitalists)}, \end{aligned}$$

[6] Assuming that price inflation follows wage inflation via instantaneous or delayed markup pricing would introduce deflationary forces originating in the labour market into this framework which however will be introduced only later on in the 4D demand side reformulation of the model.

[7] We use the term 'financial capitalist' for the suppliers of credit, for instance through commercial banks, and the term 'industrial capitalist' to denote the owners of firms that are also partly financed through retained earnings in their investment behaviour.

$$r = y^p(1-v) - \delta - \bar{i}\lambda \qquad \text{(pure rate of profit)},$$
$$v = wL^d/Y \qquad \text{(wage share)},$$
$$y = y^p = y^p K, \quad y^p = \text{const.} \qquad \text{(potential output)},$$

with $\bar{e} \in (0,1)$ a benchmark (yet not the NAIRU) rate of employment, $\bar{r} > 0$ the required rate of profit and $\bar{i} > 0$ the given rate of interest.

Given the way the model is constructed we have to assume that $C_b \geq 0, I \geq -\delta K$ hold true, which suggests (via later steady state considerations and on the intensive form level)

$$\bar{i} \geq \hat{\Lambda} \quad \text{(always)} \quad \Rightarrow \hat{\Lambda} = \hat{K} = n + n_l \text{ (in the steady state)},$$
$$r \geq \bar{r} - \delta/\alpha_1^k \text{ (always)}.$$

We note that the budget equations imply $C_w + C_b + I = Y - \delta K$, which is Say's Law in its simplest form in an economy with investment expenditures. There is thus no reason for deflation from the side of goods market behaviour in this model type (but there may of course be deflationary forces in labour market dynamics). Implicitly contained in the above equations also is the fact that the loans demanded by industrial capitalists are always supplied by financial capitalists, up to the point where $C_b = 0$ becomes binding. The credit supply is thus limited by interest income of financial capitalists in a very narrow way. Financial capitalists cannot supply new money in this 3D approach (in fact money M is totally neglected in the above formulation of the model) and thus cannot extend the above financing constraint via the mechanism $i\Lambda + \dot{M} = \dot{\Lambda} + C_b$. Could they do so, they would introduce money into the system and would then also allow aggregate demand for goods to deviate from aggregate supply, which at one and the same time would invalidate Say's Law and establish a Keynesian demand driven system in place of the above classical 3D supply side dynamics.

Equation (4.1) is based on a linear real-wage PC $\hat{\omega} = \widehat{w/p} = \beta_w(e - \bar{e})$ from which the growth rate n_l of labour productivity z has to be deducted in order to arrive at the growth law for the wage share $v = wL^d/pY = \omega/z$. Equation (4.2) says that capital stock growth has to be diminished by labour force growth n and productivity growth n_l in order to arrive at the growth law for the rate of employment (in our set-up of a fixed proportions technology z, y^p with Harrod neutral technical progress of the given rate $n_l = \hat{z}$). Equation (4.3), finally, is the new and difficult one of this extension of a Goodwin (1967) type growth model towards the inclusion of debt-financed investment. It is derived as follows:

$$\hat{\lambda} = \hat{\Lambda} - \hat{K} = \frac{\dot{\Lambda}}{K}/\lambda - \frac{I}{K} = \left(\frac{I}{K} - r\right)/\lambda - \frac{I}{K}.$$

This implies the differential equation (4.3) for the debt to capital ratio λ.

This closes the discussion of our partly debt-financed model of capital accumulation and cyclical growth. It is obvious from equation (4.3) that the dynamics of debt

accumulation as they derive from the budget constraints of firms introduce a severe non-linearity into the simple Goodwin (1967) type real growth dynamics.

Proposition 4.1 (The balanced growth path of the model)
Assume $\alpha_r^k > 1$. There is a unique interior steady state of the dynamical system (4.1)–(4.3) given by

$$e_o = \bar{e} + n_l/\beta_w > \bar{e},$$
$$\lambda_o = 1 - \frac{r_o}{n + n_l} < 1, \quad r_o = \bar{r} + (n + n_l)/\alpha_r^k > \bar{r},$$
$$v_o = \frac{y^p - \delta - r_o - \bar{i}\lambda_o}{y^p}.$$

Proof: The value for e_o follows immediately from setting equation (4.1) equal to zero ($v \neq 0$). Next, the value of r_o can be calculated from equation (4.2) for $e_o \neq 0$ by setting the right-hand side equal to zero, and solving for r_o. Furthermore, inserting the resulting value for r_o into equation (4.3), setting $g_k = n + n_l$ and solving for λ_o immediately implies the steady state value for λ, which must be smaller than 1, due to $r_o > \bar{r} > 0$. The steady state value for v_o finally follows from the definition of the pure rate of profit $r = y_p(1 - v_o) - \delta - \bar{i}\lambda_o$. □

Remark 4.1 We assume that the parameters of the model are such that the inequalities $\lambda_o, v_o > 0$ hold true. Just as in Goodwin's (1967) growth cycle model there is in addition a border steady state that is given by $v_o = e_o = 0$, and λ_o the solution of

$$\alpha_r^k(y^p - \delta - \bar{i}\lambda - \bar{r})(1 - \lambda) - (y^p - \delta - \bar{i}\lambda) = 0.$$

Remark 4.2 In order to get a positive steady state debt to capital ratio it is necessary and sufficient that $r_o < n + n_l$ holds, so that the steady growth rate $n + n_l$ exceeds the growth rate that could be generated by investing pure profits solely. The condition $r_o < n + n_l$ in turn is equivalent to $\alpha_r^k > \frac{n+n_l}{n+n_l-\bar{r}}$, that is the propensity to invest must be chosen sufficiently large for this purpose, and the larger it is the closer the minimum rate of profit $\bar{r}$ will be to the natural rate of growth $n + n_l$. Note furthermore that the steady state level of the wage share is determined after the determination of r_o, λ_o and thus residually after income distribution for industrial and financial capital has been determined. An increase in the interest rate $\bar{i}$ only reduces the wage share and is thus of no concern for the pure rate of profit in the steady state. Note finally that an increase in $\bar{r}$ reduces both interest income and wage income per unit of capital.

Proposition 4.2 (The positive contribution of debt financing)[8]
Assume $\alpha_r^k > 1$. The steady state of the dynamics (4.1)–(4.3) is always locally asymptotically stable.

[8] Note that the Goodwin (1967) growth cycle model can be obtained by assuming $\alpha_1^k = 1, \bar{r} = 0$.

Proof: Evaluating the Jacobian J of system (4.1)–(4.3) at the steady state gives for its third row the expression

$$(\alpha_r^k(1-\lambda_o)-1)(r_u, 0, r_\lambda)-(0,0,n+n_l)$$

and for its second row $\alpha_r^k(r_u, 0, r_\lambda)$. Since $1-\lambda_o = 1/\alpha_r^k + \bar{r}/(n+n_l)$ we know that $\alpha_r^k(1-\lambda_o)-1$ must be positive. The sign structure of the Jacobian is therefore given by[9]

$$J = \begin{pmatrix} 0 & + & 0 \\ - & 0 & - \\ - & 0 & - \end{pmatrix}.$$

According to the Routh–Hurwitz theorem (see e.g. Gantmacher (1959)), we have to show that

$$a_1 = -\text{trace } J > 0, \quad a_2 = J_1 + J_2 + J_3 > 0, \quad a_3 = -\det J > 0, \quad a_1 a_2 - a_3 > 0,$$

holds, where J_1, J_2, J_3 are the three principal minors of order two with the index of J indicating the row and column that is ignored in the principal minor concerned.

Obviously, trace $J = -\gamma - (\alpha_1^k - 1)\bar{i} < 0 \quad (a_1 > 0)$ holds. Furthermore, when calculating the determinant of J, the third row term

$$(\alpha_r^k(1-\lambda_o)-1)(r_u, 0, r_\lambda)$$

can be removed without change in this determinant, giving rise to

$$\det J = \begin{vmatrix} 0 & + & 0 \\ - & 0 & - \\ 0 & 0 & -(n+n_l) \end{vmatrix} < 0 \quad (a_3 > 0).$$

Next, we immediately obtain

$$J_1 = \begin{vmatrix} 0 & - \\ 0 & - \end{vmatrix} = 0, \; J_2 = \begin{vmatrix} 0 & 0 \\ - & - \end{vmatrix} = 0, \; J_3 = \begin{vmatrix} 0 & + \\ - & 0 \end{vmatrix} > 0,$$

and thus $a_2 > 0$ holds true as well. Finally, $a_1a_2 - a_3 = -(\alpha_1^k(1-\lambda_o)-1)r_\lambda > 0$ since $r_\lambda = -\bar{i} < 0$ and since $J_{33} = -(\alpha_1^k(1-\lambda_o)-1)\bar{i} - (n-n_l)$ dominates the element $-(n+n_l)$ that appears in the above calculation of det J $\quad (\lambda_o < 1!)$. □

Remark 4.3 1. Adding partial debt financing of the investment of firms therefore always turns the centre type dynamics of the Goodwin (1967) growth cycle model into convergent dynamics. Note here again that the two constraints $C_b \geq 0, I \geq -\delta K$ imply for the pure rate of profit r that it must stay in the interval $[\bar{r}-\delta, (\bar{i}\lambda + \alpha_1^k \bar{r})/(\alpha_1^k - 1)]$ which establishes a corridor for this rate that must be ensured by appropriate non-linearities when its boundary is approached. We suggest here however that the lower and upper bound on the pure rate of profit will rarely become binding.

[9] This sign structure shows that debt evolution interacts in a very specific way with the cross-dual dynamical structure of the first two laws of motion.

2. Introducing credit into the Goodwin (1967) growth cycle thus makes their dynamics convergent, and this for all $\bar{i}, \bar{r}, n+n_l > 0$, so that interest, profit and growth are needed here (if $C_b \geq 0, \lambda_o > 0$ is not imposed).

3. We assert, but do not prove this here, that the addition of a term like

$$\alpha_i^k(\bar{\lambda}-\lambda),$$

where $\bar{\lambda}$ is a target debt to capital ratio makes the determination of steady state values r_o, λ_o more involved, but does not alter the stability properties of the dynamics. Adding to the pure budget *constraint* dynamics (4.3) a direct impact of the debt to capital ratio on investment *behaviour* therefore enhances the convergence of the system back to the steady state (with a debt to capital ratio $\lambda_o \neq \bar{\lambda}$ in general).

4. Ignoring the credit constraint $\bar{i} \geq n+n_l \quad (C_b \geq 0)$ requires the interpretation that $-C_b$ commodities have to be supplied by financial capitalists in order to ensure Say's Law on the market for goods. In the present simple supply side form of the model we therefore face the difficulty of justifying the case where the interest rate $\bar{i}$ is below the steady growth rate $n+n_l$ of output and the capital stock, a case that is often stressed in its importance if the dynamics of the government budget constraint are introduced and investigated.

5. In order to avoid credit rationing of the type $\dot{\Lambda} = \bar{i}\Lambda - C_b, C_b \geq 0$ one may introduce a flexible interest rate, for example by way of the law of motion

$$\dot{i} = \beta_r(\bar{\lambda}-\lambda), \tag{4.4}$$

according to which financial capitalists attempt to steer the economy to a desired debt to capital ratio $\bar{\lambda}$. Checking the steady state conditions, however, reveals that $\bar{\lambda}$ cannot be chosen arbitrarily, but has to be determined (as before) by

$$\bar{\lambda} = 1 - \left(\frac{\bar{r}}{n+n_l} + \frac{1}{\alpha_r^k}\right).$$

Making this choice furthermore implies that the steady state rate of interest cannot be determined in a unique fashion from the remaining steady state conditions, but can in effect be arbitrarily given (only subject to certain economic boundary conditions). For the Jacobian J of the 4D dynamics (4.1)–(4.4) one furthermore gets det $J = 0$ which confirms the indeterminacy of interest rate from another angle. The Routh–Hurwitz conditions in the 4D case read

$$a_1, a_2, a_3, a_4 > 0, b_o = a_1a_2 - a_3 > 0, b_1 = a_3b_o - a_1^2a_4 > 0,$$

where a_2, a_3 consist of the sum of minors of orders 2 and 3 respectively. It is easy to show that these conditions are all fulfilled in the extension of (4.1)–(4.3) by (4.4), up to $a_4 = \det J$ which is zero here. We therefore find that the dynamics are again convergent, up to the levels of i and v which are here dependent on historical conditions and exogenous shocks as the system evolves. Let us add here also that the system

(4.1)–(4.4) is characterised by two cross-dual adjustment mechanisms of Goodwin (1967) type, one (the conventional one) for the real part of the economy

$$e \overset{+}{\longmapsto} \hat{v}, \quad v \overset{-}{\longmapsto} \hat{e},$$

and one for its financial part

$$\lambda \overset{+}{\longmapsto} \dot{i}, \quad i \overset{-}{\longmapsto} \dot{\lambda},$$

which are here coupled with each other by the definition of the pure rate of profit, namely $r = y^p(1-v) - \delta - i\lambda$. However, due to the zero-root hysteresis in the financial part of the economy these two adjustment mechanisms differ considerably in their working.

6. We stress finally that the system so far investigated is characterised by $p = 1$, $e^c = Y/Y^p = 1$, assumptions that will be successively relaxed in the following sections of the chapter.

Proposition 4.3 (The negative contribution of debt financing)[10]
We consider the situation of Proposition 4.2, but in addition that $\beta_w = 0, n_l = 0$ holds, so that there is no adjustment in the wage share occurring when the other two state variables of (4.1)–(4.3) are changing. Then for each level of the wage share v satisfying $y^p(1-v) - \delta > \bar{r}$, (which allows for minimum profitability) there exists a threshold value $\tilde{\lambda} \geq 0$ of the debt to capital ratio λ above which this ratio will increase beyond any bound according to the then isolated dynamics (4.3).

Proof: If the state variable v is stationary by assumption, we get that the third law of motion of the dynamics represents an autonomous differential equation in the variable λ and is then given by

$$\dot{\lambda} = \alpha_1^k \bar{i}\lambda^2 - [(\alpha_1^k - 1)\bar{i} + \alpha_1^k r^c]\lambda + (\alpha_1^k - 1)r^c - \bar{r},$$

with r^c being defined by $y^p(1-v) - \delta - \bar{r}$, which we can assume to be positive under normal conditions of the working of the economy. The right-hand side of this equation represents a polynomial of degree 2, $p(\lambda) = c_o\lambda^2 + c_y\lambda + c_h$ with $c_o > 0, c_y < 0$. The unique minimum of this quadratic function is at $\lambda = -c_y/(2c_o) > 0$ and it exhibits of course only positive values after the larger of its two roots has been passed (if this root is real, otherwise all values of $p(\lambda)$ are positive for all $\lambda > 0$). Initial values of the debt to capital ratio λ which lie to the right of this root (if real, and to the right of zero if not) therefore imply a purely explosive behaviour of this ratio, since $\dot{\lambda}$ is and remains positive. □

We have pointed out above that, as a minimum requirement, the side condition $r \geq \bar{r} - \delta/\alpha_1^k$ should always be fulfilled in order to allow for economically meaningful trajectories (along which gross investment should always stay non-negative). Of course, negative rates of profit r may also be excluded from consideration by economic

[10] Note that the case that is considered here can and must be approached as a limit case by reducing both n_l, β_w in size simultaneously.

reasoning. The threshold for an explosive evolution of the debt to capital ratio found to exist in Proposition 4.3 may however still be so large that explosiveness can only occur in a domain where the system is not economically viable in the above sense. In this case the proposition simply states that the dynamics will not always be globally asymptotically stable from the purely mathematical point of view, but does not yet prove that critical developments in the debt to capital ratio may also come about at initial situations to which there corresponds an economically meaningful environment. To show that such situations will indeed exist is the aim of the following Proposition 4.4.

Proposition 4.4 (Threshold values for monotonic divergence)
We consider the situation of Proposition 4.2. We assume again that $\beta_w = 0, n_l = 0$ is given and in addition that $n > \bar{i}$ holds. Then, for the steady state value of the wage share, v_o, the threshold value $\tilde{\lambda} \geq 0$ of the debt to capital ratio λ of Proposition 4.3 implies a rate of profit $r \in (0, \bar{r})$. The considered dynamics (4.1)–(4.3) therefore become divergent (e monotonically decreasing and λ monotonically increasing) for values of λ that lie in an economically meaningful part of the state space.

Proof: First, we show that the threshold value $\tilde{\lambda} > 0$ (after which $\dot{\lambda}$ is positive in the considered situation) must be larger than one if $n > \bar{i}$ holds true. To see this it suffices to show that the polynomial considered in Proposition 4.3 is still negative at $\lambda = 1$. At $\lambda = 1$ we have $\dot{\lambda} = -r = y^p(1-v_o) - \delta - \bar{i}$. This gives $\dot{\lambda} = -[r_o + (\bar{i}\lambda^o - \bar{i})]$. Inserting the steady state values for r_o, λ_o from Proposition 4.1 into this expression then implies:

$$\dot{\lambda} = -\left[\bar{r} + \frac{n}{\alpha_1^k} + \bar{i}\left(\frac{\alpha_1^k - 1}{\alpha_1^k} - \frac{\bar{r}}{n} - 1\right)\right],$$

which upon rearrangement becomes

$$\dot{\lambda} = -[\bar{r}(1 - \frac{\bar{i}}{n}) + \frac{1}{\alpha_1^k}(n - \bar{i})] < 0,$$

where the inequality is due to the assumption $n > \bar{i}$. From this result there follows immediately that the second real root of the considered polynomial, $\tilde{\lambda}$, must be larger than one (while the first coincides with the steady state due to our assumption $v = v_o$).[11]

Let us now calculate the rate of profit $\tilde{r}$ at this threshold value $\tilde{\lambda}$. Since we have $\dot{\lambda} = 0$ at this value, we get for $\tilde{r} = y^p(1-v_o) - \delta - \bar{i}\tilde{\lambda}$ the expression:

$$0 = \alpha_1^k(\tilde{r} - \bar{r})(1 - \tilde{\lambda}) - \tilde{r},$$

which in turn gives

$$\tilde{r} = \frac{\alpha_1^k \bar{r}(1 - \tilde{\lambda})}{\alpha_1^k(1 - \tilde{\lambda}) - 1} = \frac{\bar{r}}{1 - 1/(\alpha_1^k(1 - \tilde{\lambda}))},$$

[11] Note that the smaller root can be negative, meaning that firms are creditors not debtors in the steady state, if $\bar{r} > 0$ and if the parameter α_1^k is sufficiently close to one.

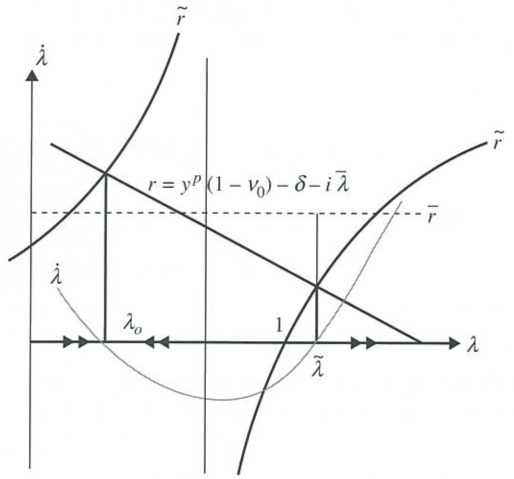


Figure 4.1 Debt dynamics around the steady state share of wages

from which we calculate that

$$\tilde{r}'(\tilde{\lambda}) > 0, \lambda \neq 1 - 1/\alpha_1^k.$$

Due to the above considerations we know however that the denominator of this expression is larger than one which implies that $\tilde{r}$ must lie in the open interval $(0, \bar{r})$ at the point $\tilde{\lambda}$. □

Note that the polynomial considered always has two positive real roots in the situation considered in Proposition 4.4, since its left one must exist and be equal to λ_o, the steady state value of the debt to capital ratio λ. Note also that the slope of the polynomial is negative there, in line with the stability result we obtained in Proposition 4.2.

The situation considered in Proposition 4.4 and its proof can be represented graphically as shown in Figure 4.1. The actual pure rate of profit r, for each value of the debt to capital ratio λ, is shown as a straight line in this figure. It cuts the $\tilde{r}$ function at $r_o(\lambda_o), \tilde{r}(\tilde{\lambda})$ where we have given either the steady state solution or the threshold after which $\dot{\lambda}$ becomes purely explosive. We stress that the figure in this form only applies to situations where $\bar{i} < n$ holds true. In the opposite case we have $\dot{\lambda} > 0$ at $\lambda = 1$ and thus for $\tilde{r}$ where $\dot{\lambda} = 0$ holds a value smaller than 1. Since the $\tilde{r}$ function does not depend in its position on the size of $\bar{i}$ we thus get that r must be negative at the second root of the given quadratic equation. The case $\bar{i} > n$ therefore implies a smaller basin of attraction for the state variable λ, but also a more rapid decline of r for increasing λ.

Should a shock throw the economy out of the steady state to a value of λ slightly above the threshold value $\tilde{\lambda}$ it will be caught in a situation where λ is monotonically increasing accompanied by a falling rate of employment e until trajectories leave the domain of economically meaningful values for these two state variables. We stress

that this result is obtained on the basis of a wage share that remains fixed at its steady state value and which therefore neither improves nor worsens the considered situation through its movements in time. This result will also hold true for all adjustments in the wage share that are sufficiently slow. At present it is however not clear whether a strongly falling wage share (based on a high value of the parameter β_w), which significantly improves the profitability of indebted firms, can lead us back to the steady state. This may depend on the size of the implied change in gross investment and its consequences for the change of the debt of firms.

For sufficiently small parameter values β_w we however know that the dynamics will produce explosiveness of the debt to capital ratio λ and impulsiveness for the rate of employment e beyond threshold values $\tilde{\lambda}, \tilde{r}$. For sufficiently high debt, measured relative to the level of the capital stock, we thus find that debt accumulation feeds itself via its impact on the budget of firms and will lead to larger and larger debt to capital ratios if there is not sufficient support for the pure rate of profit from downward changes in the wage share. Yet, as there is no price deflation, there cannot be a 'perverse' adjustment (a rise) of the wage share in such a situation of depressed profitability and high debt accumulation. Such a problematic situation comes about when there is sluggish or no downward adjustment in the level of nominal wages, but – due to insufficient goods demand, which is not yet a possibility in the considered dynamics – downward adjustment in the price level causing increases in the real wage and the wage share. This scenario will be investigated by a suitable 4D extension in the next section.

Summing up we thus have that debt financing of investment, on the one hand, turns the global centre type dynamics of the Goodwin (1967) growth cycle model into damped cycles, but this, on the other hand, only in a limited domain, outside of which the dynamics become explosive.

Figure 4.2 shows simulation runs of the 3D dynamics for small and large shocks of the debt to capital ratio. The upper figure shows the type of cycle that is implied by this debt-financed extension of the Goodwin (1967) growth cycle model and confirms what is asserted in Proposition 4.2 (for a 100 per cent shock of the steady debt to capital ratio). The lower figure adds the evolution of the ratio λ to the (v, e) phase portrait and does so for multiplicative shocks of order 4 and 5. We can see that the multiplication of the ratio λ_o by 4 still leads to convergence and thus does not yet suffice to bring the dynamics onto an explosive path, which however does come about if the factor 5 is applied in the place of only 4. Note that the wage adjustment speed is low (= 0.1), but not zero as assumed in Proposition 4.4. Note also that the other assumptions of this proposition are fulfilled in the chosen numerical example.

The parameter set for Figure 4.2 is given in Table 4.1.[12]

This basic model of debt accumulation is augmented in later sections by Rose effects in the wage-price interaction (which reveal that either wage or price flexibility must be destabilising with respect to the implied real wage adjustments), by Keynes

[12] This parameter set implies for the interior steady state of the dynamics: $e_o = 0.9, r_o = 0.055; \lambda_o = 0.45, v_o \approx 0.61$.

Table 4.1. *The parameters of the simulation of the 3D dynamics*

$y^p = 0.5$;	$\delta = 0.05$;	$\bar{e} = 0.9$;	$n = 0.1$;	$n_l = 0$
$\bar{r} = 0.03$;	$\bar{i} = 0.04$;	$\beta_w = 0.1$;	$\alpha_1^k = 4$	

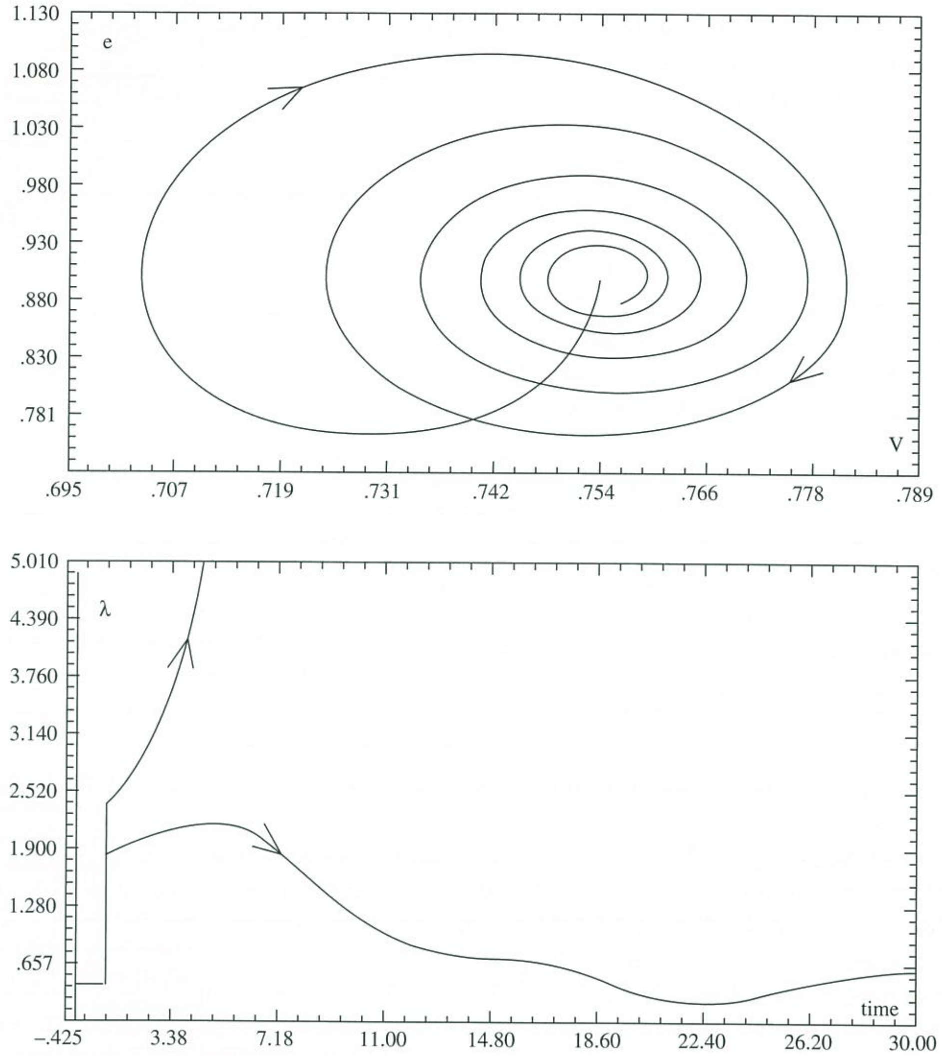


Figure 4.2 Convergence for small shocks and divergence for large shocks to λ

effects (which here are more direct than is usually the case due to the monetary policy rule assumed), by Mundell effects (which state that the interaction between price inflation and expected price deflation must be destabilising if the adaptive component of these expectations is operating with sufficient speed), by Metzler effects (which imply

accelerator type instability of the inventory adjustment mechanism when it operates with sufficient speed) and by cumulative (destabilising) effects in financial markets (if adjustments are sufficiently fast) due to positive feedback loops between expected changes and resulting actual changes of financial variables in our delayed adjustment processes towards overall interest rate parity (uniform rates of return). All these effects are of course partial in nature and must be studied in their interaction in a complete analysis of the full 16D model, which is the most general model of debt deflation that will be considered in this chapter.

We have seen in this section that the (very narrow) financing constraint $C_b \geq 0$ requires that the interest rate $\bar{i}$ is larger than the natural rate of growth $n + n_l$, while the occurrence of explosive debt accumulation for a positive rate of profit happens only when the opposite holds true. In addition to what is exemplified in Figure 4.2 we therefore conjecture (also on the basis of further numerical studies) that the considered dynamics are convergent for most initial conditions that are economically meaningful. From the economic perspective, the interior steady state of the 3D model (with only debt accumulation and its sole feedback via the budget equation of firms), is therefore asymptotically stable from a fairly global point of view. Further mechanisms are thus needed, for example a direct effect of debt on the investment behaviour of firms or a credit multiplier, in order to generate stability problems for debt financed economic growth.

Adding, as in the remark 2 on the Proposition 4.2 a term like $\alpha_i^k(\bar{\lambda} - \lambda)$, ($\bar{\lambda} = \lambda_o$ the target debt to capital ratio) to the investment function, which thus directly responds to the state of indebtedness and (not only indirectly via its consequences for the budget equation of firms) adds the expression $\alpha_i^k(\lambda_o - \lambda)(1 - \lambda)$ to the law of motion 3) for the debt to capital ratio, without modifying its interior steady state solution. Such an addition would move the $\widetilde{\lambda}$ expression in Figure 4.1 into the domain where $\dot{\lambda}$ is positive and will also increase $\dot{\lambda}$ if α_i^k is increased. The probability of an explosive evolution of the ratio λ is thereby considerably increased. However we will show in the next section that demand side additions may even be more important for the detection of important sources of cumulative instability, then mainly from a local perspective.

Summing up, we have considered in this section a limited, but useful starting point for the analysis of debt deflation where deflationary forces may be added via markup pricing,[13] where demand pressure on the market for goods is however still missing and where there is a very narrow restriction on the amount of new loans available at each moment in time, given by the interest income of financial capitalists, due to the neglect of money (creation) and to the corresponding validity of Say's Law. This

13 The addition of these forces would make the entry J_{32} in the Jacobian J of the dynamics at the steady state negative and thus introduce a negative term into the principal minors of order 2 of this Jacobian which is clearly destabilising. A downward floor to falling wages, in particular the institutional assumption that the general level of nominal wages may rise, but never falls in a noticeable way, would however remove this instability from the dynamics. Such a floor is not so obviously present in the case of price deflation to be considered in the next section.

starting point does not yet show that there may be severe consequences from high debt accumulation.

4.3 4D debt deflation

Let us thus now extend and modify the model (4.1)–(4.3) in order to include into it in a minimal, but empirically relevant, way the possibility for persistent price level deflation and thus the possibility for the occurrence of accelerating debt deflation. By the latter term we mean high levels of debt combined with declining profitability due to falling output prices, caused in turn by insufficient aggregate demand for goods. We thus have to introduce into the model on the one hand a law of motion for the price level and on the other hand a discrepancy between normal output of firms and the aggregate demand for their goods on which the required theory of price inflation is to be based. This process will turn the supply side dynamics into demand driven dynamics and thus removes the simple form of Say's Law used in the preceding section in favour of a Keynesian theory of effective demand or goods market equilibrium.

However we still assume that inflationary expectations remain fixed at their steady state level (which is zero here). On this basis we make use of the wage-price dynamics discussed in Chapter 6 of Chiarella and Flaschel (2000) and thus considerably extend the simple law of motion for nominal wages used in Section 4.2. Furthermore, we now make use of the extended investment function[14]

$$I/K = \alpha_1^k(r-\bar{i}) + \alpha_2^k(\lambda_o - \lambda) + \alpha_3^k(y/y^p - \bar{u}) + \gamma + \delta, \quad \alpha_i^k > 0, i = 1, 2, 3$$

which integrates the remarks of Section 4.2 and also introduces fluctuation of excess capacity as a further argument into investment behaviour as in the KMG growth dynamics of Section 4.4. Note that we now assume that trend investment γ is exogenously given and in fact determined by natural growth $n + n_l = \hat{L} + \hat{z}$. This gives the reason why there is no trend term in the law of motion for l^e which is to be calculated from $\hat{l}^e = \hat{L} + \hat{z} - \hat{K}$. We therefore now integrate the PC mechanisms with the growth law of the full employment labour intensity in efficiency units and the law of motion of the debt to capital ratio which as usual is given by $g_k(1-\lambda) - r$ but now with an additional term $-\hat{p}\lambda$ due to the inflationary dynamics that are now present and the definition of the debt to capital ratio by $\lambda = \Lambda/(pK)$.

This gives rise to the following type of nominal dynamics for wages w^e, prices p (adopted from the KMG growth dynamics introduced in Chiarella and Flaschel (2000) with inflationary expectations still fixed at zero) and debt λ coupled with an investment driven growth path, now represented by the dynamics of full employment labour intensity l^e.[15] The intensive form of the dynamics can be written as

$$\hat{w}^e = \kappa[\beta_w(y/l^e - \bar{e}) + \kappa_w\beta_p(y/y^p - \bar{u})], \tag{4.5}$$

$$\hat{p} = \kappa[\beta_p(y/y^p - \bar{u}) + \kappa_p\beta_w(y/l^e - \bar{e})], \tag{4.6}$$

$$\hat{l}^e = -[\alpha_1^k(r-\bar{i}) + \alpha_2^k(\lambda_o - \lambda) + \alpha_3^k(y/y^p - \bar{u})], \tag{4.7}$$

$$\dot{\lambda} = [\alpha_1^k(r-\bar{i}) + \alpha_2^k(\lambda_o - \lambda) + \alpha_3^k(y/y^p - \bar{u}) + \gamma](1-\lambda) - r - \hat{p}\lambda, \tag{4.8}$$

where the Metzlerian feedback mechanism from actually observed aggregate demand to expected demand to planned output and income and back to aggregate demand, namely

$$y^d = c_y\frac{w^e}{p}y + \alpha_1^k(r-\bar{i}) + \alpha_3^k(y/y^p - \bar{u}) + \gamma + \delta \to y^e \to y \to y^d,$$

will be simplified and specialised to the static (and again linearised[16]) relationship

$$y^d = y^e = y = y(\frac{w^e}{p}, \lambda) = \bar{u}y^p + d_1(\frac{w^e}{p} - (\frac{w^e}{p})_o) + d_2(\lambda - \lambda^o), \qquad d_w, d_\lambda \leq 0. \tag{4.9}$$

Equation (4.9) will be used in the following as a shortcut for the delayed feedback chain of the general case (and its richer concept of aggregate demand) in order to integrate the effects of price inflation and deflation into the Keen (2000) model as presented and analysed in the preceding section.[17] Note that the budget equations of the credit-giving institution (here the pure asset holders) are no longer subject to the problem we observed for the banks of the 3D Keen model, that is there may be a credit multiplier at work which always produces the amount of loans demanded by firms (and there is no need to consider the consumption demand of this sector). Note furthermore, that Goodwin type dynamics are obtained when $\bar{i}, \lambda(0), d_w, d_\lambda$ are all zero, while the more general pure Rose (1967) type of real wage dynamics requires $\bar{i}, d_\lambda = 0$ (with wage flexibility as a stabilising factor and price flexibility destabilising if $d_w < 0$ holds). Finally, the pure Fisher debt mechanism is obtained (due to $d_\lambda < 0$) by setting $\beta_w, \kappa_w, d_w = 0$. The above goods market representation therefore allows for Rose real wage effects of traditional type (where price flexibility is destabilising giving rise to adverse real wage adjustments) and for Fisher debt effects (where price flexibility is also destabilising, giving rise to unbounded increases of the real debt to capital ratio),

[14] Investment behaviour is subject to modification in various (more or less significant) ways in this chapter, which gives rise to subtle differences with respect to steady state determination. It surely represents a behavioural equation where scope for alternative specifications is given. Note in particular here that this section introduces a target debt to capital ratio in the investment behaviour of firms that must be set equal to its steady state value in order to get consistency with respect to steady state solutions. Note also that $\bar{r}$ is given by $\bar{i}$ now. Note finally that we do not yet consider credit rationing explicitly, but in fact assume that in this respect a soft budget constraint of firms is in fact subject to such forces via the debt term in the investment equation which represents the working credit limitations through the behaviour adopted voluntarily by firms themselves.

[15] Here $\bar{e}, \bar{u}$ are the NAIRU utilisation rates of the labour force and the capital stock, w^e, l^e measured in efficiency units as in Section 4.4 and Section 4.5 below.

[16] Linearised, that is, around the interior steady state solution.

[17] Note that this shortcut of the originally delayed quantity adjustment process of Metzlerian type requires that the steady state value of this function y must be equal to $y^p\bar{u}$ in order to get a steady state solution for this 4D simplification and modification of the KMG dynamics.

but it excludes Mundell effects for example (that would require at the least the inclusion of inflationary expectations into the above model).[18]

Thus we here assume (implicitly) that the propensity to invest dominates the propensity to consume with respect to the impact of real wages w^e/p on consumption and investment (the orthodox point of view) and take also from the Metzlerian feedback chain and its shown shortcut the assumption that output depends negatively on the debt to capital ratio λ. The partial derivatives of the function $y(\frac{w^e}{p}, \lambda)$ in equation (4.9) are therefore both assumed to be negative in the following (that is $d_1, d_2 < 0$, representing two channels for destabilising price flexibility, or zero when certain limit cases are considered). Since employment l^{de} per unit of capital and in efficiency units is identical to output y, due to the measurement conventions of Section 4.4 and Section 4.5 and the fixed proportions technology assumed, we have that the employment rate exhibits the same type of dependence on the real wage and the debt to capital ratio as output y. Finally we of course again have $r = y - \delta - \frac{w^e}{p}y - \bar{i}\lambda$ for the rate of pure profits r.

The above represents the simplest way to integrate from the perspective of the KMG growth dynamics of Section 4.1 the dynamics of the price level into our representation of the Keen (2000) model by abstracting from Metzlerian delayed output adjustment, from inflationary expectations, a fiscal and monetary authority, and from endogenous determination of the interest rate.

The 4D dynamics (4.5)–(4.8) can be reduced to 3D dynamics, as shown in Section 4.5, giving rise to

$$\hat{v} = \kappa[(1-\kappa_p)\beta_w(y/l^e - \bar{e}) - (1-\kappa_w)\beta_p(y/y^p - \bar{u})], \quad v = wL^d/(pY) = w^e/p, \tag{4.10}$$

$$\hat{l}^e = -[\alpha_1^k(r - \bar{i}) + \alpha_2^k(\lambda_o - \lambda) + \alpha_3^k(y/y^p - \bar{u})], \quad l^e = zL/K, \tag{4.11}$$

$$\dot{\lambda} = [\alpha_1^k(r - \bar{i}) + \alpha_2^k(\lambda_o - \lambda) + \alpha_3^k(y/y^p - \bar{u}) + \gamma](1-\lambda) - r - \hat{p}\lambda, \quad \lambda = \frac{\Lambda}{pK}, \tag{4.12}$$

supplemented by the algebraic equations

$$y = y(v,\lambda) = \bar{u}y^p + d_1(v - v_o) + d_2(\lambda - \lambda^o), \qquad d_w, d_\lambda \le 0,$$
$$r = y(v,\lambda)(1-v) - \delta - \bar{i}\lambda,$$

with an appended law for the price level dynamics

$$\hat{p} = \kappa[\kappa_p(\beta_w(y/l^e - \bar{e}) + \beta_p(y/y^p - \bar{u})]. \tag{4.13}$$

Due to the lack of changing inflationary expectations the Ordinary Differential Equation (ODE) for p does not feed back into the rest of the dynamics.[19] Our stability

[18] There is also no Keynes effect in the present formulation of the dynamics, since the nominal rate of interest is kept constant.

[19] Note however that this law has to be inserted into the law of motion for the debt to capital ratio (as shown), implying that deflationary forces are now present, based on demand pressure in the goods market and demand

analysis will be concentrated on the above 3D core dynamics (4.10)–(4.12), the stability features of which will also determine those of the price level dynamics.

Let us first calculate the interior steady state of the dynamics (4.5)–(4.8), which partly already assume knowledge of this steady state solution. This steady state is uniquely determined up to the steady level of prices p and is characterised by[20]

$$\lambda_f^o = 1 - \bar{i}/\gamma, \tag{4.14}$$
$$y_o = y^p\bar{u}, \tag{4.15}$$
$$l_o^e = y_o/\bar{e}, \tag{4.16}$$
$$r_o = \bar{i}, \tag{4.17}$$
$$v_o = \frac{y_o - r_o - \delta - \bar{i}\lambda^o}{y_o}, \tag{4.18}$$
$$p_o = \text{ determined by initial conditions}, \tag{4.19}$$
$$w_o^e = p_o(\frac{w^e}{p})_o = p_o v_o. \tag{4.20}$$

The steady state value for the debt to capital ratio is a direct consequence of equations (4.7) and (4.8) once it is shown that $r_o = \bar{i}$ must hold true, see below, since $\hat{p} = 0$ in the steady state. Equations (4.5) and (4.6) set equal to zero together imply furthermore that the two measures of demand pressure (on the market for goods and for labour) must be zero in the steady state, which determines the steady state values of y and l^e as shown. Due to the new form of the investment function[21] underlying equation (4.7), namely

$$I/K = \alpha_1^k(r - \bar{i}) + \alpha_2^k(\lambda_o - \lambda) + \alpha_3^k(y/y^p - \bar{u}) + \gamma + \delta,$$

we now have a different steady debt to capital ratio which is solely determined by trend growth γ in its deviation from the given rate of interest $\bar{i}$ on loans. Note here that the natural growth trend term $\gamma = n + n_l$ in the investment function enforces $\bar{i} = r$, in the steady state (since $y = y^p\bar{u}$ holds in the steady state, $r = \gamma(1-\lambda)$ by equation (4.12) and $\lambda_o = \bar{i}/\gamma$ by assumption). Natural growth is thus responsible for the difference to the steady state solution we have obtained for the 3D dynamics of Section 4.4 where no such trend term was present. We assume throughout that $\gamma - \bar{i} > 0$ holds in order to have a positive steady state ratio λ.

Again, the wage share v_o is determined residually, once profit and interest per unit of capital have been determined. The two demand pressure benchmarks on the labour and the goods market, $\bar{e}$ and $\bar{u}$, are the NAIRU rates of capacity utilisation on these two markets. The steady state ratios for actual and full employment labour intensity (in efficiency units), $l^{de} = y$ and l^e are thus purely supply side determined, while the

pressure in the labour market (to the extent κ_p by which wages influence prices by way of cost pressure considerations).

[20] We use $l_y^e = 1/z$ to express employment per unit of output measured in efficiency units (l_y^e a given magnitude).

[21] Which (as gross investment function) must be non-negative along the relevant trajectories of the dynamics.

demand side expression (d_w, d_λ and the α_i^k's) only enter the analysis when the stability of the steady state solution is investigated. The determination of the rate of profit through the rate of interest on loans implies a well defined level of real wages measured in efficiency units, $v = (\frac{w^e}{p})_o$, which is positive if y^p is chosen sufficiently high relative to $\gamma, \delta, \bar{i}$ and $\bar{u}$. This real wage level then determines the nominal wage level on the basis of a given price level which is determined through historical conditions and thus not uniquely determined. This is due to the fact that the price dynamics are not needed for the dynamic analysis of the evolution of the wage share, the full employment labour intensity and the debt ratio, and itself only dependent on these three state variables which implies that the determinant of the Jacobian of the 4D dynamics must be zero under all circumstances. Note in this respect that the price dynamics are needed as an equation in the determination of the interior steady state of the dynamics, but can be removed from explicit consideration in the dynamics surrounding this steady state solution.

Proposition 4.5 (Stabilising normal Rose effects)
Assume $0 < \bar{i} < \gamma, d_2 = 0$ and $\beta_p, \kappa_p = 0$,[22] so that the price level is a given magnitude. Assume furthermore a wage adjustment speed that is sufficiently high. Then the steady state (4.14)–(4.20) of the dynamical system (4.10)–(4.12) is locally asymptotically stable for all other admissible parameter values.

Proof: Note first of all that the dynamics are now truly of dimension three by assumption, since there is no longer an appended law of motion for the price level dynamics. Concerning the calculation of the determinant of the Jacobian of these reduced dynamics (4.10)–(4.12), at the steady state, we can first of all state that its third row can be reduced to $(-r_v, 0, \bar{i} - \gamma)$ by the addition of an appropriate multiple of the second row without changing its sign. This implies that this determinant can be characterised by the sign structure

$$\det J = \begin{vmatrix} - & - & 0 \\ + & 0 & + \\ + & 0 & \bar{i} - \gamma \end{vmatrix}$$

and is thus always negative if $\bar{i} < \gamma$ holds, which provides one of the Routh–Hurwitz conditions for local asymptotic stability. With respect to the sum a_2 of the principal minors of order 2, namely J_1, J_2, J_3, one furthermore gets from the full sign structure of the Jacobian matrix J in the case $d_2 = 0$ that

$$J = \begin{pmatrix} - & - & 0 \\ + & 0 & + \\ \pm & 0 & - \end{pmatrix}.$$

[22] This implies $\kappa = 1$. Note also that the assumption $\bar{i} < \gamma$ is much stronger than what is actually needed to imply $\det J > 0$.

We see that the calculation of det J involves two positive and one zero determinant and thus is unambiguously positive. Note furthermore that the entry

$$J_{33} = \bar{i} - \gamma - (\alpha_r^k \bar{i} + \alpha_i^k)(1 - \lambda_o)$$

in the Jacobian matrix J is negative and larger in absolute value than $\bar{i} - \gamma$. The trace of J is therefore indeed negative, too, since $\bar{i} < \gamma$ by assumption and since $\lambda^o < 1$ holds. The coefficients $a_1 = -$trace $J, a_2, a_3 = -\det J$ of the Routh–Hurwitz polynomial are therefore all positive and thus all support the local asymptotic stability claimed in the above proposition. Finally, we also have $a_1 a_2 - a_3 > 0$, since one of the expressions that forms det J is part of the all positive expressions contained in $a_1 a_2$ and thus cannot make the expression $a_1 a_2 - a_3$ less than or equal to zero, and since the other one depends linearly on the parameter β_w, while the component $J_{11} J_2$ of $a_1 a_2$ is a quadratic function of this parameter (with all coefficients being positive) that must dominate the value of the linear function if the parameter β_w is made sufficiently large. □

We thus have that the interior steady state (with $\lambda_o < 1, \bar{i} < \gamma$) of the reduced dynamics (4.10) – (4.12), where there is only *sluggish adjustment of prices* caused by demand pressure on the market for goods, that is where β_p, κ_p are both sufficiently small, is locally asymptotically stable if the influence of the debt to capital ratio λ on the level of output and employment, both in intensive form, is also sufficiently weak and if nominal wages adjust with sufficient speed. This outcome is due to the fact that the eigenvalues of the Jacobian of the dynamics are continuous functions of the parameters of the model, and thus cannot change sign of their real parts if the parameters that characterise price and output adjustment are chosen sufficiently small. Making the stabilising Rose effect or real wage adjustment sufficiently strong and the Fisher debt effect in the goods market and price inflation sufficiently weak thus produces stability (as was to be expected) in a world where only Rose and Fisher debt effects interact and where there is a positive amount of debt of firms in the steady state. We note that stability can get lost only in a cyclical fashion, by way of a *Hopf bifurcation* and the limit cycles they generate, when the parameter β_w is decreased, since such a change does not alter the negative sign of the determinant of the Jacobian at the steady state. This however need not hold true for the parameter β_p as we shall see below.

Proposition 4.6 (Two channels for destabilising price flexibility)[23]
Assume that $d_2 < 0$ holds. Then the steady state (4.14)–(4.20) of the dynamical system (4.10)–(4.12) loses its local asymptotic stability if the price adjustment speed β_p is sufficiently large.

[23] Assuming $d_w > 0$ and thus stabilising price flexibility from the viewpoint of the Rose effect, it can be shown that the destabilising forces of the Fisher debt effect will dominate the stabilising Rose effect, if price flexibility becomes sufficiently large.

Proof: Collecting the terms in the trace of the Jacobian J of the dynamics (4.10)–(4.12) at the steady state that depend on the parameter β_p one obtains[24]

$$-v_o\kappa(1-\kappa_w)d_w/y^p\beta_p - \kappa d_\lambda/y^p\beta_p$$

which involves positive expressions solely (up to the possibility that κ_w can be equal to one and either d_w or d_λ equal to zero). The first expression shows the strength of the destabilising Rose price flexibility effect and the second is the Fisher debt effect. Therefore the trace of J can always be made positive by choosing the parameter β_p sufficiently large. □

The local stability result for the 3D Keen model is therefore overthrown in the case where goods demand is negatively dependent on the debt to capital ratio (and the real wage) and where the price level adjusts with respect to demand pressure on the market for goods with sufficient speed. Local stability therefore gets lost for flexible price levels either through Fisher type debt deflation or through adverse Rose effects or through the joint working of these two adverse consequences of an adjustment of the price level that is sufficiently fast (and a given wage adjustment speed). In such a case, we conjecture and will test this assertion numerically, that a process of deflation will continue without end accompanied in particular by higher and higher debt ratios of firms which eventually will lead to negative profits and bankruptcy.

Proposition 4.7 (Convergent dynamics: limited basins of attraction)
We again allow $d_2 < 0, \beta_p > 0$ and assume in addition, as in Proposition 4.3, that nominal wages are completely fixed ($\beta_w = \kappa_w = 0$). Then the dynamical system (4.10)–(4.12) is monotonically explosive, implying higher and higher wage shares and debt to capital ratios, for initial debt to capital ratios chosen sufficiently high (in particular larger than 1), all wage shares above their steady state value and all positive adjustment speeds of the price level p.

Proof: The system (4.10)–(4.12) in the assumed situation can be reduced to

$$\hat{v} = -\beta_p(y/y^p - \bar{u}), \tag{4.21}$$

$$\dot{\lambda}_f = [\alpha_1^k(r-\bar{i}) + \alpha_2^k(\lambda_o - \lambda) + \alpha_3^k(y/y^p - \bar{u}) + \gamma](1-\lambda) - r - \beta_p(y/y^p - \bar{u})\lambda, \tag{4.22}$$

since l^e no longer feeds back onto the state variables of these dynamics. Since both v and λ are assumed to be larger than their steady state values, we get from the first law of motion that v must be rising further (due to falling price levels caused by $y < y^p\bar{u}$). Furthermore, since also $r - \bar{i} < 0$ and $1 - \lambda < 0$ hold, we get that $\dot{\lambda}$ must be larger than

$$\gamma(1-\lambda) - \bar{i} - \beta_p(y/y^p - \bar{u})\lambda > -\gamma\lambda - \beta_p(y/y^p - \bar{u})\lambda.$$

24 In the later 8D or 16D extensions of these dynamics the destabilising influence of price flexibility is no longer so obviously situated in the trace of the Jacobian J, but hidden in certain principal minors of J then.

If therefore $-\beta_p(y/y^p - \bar{u}) > \gamma$ has come about by choosing λ sufficiently high then $\dot{\lambda} > 0$ must be true, so that both v and λ must be rising which further strengthens the conditions for their monotonic increase. □

We thus get the same result as in Proposition 4.3, but now in an easier and more pronounced way (through the occurrence of price deflation), that there will indeed occur situations of debt *deflation* where profitability falls monotonically and where the debt of firms is increasing beyond any limit, leading to economic collapse sooner or later. Note that the above is only a first and crude estimate of such a possibility.

Proposition 4.8 (Conditions for convergence)
Assume that $\beta_p = 0, \kappa_p = 1$ holds, so that the price level is determined by cost-push considerations solely and hence by a conventional markup equation of the type

$$p = (1+a)\frac{wL^d}{Y} = (1+a)wl_y = (1+a)w^e l_y^e.$$

Assume furthermore that the given markup a is such that the implied wage share v is equal to its steady state level v_o. Assume next a given level of nominal wages (measured in efficiency units), that is $\beta_w = 0, \kappa_w = 0$.[25] *Assume finally that the investment parameters $\alpha_{1,3}^k$ are chosen such that the inequalities $\alpha_1^k > 1, \alpha_u^k > y^p(1-v)\frac{\gamma-\bar{i}}{\bar{i}}$ hold true.*[26] *Then the steady state (4.14)–(4.20) of the dynamics (4.10)–(4.12), which can then be basically reduced to adjustments of the debt to capital ratio, is locally asymptotically stable for all values of the parameter $d_\lambda < 0$.*

Proof: In the assumed situation we have $\hat{p} = 0$ due to the given level of nominal wages and thus get for the debt to capital ratio λ the single independent law of motion

$$\dot{\lambda} = [\alpha_1^k(r(\lambda) - \bar{i}) + \alpha_2^k(\lambda_o - \lambda) + \alpha_3^k(y(\lambda)/y^p - \bar{u}) + \gamma](1-\lambda) - r(\lambda).$$

We have to show that the derivative of the right-hand side of this equation is negative at λ_o for all d_λ. Note first that $r'(\lambda_o) = y'(\lambda_o)(1-v_o) - \bar{i} = d_\lambda(1-v_o) - \bar{i}$ holds. The derivative of the $\dot{\lambda}$ equation with respect to λ evaluated at the steady state reads[27]

$$-\gamma + [\alpha_1^k r'(\lambda_o) - \alpha_2^k + \alpha_3^k y'(\lambda_o)/y^p](1-\lambda_o) - r'(\lambda_o).$$

This expression can be rearranged to

$$-\gamma + (\alpha_1^k - 1)r'(\lambda_o)(1-\lambda_o) - \alpha_2^k(1-\lambda_o) + \alpha_3^k d_\lambda/y^p(1-\lambda_o) - r'(\lambda_o)\lambda_o.$$

From this expression we get through further rearrangement

$$-(\gamma - \bar{i}\lambda_o) + (\alpha_1^k - 1)r'(\lambda_o)(1-\lambda_o) - \alpha_2^k(1-\lambda_o) - d_\lambda[-\alpha_3^k/y^p(1-\lambda_o) + (1-v_o)\lambda_o]$$

25 The nominal wage is therefore then growing in line with labour productivity.
26 This inequality is equivalent to the inequality

$$\alpha_u^k > [(\gamma - \bar{i})^2 + 2(\gamma - \bar{i}) + \delta(\gamma/\bar{i} - 1)]/\bar{u}.$$

27 $\alpha_1^k(1-\lambda_o) > 1$ is already sufficient here.

with $\lambda_o = 1 - \bar{i}/\gamma, 1 - \lambda_o = \bar{i}/\gamma$. This expression must be negative, since $\bar{i} < \gamma$, $\lambda_o < 1, d_\lambda < 0, \alpha_r^k > 1, r'(\lambda_o) < 0$ and since

$$\alpha_3^k > y^p(1 - v_o)\lambda_o/(1 - \lambda_o) = y^p(1 - v_o)\frac{\gamma - \bar{i}}{\bar{i}}.$$

□

In a similar way it can also be shown that the above derivative is negative for all $\lambda \in (0, \lambda^o)$, so that there is convergence to the steady state for all positive debt to capital ratios below the steady state ratio. It is however not possible to provide an easy expression for the upper limit of the basin of attraction of the steady state (which may be less than 1). The situation that we have investigated here is indeed similar to the one considered in Figure 4.1, since the right-hand side of the above $\dot{\lambda}$ equation is again a quadratic function in λ with a positive coefficient in front of the λ^2 term and a global minimum at a positive value λ_{min} above the steady state value $\lambda_o < 1$, which of course is again one of the roots of this polynomial. The other one to the right of λ_{min} is then the limit for the basin of attraction shown to exist by Proposition 4.8.

We have formulated Proposition 4.8 in view of an intended policy application that, however, can only be sketched here. Consider the case where the debt to capital ratio λ is so large that there are cumulative forces at work (as in Proposition 4.7) that would lead to higher and higher debt and lower and lower profitability. In the case considered in Proposition 4.8 there are three possible ways to break this catastrophic tendency in the evolution of the economy:

- an increase in nominal wages w that, under the assumptions of Proposition 4.8, causes an immediate increase in the price level p and thus an immediate decrease in the ratio λ, which (if strong enough) may lead the economy back to the basin of attraction of its steady state;
- a decrease in the rate of interest $\bar{i}$ on loans, which moves the steady state of the economy to a higher sustainable debt to capital ratio;
- a decrease in the sensitivity of output y (through appropriate fiscal policies) with respect to λ, that is a parameter d_λ that is smaller in amount (which may enlarge the basin of attraction of the steady state).

There is therefore scope for economic policy to move the economy out of regions of developing debt deflation into regions where it converges back to the steady state. The details of such possibilities must however be left for future research.

What we have shown in this section is that there can be asymptotic stability under certain conditions as in the 3D dynamics considered in the preceding section. Moreover, this stability is destroyed once Fisher debt effects are present in the goods market reaction function and price level flexibility becomes sufficiently pronounced. Finally, even sluggish price levels give rise to instability outside certain corridors. It follows that certain wage-price regimes and properties of the investment function may generate

Table 4.2. *The parameters of the simulated 4D dynamics*

$\beta_w = 0.5$; $\delta = 0.05$; $\alpha_1^k = 0.5$; $\alpha_2^k = 0$; $\alpha_3^k = 0.5$;
$\bar{i} = \bar{r} = 0.05$; $\beta_p = 1$; $\kappa_p = 0; \kappa_w = 0$; $p(0) = 1$; $\gamma = 0.1$;
$l_y^e = 1$; $d_w = -0.4$; $d_\lambda = -0.4$; $\bar{u} = \bar{e} = 1$; $y^p = 0.4$;

regions of stability even for very high sensitivity of goods market positions with respect to the level of the debt to capital ratio.

We close this section by eigenvalue calculations which exemplify the stabilising and destabilising effects we have investigated for the considered 4D dynamics. To this end we perform some simulations using the parameter set in Table 4.2.

In Figure 4.3 we see first of all that wage flexibility is stabilising while price flexibility is not. Note that here again one eigenvalue of the 4D dynamics must always be zero so that stability is represented in this and the other eigenvalue diagrams by the horizontal line. We then set the parameter $\beta_w = 2$ in order to get convergent dynamics for the further evaluation of the feedback mechanisms of the dynamics by means of eigenvalue diagrams shown in the bottom six panels of Figure 4.3. In the second row of panels in Figure 4.3 we show on the left-hand side the 2D projection onto the (l^e, λ) space of a convergent trajectory of the dynamics generated by a shock of the debt to capital ratio of 10 per cent. To the right of this phase plot we then recalculate the eigenvalue diagram with respect to the parameter β_p for the now convergent dynamics and see that instability sets in much later than was the case at the parameter value $\beta_w = 0.5$.

The third row of panels shows eigenvalue diagrams that consider the role played by the Rose real wage and the Fisher debt effects. Note here the parameters d_w, d_λ are negative which means that stronger Rose and Fisher debt effects are obtained by moving to the left in the considered eigenvalue diagrams. We see that stronger Rose effects are stabilising in the considered situation (since wages are more flexible than prices), while the opposite holds for the Fisher debt effect.

In the lowest two panels of Figure 4.3, we display on the left-hand side the eigenvalue diagram for the parameter κ_p, the strength of the wage cost-push term in the price level PC. Wage flexibility is then transferred into price flexibility and thus becomes destabilising via the Fisher debt effect, overcoming the stabilising potential of wage flexibility through the Rose real wage effect. Finally, in the right-hand panel we observe that a rate of interest that is chosen sufficiently high destabilises the economy, since this increases the power of the destabilising Fisher debt effect. The opposite holds true if the parameter κ_p is increased (which is not shown here), since the stabilising Rose effect of wage flexibility is then strengthened.

We close this section with the observation that the dynamics considered in this section have been designed to display Fisher debt deflation as well as Rose real wage effects in their simplest form. This was done however in a way that is not fully consistent with the demand underlying the goods market equilibrium structure that has been employed. In the next section we represent aggregate demand and goods market adjustment processes

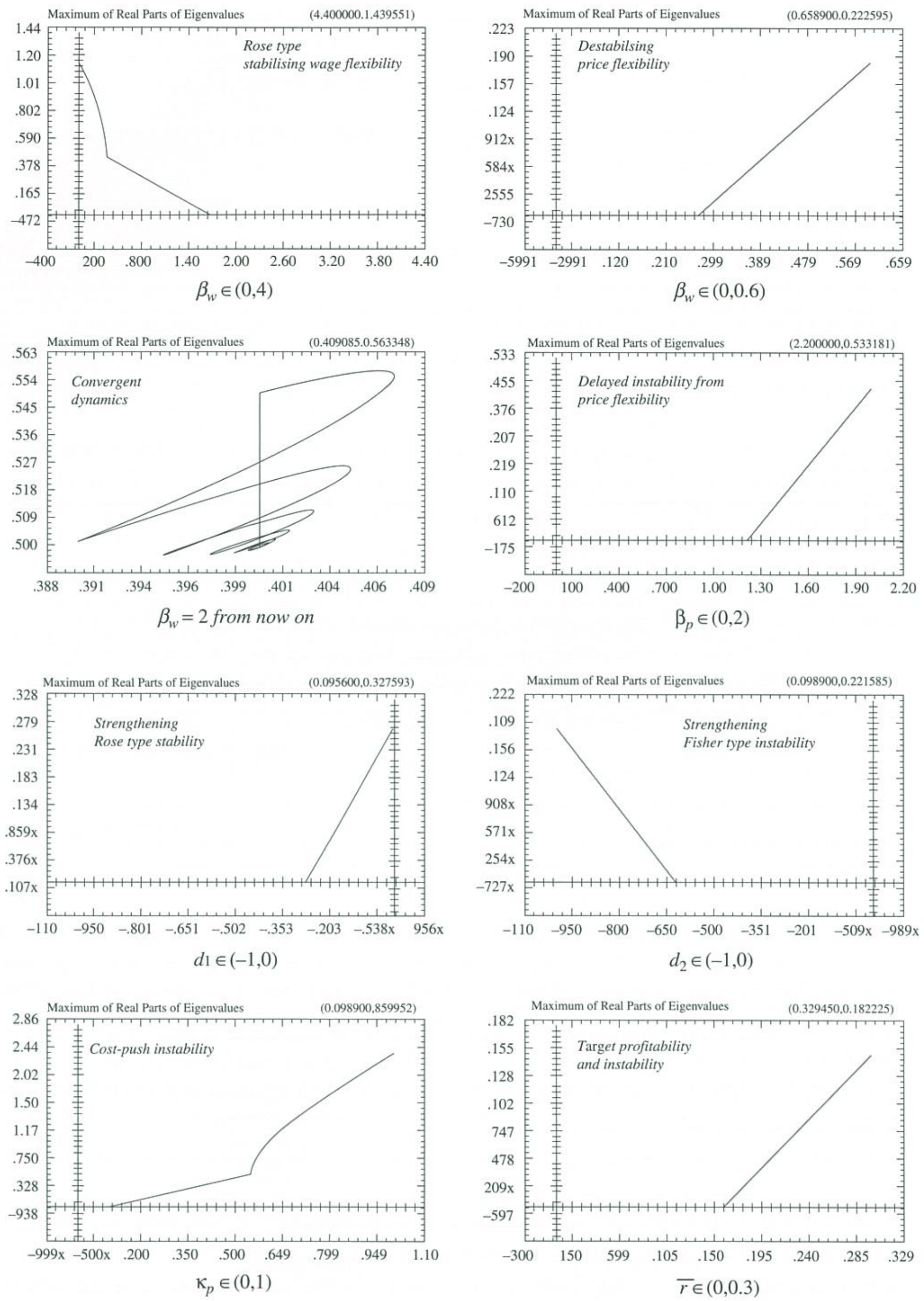


Figure 4.3 Eigenvalue diagrams for important parameters of the 4D dynamics. In the top two panels $\beta_w \in (0, 4)$ and $\beta_p \in (0, 0.6)$. In the remaining panels $\beta_2 = 2$, and other parameter values are indicated

explicitly in order to remove this inconsistency of the 4D approach and also complicated goods market equilibrium considerations. Instead, we have two further laws of motion for sales expectations and inventory adjustments as in KMG growth models. We also consider inflationary expectations and an interest rate policy rule of the monetary authority, which adds four further laws of motion to the dynamics of this section and thus increases considerably the analysis of interacting feedback chains of Keynesian disequilibrium growth.[28]

4.4 KMG real business fluctuations: the point of departure

In this section we introduce what is in our view the basic complete and coherently formulated Keynesian out-of-equilibrium macrodynamic model of the real sector of capitalist economies. The model combines on the markets for goods and labour both gradual quantity adjustment processes together with gradual price and wage adjustment. While gradual wage adjustment was always present in the old Keynesian AD-AS models of inflation and growth and while gradual price adjustment is characteristic for the New Keynesian baseline models of DSGE type, the joint consideration of both gradual wage and price adjustment processes is rarely done, not even in the old Keynesian literature. In this section we provide an introduction to the basic building blocks of the KMG model of business fluctuations and growth. We do this from an informal point of view in order to show the reader the philosophy behind this disequilibrium approach to conventional AS as well as AD analysis, which may therefore also be briefly characterised as a DAS-DAD modelling approach. Since the first publication of this model type in Chiarella and Flaschel (2000, Ch. 6) the model has been extended in numerous ways towards the treatment of the relevance of non-Keynesian regimes with supply side bottlenecks, the extension towards open economies, the calibration of this model type for the US economy, the issue of monetary policy and the inclusion of endogenous growth, to mention the most significant.

There exists in the New Keynesian approach to macrodynamics the consideration of both staggered wage and price setting, however the analysis of the resulting four dimensional dynamical system is missing in the literature.[29] Such a remark is even more applicable to the quantity side of these Keynesian approaches, both old and new, where the consideration of goods market equilibrium is the standard procedure to modelling the demand side of Keynesian AD analysis, in place of the interaction of firms' sales expectations with their inventory adjustment rules.

In view of the situation just described we shall build on the modelling philosophy of Chiarella and Flaschel (2000) and Chiarella *et al.* (2005) and their detailed investigation of models of so-called KMG type. We use continuous-time to model the data generating process of actual economies (which by and large is a daily one). In continuous-time we

28 There are two further laws of motion for stocks, deriving from the budget equations of the model, which are now all present, but (by assumption) do not all exhibit feedback effects into the dynamics of the private sector.
29 See however Flaschel *et al.* (2008) for a determinacy analysis of continuous-time limits of this type of model.

have on the quantity side sales expectations of firms on which they base their production decision (including planned inventory changes). The firms then experience actual demand for their goods which they service on the basis of their actual inventories, a process which leads to definite changes in their inventories.[30] We use reformulations of Okun's Law to relate the production decision of firms to the employment decision within firms and on the labour market.

On the price side, building on the work of Chiarella and Flaschel (1996) we have formulated a crossover wage-price spiral mechanism based on various gap measures for the labour as well as the goods market (in particular the employment rate and various utilisation rates). We have also used Blanchard and Katz (1999) type error correction mechanisms based on a wage share gap; see Chiarella *et al.* (2005) for a detailed discussion. These wage-price adjustment processes formally seem very similar to the equations used in the New Keynesian staggered-wages staggered-prices approach, however they differ significantly in their microeconomic details as well in their implications for the dynamics of the real wage. The significant outcome of this approach to the adjustment of real wages is that basically all mentioned gaps on the labour market and the goods market influence not only the law of motion of real wages, but also the formulation of a reduced form price PC (in theoretical as well as in applied inflation rate studies). This stands in stark contrast to current practice, particularly in empirical work.

4.4.1 The basic framework

We consider a closed three sector economy consisting of households (workers and asset holders), firms and the government. There exist five distinct markets: labour, goods, money, bonds and equities (which are perfect substitutes of bonds).

Our model is briefly summarised in Table 4.3, where real and nominal magnitudes are represented. The index d on a symbol refers to demand and the same symbol with no index represents supply, while the superscript index e is used to denote expectations. Table 4.3 shows the basic structure and the interaction of the sectors and the markets; the rows represent the sectors and the columns the markets. The links between the markets and sectors shown, the behavioural relationships and the dynamic adjustment processes that fill this structure have all been established in Flaschel *et al.* (1997) and in Chiarella and Flaschel (2000). They represent significant extensions of the Chiarella *et al.* (2000) baseline models in various ways. We extend this framework further by a discussion of the role of monetary policy rules and we also investigate the stability implications of kinked money-wage PCs already asserted to exist in fact by Keynes (1936).

The structure of the model is complete in the sense that it includes all major markets and sectors of a closed economy and all financing conditions and budget restrictions

[30] This scenario needs to be reformulated somewhat in the case of services, which however does not put into question the general idea of a gradual adjustment of quantities due to the observation of actual demand on the market for goods.

Table 4.3. *Sectors and markets of the economy*

	Labour market	Goods market	Money market	Bond market	Equity market
Households	L	$C = C_w + C_c$	M^d	B^d	E^d
Firms	L^d	$Y, Y^d, I + \delta K$	–	–	E
Government	–	G	M	B	–
Prices	w	p	1	1	p_e
Expectations	–	$Y^e, \pi^e = \hat{p}^e$	–	–	–

of households, firms and the government, as in Sargent (1987). However in contrast to Sargent (1987) we distinguish between workers and asset holders in the household sector, in a Kaldorian fashion. The really major difference from Sargent is however the extent of disequilibrium allowed for and the dynamical processes that follow from these disequilibria. Concerning the extent of these disequilibrium adjustment processes, firms have desired capacity utilisation rates and desired ratios of inventory to expected sales. Temporary deviations from those benchmarks are caused by unexpected changes in aggregate goods demand. We stress that a distinguishing feature of Keynesian models, in contrast in particular to equilibrium macromodels of the Sargent (1987) type, is that under- or over-utilised capital as well as an under- or over-utilised labour force are important driving factors of the economic dynamics.

The next section provides the building blocks of our KMG macrodynamics from the perspective of their fundamental adjustment mechanisms and the feedback structures that are implied. We motivate the structure of the model without presenting the many details which underlie its extensive form representation – for all of these structural details we refer the reader to Chiarella and Flaschel (2000, Ch. 6). The stability properties of the interaction of those feedback structures will be studied analytically and numerically using estimated parameters.

4.4.2 The 3D Rose type wage-price dynamics

The full dynamics, which are presented in ratio or intensive form directly, are best introduced and motivated by starting from a very basic, yet unfamiliar, wage-price module. In our first specification we follow Rose (1967, 1990) and assume two PCs in place of only one, thus providing wage and price dynamics separately, both based on measures of demand pressure $e - \bar{e}$ and $u - \bar{u}$, in the market for labour and for goods, respectively. These demand pressure influences on wage and price dynamics, or on the formation of wage and price inflation, $\hat{w}$ and $\hat{p}$, are both augmented by a weighted average of cost-pressure terms based on forward-looking myopic perfect foresight and a backward-looking measure of the prevailing inflationary climate, symbolised by π^c. Cost pressure perceived by workers is thus a weighted average of the currently evolving price inflation $\hat{p}$ and some longer-run concept of price inflation, π^c, based on past observations. Similarly, cost pressure perceived by firms is given as a weighted average of the currently evolving (perfectly foreseen) wage inflation rate, $\hat{w}$, and again the measure of the inflationary climate in which the economy is operating. Taken together we thus arrive at the following two PCs for wage and price inflation, which in this core version of the model are formulated in a fairly symmetrical way.[31]

The structural form of the wage-price dynamics is:

$$\hat{w} = \beta_w(e - \bar{e}) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c, \tag{4.23}$$

$$\hat{p} = \beta_p(u - \bar{u}) + \kappa_p \hat{w} + (1 - \kappa_p)\pi^c. \tag{4.24}$$

In the empirical application of the model we also have to take into account labour productivity growth $n_x = \hat{x}$, which from the theoretical perspective augments the cost pressure terms in the wage PC by the addition of n_x, while it reduces the wage cost pressure term $\hat{w}$ in the price PC by the same amount, as shown in calculations below.

Inflationary expectations over the medium run, π^c (the *inflationary climate* in which current inflation is operating), may be adaptively following the actual rate of inflation (for instance using an exponential weighting scheme), may be based on a rolling sample (hump-shaped weighting schemes), or may be based on other possible schemes for updating such expectations. We shall in fact simply make use of the conventional adaptive expectations mechanism in the presentation of the full model below. Besides demand pressure we thus use (as cost pressure expressions) in the two PCs weighted averages of the inflationary climate and the (foreseen) relevant cost pressure term for wage setting and price setting. In this way we get two PCs with very analogous building blocks, which despite their traditional outlook will have interesting and novel implications. In the later part of this chapter we will introduce in addition a non-linearity into the money wage PC.

Note that in the current version, the inflationary climate variable does not matter for the evolution of the real wage $\omega = w/p$. In fact nor does it matter for the wage share $v = \omega/x$ (due to the addition of productivity growth), the law of motion of which is given by

$$\hat{v} = \hat{\omega} - n_x = \kappa[(1 - \kappa_p)\beta_w(e - \bar{e}) - (1 - \kappa_w)\beta_p(u - \bar{u})].$$

[31] The use of two in place of only one PC – an unquestioned procedure during the rise of structural macroeconometric model building – see Chiarella *et al.* (2000) for a discussion of this on various levels of generality – is gathering force again, as indicated for example by the topics investigated in Cohen and Farhi (2001) and Mehra (2000). There are indeed numerous such questions that can be addressed from a closer look at the wage-price spiral in the place of the single reduced form PC of mainstream Keynesian theory, whether old or new.

The dynamics of ω follow easily from the obviously equivalent representation of the above two PCs, namely

$$\hat{w} - \pi^c - n_x = \beta_w(e - \bar{e}) + \kappa_w(\hat{p} - \pi^c),$$
$$\hat{p} - \pi^c = \beta_p(u - \bar{u}) + \kappa_p(\hat{w} - \pi^c),$$

by solving for the variables $\hat{w} - \pi^c - n_x$ and $\hat{p} - \pi^c$. The last two equations imply that the two cross-markets or *reduced form PCs* are given by:

$$\hat{p} = \kappa[\beta_p(u - \bar{u}) + \kappa_p \beta_w(e - \bar{e})] + \pi^c, \tag{4.25}$$

$$\hat{w} = \kappa[\beta_w(e - \bar{e}) + \kappa_w \beta_p(u - \bar{u})] + \pi^c + n_x. \tag{4.26}$$

Equations (4.25)–(4.26) represent *a considerable improvement* over the conventional view of a single-market price PC with only one measure of demand pressure, the one in the labour market. The traditional expectations-augmented PC formally resembles the above reduced form $\hat{p}$-equation given by (4.25) if Okun's Law holds in the sense of a strict positive correlation between $u - \bar{u}$ and $e - \bar{e}$, our measures of demand pressures on the market for goods and for labour. Yet, the coefficient in front of the traditional PC would even in this situation be a mixture of all of the $\beta's$ and $\kappa's$ of the two originally given PCs and thus represent a composition of goods and labour market characteristics (which moreover now hides the fact that myopic perfect foresight is indeed underlying these apparently only backward looking reduced form PCs). The currently prominent New Keynesian PC, see for example Gali (2000), is based on the reduced form representation for $\hat{p}$ shown above, but generally with $\beta_p = 0, \kappa_p = 1, \kappa_w = 0$ and π^c a one-period ahead forecast of the rate of price inflation. Under perfect foresight this basically implies in a continuous-time set-up a price PC of the type

$$d\hat{p}/dt = \beta_w(e - \bar{e}),$$

which provides an interesting alternative to our reduced form price PC. In the last equation the medium-run climate expression for price inflation plays no role, reducing in this way inflation dynamics to short-term expressions solely, which in our view provides one of the reasons why the New Keynesian PC behaves strangely from an empirical perspective.[32]

[32] The New Keynesian approach to the business cycle theory and in particular monetary policy is considered and evaluated in detail in King (2000). We do not go into such a discussion – on the New IS–LM model – in the present book which in this chapter in our view provides a significant alternative to the New Keynesian approach for the closed economy. A comparison with the New Keynesian IS–LM theory with its stress on microfounded, intertemporal and forward-looking behaviour is provided in Asada *et al.* (2010) where the potential and limitations of both approaches are discussed and evaluated.

Taken together our above structural approach to wage and price PCs gives rise to *three independent laws of motion* that we can write as

$$\hat{v} = \kappa[(1-\kappa_p)\beta_w(e-\bar{e}) - (1-\kappa_w)\beta_p(u-\bar{u})], \tag{4.27}$$

$$\hat{m} = \bar{\mu} - \hat{K} - \hat{p} \quad (\bar{\mu} = const.), \tag{4.28}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\hat{p} - \pi^c) = \beta_{\pi^c}\kappa[\beta_p(u-\bar{u}) + \kappa_p\beta_w(e-\bar{e})]. \tag{4.29}$$

Equations (4.27), (4.28) and (4.29) are the first three differential equations of the full 6D Keynesian dynamics to be summarised in Section 4.5 below. The essential elements in these three laws of motion are the three adjustment speeds β_w, β_p and β_{π^c} for wages, prices and the inflationary climate which strongly influence their stability properties. Note that the law of motion for the capital stock K has not yet been provided, but will be introduced when the full 6D dynamics are presented.

4.4.3 The 2D Metzlerian quantity dynamics and capital stock growth

Next, we consider the quantity dynamics of the Keynesian macromodel, where we consider goods market adjustment dynamics and capital stock growth. The resulting 3D dynamics, which provide the quantity side of our Keynesian macrodynamic model, are given by[33]

$$\begin{aligned}
Y^d &= C + I + \delta K + G,\\
\dot{Y}^e &= \beta_{y^e}(Y^d - Y^e) + (n_l + n_x)Y^e,\\
N^d &= \alpha_{n^d} Y^e,\\
\mathcal{I} &= \beta_n(N^d - N) + (n_l + n_x)N^d,\\
Y &= Y^e + \mathcal{I},\\
\dot{N} &= Y - Y^d,\\
\hat{K} &= I/K.
\end{aligned}$$

These equations represent a simple, yet consistently formulated, output and inventory adjustment process. The first equation defines aggregate demand Y^d as the sum of consumption, investment and government demand and the second equation states that expected sales Y^e follow aggregate demand in an adaptive fashion. In the third equation desired inventories N^d are then assumed to be determined as a constant fraction of expected sales, while in the fourth equation intended inventory adjustment $\mathcal{I}$ is based on the inventory adjustment process $\beta_n(N^d - N)$, with N the actual inventory holdings and β_n the speed with which the gap between desired and actual inventory holdings is closed, augmented by a term that accounts for trend growth (n_l the natural rate of growth of the labour force, $n = n_l + n_x$). In the fifth equation actual production Y must

[33] These quantity dynamics have been studied in isolation, with a non-linearity in the inventory adjustment process, in Franke and Lux (1993) and with capital stock growth in Franke (1996).

then of course be defined by the sum of expected sales and intended inventory changes, while actual inventory changes $\dot{N}$ are finally given by definition as the discrepancy between actual production and actual sales. Again, the crucial parameters in these adjustment equations are the adjustment speeds, β_{y^e} and β_n, of sales expectations and of intended inventory changes respectively. It is obvious from the above presentation of the Metzlerian inventory adjustment process that it will add two further laws of motion to those of the wage-price dynamics; these are the first two equations in the presentation of the full dynamics (4.36)–(4.41) below.

The growth dynamics of the model are based on the net investment demand of firms, as indicated in the last equation of the above quantity dynamics. We point out in addition that aggregate demand is based, on the one hand, on differentiated saving habits as far as the two groups of households of the model, workers and asset holders and their consumption functions, are concerned. On the other hand, the other part of aggregate demand, investment, is determined by the excess of the expected profit rate over the real rate of interest, on excess capacity and natural growth (including productivity growth). Moreover, there are given fiscal policy parameters for government behaviour in the intensive form of the model. We thereby in particular obtain the result that aggregate demand depends on income distribution and the wage share v, positively if consumption dominates investment and negatively if the opposite holds true. We add finally that the nominal interest rate is determined either by a conventional LM curve or by the Taylor interest rate policy rule, to be introduced below.

We already observe here that the short-run quantity dynamics are difficult to estimate; see the next section for some first results in this regard. This is partly due to the need to distinguish between output, demand and sales expectations on the one hand and between desired and actual inventory changes on the other hand. In subsequent developments of the model it would be desirable to take into account more modern cost-minimising inventory adjustment procedures on the goods market. Yet, at the present stage of development of the model a procedure that is a consistent extension of the familiar dynamic multiplier process is all that we need in order to make the model an internally coherent one.

4.4.4 Putting things together: the KMG growth dynamics

Let us finally make explicit the sixth law of motion, namely the one for economic growth, before we collect all laws of motion that are presented below. As already stated, in a Keynesian context, capital stock growth is given by net investment per unit of capital and is thus based on the assumption of an investment function of firms. This function is postulated to read

$$I/K = i_1(r^e - (i - \pi^c)) + i_2(u - \bar{u}) + n, \quad u = y/y^p, \tag{4.30}$$

with the expected rate of profit defined by

$$r^e = y^e - \delta - vy, \quad y^e = Y^e/K, \quad y = Y/K, \tag{4.31}$$

and the nominal rate of interest given by the reduced form LM equation

$$i = i_o + \frac{h_1 y - m}{h_2}. \tag{4.32}$$

Due to the assumed Metzlerian quantity adjustment process, the output to capital ratio is determined by

$$y = (1 + (n_l + n_x)\alpha_{n^d})y^e + \beta_n(\alpha_{n^d} y^e - \nu), \; y^e = Y^e/K, \; \nu = N/K. \tag{4.33}$$

The investment equation ensures that net investment depends on excess profitability with respect to the expected real rate of interest, on capacity utilisation in its deviation from desired capital utilisation and on a trend term which here has been set equal to the natural rate (including the rate of labour productivity growth) for reasons of simplicity.

The sixth state variable of our model is l, the full employment labour intensity, which in the context of Harrod-neutral technical change, with $x = Y/L^d$, $\hat{x} = n_x$, and $y^p = Y^p/K = const.$, is best represented by $l = xL/K$, where L denotes labour supply (which grows at the given natural rate of growth $n = \hat{L}$). Due to the assumed trend growth term in the investment equation shown above we find that the evolution of this state variable is given by

$$\hat{l} = -i_1(r^e - (i - \pi^c)) - i_2(u - \bar{u}).$$

We add as the final (algebraic) equation of the model the equation for aggregate demand per unit of capital that is given by

$$\begin{aligned} y^d &= (1 - s_w)vy + (1 - s_c)r^e + \gamma + I/K + \delta \\ &= (1 - s_c)y^e + (s_c - s_w)vy + \gamma + I/K + s_c\delta, \end{aligned} \tag{4.34}$$

and the defining equations for the rate of employment and the rate of capacity utilisation which we recall are

$$e = y/l \quad (= L^d/L = xL^d/xL), \quad u = y/y^p. \tag{4.35}$$

Due to our assumption of Kaldorian saving habits with $0 \leq s_w < s_c \leq 1$, we have that aggregate demand depends positively on the wage share v through consumption and negatively on the wage share through the investment component in aggregate demand. There is wage taxation and property income taxation which are assumed to be constant per unit of capital, net of interest as in Sargent (1987) and Rødseth (2000). These fiscal policy parameters as well as government expenditures per unit of capital, also assumed to be constant, are collected in the parameter γ of the aggregate demand function shown above.

We are now in a position to present the full macrodynamic model, here for brevity immediately in intensive or state variable form. We recall that the dynamic model is based on five markets: labour, goods, money, bonds and equities; and three sectors: households (workers and asset holders, with Kaldorian differentiated saving habits), firms and the government (the fiscal and monetary authority). We stress again that all

budget equations are fully specified on the extensive form level, so that all stock-flow interactions are present, although not yet fully interacting in the current version of the model.[34]

The resulting integrated six laws of motion of the dynamics to be investigated include the state variables: sales expectations $y^e = Y^e/K$; inventories $\nu = N/K$ per unit of capital; real balances per unit of capital $m = M/(pK)$; the inflationary climate π^c; the wage share $v = \omega/x$; labour intensity $l = L/K$. The laws of motion read:

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + \hat{l}y^e, \tag{4.36}$$

(the law of motion for sales expectations),

$$\dot{\nu} = y - y^d + (\hat{l} - (n_l + n_x))\nu, \tag{4.37}$$

(the law of motion for inventories),

$$\hat{m} = \bar{\mu} - \pi^c - (n_l + n_x) + \hat{l} - \kappa[\beta_p(u - \bar{u}) + \kappa_p\beta_w(e - \bar{e})], \tag{4.38}$$

(the growth law of real balances),

$$\dot{\pi}^e = \beta_{\pi^c}\kappa[\beta_p(u - \bar{u}) + \kappa_p\beta_w(e - \bar{e})], \tag{4.39}$$

(the evolution of the inflationary climate),

$$\hat{v} = \kappa[(1 - \kappa_p)\beta_w(e - \bar{e}) - (1 - \kappa_w)\beta_p(u - \bar{u})], \tag{4.40}$$

(the growth law of the wage share),

$$\hat{l} = -i_1(r^e - (i - \pi^c)) - i_2(u - \bar{u}), \tag{4.41}$$

(the growth law for labour intensity).

These equations can be easily understood from what has been stated about wage-price, quantity and investment dynamics if note is taken of the fact that everything is now expressed (with the exception of the wage share) in per unit of capital form. Inserting the algebraic equations (4.30)–(4.35) into these laws of motion one obtains a non-linear autonomous 6D system of differential equations that we will investigate with respect to the stability properties of its unique interior steady state in the remainder of the chapter.

4.5 Feedback-motivated stability analysis

As the model is formulated we can distinguish four important feedback chains which we now describe in isolation from each other. Of course these interact with each other in the full 6D dynamics and one or the other can become dominant depending on the model parameters chosen. These feedback channels are shown in bold in Figure 4.4, where also other feedback channels have been added: the Dornbusch exchange rate dynamics and other (primarily destabilising) feedback chains. Integrating these feedback channels

[34] See Chiarella and Flaschel (2000) for the details of this Keynesian working model, including the specification of all budget and behavioural equations on the extensive form level, and Chiarella *et al.* (2000) for various extensions of this model type.

into a coherently formulated Keynesian macrodynamic set-up is one of the main tasks of this book and of our future research agenda.

1. The Keynes effect: We assume IS–LM equilibrium in order to explain this well-known effect in simple terms. According to IS–LM equilibrium, the nominal rate of interest i depends positively on the price level p. Aggregate demand and thus output and the rate of capacity utilisation therefore depend negatively on the price level, implying a negative dependence of the inflation rate on the level of prices through this channel. A high sensitivity of the nominal rate of interest with respect to the price level (a low parameter h_2, the opposite of the liquidity trap) thus should exercise a strong stabilising influence on the dynamics of the price level and on the economy as a whole, which is further strengthened if price and wage flexibility increase.[35]

2. The Mundell effect: We again assume IS–LM equilibrium in order to explain this less well-known (indeed often neglected) effect. Since net investment depends (as is usually assumed) positively on the expected rate of inflation π^c, via the expected real rate of interest, aggregate demand and thus output and the rate of capacity utilisation depend positively on this expected inflation rate. This implies a positive dependence of $\hat{p} - \pi^c$ on π^c and thus a positive feedback from the expected rate of inflation on its time rate of change. Faster adjustment speeds of inflationary expectations will therefore destabilise the economy through this channel. The two effects just discussed work with further delays if Metzlerian quantity adjustment processes are allowed for.

3. The Metzler effect: In the Metzlerian quantity adjustment process, output y depends positively on expected sales y^e and this effect is stronger, the higher the speed of adjustment β_n of planned inventories. The time rate of change of expected sales therefore depends positively on the level of expected sales when the parameter β_n is chosen sufficiently large. Flexible adjustment of inventories coupled with a high speed of adjustment of sales expectations thus leads to a loss of economic stability. There will, of course, exist other situations where an increase in the latter speed of adjustment may increase the stability of the dynamics.

4. The Rose effect: In order to explain this effect we again assume for the time being IS–LM equilibrium. We know from our formulation of aggregate goods demand that output and in the same way the rate of employment and the rate of capacity utilisation may depend positively or negatively on real wages, due to their opposite effects on consumption and investment shown in equation (4.34). According to the law of motion for real wages (4.36) we thus get a positive or negative feedback channel of real wages on their rate of change, depending on the relative adjustment speed of nominal wages and prices. Either price or wage flexibility will therefore always be destabilising, depending on investment and saving propensities, i_1, s_c and s_w, with respect to the expected rate of profit and the wage share. The destabilising Rose effect (of whatever type) will be weak if both wage and price adjustment speeds β_w and β_p are low.

The effects just discussed are shown in their interaction in Figure 4.4. This figure is centred around the hypothesis that there is a downward hierarchy in the structure

35 The same argument applies to wealth effects which, however, are not yet included here.

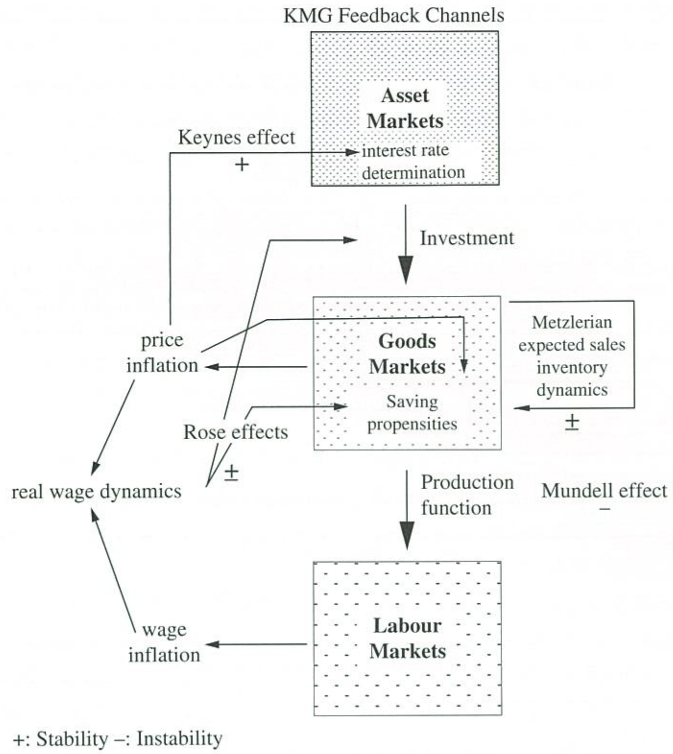


Figure 4.4 The feedback channels of the KMG modelling approach and their stabilising/destabilising tendencies

of market economies, with financial markets at the top of this hierarchy, with goods market dependence on the outcome on financial markets arising through investment behaviour and finally labour market dependence on the goods market outcome coming through the production function. However, this point of departure for macroeconomic theorising is modified in significant ways if repercussions from hierarchically lower markets are taken into account, the most prominent one being the Keynes effect, which is much favoured due to its stabilising role both with respect to wage-price increases and wage-price decreases through the change in nominal interest it implies. The Mundell inflationary expectations effect comes next in popularity, since it is generally present in Keynesian macrodynamics, whether old (through investment behaviour) or new (via the household's Euler equations) or of other contemporary type. Metzlerian inventory adjustment is generally neglected due to the consideration of IS equilibria in macroeconomics, as are other quantity adjustment processes such as the dynamic multiplier or the multiplier-accelerator process. Rose effects and the role of income distribution are rarely considered in Keynesian macrodynamics, although of course their existence is hardly surprising or difficult to grasp.

These more or less traditional feedback channels, the nominal-interest rate Keynes effect, the inflationary expectations Mundell effect, the Metzler inventory accelerator and the real-wage Rose effect, are here combined and determine in their interaction the stability of the interior steady state position of the model. If inventories are adjusted too quickly instability may arise despite the presence of a stable dynamic multiplier process, due to the fact that production is then too responsive to expected demand changes via the planned inventories channel. The Mundell effect is potentially destabilising, since inflation feeds into expected inflation, which in turn lowers the real rate of interest and further increases economic activity and thus the rate of inflation. The Rose effect can be destabilising in two ways, if aggregate demand depends positively on the real wage and the wage share in the case where wage flexibility exceeds price flexibility or in the opposite case of depressing effects of real wage increases if price flexibility exceeds wage flexibility.[36]

The only unambiguously stabilising effect is the Keynes effect whereby increasing prices and wages decrease real liquidity and thus raise nominal rates of interest which not only stops further wage-price increases, but in fact brings wages and prices back to their 'full' employment levels.

Based on the insights gained from these partial feedback chains we are now in a position to formulate Proposition 4.10 on local stability, instability and limit cycle behaviour. Our approach is based on what we term feedback-guided β-stability analysis. This methodology for the stability analysis of the high-dimensional dynamic models to which our approach leads is explained in detail in Asada *et al.* (2003).[37] This stability analysis methodology has also been applied to a variety of other situations, see in particular Asada *et al.* (2010). Here the β-stability analysis nicely confirms, for our integrated Keynesian dynamics, what has long been known (in principle) for its constituent parts.

Proposition 4.9 (KMG dynamics: basic stability results)
Assume that $0 \leq s_w < s_c \leq 1$ holds, so that the savings rate of workers is lower than that of asset holders. The following statements then hold with respect to the 6D dynamical system (4.36)–(4.41), under some further secondary assumptions on the parameters of the model:

1. *There exists a unique interior steady state of the model basically of supply side type.*
2. *The determinant of the 6D Jacobian of the dynamics at this steady state is always positive.*
3. *Assume that the parameters $\beta_w, \beta_p, \beta_{\pi^c}, \beta_n$ are chosen sufficiently small and the parameter β_{y^e} sufficiently large and assume that the Keynes effect works with sufficient strength (so that h_2 is small).*

Then, the steady state of the 6D dynamical system is locally asymptotically stable.

36 A more detailed explanation of such adverse Rose effects has to pay attention to the κ-weights in the cost-pressure terms as well.

37 See also Chiarella and Flaschel (2000, Ch. 6) and see also Köper (2000) for the first detailed presentations of such a stability investigation by means of varying adjustment speeds β_j where $j = n, \pi^c, w$.

4. *On the other hand, if any one of β_w (or β_p), β_{π^c}, β_n become sufficiently large (the latter for β_{y^e} sufficiently large), then the equilibrium is locally repelling and the system undergoes a Hopf bifurcation at an intermediate value of the relevant β parameter. In general stable or unstable limit cycles are generated close to the bifurcation value.*

Sketch of proof (the β-stability methodology). Based on our partial knowledge of the working of the four feedback channels of the considered 6D dynamics, we choose an independent 3D subsystem of the 6D Keynesian dynamics, by setting the parameters $\beta_n, \beta_{\pi^c}, \beta_w$ all equal to zero:

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + (n_l + n_x - I/K)y^e,$$
(the stable dynamic multiplier),
$$\hat{m} = \bar{\mu} - \kappa\beta_p(u - \bar{u}) - \pi_o^c - i,$$
(the stabilising Keynes effect),
$$\hat{v} = -\kappa(\kappa_w - 1)\beta_p(u - \bar{u}),$$
(sluggish price adjustment).

In this 3D system, the Keynes effect (h_2 small) and the dynamic multiplier (β_{y^e} large) dominate the outcome and imply the Routh–Hurwitz conditions for local asymptotic stability are satisfied if they operate with sufficient strength and if β_p is sufficiently small (which avoids stability problems arising from any type of Rose effect).

We then add step-by-step the further laws of motion by assuming that the adjustment speeds initially assumed to be zero are made slightly positive:

$$4D : \beta_w > 0 : \hat{l} = -i_1(r^e - (i - \pi^c)) - i_2(u - \bar{u}),$$
(labour intensity feeds back into the 3D dynamics via $e = y/l$),
$$5D : \beta_n > 0 : \dot{\nu} = y - y^d + \ldots,$$
(inventory accumulation feeds back into the 4D dynamics via y),
$$6D : \beta_{\pi^c} > 0 : \dot{\pi}^e = \beta_{\pi^c}[c_y\beta_p(u - \bar{u}) + c_h\beta_w(e - \bar{e})],$$
(inflationary climate starts moving and influencing the 5D dynamics).

Since the determinants of the Jacobian at the steady state of the sequentially enlarged dynamics always have the correct sign, as required by the Routh–Hurwitz stability conditions, we know that the eigenvalue that is departing from zero (as a result of a certain adjustment speed becoming slightly positive) must always do so through negative values. In this way, a system with at most one pair of complex eigenvalues (with negative real parts) and at least four real and negative ones is established, which proves the local asymptotic stability asserted in the proposition.[38] Since the determinant of the

38 In numerical simulations we have frequently observed monotone convergence for very small positive values of the relevant adjustment speed, indicating that the complex eigenvalues occur at higher (positive) values.

full Jacobian is always non-zero, loss of stability can only occur by way of (in general non-degenerate) Hopf bifurcations, at which eigenvalues cross the imaginary axis with positive speed.

Although intrinsically non-linear, the above 6D Keynesian growth dynamics are generally, however, too weakly non-linear in order to guarantee the boundedness of trajectories when the adjustment speeds referred to in Proposition 4.9 are chosen such that local instability comes about. Extrinsic or behavioural non-linearities thus have to be added subsequently in order to ensure boundedness for the trajectories of the dynamics.

This closes the introduction into the KMG baseline model which however is extended in the next section through the introduction of debt deflation into this baseline framework.

4.6 Debt deflation in the KMG framework

The KMG growth model of the preceding sections represents the workhorse model for the remainder of the book to which we want to add now briefly the processes of debt accumulation and debt deflation already discussed, in order to indicate how its features change through the addition of such forces. We thus modify the KMG model and include debt financing of investment in place of equity financing. Furthermore we will replace the original set-up with a money supply rule (with the resulting interest rate determination) by an interest rate Taylor policy rule for the banking sector. This will allow control of what here remains of the Keynes effect in a more direct way in the attempt to stabilise the economy in the case of debt deflation.

Investment decisions are now solely based on a term $-\alpha_3^k\lambda$ in place of $\alpha_3^k(\lambda_o - \lambda) + \gamma + \delta$ of the 4D dynamics and therefore no longer refer to steady state values of the debt to capital ratio. This modifies the interior steady state solution to some extent, but does not influence the dynamics of the model significantly.[39]

Due to the Taylor rule and the implied endogeneity of the money supply we have again that price inflation, but not inflationary expectations, can be removed from explicit consideration in the stability analysis that follows. Furthermore, since public debt is now financed through the banking system (and not through the household sector) it also has no feedback on the private sector of the economy, so that government bond accumulation is still irrelevant for the dynamics of this sector. Finally, we can of course derive a law of motion for the evolution of real balances per unit of capital, to be obtained from the budget equation of the banking system, but there is now no feedback effect from this state variable on to the other ones.

This establishes a 7D system of interdependent laws of motion to be given below, by adding the dynamics of sales expectations, inventories, inflationary expectations and

39 We now also use the normal rate of profit in place of the actual one in the investment function which makes the profitability term independent of the state of the business cycle and thus removes some correlations between the terms employed in the investment function.

the rate of interest to the core 3D dynamics considered in the preceding sections. The resulting dynamical system is thus still fairly close to the one of the preceding section and can also be investigated from the same perspective, although it is now complete and coherent in its description of goods market dynamics and the budget equations of workers, the government and the banking system, in addition to the one of firms already studied in the preceding sections from a supply side and a demand side perspective.

The complexity that the KMG approach seems to imply for the investigation of debt deflation can thus be reduced considerably by appropriate assumptions. The introduction of interdependence between money balances, government bond accumulation, and real and nominal magnitudes will be treated in Part III. The focus of interest here remains income distribution and capital accumulation, supplemented now by a Metzlerian quantity adjustment process plus the new element, the dynamics of the debt to capital ratio and the dynamics of the rate of interest on loans. In this way we obtain a 7D dynamical system that is coherent in its modelling of goods market adjustment process and is minimally complete in its treatment of the issue of debt deflation in a Keynesian set-up with sluggishly adjusting wages, prices, quantities and expectations. At the centre of the system's dynamics are the Rose effects and the Fisher debt effect of the preceding sections, now coupled with multiplier stability or instability and weak forms of stabilising Keynes and destabilising Mundell effects which here both work solely through the law of motion for the debt to capital ratio.

4.6.1 Integrating debt financing of firms

Let us first discuss the budget equations of the four sectors that interact in this extension of the KMG approach. First we consider wage earners who spend what they get (after wage taxation at rate τ_w) and who therefore do not lend, thereby contributing to the debt deflation mechanism, but exhibit the simplest type of behaviour that is possible in such a framework:

$$pC_w = (1-\tau_w)wL^d, \quad T_w^n = \tau_w wL^d \tag{4.42}$$

Next there is the sector of industrial capital which always invests its pure profits (after taxation) and takes loans (corporate debt) in addition to realising its intended investment plans. We do not discuss processes of credit rationing in this section and thus assume that loans demanded are always supplied by the banking system. Up to taxation and the fact that pure profits are now based on actual sales we have no change in this description of partially debt financed investment of firms:

$$pI = (1-\tau_c)rpK + \dot{\Lambda}, \quad rpK = pY^d - \delta pK - wL^d - i\Lambda, \quad T_c^n = \tau_c rpK. \tag{4.43}$$

Third there is the financial sector which gives loans to firms and the government (as demanded by these two sectors) and which creates or absorbs money to the extent that its interest income is different from the loans demanded by firms and the government, which therefore exhibits the soft budget constraint

$$\dot{\Lambda} + \dot{B} = i(\Lambda + B) + \dot{M}. \tag{4.44}$$

Finally, there is the government sector which finances its expenditures (goods purchases and interest payments) by means of the taxes it receives and by making loans (public debt) if necessary:

$$pG + iB = \quad T_w^n + \quad T_c^n + \dot{B}. \tag{4.45}$$

These budget equations show that there is no credit rationing by the banking system, apart from interest rate changes, so that financial capitalists accept the new issue of government bonds and also supply just the amount of new loans that is demanded by firms, through the creation or destruction of money if necessary. Money is fully endogenous in the present model and has of course inflationary consequences to the extent it feeds aggregate goods demand and thus creates demand pressure on the market for goods and for labour. Of course, due to this fact Say's Law no longer holds as in the 3D supply dynamics considered earlier.

To complete this description of budgets and spending we assume, as already discussed, that net investment of firms is determined as in the 4D case considered in this section, so that the investment function

$$I/K = [\alpha_1^k(r^n - \bar{r}) + \alpha_2^k(\frac{y}{y^p} - \bar{u})] - \alpha_3^k\lambda + \gamma \tag{4.46}$$

is now augmented by a third term and no longer makes use of a target debt to capital ratio of firms. For the final component in the aggregate demand function, government expenditure, we assume for reasons of simplicity in what follows that it is kept constant per unit of capital, in order to focus on investment and its debt financing. Government policy is thus basically characterised by a simple expenditure rule and two given tax rates and is thus kept fairly simple.

Yet, in an important respect we will depart from the representation of economic policy by given magnitudes throughout, namely with respect to monetary policy. We now assume that the monetary authority directly controls the (change in the) nominal rate of interest according to the rule

$$\dot{i} = -\beta_{i_i}(i - i_o) + \beta_{i_p}(\hat{p} - 0). \tag{4.47}$$

It therefore attempts to steer the economy to the steady state rate of interest, but deviates from this target when inflation deviates from zero inflation in an attempt to stop inflation by raising the interest rate, or stop deflation by lowering it (unless the floor of a zero rate of interest is reached). This provides a simple, but coherent description of the behaviour of the financial sector of the economy and posits implicitly that the interests of the owners of firms are more complex than just looking for the highest rate of return for the capital that they own.

4.6.2 *Enterprise debt dynamics in the KMG framework*

Putting together all the foregoing changes and adding them into the baseline KMG model of this chapter, gives rise to the following 10D (6D+4D) dynamical system in the state variables $w^e = w/z$, p, π^c, $l^e = zL/K$, $\lambda = \Lambda/(pK)$, $y^e = Y^e/K$, $\nu = N/K$, i, $b = B/(pK)$, and $m = M/(pK)$ as in the KMG model, but now with loans in place of equity financing and an interest rate policy rule in place of a money supply rule:

$$\hat{w}^e = \kappa[\beta_w(\frac{y}{l^e} - \bar{e}) + \kappa_w\beta_p(\frac{y}{y^p} - \bar{u})] + \pi^c \tag{4.48}$$

$$\hat{p} = \kappa[\beta_p(\frac{y}{y^p} - \bar{u}) + \kappa_p\beta_w(\frac{y}{l^e} - \bar{e})] + \pi^c \tag{4.49}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\hat{p} - \pi^c) \tag{4.50}$$

$$\hat{l}^e = -(g_k - \delta - \gamma) \tag{4.51}$$

$$\dot{\lambda} = (g_k - \delta)(1 - \lambda) - (1 - \tau_c)r - \hat{p}\lambda \tag{4.52}$$

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) - (g_k - \delta - \gamma)y^e \tag{4.53}$$

$$\dot{\nu} = y - y^d - (g_k - \delta)\nu \tag{4.54}$$

$$\dot{i} = -\beta_{i_i}(i - i_o) + \beta_{i_p}\hat{p} \tag{4.55}$$

$$\dot{b} = g + ib - (\tau_w(w^e/p)y + \tau_c r) - (\hat{p} + g_k - \delta)b \tag{4.56}$$

$$\dot{m} = g_k - \delta - (y^d - \delta - (1 - \tau_w)(w^e/p)y) - (\hat{p} + g_k - \delta)m \tag{4.57}$$

with the following algebraic equations supplementing these ten laws of motion ($\gamma = n + n_l$ the natural rate of growth always as trend term):[40]

$$\begin{aligned}
y &= \beta_n(\beta_{n^d}y^e - \nu) + (1 + \gamma\beta_{n^d})y^e, \\
y^d &= (1 - \tau_w)(w^e/p)y + g_k + g, \\
r &= y^d - \delta - (w^e/p)y - i\lambda, \\
r^n &= y_o^d - \delta - (w^e/p)y_o - i\lambda, \\
g_k &= \alpha_1^k(r^n - \bar{r}) + \alpha_2^k(\frac{y}{y^p} - \bar{u}) - \alpha_3^k\lambda + \gamma + \delta.
\end{aligned}$$

The laws of motion for wages in efficiency units w^e and the price level p are as before, but now augmented by inflationary expectations π^c in the form of the across market

40 Note that we now make use of a normal rate of profit in place of the actual one (which is needed in the debt accumulation dynamics) in the investment function g_k. This has the advantage – compared with the one chosen for the 4D dynamics – that the state of the business cycle is eliminated from the α_1^k expression and thus appears only once, in the α_2^k expression. Note also that the α_3^k expression no longer refers to a benchmark debt to capital ratio (which was set equal to the steady state debt to capital ratio in the 4D dynamics). The steady state debt to capital ratio is therefore now calculated in a different way than in the case of the 4D dynamics.

PCs derived in Section 4.5. Inflationary expectations are of the adaptive variety here for reasons of simplicity, and will not matter in the following analysis. Capital accumulation is represented by the full employment capital ratio (in efficiency units) driven as usual by investment demand in this Keynesian approach to economic growth. The evolution of the debt to capital ratio λ is as before and is represented by the budget equation of firms in intensive form. The Metzlerian quantity adjustment process on the market for goods, represented by sales expectations y^e and inventories ν per unit of capital, remains as in the baseline KMG growth dynamics. The Taylor rule for interest on loans i has already been explained. Finally, the laws of motion for government bonds b and money m per unit of capital have been obtained from the budget restrictions of the government and the monetary authority (here still viewed as a financial capital centre) by appropriate reformulation on the intensive form level. Output y per unit of capital in the algebraic equations is based on Metzlerian quantity adjustment as in the KMG growth dynamics of Section 4.4 and Section 4.5. Aggregate demand y^d is simply the summation of wages after taxes, gross investment and government expenditures per unit of capital. The actual pure rate of profit r is defined as before, as is investment.

4.6.3 *Analysis of the model*

The dynamical system (4.48)–(4.57) can be reduced to the following core dynamics which represent their interdependent part

$$\hat{v} = \kappa[(1-\kappa_p)\beta_w(\frac{y}{l^e} - \bar{e}) - (1-\kappa_w)\beta_p(\frac{y}{y^p} - \bar{u})], \quad v = w^e/p, \tag{4.58}$$

$$\hat{l}^e = -(g_k - \delta - \gamma), \tag{4.59}$$

$$\dot{\lambda} = (g_k - \delta)(1-\lambda) - (1-\tau_c)r - [\kappa[\beta_p(\frac{y}{y^p} - \bar{u}) + \kappa_p\beta_w(\frac{y}{l^e} - \bar{e})] + \pi^c]\lambda, \tag{4.60}$$

$$\dot{\pi}^c = \beta_{\pi^c}\kappa[\beta_p(\frac{y}{y^p} - \bar{u}) + \kappa_p\beta_w(\frac{y}{l^e} - \bar{e})], \tag{4.61}$$

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) - (g_k - \delta - \gamma)y^e, \tag{4.62}$$

$$\dot{\nu} = y - y^d - (g_k - \delta)\nu, \tag{4.63}$$

$$\dot{i} = -\beta_{i_i}(i - i_o) + \beta_{i_p}[\kappa[\beta_p(\frac{y}{y^p} - \bar{u}) + \kappa_p\beta_w(\frac{y}{l^e} - \bar{e})] + \pi^c], \tag{4.64}$$

with the defining expressions

$$y = \beta_n(\beta_{n^d}y^e - \nu) + (1+\gamma\beta_{n^d})y^e,$$

$$y^d = (1-\tau_w)vy + g_k + g,$$

$$r = y^d - \delta - vy - i\lambda,$$

$$r^n = y_o^d - \delta - vy_o - i\lambda,$$

$$g_k = \alpha_1^k(r^n - \bar{r}) + \alpha_2^k(\frac{y}{y^p} - \bar{u}) - \alpha_3^k\lambda + \gamma + \delta.$$

Note we have inserted the equation for the price level dynamics in whatever equations they appear. Due to the relation $\hat{v} = \hat{w}^e - \hat{p}$ this implies that the system could be reduced to a 7D system with the wage share v in place of w^e. The price level dynamics can therefore be removed from explicit consideration. This also implies that the 8D system with price level dynamics included must exhibit zero root hysteresis (one eigenvalue must always be zero), since the price level does not feed back into the 7D dynamics where w^e has been replaced by v. We thus obtain the result that nominal values, w^e, p, are determined by historical conditions and thus do not return to initial steady state levels as in the case of a money supply rule, $\hat{M} = \gamma$, which was considered in the original formulation of the KMG growth dynamics. This is due to the fact that money does not restrict the economy in the present extension of the KMG dynamics.

Proposition 4.10 (Existence of balanced growth)
Assume $\bar{r} < \gamma$. *There exists* a uniquely determined interior steady state *of the dynamics (4.48)–(4.57) (where* $w_o^e, l_o^e, p_o \neq 0$ *holds) which is given by*

$$y_o = l_o^{de} = \bar{u}y^p,$$

$$y_o^e = y_o^d = y_o/(1+\gamma\beta_{n^d}), \quad \nu_o = \beta_{n^d}y_o^e,$$

$$l_o^e = l_o^{de}/\bar{e} = y_o/\bar{e},$$

$$v_o = \frac{y_o^e - \delta - \gamma - g}{(1-\tau_w)y_o} \in (0,1) \quad \textit{by assumption},$$

$$\lambda_o = \frac{\alpha_1^k(\gamma - (1-\tau_c)\bar{r})}{\alpha_1^k\gamma + \alpha_3^k(1-\tau_c)} < 1,$$

$$r_o = \gamma\frac{1-\lambda_o}{1-\tau_c} \geq \bar{r},$$

$$i_o = (y_o^e - \delta - v_o y_o)/\lambda_o,$$

$$\pi_o^c = 0 \quad (= \hat{p}_o = \hat{w}_o - n_l = \hat{w}_o^e),$$

$$p_o = \textit{arbitrary}, \quad w_o^e = p_o v_o,$$

$$b_o = \frac{g - (\tau_w v_o y_o + \tau_c r_o)}{\gamma - i_o},$$

$$m_o = \frac{\gamma - (y_o^d - \delta - (1-\tau_w)v_o y_o}{\gamma}.$$

Note that the steady state value of i has to be inserted into the Taylor interest rate policy rule in order to guarantee that this policy rule is consistent with what happens in the private sector. Note also that the steady state wage share is determined via goods market equilibrium (and thus is demand determined), while the rate of profit and the capital to debt ratio have to be calculated from the two equations $\dot{\lambda} = 0$ and $g_k - \delta = \gamma$, the budget equation and the gross investment function of firms. The determination of the steady state wage share and of the steady rate of profit is thus independent of each other, while the rate of interest is determined residually on the basis of these two elements of income distribution. Output and employment by contrast are purely supply side determined and thus reflect monetarist propositions to some extent.

Setting to zero the right-hand sides of equations (4.58) and (4.61) implies that demand pressure in the market for labour and for goods must be zero, which provides the supply side expressions for y_o, l_o^e and from there, via the inventory adjustment mechanism, also the values for y_o^e, ν_o. Goods market equilibrium then gives v_o, and from there λ_o, r_o as already described above. The remaining steady state values then follow easily from what has already been determined.

Proposition 4.11 (Cyclical loss of stability)

1. *The determinant of the Jacobian J of the dynamical system (4.58) – (4.64) at the steady state is always negative.*
2. *Local asymptotic stability can only be lost by way of a Hopf bifurcation (if the speed condition on the crossing eigenvalues of the Hopf bifurcation theorem is fulfilled).*
3. *Local asymptotic stability becomes lost if the parameter β_p becomes sufficiently large, (even) if the speeds of adjustment $\beta_w, \beta_{\pi^c}, \beta_n$ are sufficiently low, the rate of interest is fixed at i_o, a stable dynamic multiplier process is given ($y^d_{y^e} < 1$) and a normal price level Rose effect prevails ($y^d_v > 0$) according to which price flexibility should be stabilising.*

Proof: 1. Similar to the proof of Proposition 4.9, since only the assumption $\beta_{i_i} = 0$ has to be made in order to perform the steps that helped to reduce the determinants to manageable expressions in that proof.
2. A direct consequence of $\det J \neq 0$, since eigenvalues can then only cross the imaginary axis with real parts not equal to zero.
3. The dynamics that remain to be investigated are again of dimension 3 (representing one principal minor of order three in the very complicated set of Routh–Hurwitz conditions of the full 7D dynamics) and read

$$\hat{v} = -\kappa(1-\kappa_w)\beta_p(u-\bar{u}),$$
$$\dot{y}^e = \beta_{y^e}(y^d(v, y^e, \lambda) - y^e), \quad y^d_v > 0,\ y^d_{y^e} < 1,\ y^d_\lambda < 0,$$
$$\dot{\lambda} = I/K(1-\lambda) - (1-\tau_c)r - \kappa\beta_p(u-\bar{u})\lambda.$$

We know in addition that $(u)_{y^e} > 0$ where $u = y/y^p$ holds true since output is strictly proportional to expected sales and only depends on this variable in the present situation.

The Jacobian of these dynamics at the steady state is characterised by

$$J = \begin{pmatrix} 0 & -\beta_p \text{ (Stable Rose Effect)} & 0 \\ + & -\beta_{y^e} \text{ (Stable Multiplier)} & - \\ - & -\beta_p \text{ (Unstable Fisher Debt Effect)} & - \end{pmatrix}.$$

We here only show the signs of the entries of the Jacobian and the parameter speeds that are present in it in order to indicate which adjustment processes are favourable for local asymptotic stability and which are not. This form of J implies for the Routh–Hurwitz conditions:

$$a_1 = -\text{trace } J > 0, \quad \text{(Basically due to the Stable Dynamic Multiplier Process)}$$

$$a_3 = -\det J - \begin{vmatrix} 0 & -\beta_p \text{ (Stable Rose Effect)} & 0 \\ + & 0 & - \\ - & 0 & - \end{vmatrix} > 0,$$

(Due to the Stabilising Rose or Real Wage Effect)

$$a_2 = J_1 + J_2 + J_3 \gtreqless 0: \quad \text{(Stabilising Rose vs. Destabilising Fisher Debt Effects)}$$

and

$$a_1a_2 - a_3 = b(-\beta_p), \ b' = const < 0.$$

The latter relationship holds, since the Rose effect appears (with the same expressions) in both a_1a_2 and a_3 and thus cancels when the term $b = a_1a_2 - a_3$ is formed, implying that only the Fisher debt effect remains present in $b = a_1a_2 - a_3$ as far as terms that depend on the parameter β_p are concerned. This effect however produces a negative and linear dependence of $b = a_1a_2 - a_3$ on the parameter β_p implying that b must become negative when the parameter β_p is chosen sufficiently large. We thus see that the destabilising Fisher debt effect must eventually overcome the stabilising Rose effect, if not in the Routh–Hurwitz condition a_2, then in the condition b where the stabilising role of price flexibility due to the real wage or Rose effect is not present. □

The isolated Fisher Debt Deflation Feedback Chain that we characterise as

Debt Ratio $\lambda \uparrow \rightarrow$ Aggregate Demand $Y^d \downarrow \rightarrow$ Output $Y \downarrow \rightarrow$
Capacity Utilisation $u \downarrow \rightarrow$ Deflationary Impulse $\hat{p} \downarrow \rightarrow \lambda\uparrow\uparrow$.

must eventually become the dominant one if instability due to large parameters $\beta_w, \beta_{\pi^c}, \beta_n$ is excluded, if interest rates do not react to the state of the economy, even if multiplier and Rose effects are favourable for stability. It is however not easy to show that this result also holds for the full 7D dynamics, since the instability result of debt deflation is now not reflected in the trace of the Jacobian J, but is in fact present in one of the numerous products of principal minors that the 7D Jacobian implies for the calculation of the Routh–Hurwitz conditions on local asymptotic stability.

We have that local stability obtains if the system is in particular sufficiently sluggish in its adjustment behaviour (up to the speed of adjustment of sales expectations). Loss of stability will occur if either β_w or β_p is made sufficiently large, β_n sufficiently large and also when β_{π^c} is sufficiently large. We note however that the full 7D system is basically shaped by destabilising Fisher debt deflation effects ($\beta_p \uparrow$), destabilising Rose real wage effects (β_p or $\beta_w \uparrow$), destabilising Metzlerian accelerator effects ($\beta_n \uparrow$), but not by destabilising real rate of interest or Mundell effects ($\beta_{\pi^c} \uparrow$). This latter mechanism represents in the present model only an accelerator effect in the Fisher debt deflation mechanism. Furthermore, the Keynes nominal rate of interest rate effect (here present in the form of an interest rate policy rule) may be only a weak stabiliser in the considered 7D dynamics, since the dynamics of real balances do not feed back into the 7D dynamics under consideration and thus do not cause stabilising shifts of LM curve type. Instead, interest rate effects only work here through the term $i\lambda$ in the definition of the pure rate of profits r. We would therefore expect that the 7D dynamics are more often characterised by local instability than by local stability.

4.7 Conclusions and outlook

In this chapter we have introduced two basic models of debt dynamics and debt deflation and have investigated not only the issue of debt accumulation in the context of a growing economy, but also the dangers of price deflation in such a framework where there is high debt inherited from the past, in particular in the sector of firms. We have then integrated these processes into the more general KMG model of Chiarella and Flaschel (2000) and Chiarella *et al.* (2005) and shown how the tendencies discussed in the basic 3D and 4D models reappear in this more general and more coherently formulated approach to Keynesian monetary growth. Issues such as debt default, firm bankruptcies and credit rationing have been touched upon, but will be studied in much more detail in Part III.

Before undertaking such an analysis we extend in Part II the KMG approach towards a small open economy with a more detailed sectoral structure, including a housing sector. Such a structure is suggested by the empirical model presented in detail in Powell and Murphy (1997) and there applied to the Australian economy. We use their approach to derive a theoretical continuous-time model comparable in extent to their applied approach in order to study again the interaction of feedback channels we have introduced in this chapter and the additional ones suggested by the Powell and Murphy (1997) modelling approach. The higher dimensional models in Part III are in fact then based on the theoretical framework we will establish in Part II, concerning the further analysis of debt deflation in the sector of firms as well as in the housing sector.

Part II

Theoretical foundations for structural macroeconometric model building