5 Keynesian macroeconometric model building: a point of departure

5.1 Introduction

In this part of the book we lay the theoretical foundations for the construction of larger macroeconometric models where large means the approximate size of, for example, the Murphy model of the Australian economy (approximately a hundred equations). In practice this implies that about twenty laws of motion have to be considered in order to describe the evolution of such an economy. Yet, in contrast to many models that are actually applied we insist here that such models must be completely specified in terms of budget equations (identities or restrictions) and the stock-flow interactions that they imply. Moreover the models should not only be formulated on the extensive form level, but must also allow for a representation in intensive terms as well (trendless variables as far as the theoretical representation of the model is concerned). This intensive form representation should then also allow the determination of at least one steady state solution, the stability of which is to be discussed from the perspective of the partial feedback structures which are included in the general formulation of the model.

In this chapter we extend the hierarchically structured continuous-time models of Keynesian monetary growth, that have been introduced and generalised in some respects in Chiarella and Flaschel (2000, Chs. 4–7), Chiarella *et al.* (2000, Chs. 4–6) and Asada *et al.* (2003) both for closed as well as open economies, along the lines of the macroeconometric Murphy model of the Australian economy. The resulting modelling framework leads towards an empirically motivated model of a small open economy with a Keynesian short and medium run and with classical or monetarist features in the medium- as well as the long-run behaviour of the economy. The Murphy model (see Powell and Murphy (1997) for its detailed description) and our theoretical reformulation of it in this chapter, therefore blend demand and supply side approaches into an integrated and coherent whole with a (from a theoretical point of view) very detailed description of the structure of a small open economy, like Australia (in the case of the Murphy model). The present chapter, however, approaches this task from the perspective of macrotheory developed in Chiarella and Flaschel (2000) and Chiarella *et al.* (2000, Chs. 4–6) and thus mirrors the approach chosen in Powell and Murphy (1997) only to a certain degree. We will go on to sketch what may be the consequences of the present approach for macroeconometric model building. In subsequent chapters we shall provide more details on the dynamic structure and analysis of the present model (its intensive form, steady state analysis, more or less complex attractors, transients, etc.). We shall also attempt to move closer to the structure of the Murphy model, by revising the equations of the model of this chapter towards the inclusion of smooth factor substitution and other flexibilities, as well as a more standard view of the money market and money supply (as contained in the Murphy model). We will continue to use a continuous-time approach (without any discrete lags) so as to allow for a compact representation and analysis of the dynamics implied by the model.

In this chapter we add to the structural form of the Murphy model a complete set of fully specified budget equations for all sectors and thereby take account of all feedback structures implied by such budget restrictions. Furthermore all equations are specified in a consistent way from the perspective of dimensional analysis and are at first chosen in as linear a fashion as possible. This allows for a discussion of intrinsic (natural) non-linearities before attempts are made to design non-linearities that may keep the economy a viable one should it depart too much from the steady state in its naturally non-linear design. This in particular means that we shall start our theoretical reconsideration of the Murphy model on the basis of a fixed proportions technology which has the additional advantage that the rate of capacity utilisation of the capital stock is easy to define and to analyse with regard to the economic consequences implied by under- or over-utilised capital.

This choice of starting point for the analysis of the dynamic properties of structural macroeconometric models of open economies thus allows us, on the one hand, to study the implications of its intrinsic or 'natural' non-linearities first, as they derive from unavoidable growth rate formulations, products or quotients of state variables and the like. We shall see in this set-up that viability or boundedness of the dynamics (and in particular convergence to the steady state) will often depend on the assumption of sufficiently low adjustment speeds for quantities, prices and expectations, while local and even global stability will normally get lost if these adjustment parameters are chosen sufficiently high.

Our approach will allow us to introduce extrinsic non-linearities into the assumed technological or behavioural relationships in a systematic way at a later stage of the analysis, as a (theoretically reflected) response of the economy to the specific local instabilities observed in the working of the basic form of the model, often already known from partial dynamic macro-models. For example, a kink in the money wage Phillips curve (PC) (which reflects the fact that money wages may rise quickly in a boom but will only fall slowly – if at all – in a depression) is often of itself sufficient to avoid the inflationary instability that derives in such models from the existence of the so-called Mundell effects (an institutional non-linearity that has been very much neglected in the theoretical and applied debate on PCs inflation and stagflation).

We therefore will attempt here and in future work to proceed step by step to a detailed and systematic theoretical and numerical analysis of the dynamic features (steady states, attractors, transients, etc.) of complete and coherently formulated structural

macroeconometric models, including those applied to actual economies, a theoretical discussion that so far has been basically lacking in the literature.[1] The reader should consult Asada *et al.* (2011) for a good overview of our modelling philosophy as well as some policy prescriptions.

Similarities of the Murphy model with the theoretical work on hierarchically structured Keynesian monetary growth models of Chiarella and Flaschel (1999a,b,c,d), became more and more apparent as work on this book progressed. Extending the working model of Chiarella and Flaschel (2000, Ch. 6) to the open economy, as in Asada *et al.* (2003), then provided the impetus for not only continuing with the hierarchical structure of Keynesian monetary growth models established in Chiarella and Flaschel (2000) towards more and more elaborate versions. This impetus also became the starting point for a further project of developing fully integrated and coherent Keynesian models from a so to speak reverse perspective, namely from the structural forms of small (or large) open economies as they are used in macroeconometric model building. Here the Murphy model was particularly useful, not only due to its many similarities with the theoretical work of Chiarella and Flaschel (2000), Chiarella *et al.* (2000, Chs. 4–6), and Asada *et al.* (2003), as already observed above, but also due to its very detailed and thorough presentation and discussion in Powell and Murphy (1997).

We use the structural model of Powell and Murphy (1997) in a simplified as well as in a more complex way. We suppress many of the lags included in the Murphy model as well as some secondary structural components. We write down for the model all sectoral identities or budget equations of agents and include all the feedbacks that they imply (in particular for asset accumulation). Finally, we modify in the present chapter (and in subsequent work) more or less the equations of the Murphy model in the light of the dynamic equations used in the main parts of Chiarella and Flaschel (2000), Chiarella *et al.* (2000), and Asada *et al.* (2003). In this way we arrive at theoretical presentations of such structural macroeconometric models which at one and the same time intend to be descriptive (to a certain extent) and theoretically consistent in the sense of dimensional analysis and of the budget equations that the various agents of the model are facing.

In the basic version of the model, we have endogenous natural rates of growth, of Harrod neutral technical change and of NAIRU employment, but exogenous output growth in the rest of the world. There is a detailed set of direct and indirect taxation schemes, including various types of wage taxation and payroll taxes. We have two types of households, pure asset holders and workers, with differentiated saving habits, where the latter group only saves in the form of savings deposits (or fixed-price bonds). Wage income (of three types of workers' households) is taxed at a different rate than interest income of asset holders (and of workers). Sluggish price dynamics, accompanied by Metzlerian quantity dynamics and varying degrees of capacity utilisation of the capital

[1] See however Barnett and He (1999) for an interesting approach to the analysis of applied macroeconometric models from the theoretical and the numerical perspective. We believe that further investigations of this type are urgently needed with respect to applied macroeconometric model building.

stock characterises the market for the non-traded domestic commodity, and we have of course also sluggish adjustment of money wages and varying employment rates on the labour market, augmented by insider and outsider considerations combined with a sluggish adjustment of the outside employment rate in view of the over- or under-employment of the employed workforce (the insiders). We have as a module of the model the market for housing services and consider investment in housing besides the investment of firms in fixed business capital, both depending on profitability and rates of capacity utilisation. There is a detailed description of the government sector with respect to tax receipts and expenditures and – based on that and the pure equity financing of firms that is still assumed in this chapter – also a full set of asset accumulation equations accompanied by asset price or interest rate dynamics (in the place of a full portfolio approach to asset markets). Pensions and unemployment benefits and their financing via payroll taxes are treated explicitly. There is no role for money in the model at present, but only an interest rate policy rule of the monetary authority which fixes the short-term rate of interest of domestically traded bonds in the light of certain measures of economic activity adopted by the central bank. There are finally exports of finished goods and imports of raw materials or semi-finished goods and there is (although still somewhat limited) international trade in financial assets, and a foreign exchange market that is always cleared by the actions of the private sector and the government despite the assumption of only a finite speed of adjustment of the rate of exchange.

The foregoing brief list of some structural components of the model of monetary growth of this chapter indicates that it will include various elements that are of importance in the current discussion of the macroeconomic problems that governments confront. It goes without saying, however, that there are still important components of a macroeconomy that are missing in the present theoretical reformulation of the Murphy model. The model of the present chapter therefore only represents a first step in the direction of formulating an integrated model of monetary growth for an open economy, which, on the one hand, is related to empirical work and which, on the other hand, allows for a complete computation of the steady state values of the model as well as numerical and sometimes also theoretical analyses of the behaviour of the dynamics around the steady state of such a high dimensional dynamical system.

In the next section we provide an overview on the structure of the real and the financial parts of the model, characterising the sectors, markets and activities that will be included in our model. Section 5.3 then presents this structure from the viewpoint of the system of national accounts and provides thereby a detailed introduction to the notation that is used in this chapter. In Section 5.4 we present the structural equations of the model (in their extensive form) by way of an appropriate subdivision into important modules that build up the model. Section 5.5 finally gives an outlook on what to do with this model type when it is reduced from extensive form to per unit of capital expressions or intensive form, to the laws of motion for the state variables of the model. We will find that there is a (basically) uniquely determined interior steady state for this dynamical system which appears to be locally attracting for low adjustment speeds, but which generally is surrounded by centrifugal forces leading to limit cycles or more complex

attractors or to pure explosiveness for large displacements from the steady state or for adjustment speeds that are chosen sufficiently large. Further (extrinsic) non-linearities are therefore needed in general to get bounded dynamics for a larger set of parameter constellations.

5.2 The real and the financial part of the economy

The two tables below provide a survey of the structure of the economy to be modelled that is related, but not identical, to the description of the Australian economy given in Powell and Murphy (1997). Note in this respect that the aim of the present chapter is to establish an integrated continuous-time model, leading to an autonomous system of differential equations, where all sectors are fully specified with respect to their behaviour and their budget constraints from the viewpoint of complete theoretical models of monetary growth. A bridge will thereby be provided between the Keynes–Metzler type monetary growth models of Chiarella and Flaschel (2000), Chiarella *et al.* (2005) and the Powell and Murphy (1997) approach. This perspective of the economy will allow us to highlight where we deviate from the framework given in Powell and Murphy (1997). In Section 5.4 we will briefly comment on the differences between their discrete time macroeconometric model and our continuous-time one.[2]

5.2.1 The structure of the real part

Let us start with a presentation of the variables that comprise the real part of the economy. Table 5.1 provides data on the temporary equilibrium position of the economy, based on given prices and expectations, and also shows real stocks and their rates of growth.

Table 5.1 describes the real sector of the economy. We have a labour market, three commodity markets and the housing market. Domestic production Y concerns one good that is only domestically used (for all private consumption $C_w + C_c$, all investment $I, I_h, \mathcal{I}$, also in housing, and all government consumption G and which uses up all the imports J^d as intermediate goods) and one that is only used for exports X. There is thus only a single commodity used in domestic absorption, apart from the housing services C_h^d demanded by workers.

Our model exhibits three domestic sectors: households, firms and the government, but with heterogeneous agents in the household sector, workers and (pure) asset holders, the former supplying their labour L at the wage level w^b (which includes payroll taxes) and the latter the housing services C_h^s for the workers as far as real flows are concerned.[3] Firms produce a non-traded domestic and an exported commodity and employ labour L_f^w (with varying rates of utilisation L_f^d) and imports J^d besides their capital stock K

[2] The real-financial market interaction is also studied in Chiarella *et al.* (2009a) from a quite different perspective. The reader may also consult this reference for applied work in this context.

[3] Powell and Murphy (1997) do not have an explicit description of heterogeneous agents in their household sector, but basically use a uniform life-cycle hypothesis for the modelling of the consumption demand of this sector; see however their p. 117 for a brief remark on income distribution.

Table 5.1. *The real part of the economy (foreign country data: γ, p_x^*, p_m^*, $\tau_c^* = \tau_c$)*

	Labour	Non-traded Goods	Exports	Imports	Dwellings
Workers	$L = \alpha_l L_1$	C_w	–	–	C_h^d
Asset holders	–	C_c	–	–	C_h^s, I_h
Firms	L_f^d, L_f^w	$Y^p, Y, I, \mathcal{I}$	X	J^d	–
Government	L_g^d	G	–	–	–
Prices	$w^b, w[w^r, w^u]$	$p_v = (1+\tau_v)p_y$	$p_x = sp_x^*$	$p_m = (1+\tau_m)$ sp_m^*	p_h, p_y
Expectations	$\pi^e = \hat{p}_v^e$	$\pi^e = \hat{p}_v^e$	–	–	$\pi^e = \hat{p}_v^e$
Stocks	L_1	K, N	–	–	K_h
Growth	$\hat{L}_1 = n$	$\hat{K} = I/K - \delta_k$ $\dot{N} = Y - Y^d$	–	–	$\hat{K}_h = \frac{I_h}{K_h} - \delta_h$

for these purposes, and invest in fixed business capital I and inventories $\mathcal{I}$. Government finally provides public consumption goods G, pays rents w^r and unemployment benefits w^u and also employs part of the workforce L_g^d. There are a number of variables needed to describe the laws of motion of the quantities, prices p_v (including value-added taxes), and expectations about their rates of change, which will be explained in detail when we turn to the description of the various equations of the model in Section 5.4. There is endogenous growth n of the potential labour force L_1, of the capital stock K, and of the stock of housing K_h (supplied at price p_h for rental services) and also actual change of inventories N that is different from their desired rate of change $\mathcal{I}$.

5.2.2 The structure of the financial part

Let us next consider the financial part of the economy. Table 5.2 provides data on the changes in financial stocks, corresponding prices, and the growth of stocks in the financial part of the economy.

The first column in Table 5.2 shows that we do not consider money holdings. Cash management and transactions money are introduced in Chiarella *et al.* (1999a,b), in the usual form of an aggregate Cagan money demand function and also in a more disaggregated form than in the Murphy type model. We exclude money holdings in our basic modelling framework, by assuming that money is a costless medium of exchange for firms, household and the government that returns at the 'end' of each point in time t to the 'local branches' of the central bank by the balancing of the budget restrictions of these sectors.[4] At present there are only (four) interest-bearing financial assets in our model that can be held by the (pure) asset owners and by the workers of our economy (as shown in Table 5.2). As in the Keynes–Metzler model of monetary growth of Chiarella and Flaschel (2000) we here assume, in order to start with a simple representation of

[4] There are no commercial banks in the model of this chapter.

Table 5.2. *The financial part of the economy (foreign country data: i^*)*

	Money	Short-term Bonds	Long-term Bonds	Equities	Foreign Bonds
Workers	–	$\dot{B}_w$	–	–	–
Asset holders	–	$\dot{B}_c$	$\dot{B}_1^l$	$\dot{E}$	$\dot{B}_2^l$
Firms	–	–	–	$\dot{E}$	–
Government	–	$\dot{B}$	$\dot{B}^l$	–	–
Prices	1	1 $[i]$	$p_b = 1/i_l$	p_e	$sp_b^* = s \cdot 1/i_l^*$
Expectations	–	–	$\pi_b = \hat{p}_b^e$	$\pi_e = \hat{p}_e^e$	$\epsilon_s = \hat{s}^e$
Stocks	–	$B = B_w + B_c$	$B^l = B_1^l + B_1^{l*}$	E	B_2^l
Growth	–	$\hat{B}, \hat{B}_w, \hat{B}_c$	$\hat{B}^l, \hat{B}_1^l$	$\hat{E}$	$\hat{B}_2^l$

financial flows, that bonds are only issued by the government, that firms use only equity financing and pay out expected earnings as dividends, and that there exist also long-term bonds issued by the 'foreign government'. Financial flows between the sectors of our economy are therefore very narrowly defined (in order to simplify the flow budget restrictions to a sufficient degree). The laws of motion of the real part of our economy do not yet depend too severely on this financial structure of the economy since, as in Powell and Murphy (1997), we do not use a full portfolio approach towards the description of the stock equilibria of the economy. Rather we determine asset prices and asset returns through certain simple laws of motion, while the inflow of financial assets is basically determined from the supply side.[5] This is done in a way that implies equilibrium on the market for foreign exchange with respect to the flows appearing in the current as well as in the capital account so that there is no change in reserves held by the central bank and thus no need to consider this item explicitly in the balance of payments to be discussed in the next section.

For initial work on the situation where loans to firms, inside debt of the household sector, supply side rationing, market imperfections and other realistic features may be introduced into the Keynes–Metzler framework of Chiarella and Flaschel (2000) we refer the reader to Chiarella *et al.* (2000).[6]

Note that we allow for savings out of wages (in a Kaldorian way) and that workers save only in the form of short-term debt (interest-bearing saving deposits[7] held at the local branches of the central bank). All other assets (plus the remainder of short-term debt) are exclusively held by the (pure) asset holders of our model. We stress that this approach serves the purpose of simplifying the budget constraints of the agents, but needs refinement in future reformulations of the model. Note that the government sector

[5] Powell and Murphy (1997) use perfect substitute assumptions, as for example the interest rate parity condition, and rational expectations to describe the behaviour of the asset markets, while we use certain delayed adjustment processes towards such an outcome and thus avoid use of the jump variable technique for the description of the financial part of the economy.

[6] See also Franke and Semmler (1999) for a full portfolio approach to asset market behaviour.

[7] These can be thought of as fixed-price bonds, which are thus perfectly liquid.

includes the activities of the central bank (and its branches), which in the following model type boil down to setting the interest rate of (only domestically held) short-term government bonds according to some type of Taylor rule.[8]

5.3 The structure of the economy from the viewpoint of national accounting

We shall consider in this section the production accounts, income accounts, accumulation accounts and financial accounts of the four internal agents in our economy:[9] firms, workers, asset holders and the government (including the monetary authority). These accounts, plus the balance of payments, provide basic information on what is assumed for these four sectors as well as which of their activities are excluded from the present theoretical framework. These accounts furthermore serve the purpose of checking that all ex post results of the economy are consistent with each other and showing how the usual basic identities of national accounting (concerning output and income, savings and investment) can be derived from them.

5.3.1 The four sectors of the economy

We start with the accounts of the sector of firms (shown in Table 5.3) that organise production Y, employment L_f^d of their workforce L_f^w and gross business fixed investment I and that use (in the present formulation of the model) only equities E as financing instrument (no debt in the form of bank loans or bonds issued by firms). There are value-added taxes τ_v on consumption goods, import taxes τ_m and payroll taxes τ_p with respect to hours worked L_f^d, but no further taxation in the sector of firms and there are no subsidies.

All accounts are expressed in terms of the domestic currency. Firms build dwellings, which are of the same type as all other domestic production, and sell them to the asset holders (as investors) and thus have no own investment in the housing sector. They sell consumption goods to workers, asset holders and the government, export goods to the world economy, organise fixed gross investments with respect to their capital stock (as well as voluntary inventory changes $\mathcal{I}$ with respect to finished goods) and experience involuntary inventory changes $Y - Y^d$ due to the deviation of aggregate demand Y^d from output Y (which is based on expected sales Y^e and planned inventories $\mathcal{I}$).[10]

Firms use up all imports as intermediate goods which thereby become part of the unique homogeneous good that is produced for domestic purposes. They have replacement costs with respect to their capital stock, pay indirect taxes and wages including payroll taxes. Their accounting profit is therefore equal to expected profits (based on

[8] Such a mechanism is in the place of the indirect steering of this rate of interest through a monetary supply rule and money market equilibrium as in Powell and Murphy (1997).

[9] The fifth agent, the foreign economy, is represented by the balance of payments at the end of this section and later on will be confined to steady state behaviour. All demands of the foreign sector are indexed by $*$, while its supply of long-term bonds B_2 to domestic residents is indexed by 2.

[10] No other type of inventory holding is considered in the model of this chapter.

Table 5.3. *The production, income, accumulation and financial accounts of firms*

Uses	Sources
Production Account of Firms:	
Imports $sp_m^* J^d$	Consumption $p_v C_w$
Depreciation $p_y \delta_k K$	Consumption $p_v C_c$
Indirect Taxes $\tau_v p_y (C_w + C_c + G) + \tau_m s p_m^* J^d$	Consumption $p_v G$
Wages (including payroll taxes) $w^b L_f^d$	Exports $p_x X$
	Gross Investment $p_y I$
	Durables (Dwellings) $p_y I_h$
Profits $\Pi = r^e p_y K + p_y \mathcal{I} = r^a p_y K + p_y \dot{N}$	Inventory Investment $p_y \dot{N}$
Income Account of Firms:	
Dividends $r^e p_y K$	Profits Π
Savings $S_f^n = p_y \mathcal{I}$	
Accumulation Account of Firms:	
Gross Investment $p_y I$	Depreciation $p_y \delta_k K$
Inventory Investment $p_y \dot{N}$	Savings S_f^n
	Financial Deficit FD
Financial Account of Firms:	
Financial Deficit FD	Equity Financing $p_e \dot{E}$

sales expectations and paid out as dividends to equity owners) and retained profits (equal to planned inventories). As is obvious from the narrow income account of firms, firms thus only save an amount equal to their intended inventory changes. The accumulation account is self-explanatory as is the financial account which repeats our earlier statement that the financial deficit of firms is financed solely by the issuing of new equities.

Note that all investment is valued (and performed) net of value-added tax and thus at producer prices p_y in the place of the consumer prices $p_v = (1+\tau_v)p_y$. Indirect taxes (value-added taxes)[11] only fall on consumption activities and not on gross investment, thus not on housing investments and the inventory investment of firms. There are furthermore no direct (capital) taxes in the sector of firms, neither on property nor on profits, since all expected profits are distributed to asset holders and since there are no taxes on windfall profits (unexpected retained earnings – or losses – of firms that help to finance investment). Note however that the wages w^b paid by firms include payroll taxes $\tau_p w$ (for unemployment insurance, health and other social insurance, and retirement pensions) and that wage income w of workers is taxed at the rate τ_w. Note finally that the accumulation account of firms is based on realised magnitudes and thus does not refer explicitly to their intended inventory changes.

[11] There is however a tax on the imports made by the firms.

Table 5.4. *The production, income, accumulation and financial accounts of asset holders*

Uses	Sources
Production Account of Households (Asset Holders/Housing Investment):	
Depreciation $p_y \delta_h K_h$	Rent $p_h C_h^d$
Earnings Π_h	
Income Account of Households (Asset Holders):	
Tax Payment $\tau_c i B_c$	Interest Payment $i B_c$
Tax Payment $\tau_c B_1^l$	Interest Payment B_1^l
Taxes $\tau_c(p_h C_h^d - p_y \delta_h K_h)$	Interest Payment $s(1-\tau_c^*)B_2^l$
Tax Payment $\tau_c r^e p_y K$	Dividend Payment $r^e p_y K$
Consumption $p_v C_c$	Earnings Π_h
Savings S_c^n	
Accumulation Account of Households (Asset Holders):	
Gross Investment $p_y I_h$	Depreciation $p_y \delta_h K_h$
Financial Surplus FS	Savings S_c^n
Financial Account of Households (Asset Holders):	
Short-term Bonds $\dot{B}_c$	Financial Surplus FS
Long-term Bonds $p_b \dot{B}_1^l$	
Foreign Bonds $s p_b^* \dot{B}_2^l$	
Equities $p_e \dot{E}$	

Consider next the sector of asset holders (see Table 5.4). Investment in housing as well as the supply of housing services has been exclusively allocated to this sector. The production account thus shows the actual sale (not the potential sale) of housing services (= demand for housing services by assumption) which is divided into replacement costs and actual earnings or profits on the uses side of the production account.

The income of asset holders comes from various sources: interest payments on short- and long-term domestic bonds and on long-term foreign bonds (net of tax payments which must be paid abroad), dividend payments of firms (based on their expected profit) and profits from housing rents. All domestic profit income is subject to tax payments at the rate τ_c and after tax income by definition is divided into the consumption of domestic commodities (including houses, but not housing services) and the nominal savings of asset owners.

The accumulation account shows the sources for gross investment of asset holders in the housing sector, namely depreciation and savings, the excess of which (over housing investment) is then invested in financial assets as shown in the financial account. Note here that short-term bonds are fixed-price bonds $p_b = 1$ (which are perfectly liquid), while long-term bonds have the variable price[12] $p_b = 1/i_l$ (and fixed nominal interest

[12] These bonds are thus not perfectly liquid, since there is no 'money back' guarantee here for the sector of asset owners as a whole.

Table 5.5. *The production, income, accumulation and financial accounts of worker households*

Uses	Sources
Production Account of Households (Workers):	
–	–
Income Account of Households (Workers):	
Taxes $\tau_w[wL^d + w^u(L - L^w) + w^r \alpha_l L_2] + \tau_c i B_w$	Wages wL^d
Consumption $p_v C_w + p_h C_h^d$	Unemployment Benefits $w^u(L - L^w)$
–	Pensions $w^r \alpha_l L_2$
Savings S_w^n	$i B_w$ Interest Payments
Accumulation Account of Households (Workers):	
Financial Surplus FS	Savings S_w^n
Financial Account of Households (Workers):	
Short-term Bond Accumulation $\dot{B}_w$	Financial Surplus FS

payments of one unit of money per period) which shows that they are akin to consols or perpetuities (the same holds true for imported foreign bonds, which are of long-term type solely).[13]

There is no taxation of financial wealth (held or transferred) in the household sector. Furthermore, although asset holders will consider expected gross rates of return on financial markets in their investment decision, there is no taxation of capital gains on these markets, which descriptively seems realistic.

The next set of accounts, the ones of worker households in Table 5.5, is fairly simple and easy to explain. First, there is no production account in this sector. Income of the members of the workforce, which may be employed, unemployed or retired, thus derives from wages, unemployment benefits or pension payments where $L = \alpha_l L_1$ denotes the total number of persons in the current workforce (L^w the part that is employed) and $\alpha_l L_2$ the number of retirees who have access to pension funds (α_l=const. the participation rate of the potential workforce L_1). To this we have to add the interest income on saving deposits (short-term bonds) which is taxed at the general rate used for financial asset income. All wage type incomes are subject to taxation at the rate τ_w and are again by definition divided into nominal consumption (consumption goods and housing services) and savings. Note here that the employment L^d of the employed L^w can differ from their normal employment which is measured by L^w, the number of persons who are employed. Note also that wages w are net of payroll taxes (used to finance unemployment benefits, social insurance and pensions in particular).

[13] Due to the assumption of a given nominal rate of interest on foreign bonds, these bonds can be liquidated if this is desired by domestic residents, but they are of course subject to exchange rate risk. Foreign bond purchases by domestic residents will be treated as a residual in the wealth accumulation decisions of the asset holders.

Table 5.6. *The production, income, accumulation and financial accounts of the monetary and fiscal authorities*

Uses	Sources
Production Account of Fiscal and Monetary Authorities:	
Government Expenditure for Goods $p_v G$	Costless Provision
Government Expenditure for Services $w^b L_g^d$	of Public Goods
Income Account of Fiscal and Monetary Authorities:	
Interest Payment iB	Wage Income Taxation $\tau_w[wL^d + w^u(L - L^w) + w^r \alpha_l L_2]$
Interest Payment B^l	Profit and Interest Taxation $\tau_c[r^e p_y K + iB + B^l]$
Pensions $w^r \alpha_l L_2$	Rent Income Taxation $\tau_c(p_h C_h^d - p_y \delta_h K_h)$
Unemployment Benefits $w^u(L - L^w)$	Payroll Taxes $\tau_p w L^d$
Government Consumption $p_v G$	Value Added Tax $\tau_v p_y (C_w + C_c + G)$
Salaries $w^b L_g^d$	Import Taxes $\tau_m s p_m^* J^d$
Savings S_g^n	
Accumulation Account of the Fiscal Authority:	
	Savings S_g^n
	Financial Deficit FD
Financial Account of Fiscal and Monetary Authorities:	
Financial Deficit FD	Short-term Debt $\dot{B}$
	Long-term Debt $p_b \dot{B}^l$

We assume in the following that workers have a positive savings rate and that they hold their savings in the form of short-term bonds solely, which is mirrored here in the accumulation and financial account in a straightforward way.

There are finally the accounts of the fiscal and monetary authorities (see Table 5.6), which due to the many taxation schemes and transfer payments that are assumed are more voluminous than the preceding accounts – at least with respect to the income account. There is first however a fictitious production account where the supply of public goods is valued at production costs which consist of government expenditures for goods and labour.

The sources of government income consist of taxes on the various forms of workers' income (taxed at a uniform rate), of taxes on the various forms of profit, interest and rental income (again taxed at a uniform rate), payroll taxes, value-added taxes and import taxes. Uses of the tax income of the government are interest payments, transfers to the unemployed and retirees, and the costs of the aforementioned government 'production'. In general all these uses of the tax income of the government will exceed its income so that there will result a negative amount of nominal savings S_g^n which balances the income account of the government.

There is no accumulation of real assets in the government sector, which means that we only have to look into the financial account of the government to see how the excess of government outlays over government revenue is financed through short- or long-term debt. Note that there is some type of accounting money in the economy that however only fuels the economy during the transactions period, but does not appear as flow in the financial accounts of asset owners, and workers, and the government, but instead returns to the banking sector at the end of each transaction period t by the settlement of all budget restrictions in the economy. In striking contrast to a cash in advance constraint we thus assume in this chapter that agents can obtain all money they need for transaction purposes during the transaction period t (as intra-day credit in one form or another), but that they have to satisfy their budget constraint at the end of each such period t where money holdings are not needed and are thus not present. Instead all liquid asset holdings concern the short-term bonds of the government as some form of interest bearing saving deposit.

There is a variety of further types of taxes that could have been included into the structure of the model as we have discussed it so far. The most important types from a macroeconomic point of view are probably: corporate profit taxation; investment taxes (or subsidies) – fixed business investment and housing investment – including a treatment of depreciation allowances; financial wealth taxation and inheritance taxes; capital gains taxation; real property taxation; taxes on the rents imputed in the case of asset holders.

In the model of this chapter corporate income taxation would reduce to taxation of the windfall profits of firms, since all expected earnings of firms are distributed to workers and as dividend payments to asset holders. We leave (gross) investment as untaxed in order to stress that this type of activity is to be supported for the future development of the economy by the government. Financial wealth taxation will be considered in future work where a more advanced and much more interdependent structure for the small open economy to be introduced in Section 5.4 is considered. Similarly capital gains are not taxed in order to stimulate financial investment in risky assets (and since it is also difficult to treat in its exact amount and dating and, from an institutional perspective, with respect to capital losses). Real property taxation is probably a major item in many countries, but is here left aside for simplicity as are the imputed rents of the housing services consumed by the asset owners of our economy.

There are further taxes on the state level and on the local level of government administration (which are here completely ignored), taxes on private insurance and pensions (which do not exist in our model), product specific taxes, subsidies for employment and investment on the margin, and turnover taxes, which may be important from a partial microeconomic analysis of public economics, but which have to be left aside here in our broad picture of the macroeconomy. Furthermore, we do not impose internal constraints on the uses of the taxes that are actually received by the government of our macroeconomy and which might restrict the use of certain taxes to certain expenditures or transfers made by the government.

Table 5.7. *The external account*

Uses	Sources
External Account:	
sp^*X	sp^*J^d
$s(1-\tau_c^*)B_2^l$	$(1-\tau_c)B_1^{l*}$
$\dot{B}_1^{l*}/i_l$	$s\dot{B}_2^l/i_{l*}$

Let us finally describe the balance of payments of the economy under consideration. This will be done from the viewpoint of the foreign sector which can be viewed as a fifth agent of the economic structure considered in this chapter. The description of the behaviour of this agent will however be confined to steady state behaviour in the subsequent presentation of the structural equations of the model.

Table 5.7 is viewed from the perspective of the foreign economy and in terms of the domestic currency.

The external, or balance of payments, account (see Table 5.7) shows the trade account (exports X and imports J^d), the international component of interest payments (to foreigners and from abroad) that are all assumed to cross borders and the outflow and the inflow of new capital (long-term bonds). Note that this account does not show any reserve changes of the central bank due to foreign exchange market operations. This is possible in the approach chosen in this chapter, despite a temporarily given exchange rate s, since on the one hand the supply of bonds of the government domestically and abroad of the equities of firms are channelled into the savings decisions of households without readjustments. On the other hand the excess of domestic private savings is going into foreign bonds, which in turn implies that the balance of payments must be balanced without any intervention from the central bank. This will be checked in the next subsection from the viewpoint of the ex post equality of aggregate savings with aggregate investment plus the current account balance on the one hand and with aggregate investment plus the capital account balance on the other hand, where both equalities can be established without any interference from the central bank. The result obtained is basically due to the fact that all foreign exchange market operations can be settled without any help from the central bank, and without any rationing processes, since the residually determined item $s\dot{B}_2^l/i_{l*}$ just provides the balancing item for this account.

This concludes our description of the four accounts of the three typical sectors of a small open economy (with heterogeneous agents in the household sector) plus a foreign sector that is here represented solely via the balance of payments.

5.3.2 *Gross domestic product, savings, investment and further aggregates*

In this subsection we derive some basic concepts of national accounting in the specific form they receive in our model economy and also the relationships between nominal aggregate savings and nominal total investment. Considering nominal gross domestic product first, we see that this concept aggregates (with respect to uses) total private and government consumption, net exports, total investment (including housing and all inventory investment $\dot{N}$) and finally the services of housing actually demanded and supplied. The sources of these expenditures are the depreciation of the capital stock of firms and of the housing stock, indirect taxes, wage payments of firms and profits in production and in housing supply.

Gross Domestic Product (GDP):[14]

$$p_y\delta_k K + p_y\delta_h K_h + \tau_v p_y(C_w + C_c + G) + \tau_m s p_m^* J^d + w^b L_f^d + \Pi + \Pi_h$$
$$= p_v(C_w + C_c + G) + p_x X - s p_m^* J^d + p_y I + p_y I_h + p_y \dot{N} + p_h C_h^d.$$

Net domestic product is then obtained (also in nominal terms and at market prices) by moving the depreciation items from the left-hand side of the preceding equation to its right-hand side and thus by deducting them from the corresponding gross investment, giving rise to net investment descriptions for firms as well as for the housing sector of the economy.

Net Domestic Product at market prices (NDP):[15]

$$\tau_v p_y(C_w + C_c + G) + \tau_m s p_m^* J^d + w^b L_f^d + \Pi + \Pi_h$$
$$= p_v(C_w + C_c + G) + (p_x X - s p_m^* J^d) + p_y(I - \delta_k K)$$
$$+ p_y(I_h - \delta_h K_h) + p_y\dot{N} + p_h C_h^d.$$

Net domestic product at factor costs follows from net domestic product at market prices by deducting indirect taxes from both sides of the preceding equation which simply leads to a revaluation of consumption goods and imports, both measured now at prices without value-added taxes and without import taxes.

Net Domestic Product at factor costs (NDP-F):[16]

$$w^b L_f^d + \Pi + \Pi_h$$
$$= p_y(C_w + C_c + G) + (p_x X - p_m J^d) + p_y(I - \delta_k K) + p_y(I_h - \delta_h K_h)$$
$$+ p_y\dot{N} + p_h C_h^d$$

On the basis of the uses of nominal savings of the four sectors considered (see their accumulation and financial accounts), one furthermore obtains via their aggregation the result

$$S^n = S_w^n + S_c^n + S_f^n + S_g^n = I^{na} + [s\dot{B}_2^l/i_l^* - (\dot{B}^l - \dot{B}_1^l)/i_l]$$

[14] Gross National Product (GNP) = GDP + $s(1-\tau_c^*)B_2^l - (1-\tau_c)(B^l - B_1^l)$.
[15] Net National Product (NNP) = NDP + $s(1-\tau_c^*)B_2^l - (1-\tau_c)(B^l - B_1^l)$.
[16] National Income is defined on this basis by NDP-F $+s(1-\tau_c^*)B_2^l - (1-\tau_c)(B^l - B_1^l)$.

with

$$I^{na} = p_e \dot{E} + p_y \mathcal{I} + p_y(I_h - \delta_h K_h) = p_y(I - \delta_k K) + p_y \dot{N} + p_y(I_h - \delta_h K_h).$$

We here see that total nominal savings are ex post always equal to total nominal net investment plus net capital exports. Note that the home country's capital exports are equal (in value) to the import $\dot{B}_2^l$ of foreign bonds and that its capital imports are given by the value of the export of the home country's long-term bonds $\dot{B}^l - \dot{B}_1^l$. Note also that actual net investment consists of business fixed investment, of actual inventory changes and of net investment in the supply of dwellings. This important identity of national accounting is based on the four identities that relate the nominal savings of the various sectors to the uses made of these savings. Approaching aggregate nominal savings from the definitions of the various savings items, that is from the income side, by contrast gives rise to

$$\begin{aligned} S^n &= S_w^n + S_c^n + S_f^n + S_g^n \\ &= I^{na} + [p_x X - s p_m^* J^d] + [s(1 - \tau_c^*)B_2^l - (1 - \tau_c)(B^l - B_1^l)], \\ I^{na} &= p_y(I - \delta_k K) + p_y \dot{N} + p_y(I_h - \delta_h K_h), \end{aligned}$$

so that aggregate nominal savings equals aggregate nominal actual net investment plus nominal net exports plus international nominal net transfers. The identities just discussed thus in sum show that basic concepts of the system of national accounts can already be quite complicated in our model economy.

We digress to add a detailed calculation to what has just been asserted and in particular shows that there indeed is no intervention needed from the monetary authority on the market for foreign exchange, due to the assumed budget restrictions for households, firms and the government.

Starting from the definitions of the nominal savings of the four sectors the identity on nominal savings and investment just stated can also be shown as follows:

$$\begin{aligned} S^n = {} & Y_w^{Dn} - p_v C_w - p_h C_h^d \\ & + Y_c^{Dn} - p_v C_c + p_y \mathcal{I} \\ & + T^n - w^u(L - L^w) - w^r \alpha_l L_2 - (iB + B^l) - (p_v G + w^b L_g^d), \end{aligned}$$

which gives rise to

$$\begin{aligned} S^n = {} & wL^d + w^u(L - L^w) + w^r \alpha_l L_2 + iB_w - p_v C_w - p_h C_h^d \\ & + r^e p_y K + iB_c + B_1^l + p_h C_h^d - p_y \delta_h K_h + s(1 - \tau_c^*)B_2^l - p_v C_c + p_y \mathcal{I} \\ & + \tau_m s p_m^* J^d + \tau_c(B^l - B_1^l) + \tau_p w L^d \\ & + \tau_v p_y(Y^d - I - I_h) - w^u(L - L^w) - w^r \alpha_l L_2 - (iB + B^l) - (p_v G + w^b L_g^d). \end{aligned}$$

These expressions can be rearranged as

$$\begin{aligned} & w^b L^d - p_v C_w + r^e p_y K + iB + B_1^l - p_y \delta_h K_h + s(1 - \tau_c^*)B_2^l - p_v C_c \\ & \quad + p_y \mathcal{I} + \tau_m s p_m^* J^d + \tau_c(B^l - B_1^l) \\ & \quad + \tau_v p_y(C_w + C_c + G) - (iB + B^l) - (p_v G + w^b L_g^d) \\ & = w^b L_f^d - p_y C_w + r^e p_y K - (1 - \tau_c)(B^l - B_1^l) - p_y \delta_h K_h + s(1 - \tau_c^*)B_2^l - p_y C_c \\ & \quad + p_y \mathcal{I} + \tau_m s p_m^* J^d - p_y G \\ & = p_y(Y^e - \delta_k K) + p_x X - s p_m^* J^d - (p_y C_w + p_y C_c + p_y G) \\ & \quad - (1 - \tau_c)(B^l - B_1^l) + s(1 - \tau_c^*)B_2^l - p_y \delta_h K_h + p_y \mathcal{I} \\ & = p_y(Y - Y^d) + p_y Y^d - (p_y C_w + p_y C_c + p_y G) - p_y \delta_k K + p_x X - s p_m^* J^d \\ & \quad + s(1 - \tau_c^*)B_2^l - (1 - \tau_c)(B^l - B_1^l) - p_y \delta_h K_h \\ & = p_y \dot{N} + p_y(I - \delta_k K) + p_y(I_h - \delta_h K_h) \\ & \quad + p_x X - s p_m^* J^d + s(1 - \tau_c^*)B_2^l - (1 - \tau_c)(B^l - B_1^l) \\ & = I^{na} + s(1 - \tau_c^*)B_2^l - (1 - \tau_c)(B^l - B_1^l). \end{aligned}$$

This proves the asserted identity from the viewpoint of the definitions of nominal savings. Note here that $B^l - B_1^l = B_1^{l*}$ holds by definition for the international allocation of domestic long-term bonds and that aggregate goods demand is defined by the expression $Y^d = C_w + C_c + I + I_h + G$.

Having presented the model from the ex post point of view by means of structured tables and the system of national accounts we now turn to the structural form of the model and present in the following section its technological foundations, its behavioural relationships, various definitions and the budget equations of the four agents of the domestic economy, and finally also its laws of motion for quantities, prices and expectations.

5.4 The model

In this section we develop the extensive form equations of our model based on the structure laid out in Section 5.3. We reformulate the Murphy model for the Australian economy, as presented in Powell and Murphy (1997), from a macrotheoretic perspective, by making it a continuous-time dynamic model of monetary growth, suppressing all discrete lag structures of their quarterly period model in particular. The present reformulation of the Murphy model is furthermore based on the experience gained in Chiarella and Flaschel (1999b,c) in the modelling of integrated Keynes–Metzler models of monetary growth for closed as well as open economies. Certain features of this dynamical system approach to growth and fluctuations are therefore retained in the formulation of our continuous-time version of the Murphy model, which of course means that its dynamical structure will differ from that of the Murphy model to a certain degree.[17]

Our aim in this section is not so much to fully mirror the dynamical structure and implications of the Murphy model. Rather our aim is to formulate and to investigate, to a first approximation of this 100 equations approach to macroeconometric model building, the set of the most prominent feedback structures of macrodynamic theory it basically contains and the role they play for stability analysis. As we have stated, we set up an integrated macrotheoretic monetary growth framework that in its generality is comparable to this type of macroeconometric model building. The current section therefore attempts to build a bridge between empirically motivated work on structural model building (where there generally is no analysis of the mechanisms that are hidden in the formulated structure) and theoretical investigations of reasonably large representation of concrete economies, where the interest is to see what the steady state of such economies will look like in all of its details and what stabilising (or destabilising) effects are present around it, or have to be added far off the steady state in order to ensure the boundedness of the considered dynamics.

5.4.1 *Preliminaries*

Let us start with some notation to be used in the structural equations we shall employ in our approach to Keynesian monetary growth. Module 1 of the model provides definitions of important rates of return r^e, r^a, r^n, r_h, of nominal wealth W^n[18] and of hourly wages including payroll taxes, w^b, prices p_v including value-added tax, of pension payments per retired worker (in the workforce) per time unit, w^r, and unemployment benefits per unemployed worker (of the workforce) per time unit, w^u, with w denoting the money wage exclusive of payroll taxes but still including wage income taxes. We here in particular define the currently expected rate of profit based on the sales expectations Y^e of firms (net of depreciation $\delta_k K$) and on actual exports $X = x_y Y$, imports $J^d = j_y Y$ and the actual employment $L^d = l_y Y$ of the workforce of the firms. In a similar fashion also the actual and the normal rate of return on business fixed investment, r^a, r^n, based on actual sales and normal rates of capacity utilisation of the capital stock. Our choice of notation of production coefficients already indicates that we are assuming a technology with fixed input/output coefficients where export supply is in fixed proportion to actual output Y, as is import demand and labour demand. Furthermore, potential output is defined on the basis of a given capital stock as $Y^p = y^p K,\ y^p = const$, and is used in the definition of normal profits in a specific way that has still to be explained. We use fixed coefficients technology for the same reasons as in Chiarella and Flaschel (2000), namely that it allows clearer insight into the dynamic feedback structure of the

[17] This remark in particular applies to our treatment of financial markets where we attempt to avoid the so-called jump variable technique of models with only rational expectations by allowing for heterogeneous expectations formation and somewhat delayed adjustments towards interest rate parity conditions.

[18] Wealth effects will however only be studied in future extensions of the model of this chapter, but should of course be kept in mind when interpreting the behaviour of the present model.

model without altering the qualitative features of the dynamics under smooth factor substitution technology.

1. Definitions (Rates of Return, Nominal Wealth, Wages and Prices):

$$r^e = \frac{p_y Y^e + p_x x_y Y - w^b l_y Y - p_m j_y Y - p_y \delta_k K}{p_y K}, \tag{5.1}$$

$$r^a = \frac{p_y Y^d + p_x x_y Y - w^b l_y Y - p_m j_y Y - p_y \delta_k K}{p_y K}, \tag{5.2}$$

$$Y^{dp} = \frac{\bar{u} Y^p}{1 + \gamma \beta_{n^d}}, \qquad Y^n = \bar{u} Y^p, \tag{5.3}$$

$$r^n = \frac{p_y Y^{dp} + p_x x_y Y^n - w^b l_y Y^n - p_m j_y Y^n - p_y \delta_k K}{p_y K}, \tag{5.4}$$

$$r_h = \frac{p_h C_h^d - p_y \delta_h K_h}{p_y K_h}, \tag{5.5}$$

$$i^r = (1 - \tau_c) i_l - \pi^c, \tag{5.6}$$

$$W^n = B + B_1^l / i_l + s B_2^l / i_l^* + p_e E + p_y K_h, \tag{5.7}$$

$$w^b = (1 + \tau_p) w, \tag{5.8}$$

$$w^r = \alpha^r w, \tag{5.9}$$

$$w^u = \alpha^u w, \tag{5.10}$$

$$p_v = (1 + \tau_v) p_y. \tag{5.11}$$

Note that the various rates of profits are defined on the basis of output prices p_y net of value-added tax, since they measure what can actually be distributed to equity owning households (with the rate r^e measuring actual dividend payments at each moment in time, while r^a measures the actual rate of profit of firms based on their actual sales).

Firms have a desired rate of capacity utilisation $\bar{u} < 1$ (which is not endogenised in the present model) and thus plan a normal output $Y^n = \bar{u} Y^p$ less than potential output in order to have capacity reserves in the case of unforeseen demand shocks. Furthermore they have to hold inventories $N^d = \beta_{n^d} Y^n$ which have to grow at the given world growth rate γ in the steady state which means that the demand Y^{dp} they consider as adequate in the light of their potential output (or as satisfying in the steady state) must be less than normal production since part of the latter is going into inventories). The normal rate of profit, r^n, is then defined on the basis of this concept of normal output as the other rates of profits just discussed. The rate of return r_h, finally, refers to the housing sector and its actual sale of (= demand for) housing services C_h^d at price p_h. It is diminished through the depreciation of dwellings at rate δ_h and set in relation to the net value of the capital stock K_h in the housing sector. All capital goods are (or have been) purchased at price p_y in the market for non-traded (domestic) goods, since value-added taxes only concern consumption expenditures in the present model.

We often compare profitability as measured by the above rates of return with required profitability given by the expected long-term real rate of interest $(1-\tau_c)i_l - \pi^c$ which is related to the price of long-term bonds (consols or perpetuities) in the following well known way $p_b = 1/i_l$, π^c a weighted (long-term) expected rate of inflation to be defined in module 5b of the model. Note that we calculate this required rate of return net of interest taxation, while all other rates in this block are gross rates of return.

Aggregate nominal wealth of asset holders and workers (the latter only hold short-term debt of the government as saving deposits) is composed of short-term fixed-price (= perfectly liquid) bonds of the domestic government, B, held only domestically, long-term bonds issued by the domestic and the foreign government in the amounts held by domestic residents, B_1^l, B_2^l, equities E and the value of the housing capital stock K_h, again measured at producers' prices. We assume that there is no resale market for houses (and goods in general) and thus do not have a secondary market in this segment of the economy in order to keep things simple in our one domestic good economy (once these commodities are sold they do not reappear on the market). Financial assets by contrast are traded in secondary markets and give rise to certain price adjustment equations that are implicitly based on stock reallocations in the financial markets considered. Asset markets are therefore still treated in a preliminary way, since portfolio decisions are not yet modelled explicitly (see module 6 of the model).

Note that government bonds are treated as net wealth.[19] However, since wealth effects are still excluded from the behavioural equations to be introduced later on, this concept of wealth is here presented solely in order to point to the necessity of treating such wealth effects in future extensions of the model. Note furthermore that central bank money is not treated as a component of financial wealth in the present chapter. Such money is here assumed to be used for intra-day transaction purposes solely and is supplied by branches of the central bank without user costs (cash, ATM and credit cards) for the public during the day, but which however transfers this type of money back to these branches at the end of each 'trading period', by fulfilling their budget equations and due to the loss of interest rate payments that would otherwise arise. Households, firms and the government thus do not need to hold money balances for intra-day trading (due to flexibilities in the management of their intra-day income accounts) and are thus not forced to devote part of their asset holdings to pure cash holdings. Paper money fuels the economy within each period, but is simply stored in the banking sector at the end of it, while all savings decisions go into interest-bearing assets solely.[20] A

[19] A Ricardian equivalence argument might lead to the exclusion of workers' saving deposits B_w from aggregate wealth, since the wage taxation rate is endogenous and is varied by the government in order to establish a desired 'government debt/GDP ratio' in the economy.

[20] Since there is no need to hold non-interest-bearing cash balances in our type of economy.

Keynesian liquidity preference function, if it were explicitly present in our model, thus would concern the allocation of wealth (of wealth owners) between liquid short-term and illiquid long-term bonds and is thus not related to the specific treatment of cash or the means of exchange management chosen in the present chapter.

The remaining equations in this module define (on the basis of before tax money wages w) gross wages w^b that include payroll taxes (as the intended basis for government transfers to the unemployed, the retired, etc.), pensions w^r and unemployment benefits w^u, which are all in constant proportion to money wages w. Finally, p_v is the consumer price of the domestic good that is assumed to be in fixed proportion to the producer price p_y on the basis of a given value-added tax rate τ_v.

Module 1 finally provides the definitions of unemployment benefits, rents paid to retired worker households and consumer prices (producer prices plus value-added tax) which are not explicitly represented in the consolidated list of equations that Powell and Murphy (1997) supply, but which are not different from their use of these concepts. Crucial differences in the equations considered so far therefore basically concern the use by Powell and Murphy of model consistent inflations in the calculation of the required rate of return used by investors and their definition of private wealth that they use in the single optimal consumption function of their model.

Module 2 concerns the household sector where two types of households are distinguished, pure workers and pure asset holders or wealth owners. Of course, these two types of households are only polar opposite cases in the actual distribution of household types. Nevertheless we believe that it is useful to start from such polar opposite household types before intermediate cases are introduced and formalised. Powell and Murphy (1997) consider only one type of household explicitly (although they briefly refer to the effects of income distribution implicitly contained in their formulation of a consumption function) the consumption behaviour of which is based on the life-cycle hypothesis with respect to wage income and wealth. We shall use differentiated saving habits for the two types of households instead (as they can be derived from Cobb–Douglas utility functions[21]) and we will ignore wealth as a direct argument in our consumption functions (leaving this issue for later reformulations of the model).

5.4.2 Households

We consider the behavioural equations of worker households first:

2*a*. Households (Workforce):

$$Y_w^{Dn} = (1-\tau_w)[wL^d + w^u(L-L^w) + w^r\alpha_l L_2] + (1-\tau_c)iB_w$$

$$= Y_{w1}^{Dn} + (1-\tau_c)iB_w \tag{5.12}$$

$$L^w = L_f^w + L_g^w \tag{5.13}$$

[21] Note in this respect also that the relative price p_v/p_h does not yet play a role in the consumption decisions of workers.

$$L^d = L_f^d + L_g^d = L_f^d + L_g^w \tag{5.14}$$

$$C_w^{on} = c_y(1-\tau_w)[wL^d + w^u(L - L^w) + w^r \alpha_l L_2], \quad C_w^o = C_w^{on}/p_v \tag{5.15}$$

$$\hat{C}_w = \alpha_1^w (C_w^o / C_w - 1) + \alpha_2^w (e - \bar{e}) + \gamma \tag{5.16}$$

$$p_h C_h^{do} = c_h(1-\tau_w)[wL^d + w^u(L - L^w) + w^r \alpha_l L_2] \tag{5.17}$$

$$\hat{C}_h^d = \alpha_r^h (C_h^{do}/C_h^d - 1) + \alpha_i^h (e - \bar{e}) + \gamma \tag{5.18}$$

$$S_w^n = Y_w^{Dn} - p_v C_w - p_h C_h^d = \dot{B}_w \tag{5.19}$$

$$\hat{L}_1 = \hat{L}_2 = \hat{L}_0 = n \qquad (L_0(0), L_1(0), L_2(0) \quad \text{given}) \tag{5.20}$$

$$\dot{n} = \beta_{n_w}(\widetilde{n} - n), \quad \widetilde{n} = \widetilde{n}(e, \hat{e}) \tag{5.21}$$

$$L = \alpha_l L_1 \tag{5.22}$$

The first equation in this module defines the aggregate disposable income of workforce households by the sum of the wage incomes of the employed, unemployment benefits for the unemployed, unemployment being measured by $\alpha_l L_1 - L^w$,[22] and the pensions of retirees, after taxes (with the tax rate τ_w uniformly applied to these three types of workers' incomes). Furthermore workers as a group also have interest income from their holding of saving deposits which is taxed as all other interest payments (which goes to pure asset owners) by means of the rate τ_c. Note that retirees L_2 receive pension payments in an amount that is scaled down by the given participation rate α_l (of the persons L_1 between 16 and 65) which is constant in the present model. Pensions are thus paid to both employed and unemployed workers in the workforce once they retire.[23]

Next we consider the number of employed workers L^w who are working in the sector of firms, L_f^w, or for the government, L_g^w, there providing public services. In contrast to L^w we denote by L^d the actual employment of the employed which can be larger or smaller than the normal hours of work L^w of the employed workforce due to over- or undertime work (such situations by assumption only occur in the firm sector, but not in the government sector, see equation 5.14).

Desired consumption (in nominal and in real terms) C_w^{on}, C_w^o of workers is proportional to their nominal and real wage income, respectively, with c_y denoting the uniform marginal propensity to consume of both employed and unemployed workers as well as for retirees. Note that we always use consumer prices p_v when going from nominal to real magnitudes (we thus ignore the influence of the price p_h of housing services here and later on). Note also that the interest income of worker households does not influence their consumption plans here, since we assume that all of their interest income is saved in order to simplify the feedback from asset accumulation into the

[22] α_l the participation rate of the workforce which is endogenously determined in the Murphy model for the Australian economy.

[23] The participation rate is also applied – for reasons of simplicity – to the growth rate in pension-receivers that is caused by the assumed migration of whole families.

real part of the model. Following Powell and Murphy (1997) we assume that actual consumption plans C_w of all types of workers adjust towards desired consumption with a time delay that depends on their deviation from desired consumption and on the state of the labour market which is measured by the rate of employment e (plus a trend term γ that ensures the existence of steady growth paths later on). The demand for desired housing services is treated in the same way as the demand of workers for consumption goods which means that we assume for this type of consumption: $p_h C_h^{do} = c_h(1-\tau_w)[wL^d + w^u(L - L^w) + w^r \alpha_l L_2]$, with c_h as marginal propensity to consume these services. Note that we have to use the price for these services on the left-hand side of this consumption function. Note also that adjustment towards desired levels is of the same type as the one for the consumption of the domestic goods produced by firms.

It is of course questionable whether the considered marginal propensities to consume are really uniform with respect to the three types of situations adult workers may be in and whether they all use the actual rate of employment as an expression on the prospects of their future incomes. Introducing different behaviour in this place is easily possible, but should be left to investigations with a more pronounced empirical orientation.

The next equation defines nominal savings S_w^n of workers and states that these savings are held in the form of short-term bonds solely. As already explained in the preceding section money is assumed to fuel real transactions via its circulation, since there is no cash or income in advance constraint for obtaining such means of payments for intra-day trading.[24] It is thus considered to be stored in the banking sector (here only the branches of the central bank) after each round of transactions. Note that workers do not accumulate wealth in the form of real estate which is of course not true for example for the Australian economy. Including this into the present module and adding a resale market for houses is thus left here for future extensions of the model.[25]

When we say 'workers' we have, as already noted above, three groups of persons in mind: L_1, the potential workforce, L_0 the young people (below 16 years) and L_2, the retirees (above 65 years). All three components of the workforce households grow – via migration into the considered country (and possibly also by reasons internal to the economy) – at the same rate n, for reasons of simplicity and for the purpose of later steady state analysis. Note that these new members of the workforce are immediately treated as the residents of the country under consideration. This rate n follows with a delay the growth rate $\widetilde{n}$ which represents the desire to migrate (with constant population shares) into the labour market of our economy and which is here endogenously determined through the state of the labour market e and its rate of change $\hat{e}$.

[24] Note again that the temporal budget equations of all agents in the economy must be fulfilled at each end of the trading period t.

[25] It is however possible to assume that the consumption of the domestic good through workforce households is partly going into the purchase of houses if it is assumed that goods purchased cannot be sold anymore at a later point in time to another sector of the economy.

Actual labour supply (in terms of persons), finally, is given by $\alpha_l L_1$ with α_l the participation rate, and is divided according to the state of the economy into employed and unemployed people, L^w, $L - L^w$. Note again that we have assumed that the participation rate is constant in time and thus do not make any use of the encouraged/discouraged worker effect as in Powell and Murphy (1997, Ch. 6.7).

Summing up the module 2a thus basically describes the two consumption decisions of workers' households based uniformly on their various sources of wage income. It is easy to derive such consumption functions by assuming Cobb–Douglas utility functions. Powell and Murphy (1997) make use of a life-cycle approach in the place of our description of the consumption behaviour of workers and thus immediately include wealth effects into the consumption decisions of their single type of household. We shall apply their approach to consumption behaviour in our two agents framework in another paper, see Chiarella *et al.* (1999a,b), and will then study the role of wealth effects in such an extended framework.

Next, we consider the other type of household of our model, the (pure) asset owners who desire to consume C_c (goods and houses as supplied by firms through domestic production Y) at an amount that is growing exogenously at the rate γ and which is thus in particular independent of their current nominal disposable income Y_c^{Dn}. The consumption decision is thus not an important decision for pure asset holders. Their nominal income diminished by the nominal value of their consumption $p_v C_c$ is then spent on the purchase of financial assets (three types of bonds and equities) as well as on investment in housing supply (for worker households). Note here that the one good view of the production of the domestic good entails consumption goods proper and houses (both at consumer prices p_v) so that asset holders buy houses for their consumption as well as investment purposes.

2*b*. Households (Asset Holders):

$$Y_c^{Dn} = (1-\tau_c)[r^e p_y K + iB_c + B_1^l + p_h C_h^d - p_y \delta_h K_h] + s(1-\tau_c^*)B_2^l \tag{5.23}$$

$$\hat{C}_c = \gamma \tag{5.24}$$

$$S_c^n = Y_c^{Dn} - p_v C_c \tag{5.25}$$

$$= \dot{B}_c + \dot{B}_1^l/i_l + s\dot{B}_2^l/i_{l*} + p_e \dot{E} + p_y(I_h - \delta_h K_h), \quad \dot{B}_c = \dot{B} - \dot{B}_w$$

$$C_h^s = K_h \quad [\hat{C}_h^d = \alpha_r^h (C_h^{do}/C_h^d - 1) + \alpha_i^h (e - \bar{e}) + \gamma] \tag{5.26}$$

$$g_h^d = (I_h/K_h)^d$$

$$= \alpha_r^h((1-\tau_c)r_h^l - i^r) + \alpha_i^h(i_l - (i+\xi)) + \alpha_u^h\left(\frac{C_h^d}{C_h^s} - \bar{u}_h\right) + \gamma + \delta_h \tag{5.27}$$

$$\dot{r}_h^l = \beta_{r_h^l}(r_h - r_h^l) \tag{5.28}$$

$$\dot{g}_h = \beta_{g_h}(g_h^d - g_h), \quad g_h = I_h/K_h \tag{5.29}$$

$$\hat{p}_h = \beta_{p_h}\left(\frac{C_h^d}{C_h^s} - \bar{u}_h\right) + \kappa_h \hat{p}_v + (1-\kappa_h)\pi^c \tag{5.30}$$

$$\hat{K}_h = I_h/K_h - \delta_h \tag{5.31}$$

Equation (5.23) defines the disposable income of asset holders that consists of dividend payments of firms (which distribute their whole expected profit to equity holders), interest on government bonds, $iB_c + B_1^l$, insofar as they are held by domestic residents, rents for housing services net of depreciation, and interest payments on foreign bonds held by domestic households (after foreign taxation and expressed in domestic currency by means of the exchange rate s). Private savings of asset holders S_c^n concerns short-term and long-term bonds (domestic and foreign ones with respect to the latter), equities and net housing investment.

We assume in the following that the amount of savings of asset holders that goes into short-term bonds, $\dot{B}_c$ is given by $\dot{B} - \dot{B}_w$, which means that asset holders passively accept the inflow (or even the outflow) of short-term bonds that is implied for them by independent decision of the government on its short-term debt policy and by the savings decision of workers (that only concerns short-term bonds). This is clearly a very restrictive assumption which – together with the treatment of the other flows of financial asset accumulation – must be improved in further elaborations of the asset market dynamics of the model.

Note that there is no inside debt of the household sector (lending of asset owners to worker households). Note also that we have supplied – and not only here – a full treatment of budget equations including all feedbacks on asset accumulation that are implied by them, a degree of completeness which is missing in the Powell and Murphy (1997) model.

The supply of housing services C_h^s is assumed to be proportional to the existing stock of houses that is devoted to the supply of such services (there are no maintenance costs in the housing sector as in Powell and Murphy (1997)). We assume for simplicity that there is no resale market for dwellings. Note again that the production of dwellings is part of the production activities of firms and thus part of the homogeneous supply of the domestic (non-traded) output.

The demand for housing services has already been defined in module 2a. We assume that housing demand is always served and we can guarantee this in general – up to certain extreme fluctuations in the demand for housing services – by assuming that house owners voluntarily hold excess capacities as measured by the exogenously given desired rate of capacity utilisation $\bar{u}_h$ of the housing service supply. We have assumed in the workforce sector that their demand for housing services grows beside short-term influences with trend rate γ (underlying the steady state of the model). This implies that housing services per household grow with trend rate $\gamma - n$, where n is the natural rate of growth of the workforce. Therefore, over the growth horizon of the economy, we have that worker households consume more and more housing services (measured by square metres per housing unit for example).[26]

Equation (5.27) of module 2b describes the desired rate of gross investment of asset holders, which depends on the (expected!) long-run profit rate r_h^l in the housing sector compared with the required rate of return, measured in reference to government consols by $i^r = i_l - \pi^c$ (via Tobin's q as relative profitability measure), on the interest spread $i_l - (i + \xi)$ as a measure for the tightness of monetary policy (here based on an interest rate policy rule) and its perceived (or only believed) effects on the level of economic activity and employment,[27] on the actual rate of capacity utilisation[28] with respect to housing services (representing the demand pressure in this investment behaviour), $\frac{C_h^d}{C_h^s} - \bar{u}_h$, on the trend rate of growth γ and on the rate of depreciation δ_h in the housing sector. We assume that the actual rate of investment g_h in houses follows the desired one, g_h^d, with some delay.[29] Furthermore, the long-term rate of profit r_h^l in the housing sector follows the actual profit rate in this sector, r_h, with a delay, an approach towards long-run views in investment behaviour that will also be used in the description of the dynamics of the capital stock of firms.

The rate of inflation of the rental price in housing, $\hat{p}_h$, depends as investment on the rate of capacity utilisation in the housing sector (the demand pull component) and on a weighted average formed by the actual rate of inflation of consumer or producer prices in the production of the domestic good and on the level of this inflation that is expected as a long-term average, the rate π^c, whose law of motion will be provided later on (the cost-push components).[30] Finally actual gross investment plans are always realised and thus determine the rate of growth of the housing stock by deducting depreciation from them.

Summing up we can state that consumption decisions of asset owners are basically driven by exogenous habits that are independent of their income and wealth position and that their investment decision into the housing sector is preceding the other asset accumulation decisions as they derive from their choice of nominal savings. These latter decisions are in the present framework governed by supply side forces based on the new issuing of bonds by the domestic government and of equities by firms. Note here that asset holders accumulate or decumulate short-term bonds depending on the difference between their flow supply by the government and the flow demand of workers. Asset holders are thus simply adapting themselves to the decisions of these two other agents. Furthermore, their choice of accumulating or decumulating foreign long-term bonds is here determined as the residual to all these flows in or out of short-

[26] Such a construction is needed for the discussion of steady states of the considered economy.

[27] ξ a liquidity and risk premium with respect to long-term bond holdings.

[28] Powell and Murphy (1997) use the rate of employment on the labour market in the place of this rate which is a more indirect way of expressing the demand conditions on the market for housing services.

[29] Related to but also different from the approach chosen in Powell and Murphy (1997), which introduces some inertia into the housing investment decisions of asset owners.

[30] This adjustment equation for rental prices in housing differs considerably from the one chosen in Powell and Murphy (1997).

and long-term domestic debt of the government and the flow of new equities issued by firms and is thus determined as a last step in the savings decision of asset holders. The essential decisions in this block of the model are therefore the housing investment decision and the pricing rule for housing services which is based on demand pressure as well as cost-push elements.

5.4.3 *Firms*

In the following module 3 of the model we describe the sector of firms, whose planned investment demand is also assumed to be always served, just as all other consumption and investment plans. We thus assume for the short run of the model that it is always of a Keynesian nature since aggregate demand is never rationed, due to the existence of excess capacities, inventories, overtime work and other buffers that exist in real market economies. There is thus only one regime possible, the Keynesian one, for the short run of the model, while supply side forces come to the surface only in the medium and the long run of the model. Up to certain extreme episodes in history this may be the appropriate modelling strategy for the macro-level of a market economy. This is shown in more detail in Chiarella *et al.* (2000, Ch. 5) for an integrated Keynes–Metzler model of monetary growth of a closed economy.

3. <u>Firms (Technology, Production, Employment and Investment):</u>

$$Y^p = y^p K, \quad y^p = const \tag{5.32}$$

$$J^d = j_y Y, \quad j_y = const \tag{5.33}$$

$$X = x_y Y, \quad x_y = const \tag{5.34}$$

$$L_f^d = l_y \exp(-n_l t) Y, \quad l_y = 1/z = const \tag{5.35}$$

$$\dot{n}_l = \beta_z(\widetilde{n}_l - z), \quad \widetilde{n}_l = \widetilde{n}_l(g_k) \tag{5.36}$$

$$u = Y/Y^p \tag{5.37}$$

$$\dot{L}_f^w = \beta_l(L_f^d - \bar{u}_f^w L_f^w) + (\gamma - n_l)L_f^w, \quad \bar{u}_f^w \in (0,1) \tag{5.38}$$

$$\begin{aligned} g_k^d &= (I/K)^d \\ &= \alpha_1^k((1-\tau_c)r^e - i^r) + \alpha_2^k(i_l - (i+\xi)) + \alpha_3^k(u - \bar{u}) + \gamma + \delta_k \end{aligned} \tag{5.39}$$

$$\dot{r}^l = \beta_{r^l}(r^e - r^l) \tag{5.40}$$

$$\dot{g}_k = \beta_{g_k}(g_k^d - g_k), \; g_k = I/K \tag{5.41}$$

$$Y_f = Y - Y^e = \mathcal{I} \tag{5.42}$$

$$S_f^n = p_y Y_f \tag{5.43}$$

$$p_e \dot{E} = p_y(I - \delta_k K) + p_y(\dot{N} - \mathcal{I}) \tag{5.44}$$

$$I^a = I + \dot{N} \tag{5.45}$$

$$\hat{K} = I/K - \delta_k = g_k - \delta_k \tag{5.46}$$

As already stated we assume in the sector of firms a fixed proportions technology, with respect to the three inputs, labour L_f^d, imports (raw materials) J^d, and capital K, and its two outputs (internationally), non-traded and traded goods, Y, X (which are not constrained on the world markets for these two goods). Imports and exports are thus inelastically demanded and supplied by domestic firms. In addition we have endogenous Harrod neutral technological progress at the rate z with respect to labour productivity $z = Y/L^d$ – which follows the rate of innovations $\tilde{n}_l(g_k)$ with some delay. We stress that the capital stock is used to measure potential output $Y^p = y^p K$ in the following, while all other magnitudes are provided by the Keynesian regime and its demand determined output rate Y. The rate of capacity utilisation u is defined on the basis of this concept of potential output and will receive importance when describing the investment behaviour and the pricing policy of firms. Firms employ a labour force of amount L_f^w which supplies labour effort of amount L_f^d as determined by the present state of sales expectations (plus voluntary inventory production). This labour force of firms is adjusted in a direction that reduces the excess or deficit in the utilisation of the employed labour force, $L_f^d - \bar{u}_f^w L_f^w$, which means that firms intend to return to the normal usage of their labour force thereby.[31] An additional growth term for the employed labour force takes account of the trend growth γ of domestic output, but is diminished by the effect of Harrod neutral technical change which when working in isolation would allow to reduce the workforce of the firms.

Next there is the formulation of the desired gross rate of capital stock accumulation of firms which depends on four factors. First, relative profitability, measured by the deviation of the long-term rate of profit r^l from the required rate of interest $i^r = i_l - \pi^c$ via the type of calculations underlying Tobin's q. Second, on the interest rate spread $i_l - (i + \xi)$, again representing the tightness of monetary policy and its believed effects on economic activity and employment. Third, on the rate of capacity utilisation u of the capital stock of firms in its deviation from the desired rate of capacity utilisation[32] $\bar{u}$, which is given exogenously. Fourth, on trend growth γ and the rate of depreciation δ_k of business fixed investment. As in the case of housing investment, we assume that the actual rate of accumulation g_k is following the desired one with some time delay. Furthermore, also the expected long-term rate of profit r_l is adjusted towards the currently expected rate of profit r^e with some time delay.

Firms produce output to cover expected demand for it and intended additions to inventories. Expected sales are also the basis of the dividend payments of firms and thus do not allow for retained earnings of firms, whose income Y_f is by definition equal

[31] Note that this normal usage includes a certain amount of absentism and is thus less than the full normal usage of this labour force ($\bar{u}_f^w < 1$).

[32] We could include here a dependence of the gross rate of investment on the rate of change of the rate of capacity utilisation which would add Harrod's accelerator to the present framework (which is in fact done in a similar fashion in Powell and Murphy (1997)).

to their output Y minus the expected sales Y^e, which in turn must be equal to desired inventory changes, to be defined below. Valued at producer prices p_y these inventory changes thus also represent the nominal savings of firms.

Due to these assumptions, on the dividend policy of the firm in particular, and due to our assumption that firms only use equities for financing their expenditures, we get as budget equation of firms

$$p_e \dot{E} = p_y(I - \delta_k K) + p_y(\dot{N} - \mathcal{I}),$$

implying that firms finance net investment and unintended inventory changes by issuing new equities (no bonds and no bank loans are allowed at present). Note here that unintended inventory disinvestment gives rise to windfall profits to firms which are retained, not subject to taxation and used to finance part of the fixed business investment as shown in the above equation. We stress once again that this particular financing rule is not crucial for the dynamic evolution implied by the model, but should of course give way to more realistic financing conditions in later reformulations of the model. The last two equations of module 3 then define the total actual investment of firms (for national accounting purposes) and the growth rate of the capital stock which is determined by the net rate of capital accumulation planned by firms.

Powell and Murphy (1997) allow for substitution in production by using a nested input technology of Constant Elasticity of Substitution (CES) type and CES transformation curves with respect to the two outputs that are produced by firms. With respect to such smooth transformation functions they then define medium-run marginal cost and revenue pricing procedures which act as attractors for the development of short-run prices in a particular way. We reconsider their approach to substitution and competitive pricing in Chiarella *et al.* (1999a,b). Furthermore, Powell and Murphy (1997) do not distinguish between actual working time and the normal working time of the employed, but use a single employment equation in their place which differs from our definitions of (efficient) employment L^d of the employed L^w and which is not backed up by budget equations. Finally there are some minor differences in the description of the investment behaviour of firms which, however, do not matter very much (it can be shown that the medium-run target prices of Powell and Murphy (1997) can be reformulated in terms of rates of capacity utilisation; see Chiarella *et al.* (1999a,b) in this regard).

Note finally that there is here no value-added tax on depreciation, investment and planned or unplanned inventories as in the housing sector considered beforehand and that there is no direct taxation of firms. Summing up the above module of the model basically provides descriptions of the output, the employment and the investment decisions undertaken in the sector of firms and this on the basis of various delays concerning employment, investment and underlying profitability measures (the delayed output adjustment decision is described under the heading 'quantity adjustment' in module 5a and the price adjustments undertaken by firms are considered in block 5b of the model).

Next, import and export prices are treated in the simplest way possible by assuming that they are fixed in terms of the foreign currency and thus need only to be multiplied with the exchange rate in order to arrive at domestic producer prices. There is no subsidy or tax on exports, but there is a tax rate on imported commodities of size τ_m. This module of the model basically impacts the profitability of firms as measured by the expected rate of profit r^e in the first block of our model.

3*a*. Export Prices and Import Prices in Domestic Currency

$$p_m = (1+\tau_m)sp_m^* \tag{5.47}$$

$$p_x = sp_x^* \tag{5.48}$$

In contrast to Powell and Murphy (1997) there are here no inventories held with respect to imports or exports. Since imports only serve as intermediate inputs of firms there is also no need to represent them in the consumer price index of the domestic country. Note furthermore that imports are demanded and exports supplied independently of their price changes, since they are in fixed proportions to the output Y of the domestic commodity. Note finally that the Purchasing Power Parity (PPP) theory cannot be valid here, since there is no common basket of goods that is produced and used internationally.

Powell and Murphy (1997) allow for certain price responses on the world market – due to varying imports and exports of the Australian economy – which we briefly consider in Chiarella *et al.* (1999a,b). Furthermore, they divide exports (in fixed proportions) into agricultural and non-agricultural exports which from a theoretical perspective does not contribute much to the generality of the model, but which may provide extra descriptive relevance.

5.4.4 *The government*

In module 4 we describe the public sector of the economy in a way that allows for government debt in the steady state and for a monetary policy that fixes the rate of interest on short-term debt in view of the level of the long-term world rate of interest, the domestic rate of inflation and the domestic level of activity of firms.

4. Government (Fiscal and Monetary Authority):

$$\begin{aligned} T^n &= \tau_w[wL^d + w^u(L-L^w) + w^r\alpha_l L_2] + \tau_p wL^d + \tau_v p_y(C_w + C_c + G) \\ &\quad + \tau_c[r^e p_y K + iB + B^l + p_h C_h^d - \delta_h K_h] + \tau_m s p_m^* J^d \end{aligned} \tag{5.49}$$

$$p_v G = g p_v Y^e, \quad g = \text{const.} \tag{5.50}$$

$$L_g^d = L_g^w = \alpha_g G/\exp(\hat{z}t) \tag{5.51}$$

$$\dot{i} = -\beta_{i_i}(i + \xi - i_{l*}) + \beta_{i_p}(\hat{p}_v - \bar{\pi}) + \beta_{i_u}(u - \bar{u}) \tag{5.52}$$

$$\hat{\tau}_w = \alpha_{\tau_{w1}}(d/\bar{d} - 1) + \alpha_{\tau_{w2}}\hat{d}, \qquad d = \frac{B + B^l/i_l}{p_v Y^e} \tag{5.53}$$

$$\hat{\tau}_m = \alpha_{\tau_m}\frac{p_x X - p_m J^d}{p_x X} \tag{5.54}$$

$$S_g^n = T^n - w^u(L-L^w) - w^r\alpha_l L_2 - (iB + B^l) - (p_v G + w^b L_g^d) \tag{5.55}$$

$$\begin{aligned} \dot{B} &= \alpha_b^g(p_v G + iB + B^l \\ &\quad - T^n + w^u(L-L^w) + w^r\alpha_l L_2 + w^b L_g^d) \end{aligned} \tag{5.56}$$

$$\begin{aligned} \dot{B}^l/i_l &= (1-\alpha_b^g)(p_v G + iB + B^l \\ &\quad - T^n + w^u(L-L^w) + w^r\alpha_l L_2 + w^b L_g^d) \end{aligned} \tag{5.57}$$

$$\dot{B}_1^l = \alpha_{b1}^g \dot{B}^l \tag{5.58}$$

$$\dot{B}_1^{l*} = (1-\alpha_{b1}^g)\dot{B}^l \tag{5.59}$$

The first equation in the government module describes the tax collection by the government which consists of taxes on wages, unemployment benefits and pensions, payroll taxes as the basis of state transfers to worker households, value-added taxes on consumption goods, capital taxes on profit, interest and rent (net of depreciation), and import taxes. Note that – symmetric to the treatment of interest payments received and taxed abroad – we have here that all interest payments of the government (to domestic residents or foreigners) are taxed domestically and thus contribute to a reduction in domestic government debt. Note also that there are no taxes on wealth, investment, depreciation and inventories and of course none on firm income which is equal to intended inventories plus windfall profits or losses, $Y^d - Y^e$, solely.

Government expenditures are assumed to be a fixed proportion of expected sales[33] (at consumer prices) and employment in the government sector is a fixed proportion of real government expenditure. In view of later steady state calculations we assume that this employment relationship is also subject to Harrod neutral technical change (of the same type as in the sector of firms), but that government employees have fixed normal working hours and thus are never over- or under-employed as is the case for the workers in the sector of firms. Note that this implies that there is no lag in the employment policy of the government in view of its employment function shown above.[34]

With respect to monetary policy we assume that monetary authorities determine by legislature (the change in) the nominal rate of interest on short-term bonds (which are not traded internationally) by means of a Taylor type policy rule.[35] There are no money holdings in the private sector of the economy and there is therefore no need to specify the new supply of money, used for open market operations as well as foreign exchange market operations. With respect to the first type of operation we observe

[33] An easy extension of this rule for government expenditures would be to assume the ratio g depends negatively on short- and long-term interest rates i, i_l, which could also be extended to the consumption decision of workforce households and which thereby give extra power to the interest policy rule of the central bank to be considered below. Note also that we do not consider delays in the adjustment of government expenditure (and the employment decisions that accompany it).

[34] Note here that the employment level in this part of the economy is much larger than any other employment level within each firm and may therefore allow for such a direct employment policy simply due to retirement effects and the like.

[35] See Flaschel *et al.* (2001) with respect to this particular choice of an interest rate policy rule.

that there is no need for it in an economy where the short-term rate of interest is directly set by the monetary authority and where transactions are performed by costless temporary credit by the branches of the central bank that must be settled in accordance with the budget constraints at the end of each day. With respect to foreign market operations we have shown in the preceding section that they are not needed as long as the private sector just absorbs the inflows of domestic bonds and equities and invests the remaining savings into long-term foreign bonds. This considerably simplifies the feedback structure between the real and the financial sector of the economy, but of course should give way to more realistic descriptions of the financial markets in future extensions of the model.

With respect to the interest rate policy of the central bank we assume that it attempts to move the actual rate of interest, i, toward the steady state short-term rate of interest, i_o,[36] as it is determined by the world rate of interest on long-term bonds minus the liquidity premium that applies to them, but that it at the same time aims at moving the actual rate of inflation, $\hat{p}$, toward some target rate, $\bar{\pi}$, for example from above by raising the rate of interest in order to reduce economic activity (as measured by the rate of capacity utilisation u) and thus the demand pressure on the rate of inflation. Of course, high levels of economic activity u will make this decision for a tight monetary policy more pronounced than low levels of activity which explains the third term in our interest rate policy rule. In view of the fact that we will not consider inflationary processes in the world economy, but shall assume given world market prices for imports and exports in module 8 of the model, we set the target rate of inflation $\bar{\pi}$ of the central bank equal to zero throughout this chapter (in order to simplify the presentation of the interior steady state of the model).[37]

Note again that money is not held as a store of liquidity and wealth by the private sector of the economy, but is only used as means of transactions that flow from banks (here only branches of the central bank) to households, firms and the government and then back to the banking sector on each trading day, without any income-in-advance restrictions.

We use d to denote the ratio between government debt and expected sales (debt-GDP ratio) at consumer prices and assume as policy rule for the tax rate on wages that this rate is adjusted such that government moves debt into the direction of a desired debt-GDP ratio $\bar{d}$ augmented by a term that describes reactions to the rate of change of the debt-GDP ratio d as in a derivative control feedback loop. The burden of too high debt thus falls entirely on wage income which supports our view that government bonds are net wealth. In addition, the import tax rate τ_m is adjusted in order to reduce any possible surplus or deficit in the trade balance in terms of the domestic prices for exports and imports (which include import taxation).

[36] Or at least attempts to not let it go too far away from it.

[37] We shall show in future work however that this rate should be chosen positive in order to avoid certain problems caused by the actual behaviour of money wages (and that central banks actually generally have a target that is greater than zero).

The next two equations, on the debt financing of the government, are based on their left-hand sides on the actual government deficit or surplus. Government revenue is based on nominal taxes T^n and is used to finance nominal government expenditures $p_v G$, interest payments on short- and long-term debt $(iB+B^l)$, unemployment benefits, pensions and the wage sum of state employees. The deficit that generally will come about in this way is then financed by issuing new short-term or long-term debt. We here assume that the portion α_b^g of the new government debt is financed short term, while the remainder is financed long term.

For accounting purposes we have added the definition of nominal government savings S_g^n which – if negative – is financed in the just stated way through short- and long-term debt. Finally we have to state how the new long-term government debt is distributed in the world. As before we here too assume that this is done in constant proportions with respect to the domestic and the foreign market for domestic long-term debt.

New assets are therefore distributed to asset owners in fairly rigid proportions (on primary asset markets) supplemented by a procedure whereby we will only introduce laws of motion for the various asset prices in the following, but will not develop a full portfolio approach to the determination of asset prices (or their rates of change) and the implied portfolio adjustments on secondary as well as on primary asset markets. Hence, asset markets are represented here solely by way of certain interest rate adjustment processes (and their impact on the investment decisions of firms and in dwellings). Asset markets are in this chapter thus surely modelled less complicated than in a full portfolio approach (and the liquidity preference schedule this approach would imply for the holding of short-term debt). More or less assets therefore just flow into the private sector of the economy in proportions that are determined by the government and the firms, which represents a very tranquil way of asset absorption. It is therefore quite obvious that asset markets are the substructure of the model that need improvement most urgently; see also the module on asset price dynamics.[38]

For the moment we justify this approach to asset accumulation and their price dynamics by the fact that we at least provide by it a complete – although not yet really convincing – description of the dynamics of asset markets, which must be improved later on by a static or dynamic portfolio approach to the behaviour of these markets that gives more role to the demand side. At present however supplies of new assets just flow into the economy – up to the foreign investment of asset holders – and lead through some type of not explicitly formulated process to interest rate differentials and the dynamics of asset prices and expectations about them as they are described in Section 5.4.

Note that this description of the government sector excludes open market operations as well as foreign exchange operations of the central bank. The first type of policy is not

[38] Köper and Flaschel (2000) integrate a portfolio approach into the real dynamics of the 6D Keynes–Metzler model of Chiarella and Flaschel (2000) and find that the implications of this portfolio approach to the real-financial interaction share many similarities with model types where the present approach to asset market dynamics is used instead.

needed in an economy where the interest rate on short-term bonds is set by the central bank and where accounting money only serves the purpose of intra-day trading until all budget equations are settled again. The second type of policy is not needed since the supply side description of asset markets and the accommodating behaviour of asset holders with respect to foreign bonds always clears the market for foreign exchange as we have seen in the preceding section.

In sum we have a target rate of inflation $\bar{\pi}$ of the central bank which is here zero by assumption, a debt target per unit of expected nominal GDP which is given by $\bar{d}$, government's expenditures which are a given share in expected GDP, and the attempts of the government to establish external trade equilibrium via import taxation. In a sense the behaviour of the government is therefore still fairly neutral, although we allow for steady state debt and deficit according to certain rules. There are certain similarities between our description of the government sector and the one of Powell and Murphy (1997), in particular with respect to the wage tax rate adjustment rule. It is not difficult to add further policy rules to this module of the model, as e.g. an anticyclical government expenditure and employment policy rule, an anticyclical behaviour of payroll taxes or other formulations of the Taylor interest policy rule. Later extensions and modifications of the model should concern the introduction of a banking industry (which transforms short-term debt into long-term debt, issues loans to firms and the like) and a less rigid diversification and distribution scheme for the allocation of government debt to the various other agents of the model (where we in principle have followed Powell and Murphy (1997) for the time being).

5.4.5 Quantity and price adjustment processes

We now come to the description of the dynamics of quantities (module 5a) and prices (module 5b). Module 5a of the model basically describes a Metzlerian inventory adjustment process for the non-traded good produced by firms.[39] Module 5b describes the nominal adjustments in the goods and in the labour market, as well as the adjustment of long-term inflationary expectations π^c.

5a. Quantity Adjustments in the Production of the Domestic (Non-traded) Good:

$$Y^e \neq Y^d = C_w + C_c + I + I_h + G \tag{5.60}$$

$$S^n = S_p^n + S_f^n + S_g^n = I^{na} + NCX^n = I^{na} + NX^n + NFX^n \tag{5.61}$$

$$I^{na} = p_y(I - \delta_k K) + p_y(I_h - \delta_h K) + p_y\dot{N} \tag{5.62}$$

$$N^d = \beta_{n^d} Y^e \tag{5.63}$$

$$\mathcal{I} = \beta_n(N^d - N) + \gamma N^d \tag{5.64}$$

$$Y = Y^e + \mathcal{I} \tag{5.65}$$

[39] There are no sales and delivery constraints for traded goods and there is thus no direct need to consider inventory adjustment processes in their case.

$$\dot{Y}^e = \beta_{y^e}(Y^d - Y^e) + \gamma Y^e \tag{5.66}$$

$$\dot{N} = Y - Y^d \tag{5.67}$$

The first equation in 5a contrasts expected sales Y^e with aggregate demand and actual sales Y^d of the non-traded good for our Keynesian description of the short run of the model. Actual sales = aggregate demand consists of five different items here (two types of consumers' demand, two types of investors' demand and the government's demand for domestic goods). Next we consider once again (for consistency reasons) the accounting identity for actual total savings, actual total investments and the balance in the current or the capital account, where nominal actual total investment I^{na} is defined by net fixed business investment and net investment in houses and by total inventory changes – everything valued at producers' prices. This equation provides an important consistency check for our analysis of goods market disequilibrium in the context of a small open economy. It also implies, see Section 5.3.2, that the flows of new assets supply are equal to the absorption of these supplies by the household sector (workers and asset holders).

The remaining five equations describe the inventory adjustment process. Desired inventories N^d are a constant fraction of expected sales Y^e. Intended inventory changes $\mathcal{I}$ are proportional to the gap between desired inventories and actual ones, N, plus a term that accounts for the fact that inventory formation takes place in a growing economy with trend growth γ. Output decisions Y are based on the sum of expected sales and intended inventory changes, while sales expectations Y^e are changed in an adaptive way through the observation of the discrepancy between actual sales Y^d and the expected ones Y^e, again augmented by a term γY^e that accounts for the trend growth underlying the evolution of this economy. Finally, actual inventory changes $\dot{N}$ are just given by the difference between actual output and actual sales, which once again gives expression to our general assumption that the short run of our economy is always of a Keynesian nature and not perfectly foreseen by the agents of our economy. This inventory adjustment process is the same as the one in Powell and Murphy (1997) with the exception that sales expectations are always correct in the Murphy model.

Next we consider the wage-price dynamics of the model. This type of dynamics is receiving more and more attention in recent studies of primarily empirical orientation[40] and thus represents an important module of the present stage of modelling the details of a small open economy with an integrated treatment of its short-, medium- and long-run behaviour. We stress however that we do not yet treat consumer price indices and the role of import prices in the formation of the money wage and the price level PCs respectively; see Chiarella *et al.* (1999a,b) in this regard.

5b. Wage-Price Adjustment Equations, Expectations:

$$\hat{w}^b = \beta_{w_e}(e - \bar{e}) + \beta_{w_u}(u_f^w - \bar{u}_f^w) + \kappa_w(\hat{p}_v + \tilde{n}_l) + (1 - \kappa_w)(\pi^c + \tilde{n}_l) \tag{5.68}$$

[40] See Fair (1997, 2000), and Stock and Watson (1999).

$$\hat{p}_v = \hat{p}_y = \beta_p(u - \bar{u}) + \kappa_p(\hat{w}^b - \widetilde{n}_l) + (1 - \kappa_p)\pi^c \quad (5.69)$$

$$\dot{\pi}^c = \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p}_v - \pi^c) + (1 - \alpha_{\pi^c})(\bar{\pi} - \pi^c)) \quad (5.70)$$

$$L^w = L_f^w + L_g^w = L_f^w + L_g^d \quad (5.71)$$

$$e = L^w/L = e_f + e_g = L_f^w/L + L_g^w/L \quad (5.72)$$

$$u_f^w = L_f^d/L_f^w, \qquad [u_g^w = L_g^d/L_g^w = 1] \quad (5.73)$$

$$\dot{\bar{e}} = \beta_{\bar{e}}(e - \bar{e}) \quad (5.74)$$

Wage inflation $\hat{w}^b = \hat{w}$ is nearly of the same type as in Powell and Murphy (1997). Wage inflation responds in the traditional PC manner; to the state of the demand pressure in the labour market as measured by the deviations of the rate of employment e from its NAIRU level $\bar{e}$; to the deviation of the employment rate u_f^w of the employees of firms from their norm (including absentism) which is measured by $\bar{u}_f^w$ (and which corresponds to the derivative term for the rate of employment, $\dot{e}$, that Powell and Murphy (1997) employ in this place); and to the usual accelerator term of price inflation which is here measured as a weighted average of actual price inflation based on short-term perfect foresight (plus the actual rate of productivity growth) and expected long-term price inflation (plus the long-run rate of productivity growth) in the place of the simple adaptive scheme used by Powell and Murphy (1997). Wage inflation is therefore governed by demand pull terms augmented by a weighted average of cost-push expressions.

The law of motion for consumer prices p_v of the non-traded commodity is formulated in a similar way, as a second type of PC. In the place of the concept of medium-run prices used by Powell and Murphy (1997) we use the demand pressure measure $u - \bar{u}$, the deviation of actual capacity utilisation from its norm, as one cause of price inflation. In Chiarella *et al.* (1999a,b) we show that this measure is closely related to the medium-run price concept of Powell and Murphy (1997) in the case of smooth factor and output substitution. The cost-push term in the price inflation equation is given as a weighted average of current wage inflation and the one expected for the long run (both made less severe in their influence on price inflation by the existence of a positive growth rate of labour productivity, now and in the longer run).

Expected long-term inflation π^c in turn is based on a weighted average of two expectations mechanisms, an adaptive one with weight α_π and a forward-looking one with weight $1-\alpha_\pi$. Forward-looking expectations are here simply based on the inflation target of the central bank $\bar{\pi}$, in the usual way of a regressive scheme of expectations revision. Inflationary expectations are thus following a weighted average of actual inflation and the target rate of the monetary authority.

This description of the wage-price spiral is based on formulations used extensively in Chiarella and Flaschel (2000) and is therefore not explained in more detail here.

Equations (5.71) to (5.73) describe some definitions concerning total employment (through firms and the government) and the outside rate of employment e as well as the inside rate of employment u^w of the employed. The last equation, finally, assumes

that the NAIRU rate of employment follows the actual rate of employment with some delay. We here deviate again from Powell and Murphy (1997) who consider this rate of employment as being determined exogenously. Note that firms follow the rate u^w when deciding on the change in the workforce they employ.

We can see from the above description that only the inflation rate of non-traded domestic goods matters in the wage-price module of our economy. Housing, meaning the rental price of dwellings (and its rate of change $\hat{p}_h$), is thus completely ignored in this description of the wage-price interaction. This simplifies the feedback structure of the model, but should give way to a domestic price index of the form $p_c = p_y^a p_h^{1-a}$ and its rate of change in the wage equation in future reformulations of the model.

5.4.6 *The dynamics of asset market prices and expectations*

The sixth module lists the dynamic adjustment equations we assume to hold for the asset prices of our model: long-term domestic bonds, p_b, equities, p_e, and for the exchange rate (in view of the given US $ rate of return on foreign bonds). We stress that reallocations of the stock of wealth are not considered explicitly in the present version of the model (which also does not yet allow for wealth effects in the behavioural assumptions that are employed).

As already discussed in the preceding descriptions of the modules of the model, asset flows and asset accumulation are determined by supply side conditions in the main in the present form of the model and are thus just absorbed by asset holders, at least as far as short-term bonds (leaving aside those already purchased by worker households), long-term domestic bonds and equities are concerned. In contrast to this, asset holders are supplied with an investment demand function as far as their housing investment is concerned (which is never rationed) and they balance their savings account thereafter by purchasing or selling foreign bonds on the world market. This is surely only a preliminary approach to the accumulation of financial wealth and its distribution to the two household sectors we consider. We return to this question in Chiarella *et al.* (1999a,b) where money holdings are added to the model and are adjusted in time via changes in the short-term rate of interest.

The adopted approach to asset accumulation is acceptable in a continuous-time framework, if there is subsequent stock reallocation according to a specified money demand function and if all other assets can be considered as perfect substitutes for each other, since asset holders are in such a case indifferent with respect to their holding of interest bearing bonds, see Sargent (1987) for example. In the present approach there is however no stock demand for transaction balances and thus no explicit reallocation of stock positions that have been changed by flows of new assets into the asset markets. Furthermore, due to somewhat delayed responses of asset prices to expected interest rate differentials we depart in the following from the perfect substitutability assumption. Thus we have to acknowledge that asset market dynamics are not yet well-founded as far as conceivable behaviour of individual holders of financial assets is concerned.

Franke and Semmler (1999) provide a portfolio approach with imperfect substitutabilities to the determination of the temporary structure of interest rates of the economy which will be adapted to the present framework in either stock or flow form in future extensions of the model.

6. Asset Prices, Expectations and Interest Rate Adjustments:

$$\hat{p}_b = \beta_{p_b}[(1-\tau_c)i_l + \pi_b - ((1-\tau_c)i + \xi)], \quad \hat{p}_b = -\hat{i}_l \tag{5.75}$$

$$\hat{p}_e = \beta_{p_e}[(\frac{(1-\tau_c)r^e p_y K}{p_e E} + \pi_e) - ((1-\tau_c)i_l + \pi_b)] \tag{5.76}$$

$$\hat{s} = \beta_s[(1-\tau_c^*)i_{l*} + \epsilon_s - ((1-\tau_c)i_l + \pi_b)] \tag{5.77}$$

$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}), \qquad p_b = 1/i_l \tag{5.78}$$

$$\pi_{bc} = \hat{p}_b \tag{5.79}$$

$$\pi_b = \alpha_s \pi_{bs} + (1-\alpha_s)\pi_{bc} \tag{5.80}$$

$$\dot{\pi}_{es} = \beta_{\pi_{es}}(\hat{p}_e - \pi_{es}) \tag{5.81}$$

$$\pi_{ec} = \hat{p}_e \tag{5.82}$$

$$\pi_e = \alpha_s \pi_{es} + (1-\alpha_s)\pi_{ec} \tag{5.83}$$

$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s) \tag{5.84}$$

$$\epsilon_c = \hat{s} \tag{5.85}$$

$$\epsilon = \alpha_s \epsilon_s + (1-\alpha_s)\epsilon_c \tag{5.86}$$

Note first of all with respect to the three laws of motion for the bond price, the share price and the nominal rate of exchange, that they have to be based on interest rate differentials after taxes, but that there is no taxation of actual capital gains in the model, and thus no tax term applied to expected capital gains in the formulae shown above.

Instead of a full portfolio approach to asset market equilibria it is assumed in the above adjustment equations for asset prices p_b, p_e and the exchange rate s that stocks in asset markets give rise to forces that imply certain laws of motion for their prices. The law of motion for the price of long-term domestic bonds, $p_b = 1/i_l$, for example, states that the rate of change of p_b is determined by the differential between the net rate of return $(1-\tau_c)i_l + \pi_b$ on long-term bonds (including expected capital gains π_b) and the short-term rate of interest $(1-\tau_c)i$ (of fixed-price bonds and after taxes) augmented by a liquidity and risk premium ξ that is exogenously given. In the limit, $\beta_{p_b} = \infty$, we interpret this dynamic law as an equilibrium relationship: $(1-\tau_c)i_l + \pi_b = ((1-\tau_c)i + \xi)$ which could then be used as in Blanchard (1981) to study the conventional type of saddlepoint adjustment processes based on the jump variable technique. Yet it is not at all clear under which circumstances fast, but finite adjustments of consols prices will lead to dynamics that mirror such saddlepoint dynamics obtained in the limit. On the contrary, considerations of small size models as in Flaschel *et al.* (1997) have shown that nothing of this type can be expected in general. Therefore, we stick to the assumption

that the rate of interest $i_l = 1/p_b$ of long-term bonds follows the movement of the short-term rate of interest i (in the above assumed way) with some delay which may be very short, but which is larger than zero.

The next equation describes the evolution of equity prices, p_e, in a similar way. Their rate of change, p_e, is driven by the discrepancy between the rate of return on equities (dividend payments of firms after taxes and expected capital gains π_e per equity value) and the rate of return on long-term bonds after taxes (including expected capital gains). Note here that $\frac{(1-\tau_c)r^e p_y K}{p_e E}$ describes the actual dividend payments per unit of equity value and represents a nominal rate of return which – leaving capital gains aside – has to be compared with the nominal rate of interest on bonds $(1-\tau_c)i_l$ in order to make the correct rate of return comparison. Assuming $\beta_{p_e} = \infty$ would again imply the often used assumption that long-term bonds and equities are considered as perfect substitutes and would thus lead to asset market representations of a more conventional type.

Finally, we assume that the dynamic of the exchange rate, $\hat{s}$, is also based on an expected interest rate differential, namely between domestic and foreign long-term bonds (both after taxes), the latter augmented by the expected capital gains from possible devaluations, ϵ_s, of the domestic currency and the former by expected capital gains on domestic bonds. Note here that the foreign rate of interest is exogenously given so that there is no comparable capital gain on long-term foreign bonds. An increase in the above differential makes foreign bonds more attractive which leads to a capital outflow and thus an increase of the demand for the foreign currency which is the (here implicit) cause for the increase of the exchange rate s implied by equation (5.77). Assuming $\beta_s = \infty$ would lead us to the limit assumption of uncovered interest rate parity (UIP) – often employed in the literature – which in this chapter is however subject to some time delay.

With respect to expected capital gains on long-term bonds and equities we assume heterogeneous expectation formation. On the one hand, there is technical or time series analysis (for a certain group of asset owners), which here boils down to an exponentially weighted formula based on past observations or simply an adaptive formation of expectations with speeds of adjustment $\beta_{\pi_{bs}}$, $\beta_{\pi_{es}}$ (less ambitious 'retired' or less well-informed asset owning agents which we here identify with the fraction of elderly people among the asset holders). On the other hand, there exists a portion of asset owners with correct expectations $\hat{p}_b$, $\hat{p}_e$ (ambitious 'younger' or well-informed agents who achieve myopic perfect foresight by sacrificing leisure time).

We thus assume that the establishment of myopic perfect foresight is very time consuming (reducing the leisure time of the ambitious wealth owners significantly). There is thus only a certain fraction, the 'younger' ones of the population of all pure asset owners, who devote themselves and their leisure time to this formidable task (represented by $1-\alpha_s$), while the other (the less ambitious ones among the wealth owners) rely on less time consuming time series analysis in order to make their predictions of temporary asset price changes. The market opinion is then simply reflected on the macroeconomic level by the average of these two expectations generating mechanisms formed by means of the weight α_s, $1-\alpha_s$ in each case (long-term bonds and equities).

In line with the dominant view of currently prevailing economic theory we consider the less ambitious agents as the stupid ones and the ambitious agents as the clever ones.

Due to their myopic perfect foresight clever agents of course perform better than the stupid ones and thus will have a higher overall rate of return than this latter group. Yet, since they will change their behaviour in later parts of their lives (due to changing habits and obligations of people that get older) they will only temporarily outperform the market and accumulate wealth at a higher pace. The overall effect of the existence of these two groups of people among the pure asset holders is that markets would adjust their prices according to the interest rate differentials perceived by the stupid agents, but that the existence of clever agents and their perfect short-run expectations works such that the former interest rate differentials are transformed and corrected to some extent into the directions the clever agents see them to be. This is due to the redirection of portfolio demands as they come from this second group of agents which are not made explicit here. Adaptive expectations that are too high with respect to actual changes in asset prices are thereby made less severe in their impact on asset demands and resulting asset price changes.

In order to justify this approach to expectations formation further we now insert this average expectation on asset price or exchange rate changes into the corresponding price formation rule which in the case of long-term bonds, for example, implies the following final form for the adjustment of bond prices:

$$\hat{p}_b = \frac{\beta_{p_b}}{1 - \beta_{p_b}(1 - \alpha_s)}[(1 - \tau_c)i_l + \alpha_s \pi_{bs} - ((1 - \tau_c)i + \xi)]$$

together with equation (5.78) for π_{bs}. Increases in the population and the weight $1 - \alpha_s$ of the clever ones among the asset owners (with their time-consuming establishment of myopic perfect foresight), starting from $\alpha_s = 1$ therefore increases the volatility of bond prices and the difficulties to predict them perfectly, since it increases the adjustment speed with which the interest rate differentials as viewed by stupid agents (weighted by their fraction in the total population of asset owners) is transferred into bond price changes.

There is an absolute upper limit with respect to this increase in volatility which is represented by the critical proportion α_s^* of 'less ambitious' asset owners given by:

$$\alpha_s^* = (\beta_{p_b} - 1)/\beta_{p_b} < 1.$$

At α_s^* the speed of adjustment of bond prices has become infinite while it is still finite (and working into the right direction) for all admissible α_s that are larger than α_s^*. Decreasing the number of less ambitious asset owners therefore is bounded by this critical value α_s^* where the young workaholics must finally lose sight of the true behaviour of asset prices and the exchange rate. We do not however investigate in this chapter the adjustments that may take place in the share α_s of time series based expectations according to some switching mechanism between the two groups of asset owners here considered, but assume instead that this proportion is constant in time at a 'balancing age' where asset owners switch from ambitiousness to laziness. Endogenous changes

in this dividing line and also other reasons for such a switch should be incorporated at a later stage.[41]

Whatever the outcome of such a discussion may be, we thus here simply assume that there is a mechanism at work that creates heterogeneous asset owner behaviour and heterogeneous expectation formation of a type and extent that prevents the model converging to situations of overall myopic perfect foresight in the financial markets. Rather a situation is reached where asset price reactions to interest rate differentials are still normal with respect to direction and are finite.

Of course, these considerations of long-term bond price dynamics apply to the dynamics of equities in the same way and as the above module of asset prices shows also to the dynamics of the exchange rate and the expectations mechanisms there assumed. In all three cases we assume therefore a lower limit for the proportion or market share of 'less ambitious' or 'stupid' asset owning households given by

$$\alpha_x^* = (\beta_x - 1)/\beta_x$$

where x stands for the asset market under consideration. Note that we must assume that α_s is larger than all three critical ratios that are generated on the three considered asset markets in order to have normal reactions of asset prices and exchange rates on all three markets.

We stress that all interest rate comparisons are made with respect to gross levels of these rates (not net of taxes at the rate τ_c), so that the tax rate τ_c does not show up in these laws of motion for interest rates, asset prices and the rate of exchange. Taking net rate in the place of gross ones would only complicate the above formulae without any change in substance.

This closes the description of the behaviour we assume for the asset markets of the economy. Powell and Murphy (1997) assume only 'clever' agents or 'workaholics' to exist in their formulation of asset market behaviour and assume in addition that adjustment speeds of asset revaluations are always infinite, leading them to the usual interest rate parity conditions as for example for the comparison of domestic and foreign long-term bonds: $(1 - \tau_c)i_{l*} + \hat{s} = (1 - \tau_c)i_l$. Their model thus is based on the limiting case of myopic perfect foresight in the asset markets which leads them to the then usual jump variable technique as assumed representation of the forward-looking behaviour of agents and restricts the dynamics of the model to its stable manifold (thereby removing *all* local instability from sight). We do not follow this procedure in our formulation of the dynamics of the model, which only partly incorporates forward-looking behaviour on the asset markets, whose dynamical implications are – as will be seen – radically different from those with complete myopic foresight on the asset markets and perfect adjustments of asset price (no interest rate differentials).

41 See Brock and Hommes (1997), Chiarella and Khomin (1999) and Sethi (1996) for examples of analysis of the implication of such switching mechanisms.

The consequence is of course that we will have differentiated rates of return at each moment in time, without formulating a full portfolio approach to take account of the non-uniformity of these rates. In sum, we must state that the description of the asset market adjustment processes represents the module of our model where future improvements are needed the most. At present 'causality' runs from the interest rate policy of the central bank (with respect to short-term debt which is not traded internationally) to adjustments in the rate of interest of long-term debt to adjustments of the exchange rate (in view of internationally traded long-term bonds) to changes in the rate of profit expected by firms. There are also adjustments in equity prices based on the rate of return for long-term bonds but these adjustments do not feed back into the model due to the lack of wealth effects and more advanced financing rules for firms (and due to the hierarchy chosen for the adjustment of asset prices). With the exception of the latter type of dynamics we have effects of the above changes in asset markets on business fixed investment and housing investment, but this is basically all that relates the real and the financial part of our economy. Note here also that the monetary authority is steering the short-term rate of interest basically from an anti-inflationary perspective and that it can do so to some extent since short-term debt is not traded internationally. Of course, it has to accept then the consequences that result from the adjustments of the long-term rate of interest and nominal exchange rates.

5.4.7 External accounts and foreign country data

The next module, 7, describes the various items that appear in the balance of payments Z, nominal net exports NX^n, nominal net (international) interest payments NFX^n and nominal net capital exports NCX^n. Concerning nominal net interest payments, which are normally interpreted as net 'factor' exports NFX^n and which need not cross borders and thus need not appear as an item in the current account, we have in fact assumed that they do cross borders. They are fully present in the calculation of the disposable income of wealth owners and also in the current account of the balance of payments Z. We stress again that the balance of payments must be balanced in our model due to assumed behaviour of asset holders with respect to the domestic supply of debt and equities and the international adjustments that residually follow from them.

7. Balance of Payments:

$$NX^n = EX^n - IN^n = sp_x^*X - sp_m^*J^d \tag{5.87}$$

$$NFX^n = s(1-\tau_c^*)B_2^l - (1-\tau_c)B_1^{l*} \tag{5.88}$$

$$NCX^n = s\dot{B}_2^l/i_{l*} - \dot{B}_1^{l*}/i_l \tag{5.89}$$

$$Z = NX^n + NFX^n - NCX^n = 0 \tag{5.90}$$

Module 8 finally provides the data needed from the 'foreign' economy in the simplest form possible. It is assumed that the modelling of the foreign economy is based on the same qualitative principles we used for the description of the domestic economy and

that it is inflation free, exhibits a constant rate of growth and a constant rate of interest on long-term bonds.

8. Foreign Country Data:

$$i_{l*} = const. \quad \text{(world interest rate)} \tag{5.91}$$

$$\tau_c^* = const. \quad \text{(foreign tax rate} = \tau_c \text{ by assumption)} \tag{5.92}$$

$$\gamma = const. \quad \text{(world growth rate)} \tag{5.93}$$

$$p_x^* = const. \quad \text{(world price level of the export good)} \tag{5.94}$$

$$p_m^* = const. \quad \text{(world price level of the import good)} \tag{5.95}$$

This closes the description of the extensive or structural form of the model of a small open economy and its detailed comparison with the structure of the Murphy model for the Australian economy.

We stress once again that the short run of the model is Keynesian throughout which means that supply bottlenecks can either be avoided through appropriate buffers or have to be added still for larger deviations of the economy from its steady state behaviour as described in Chiarella *et al.* (1999a,b).[42]

Summarising our comparison with the Murphy model as presented in Powell and Murphy (1997) we can state that their model basically differs in the range of assets they allow in the financial part of the economy where we use a disequilibrium approach to asset market dynamics and expectations while the Murphy model rests on interest rate parity conditions coupled with perfect foresight of investors, both with respect to financial as well as real investment which is a limit case of the approach we have adopted. It may be that this limit case is the only convincing case of the situations we allow for asset market dynamics as far as a pure flow treatment of these markets is concerned. In our view this would imply that the asset market module of our model must be replaced by a full portfolio approach in later reformulations of the model as it is presented in Franke and Semmler (1999).

The largest difference is the difference in the treatment of production as far as formal difficulties are concerned. Powell and Murphy (1997) have to solve an eight dimensional non-linear equation system in their treatment of the objectives of firms on the background of their nested CES technology while we have only explicit linear expressions for the same procedures in the case of a fixed proportions technology of the same type. We show in Chiarella *et al.* (1999a,b), however, how the approach chosen for the Murphy model, its so-called neoclassical heart, can be integrated into the model of the present chapter and that the qualitative dynamic behaviour of the model remains the same.

Further differences concern household behaviour where we show in Chiarella *et al.* (1999a,b) how the representative life-cycle approach of the Murphy model can be

42 See Chiarella *et al.* (2000) for the details of such an extension which explicitly includes the possibility of and the reaction to supply bottlenecks.

integrated into our heterogeneous household framework. There we also allow again for the conventional type of money holding and a Cagan type money demand function for the two types of households we consider. There are also many further, but generally minor, differences to the Murphy model which will not be investigated here. In view of this we would nevertheless claim, even at the present stage of the investigation, that the model of this chapter and the Murphy model are very similar in spirit, although of course different in purpose still, with respect to the weights these two model types give to theoretical or applied considerations.

5.5 The next steps

We have introduced and discussed in this chapter in great detail an integrated macrotheoretical model of monetary growth for a small open economy in extensive or level form which has many features in common with the macroeconometric model for the Australian economy as presented in Powell and Murphy (1997). Our aim in this chapter was to provide a starting model which can provide macrofoundations to applied models of the Murphy type, in the twofold sense that all budget equations of all agents that are considered be spelt out in their details and in their consequences (the assumed behaviour of economic agents of course being consistent with these budget restrictions) and second that there be a fully specified steady state solution to be used as a consistency check and as a reference path for the dynamics implied by the model.

For the moment we have used fairly conventional macro descriptions for the behaviour of households, firms and the government without demonstrating how they can and have been microfounded in the literature. Specific microfoundations may be provided later on and may change some of the modules we have presented here to a certain degree, but we expect that they will not change the general outlook and type of investigation of the presented description of labour, goods, and asset market dynamical (disequilibrium) adjustment processes. In our view, macrofoundations (or macroperspectives) come first (before microfoundations), since they provide the overview across the modules of the structure to be studied in their dynamic interdependence, while microfoundations are needed later on to obtain hopefully a firmer basis and more convincing formulation of the modules used in the initial macrostructure.

Our next steps in pursuing the project of macrofoundations and macro analysis in this way will aim at obtaining first of all a thorough presentation of the intensive or state variable form of the model (explaining its characteristics in detail also on this level). We then calculate (with respect to real magnitudes) on this basis a uniquely determined interior steady state solution of the model and study the comparative dynamic implications to which it gives rise. This will lead us to a 34 dimensional non-linear dynamical model and its steady state solution which in this general form is difficult to understand with respect to the many economic features it contains and which generally can only be investigated from the numerical point of view.

In order to approach the understanding of such large disequilibrium growth models in a systematic way we shall simplify them in various ways in the subsequent chapters.

We shall make use of a core 18D model that is obtained from the general version by suppressing certain secondary feedback structures of the full 34D dynamics. This 18D model can be further reduced to a basic 6D Keynes–Metzler–Goodwin type real dynamical model of a closed economy as it was introduced in Chiarella and Flaschel (2000, Ch. 6). Starting from this model type various routes for extending it back to the 18D structure will then be investigated and compared in their numerical behaviour, see Chiarella *et al.* (2003b) for details.

Next, from the theoretical point of view, we shall isolate all the partial feedback mechanisms that are contained in the 18D core case in order to discuss their stabilising or destabilising potential from a theoretical point of view. This will add extra insights to the numerical investigations already carried out and will often allow us to predict how the full 18D model behaves when some of these feedback mechanisms become more pronounced; see Chiarella and Flaschel (1999b) for details. In this way we will arrive at a method of understanding large theoretical (but small applied) macrodynamical models that is quite new, since these models have rarely been studied in the literature from the theoretical perspective, although Barnett and He (1999) is one important exception. It is our opinion that there is an urgent need for similar investigations of the dynamical features of applied or applicable integrated macrodynamical models and that tools are now indeed available for the achievement of progress at this frontier. We expect that one outcome of this analysis will be that applied structural disequilibrium models of monetary growth will exhibit a rich menu of attractors (points, limit cycles, quasi periodic orbits and also more complex ones) and also interesting transient behaviour towards such attractors that will severely question the narrow, but still prevalent, view of only steady state attractors as far as the deterministic part of published macroeconometric models is concerned. In our view this understanding will drastically change the way such models are conceived and utilised in theory as well as in applications in the future.

6 Intensive form and steady state calculations

6.1 Introduction

In this chapter we derive and investigate the 34D intensive (state variable) form of the applied structural model of disequilibrium growth we have introduced and discussed in its originally extensive form level in great detail in Chiarella and Flaschel (1999b) and in the preceding chapter. We will represent the resulting 34 dimensional dynamical system from various perspectives, providing compact intensive form representations of the real and the financial sector of this economy in tabular form and also in the form of a system of national accounts. We will then discuss to some extent the economic content of the resulting laws of motion from their intensive form perspective, thereby showing that the model can be understood from the outset on the intensive form level.

Presenting the system from these various perspectives serves the purpose of making the reader acquainted with the notation and the relationships that apply on the intensive form level of the model. We hope that this approach will increase the readability of the laws of motion for quantities (including rates of growth), for prices (including wages, asset prices and also expectations), financial asset accumulation and feedback fiscal and monetary policy rules to be presented and discussed in Section 6.3. Section 6.4 then calculates the (up to the determination of nominal variables) uniquely determined steady state solution of this dynamical system and briefly considers its comparative dynamic properties which are generally very simple in nature. We then go on and show that the dimension of the dynamics can be significantly decreased by only a few simplifying assumptions (leading us from 34D to 18D dynamics) whereby we obtain what we will call the 18D core dynamics of our approach to disequilibrium growth.

We shall briefly compare these dynamics in Section 6.6 with the fourteen equations second order system of Bergstrom *et al.* (1994), a prominent example from the literature on continuous-time macroeconometric model building and testing.[1] We then use an approach similar to the one by Barnett and He (1999), who reconsider the fourteen-equation model just mentioned from the numerical perspective, in order to study the numerical properties of our 18D core dynamics in particular with respect to the role

[1] See also Bergstrom and Nowman (2007) for a survey on this literature.

played by speeds of price and quantity adjustment. In the present chapter, however, we shall for the time being use only eigenvalue calculations based on one-parameter changes in order to see which adjustment speeds (and their corresponding feedback chains) are stabilising in the full 18D dynamics, and which are destabilising when they are increased. In future work we will also calculate, as in Barnett and He (1999), bifurcation boundaries in various two-parameter spaces (which bound the regions of local asymptotic stability of the system) and will then show that the bifurcations that occur are essentially of the Hopf type (which at present is only a conjecture based on earlier work on such disequilibrium growth dynamics).

6.2 The real and the financial structure on the intensive form level

Tables 6.1 and 6.2 provide a survey of the structure of the economy to be investigated in the following and they do so on the basis of what has been presented and discussed in Chiarella and Flaschel (1999b) with respect to the extensive structural form of a general disequilibrium monetary growth model by transferring this discussion to the intensive form level and related steady state calculations.[2] This chapter therefore continues the analysis begun in Chiarella and Flaschel (1999b) by showing that this model type has a well-defined intensive form state variable representation and also a basically (up to the level of nominal variables) uniquely determined interior steady state or balanced growth path solution.

6.2.1 The real part of the economy

Let us start with a presentation of the variables that comprise the real part of the economy to be considered which, as already stated, are all recalculated here in per unit of capital form as far as the side of quantities is concerned, plus in efficiency units in the case of labour, and also in efficiency units in the case of wage rates, since these variables also would exhibit a positive trend otherwise (since they rise with labour productivity on average). Price levels, however, are at present without trend in the considered model, since it is assumed that the central bank follows an interest rate policy rule with a zero target rate of inflation, which restricts the steady state solution of the dynamics to zero.

Table 6.1 describes the real sector of the considered economy. We have a labour market, three commodity markets and the housing market. Domestic production $y = Y/K$, per unit of capital, concerns one good that is only domestically used (for all private consumption $c_w + c_c$, all investment g_k^d, g_h^d, $\mathcal{I}/K$, also into housing, and all government consumption $g = G/K$ and which uses up all the imports j^d as intermediate goods) and one good that is only used for exports x. There is thus only a single commodity used in domestic absorption – up to the housing services c_h^d demanded by workers. We denote the demand for this domestically produced and absorbed commodity by $y^d (= Y^d/K)$.

[2] In order to clarify the notation used and the contents it represents the reader should therefore utilise this original presentation of the model.

Table 6.1. *The real part of the economy*

	Labour	Non-traded Goods	Exports	Imports	Dwellings
Workers	$l^e = \alpha_l l_1^e$	c_g^o	–	–	c_h^o
Asset holders	–	g_h^d	–	–	c_h^s, g_h^d
Firms	l_f^{de}, l_f^{we}	$y^p, y, g_k^d, \mathcal{I}/K$	x	j^d	–
Government	$l_g^{de} = l_g^{dw}$	g	–	–	–
Prices	$w^e, w^{re}, w^{be}, w^{ue}$	$p_v = (1+\tau_v)p_y$	$p_x = sp_x^*$	$p_m = (1+\tau_m)sp_m^*$	p_h, p_y
Expectations	$\pi^e = \hat{p}_v^e$	$\pi^e = \hat{p}_v^e$	–	–	$\pi^e = \hat{p}_v^e$
Stocks	l_1^e	ν=N/K	–	–	k_h
Growth	n	$\hat{K} = g_k^d - \delta_k$	–	–	$\hat{K}_h = g_h^d - \delta_h$
		$\hat{N} = (y - y^d)/\nu$			

Our model exhibits three domestic sectors: households, firms and the government, but with heterogeneous agents in the household sector, workers and (pure) asset holders, the former supplying their labour l^e (measured here in efficiency units) at the gross wage level w^{be} (which includes payroll taxes) and the latter the housing services c_h^s for the workers. Firms produce a non-traded domestic and an exported commodity and employ labour l_f^{we} (with varying rates of utilisation l_f^{de}) and imports j^d (besides their capital stock K) for these purposes, and invest into fixed business capital g_k^d (per unit of capital) and inventories $\mathcal{I}/K$. Government finally provides public consumption goods g, pays rents w^{re} and unemployment benefits w^{ue} and also employs part of the workforce l_g^{de}. There is endogenous growth n of the potential labour force L_1, of the capital stock K, by $g_k^d - \delta_k$ and of the stock of housing K_h, by $g_h^d - \delta_h$ (supplied at price p_h for rental services) and also actual change of inventories $\nu = N/K$ that is different from their desired rate of change $\mathcal{I}/K$.

6.2.2 *The financial part of the economy*

Let us next consider the financial part of the economy. Note that all stock variables B, B_w, B_c, B^l, B_1^l, B_2^l, E (and their rates of change) appearing here are measured relative to the gross value of the capital stock p_vK based on prices p_v that include value-added tax. They are then denoted by lower case Latin letters (and by ε in the case of equities E).

The first column in Table 6.2 shows that we do not consider money holdings in the model of this chapter; see Chiarella and Flaschel (1999b) for details. At present there are only (four) interest-bearing financial assets in our model that can be held by the (pure) asset owners and by the workers of our economy (as shown in Table 6.2). As in the Keynes–Metzler model of monetary growth of closed and open economies – see Chiarella and Flaschel (1999a,b,c,d, 2000) – we here assume, in order to start with a

Table 6.2. *The financial part of the economy*

	Short-term Bonds	Long-term Bonds	Equities	Foreign Bonds	
Workers	$\dot{B}_w/(p_vK) = \hat{B}_w b_w$	–	–	–	
Asset holders	$\dot{B}_c/(p_vK) = \hat{B}_c b_c$	$\dot{B}_1^l/(p_vK)$	$\dot{E}/(p_vK)$	$\dot{B}_2^l/(p_vK)$	
Firms	–	–	–	$\dot{E}/(p_vK)$	–
Government	$\dot{B}/(p_vK) = \hat{B}b$	$\dot{B}^l/(p_vK)$	–	–	
Prices	1 [i]	$p_b = 1/i_l$	p_e	$sp_b^* = s \cdot 1/i_l^*$	
Expectations	–	$\pi_b = \hat{p}_b^e$	$\pi_e = \hat{p}_e^e$	$\epsilon_s = \hat{s}^e$	
Stocks	$b = B/(p_vK)$	$b^l = B^l/(p_vK)$,	$\varepsilon = E/(p_vK)$	$b_2^l = B_2^l/(p_vK)$	
		$b_1^l = B_1^l/(p_vK)$			
Growth	$\hat{B}$	$\hat{B}^l, \hat{B}_1^l$	$\hat{E}$	$\hat{B}_2^l$	

simple representation of financial flows, that bonds are only issued by the government, that firms use only equity financing and pay out expected earnings as dividends, and that there exist also long-term bonds issued by the 'foreign government'. Financial flows between the sectors of our economy are therefore very narrowly defined. Note that we allow for savings out of wages in the present model (in a Kaldorian way) and that workers save only in the form of short-term debt (interest-bearing saving deposits[3] held at the local branches of the central bank).[4] All other assets (plus the remainder of short-term debt) are exclusively held by the (pure) asset holders of our model. We stress that this formulation has served the purpose of simplifying the budget constraints of the agents in Chiarella and Flaschel (1999b), but should be extended in future developments of the model.

This is the basic structure we assume for our economy which will be further explained in the next section from the viewpoint of national accounting before we present and discuss the intensive form of the model of Chiarella and Flaschel (1999b).

6.3 The implied 34D dynamics

In order to study the dynamics of our stylised disequilibrium growth model analytically and numerically it is necessary to reduce the equations of the model presented in Chiarella and Flaschel (1999b) to intensive or per (value) unit of capital form. This has already been indicated and discussed in the preceding section from the viewpoint of the system of national accounts by dividing all (nominal) level magnitudes through (the value of) the capital stock K of firms (measured at consumer prices p_v) and by taking note of the fact that the model exhibits Harrod neutral technological change

3 Or fixed-price bonds, which are perfectly liquid, while the other type of bonds, long-term bonds (here consols or perpetuities, held by asset holders), cannot be redeemed at a given price from the viewpoint of the sector of asset holders as a whole.

4 For the purpose of financing government expenditures with no explicit reserve requirements.

which means that all variables involving labour must be measured in efficiency units, so they are to be multiplied by the term $\exp(n_l t)$ in order to remove the trend in labour productivity from them. Note that this procedure must also be applied to the nominal wage w which is to be replaced by the term $w^e = w/\exp(n_l t)$ since nominal wages w will rise with labour productivity in the steady state and must therefore be detrended and replaced by the wage rate per efficiency unit of labour in order to get a variable that in principle allows for stationarity ($\hat{w}^e = \hat{w} - n_l$).

Note here again that the model has been formulated in a way that implies zero price inflation in the steady state if it is assumed that the target inflation rate of the central bank, $\bar{\pi}$, equals zero – an assumption that will be made for the remainder of the chapter. The variables such as w^e, p_y, p_h therefore need not be detrended any further, but represent state variables of the dynamical system to be formulated below. The first two laws of motion of these state variables are easily obtained from module 5b of Chiarella and Flaschel (1999b) by inserting there the definitions for $u_f^w = l_f^{de}/l_f^{we}, u = y/y^p$ in particular and by replacing level variables by their intensive form measures. In the same way we obtain the dynamical laws for long-run inflationary expectations π^c by making use of $\hat{Y}^p = \hat{K}$, and also the law of motion for the price of dwelling services p_h, see module 2b of Chiarella and Flaschel (1999b).

Note finally that the magnitude $c_c = C_c/K$, the consumption of asset holders of the domestic good per unit of capital, is a given magnitude in the steady state of the model (but not off the steady state), due to the assumption $\hat{C}_c = \gamma$ made in Chiarella and Flaschel (1999b).

6.3.1 The laws of motion

Let us start our presentation of the model in intensive form by first considering the quantity dynamics it implies,[5] which are given by

1. The Quantity Dynamics (seven laws of motion):

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + (\gamma - (g_k - \delta_k))y^e, \tag{6.1}$$

$$\dot{\nu} = y - y^d - (g_k - \delta_k)\nu, \tag{6.2}$$

$$\hat{c}_w = \alpha_1^w(c_w^o/c_w - 1) + \alpha_2^w(e - \bar{e}) + \gamma - (g_k - \delta_k), \tag{6.3}$$

$$\hat{c}_h^d = \alpha_r^h(c_h^{do}/c_h^d - 1) + \alpha_i^h(e - \bar{e}) + \gamma - (g_k - \delta_k), \tag{6.4}$$

$$\hat{c}_c = \gamma - (g_k - \delta_k), \tag{6.5}$$

$$\hat{l}_f^{we} = \beta_l(u_f^w - \bar{u}_f^w) + \gamma - (g_k - \delta_k), \tag{6.6}$$

$$\dot{\bar{e}} = \beta_{\bar{e}}(e - \bar{e}). \tag{6.7}$$

These formulae are obtained from the extensive form presented in Chiarella and Flaschel (1999b) by the usual growth rate formula for intensive expressions, for example

[5] These dynamics, as well as the growth dynamics, will be considerably more complicated if substitution is allowed for in the production possibilities of firms: see Chiarella *et al.* (1999a,b) on this matter.

$\widehat{Y^e/K} = \hat{Y}^e - \hat{K}$ with $\hat{K} = g_k - \delta_k$ by reformulating such expression in terms of time derivatives whenever necessary. The dynamical laws for quantities describe sales expectations dynamics, actual inventory dynamics, three types of consumption demand dynamics for workers (domestic goods and housing services) and asset holders (domestic goods including houses), the dynamic employment policy of firms and finally the dynamics of the NAIRU rate of employment.

Next we describe the price dynamics as far as real markets are concerned:

2. Wage/Price Dynamics (four laws of motion):

$$\hat{w}^e = \beta_{w_e}(e - \bar{e}) + \beta_{w_u}(u_f^w - \bar{u}_f^w), \tag{6.8}$$
$$+ \kappa_w(\hat{p}_y + n_l) + (1 - \kappa_w)(\pi^c + \tilde{n}_l) - n_l,$$

$$\hat{p}_y = \beta_p(y/y^p - \bar{u}) + \kappa_p\hat{w}^e + (1 - \kappa_p)\pi^c, \tag{6.9}$$

$$\dot{\pi}^e = \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p}_y - \pi^c) + (1 - \alpha_{\pi^c})(0 - \pi^c)), \tag{6.10}$$

$$\hat{p}_h = \beta_h\left(\frac{c_h^d}{k_h} - \bar{u}_h\right) + \kappa_h\hat{p}_y + (1 - \kappa_h)\pi^c. \tag{6.11}$$

These equations for wage and price dynamics (including medium-run inflationary expectations) and rental price dynamics are straightforward consequences of the laws of motion as they were formulated in Chiarella and Flaschel (1999b).

We have next the dynamics of asset prices, expectations about the dynamics and the dynamics of certain long-run concepts of interest and profits. Note that we here make use of Tobin's $q = \frac{p_e E}{p_y K}$ as an aggregate expression for the joint dynamics of equity prices p_e and the number of equities E. Given the formulation of the model in Chiarella and Flaschel (1999b) it suffices to describe the dynamics of q in the intensive form which moreover, due to the lack of wealth effects and the like, does not feed back into the rest of the dynamics. Note however that the expression for $\hat{p}_e$ from Chiarella and Flaschel (1999b),

$$\hat{p}_e = \frac{\beta_{p_e}}{1 - \beta_{p_e}(1 - \alpha_s)}[(1 - \tau_c)r^e/q + \alpha_s\pi_{es} - ((1 - \tau_c)i_l + \pi_b)], \tag{6.12}$$

with aggregate expectations π_b being determined by $\alpha_s\pi_{bs} + (1 - \alpha_s)\hat{p}_b$, has to be inserted into the law of motion for Tobin's q in order to get a description of these dynamics that is complete. Due to the isolated nature of these dynamics it is however not necessary here to go into more detail.

3. Asset Prices and Expectations (eight laws of motion):

$$\hat{p}_b = \frac{\beta_{p_b}}{1 - \beta_{p_b}(1 - \alpha_s)}[(1 - \tau_c)i_l + \alpha_s\pi_{bs} - ((1 - \tau_c)i + \xi)], \quad (p_b = 1/i_l), \tag{6.13}$$

$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}), \tag{6.14}$$

$$\hat{q} = \hat{p}_e - \hat{p}_y + \frac{g_k - \delta_k + y - y^d - (\beta_n(\beta_{n^d} y^e - \nu) + \gamma \beta_{n^d} y^e)}{q} - (g_k - \delta_k), \tag{6.15}$$

$$\dot{\pi}_{es} = \beta_{\pi_{es}}(\hat{p}_e - \pi_{es}), \tag{6.16}$$

$$\hat{s} = \frac{\beta_s}{1 - \beta_s(1 - \alpha_s)}[(1 - \tau_c^*)i_l^* + \alpha_s \epsilon_s - ((1 - \tau_c)i_l + \pi_b)], \tag{6.17}$$

$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s), \tag{6.18}$$

$$\dot{r}^l = \beta_{r_l}(r^e - r^l), \tag{6.19}$$

$$\dot{r}_h^l = \beta_{r_h^l}(r_h - r_h^l). \tag{6.20}$$

We have first the law of motion for the price of long-term bonds, $p_b = 1/i_l$, which is here expressed in terms of the interest rate that these bond prices (consols) represent. This interest rate adjusts in the direction of the risk free interest rate on short-term bonds after taxes $(1 - \tau_c)i$, augmented by a risk and liquidity premium ξ for long-term bonds. Note that we have removed the perfect foresight expectations from the left side of this adjustment equation which, as shown in Chiarella and Flaschel (1999b), gives rise to the fraction in front of the shown formula. Note furthermore that only the law of motion of 'less ambitious' expectations is then needed in order to make this substructure determinate, but that aggregate expectations on these bond price changes π_b are needed in the subsequent laws of motion of asset prices.

Making use of the formula for the rate of change of equity prices π_e expected on average we can transform the law of motion for equity prices just as the law of motion for long-term bond prices and also remove the explicit representation of 'ambitious' agents, as shown in the representation of the p_e dynamics. This law is again to be supplemented by the law of motion for the expectations of 'less ambitious' agents.

The next law of motion concerns Tobin's q which, as already shown, is measured by the ratio between the value of equity stock and the producer price of the existing capital stock, that is $q = \frac{p_e E}{p_y K}$. We have $\hat{q} = \hat{p}_e - \hat{p}_y + \hat{E} - \hat{K}$ where the first two inflation rates have already been determined in equations (6.12) and (6.9). For the remaining expression $\widehat{E/K} = \hat{E} - \hat{K}$ we have

$$\widehat{E/K} = \hat{E} - \hat{K} = \frac{p_e \dot{E}}{p_y K} \cdot \frac{p_y K}{p_e E} - \hat{K}$$
$$= [g_k - \delta_k + y - y^d - (\beta_n(\beta_{n^d} y^e - \nu) + \gamma \beta_{n^d} y^e)]/q - (g_k - \delta_k),$$

which yields the law of motion for Tobin's q shown in module 3. Note again that the expression for $\hat{p}^e$ can be inserted into the q dynamics and thus gives rise to one law of motion in Tobin's q solely.

The method used to describe the dynamics of p_e also applies to the law of motion for the exchange rate s by removing again the correct expectations of the 'ambitious' agents from its right-hand side after having inserted the expression for average expectations $\epsilon = \alpha_s \epsilon_s + (1 - \alpha_s)\hat{s}$ into this formula. Again this is to be supplemented by the law that describes the evolution of the expectations of 'less ambitious' agents in the postulated adaptive way.

There follow the two laws of motion for expected long-run profitability, r^l, which is used in the investment equation for the capital stock, and r_h^l, which is used in the investment equation for the capital stock in housing. Both of these measures follow their short-run equivalents with some time delay.

Next we consider the laws of growth that apply to the economy under consideration:

4. Growth Dynamics (six laws of motion):

$$\dot{n} = \beta_{n_w}(\widetilde{n} - n), \quad (\widetilde{n} = \widetilde{n}(e, \hat{e})), \tag{6.21}$$

$$\hat{l}^e = n + n_l - (g_k - \delta_k), \tag{6.22}$$

$$\dot{g}_k = \beta_{g_k}(g_k^d - g_k), \tag{6.23}$$

$$\hat{k}_h = g_h - \delta_h - (g_k - \delta_k), \tag{6.24}$$

$$\dot{g}_h = \beta_{g_h}(g_h^d - g_h), \tag{6.25}$$

$$\dot{n}_l = \beta_{n_l}(\widetilde{n}_l - n_l), \quad \widetilde{n}_l = (\widetilde{n}_l(g_k)). \tag{6.26}$$

The growth equations represent the time rate of change of the so-called natural rate of growth, the law of motion for the full employment labour intensity (in efficiency units), the time rate of change of gross investment per unit of capital (also for the housing sector), the growth rate of the relative magnitude of the stock of houses to the capital stock employed by firms and finally the time rate of change of the rate of Harrod neutral technological change. There is no further comment needed with respect to the above presentations of the growth laws of the economy which again use the formula $\hat{K} = g_k - \delta_k$ in the formulation of intensive expressions.

Next the dynamic feedback rules for government behaviour are collected. These concern the steering of the short-term nominal rate of interest by the central bank, the dynamic wage taxation rule based in reaction to the evolution of government debt d and the motion of the tax rate on imports, which is here used to establish a balanced trade account in the steady state.

5. Monetary and Fiscal Policy Rules (three laws of motion):

$$\dot{i} = -\beta_{i_i}(i + \xi - i_l^*) + \beta_{i_p}(\hat{p}_y - \bar{\pi}) + \beta_{i_u}(y/y^p - \bar{u}), \quad (\bar{\pi} = 0), \tag{6.27}$$

$$\hat{\tau}_w = \alpha_{\tau_{w1}}(d/\bar{d} - 1) + \alpha_{\tau_{w2}}\hat{d}, \qquad \left(d = \frac{b + b^l/i_l}{y^e}\right), \tag{6.28}$$

$$\hat{\tau}_m = \alpha_{\tau_m}\frac{p_x x - p_m j^d}{p_x x}. \tag{6.29}$$

Note with respect to module 5 that the aggregate accumulation of government bonds cannot be divorced from the real sector (even though wealth effects are not yet included in the model) due to the assumed wage income taxation rule whereby the government attempts to steer a certain ratio for government debt to a desired ratio $\bar{d}$.

There remain the dynamics of aggregate and individual asset holdings which represent the most involved block in our dynamical system. We have already stressed that the individual allocation of government bonds (between workers and asset holders and also throughout the world) does not feed back into the remaining dynamics, since only total government bonds matter in the present context due to the absence of wealth effects in consumption and due to the independence of consumption of workers and asset holders from their interest income. In the current version of the dynamics only the laws of motion for b and b^l feed back into the real part of the dynamics via the wage tax collection rule of the government.

6. Assets Dynamics (six laws of motion):

$$\begin{aligned}
\dot{b} &= \alpha_b^g[gy^e + ib + b^l \\
&\quad - t^n + (w^{ue}/p_v)(l^e - l^{we}) + (w^{re}/p_v)\alpha_l l_2^e + (w^{be}/p_v)l_g^{de}] \\
&\quad - (\hat{p}_y + g_k - \delta_k)b, \qquad (6.30)\\
\dot{b}^l &= i_l(1-\alpha_b^g)[gy^e + ib + b^l \\
&\quad - t^n + (w^{ue}/p_v)(l^e - l^{we}) + (w^{re}/p_v)\alpha_l l_2^e + (w^{be}/p_v)l_g^{de}] \\
&\quad - (\hat{p}_y + g_k - \delta_k)b^l, \qquad (6.31)\\
\dot{b}_w &= y_w^D - c_w - \frac{p_h}{p_v}c_h^d - (\hat{p}_y + g_k - \delta_k)b_w, \qquad (\dot{b}_c = \dot{b} - \dot{b}_w), \qquad (6.32)\\
\dot{b}_1^l &= i_l\alpha_{b1}^g(1-\alpha_b^g)[gy^e + ib + b^l \\
&\quad - t^n + (w^{ue}/p_v)(l^e - l^{we}) + (w^{re}/p_v)\alpha_l l_2^e + (w^{be}/p_v)l_g^{de}] \\
&\quad - (\hat{p}_y + g_k - \delta_k)b_1^l, \qquad (6.33)\\
\dot{b}_1^{l*} &= i_l(1-\alpha_{b1}^g)(1-\alpha_b^g)[gy^e + ib + b^l \\
&\quad - t^n + (w^{ue}/p_v)(l^e - l^{we}) + (w^{re}/p_v)\alpha_l l_2^e + (w^{be}/p_v)l_g^{de}] \\
&\quad - (\hat{p}_y + g_k - \delta_k)b_1^{l*}, \qquad (6.34)\\
\dot{b}_2^l &= \dot{B}_2^l/(p_vK) - (\hat{p}_y + g_k - \delta_k)b_2^l, \qquad (6.35)
\end{aligned}$$

where we have

$$\begin{aligned}
\frac{\dot{B}_2^l}{p_vK} &= \frac{i_l^*}{s}[y_c^D - c_c - \frac{\dot{B}_c}{p_vK} - \frac{\dot{B}_1^l/i_l}{p_vK} - \frac{p_e\dot{E}}{p_vK} - \frac{p_y}{p_v}(g_h - \delta_h)k_h] \\
&= \frac{i_l^*}{s}\{y_c^D - c_c - (\alpha_b^g + (1-\alpha_b^g)\alpha_{b1}^g)[gy^e + ib + b^l - t^n
\end{aligned}$$

$$\begin{aligned}
&+ (w^{ue}/p_v)(l^e - l^{we}) + (w^{re}/p_v)\alpha_l l_2^e + (w^{be}/p_v)l_g^{de}] \\
&- (p_y/p_v)[g_k - \delta_k + y - y^d - (\beta_n(\beta_{n^d}y^e - \nu) + \gamma\beta_{n^d}y^e)] \\
&- (p_y/p_v)(g_h - \delta_h)k_h)\}
\end{aligned}$$

according to the flow budget constraint of asset holders.

This part of the dynamics is to some extent missing in the Murphy model; see Powell and Murphy (1997), with which we compared our model in detail in Chiarella and Flaschel (1999b), due to the lack of a complete treatment of the budget equations of the three sectors that form the basis of this model. As indicated above the laws of motion of the individual assets that are held by the household sector in our model however do not feed back into the rest of the dynamics, since they do not show up in the real part of the economy. Therefore, only the two laws of motion for short- and long-term government debt are really needed at present in the discussion of the growth pattern and the fluctuations that may occur around them which are implied by the disequilibrium growth model under consideration.

Note with respect to the right-hand sides of these stock accumulation equations that they are based on a fixed ratio α_b^g describing the allocation between the short- and long-term financing of government debt done by the government (as in the Murphy model) and that the term in square brackets, the government budget equation

$$gy^e + ib + b^l - t^n + (w^{ue}/p_v)(l^e - l^{we}) + (w^{re}/p_v)\alpha_l l_2^e + (w^{be}/p_v)l_g^{de},$$

in both cases represents the sum of government expenditure for goods, labour, interest and transfer payments to the unemployed and retirees minus t^n, the sum of all taxes that are raised by the government (per value unit of capital).[6]

The total amount of debt financing is thus represented through the expression shown above, leading to $\dot{b} + \dot{b}_l/i_l$, and it is split via the weights $\alpha_b^g, 1-\alpha_b^g$, applied to the Government Budget Restraint (GBR) in the above intensive form, into short-term financing $\dot{b}$ (equation (6.30)) and long-term financing $\dot{b}_l/i_l$ (equation (6.31)). Note that $(\hat{p}_y + g_k - \delta_k)b$ and $(\hat{p}_y + g_k - \delta_k)b^l$ have to be deducted from the resulting expression, due to the fact that these bond variables are in intensive form and are thus divided by p_vK.

The dynamic equation (6.32) for real savings of workers per unit of capital, $b_w = B_w/(p_vK)$, follows from the definition of this expression and from the definition of the disposable income of worker households and their saving plans, while the law of motion for short-term debt of asset holders per capital, $b_c = B_c/(p_vK)$, is a simple consequence of the two laws of motion assumed for the expressions b and b_w.

The next two dynamic laws (6.33) and (6.34) for the distribution of new domestic long-term debt throughout the world basically follow again from the budget restriction of the government shown above (expressed in intensive form), since there is also a

[6] Note that all wage concepts in the above intensive form of the GBR are in efficiency units (to allow for stationarity) and are deflated by consumer prices p_v.

fixed proportion α^g_{b1} assumed to apply with respect to the distribution of long-term debt between domestic and foreign asset holders. The expressions for the proportions of long-term financing that go to domestic and foreign residents, $\dot{b}^l_1, b^l_1 = B^l_1/(p_v K)$ and $\dot{b}^{l*}_1, b^{l*}_1 = B^{l*}_1/(p_v K)$, are thus obtained by applying the weights $\alpha^g_{b1}, 1-\alpha^g_{b1}$ to $\dot{b}^l$ and again noting the fact that now $(\hat{p}_y + g_k - \delta_k)b^l_1$ and $(\hat{p}_y + g_k - \delta_k)b^{l*}_1$ have to be deducted from the resulting expressions (due to the intensive form formulation) in the place of the former expression $(\hat{p}_y + g_k - \delta_k)b^l$.

There remains the law of motion of foreign assets held by domestic residents which, due to the definition of the intensive variable $b^l_2 = B^l_2/(p_v K)$, basically demands the determination of the variable $\dot{B}^l_2/(p_v K)$ in terms of intensive expressions. This task is solved residually by referring to the fact that $\dot{B}^l_2$ is given by the disposable income of asset holders minus their consumption minus all other asset accumulation that these households undertake. This provides the last law of motion shown in the above block of asset accumulation equations, which is thus purely residual in nature.

6.3.2 *Static relationships*

There is a variety of definitions and static relationships used in the above collection of the laws of motion of our disequilibrium model of monetary growth. These abbreviations are collected in the next six blocks of the intensive presentation of the model and are generally immediate consequences of the corresponding equations in extensive form presented in Chiarella and Flaschel (1999b).

7. Output and Demand on the Market for Goods (including Housing Services):

$$y^d = c_w + c_c + g_k + g_h k_h + g y^e, \tag{6.36}$$

$$y = y^e + \beta_n(\beta_{n^d} y^e - \nu) + \gamma\beta_{n^d} y^e, \tag{6.37}$$

$$x = x_y y, \tag{6.38}$$

$$j^d = j_y y, \tag{6.39}$$

$$c^o_w = c_y(1-\tau_w)[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l^e_2]/p_v, \tag{6.40}$$

$$c^{do}_h = c_h(1-\tau_w)[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l^e_2]/p_h. \tag{6.41}$$

Note with respect to block 7 of the static equations that all variables are obtained here by dividing the extensive expressions by K, giving rise for example to $k_h = K_h/K$. The same procedure applies to block 8 of the algebraic equations underlying our dynamic model:

8. Employment, Labour Supply and Retirees:

$$l^{de}_f = l_y y, \quad \text{(see block 1 for the law of motion for } l^{we}_f), \tag{6.42}$$

$$l^{de}_g = l^{we}_g = \alpha_g g y^e, \tag{6.43}$$

$$l^{de} = l^{de}_f + l^{de}_g, \quad (u^w_f = l^{de}_f/l^e), \tag{6.44}$$

$$l^{we} = l^{we}_f + l^{we}_g, \quad (e = l^{we}/l^e), \tag{6.45}$$

$$l^e_2 = (L_2(0)/L_1(0))l^e/\alpha_l. \tag{6.46}$$

Note again that all magnitudes concerning labour inputs and supply are expressed in efficiency units – due to the technological condition $L^d_f = l_y \exp(-n_l t)Y$ that represents Harrod neutral technical change. When multiplied with wages measured in efficiency units the term $\exp(-n_l t)$ just cancels from the resulting expression and gives the corresponding wage payments in both efficiency units and original levels. Note, finally, that we need initial conditions in order to relate the sizes of the intensive expression for potential labour supply and retirees.

9. Desired Growth Rates of the Capital Stocks (Firms/Dwellings):

$$g^d_k = \alpha^k_1((1-\tau_c)r^e - i^r) + \alpha^k_2(i_l - (i+\xi)) + \alpha^k_3(y/y^p - \bar{u}) + \gamma + \delta_k, \tag{6.47}$$

$$g^d_h = \alpha^h_r((1-\tau_c)r^l_h - i^r) + \alpha^h_i(i_l - (i+\xi)) + \alpha^h_u(\frac{c^d_h}{k_h} - \bar{u}_h) + \gamma + \delta_h. \tag{6.48}$$

The growth rates of the capital stocks for fixed investment of firms and housing investment of asset holders (here shown as gross rates) are an immediate consequence of their original formulation in Chiarella and Flaschel (1999b), and simply state that these ratios are assumed to be influenced by long-run profitability measures, by the interest rate spread and by the rate of capacity utilisation. The same immediate correspondence to what has been introduced in Chiarella and Flaschel (1999b) holds true for the following definitions of rates of return needed for the dynamical laws or just for the discussion of the steady state of the model.

10. Rates of Return on Real and Financial Assets:

$$r^e = y^e - \delta_k + (p_x/p_y)x - (w^{be}/p_y)l^{de}_f - (p_m/p_y)j^d, \tag{6.49}$$

$$y^{dp} = \frac{\bar{u}y^p}{1+\gamma\beta_{n^d}}, \quad (y^n = \bar{u}y^p),$$

$$r^n = y^{dp} - \delta_k + (p_x/p_y)x_y y^n - w^{be} l_y y^n - (p_m/p_y)j_y y^n, \tag{6.50}$$

$$i^r = (1-\tau_c)i_l - \pi^c, \tag{6.51}$$

$$r_h = (p_h/p_y)c^d_h/k_h - \delta_h. \tag{6.52}$$

The following definitions of the various prices we use in our model in addition to the prices that appear as state variables in the dynamics are also an immediate consequence of their definitions in the extensive form of the model in Chiarella and Flaschel (1999b) when account is again taken of the fact that wages have to be expressed in efficiency units in order to ensure their stationarity in the steady state: $w^e = w/\exp(n_l t)$.

11. Consumer Prices, Gross Wages and Transfers:

$$p_x = sp_x^*, \tag{6.53}$$

$$p_m = (1+\tau_m)sp_m^*, \tag{6.54}$$

$$w^{be} = (1+\tau_p)w^e, \tag{6.55}$$

$$w^{ue} = \alpha^u w^e, \tag{6.56}$$

$$w^{re} = \alpha^r w^e, \tag{6.57}$$

$$p_v = (1+\tau_v)p_y. \tag{6.58}$$

Finally the short-cut expressions for the intensive forms of disposable incomes and taxes are easily obtained from their extensive form analogues and are of the form shown below. Note that the expressions for disposable income of workers and asset holders are only needed in their bond accumulation equations and, as the model is currently formulated, are not yet involved in the consumption decisions of the two types of households considered.

12. Disposable Incomes and Government Taxes per Value Unit of Capital:

$$y_w^D = (1-\tau_w)[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l_2^e]/p_v + (1-\tau_c)ib_w, \tag{6.59}$$

$$y_{w1}^D = (1-\tau_w)[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l_2^e]/p_v, \tag{6.60}$$

$$\begin{aligned} y_c^D &= (1-\tau_c)[r^e(p_y/p_v) + ib_c + b_1^l + (p_h/p_v)c_h^d - (p_y/p_v)\delta_h k_h] \\ &\quad + s(1-\tau_c)b_2^l, \end{aligned} \tag{6.61}$$

$$\begin{aligned} t^n &= \tau_w[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l_2^e]/p_v + \tau_p(w^e/p_v)l^{de} \\ &\quad + \tau_v(y^d - g_k - g_h k_h)(p_y/p_v) \\ &\quad + \tau_c[r^e(p_y/p_v) + ib + b^l + (p_h/p_v)r_h k_h] + \tau_m s(p_m^*/p_v)j^d. \end{aligned} \tag{6.62}$$

In closing the discussion of the intensive form of our disequilibrium model of monetary growth we stress again that the law of motion for equity prices p_e is not needed, since it can be substituted into the law of motion for Tobin's q. It can thus be removed from explicit representation, since there is here no feedback from Tobin's q on the real part of the economy, due to the lack of wealth effects in the current version of the model.

6.4 Steady state analysis

In this section we show that there is, up to the level of nominal variables, a uniquely determined economically meaningful balanced growth path or steady state solution of our model. This steady state provides us with a useful reference path for the dynamical evolution implied by the model, which may or may not converge to this steady state solution, not even if long-run moving averages are used in the place of the temporary positions that the economy will pass through.

The calculation of this interior, economically meaningful, steady state[7] of the full model (which up to the level of nominal magnitudes for wages and goods prices is uniquely determined) is in many respects simple due to the given growth rate of the world economy and the given interest rate (on consols) abroad. Note that we only consider expressions for the total supply of domestic bonds in the following, and not their distribution at home and abroad which can be easily obtained from the savings decisions of workers and pure asset owners.

To simplify subsequent presentations of the dynamics of the model, and also its steady state solution, we assume in the remainder of this chapter for the consumption of asset owners that $C_c = 0$ and for the liquidity premium applied to long-term debt that $\xi = 0$. These two assumptions do not restrict the dynamical behaviour of the system in any significant way.

The first set of steady state conditions presented below concerns the growth rates of our small open economy, and are given by

$$\gamma = g_k - \delta_k = g_k^d - \delta_k \quad \Rightarrow g_k = g_k^d = \gamma + \delta_k, \tag{6.63}$$

$$\gamma = g_h - \delta_h = g_h^d - \delta_h \quad \Rightarrow g_h = g_h^d = \gamma + \delta_h, \tag{6.64}$$

$$\gamma = \tilde{n}(\bar{e}) + \tilde{n}_l(g_k) = \tilde{n}(\bar{e}) + \tilde{n}_l(\gamma + \delta_k) \quad \Rightarrow \bar{e}, \quad (n = \tilde{n}, n_l = \tilde{n}_l). \tag{6.65}$$

These equations state that capital (and thus also output) will grow with the external rate γ, to which also the natural rate of growth adjusts. This means that the steady state value of NAIRU rate of employment $\bar{e}(=e)$ has to adjust such that $\gamma = \tilde{n}(\bar{e}) + \tilde{n}_l(g_k)$ holds. This determines a unique NAIRE $\bar{e} = e$ for the long run of the model under suitable (but simple) assumptions on the function $\tilde{n}$.

The next set of steady state conditions concerns inflation and expected inflation (for all prices that exist in our model, except wage rates) and also the various rates of interest and profit of our model:

$$\hat{p}_v = \hat{p}_y = \hat{p}_h = \pi^c = \pi_{bs} = \pi_b = \pi_{es} = \pi_e = \epsilon_s = \epsilon = 0, \tag{6.66}$$

$$i_l^* = i_l = i^r = i = 1/p_b, \tag{6.67}$$

$$i_l^* = r^e = i_l = r^n = r_h = r_h^l. \tag{6.68}$$

These conditions state that there is no steady state inflation and thus no non-zero expectations of it, which is due to the interest rate policy rule of the central bank. Furthermore, all (expected) rates of return are equalised in the steady state and determined by the world rate of interest on long-term bonds i_l^*.

The next block concerns the steady state determination of various quantities, of the steady state ratio of government debt to aggregate demand and of the tax rate on

[7] We thus neglect as alternative steady state positions all zeros which can be obtained mathematically from the growth law formulations that our model employs. Note also the steady state depends parametrically on the initial conditions for $\frac{L_2(0)}{L_1(0)}, p_v, \frac{C_c(0)}{L_1(0)}$ that characterise the initial composition of the labour force, the initial output price level (including value-added taxes) and a relative expression for the consumption of asset owners.

imports:

$$y = y^p \bar{u}, \quad c_h^d = k_h \bar{u}_h, \tag{6.69}$$

$$l_f^{de} = l_f^{we} = l_y y, \tag{6.70}$$

$$l_g^{de} = l_g^{we} = \alpha_g g y^e, \tag{6.71}$$

$$l^{we} = l^{de} = l_f^{we} + l_g^{we}, \tag{6.72}$$

$$l^e = l^{de}/\bar{e} \quad [e = \bar{e}],\ l_1^e = l^e/\alpha_l,\ l_2^e = (L_2(0), L_1(0)) l_1^e, \tag{6.73}$$

$$x = x_y y, \quad j^d = j_y y, \tag{6.74}$$

$$d = (b + b^l/i_l^*)/y^e = \bar{d}, \tag{6.75}$$

$$\tau_m = \frac{p_x^* x - p_m^* j^d}{p_m^* j^d}. \tag{6.76}$$

These equations still depend on the steady state value of y^e, which will be given below, and they reveal certain supply side influences on the steady behaviour of our economy.

Further steady state relationships on the side of quantities are

$$y^e = \frac{y}{1 + \gamma \beta_{n^d}}, \tag{6.77}$$

$$y^d = y^e, \quad \nu = \beta_{n^d} y^e, \tag{6.78}$$

$$c_w^o = c_y y_{w1}^D, \quad c_h^{do} = c_h p_v y_{w1}^D / p_h, \tag{6.79}$$

$$y_{w1}^D = (1 - \tau_w)[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re} \alpha_l l_2^e]/p_v. \tag{6.80}$$

By using the distribution laws for government bonds we furthermore find that

$$b = \alpha_b^g \bar{d} y^e, \tag{6.81}$$

$$b^l = i_l^* (1 - \alpha_b^g) \bar{d} y^e, \tag{6.82}$$

from which the individual distribution of bonds can be derived if desired.

Next, one can determine the nominal steady state expressions on the basis of an (arbitrarily) given price level p_v. This indeterminacy of the general price level of domestically produced goods is due to the fact that the central bank has adopted an interest rate policy rule and allows for intra-day deviations from the budget constraints of the two types of households, the firms and the government and that therefore, in our present model, money is not held as cash balances by the agents of our economy.

$$p_v = \text{undetermined and thus a parameter}, \tag{6.83}$$

$$p_y = p_v/(1 + \tau_v), \tag{6.84}$$

$$w^{be} \quad via \quad i_l^* = y^e - \delta_k - \frac{w^{be}}{p_y} l_f^{de}: \quad w^{be} = \frac{y^e - \delta_k - i_l^*}{l_y y} p_y, \tag{6.85}$$

$$w^e = w^{be}/(1 + \tau_p), \ w^{ue} = \alpha^u w^e, \ w^{re} = \alpha^r w^e. \tag{6.86}$$

The equation for the determination of gross wages in efficiency units holds since we have balanced trade in the steady state (see the above), and thus no influence of the international trade in goods on the rate of profit in the steady state. Note here that net real wages ω^e (excluding payroll taxes, but before wage taxation), measured in terms of consumer prices p_v and efficiency units, are given by

$$\omega^e = \frac{w^e}{p_v} = \frac{1}{(1 + \tau_v)(1 + \tau_p)} \frac{y^e - \delta_k - i_l^*}{l_y y}$$

and thus do not depend on the arbitrarily determined nominal price level p_v. This observation also applies to all other real magnitudes implying that there are no real effects of shocks which lead to a different consumer price level p_v in the long run.

The equations $c_h^d = \bar{u}_h k_h$ and $i_l^* = p_h c_h^d/(p_y k_h) - \delta_h$, on the one hand, and $c_h y_{w1}^D = p_h c_h^d / p_v$, $y^e = c_y y_{w1}^D + \gamma + \delta_k + (\gamma + \delta_h) k_h + g y^e$, on the other hand,[8] allow us to determine the price ratio p_h/p_y and k_h, y_{w1}^D, τ_w according to

$$p_h/p_y = (i_l^* + \delta_h)/\bar{u}_h, \tag{6.87}$$

$$k_h = \frac{y^e(1 - g) - \gamma - \delta_k}{c_y(i_l^* + \delta_h)/(c_h(1 + \tau_v)) + \gamma + \delta_h}, \tag{6.88}$$

$$y_{w1}^D = p_h \bar{u}_h k_h / (c_h p_v), \tag{6.89}$$

$$\tau_w = 1 - \frac{y_{w1}^D}{[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re} \alpha_l l_2^e]/p_v}. \tag{6.90}$$

Note also that this determination of y_{w1}^D and k_h must be used above for the determination of consumption plans per unit of capital.

There remains the determination of the steady state value of the exchange rate s,[9] since the price of long-term bonds, $p_b = 1/i_l^*$, has already been determined and since

[8] Note that the last equation represents goods market equilibrium in the steady state.
[9] This in turn determines the prices p_x, p_m.

the price of equities does not matter in the core 34D dynamics of the model.[10] The calculation of the steady state exchange rate is economically complex, but mathematically quite simple. Mathematically it is provided by the implicit equations for the variables t^r, s on the basis of the expression for t^c, namely

$$0 = gy^e + ib + b^l - t^w - t^c + [w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l_2^e + w^{be} l_g^{de}]/p_v - \gamma b/\alpha_b^g,$$

$$0 = \tau_w y_{w1}^D/(1-\tau_w) + \tau_p(w^e/p_v)l^{de} + \tau_v(p_y/p_v)(y^d - g_k^d - g_h^d k_h) + \tau_m s p_m^* j^d/p_v - t^r,$$

where

$$t^c = \tau_c[r^e(p_y/p_v) + i_l^* b + b^l + (p_h/p_v)c_h^d - (p_y/p_v)\delta_h k_h] = \tau_c[(r^e + i_l^* k_h)/(1+\tau_v) + i_l^* b + b^l].$$

These equations lead to

$$t^r = gy^e + ib + b^l - t^c + [w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l_2^e + w^{be} l_g^{de}]/p_v - \gamma b/\alpha_b^g, \tag{6.91}$$

$$s = \frac{t^r - [\tau_w y_{w1}^D/(1-\tau_w) + \tau_p(w^e/p_v)l^{de} + \tau_v(p_y/p_v)(y^d - g_k^d - g_h^d k_h)]}{\tau_m p_m^* j^d/p_v}. \tag{6.92}$$

We see that the long-run rate of exchange is neither determined in the market for goods, nor through the trade balance, nor by international capital flows, but rather is a complicated expression of many parameters and steady state values of the model and is in particular heavily dependent on the form of the GBR and its components.

Note finally that the dynamics of equity prices imply that $q = 1$ must hold true in the steady state, so that $p_e E = p_y K$ and that the steady state distribution of long-term bonds can be derived from block 6 of Chiarella and Flaschel (1999b), while the steady state value of b_w (and b_c) follows by setting (6.32) equal to zero. This gives rise to[11]

$$b_w = \frac{(1 - c_y - (p_h/p_v)c_h)y_{w1}^D}{\gamma - (1-\tau_c^*)i_l^*},$$

due to the difference that exists between the definitions of y_{w1}^D and y_w^D.

Summarising, we see that the steady state of the considered economy depends heavily on the data assumed to apply to the rest of the world (generally in a fairly straightforward and simple way) and that the steady values of the wage taxation rate and in particular the exchange rate are complicated functions of the parameters and various other steady

[10] The same holds true for the disposable income of asset holders y_c^D.
[11] This solution implies by economic reasoning that $\gamma > (1-\tau_c^*)i_l^*$ should hold true in the world economy (because of the particular interest income policy of workers).

state solutions of the dynamics. Furthermore, since the steady rate of profit r^e and thus also the real wage rate $\omega^e = w^e/p_v$ (see the above expressions) are determined through the foreign rate of interest, we obtain for the real wage ω^{ne} (after taxes, at consumer prices and in efficiency units) the expression

$$\omega^{ne} = (1-\tau_w)\omega^e = \frac{1-\tau_w}{(1+\tau_v)(1+\tau_p)} \frac{y^e - \delta_k - i_l^*}{l_y y},$$

which implies that all increases in the three tax rates shown (value-added tax, payroll taxes and – endogenously determined – wage income taxes) fall on real wages of workers (measured in this chapter solely by the consumer price of domestic goods). There is however also an influence of the value-added tax on overall interest rate income per value unit of capital, besides of course a dependence of this income on the capital taxation rate τ_c.

Output levels (per unit of capital) and also steady employment (per unit of capital and measured in efficiency units) are basically determined through supply side considerations (technology and desired steady rates of capacity utilisation), but the former also depend on government expenditures g per unit of capital and thus will change with this ratio.

We stress again that the external growth rate γ determines the NAIRU rate of employment via

$$\gamma = \tilde{n}(\bar{e}) + \tilde{n}_l(\gamma + \delta_k),$$

which implies that an increase in γ will increase the NAIRU rate of employment $\bar{e}$ if $\tilde{n}' > 0, \tilde{n}_l \in (0,1)$ holds.

The above has also shown that no inflation occurs in the steady state of the 34D dynamics as far as goods and asset prices are concerned. It is of course important to also consider economies which allow for (moderate) price inflation in the steady state, in the place of stationary price levels that here prevail in the steady state. Just as rates of growth, the steady levels of interest and profit rates are here all fixed and given through the external rate of interest i_l^* so that there is no possibility that the considered country may exhibit an extraordinary level of profitability in the steady state.

6.5 The 18D core dynamics of the model

In order to get (as a starting point for our dynamical investigations of the model in subsequent chapters) a dynamical system that is, on the one hand, as close as possible in spirit to the one of the general model of this chapter and, on the other hand, also as low dimensional as possible, we consider in this section a simplified structure for our Keynesian disequilibrium dynamics of monetary growth which reduces its 34D representation to the following 18D core dynamics. This reformulation makes use of the assumptions $C_c = 0$ and $\xi = 0$ that have already been used in the preceding section (there to simplify slightly the calculation of the interior steady state of the model), and it removes furthermore certain delayed adjustment processes from the considered dynamics, which gives the dynamic model an outlook that is not too far from the theoretical models introduced and analysed in Chiarella and Flaschel (2000, Chs. 6, 7). Furthermore, the natural rate of growth, of Harrod neutral technological change, and of employment are assumed as constant in the following analysis. Finally, the parameter $\alpha_{\tau_{wl}}$ is set equal to zero, implying that there is only a proportional influence of government debt on the wage taxation rate (and no longer an additional derivative one).

6.5.1 The laws of motion

1. The Quantity Dynamics (three laws of motion):

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + (\gamma - (g_k^d - \delta_k))y^e, \tag{6.93}$$

$$\dot{\nu} = y - y^d - (g_k^d - \delta_k)\nu, \tag{6.94}$$

$$\dot{l}_f^{we} = \beta_l(l_f^{de} - \bar{u}_f^w l_f^{we}) + [\gamma - (g_k^d - \delta_k)]l_f^{we}. \tag{6.95}$$

In the quantity dynamics we have removed the adjustment equations (6.3), (6.4), (6.5) for the actual consumption of workers (for goods and housing services) and replaced these consumption plans by the desired consumption targets which are now immediately realised (without any lag). Furthermore, the NAIRU rate of employment, $\bar{e}$, is no longer considered as endogenously determined by (6.7), but is now a given parameter of the model. There remain the equations for the adjustment of sales expectations, for inventories and for the workforce employed by firms.

2. The Wage/Price Dynamics (four laws of motion):

$$\hat{w}^e = \beta_{w_e}(l^{we}/l^e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - \bar{u}_w^f) + \kappa_w\hat{p}_y + (1-\kappa_w)\pi^c, \tag{6.96}$$

$$\hat{p}_y = \beta_p(y/y^p - \bar{u}) + \kappa_p\hat{w}^e + (1-\kappa_p)\pi^c, \tag{6.97}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p}_y - \pi^c) + (1-\alpha_{\pi^c})(0 - \pi^c)), \tag{6.98}$$

$$\hat{p}_h = \beta_h\left(\frac{c_h^{do}}{k_h} - \bar{u}_h\right) + \kappa_h\hat{p}_y + (1-\kappa_h)\pi^c. \tag{6.99}$$

There is no direct change to the dynamics of the various types of price adjustment rules of the model, on the market for labour, goods and housing services (including the expectations formation mechanism for medium-run inflation rate of the domestically produced good), but an indirect one due to the assumptions $\gamma = n + n_l = \tilde{n} + \tilde{n}_l\bar{e}$, on natural growth and employment (which are now considered as given and together with all constant).

3. The Growth Dynamics (two laws of motion):

$$\hat{l}^e = \gamma - (g_k^d - \delta_k), \tag{6.100}$$

$$\hat{k}_h = g_h^d - \delta_h - (g_k^d - \delta_k). \tag{6.101}$$

In block 3 we assume that the natural rates of growth and of technical change, n, n_l, are given exogenously and equal in sum to γ. Furthermore actual accumulation rates are assumed to adjust with infinite speed to their desired targets and thus are no longer represented as lagged adjustment rules. Moreover we have removed here the adjustment equations for the rates of return r^l, r_h^l by assuming that these rates adjust with infinite speed to their short-run equivalents, which are now used in the corresponding behavioural equations in their place.

Next, we present the set of equations that represent the dynamics of asset accumulation and asset prices that are needed for the analysis of the real part of the dynamics of the model:

4. Asset Market Dynamics (six laws of motion):

$$\begin{aligned}\dot{b} = {} & \alpha_b^g[(gy^e + ib + b^l \\ & - t^w - t^c + (w^{ue}/p_v)(l^e - l^{we}) + (w^{re}/p_v)\alpha_l l_2^e + (w^{be}/p_v)l_g^{de}] \\ & - (\hat{p}_y + g_k^d - \delta_k)b,\end{aligned} \tag{6.102}$$

$$\begin{aligned}\dot{b}^l = {} & i_l(1-\alpha_b^g)[gy^e + ib + b^l \\ & - t^w - t^c + (w^{ue}/p_v)(l^e - l^{we}) + (w^{re}/p_v)\alpha_l l_2^e + (w^{be}/p_v)l_g^{de}] \\ & - (\hat{p}_y + g_k^d - \delta_k)b^l,\end{aligned} \tag{6.103}$$

$$\hat{p}_b = \frac{\beta_{p_b}}{1 - \beta_{p_b}(1-\alpha_s)}[(1-\tau_c)i_l + \alpha_s\pi_{bs} - ((1-\tau_c)i)], \quad \left(i_l = \frac{1}{p_b}\right), \tag{6.104}$$

$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}), \qquad (\pi_b = \alpha_s\pi_{bs} + (1-\alpha_s)\hat{p}_b),$$

$$\hat{s} = \frac{\beta_s}{1 - \beta_s(1-\alpha_s)}[(1-\tau_c)i_l^* + \alpha_s\epsilon_s - ((1-\tau_c)i_l + \pi_b)], \tag{6.105}$$

$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s). \tag{6.106}$$

The price adjustment rules in block 4 concern the nominal value of long-term bonds p_b and the nominal exchange rate s and they are based (as explained in Chiarella and Flaschel [1999b] and also in this chapter) on heterogeneous expectations of the pure wealth owners of the model. Note that, as in the larger model, the dynamics of equity prices and of Tobin's q are not needed in the investigation of the core dynamics of the model.

Next, the dynamic policy rules are presented which are basically the same as in the larger model. Note however that we have removed the derivative term from the right-hand side of the wage tax rate dynamics (6.28) to obtain (6.108).

5. The Feedback Policy Rules (three laws of motion):

$$\dot{i} = -\beta_{i_i}(i - i_l^*) + \beta_{i_p}(\hat{p}_y - 0) + \beta_{i_u}(y/y^p - \bar{u}), \tag{6.107}$$

$$\hat{\tau}_w = \alpha_{\tau_{wl}}(d/\bar{d} - 1), \quad \left(d = \frac{b + b^l/i_l}{y^e}\right), \tag{6.108}$$

$$\hat{\tau}_m = \alpha_{\tau_m} \frac{p_x x - p_m j^d}{p_x x}. \tag{6.109}$$

Summing up we have thus arrived at an 18D dynamical system by setting certain adjustment speeds equal to infinity or equal to zero, by cutting certain feedback effects of the individual distribution of bonds on aggregate demand (based on the facts that $C_c = 0$ holds and that workers save all their interest income), and by assuming constant 'natural' rates of growth as well as of employment. Note that the housing services sector feeds back into this core dynamics of the model through the investment demand for dwellings (on the market for domestic goods) and through its rate of return which is in particular determined by the law of motion for the rent price of housing services.

6.5.2 *Static relationships*

As abbreviations and static relationships we now have the following reduced and modified list of equations underlying this 18D dynamical system.

1. Output and Demand:

$$y^d = c_w^o + g_k^d + g_h^d k_h + g y^e, \tag{6.110}$$

$$c_w^o = c_y y_{w1}^D, \tag{6.111}$$

$$c_h^{do} = p_v c_h y_{w1}^D / p_h, \tag{6.112}$$

$$y = y^e + \beta_n(\beta_{n^d} y^e - \nu) + \gamma\beta_{n^d} y^e, \tag{6.113}$$

$$x = x_y y, \tag{6.114}$$

$$j^d = j_y y. \tag{6.115}$$

2. Employment and Labour Supply:

$$l_f^{de} = l_y y, \tag{6.116}$$

$$l_g^{de} = l_g^{we} = \alpha_g g y^e, \tag{6.117}$$

$$l^{de} = l_f^{de} + l_g^{de}, \tag{6.118}$$

$$l^{we} = l_f^{we} + l_g^{we}, \tag{6.119}$$

$$l_2^e = (L_2(0)/L_1(0))l_1^e = (L_2(0)/L_1(0))l^e/\alpha_l. \tag{6.120}$$

3. Growth Rates of the Capital Stocks of Firms and Asset Owners:

$$g_k^d = \alpha_1^k((1-\tau_c)r^e - i^r) + \alpha_2^k(i_l - i) + \alpha_3^k(y/y^p - \bar{u}) + \gamma + \delta_k, \tag{6.121}$$

$$g_h^d = \alpha_r^h((1-\tau_c)r_h - i^r) + \alpha_i^h(i_l - i) + \alpha_u^h\left(\frac{c_h^d}{k_h} - \bar{u}_h\right) + \gamma + \delta_h. \tag{6.122}$$

4. Rates of Return:

$$r^e = y^e - \delta_k + (p_x/p_y)x - (w^{be}/p_y)l_f^{de} - (p_m/p_y)j^d, \tag{6.123}$$

$$i^r = (1-\tau_c)i_l - \pi^c, \tag{6.124}$$

$$r_h = (p_h/p_y)c_h^{do}/k_h - \delta_h. \tag{6.125}$$

5. Prices, Wages and Transfers:

$$p_x = s p_x^*, \tag{6.126}$$

$$p_m = (1+\tau_m)s p_m^*, \tag{6.127}$$

$$w^{be} = (1+\tau_p)w^e, \tag{6.128}$$

$$w^{ue} = \alpha^u w^e, \tag{6.129}$$

$$w^{re} = \alpha^r w^e, \tag{6.130}$$

$$p_v = (1+\tau_v)p_y. \tag{6.131}$$

6. Disposable Income of Workers and Taxes Per Value Unit of Capital:

$$y_{w1}^D = (1-\tau_w)[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l_2^e]/p_v, \tag{6.132}$$

$$y_w^D = y_{w1}^D + (1-\tau_c)ib_w, \tag{6.133}$$

$$t^w = \tau_w[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l_2^e]/p_v + \tau_p(w^e/p_v)l^{de} + \tau_v(p_y/p_v)(y^d - g_k^d - g_h^d k_h) + \tau_m s p_m^* j^d/p_v, \tag{6.134}$$

$$t^c = \tau_c[r^e(p_y/p_v) + ib + b^l + (p_h/p_v)c_h^d - (p_y/p_v)\delta_h k_h]. \tag{6.135}$$

The steady state of the model is the same as in the preceding section. The dimension of this dynamical system can be further reduced, to dimension 16, if the housing sector is removed from the model.

6.6 Outlook: feedback structures and stability issues

We have presented a structural model of disequilibrium growth which is fairly complete with respect to markets, sectors and agents. We believe it is sufficiently detailed to capture the essential dynamic features of modern macroeconomies, while at the same time abstracting from the welter of detail that inevitably must characterise large-scale macroeconometric models such as the Fair model for the US economy or the Murphy model for the Australian economy.

We considered the model from the point of view of national accounts, and then discussed the development of its extensive form and finally expressed its dynamic structure in terms of intensive form state variables. We saw in particular that in intensive form we are dealing with a 34D dynamical system. We further found that with a small number of further assumptions (concerning consumption of asset holders and certain secondary delayed adjustment processes) the dynamics reduce to an 18D dynamical system which we call the core model.

There turn out to be eight main partial feedback mechanisms contained in the 18D core model. First, the labour and goods market interaction, the tendency of which to become destabilising is determined by an interplay between wage and price flexibility. Second, the expected sales and inventory accumulation interaction. The tendency to instability of this mechanism is determined by the relative values of speeds of adjustment of expected sales and inventory changes. Third, the dynamics of the housing sector are determined largely by the strength of investment into this sector and the speed of adjustment of the prices for housing services. This partial feedback mechanism is always stabilising. Fourth, the dynamic interaction between the level of economic activity and the nominal interest rate (the so-called Keynes effect). The stabilising/destabilising tendency of this mechanism is very dependent on the sensitivity of the nominal interest rate to the price level. We note that when considered in conjunction with the Taylor-style interest rate rule used in our model this mechanism is by and large stabilising. Fifth, the inflation/expected inflation mechanism. This is essentially a destabilising effect (associated with the names of Mundell and Cagan) determined by the interplay of speeds of adjustment of prices and inflationary expectations. Sixth, the bond and stock market dynamics (originally considered by Blanchard (1981)), first in isolation and then in their interaction, are driven by rates of return and expectations feedback. The mainly destabilising tendency of this mechanism is driven by the speeds of adjustment of bond prices and expectations of bond price inflation. Seventh, the dynamics of the GBR. This mechanism, when considered in isolation, is stabilising provided the rate of growth of government debt is restricted in certain ways. Eighth, the exchange rate/expected exchange rate (Dornbusch) mechanism where stabilising/destabilising tendencies essentially depend upon the relationship between the speed of adjustment of exchange rates and exchange rate expectations.

The 18D core model of this chapter has been studied in a series of papers; see Chiarella and Flaschel (1999b,c,d), Chiarella *et al.* (1999a,b, 2003b). These papers discuss the feedback structure of the 18D core dynamics in great detail with respect to the partial stabilising or destabilising feedback chains that are present. They also present further numerical investigations of the model both from the local and the global point of view (adding also extrinsic non-linearities in order to achieve global boundedness) and extending the theoretical basis of the disequilibrium growth model employed so far (by allowing for smooth input and output substitution and other flexibilities).

Prototype subdynamics of the 18D system which are often discussed in the literature in isolation, and which we have briefly discussed above, will be derived and analysed in subsequent chapters. The current chapter provides more insights into the stability properties of the 18D dynamics by investigating the partial feedback chains it contains (often well known from comparative static analysis) with respect to the partial dynamics to which they give rise. The interaction of these partial dynamics has been studied in Chiarella *et al.* (2003b), there basically from the numerical point of view as in Barnett and He (1999) who as we have seen in Section 6.1, use a 14D second order dynamical representation of the UK economy to study the bifurcation loci to which such a model type can give rise.

We close this chapter by giving a graphical representation of the disequilibrium growth approach that will be considered in the subsequent discussion on the intensive

form level. Figure 6.1 shows the various points we wish to stress in this book in different fonts. The figure presents as a starting point the basic market hierarchy of Keynes' (1936) General Theory as we interpret it. This view asserts that asset markets dominate (in a casino-like fashion) the outcome in the real markets basically by way of the investment decision of firms that here still simply depend on financial markets and the structure of nominal interest rates that they generate, but does not exercise a feedback influence on them. Effective demand (with a Metzlerian feedback mechanism included) that derives from the investment decision determines the outcome on the market for goods which is therefore purely demand determined. Finally, at the lowest level of the hierarchy, there are the labour markets which just have to accept the amount of employment that is generated by (expected) goods demand and the technology with which firms produce their demand determined supply of goods (including intended inventory changes).

This 'prejudice' on the fundamental macroeconomic forces that may (or may not) shape the temporary equilibrium position of the economy represents in our view a way of thinking that allows one to approach the full situation in a systematic hierarchically ordered way, and which does not just state that everything depends on everything else as far as financial decisions, and quantity and price determination, are concerned. Of course, one has to address the question of which feedback mechanisms have to be added to this picture (and those which may be left aside due to their minor importance) and in what way they would alter the temporary equilibrium position (or its evolution) as determined by the fundamental 'causal nexus' from the asset market to the labour market shown by the grey boxes in Figure 6.1. Therefore, the proposed feedback mechanisms (or repercussions) have to be discussed as to their importance for full employment positions or (if they work with a delay) as to their role as a stabilising or destabilising mechanism in the evolution of the temporary equilibrium positions of the economy. Of course, there may be feedback mechanisms of great importance that are still missing in Figure 6.1. An example for this is given by the Fisher debt effect relating firms with households or banks which is still missing in our model due to the limited (supply side oriented) treatment of asset markets and the financial decisions made by firms.[12] By contrast, it may be questioned whether for example the Keynes short-term interest rate effect on investment or the Pigou effect on consumption demand are really of importance in the picture of the real and financial interaction shown by the black portions in Figure 6.1. There are also feedback mechanisms that are internal to the asset markets and that may create cumulative processes in them that are bounded by certain switches in expectational regimes.

An important controversial aspect of Figure 6.1 is given by its presentation of supply side influences. In our view this is basically the question of the wage-price interaction (including expectations) which should be approached from the viewpoint of a separate

[12] Note however that we have a Fisher debt effect in the present model as far as the sector of households is concerned where we can have the situation that workers with a high marginal propensity to consume become debtors to asset holders (with a low or zero marginal propensity to consume) which in this way implies that consumption will be depressed in deflationary periods and will accelerate in periods of inflation.

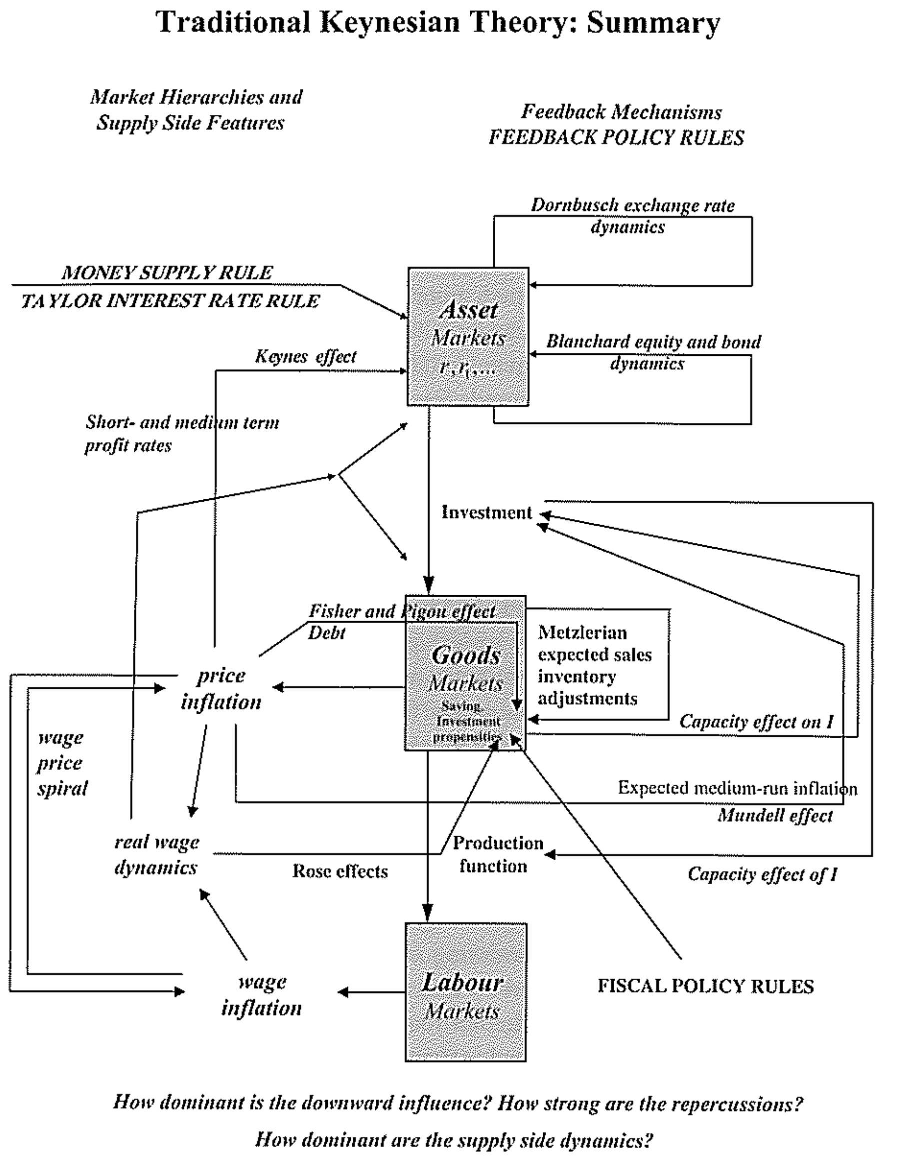

Figure 6.1 Advanced traditional disequilibrium growth dynamics: graphical summary. Different fonts and shading represent the market hierarchies, supply side features, feedback mechanisms and feedback policy rules

treatment of Phillips curves (PCs), one for the nominal wage and one for the price level. Such an approach allows the analysis to escape from the narrow perspective of a unique monetarist type of expectations augmented price inflation labour market PC which does not really allow for the discussion of wage-price spirals, which is needed in a full approach to disequilibrium growth where both labour and capital can be over- or

under-utilised. Such a view on the working of the wage-price mechanism, which allows for a sluggish adjustment of both of these nominal magnitudes, will include Rose (1967) effects in the interaction of the goods and the labour markets, which generally imply that either wage or price flexibility will lead to instability in the process that is usually considered as the medium run of macrodynamic model building.

Finally we assume that certain policy feedback rules interact with the dynamics of the private sector. Such rules are present in the approach investigated in the present chapter, but are not yet at the centre of interest in the present stage of the investigation.

Figure 6.1 therefore summarises and contrasts the main substructures we have introduced in greater detail in Chiarella *et al.* (1999a,b) and which we describe on the intensive form level in this chapter and also investigate further in subsequent chapters. At the bottom of Figure 6.1 we also see a summary of the main questions that should find some answers in the course of the investigations of this general approach to disequilibrium growth. First, we have the question to what extent the asset markets dominate the outcome of the real/financial interaction (which, as we shall see, is here still of a very particular type and thus demands further extensions of the model if the dynamics of asset prices are to be linked more closely to the stocks supplied and demanded on these markets). Second, there is the question as to what the various feedback mechanisms shown add to the real/financial interaction and to what extent they will contribute to or undermine, when working together, the local stability of the balanced growth solution(s) of the model. Third, the dynamics of income distribution, as they find expression in the wage-price spiral, have to be investigated, in particular in their role of shaping the long-run outcome of this model which has a strictly Keynesian short-run outcome. Finally, the perspective of our approach is of course to contribute to the analysis of policy issues which due to the fact that we want to treat medium- and long-run dynamics as well is more oriented to the treatment of monetary policy rules than to a treatment of the consequences of isolated fiscal or monetary shocks that occur only once in time. In the present chapter we have only set the stage for such investigations, while subsequent work is needed, starting from special cases of the model as in Chiarella *et al.* (2003b), in order to understand in a systematic way the implications that are contained in our approach to disequilibrium growth.

7 Partial feedback structures and stability issues

7.1 Introduction

In this chapter we continue the analysis of our applied structural model of disequilibrium growth initiated in Chiarella and Flaschel (1999b,c,d). In those earlier papers we developed a model of disequilibrium growth which is fairly complete with respect to markets, agents and sectors and consistent with respect to the various budget constraints between them. We showed in Chiarella and Flaschel (1999b,c,d) that the model could be expressed as a dynamical system of 34D intensive state variables together with a number of static relationships. We further showed how a small number of not unreasonable simplifying assumptions reduced this dynamical system to one of eighteen laws of motion solely, which we have dubbed the core model of this approach with fixed proportions in production. Our aim in this, and in the next two chapters, is to analyse in quite some detail the properties of this 18D core model. In this chapter in particular we focus on the basic partial feedback structures of the core model and their stability characteristics.

We can distinguish qualitatively at least seven important feedback chains (plus stabilising or destabilising policy reaction functions), which we will describe below in isolation from each other. These will of course interact with each other in the full 18D dynamics of the core model, so that one or another may become dominant when the parameters of the model are chosen appropriately.[1]

The structure of the chapter is as follows: Section 7.2 describes the accounts of the various agents of our modelling framework. In Section 7.3 we summarise the special 18D case of the preceding chapter from the perspective of the various accounts of the sectors of our structural macroeconometric model of Chapter 5. In Section 7.4 we then briefly recapitulate the obtained 18D core dynamics subject to the 'scissor and paste' methodology we use in the present chapter to study the feedback chains it contains. In Section 7.4 and Section 7.5 we then start the investigation of the partial feedback mechanisms contained in the core dynamics of our model by isolating the dynamic

[1] To simplify the presentations in this chapter we assume in the following without loss of generality $\tau_c = 0$, $\xi = 0$, $c_c = 0$; see the following compact representation of the 18D core dynamics.

interaction of income distribution with various measures of economic activity on the market for labour and for goods which will push income distribution in favour of labour or of asset holders depending on marginal propensities to demand goods for consumption and investment purposes and on the speeds of adjustment of nominal wages and prices in the market for labour and goods, respectively. This multi-faceted feedback mechanism is related to early (and later) work of Rose; see Rose (1966, 1990) in particular, and also to Goodwin's (1967) growth cycle model, but there in a less multi-faceted way due to the simpler real wage mechanism of this model type. Goodwin's (1967) growth cycle model nevertheless constitutes the core dynamics that, married with goods market adjustment processes, will provide the basic 5D situation from which our numerical analysis of the model of Chiarella *et al.* (2003b) will start.

In Section 7.6 we consider the Metzlerian inventory mechanism describing the dynamic interaction of expected sales and inventory accumulation. We also discuss in this section a certain non-linearity (in the adjustment speed of inventories) which can 'tame' the instabilities that this mechanism is capable of generating. Section 7.7 discusses the dynamics of the housing sector, while Section 7.8 analyses the well-known Keynes effect which describes the interaction between economic activity and the nominal interest rate. Section 7.9 describes the Mundell–Tobin mechanism of the interaction between inflation and expected inflation, and discusses how an interest rate policy rule may help to overcome this largely destabilising adjustment process.

Section 7.10 outlines the infrequently discussed Blanchard mechanism for the dynamic adjustments on, and the dynamic interaction between, bond and stock markets. This section also discusses some non-linear mechanisms which can tame the rapid tendency to instability in the price dynamics of long-term bonds (or equities). Section 7.11 describes the dynamics of the government budget constraint, which, unlike in other studies and due to the assumptions made, we find to be locally stabilising at least when considered in isolation. Section 7.12 considers briefly the dynamics of import taxation. Section 7.13 discusses in some detail the Dornbusch mechanism for the dynamic interaction between the exchange rate and expectations of its appreciation (or depreciation), identifying the factors leading to stability and instability of this mechanism. Section 7.14 draws some conclusions and points to future directions in the research agenda initiated by this chapter.

7.2 National accounting (in intensive form)

The structure of the considered economy from the viewpoint of national accounting is shown in Table 7.1 (everything being measured in nominal domestic currency units per gross value of the capital stock).

7.2.1 Firms

The firms produce two kinds of output, the pure export good which is tradable only on the world market and the domestic good which can only be sold in the domestic

Table 7.1. *The accounts of firms*

Uses	Resources
Production Account of Firms:	
Imports $sp_m^* j^d / p_v$	Consumption c_g^o
Depreciation $\delta_k p_y / p_v$	–
Value-added Taxes $\tau_v (c_g^o + g) p_y / p_v$	Consumption g
Taxes on Imports $\tau_m sp_m^* j^d / p_v$	Exports $sp_x^* x / p_v$
Wages (excluding payroll taxes) $w^e / p_v l_f^{de}$	Gross Investment $g_k^d p_y / p_v$
Payroll Taxes $\tau_p w^e / p_v l_f^{de}$	Durables (Dwellings) $g_h^d p_y / p_v$
Profits $(r^e + \mathcal{I}/K) p_y / p_v$	Inventory Investment $p_y \dot{N} / (p_v K) = p_y (y - y^d) / p_v$
Income Account of Firms:	
Dividends $r^e p_y / p_v$	Profits $(r^e + \mathcal{I}/K) p_y / p_v$
Savings $\mathcal{I}/K p_y / p_v$	
Accumulation Account of Firms:	
Gross Investment $g_k^d p_y / p_v$	Depreciation $\delta_k p_y / p_v$
Inventory Investment $\dot{N}/K p_y / p_v$	Savings $S_f^n / (p_v K)$
	Financial Deficit $FD / (p_v K)$
Financial Account of Firms:	
Financial Deficit $FD / (p_v K)$	Equity Financing $p_e \dot{E} / (p_v K)$

economy. The domestic good serves as the consumption good for the workforce and the government (in our simplified 18D dynamical version of the model). It can also be used for investment in inventories, in business fixed capital and in housing. Firms use three kinds of inputs for their production: imports, capital and labour. The capital stock in the sector of firms depreciates by a given rate δ_k. Value-added taxes (on consumption goods solely) appear on the left side of the production account and have to be paid to the government. The balance of this account is the profit of the sector of firms. Note that all expressions are in intensive form (they have all been measured in domestic currency units in Chiarella and Flaschel (1999b,c,d)) and are here divided uniformly by $p_v K$, the value of the capital stock (including value-added taxation by assumption).[2] We stress that the profits are not subject to any direct tax. By assumption profits are only used to be paid as dividends to asset holders (and then taxed) or to be used for planned inventory investments. One can clearly see this in the income account. The accumulation account displays again that investments in business fixed capital and in inventories are the only stocks which can be accumulated by firms. There is no

[2] Note that all investment and thus also the value of the capital stock and the measure of the rate of profit based on it are in prices p_y net of value-added tax, since these taxes are only applied to consumption purchases and not to investment purchases in the present model. Note also that the following uniform intensive form representation of the model does not immediately apply to the structural form of the model in intensive form, since we do not need accounting homogeneity in this structural form as is necessary in the present subsection.

possibility of accumulating financial stocks, that is no holding of bonds by firms in the present context. The financial deficit of firms must be financed in our present model by selling new equities. This assumption is of course not very realistic, and thus should be modified in future reconsideration of the model to allow in particular for bond financing and loans of firms.

7.2.2 *Asset holders*

While firms produce and sell two types of goods, the sector of the private asset holders sells dwelling services. Hence there is a production account for this sector. The income of this sector consists of interest payments (long- and short-term bonds, the former also from abroad), dividend payments from the sector of firms and the profits from selling dwelling services. This income is reduced through profit income taxation. The remaining amount is the saving of this sector (since asset holders do not consume in the 18D core dynamics of our general model to be considered in this chapter). Savings plus depreciation is split into gross investment in housing and the financial surplus in the following account. The financial surplus is distributed by asset owners to all kinds of financial assets that exist in our model. See Table 7.2 for the accounts of asset holders.[3]

7.2.3 *Workers*

This sector does not take part in private ownership production, but only provides the labour input for firms. Therefore the production account remains empty. The income account includes wages, unemployment benefits and pensions. Workers' income is allocated to income taxes and consumption and savings. All savings is allocated to short-term bonds. See Table 7.3 for the accounts of workers.

7.2.4 *Fiscal and monetary authorities*

The government sector's production account takes up the costless provision of public goods which is defined to be identical to consumption of the government. In order to provide the economy with public goods the government has to buy goods and pay wages to the workers it employs.

The only sources of income for the government are the various taxes, which are used for interest payments, pensions, unemployment benefits and salaries. The balance of this account are the savings of the government. Generally these savings are negative, hence there is a financial deficit in the accumulation account, rather than a financial surplus in general.

In financial accounting of the government one can see the sources from which the deficit is financed, namely by issuing short- and long-term bonds. See Table 7.4.

[3] Expressions such as $\hat{B}b (= \dot{B}/(p_v K))$ are used to indicate the way the law of motion, of here $b = B/(p_v K)$, has to be derived.

Table 7.2. *Accounts of households (asset owners)*

Uses	Resources
Production Account of Households (Asset Owners/Housing Investment):	
Depreciation $\delta_h k_h p_y/p_v$	Rent $p_h c_h^o/p_v$
Earnings $\Pi_h/(p_v K)$	
Income Account of Households (Asset Owners):	
Tax Payment $\tau_c ib$	Interest Payment ib
Tax Payment $\tau_c b_1^l$	Interest Payment b_1^l
Taxes $\tau_c(p_h c_h^o/p_v - \delta_h k_h p_y/p_v)$	Interest Payment $s(1-\tau_c^*)b_2^l$
Tax Payment $\tau_c r^e p_y/p_v$	Dividend Payment $r^e p_y/p_v$
Savings $S_c^n/(p_v K)$	Earnings $\Pi_h/(p_v K)$
Accumulation Account of Households (Asset Owners):	
Gross Investment $g_h^d p_y/p_v$	Depreciation $\delta_h k_h p_y/p_v$
Financial Surplus $FS/(p_v K)$	Savings $S_c^n/(p_v K)$
Financial Account of Households (Asset Owners):	
Short-term Bonds $\hat{B}b$	Financial Surplus $FS/(p_v K)$
Long-term Bonds $p_b \hat{B}_1^l b_1^l$	
Foreign Bonds $s\hat{B}_2^l b_2^l/i_l^*$	
Equities $p_e \hat{E}\varepsilon$	

Table 7.3. *Accounts of households (workers)*

Uses	Resources
Production Account of Households (Workers):	
–	–
Income Account of Households (Workers):	
Taxes $\tau_w[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re} l_2^e]/p_v$	Wages $w^e l^{de}/p_v = (w^e l_f^{de} + w^e l_g^{de})/p_v$
Consumption $c_g^o + p_h c_h^o/p_v$	Unemployment Benefits $w^{ue}(l^e - l^{we})/p_v$
Savings $S_w^n/(p_v K)$	Pensions $w^{re} l_2^e/p_v$
Accumulation Account of Households (Workers):	
Financial Surplus $FS/(p_v K)$	Savings $S_w^n/(p_v K)$
Financial Account of Households (Workers):	
Short-term Bond Accumulation $\hat{B}_w b_w$	Financial Surplus $FS/(p_v K)$

7.2.5 *International relationships*

The external account contains all transactions with the foreign countries. It exhibits the amounts of goods, capital and interest payments that cross the borders. See Table 7.5. This closes this section on the national accounts of the model that will be investigated numerically in the following sections.

Table 7.4. *Accounts of the fiscal and monetary authorities*

Uses	Resources
Production Account of Fiscal and Monetary Authorities:	
Government Expenditure for Goods g	Costless Provision of
Salaries $w^{be} l_g^{de}/p_v = (w^e l_g^{de} + \tau_p w^e l_g^{de})/p_v$	Public Goods = Self Consumption
Income Account of Fiscal and Monetary Authorities:	
Interest Payment ib	Wage Income Taxation
	$\tau_w[w^e l^{de} + w^{ue}(l^e - l^{we}) + w^{re} l_2^e]/p_v$
Interest Payment $b_1^l + b_1^{l*}$	Profit and Interest Taxation
	$\tau_c r^e p_y/p_v + \tau_c ib + \tau_c b_1^l + \tau_c b_1^{l*}$
Pensions $w^{re} l_2^e/p_v$	Rent Income Taxation $\tau_c(p_h c_h^o/p_v - \delta_h k_h p_y/p_v)$
Unemployment benefits $w^{ue}(l^e - l^{we})/p_v$	Payroll Taxes $(\tau_p w^e l_f^{de} + \tau_p w^e l_g^{de})/p_v$
Self consumption g	Value-added Tax $\tau_v(c_g^o + g)p_y/p_v$
Savings $S_g^n/(p_v K)$	Import Taxes $\tau_m s p_m^* j^d/p_v$
Accumulation Account of the Fiscal Authority:	
	Savings $S_g^n/(p_v K)$
	Financial Deficit $FD/(p_v K)$
Financial Account of Fiscal and Monetary Authorities:	
Financial deficit $FD/(p_v K)$	Short-term Debt $\hat{B}b$
	Long-term Debt $\hat{B}^l b^l/i_l$

Table 7.5. *International relationships*

Uses	Resources
External Account:	
Exports $sp_x^* x/p_v$	Imports $sp_m^* j^d/p_v$
Factor Income from Abroad $s(1-\tau_c^*)b_2^l$	Factor Income to Abroad $(1-\tau_c)b_1^{l*}$
Capital Imports $\hat{B}_1^{l*} b_1^{l*}/i_l$	Capital Exports $s\hat{B}_2^l b_2^l/i_l^*$

7.3 The core 18D dynamical system: a recapitulation

We will base our subsequent numerical investigation of the 18D core model of Chiarella and Flaschel (1999b,c,d) in this chapter on the following condensed form of its eighteen laws of motion (which is the one used for the simulations reported below) and the unique interior steady state that this dynamical model exhibits (up to the level of nominal magnitudes). In order to simplify the notation to some degree we modify the model of the previous chapters by assuming in the following for the risk and liquidity premium $\xi = 0$ and thus will have $r^e = i = i_l = i_{l*}$ in the steady state. For the same reason we also assume for the normal employment rate $\bar{u}_f^w = 1$, and also $C_c = 0$, thus there is no consumption goods demand of asset holders who thus save all of their income. All these assumptions have only slight influences on the steady state position of the economy, and do not alter at all the dynamics around the steady state.

We consider the steady state values of the model first. All these values should have an index 'o' (denoting their steady state character). In order not to overload the notation, we do not add this index to the list of steady state values (7.1)–(7.18). Note again that all steady state values are expressed in per unit of capital form and if necessary in efficiency units:

$$y^e = \frac{y^p \bar{u}}{1+\gamma\beta_{n^d}}, \quad [y = y^p \bar{u}], \tag{7.1}$$

$$\nu = \beta_{n^d} y^e, \tag{7.2}$$

$$l_f^{we} = l_f^{de} = l_y y^p \bar{u} \quad [\text{total employment: } l^{we} = l_f^{we} + l_g^{we}, l_g^{we} = \alpha_g g y^e], \tag{7.3}$$

$$l^e = (l_f^{we} + \alpha_g g y^e)/\bar{e}, \tag{7.4}$$

$$p_y = \frac{p_v}{1+\tau_v}, \quad [p_v \quad \text{arbitrarily given}], \tag{7.5}$$

$$w^e = \frac{\omega^{be} p_y}{1+\tau_p}, \quad \left[\omega^{be} = \frac{w^e}{p_y} = \frac{y^e - \delta_k - i_l^*}{l_f^{we}}\right], \tag{7.6}$$

$$\pi^c = 0, \tag{7.7}$$

$$p_h = p_y(i_l^* + \delta_h)/\bar{u}_h, \tag{7.8}$$

$$k_h = \frac{c_h(y^e(1-g) - (\gamma+\delta_k))}{c_y(i_l^* + \delta_h)/(1+\tau_v) + (\gamma+\delta_h)c_h}, \tag{7.9}$$

$$b = \alpha_b^g \bar{d} y^e, \tag{7.10}$$

$$b^l = i_l^*(1-\alpha_b^g)\bar{d} y^e, \tag{7.11}$$

$$p_b = 1/i_l^*, \tag{7.12}$$

$$\pi_{bs} = 0, \tag{7.13}$$

$$\epsilon_s = 0, \tag{7.14}$$

$$i = i_l^* \quad [= r^e], \tag{7.15}$$

$$\tau_m = \frac{p_x^* x_y - p_m^* j_y}{p_m^* j_y}, \tag{7.16}$$

$$\tau_w = 1 - \frac{p_h \bar{u}_h k_h}{c_h(1+\tau_v) p_y y_{w1}}, \tag{7.17}$$

$$s = \frac{s_o - [\tau_w y_{w1} + \frac{\tau_p}{1+\tau_v}\frac{w^e}{p_y} l^{we} + \frac{\tau_v}{1+\tau_v}(y^e - (\gamma+\delta_k) - (\gamma+\delta_h)k_h)]}{\tau_m p_m^* j_y y/((1+\tau_v)p_y)}. \tag{7.18}$$

With respect to the last two of the above equations, for the taxation rate τ_w and for the rate of exchange s of the model, we have to apply (besides the above definitions of y, l^{we} and ω^{be}, see the above) the further defining expressions

$$c_h^o = \bar{u}_h k_h,$$

$$t_o^c = \tau_c[i_l^*/(1+\tau_v) + ib + b^l + (p_h/p_y)c_h^o/(1+\tau_v) - \delta_h k_h/(1+\tau_v)],$$

$$s_o = g y^e + ib + b^l - t_o^c + \frac{w^e}{(1+\tau_v)p_y}[\alpha^u(l^e - l^{we}) + \alpha^r L_2(0)/L_1(0) l^e]$$
$$+ (1+\tau_p)\frac{w^e}{(1+\tau_v)p_y}\alpha_g g y^e - \frac{\gamma b}{\alpha_b^g},$$

$$y_{w1} = w^e[l^{we} + \alpha^u(l^e - l^{we}) + \alpha^r L_2(0)/L_1(0) l^e]/((1+\tau_v)p_y)$$

in order to have a determination of the steady state that is complete.

Note that the value of the exchange rate s will be indeterminate when we have $\tau_m = 0$ in the steady state, in which case the above formula for s cannot be applied. Note furthermore that the parameters of the model have to be chosen such that k_h, τ_w, s are all positive in the steady state.[4] Note finally that the parameter α_s must always be larger than $1 - 1/\beta_x$ for $x = p_b, s, p^e$ in order to satisfy the restrictions established in Chiarella and Flaschel (1999b,c,d).

Equation (7.1) gives expected sales per unit of capital K, while equation (7.2) provides the steady inventory-capital ratio N/K. Equation (7.3) provides the amount of workforce employed by the firms which in the steady state is equal to the hours worked by this workforce (assuming that the normal working day or week is represented by 1). It also shows total employment per K where account is taken of the employment in the government sector in addition. Equation (7.4) is the full employment labour intensity. Equation (7.5) provides the price level (net of value-added tax) and equation (7.6) gives the wage level (net of payroll taxes) based on the steady state value for the real wage ω^{be}. The steady state value of the inflation rate expected to hold over the medium run is zero, since the inflationary target of the central bank is zero.

Next we have the price level for housing rents (in equation (7.8)) and the stock of houses per unit of the capital stock K (in equation (7.9)). There follows the steady state value of $b = B/(p_v K)$ as well as the one for long-term domestic bonds. The price of these bonds is given by the given price $1/i_l^*$ of foreign long-term bonds in the steady state; see equation (7.12). The steady state value of the short-term rate of interest settles at its long-run equivalent as there is no risk or liquidity premium allowed for.

Import taxes τ_m just balance the trade balance in the steady state, see equation (7.16), while the wage tax rate τ_w must be calculated from gross steady wage income y_{w1} and the marginal propensity to spend for housing services, see equation (7.17). Equation

4 There are further simple restrictions on the parameters of the model due to the economic meaning of the variables employed.

(7.18), finally, provides the steady state value of the rate of exchange which depends on nearly all parameters of the model, due to the definitional terms shown (to be inserted into the expression for s shown in equation (7.18)).

Next we present the eighteen laws of motion of our dynamical system which have been derived in Chiarella and Flaschel (1999b,c,d) and which of course also make use of the state variables we have just discussed.

Making use of the formula

$$\Delta\hat{p}_y = \hat{p}_y - \pi^c = \kappa[\kappa_p(\beta_{w_e}(e-\bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - 1)) + \beta_p(y/y^p - \bar{u})]$$

for the deviation of the actual inflation rate from the expected one, the laws of motion around the above steady state solutions of the dynamics read as

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + (\gamma - (g_k^d - \delta_k))y^e, \tag{7.19}$$

$$\dot{\nu} = y - y^d - (g_k^d - \delta_k)\nu, \tag{7.20}$$

$$\dot{l}_f^{we} = \beta_l(l_f^{de} - l_f^{we}) + [\gamma - (g_k^d - \delta_k)]l_f^{we}, \tag{7.21}$$

$$\hat{l}^e = \gamma - (g_k^d - \delta_k), \tag{7.22}$$

$$\hat{w}^e = \pi^c + \kappa[\beta_{w_e}(l^{we}/l^e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - 1) + \kappa_w\beta_p(y/y^p - \bar{u})], \tag{7.23}$$

$$\hat{p}_y = \pi^c + \kappa[\kappa_p(\beta_{w_e}(l^{we}/l^e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - 1)) + \beta_p(y/y^p - \bar{u})], \tag{7.24}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\alpha_{\pi^c}\Delta\hat{p}_y + (1-\alpha_{\pi^c})(0 - \pi^c)), \tag{7.25}$$

$$\hat{p}_h = \beta_h\left(\frac{c_h^o}{k_h} - \bar{u}_h\right) + \kappa_h\Delta\hat{p}_y + \pi^c, \tag{7.26}$$

$$\hat{k}_h = g_h^d - \delta_h - (g_k^d - \delta_k), \tag{7.27}$$

$$\dot{b} = \alpha_b^g[gy^e + ib + b^l - t^a - t^c + g^a] - (\Delta\hat{p}_y + \pi^c + g_k^d - \delta_k)b, \tag{7.28}$$

$$\begin{aligned}\dot{b}^l &= (1-\alpha_b^g)/p_b[gy^e + ib + b^l - t^a - t^c + g^a]\\ &\quad - (\Delta\hat{p}_y + \pi^c + g_k^d - \delta_k)b^l,\end{aligned} \tag{7.29}$$

$$\hat{p}_b = \frac{\beta_{p_b}}{1 - \beta_{p_b}(1-\alpha_s)}[(1-\tau_c)i_l + \alpha_s\pi_{bs} - (1-\tau_c)i], \quad i_l = 1/p_b, \tag{7.30}$$

$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}), \tag{7.31}$$

$$\dot{i} = -\beta_{i_i}(i - i_l^*) + \beta_{i_p}(\Delta\hat{p}_y + \pi^c) + \beta_{i_u}(y/y^p - \bar{u}), \tag{7.32}$$

$$\hat{\tau}_m = \alpha_{\tau_m}\frac{p_x^*x - (1+\tau_m)p_m^*j^d}{p_x^*x}, \quad (x = x_y y,\ j^d = j_y y), \tag{7.33}$$

$$\hat{\tau}_w = \alpha_{\tau_{w1}}\left(\frac{d}{\bar{d}} - 1\right), \quad d = \frac{b + p_b b^l}{y^e}, \tag{7.34}$$

$$\hat{s} = \frac{\beta_s}{1-\beta_s(1-\alpha_s)}[(1-\tau_c)i_l^* + \alpha_s\epsilon_s - ((1-\tau_c)i_l + \pi_b)], \quad (i_l = 1/p_b), \tag{7.35}$$

$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s). \tag{7.36}$$

These laws of motion make use of the supplementary definitions and abbreviations

$$y = y^e + \beta_n(\beta_{n^d}y^e - \nu) + \gamma\beta_{n^d}y^e,$$

$$l_f^{de} = l_y y,$$

$$l_g^{de} = l_g^{we} = \alpha_g g y^e,$$

$$l^{de} = l_f^{de} + l_g^{de},$$

$$l^{we} = l_f^{we} + l_g^{we},$$

$$y_{w1} = w^e[l^{de} + \alpha^u(l^e - l^{we}) + \alpha^r\frac{L_2(0)}{L_1(0)}l^e]/[(1+\tau_v)p_y],$$

$$c_g^o = c_y(1-\tau_w)y_{w1},$$

$$c_h^o = (1+\tau_v)p_y c_h(1-\tau_w)y_{w1}/p_h,$$

$$r^e = y^e - \delta_k + (sp_x^*/p_y)x_y y - ((1+\tau_p)w^e/p_y)l_f^{de} - ((1+\tau_m)sp_m^*/p_y)j_y y,$$

$$\begin{aligned}g_k^d &= \alpha_1^k((1-\tau_c)r^e - ((1-\tau_c)i_l - \pi^c)) + \alpha_2^k(i_l - i),\\ &\quad + \alpha_3^k(y/y^p - \bar{u}) + \gamma + \delta_k, \quad i_l = 1/p_b,\end{aligned}$$

$$\begin{aligned}g_h^d &= \alpha_r^h((1-\tau_c)((p_h/p_y)c_h^o/k_h - \delta_h) - ((1-\tau_c)i_l - \pi^c)) + \alpha_i^h(i_l - i)\\ &\quad + \alpha_u^h\left(\frac{c_h^o}{k_h} - \bar{u}_h\right) + \gamma + \delta_h, \quad i_l = 1/p_b,\end{aligned}$$

$$y^d = c_g^o + g_k^d + g_h^d k_h + gy^e,$$

$$\pi_b = \alpha_s\pi_{bs} + (1-\alpha_s)\hat{p}_b,$$

$$g^a = w^e\left[\alpha^u(l^e - l^{we}) + \alpha^r\frac{L_2(0)}{L_1(0)}l^e + (1+\tau_p)l_g^{de}\right]/(1+\tau_v)p_y,$$

$$\begin{aligned}t^a &= \tau_w w^e\left[l^{de} + \alpha^u(l^e - l^{we}) + \alpha^r\frac{L_2(0)}{L_1(0)}l^e\right]/((1+\tau_v)p_y) + \tau_p w^e l^{de}/((1+\tau_v)p_y)\\ &\quad + \frac{\tau_v}{1+\tau_v}(y^d - g_k^d - g_h^d k_h) + \tau_m sp_m^* j_y y/((1+\tau_v)p_y),\end{aligned}$$

$$t^c = \tau_c[r^e/(1+\tau_v) + ib + b^l + (p_h/p_y)c_h^o/(1+\tau_v) - \delta_h k_h/(1+\tau_v)].$$

Inserting these equations into the above eighteen laws of motion gives an explicit system of eighteen autonomous non-linear differential equations in the eighteen state variables (7.19)–(7.36) shown above. Note that we have to supply as initial conditions the relative magnitude $\frac{L_2(0)}{L_1(0)}$ in order to get a complete characterisation of the dynamics and that the evolution of price levels is subject to hysteresis, since it depends on historical conditions due to our assumptions on costless cash balances for the behaviour of the four agents of the model.

Our strategy for analysing the 18D core model is to discuss, in the subsequent sections of this chapter, the various partial feedback mechanisms it contains. In Chiarella *et al.* (2003b) we have considered the interaction of these partial mechanisms, and via a series of partial analyses built up to an analysis of the full 18D dynamics. We refer the reader to Chiarella *et al.* (2009b) for a more complete description of the business fluctuations and long-phased cycles that the 18D core model is capable of generating.

7.4 A Goodwin wage income/insider-outsider labour market dynamics

In order to isolate this extended feedback mechanism between functional income distribution and capital accumulation we assume for the parameter κ_w that it equals 1. Furthermore we abstract from the government sector and its employment decisions and from exports and imports and also assume that fixed business investment is simply given by $g_k^d = \alpha_1^k(r^e - \bar{i}^r) + \gamma + \delta_k$ based on the assumptions $\alpha_2^k = 0$, $y = y^p$, implying that there is no impact of interest rates on fixed business investment and firms also face no demand constraint, but produce at full capacity $Y^p = y^p K$ throughout.[5]

It is easy to show then that the law of motion of real wages ω^e is in this case given by

$$\hat{\omega}^e = \beta_{w_e}(e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - \bar{u}_f^w),$$

so that the demand pressure on outside and inside labour markets are here the sole determinants of the dynamics of real wages (since the short-run inflation rate is fully reflected in the adjustment of money wages).[6] The obtained law of motion for real wages is easily rewritten as

$$\hat{\omega}^e = \beta_{w_e}(l^{we}/l^e - \bar{e}) + \beta_{w_u}(u_f^w - \bar{u}_f^w), \tag{7.37}$$

with $u_f^w = l_f^{de}/l_f^{we}$ ($l_f^{de} = l_y y^p$) and $e = l^{we}/l^e$. The laws of motion of these latter two variables can then be obtained from the complete model of the preceding section, in fact specifically from the equations

$$\hat{l}^e = \gamma + \delta_k - \alpha_1^k(r^e - \bar{i}^r),$$

$$\hat{l}_f^{we} = \beta_l(l_y y^p/l_f^{we} - \bar{u}_f^w) + \gamma + \delta_k - \alpha_1^k(r^e - \bar{i}^r),$$

[5] Aggregate demand may and will here differ from aggregate supply, but is assumed to have effects on the rate of price inflation solely with no feedback on the growth dynamics considered below.

[6] See the next section for a more complicated real wage mechanism which also takes account of the imbalance that exists in the market for goods.

with r^e given by $y^e - \delta_k - (1 + \tau_p)\omega^e l_f^{de}$. After some manipulations the differential equations for u_f^w and e can be written as

$$\hat{u}_f^w = \alpha_1^k(r^e - (\gamma + \delta_k) - \bar{i}^r) - \beta_l(u_f^w - \bar{u}_f^w), \tag{7.38}$$

$$\hat{e} = \beta_l(u_f^w - \bar{u}_f^w). \tag{7.39}$$

These laws of motion basically represent the assumed investment behaviour and the employment policy of firms which indeed influence both of these dynamics. On the basis of the above assumptions we thus obtain a 3D dynamical system in the state variables ω^e, u_f^w, e, of the real wage in efficiency units, of the rate of employment of the employed and of the outside rate of employment.

Proposition 7.1 *1. The dynamical system (7.37), (7.38), (7.39) for ω^e, u_f^w and e has a unique interior steady state given by*

$$\omega_o^e = \frac{y^p - \delta_k - \bar{i}^r}{(1 + \tau_p)l_y y^p}, \quad u_{f0}^w = \bar{u}_f^w, \quad e_o = \bar{e}.$$

2. The steady state is locally asymptotically stable if $\beta_{w_u}\bar{u}_f^w > \beta_{w_e}\bar{e}$ holds.

3. At the value $\beta_{w_u}^H = \beta_{w_e}\bar{e}/\bar{u}_f^w$ of the parameter β_{w_u} there occurs a Hopf bifurcation, a cyclical loss of stability, of either subcritical, supercritical or degenerate type.

Proof:

1. Obvious.

2. The Jacobian of the dynamics at the steady state is given by

$$J = \begin{pmatrix} 0 & \beta_{w_u}\omega_o & \beta_{w_e}\omega_o \\ -\alpha_1^k(1 + \tau_p)l_y y^p u_o^w & -\beta_l u_o^w & 0 \\ 0 & \beta_l u_o & 0 \end{pmatrix}$$

Therefore, $-a_1 =$ trace $J = -\beta_l u_o^w < 0$ and $-a_3 =$ det $J = -\beta_{w_e}\omega_o\alpha_1^k(1+\tau_p)l_y y^p u_o^w \beta_l e_o < 0$. For a_2 (the sum of the principal minors) we get $a_2 = \beta_{w_u}\omega_o\alpha_1^k(1 + \tau_p)l_y y^p u_o^w > 0$. According to the Routh–Hurwitz conditions – see Gantmacher (1959) – we have to consider in addition the positivity of

$$a_1 a_2 - a_3 = \beta_l u_o^w \omega_o \alpha_1^k(1 + \tau_p)l_y y^p(\beta_{w_u}\bar{u}_f^w - \beta_{w_e}\bar{u}_o).$$

Hence $a_1 a_2 - a_3 > 0$ if $\beta_{w_u}\bar{u}_f^w > \beta_{w_e}\bar{u}_o$. The assertion of a Hopf bifurcation at $\beta_{w_u} = \beta_{w_e}\bar{e}_o/\bar{u}_f^w$ is then proved by means of the above expression for $a_1 a_2 - a_3$ as in Benhabib and Miyao (1981).

3. As in Benhabib and Miyao (1981), due to the above expression for $a_1 a_2 - a_3$. □

We thus in particular have that fast inside wage adjustments speeds, β_{w_u}, are enhancing local asymptotic stability, while the opposite holds true for the adjustment speed of outside wage claims, β_{w_e}. All other parameters of these dynamics (up to the levels of the employed rates of Non-Accelerating Inflation Rate of Unemployment (NAIRU) type) do not matter for the stability of this partial dynamics between real wages and the inside and outside rate of employment. This holds in particular for the speed of adjustment β_l of the hiring and firing policy of the firms. If the steady state is locally explosive, because of a high adjustment speed with respect to outside labour demand pressure, it is easy to establish global stability or boundedness of the dynamics through simple further (extrinsic) non-linearities as in Flaschel (2000), the main results of which read as follows:

Proposition 7.2 *1. The interior steady state of the dynamical system (7.37)–(7.39), with smooth factor substitution*[7] $z = f(k)$ *in the place of fixed coefficients in production, is of the same type as before, but now with the endogenous determination of the steady state ratios* $x_o = f(k_o)$, $y_o = f(k_o)/k_o$ *and* $k_o = k(\omega_o)$, *where* ω_o *is given by the solution of the equation* $\gamma/\alpha_r^k = f'(k(\omega_o))$.

2. This steady state is locally asymptotically stable if $\beta_{w_u} - \beta_{w_e}\bar{e} - c > 0$ *holds for some suitably chosen* $c > 0$. *The size of c can be chosen the larger the term* $\epsilon_s(\omega) = k'(\omega)\omega/k(\omega)$, β_l, β_{w_u} *becomes.*

Proof: See Flaschel (2000). □

Proposition 7.3 *1. The dynamical system (7.37)–(7.39) with smooth factor substitution and supply constraints exhibits the sub domain* $(0, \infty) \times (0, 2) \times (0, 1)$ *of its phase space as invariant subset which it therefore cannot leave. The dynamics can therefore be bounded to an economically meaningful domain and may, depending on parameters and functional shapes, give rise in this domain to a variety of simple or complex motions.*

2. The so-called classical regime of non-Walrasian disequilibrium analysis – see Flaschel (2000) for its definition – is the only regime that is possible in this domain.

Proof: See Flaschel (2000). □

Figure 7.1 provides an example for this type of bounded dynamics where the attracting set that is shown is still of a simple limit cycle type. The figure top left shows the stable limit cycle of the dynamics with the additional non-linearity in the growth rate of the labour supply assumed in Flaschel (2000), while the two cycles that border this figure show its projection into the adjacent planes, now with the variable v, the share of wages, in the place of ω, the real wage in order to show that profits remain positive along the cycle (and on the way to it). The figure bottom right finally shows the time series for the outside and inside rate of employment (with the full employment ceiling for

[7] Denoting by k the actually prevailing capital intensity K/L we know from the Solow model of neoclassical growth that this gives rise to equations of the type

$$z = f(k) \quad [y = f(k)/k], \ \omega = f(k) - f'(k)k = g(k).$$

For the function g there then holds

$$g'(k) = f'(k) - f'(k) - f''(k)k = -f''(k)k > 0$$

so that the function g is strictly increasing (due to decreasing marginal products of labour). We denote by $k = k(\omega)$ the inverse of g and by $\epsilon_s(\omega) = k'(\omega)\omega/k(\omega) > 0$ the elasticity of this function k.

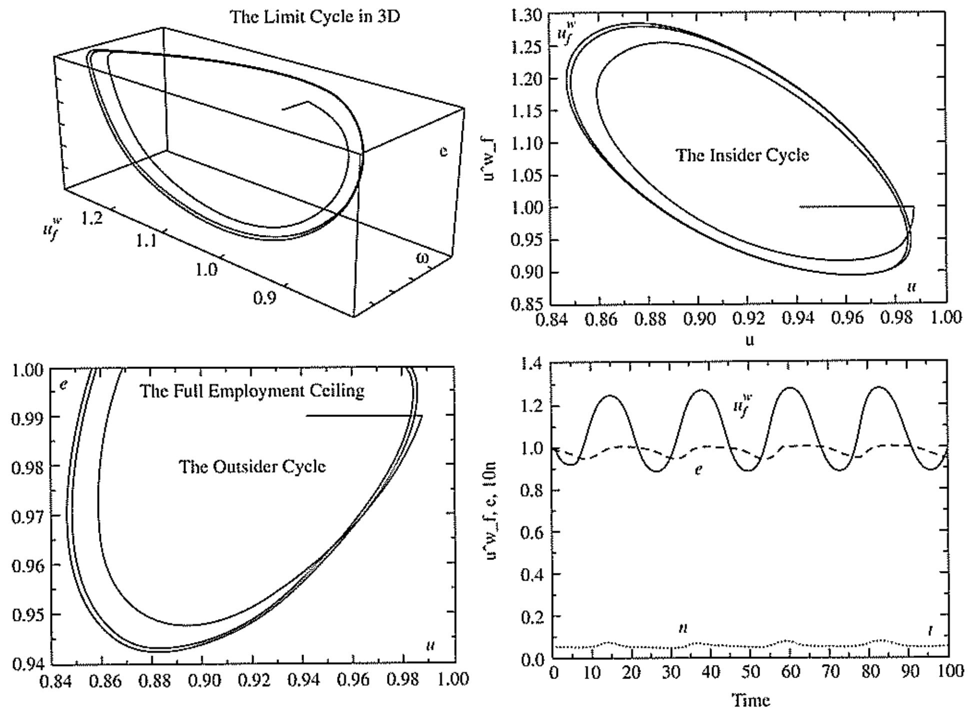

Figure 7.1 A limit cycle of the dynamics (7.37), (7.38), (7.39) showing the full employment ceiling

the first rate sometimes in operation and with a rate of employment of inside workers that stays below 130 per cent). When inside employment approaches this level it is furthermore clearly visible that the rate of growth of labour supply responds to this as assumed in Flaschel (2000) but as in the case of the inside rate of employment only in a moderate way in order to create the volume of labour supply and its rate of growth that is demanded by firms.

The result established by this section therefore is that labour demand pressure within the firms, when operating on wage adjustment in a sufficiently strong way, can destabilise the interior steady state of the model, but only locally if smooth factor substitution and certain labour supply adjustment processes are added to the model. This global aspect of the considered dynamics is investigated in detail in Flaschel (2000) and is therefore not considered here again, where local feedback mechanisms are at the centre of interest.

7.5 Adding the Rose real wage feedback chain

In order to sketch the details of this further economic feedback chain which now integrates goods market dynamics into the growth cycle dynamics of the preceding section we have to derive anew the law of motion for real wages from the wage-price dynamics first. Starting from the corresponding equations of the general 18D dynamics we have

$$\hat{w}^e - \pi^c = \beta_{w_e}(e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - \bar{u}_f^w) + \kappa_w(\hat{p}_y - \pi^c), \tag{7.40}$$

$$\hat{p}_y - \pi^c = \beta_p(y/y^p - \bar{u}) + \kappa_p(\hat{w}^e - \pi^c). \tag{7.41}$$

We can see that these equations form a linear equation system in the two unknowns: $\hat{w}^e - \pi^c$, $\hat{p}_y - \pi^c$. This system can be uniquely solved if $\kappa = 1 - \kappa_w\kappa_p \neq 0$ holds true for $\kappa_w, \kappa_p \in [0, 1]$, which is so if both of these parameter values are not equal to 1, meaning that the cost-push terms in both the wage and the price dynamics are not solely based on currently observed price and wage inflation rates. The explicit solution of this equation system is

$$\hat{w}^e - \pi^c = \kappa[\beta_{w_e}(e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - \bar{u}_f^w) + \kappa_w\beta_p(y/y^p - \bar{u})],$$

$$\hat{p}_y - \pi^c = \kappa[\kappa_p(\beta_{w_e}(e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - \bar{u}_f^w)) + \beta_p(y/y^p - \bar{u})],$$

which in turn implies for the real wage, $\omega^e = w^e/p_y$, measured in efficiency units and relative to producer prices, the expression

$$\hat{\omega}^e = \kappa[(1 - \kappa_p)(\beta_{w_e}(e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - \bar{u}_f^w)) - (1 - \kappa_w)\beta_p(y/y^p - \bar{u})].$$

The dynamics of the real wage therefore depend positively on the demand pressure in the market for labour and negatively on the demand pressure in the market for goods, while the cost-push terms of the nominal dynamics have neutralised themselves in this relative expression for the wage dynamics. The economic reason for and the meaning of this result is easy to understand, since real wages should generally also depend on what happens in the market for goods. It is therefore astonishing to see that studies of Phillips curves (PCs), that integrate labour market phenomena with price inflation, are often built on only one of these demand pressures (the one in the labour market) in the theoretical as well as in the empirically oriented literature.

On the basis of the foregoing discussion we can now describe the feedback chain of real wage increases onto their rate of change implied by the core 18D model of this chapter. Increases in real wages will either increase or decrease aggregate demand y^d for the domestic good (per unit of capital) depending among others on the consumption propensity c_y of workers in comparison with the marginal propensity to invest i_1 that mirrors the influence of the expected profit rate

$$r^e = y^e - \delta_k + (p_x/p_y)x - (1 + \tau_p)\omega^e l_f^{de} - (p_m/p_y)j^d$$

in the investment demand function of firms.

Let us consider first the situation where economic activity y or $u = y/y^p$ is reduced through this channel by real wage increases. We know from the model that this decreases the employment of the employed and with some time delay also the rate of outside employment e. According to the above dynamical law for real wages we thus get that real wage increases are slowed down if wage flexibility is high and price flexibility is low, since the money wage will then react more strongly than the price level to this

reduction in economic activity and will thus dominate the response of real wages to reduced economic activity. In this situation, the interaction between economic activity and real wages is therefore stabilising, since real wage increases are then checked by decreases in economic activity through their impact on real wages. However, in the opposite case of high price flexibility and low wage flexibility real wages will increase in the case of a reduction of economic activity and will thus amplify the initial increase in real wages, which creates a destabilising feedback chain between real wages and economic activity.

Consider next the case where economic activity increases with real wage increases, since consumption demand responds more strongly than investment demand of firms to changes in the real wage. Of course, we then get the opposite conclusions to the case just considered. Price flexibility will now be enhancing economic stability while wage flexibility will be detracting from it. We thus find that the real wage/economic activity interaction depends crucially on the parameters that characterise the market for goods and the labour market.

Either price or wage flexibility must however always be destabilising. The destabilising Rose effect (of whatever type) will be weak if both wage and price adjustment speeds $\beta_{w_e}, \beta_{w_u}, \beta_p$ are low, at least as far as situations of a depressed economy are concerned. The working of the Rose mechanism in integrated models of monetary growth is explored in detail in Flaschel *et al.* (1997) and Chiarella and Flaschel (2000), where also the original approach of Rose (1967) is reconsidered and discussed.

We now go on to consider the situation in which we add growth dynamics to the above considerations as in the considered extended Goodwin (1967) case, but will now neglect inside employment adjustments ($\beta_{w_u} = 0$) so that increases in the output of firms are immediately transferred to new employment with respect to the external labour market. We thus assume $l_f^{de} = l_f^{we}$ and again neglect any employment in the government sector. This gives rise to the following growth dynamics

$$\hat{\omega}^e = \kappa[(1 - \kappa_p)(\beta_{w_e}(l_f^{de}/l^e - \bar{e}) - (1 - \kappa_w)\beta_p(y/y^p - \bar{u})],$$

$$\hat{l}^e = \gamma + \delta_k - \alpha_1^k(r^e - \bar{i}^r) - \alpha_3^k(y/y^p - \bar{u}),$$

where $l^{de} = l_y y$ and $r^e = y - \delta_k - (1 + \tau_p)\omega^e l_f^{de}$ and where the second law of motion is derived as usual from the growth law for the capital stock. Note here that we have included now the third term $\alpha_2^k (> 0)$ of the fixed business investment function again, since the rate of capacity utilisation is variable in the Rose type labour and goods market interaction. Note also that we do not distinguish here between output and the (expected) demand for goods and thus ignore the quantity adjustment process of firms and the details of the formation of aggregate demand. Instead we shall now simply assume that output per capital y is a function of real wages ω^e measured in efficiency units and that this function is increasing if we assume that the impact of real wage changes on y is positive (if consumption demand is more sensitive than investment demand to real wage changes) while it is decreasing in the opposite case.

We thereby arrive at the autonomous non-linear system of differential equations of dimension 2,

$$\hat{\omega}^e = \kappa[(1-\kappa_p)\beta_{w_e}(l_y y(\omega^e)/l^e - \bar{e}) - (1-\kappa_w)\beta_p(y(\omega^e)/y^p - \bar{u})], \quad (7.42)$$

$$\hat{l}^e = \gamma + \delta_k - \alpha_1^k(y(\omega^e) - \delta_k - (1+\tau_p)\omega^e l_y y(\omega^e) - \bar{i}^r) - \alpha_3^k(y(\omega^e)/y^p - \bar{u}), \quad (7.43)$$

in the two state variables ω^e, l^e. We consider here only the case where the rate of profit $r^e = y(\omega^e) - \delta_k - (1+\tau_p)\omega^e l_y y(\omega^e)$ depends negatively on the real wage ω^e, namely where the mass purchasing effect of real wage increases is not so large that it outweighs the wage cost effect on the rate of profit.[8] In this case we get for the Jacobian J of the above 2D dynamics at the steady state the following sign structure (that is typical for the Rose (1967) employment cycle mechanism)

$$J = \begin{pmatrix} \kappa[(1-\kappa_p)\beta_{w_e} l_y y(\omega^e)/ \\ l^e - (1-\kappa_w)\beta_p y'(\omega^e)/y^p] & -\kappa(1-\kappa_p)\beta_{w_e} l_y y(\omega^e)/(l^e)^2 \\ -\alpha_1^k (r^e)'(\omega^e) - \alpha_3^k y'(\omega^e)/y^p & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \pm & - \\ + & 0 \end{pmatrix}.$$

The sign of J_{11} in the trace is therefore the decisive element that determines the local stability or instability of the Rose real wage mechanism in isolation as well as its interaction with economic growth.

A wage mechanism of the extrinsically non-linear type shown in Figure 7.2, as it was used in Rose (1967),[9] will therefore generally only be successful in bringing boundedness to the dynamics of the economy for a given speed of adjustment β_p of the price inflation rate when the sign in J_{11} is negative. If the opposite holds true, then one would need an extrinsic non-linearity with respect to price adjustment, not wage adjustment, which would need to be assumed to be limited in its speed, in order to get boundedness for the dynamics in this case. Depending on propensities to consume and to invest we therefore have to assume more flexible adjustments in the labour market or in the market for goods in order to get boundedness of the dynamics. Not knowing which situation in fact prevails it may therefore be best to assume that both speeds of adjustment are fairly low in which case the entry J_{11} in the trace of J will be small, but positive or negative. It can then be hoped that this germ of instability – which in the general 18D dynamics need not and will not appear in the trace of their Jacobian – is overcome by the other stabilising mechanisms in the dynamics.

8 In addition the parameter α_3^k needs to be chosen sufficiently small.
9 The function displayed, $\beta_{w_e}(e)$, replaces the linear term $\beta_{w_e}(e-\bar{e})$ in equation (7.40).

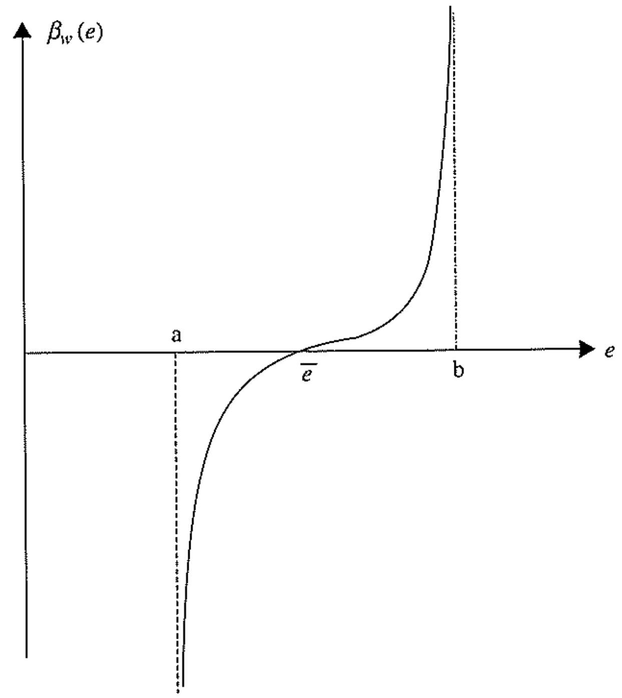

Figure 7.2 A non-linear law of demand in the labour market

If we however are sure that y and therefore also r^e depend negatively on the real wage ω^e, the above dynamical system (7.42), (7.43) will imply the phase portrait representation shown in Figure 7.3.[10]

This diagram indicates that the growth dynamics of this section are globally asymptotically stable if price flexibility is sufficiently low such that the steady state of these dynamics is locally asymptotically stable. Then – as we shall assert without proof – the growth dynamics are also globally asymptotically stable with respect to the triangular domain shown in Figure 7.3 and only globally stable. The dynamics will give rise to stable limit cycles around the steady state if the steady state is locally a repeller (which could occur for a given value of price flexibility that is chosen sufficiently high).[11]

10 We neglect technical change and therefore efficiency units in Figure 7.3.
11 The isoclines $\dot{\omega}^e = 0, \dot{l}^e = 0$ of (7.42), (7.43) are given by

$$l^e = l_y y(\omega^e)\left(\beta_w^{-1}\left[\frac{1-\kappa_w}{1-\kappa_p}\beta_p\right] + \bar{e}\right)^{-1},$$

$$\omega^e = \omega_o^e$$

where ω_o^e is given by that level of real wages that implies the required rate of profit $\bar{i}^r$. Due to the assumed shape of the β_{w_e} function we know that the first expression is always well defined and must always lie between $y(\omega^e)/b$ and $y(\omega^e)/a$; see Figure 7.2. The above two isoclines then divide the phase space as shown in Figure 7.3.

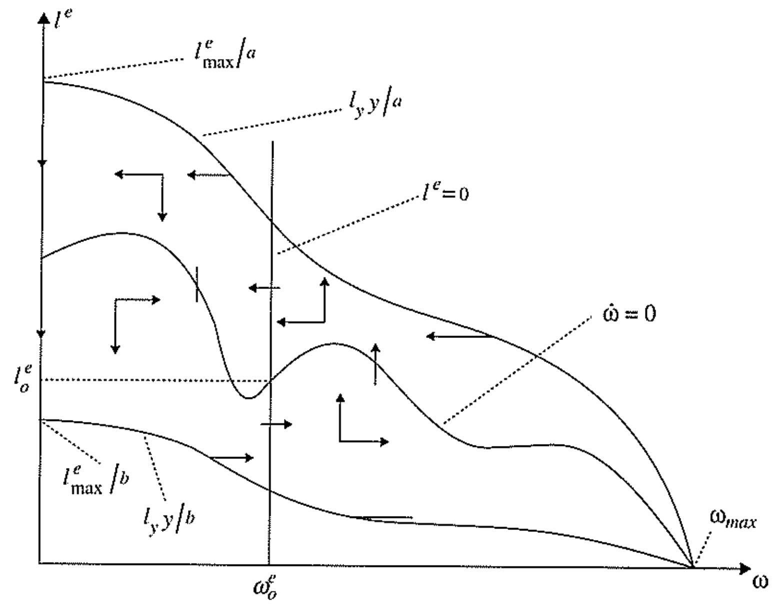

Figure 7.3 The viability domain of the Rose dynamics for $y'(\omega^e) < 0$

The details underlying the construction of the viability domain shown in Figure 7.3 are given in also Chiarella and Flaschel (2000, Ch. 5). We conclude from the above that large values of β_{w_u} and either β_{w_e} or β_p provide problems for the local asymptotic stability of the interior steady state of the dynamics, but that extrinsic non-linearities in fact can be tailored so that they tame local explosiveness and bound the considered growth dynamics to an economically meaningful domain.

7.6 The Metzlerian expected sales/inventory dynamics

In order to isolate this mechanism we assume that fixed business investment is given by its trend component solely: $g_k^d = \gamma + \delta_k$. In this case we get for the interaction of expected sales y^e and actual inventories ν from the 18D core dynamics of the general model, the equation system

$$\dot{y}^e = \beta_{y^e}(y^d - y^e),$$
$$\dot{\nu} = y - y^d - \gamma\nu$$
$$y^d = c_w^o + \gamma + \delta_k + \gamma k_h + \delta_h + g y^e,$$
$$c_w^o = c_y y_{w1}^D,$$
$$y_{w1}^D = (1-\tau_w)[w^e l_y y + w^{ue}(l^e - l^{we}) + w^{re}\alpha_l l_2^e]/p_v,$$
$$y = y^e + \beta_n(\beta_{n^d} y^e - \nu) + \gamma\beta_{n^d} y^e.$$

These equations provide us with two linear differential equations in the state variables expected sales and actual inventories (per unit of capital). To simplify our argument we have ignored here the delay in the firms' employment policies and of course do not take wage-price reactions to the change in economic activity into account in the investigation of this partial feedback structure. It is then easy to see that a sufficiently large adjustment parameter value β_n (which can approach infinity in continuous-time if this is needed) implies that the dependence of y^d on y and thus on y^e obtains a slope that is larger than one, in which case the law of motion for y^e then depends positively on the size of y^e, or in other words, the entry J_{11} of the Jacobian J of the above dynamics at the steady state becomes positive under these circumstances. We conclude that the trace of J must then become positive if the parameter β_{y^e} is chosen sufficiently large in addition, since this parameter is not involved in the second component that defines the trace of J.

The above equations for the 2D inventory dynamics thus show that output y depends positively on expected sales y^e and this more and more strongly the higher the speed of adjustment β_n of planned inventories becomes. The time rate of change of expected sales therefore depends positively on the level of expected sales when the parameter β_n is chosen sufficiently large. Flexible adjustment of inventories coupled with a high speed of adjustment of sales expectations are thus bad for obtaining economic stability. There will, however, exist other situations (with a low inventory accelerator) where an increase in the latter speed of adjustment may increase the stability of the dynamics. The question of local explosiveness and global boundedness of this inventory cycle dynamics has been discussed in detail in Franke and Lux (1993), Franke (1996) and Chiarella and Flaschel (2000, Ch. 6).

In view of these contributions we add the following further simplified modelling and analysis to the observations just made. We now assume, as a simple expression for aggregate demand in the place of the static equations employed above, that there holds

$$y^d = d_1 y + d_0, \quad \text{with} \quad d_0 > 0, d_1 \in (0,1)$$

and also set the exogenous growth rate γ of the general 18D model equal to zero. On this basis, the isolated inventory dynamics of the model read

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) = \beta_{y^e}(d_1 y + d_0 - y^e), \tag{7.44}$$
$$\dot{\nu} = y - y^d = (1-d_1)y - d_0, \tag{7.45}$$

where the output y produced by firms is again given by

$$y = y^e + \beta_n(\beta_{n^d} y^e - \nu), \tag{7.46}$$

that is as the sum of expected sales and intended inventory changes. The steady state of the model (7.44)–(7.46) of sales and inventory dynamics is given by

$$y_o^d = y_o^e = y_o = \frac{1}{1-d_1} d_0, \qquad \nu_o = \beta_{n^d} y_o.$$

Inserting (7.46) into (7.44), (7.45) one obtains a linear system of two differential equations with the matrix of partial derivatives

$$J = \begin{pmatrix} (d_1 - 1 + d_1\beta_n\beta_{n^d})\beta_{y^e} & -d_1\beta_n\beta_{y^e} \\ (1-d_1)(1+\beta_n\beta_{n^d}) & -(1-d_1)\beta_n \end{pmatrix}.$$

For the determinant of this matrix we find

$$|J| = \beta_{y^e}(1-d_1)\beta_n > 0.$$

and for the trace

$$\text{trace } J = -\beta_{y^e}(1 - d_1 - \beta_n\beta_{n^d}d_1) - \beta_n(1-d_1).$$

From these expressions we see that the dynamics (7.44), (7.45) can only be unstable if $1 - d_1 - \beta_n\beta_{n^d}d_1 < 0$ holds and then if and only if

$$\beta_{y^e} > \beta_{y^e}^H = -\frac{\beta_n(1-d_1)}{1 - d_1 - \beta_n\beta_{n^d}d_1}$$

applies. Since we want to show in this section the existence of inventory oscillations and in the limit also of relaxation oscillations for inventory dynamics of sales expectations, we assume in the following that the adjustments speed β_n for inventory changes satisfies

$$\beta_n > \frac{1-d_1}{d_1\beta_{n^d}} > 0$$

at the steady state, which means that the above necessary condition for instability is fulfilled at the steady state. In this case, the dynamics (7.44), (7.45) would be totally unstable when sales expectations are adjusted with a sufficiently high speed. In view of such a situation, Franke and Lux (1993) assume that the inventory adjustment speed that firms choose is slowed down the further the economy departs from the steady state (since firms become more cautious then). We add to this assumption the motivation that firms slow down their inventory adjustment far off the steady state, since they expect a turn in economic activity which would by itself give rise to the desired direction of inventory changes. In making this assumption the action of firms will then in fact lead to such a turn in economic activity which thus confirms the cautious policy adopted by firms. In their paper, Franke and Lux (1993) present a set of related assumptions which in sum allow them to show that despite local instability, such an inventory dynamics will be globally stable or viable and give rise to persistent oscillations (or relaxation oscillations when expectations tend to myopic perfect foresight).

In the following, we will not reproduce the details of such an analysis, but only sketch in an intuitive way[12] how such global boundedness of the dynamics and the implied limit cycle (or limit limit cycle in the limit case of relaxation oscillations) can be obtained in principle.

[12] We also appeal to the special example of the non-linearity introduced by Franke and Lux in the adjustment speed of inventories.

To this end, we choose for the adjustment speed β_n of inventories the functional form of their dependence on sales expectations (per unit of capital) given by

$$\beta_n = (\beta_n^* - \beta_n^0)\exp(-\beta_n^1(y^e - y_o^e)^2) + \beta_n^0, \tag{7.47}$$

where $\beta_n^* - \beta_n^0 > \frac{1-d_1}{d_1\beta_{n^d}}$ and $0 < \beta_n^0 < \frac{1-d_1}{d_1\beta_{n^d}}$ holds.

On the basis of this modification of the model, the dynamics (7.44), (7.45) become non-linear, with a steady state that is identical to the one of the linear system and which must be unstable for a high adjustment speed of sales expectation since the Jacobian J is the same for the linear and the non-linear system at the steady state. It is not difficult to show that this non-linear version of the inventory dynamics implies again the existence of persistent fluctuations or even relaxation oscillations, as in Kaldor (1940) and by means of diagrams as in Chiarella and Flaschel (2000) and Asada *et al.* (2003). This analysis is here simply exemplified for the limit case of relaxation oscillations by means of the numerical simulation shown in Figure 7.4.

The bottom left panel of Figure 7.4 shows the relaxation oscillations in sales expectations y^e. Inventories ν, on the other hand, exhibit no jump in their levels (as is reasonable), but of course their growth rate is subject to such jumps whenever a regime switch occurs in the perfect foresight regime from optimistic (nearly perfect) sales expectations to pessimistic ones and vice versa. The bottom right panel of Figure 7.4 however reveals that such sales expectations are not always perfect, since sales expectations may overshoot aggregate demand during a regime switch – at least for the discretisation we have chosen here to simulate this model.

Figure 7.4, at the top right, shows the development of output as compared with sales expectations (and aggregate demand). Of course, the path of output must depart in a systematic way from that of expected sales, since firms pursue an active inventory policy. Finally, the top-left panel of Figure 7.4 shows again the relaxation cycle in the phase space, revealing part of the y^e isocline as well as of the nearly horizontal adjustments that occur in sales expectations when phases of boom give way to phases of recession or depression by way of a regime switch in sales expectations.

The panels of Figure 7.4 also show that the cycle period is approximately three years. We note that the phase length of this cycle can be decreased if the parameter β_{n^d} is reduced in size. Of course, the amplitude of the cycle is completely determined by the shape of the non-linearity that has been assumed for its generation; see equation (7.47) for the parameter β_n.

The discussion of this section may be summarised in the following proposition:

Proposition 7.4 *The dynamics of the Metzlerian inventory feedback mechanism are dominated by a trade-off between β_{ye} (speed of adjustment of sales expectations) and β_n (speed of adjustment of planned inventories). At low values of both of these parameters, this mechanism is locally stable. For β_n larger than a certain value, β_{ye} acts as a bifurcation parameter, giving rise to local instability and limit cycles via a Hopf bifurcation beyond a certain critical value.*

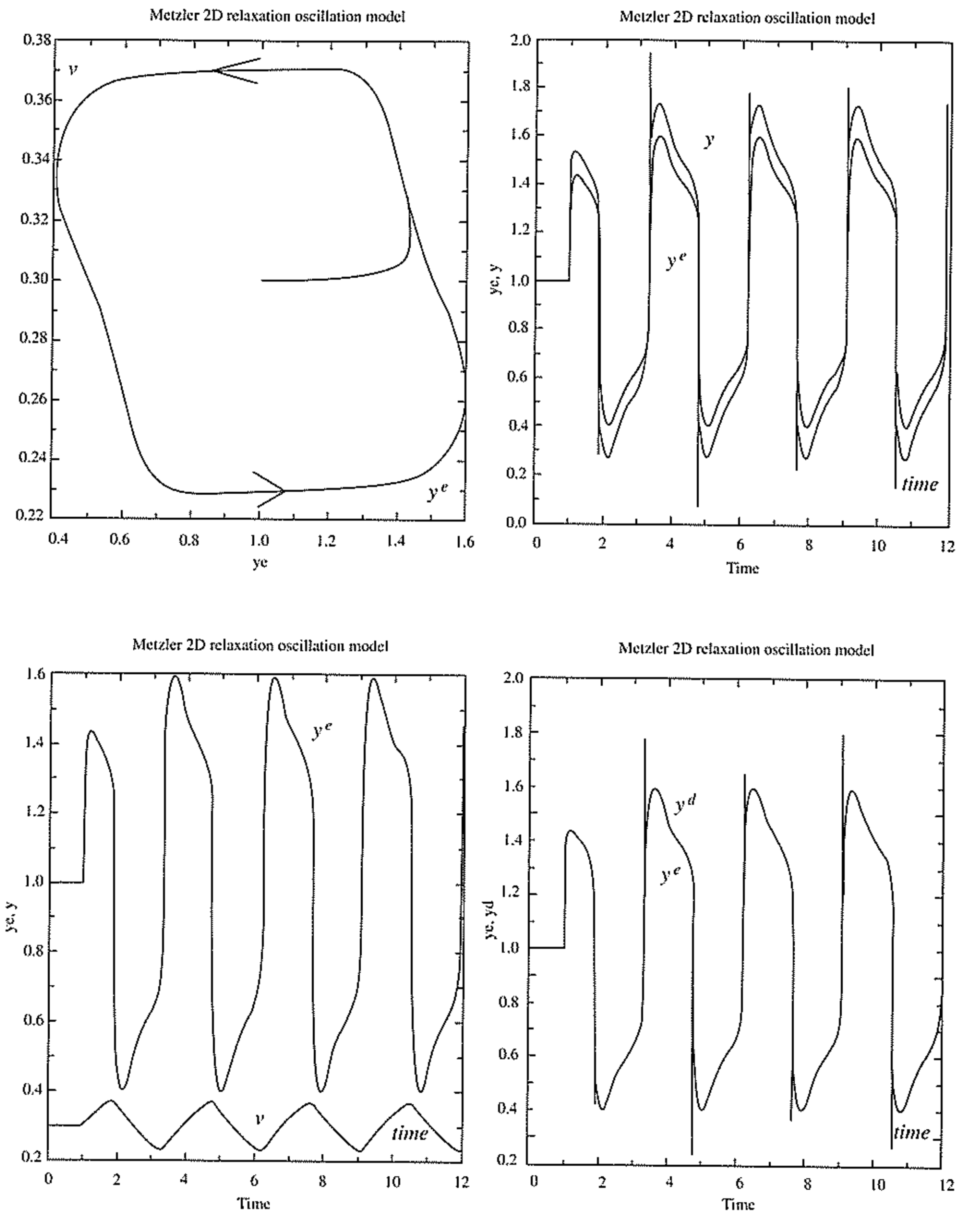

Figure 7.4 A numerical representation of the limiting relaxation oscillations in the Metzlerian 2D dynamics

7.7 The dynamics of housing supply

In order to study the dynamics of housing investment and housing rents in isolation we assume that the capital stock of firms grows with constant rate $\gamma = g_k^d - \delta_k$, that the rate of investment in dwellings does not depend on bond interest rates and that the price

level of domestic goods (and thus of the supply of dwellings) p_y is a given magnitude. Furthermore, total wage income of workers (per unit of capital) is held constant at $\bar{y}_{w1}^D$. The laws of motions in the housing sector are in this case given by[13]

$$\hat{k}_h = g_h^d - \delta_h - \gamma = \alpha_r^h(r_h - \bar{i}^r) + \alpha_u^h\left(\frac{c_h^{do}}{k_h} - \bar{u}_h\right), \qquad (7.48)$$

$$\hat{p}_h = \beta_h\left(\frac{c_h^{do}}{k_h} - \bar{u}_h\right), \qquad (7.49)$$

with

$$r_h = c_h\bar{y}_{w1}^D/k_h - \delta_h,$$
$$c_h^{do} = \bar{p}_v c_h \bar{y}_{w1}^D/p_h.$$

This system of equations can be reduced to two autonomous differential equations in the state variables k_h, p_h with a uniquely determined point of rest in the positive domain of the real plane, given by

$$k_{h0} = c_h\bar{y}_{w1}^D/(\bar{i}^r + \delta_h), \quad p_{h0} = \bar{p}_v c_h\bar{y}_{w1}^D/(k_{h0}\bar{u}_h),$$

due to
$$r_h = \bar{i}^r = c_h\bar{y}_{w1}^D/k_h - \delta_h.$$

Proposition 7.5 *The interior steady state of the dynamics of the housing sector, given by (7.48) and (7.49), is always locally asymptotically stable with monotonic convergence back to the steady state for small displacements out of the steady state, so that it is a stable node.*[14]

Proof: Let us first consider the above dynamics in the case $\alpha_3 = 0$. In this case we get for the Jacobian of the dynamics at the steady state the sign structure

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} - & 0 \\ - & - \end{pmatrix}.$$

This implies the assertion of Proposition 7.5 in this special case, since such a dynamical system has only real eigenvalues with negative sign due to

$$(\text{trace } J)^2/4 - \det J = (J_{11} - J_{22})^2/4 \geq 0.$$

In the case $\alpha_3 > 0$ it is however easy to see that the trace will become more negative than compared with the situation when $\alpha_3 = 0$ while the determinant remains unchanged, which implies local asymptotic stability also in this case, and of course

$$(\text{trace } J)^2/4 - \det J = (J_{11} - J_{22})^2/4 > 0.$$

□

[13] All tax rates are set equal to zero in the present subsystem of the general 18D dynamics.
[14] We here only assert that these dynamics are also globally stable and that this can be proved by means of Olech's theorem in a similar way to what is presented in Flaschel (1984).

The adjustments that take place in the stock of dwellings and the price of dwelling services thus do not give rise to the results we derived for business fixed capital investment (as in Goodwin (1967)) and for the Tobin (1975) nominal dynamics considered below, since there is no labour market involved in this subdynamics and also no inflationary expectations mechanism with respect to p_h. The latter may be considered a shortcoming of the present model type which should find better treatment in later reformulations of this macroeconomic dynamics.

Adjustment processes in this segment of the economy therefore do not cause problems for economic stability, at least as the model is presently formulated.

7.8 The Keynes effect

In order to discuss this effect in the conventional way one has to reformulate the model by means of a money supply rule and IS–LM equilibrium (in the place of the Taylor interest rate policy rule that we have used in the general 18D dynamics of Section 7.2). According to conventional LM equilibrium the nominal rate of interest i depends positively on the price level p, with all other variables kept fixed. Aggregate demand and thus output and the rates of capacity utilisation therefore depend negatively on the price level implying a negative dependence of the inflation rate on the level of prices through this channel. A high sensitivity of the nominal rate of interest with respect to the price level, the opposite of the liquidity trap, should thus exercise a strong stabilising influence on the dynamics of the price level and on the economy as a whole, which is further strengthened if price and wage flexibility increase. We expect that this effect is also present (in modified form) in the case of the interest rate policy rule, as we shall show succinctly below.

Monetary policy rules that attempt to control money supply in order to achieve inflationary and real stability may for example be formulated as

$$\hat{M} = \mu = \mu_o + \beta_m(\bar{\pi} - \hat{p}), \tag{7.50}$$

$$\hat{M} = \mu = \mu_o + \beta_m(\mu_o - n - \hat{p}), \tag{7.51}$$

$$\hat{M} = \mu = \mu_o + \beta_m(\bar{\pi} - \hat{p}^*). \tag{7.52}$$

The first rule (equation (7.50)) states that the growth rate of money supply is adjusted in view of the discrepancy that exists between the target rate of inflation of the central bank and the actual rate of inflation, implying for example that the growth rate of the money supply is reduced below its desired level μ_o if the actual rate of inflation exceeds the target rate (based on the assumption that this will dampen economic activity and the inflationary pressure that derives from it). The second rule (equation (7.51)) can be considered a special case of the first one, since it uses the steady state rate of inflation as the target rate. The third rule (equation (7.52)) assumes the so-called p^* theory of the rate of inflation, which assumes that the actual rate of inflation will tend to the p^* rate $\hat{p}^*$ as its centre of gravity where p^* is given by the application of the quantity

theory of money to the case where the economy operates at its potential output level (or a certain fixed percentage below it). This would therefore give

$$p^* = vM/Y^p, \quad \text{so that,} \quad \hat{p}^* = \mu - \hat{K},$$

where v denotes the constant velocity of money[15] and Y^p the potential output of firms which is a fixed multiple of the capital stock K in our model. Of course, we could have also used the rate π^c, expected to hold for the medium run, in the place of the actual rate of inflation or the p^* rate (where π^c is in turn partly based on the knowledge of $\hat{p}^*$). Equations (7.50), (7.51) and (7.52) show that there is a variety of possibilities by which a money supply rule that pays attention to economic activity and inflationary pressure can be formulated.

Yet, in our general model, presented in Section 7.2 and in Chiarella and Flaschel (1999b,c,d), we have adopted a different approach to monetary policy that is based on a direct setting of the short-run rate of interest (or its time rate of change) in the place of the more indirect approach that attempts to influence this rate of interest via changes in the supply of money as described above. We believe that this latter approach is, on the one hand, less ambiguous as far as the definition of the instrument that is being used is concerned and, on the other hand, more successful in the attempt to stabilise the economy and to remove inflationary pressure from it. Furthermore in Chiarella and Flaschel (1999b,c,d) we have treated money in a way that makes it redundant in the presentation of the structural equations of the model.

In order to show this last point in as simple a way as possible consider again our formulation of the Taylor interest rate policy rule, given by

$$\dot{i} = -\beta_{i_i}(i - i_l^*) + \beta_{i_p}(\hat{p}_y - 0) + \beta_{i_u}(y/y^p - \bar{u})$$

in the light of the definition of aggregate demand, which we recall in

$$\begin{aligned}
y^d &= c_w^o + g_k^d + g_h^d k_h + g y^e, \\
g_k^d &= \alpha_1^k(r^e - i^r) + \alpha_2^k(i_l - i) + \alpha_3^k(y/y^p - \bar{u}) + \gamma + \delta_k, \\
g_h^d &= \alpha_r^h(r_h - i^r) + \alpha_i^h(i_l - i) + \alpha_u^h\left(\frac{c_h^{do}}{k_h} - \bar{u}_h\right), \\
i^r &= i_l - \pi^c.
\end{aligned}$$

Keeping all variables fixed at their steady state values, with the exception of the short-run rate of interest rate i, immediately implies the conventional situation that aggregate demand depends negatively on this nominal rate of interest – due to investment behaviour of firms and in dwellings. This implies again that expected sales and actual output of firms will also respond (*ceteris paribus*) negatively to increases in the

15 Here assumed to be constant for simplicity of exposition.

nominal interest rate i, although in a somewhat delayed fashion, depending on adjustment speeds of the sales expectations of firms. Furthermore, domestic price inflation $\hat{p}_y$ depends positively on the degree of capacity utilisation of firms.

Taken together, this implies that there are only negative feedback mechanisms involved in the above formulation of the Taylor rule, so that inflation or capacity utilisation rates above the norm will be diminished through this interest rate policy of the central bank (the opposite is of course true in the case of price deflation and less than normal activity of firms). The Taylor rule therefore has the same stabilising features as is known for the Keynes effect, and it has the additional advantage that it incorporates these features in the form of a more direct steering of the economy than a rule that attempts to control the money supply in a counter-cyclical fashion. Note again that this stabilising adjustment process will not simply appear as a negative trace component in the local stability analysis of the full 18D dynamical system.

7.9 The Mundell–Tobin effect

The assumption that $\kappa_p = 0, \alpha_{\pi^c} = 1$ in the 18D model of Section 7.3 implies that equation (7.25) for the evolution of adaptively formed inflationary expectations π^c becomes

$$\dot{\pi}^c = \beta_{\pi^c}\beta_p(y/y^p - \bar{u}).$$

In order to derive from this formula the basic way in which $\dot{\pi}^c$ is influenced by the level of π^c, we make the further assumption that there holds the simplified equations

$$\begin{aligned} y^d &= const. + g_k^d + g_h^d k_h, \\ g_k^d &= \alpha_1^k(\bar{r}^e - (\bar{i}_l - \pi^c)) + \gamma + \delta_k, \\ g_h^d &= \alpha_r^h(\bar{r}_h - (\bar{i}_l - \pi^c)) + \gamma + \delta_h. \end{aligned}$$

In these expressions we consider only investment expenditures as variable and only in their dependence on the expected rate of inflation π^c, that is we allow only for profitability effects in investment behaviour by freezing the rate of profit r^e and the long-term rate of interest i_l at their steady state values (k_h is also considered a given magnitude). On the basis of these simplifications we see that aggregate demand (per unit of capital) depends positively on the expected rate of inflation, since an increase in this rate improves the profitability differential and thus increases the rate of investment, both for fixed business investment and housing. Since the sales expectations mechanism implies that sales expectations y^e (and thus also output y) follow aggregate demand y^d with a time delay, we get from that a (delayed) positive response of output to increases in inflationary expectations and hence (according to the above law of motion for these expectations) a positive feedback of inflationary expectations on their time rate of change. Due to the delays just discussed this dependence is an indirect one, so that it does not show up in the trace of the Jacobian of the dynamics at the steady state, but is distributed in a specific way in the Jacobian's off-diagonal elements.

Nevertheless, we have that increases in inflationary expectations have a (somewhat delayed) positive effect on economic activity and, due to the additional inflationary pressure this creates, a positive impact effect on their time rate of change. This effect has come to be known as the Mundell effect in the literature. Its working in a Keynesian dynamic multiplier/money market equilibrium framework with a Friedmanian type of PC has been investigated in a 3D approach to such Keynesian/Monetarist dynamics in a prominent paper by Tobin (1975). In that paper a critical level of the adjustment speed of inflationary expectations is determined at which the system loses its local stability by way of a Hopf bifurcation, similar to Proposition 7.1 that we have established in Section 7.4 for the adjustment speeds in the wage formation process. We claim – but do not prove here – that the destabilising Mundell effect becomes dominant also in our 18D core dynamics of Section 7.3 if the parameter β_{π^c} is chosen sufficiently large (for $\alpha_{\pi^c} > 0$.)

The destabilising Mundell effect can be checked if an interest rate policy rule is chosen that attempts to steer the expected real rate of interest as in Flaschel and Groh (1997). Such a feedback policy rule counteracts the source of the inflationary process which lies in the expansionary forces that are created by inflation and which in turn further stimulate the inflationary climate already in existence. We shall demonstrate this briefly here for the type of interest rate policy rule we have introduced in the preceding section.

For this purpose we assume that there is no delayed quantity adjustment process of Metzlerian type, but that there is goods market equilibrium $y = y^e = y^d$ throughout, based on the aggregate demand function y^d of this section, which is now however augmented by the terms that describe the sensitivity of investment with respect to short-term interest and the monetary policy rule applied by the central bank. In sum this gives rise to the dynamical system

$$\begin{aligned} \dot{\pi}^c &= \beta_{\pi^c}\beta_p(y^d/y^p - \bar{u}), \\ y^d &= const. + g_k^d + g_h^d k_h, \\ g_k^d &= \alpha_1^k(\bar{r}^e - (\bar{i}_l - \pi^c)) + \alpha_2^k(\bar{i}_l - i) + \gamma + \delta_k, \\ g_h^d &= \alpha_r^h(\bar{r}_h - (\bar{i}_l - \pi^c)) + \alpha_i^h(\bar{i}_l - i) + \gamma + \delta_h, \\ \hat{p}_y &= \beta_p(y^d/y^p - \bar{u}) + \pi^c, \\ \dot{i} &= -\beta_{i_i}(i - i_o) + \beta_{i_p}(\hat{p}_y - 0) + \beta_{i_u}(y^d/y^p - \bar{u}). \end{aligned}$$

This system can be reduced to a two-dimensional system in the two state variables π^c, i as

$$\dot{\pi}^c = \beta_{\pi^c}\beta_p(y^d(\pi^c, i)/y^p - \bar{u}) \tag{7.53}$$

$$\dot{i} = -\beta_{i_i}(i - i_o) + (\beta_{i_p}\beta_p + \beta_{i_u})(y^d/y^p - \bar{u}) + \beta_{i_p}\pi^c \tag{7.54}$$

where $y^d(\pi^c, i)$ is given by the linear function

$$y^d(\pi^c, i) = const. + \alpha_1^k \pi^c - \alpha_2^k i + \alpha_r^h \bar{k}_h \pi^c - \alpha_i^h \bar{k}_h i.$$

The system (7.53), (7.54) is thus a linear system of differential equations with a system matrix J that obviously has a positive determinant if β_{i_i} is zero (or chosen sufficiently small).[16] For the trace of this matrix J in the case of $\beta_{i_i} = 0$ one finds

$$-(\beta_{i_p}\beta_p + \beta_{i_u})(\alpha_2^k + \alpha_i^h)/y^p + \beta_{\pi^c}\beta_p(\alpha_1^k + \alpha_r^h)/y^p,$$

which implies that any instability that is caused by the positive term J_{22} of the matrix J can in principle (but maybe not in practice) be overcome and neutralised by choosing the policy parameters β_{i_p}, β_{i_u} sufficiently large – if and only if investment is influenced by short-term interest rate changes. We thus see how the destabilising Mundell effect of inflationary expectations may be overcome by a policy that makes the usual Keynes effect of models of IS–LM type sufficiently large by way of an appropriately tailored monetary policy rule.

7.10 The Blanchard bond and stock market dynamics

Blanchard (1981) has investigated the dynamic adjustment processes in the market for long-term bonds and for equities on the basis of myopic perfect foresight and perfect asset substitutability by means of the saddlepath dynamics that is then present and the jump variable technique that is then typically applied in order to have asymptotically stable adjustment processes after the occurrence of unanticipated shocks or changes in the expectations of future events.

We have assumed in the presentation of the structural form of our model that rate of return differentials are not instantaneously removed, but give rise to somewhat delayed adjustments in asset prices. We have also argued that there are always heterogeneous expectations present, here of asset holders who fall into two groups – ambitious agents who devote significant parts and their time (and resources) to the effort of forming perfect anticipations, and less ambitious (or perhaps less well-informed) asset holders who behave in an adaptive fashion. We have argued furthermore that the market share of the latter agents, despite their less accurate predictions of asset price dynamics, does not tend to zero due to the fact that all asset owners have a life-cycle profile that lets them act in an ambitious fashion when they are young and in a less ambitious fashion when they become old (due to changes in their preference relations). Although ambitious agents have more profitable investments (a fact that is only implicitly present in our model) their influence is bounded since they become less ambitious later on.

16 It is however interesting to see that a parameter β_{i_i} that is chosen too large may lead to saddlepath instability of the steady state solution.

For the laws of motion for the price of long-term bonds and expectations about its rate of change we have assumed

$$\hat{p}_b = \frac{\beta_{p_b}}{1-\beta_{p_b}(1-\alpha_s)}\left[\frac{1}{p_b} + \alpha_s\pi_{bs} - \bar{i}\right], \tag{7.55}$$

$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}). \tag{7.56}$$

Note that the short-term rate of interest i is considered as given in this partial analysis of the market for consols. Insertion of (7.55) into (7.56) yields

$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}\left[\left(\frac{\alpha_s\beta_{p_b}}{1-\beta_{p_b}(1-\alpha_s)} - 1\right)\pi_{bs} + \frac{\beta_{p_b}}{1-\beta_{p_b}(1-\alpha_s)}\frac{1}{p_b} + const.\right]. \tag{7.57}$$

We see that the trace of the Jacobian J of the 2D dynamical system (7.55), (7.57) at the steady state can be made as positive as desired. This is so, since the parameter β_{p_b} can always be chosen to make J_{22} positive, then $\beta_{\pi_{bs}}$ can be chosen so as to scale up J_{22} in the trace to an arbitrarily large value without changing the other coefficient J_{11} of the trace. It is easy to show that the determinant of the Jacobian J of the full 2D dynamics shown above is always positive and that the system switches from stable nodes to stable foci to unstable foci to unstable nodes when the adjustment speed of expectations of less ambitious agents is increased from zero towards infinity. Therefore all local stability scenarios – apart from saddlepoint dynamics – are possible, depending on the adjustment speed of adaptively formed expectations.

In sum, the foregoing analysis implies that there is a tendency for the price dynamics of long-term bonds to become at least locally explosive when the adjustment speed of bond prices becomes sufficiently large and when the expectations adjustment speed of less ambitious asset owners approach the limit case of myopic perfect foresight $\beta_{\pi_{bs}} \to \infty$. We stress that the bond rate dynamics influence investment behaviour of firms and of asset holders and thus will transfer its instability to the rest of the full 18D dynamical system.

Let us next investigate the isolated bond market dynamics shown above in more detail, again on the basis of a given short-term interest-rate $i = i_o$. To simplify the notation we rewrite the system as

$$\dot{p}_b = \beta_1(1 + (\alpha_s\pi_{bs} - i_o)p_b),$$

$$\dot{\pi}_{bs} = \beta_2(\hat{p}_b - \pi_{bs}) = \beta_2\left(\beta_1\left(\frac{1}{p_b} + \alpha_s\pi_{bs} - i_0\right) - \pi_{bs}\right),$$

where for convenience we set $\beta_1 = \beta_{p_b}(1 - \beta_{p_b}(1-\alpha_s))$ and $\beta_2 = \beta_{\pi_{bs}}$. For local stability analysis we have to calculate the determinant and the trace of the Jacobian J of this system at the steady state $p_b = 1/i_{lo}, \pi_{bs} = 0$. The Jacobian J is given by

$$J = \begin{pmatrix} -i_o\beta_1 & \beta_1\alpha_s/i_o \\ -\beta_2\beta_1/i_o^2 & \beta_2\beta_1\alpha_s - \beta_2 \end{pmatrix}.$$

Therefore, $\det J = i_o\beta_1\beta_2 > 0$ and $\text{trace } J = -i_o\beta_1 + \beta_2\beta_1\alpha_s - \beta_2 = \beta_2[\beta_1\alpha_s - 1] - i_o\beta_1$. The critical condition for stability thus is $\beta_2^H = i_o\beta_1/(\beta_1\alpha_s - 1) =$

$i_o/(\alpha_s - 1/\beta_1)$.[17] Below this value for β_2, J has a negative trace and thus the dynamics display a stable node or focus, and above it J has a positive trace and the dynamics display either an unstable focus or a node. In the latter case of an explosive motion of asset prices on the bond market we have to ask ourselves of course what can limit these dynamics and thus prevent economic collapse.

We propose the following stylised solution to this problem. Assume that there are subjective thresholds for the adaptive expectations mechanism, based on deviations of the long-term rate of interest i_l from the given international rate i_l^*, beyond which (for large deviations) the adjustment parameter β_2 is significantly reduced, since the agents who form their expectations in this way believe that the market will not deviate much further from the norm i_l^*. Should the market, however, continue to do so, they slow down their response to this fact by following this development with a much lower error correction speed β_2, becoming more cautious and thus responding in a more reserved way to such a development (by lowering β_2). We shall see that this in turn will indeed stop the explosive motion and thus confirm the reasons on which this response is based.

To provide a simple example for this (which can, however, be modified in many ways) assume now that β_2 is a function of the rate of interest i_l in its deviation from the rate i_l^* of the simple form displayed in Figure 7.5, where β_2^H is the critical parameter

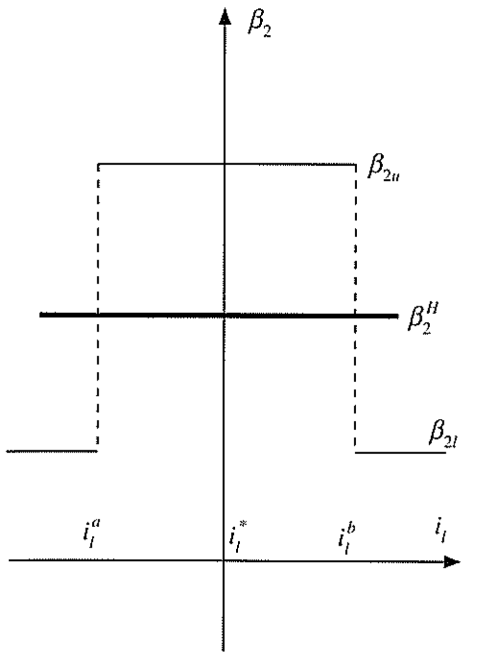

Figure 7.5 Variable speed of adjustment of expected bond price inflation

[17] We assume that $\beta_1\alpha_s > 1$ holds true.

value at which the dynamics investigated above turn from local asymptotic stability to instability.

Outside the interval $[i_l^a, i_l^b]$ adaptive expectations are thus made in a cautious way, as further deviations are considered as suspect and thus followed with lower speed, whereas inside this interval there is a rapid adjustment towards observed changes in i_l.

Calculating the isoclines of the bond price dynamics with the above threshold behaviour gives rise to the formulae

$$\dot{p}_b = 0: p_b = \frac{1}{i_o - \alpha_s \pi_{bs}}$$

$$\dot{\pi}_{bs} = 0: p_b = \frac{1}{i_o - (\alpha_s - 1/\beta_1)\pi_{bs}} = \frac{1}{i_o - \alpha_s\pi_{bs} + \frac{1}{\beta_1}\pi_{bs}}$$

and the phase diagram displayed in Figure 7.6.

The assumed threshold behaviour of Figure 7.5 makes the dynamics shown in the phase portrait of Figure 7.6 stable for large deviations of i_l from i_l^* and explosive in the vicinity of i_l^*. We conjecture that this creates a limit cycle when these aspects are combined with each other and will look for numerical confirmations of this conjecture in further extensions of this chapter, also in combination with the impact this has on the real part of the economy.

The claim just made can be more easily shown if an alternative non-linearity is added to the dynamics of long-term interest rates. To show this we now assume (for a given speed of adjustment β_2 of bond price expectations) that the parameter β_p (we assume further that $\alpha_s = 1, \beta_2 > i_o$, for simplicity) depends on the deviation of long-term interest i_l from the steady state rate i_l^* in the way displayed in Figure 7.7.

The functional form shown in Figure 7.7 may be justified by stating that the bond price dynamics slow down far off the steady rate of interest due to a slowdown in

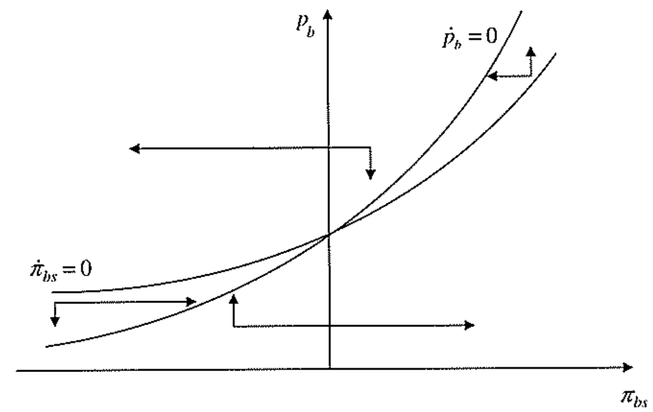

Figure 7.6 The phase diagram of the bond price dynamics with the assumed threshold behaviour in Figure 7.5

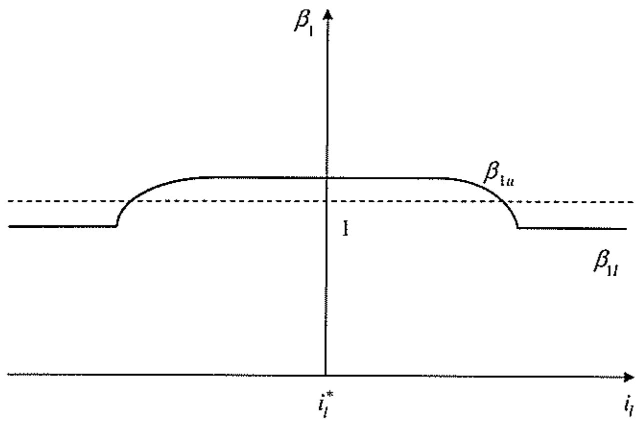

Figure 7.7 A variable speed of bond price adjustment

capital movements. We admit however that this type of occurrence is more difficult to rationalise in the heterogeneous agent framework we have postulated for our general model. We assume that β_2 is chosen such that the speed β_{1u} (β_{1l}) leads to local instability (stability) when combined with this adjustment speed of inflationary expectations. We know furthermore from the above local stability analysis that we have asymptotic stability for all parameter values β_2 if $\beta_1 < 1/\alpha_s$. We therefore again have the situation that the 2D bond price dynamics are locally unstable, but subject to stabilising forces when they departs too much from the steady state i_l^* of the foreign rate of interest.

If such a threshold behaviour exists, it would give rise to the type of phase diagram shown in Figure 7.8. In this phase diagram we have in fact already assumed that the adaptive revision of expectations is very fast so that there is a nearly horizontal movement in situations of the perfect foresight line $\dot{\pi}_{bs} = 0$. In this situation it can be judged from the phase portrait shown that there is a single attracting limit cycle for this type of dynamics (in fact a limit limit cycle or a so-called relaxation oscillation in the case $\beta_2 = \infty$). We assert that there will also exist a unique attracting limit cycle in situations where expectations are fast but not infinitely so.

Markets that slow down in their adjustment behaviour far off the steady state (in the expectation of turning points of the considered dynamics or simply in their speed of adjustment) may therefore stabilise what is in fact a cumulative process close to the steady state and thus induce in fact the turning points that are expected by less ambitious (adaptively behaving) asset owners or the market as a whole. We have therefore at least two possibilities at our disposal by which we can generate bounded asset market dynamics and on this basis also bounded dynamics in the real part of the economy, as long as these latter dynamics are bounded by themselves.

It is not difficult to show that the results on long-term bond price dynamics hold also for the dynamics of stock market prices p_e; see Asada *et al.* (2003), which (when

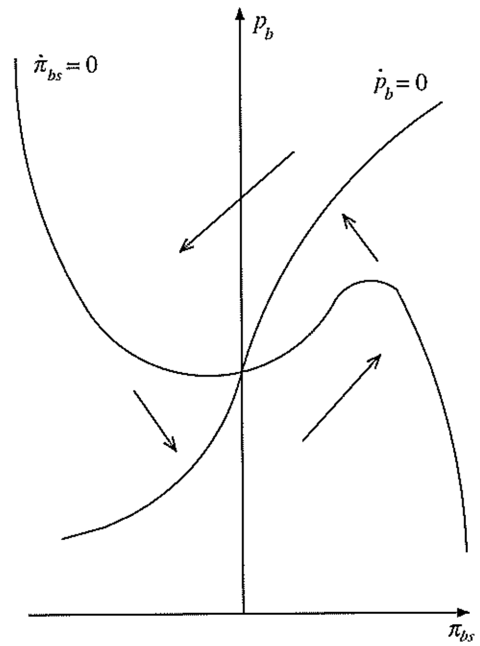

Figure 7.8 The phase diagram for variable speed of bond price adjustment

formulated in isolation) can be described as

$$\hat{p}_e = \frac{\beta_{p_e}}{1 - \beta_{p_e}(1-\alpha_s)}[r^e/q + \alpha_s \pi_{es} - (i_l + \pi_b)] \qquad \left(q = \frac{p_e E}{p_y K} \text{ Tobin's } q\right),$$

$$\hat{q} = \hat{p}_e - \hat{p}_y + (g_k - \delta_k + y - y^d - (\beta_n(\beta_{n^d} y^e - \nu) + \gamma \beta_{n^d} y^e))/q - (g_k - \delta_k),$$

$$\dot{\pi}_{es} = \beta_{\pi_{es}}(\hat{p}_e - \pi_{es}),$$

and which is here thus to be investigated on the basis of frozen (steady state) values for the bond market and for the expected rate of profit of firms, due to the partial perspective adopted in this chapter. From these equations we get for the isolated adjustment of equity price change expectations the differential equation

$$\dot{\pi}_{es} = \beta_{\pi_{es}}\left[\left(\frac{\beta_{p_e}\alpha_s}{1 - \beta_{p_e}(1-\alpha_s)} - 1\right)\pi_{es} + \frac{\beta_{p_e}}{1 - \beta_{p_e}(1-\alpha_s)} r_o^e/q + const.\right]$$

which allows us to draw the same conclusions on the trace of the q, π_{es} dynamics as we have obtained for the dynamics of the price of long-term bonds. High adjustment speeds of expectations in the stock market therefore give rise to unstable spirals or unstable nodes as in the case of bonds, if the adjustment speed of equity prices to rates of return differentials is chosen sufficiently high. Again, the dynamics can be made viable or bounded by assuming changes in adjustment speeds as in the case of long-term bonds. We however do not go into details here as the dynamics of equity prices do not feed back into the 18D dynamics whose theoretical and numerical analysis is the theme of this chapter.

7.11 The dynamics of the government budget constraint

In order to isolate the dynamics of government debt from the rest of the dynamics we assume that all of its state variables are frozen at the steady state with the exception of the variables b, b^l, τ_w which describe the evolution of short- and long-term government debt together with the adjustments in the wage taxation rate that have been assumed to take place in view of the deviation of government debt from a certain target ratio (per unit of expected sales). The remaining dynamics then can be expressed as

$$\dot{b} = \alpha_b^g \left[i_o b + b^l - \tau_w c^1 + c^2 \right] - \gamma b, \tag{7.58}$$

$$\dot{b}^l = i_{lo}(1 - \alpha_b^g) \left[i_o b + b^l - \tau_w c^1 + c^2 \right] - \gamma b^l, \tag{7.59}$$

$$\hat{\tau}_w = \alpha_{\tau_{w1}} \left(\frac{b + b^l / i_{lo}}{y^e \bar{d}} - 1 \right), \tag{7.60}$$

where $c^1 > 0$, c^2 denote certain constants and where $i_o^l = i_o$.

Proposition 7.6 *The interior steady state of the government debt dynamics (7.58), (7.59), (7.60) is locally asymptotically stable if*

$$\alpha_b^g i_o - \gamma \quad \textit{and} \quad i_{lo}(1 - \alpha_b^g) - \gamma$$

are both smaller than zero.

Proof: The Jacobian of the dynamical system (7.58), (7.59), (7.60) at the steady state reads

$$J = \begin{pmatrix} \alpha_b^g i_o - \gamma & \alpha_b^g & -c^1 \alpha_b^g \\ i_{lo}(1 - \alpha_b^g) i_o & i_{lo}(1 - \alpha_b^g) - \gamma & -c^1 i_{lo}(1 - \alpha_b^g) \\ \alpha_{\tau_{w1}} / (y^e \bar{d}) & \alpha_{\tau_{w1}} / (i_{lo} y^e \bar{d}) & 0 \end{pmatrix}.$$

Due to the assumptions made we immediately have the result that the trace of the matrix J is negative and thus $a_1 > 0$.[18] Multiplying the second row of this matrix J by

$$-\frac{\alpha_b^g}{i_{lo}(1 - \alpha_b^g)}$$

and adding the resulting vector to the first row does not alter the determinant of J, which is therefore based on the sign structure

$$J = \begin{pmatrix} - & 0 & 0 \\ + & - & - \\ + & + & 0 \end{pmatrix}.$$

The determinant of J is therefore negative. We thus also have $a_3 > 0$.

[18] We recall that a_1, a_2, a_3 refers to the Routh–Hurwitz coefficients and thus we require $a_1 > 0, a_2 > 0, a_3 > 0$, $a_1 a_2 - a_3 > 0$ for local asymptotic stability.

Next we show that the principal minors of J are all positive so that also $a_2 > 0$ holds true. This is easily obtained from the full sign structure of the matrix J which is given by

$$J = \begin{pmatrix} - & + & - \\ + & - & - \\ + & + & 0 \end{pmatrix}.$$

First, $J_{11}J_{33} - J_{13}J_{31}$ and $J_{22}J_{33} - J_{23}J_{32}$ are obviously positive, since only the off-diagonal elements are of importance in these cases. Furthermore, also $J_{11}J_{22} - J_{12}J_{21}$ must be positive as can be seen by means of the same row operation we have used above for the calculation of the sign of the determinant of the matrix J, again based on the factor $-\frac{\alpha_b^g}{i_{lo}(1-\alpha_b^g)}$. The sum a_2 of these three expressions is therefore unambiguously positive.

Finally, also the expression $a_1 a_2 - a_3$ must be positive since the $-a_3$ expression, the determinant of J (due to the above calculation) is neutralised by one of the products in the expression $a_1 a_2$ which are all positive. □

The assumptions of Proposition 7.6 compare steady rates of return with the steady rate of growth and thereby restrict the rate with which government debt grows due to the debt service that has to be made. The two conditions of the proposition weaken the assumption that is normally made if only one debt financing instrument is considered. However, we state without proof that there is also one aggregate stability condition that is sufficient for the stability result of the proposition, namely $i_o - \gamma < 0$. Note in this respect also that one can of course assume that the government relies only on debt instruments, in which case the dynamical system (7.58), (7.59), (7.60) becomes two-dimensional. Finally, if Proposition 7.6 does not apply and if furthermore the dynamics are even monotonically explosive we get, due to the assumed tax rate policy rule, that the tax rate τ_w tends monotonically to either zero or one. This suggests that the dynamics are only valid over a restricted domain in such a case and that further adjustments will come into operation if certain thresholds in wage taxation are crossed. One possibility that may help to avoid the occurrence of such a situation is given by adding a derivative term for debt evolution based on a parameter $\alpha_{\tau_{w2}} > 0$ to the tax rate adjustment rule of the general model of Section 7.2. We assert that the addition of such a term, a derivative control in fact, will improve the stability properties in the fiscal policy part of the model. Nevertheless it may be necessary to add further or other adjustment mechanisms, in this module of the model, in order to really get dynamics that stay at least bounded if they are not locally asymptotically stable.

Proposition 7.6 shows that the evolution of government debt (due to the interest payments that have to be made, to the steady state size of government expenditures and revenues and also due to the taxation rule that has been assumed for wage taxation) contributes to the local asymptotic stability of the 18D dynamical system. The dynamic instability found in other studies of the evolution of the GBR is thus not a problem in the present formulation of government debt and its evolution, at least when considered in isolation. The extent to which this result holds is dependent on the suppression of other feedback mechanisms that are involved in the GBR, which may come into operation when the general 18D core dynamics of Section 7.3 are investigated. We refer the reader to the much further reaching generalisations of this model type considered in Chiarella *et al.* (1999a,b) and also Chiarella *et al.* (2000).

The monetary and fiscal policy rules we have considered so far have been of a fairly orthodox type: an anti-inflationary interest rate policy rule, a target debt fiscal policy rule and an import taxation rule that attempts to reduce the impact of nominal exchange rate changes on the business sector of the economy. There exist however a variety of other policy rules that might be helpful in reducing the disequilibria that can occur in our model economy. At present we only wish to point to four further extensions of a rule determined behaviour of the government, without attempting to integrate these features into the general 18D core dynamics model of this chapter or even to discuss the role of these rules for economic stability and the like. These extensions are:

1. A counter-cyclical adjustment rule for payroll taxes τ_p.
2. A labour market policy rule that attempts to raise the NAIRE level $\bar{e}$.
3. A counter-cyclical policy rule for government expenditures g per unit of social product (appropriately measured).
4. A counter-cyclical employment policy in the government sector which endogenises the parameter α_g.

This brief list must here suffice to indicate that there is a lot of room in our 18D core model for designing, testing and evaluating various fiscal (and monetary) policy rules.

7.12 Import taxation

The evolution of import taxation, when treated in isolation, is particularly easy, since its law of motion

$$\hat{\tau}_m = \alpha_{\tau_m} \frac{p_x x - p_m j^d}{p_x x}$$

can be reduced to

$$\hat{\tau}_m = \alpha_{\tau_m} \frac{p_x^* x - (1+\tau_m) p_m^* j^d}{p_x^* x}$$

where p_m^*, p_x^*, x, j^d are given magnitudes of the model.

We thus have only to consider one law of motion here, which is based on a negative feedback of the rate of import taxation on to itself and therefore is globally asymptotically stable and establishes balanced trade in the limit. This simply serves the important purpose of making the solution of the private sector of the economy independent of trade flows and the revenues and costs these trade flows generate for the sector of firms (off the steady state). We expect that this subsector of the model needs further improvements in future reformulations. At present international trade of goods has only limited but well defined effects on the behaviour of firms, but not on the domestic goods market or Keynesian aggregate demand in general.

7.13 The Dornbusch exchange rate dynamics mechanism

The evolution of the exchange rate and expectations about its behaviour can be reduced to an independent 2D subsystem of the general 18D dynamics of Section 7.3 if the data concerning bond price dynamics are considered as given for the time being. In that case the dynamics of s and ϵ_s read (see equations (7.35), (7.36)).

$$\hat{s} = \frac{\beta_s}{1-\beta_s(1-\alpha_s)}[i_l^* + \alpha_s \epsilon_s - (i_l + \pi_b)],$$

$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s).$$

To study the resulting dynamics in isolation we again assume that the other asset market situations are frozen at their steady state values which fixes the expression $(i_l + \pi_b)$ involved in the above equations to i_{lo}. From this we thus derive as law of motion for the change in exchange rate expectations of the less ambitious agents

$$\dot{\epsilon}_s = \beta_{\epsilon_s}\left(\frac{\beta_s \alpha_s}{1-\beta_s(1-\alpha_s)} - 1\right)\epsilon_s,$$

which clearly provides (trivial) monotonically explosive dynamics if the parameters in the fraction are chosen such that it becomes larger than one. Increasing the parameter β_{ϵ_s} beyond any bound then makes this process as explosive as desired and thus will significantly contribute to local instability of the full 18D dynamics. Compared with the isolated dynamics for long-term bonds and equities considered in Section 7.10 we therefore here find a particularly simple representation of the centrifugal forces that surround asset market dynamics in our approach to their behaviour.

Increasing the parameters β_s for exchange rate flexibility will increase the positive influence of the expected exchange rate changes ϵ_s on the actual rate of change of the exchange range without bound. For positive α_s we get in this way a positive feedback of exchange rate expectations on their time rate of change which becomes the more destabilising the faster these expectations are adjusted. This effect is similar to the Mundell effect we considered previously.

The influence of the dynamics of the nominal exchange rate on the rest of the dynamics is limited in the model of monetary growth investigated in this chapter, since it only works through the rate of profit of firms which depends on the rate s via exports and imports and the tax revenue that is generated from import taxation.

The above extremely one-sided situation in the adjustments of the exchange rate is partly due to our hierarchical treatment of the asset market dynamics, where we have assumed that the short-run rate of interest is set by the monetary authority, where the long-term rate of interest adjusts into the direction of this short-term interest rate and where the exchange rate is driven by the differential that expectations about its rate of change create between the expected rate of return on foreign as compared with the expected rate of return on domestic long-term bonds. The positive feedback mechanisms that exist in the dynamics of asset prices and the exchange rate are therefore built on a sequential reasoning in our model and lead to an extreme type of instability when the foreign exchange market is considered in isolation – without its feedback through the real part of the economy.

A similar observation is not so obvious, if we allow the exchange rate s to influence the evolution of the real part of the dynamics, by removing the assumption that the rate of import taxation is always set such that the trade account of firms is balanced (when measured in domestic prices). In this latter case, the expected rate of profit of firms does not depend on their export and import levels and thus on exchange rate changes. As long as there are no wealth effects in the model and as long as the individual allocation of bonds on the various sectors does not matter, there is indeed only this one channel through which the nominal exchange rate can influence the real economy (besides of course through the GBR which includes the tax income of the government) that derives from import taxation, but which does not play a role for the real part of the model unless wage taxation is responsive to the evolution of government debt (as we have seen in the preceding section). To have this influence of the exchange rate we thus have to extend the 9D real dynamics by the three laws of motion[19]

$$\hat{\tau}_m = \alpha_{\tau_m} \frac{p_x^* x - (1+\tau_m) p_m^* j^d}{p_x^* x}, \tag{7.61}$$

$$\hat{s} = \frac{\beta_s}{1-\beta_s(1-\alpha_s)} \left[i_l^* + \alpha_s \epsilon_s - \left(\frac{1}{p_b} + \pi_b \right) \right], \tag{7.62}$$

$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s). \tag{7.63}$$

The exchange rate dynamics are now more difficult to analyse, since their two laws of motion require the influence of the bond dynamics in order to be meaningful. Otherwise the two laws of motion (7.62), (7.63) would imply monotonic implosion or explosion of exchange rate expectations and the actual exchange rate depending on whether the adjustment speed of the exchange rate is smaller or larger than one (for $\alpha_s = 1$), as we have seen earlier. The financial dynamics are therefore in this respect immediately of dimension 5 and one also needs input from the real dynamics in order to get the effects from the exchange rate s on bond prices p_b and thus an interdependent dynamics and not one of the appended monotonic form just discussed. Yet, the effect of changes in s via the rate of profit r^e of firms and the investment decisions that are based on it needs to reach a long way in order to influence the market for long-term bonds. Changes in investment lead to changes in aggregate goods demand and thus to changes in sales expectations and actual output. This leads to changes in capacity utilisation of firms and domestic price inflation which (if and only if monetary policy responds to them) are transferred to changes in the short-term rate of interest and thus to changes in the long-term rate of interest. In this way there is a feedback of a change in the exchange rate on its rate of change, which has to be analysed if the full dynamics are to be investigated.

For the moment we consider that this feedback chain is too long and complicated for a first discussion of the dynamics of asset markets that integrate exchange rate dynamics.

[19] Note that the first law is independent of the changes in the exchange rate.

We thus consider the following simplification of this feedback mechanism, which here serves to demonstrate that there is some similarity between the isolated bond price dynamics and the dynamics of the exchange rate. In models of the Dornbusch (1976) type of overshooting exchange rates there is generally a very determinate mechanism that leads to a positive impact effect of an increase in the exchange rate s on the nominal rate of interest i, via increasing exports and decreasing imports, the resulting increases in economic activity and in the price level and thus to i increases via an LM theory of the money market. This mechanism is here used as a basis for the design of a monetary policy rule that copies this feedback chain according to

$$i = i_o + \beta_i(s - s_o), \quad i_o = i_l^*,$$

where i_o, s_o denote the steady state values of i, s. In a first application of this rule we in addition assume that it applies immediately to the long-term rate of interest $i_l = 1/p_b$.

The dynamical equations for the exchange are thereby transformed to[20]

$$\hat{s} = \beta_s[i_l^* + \epsilon_s - (i_o + \beta_i(s - s_o) + \pi_{bs})],$$
$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s).$$

In order to consider these dynamics in the simplest possible way we finally assume $\pi_{bs} \equiv 0$. Rearranging terms we then obtain

$$\hat{s} = \beta_s[-\beta_i s + \epsilon_s + \beta_i s_o],$$
$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s).$$

In this form the system is of the same type as the one for the long-term bond dynamics shown above. It therefore will give rise to the same conclusions as the bond dynamics that we have considered in Section 7.10.

Yet this similarity is based on a number of artificial assumptions as far as our original 18D dynamics are concerned. Therefore using (as is necessary) as policy rule the short-term interest rate policy of the 18D dynamics would imply for example the extended dynamics (again assume for $\alpha_s = 1$ for simplicity)

$$\hat{p}_b = \beta_{p_b}\left[\frac{1}{p_b} + \pi_{bs} - i\right],$$
$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}),$$
$$\hat{s} = \beta_s\left[i_l^* + \epsilon_s - \left(\frac{1}{p_b} + \pi_{bs}\right)\right],$$
$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s),$$
$$i = i_o + \beta_i(s - s_o)$$

[20] We again assume $\alpha_s = 1$ for simplicity. Note that import taxation was of no importance in the presently considered dynamics.

which is already a 4D dynamical system (as can be seen by inserting the last equation into the first one) representing in isolation those financial markets that will have impact on the real part of the economy. The question then arises as to what extent the 2D analysis of the market for long-term bonds also applies to this two asset approach and its four laws of motion from the local as well as from the global perspective (giving rise again to limit cycles or limit limit cycles or possibly also more complex types of attractors).

Rearranging the above system slightly reduces it to

$$\hat{p}_b = \beta_{p_b}\left[\frac{1}{p_b} + \pi_{bs} - \left(i_l^* + \beta_i(s - s_o)\right)\right], \tag{7.64}$$

$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}), \tag{7.65}$$

$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s), \tag{7.66}$$

$$\hat{s} = \beta_s\left[i_l^* + \epsilon_s - \left(\frac{1}{p_b} + \pi_{bs}\right)\right]. \tag{7.67}$$

The interior steady state of these dynamics is given by $p_b = 1/i_l^* = 1/i_o^*, \pi_{bs} = 0, s = s_o, \epsilon_s = 0$. The Jacobian of the dynamics at the steady state in the case $\beta_{\pi_{bs}}, \beta_{\epsilon_s} < 1$, has the sign distribution

$$J = \begin{pmatrix} - & + & 0 & - \\ - & - & 0 & - \\ + & - & - & 0 \\ + & - & + & 0 \end{pmatrix}.$$

It is easy to see from this form that the dynamics must be locally asymptotically stable if the parameter β_s is chosen sufficiently small. Setting this parameter equal to zero and considering only the remaining 3D dynamical system one obtains for the Jacobian the sign structure

$$J = \begin{pmatrix} - & + & 0 \\ - & - & 0 \\ + & - & - \end{pmatrix}.$$

It is fairly obvious that this matrix must fulfil the Routh–Hurwitz conditions for local asymptotic stability, since this is obviously true for the upper principal minor

$$\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}.$$

Thus the system is obviously composed of two stable subdynamics with all three eigenvalues having negative real part and determined only by their respective subsystem.

Furthermore as far as the determinant of the full Jacobian is concerned we can easily remove linear dependencies from this Jacobian in order to get for its determinant

$$\det J = \det \begin{pmatrix} - & + & 0 & - \\ 0 & - & 0 & 0 \\ 0 & 0 & - & 0 \\ + & - & + & 0 \end{pmatrix},$$

which is easily calculated to be positive. Due to the continuity of eigenvalues with respect to the entries of the matrix J we see that a negative eigenvalue must be born adding to the three of the 3D dynamics when the parameter β_s is changed from zero to small positive values. We conjecture that this remains valid as long as the parameter β_s remains less than one. We thus have the result:

Proposition 7.7 *1. The dynamical system (7.64), (7.65), (7.66), (7.67) for bond price/exchange rate dynamics has a unique interior steady state which is locally asymptotically stable for adjustment speeds of prices and expectations sufficiently low.*

2. The steady state will generally lose its stability in a cyclical fashion by way of a Hopf bifurcation if in particular the adjustment speeds of the adaptively formed expectations are chosen sufficiently high.

The second part of Proposition 7.7 holds true since we have shown that the determinant of the full Jacobian J is always positive, so that eigenvalues cannot become zero (in particular when the imaginary axis in the complex phase plain is crossed. We conjecture but cannot show this here that all eigenvalues will become real and will thus give rise to saddlepath dynamics with monotonic features if the adjustment speeds in the above model are all chosen sufficiently high.

If all adjustment speeds are set equal to infinity the dynamical system (7.64)–(7.67) reduces to the 2D form

$$\hat{s} = \beta_i(s - s_o),$$

$$\hat{p}_b = i_l^* - \frac{1}{p_b} + \beta_i(s - s_o).$$

In this representation the order of dependence of the two asset markets is reversed and the saddlepath dynamics that are then implied are clearly visible. In the case of unanticipated shocks that concern these dynamics, the traditional mode of analysis would then be to apply the jump variable technique which would here imply that the system immediately jumps to its new steady state values $(s_o, 1/i_l^*)$ when these values have been moved to a new location through the assumed type of shock.

We thus see that the rational expectations solution in our deterministic model is a limit case of our approach that demands a new solution technique if one wants to suppress the explosiveness of the asset market subdynamics under perfect foresight and infinitely fast price adjustments as far as the obtained 2D dynamics are concerned. We believe however that the considered limit case is too ideally chosen and that one should apply the relaxation oscillation methodology of Section 7.6 in the derivation of the limiting properties instead of simply setting all adjustment speeds equal to infinity and of only discussing the consequences of replacing certain laws of motion by simple algebraic conditions when deriving this ideal situation of perfect flexibility and perfect anticipation of asset market prices. We admit however that the relaxation oscillation methodology is difficult to apply to the 4D dynamics considered above due to the assumed hierarchical structure in the interaction of long-term bond prices with the rate of exchange. The task would be much easier if the expected rate of return on foreign bonds could be compared with the short-term rate of interest in the law of motion for the exchange rate, which however would demand a reformulation of the asset market structure of our general model.

7.14 Conclusions

Summing up the isolated stability or instability results we have obtained in this chapter we can state the following conclusions.

We have found that either wage or price level flexibility is bad for local economic stability. Whenever wage flexibility is good its flexibility on the outside labour market will nevertheless be bad for local stability if it becomes too large compared with the demand pressure that originates from the inside labour market. Next, a very flexible adjustment speed of planned inventory holdings has been shown to lead to local instability when coupled with a rapid adjustment of sales expectations. We have in addition described situations where some of these local instabilities can be overcome by certain bounds on the behaviour of these subdynamics.

On the nominal side there was still more room for the occurrence of centrifugal forces around the steady state of the model, since the price level of goods and long-term bonds as well as equity prices and the nominal exchange rate all gave rise to local explosiveness if adjustment speeds in these markets are sufficiently high and coupled with a rapid adjustment of the expectations of workers (in the market for goods) and less ambitious asset holders (in the remaining markets).

Stability only came about, first, through the Keynes effect and the nominal interest rate policy of the central bank that derives from it; second, through assumptions on the size of steady rates of returns as compared with the size of steady economic growth coupled with a stabilising feedback rule between government debt and wage taxation; and third, through a simple adjustment rule for import taxes. Stabilising forces therefore mainly originate in the behaviour of the government and the central bank, unless the relaxation oscillations mechanism considered in Section 7.6 and Section 7.10 can be successfully applied to the expected rate of inflation, sales expectations, long-term bond price expectations and expectations on exchange rate changes as far as the adaptive component of these expectations is concerned.

This brief discussion of the basic partial feedback mechanisms of our full 18D dynamics on balance suggests that increases in the speeds of adjustments of the dynamics will generally be bad for economic stability or viability. Exceptions to this rule are given

by either wage or price flexibility and by the sales expectations mechanism, in the case where inventories are adjusted in a sufficiently slow fashion. Mathematically speaking it should be noted finally that the destabilising effects we have discussed in this chapter will generally not appear as obviously destabilising mechanisms in the guise of positive entries in the trace of the Jacobian of the system at the steady state. Rather such destabilising effects will be hidden somewhat in the many principal minors that underlie the calculation of the Routh–Hurwitz conditions for local asymptotic stability in high dimensional dynamical models.

Due to our simple formulation of the investment and pricing behaviour with respect to dwellings and housing services we have also found local asymptotic stability in this part of the private sector. The overall impression nevertheless surely is that the steady state of the private sector is more likely to be subject to centrifugal forces than to centripetal ones, which moreover generally will not remain bounded to an economically meaningful domain around this steady state if the 18D dynamics are considered from a more global perspective. This is in particular shown by the numerical simulations in Chiarella *et al.* (2003b). Extrinsic non-linearities, such as the assumptions underlying relaxation oscillations, therefore have to be added, at least far off the steady state, in order to obtain economic boundedness for the considered dynamics. Further important and still very fundamental candidates in this respect are downward inflexibilities of nominal wages and/or prices, supply bottlenecks as in non-Walrasian macroeconomic theory, further non-linearities in the inventory mechanism and in investment behaviour, and the like. Such extrinsic non-linearities have to be added later on to the intrinsic ones that are 'naturally' present in the dynamics we have considered so far[21] in order to obtain a dynamic model that can generate viability for the orbits it implies. These are studied in a systematic way in Chiarella *et al.* (2003b).

[21] Due to growth rate expressions, products or quotients of state variable as they derive from multiplicative value magnitudes ($w^e l_f^{de}$ for example) or certain quantity ratios (l_f^{de}/l_f^{we} for example), and the like.