Part III

Debt crises: firms, banks and the housing markets

8 Debt deflation: from low to high order macrosystems

8.1 Introduction

At the beginning of the current century, in the public debate on problems of the world economy, 'deflation' or more specifically 'debt deflation' has once again become an important topic. The possible role of debt deflation in triggering the Great Depression of the 1930s has long been the subject of academic studies. It has been observed that there are similarities between recent global trends and the 1930s, namely the joint occurrence of high levels of debt and falling prices: the dangerous downside to cheaper credit when debt is high. Debt deflation thus concerns the interaction of high nominal debt of firms, households and nations and shrinking economic activity due to falling output prices and therefore increasing real debt.

There is often another mechanism accompanying the first one. That other mechanism deals with how large debt may exert its impact on macroeconomic activity, and works through the asset market. Asset price inflation during economic expansions normally gives rise to generous credit extension and lending booms. Assets with inflated prices serve as collateral for borrowing by firms, households or nations. On the other hand when asset prices fall the borrowing capacity of economic agents shrinks, financial failures may set in, macroeconomic activity decreases and consequently large output losses may occur.

Countries that have gone through such booms and busts are Asian countries, in particular Japan, as well as Russia and Brazil in the years 1998 and 1999. In all of those countries as well as during the financial crisis in Mexico in 1994 asset price inflation and lending booms entailed subsequent debt crises and asset price deflation. Thus, usually the mechanism of debt deflation due to falling output prices has been accompanied by the asset price deflation mechanism.[1]

Concerning the public debate on problems of the world economy, 'debt deflation' is surely one of the key expressions that has significantly shaped this discussion, although it is now much less debated than the current subprime crisis. The behaviour of firms

[1] For a detailed study employing asymmetric information theory, see Mishkin (1998).

relying on rounds of downsizing and cost-cutting from the perspective of short-run profitability solely (short-term maximisers), thereby demolishing their productivity over the medium run, was noted as a dangerous strategy caused by their dependence on financial markets. Criticism however has also been raised with respect to the single-minded preoccupation of certain central banks with inflation and it has been suggested that some inflation could be of help in preventing a global economic crisis. The viewpoint of the FED, and of the government in the USA, of course received particular attention in this respect and the former chairman of the FED, Alan Greenspan, was quoted with passages such as:[2]

> Deflationary forces that emerged a year ago were expanding 'and there's no evidence of which I am aware which suggests that the process ... has stabilized'.

Moreover, global growth strategies, and the elements they should contain, continue to be discussed in academic and policy circles. The need for a fundamental restructuring of the IMF and World Bank and a new financial architecture is continuously stressed based on the judgement that the world has, since the years 1998 and 1999 and particularly in the last two years, faced its biggest financial challenge since the 1930s. Debt deflation and its destabilising potential therefore appears to be an important problem that the world economy is still facing.

Deflation, at least in certain sectors of the economy, combined with high indebtedness of firms, although currently not the focus of interest, therefore appears to be an important problem for the world economy, and – as is sometimes stressed – one that it will continue to face for a considerable period into the future; see in particular Shilling's (1999) detailed study of the long-run forces driving deflation. The destabilising potential of debt deflation without and with its interaction with other economic feedback mechanisms that concern the danger of deflationary processes (to be discussed in this chapter) should therefore be modelled and investigated thoroughly. One should also take into consideration the possibilities for monetary and fiscal policies that allow a cessation of the processes of rising debt and falling output prices that can lead to depression or in the extreme even economic breakdown as in the Great Depression.

Modern macroeconomic theory, as it has evolved since the Second World War, has paid scant attention to the above described mechanism of debt deflation. No doubt this is due to the fact that during that time the major economies in the world experienced a long period of growth followed by a long period of inflation from which we have recently emerged. The classic study of debt deflation remains Fisher (1933), though Minsky (1975, 1982) in his writings on the financial instability hypothesis continued to warn of the dangers of another great depression. There is therefore an urgent need for economists to model the process of debt deflation in its interaction with monetary and fiscal policies that may stop the process of rising debt, falling output and asset prices and a subsequent collapse into depression. We here note that the current subprime crisis at present primarily concerns the financial sector of the economies involved, but may

[2] *The Sydney Morning Herald*, 21 September 1998.

easily give rise to subsequent processes of debt deflation if it spreads into the real part of these and other economies.

In this chapter we embed the process of debt accumulation and debt deflation into a fully integrated and consistent – with respect to budget constraints – macroeconomic model as it has already been formulated in Chiarella *et al.* (2001a,b); see Part II for a detailed formulation of the underlying structure. At the core of the model will be firms that finance fixed investment not from retained earnings, but by loans from the credit market. In the current chapter we therefore neglect the equity financing of the earlier approach of Part II. Our model will thus focus mainly on the first mechanism of the debt deflation process, the destabilising role of flexible wages and prices in economies with high nominal debt, while the destabilising role of asset prices will be neglected here.[3]

Our macroeconomic model contains a sufficient number of agents and markets to capture the essential dynamic features of modern macroeconomies, and stresses the dynamic interaction between the main feedback loops of capital accumulation, debt accumulation, price and wage inflation (and deflation), income distribution, inventory accumulation and government monetary and fiscal policies. Our modelling framework relies on previous work by the authors and contributions by other co-authors underlying Part II. The essential difference is that we here focus on debt-financed investment of firms in place of pure equity financing. We will thus add a further important feedback loop that was missing in our earlier approach to macro modelling, namely, from a partial point of view, the destabilising Fisher debt effect of deflationary (or inflationary) phases of capital accumulation arising from the creditor–debtor relationship between banks and firms.

The Fisher debt deflation mechanism is easily described, for example by means of the diagram shown in Figure 8.1. This diagram shows that price (and wage) deflation, caused by depressed markets for goods (and for labour), increase the real debt of firms (and indebted households), thereby leading to reduced investment (and consumption), which gives further impetus to the depression already under way and its subsequent consequences for further price (and wage) deflation. This partial reasoning thus suggests that debt deflation may end up in a deflationary spiral and economic breakdown, if this downward movement in prices (and wages) cannot be stopped.

Further issues in this context concern subjects that can aggravate the development of debt deflation (stock price collapses, credit rationing, large scale bankruptcy, banking and foreign exchange crisis and domestic or foreign policy intervention). These issues will be approached to a certain degree in the chapters of Part III, but remain to be integrated and investigated into the model of the present chapter in future work in order so as to allow a full treatment of the dangers of the joint occurrence of debt and deflation in certain regions of the world economy or even on a worldwide scale.

[3] For work on the credit market, economic activity and the destabilising role of asset price inflation and deflation, see Minsky (1975).

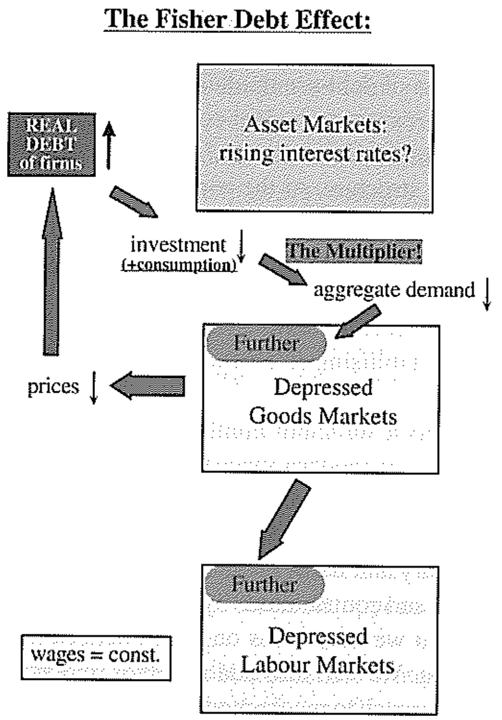


Figure 8.1 The Fisher debt deflation effect

However, such deflationary spirals do not work in isolation and may be counteracted by well-known Keynes or Pigou effects. This chapter will, however, primarily pay attention to another aspect of falling wages and prices, namely the so-called normal or adverse Rose (1967) or real wage effects. The working of such effects is explained by means of the following two diagrams. Considering normal Rose effects first, there are two possibilities for their occurrence:

1. The case where aggregate demand depends positively on the real wage and a price flexibility which is sufficiently higher than wage flexibility, in which case real wages fall in a boom and rise in a depression, which is stabilising.
2. The case where aggregate demand depends negatively on the real wage and a wage flexibility which is sufficiently higher than price flexibility in which case real wages also fall in a boom and rise in a depression, which is again stabilising.

Such stabilising real wage adjustments are exemplified in Figure 8.2 and they would – just as do Keynes or Pigou effects – work against the depressing forces of the Fisher debt deflation mechanism. The question however is whether such stabilising forces can really overcome the depressing effects of rising real debt. This question will be investigated in Section 8.3.

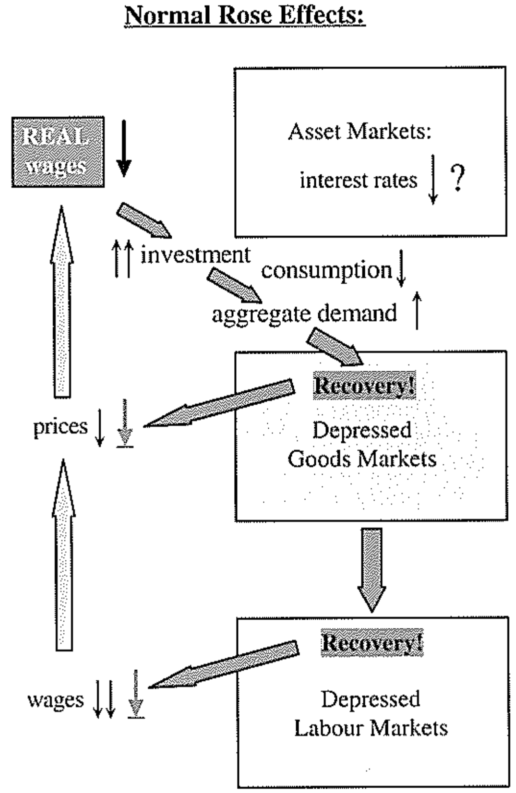


Figure 8.2 Normal Rose effects

However, Rose effects can also be adverse or destabilising, namely when in the first of the above considered two situations wage flexibility is sufficiently high or in the second case when price flexibility is sufficiently high. These adverse Rose effects are exemplified in Figure 8.3. Price and wage flexibility may, therefore, be destabilising through two channels, the Fisher debt effect and the adverse Rose real wage effects. These two mechanisms are at the core of the high dimensional analysis of AS–AD disequilibrium growth type which this chapter provides, while further traditional mechanisms such as the Keynes effect or the dynamic multiplier process are also addressed, but are of secondary importance here.

In this chapter we therefore start a series of investigations that attempt to apply the general approach to disequilibrium growth theory of Part II of this book to contemporary topics of applied economic analysis and to policy issues that are debated in the economics literature and amongst the public. These issues were not included in or not sufficiently stressed in our approach in Part II to disequilibrium growth. The main purpose of the present and of subsequent applications and extensions of our integrated disequilibrium approach to economic growth is therefore to bring the models used by this approach closer to the applied macroeconomic literature, by considering the (often)

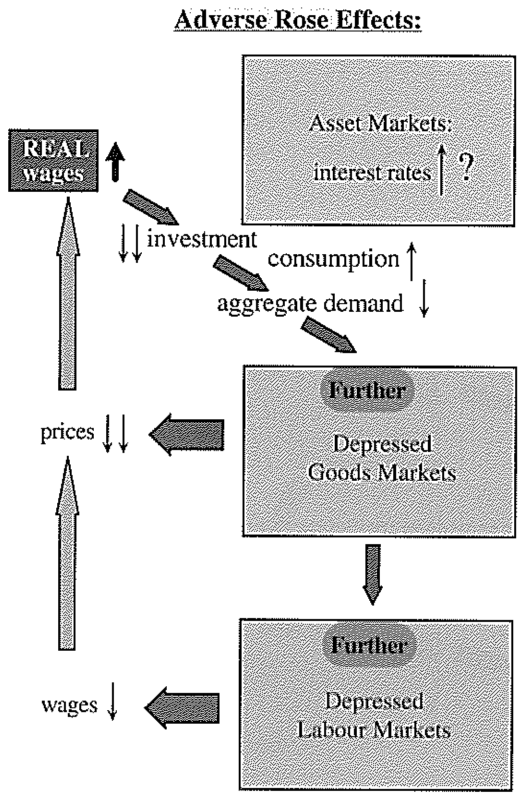


Figure 8.3 Adverse Rose effects

partial models of this literature and their theoretical or empirical results from the general perspective and model formulation reached in Part II.

Topics that will in this way be integrated or be the focus of our investigations are (among others): situations of high indebtedness of firms (bank loans) and households (mortgages) and the danger of deflationary processes in such an environment; other aspects of financial markets and their implications for fixed business investment, in particular for the housing sector (see Chapter 11) as well as the commercial office market in view of the fluctuations that are there observed. Such topics will be integrated one after another into our general approach to disequilibrium growth and then investigated with respect to the changes their inclusion may imply for the dynamics of the whole economy and, in turn, what the general approach to fluctuating growth chosen in Part II may imply for the outlook of the new partial mechanism that is imbedded into it and investigated from this perspective. In this way we believe we can demonstrate to the reader the power of our approach to treat those real world applications or topics that have received a great deal of attention from the viewpoint of applied macroeconomics.

Keen (2000) has investigated the Fisher debt effect (between firms and financial intermediaries) in the context of an augmented Goodwin growth cycle model and has

found that such an effect may imply local asymptotic stability for the overshooting mechanism of the Goodwin growth cycle, but can lead to instability (for high debt) outside a corridor around the steady state of the model. His paper in addition provides an interesting discussion of Fisher's vision of the interaction of over-indebtedness and deflation and also of Minsky's financial instability hypothesis. It then extends the proposed model of the interaction of indebted firms and income distribution to an inclusion of the role of government behaviour in such an environment and of nominal adjustment processes in place of the real ones of the Goodwin model. Details of his approach to debt deflation will be mentioned in the following sections of the chapter.

To introduce such a debt effect into our model of integrated disequilibrium growth demands, of course, that firms finance their fixed business investment expenditures not only by issuing new equities, as was the case in the approach chosen in Part II, but also by loans which they obtain (via certain intermediaries, not explicitly considered in this chapter; see however the subsequent ones) from pure asset owners, so that they finance these expenditures by a combination of equity (or retained earnings) and debt. Such a situation however calls for some rule by which firms split their financial needs into new equity supply and loan demand. There is a variety of possibilities for formulating such a rule which however makes the discussion of debt deflation more involved than is really necessary in a first treatment of its occurrence in a fully specified macrodynamic model. We therefore assume in this chapter for reasons of simplicity that firms finance their investment decisions (fixed business investment as well as involuntary inventory investment) exclusively via loans – apart from pure profits, as will be made clear below. The accumulation of debt is thus a simple consequence of the budget constraint of firms which only needs to be transferred to per unit of capital terms in order to provide one of the two new laws of motion of this chapter compared with the 18D case developed in Part II (the other new law of motion concerns the dynamics of the rate of interest on loans).

Introducing debt financing and removing equity financing from the general approach of Part II has the further implication that there are now fluctuations in the income of firms that go beyond the windfall losses or profits caused by disappointed or over-satisfied sales expectations. There are now also pure profits (or losses) to be considered as they will result from systematic deviations of actual (or expected) sales from the factor costs of firms now including interest payments besides wage and import costs. The budget equations and financing behaviour of firms, and their impact on their investment behaviour, therefore have to be reformulated in an appropriate way in order to take account of this deviation between total factor costs and the total proceeds of firms and the retained earnings based upon them.

Such a revision is however not needed if it were assumed that there are further investment expenditures (over and above those based on debt financing) that are financed by issuing new equities, as in Chiarella *et al.* (1999a,b),[4] and that all expected (pure) profits, based on total factor costs (including interest payments) and on the sales expectations

[4] In this regard see also Chiarella *et al.* (2000).

of firms, are paid out as dividends to asset owners (who besides providing the loans, also hold the total stock of equities that was issued in the past). This is a second way of investigating the role of debt of firms in inflationary or deflationary environments (not yet pursued in this chapter). As already stated it requires, in contrast to the case of a pure loan financing, some rule describing how firms choose between debt and equity financing in view of their intended fixed business investment.

In Section 8.2 we briefly present the changes to the model of Part II that are needed for a discussion of debt deflation from the perspective of national accounting. Section 8.3 then provides the new equations of the debt deflation model on the extensive form level and discusses these changes in comparison with the extensive form model of Part II and also their relationships to the work of Keen (2000). Section 8.4 gives a short description of the interior steady state of the model, its twenty laws of motion (now including loans and interest on loans) for its intensive form state variables, including various algebraic equations that supplement the dynamical laws.

Section 8.5 then approaches the issue of debt deflation by first starting from the basic 3D model of Keen (2000) which only allows for debt accumulation, but not yet for deflationary processes, by presenting some propositions relevant to this starting situation. We then extend the model to dimension 4 by including nominal price dynamics and again derive certain propositions on this extended situation with nominal price level adjustments. Section 8.6 then considers the 3D, the 4D and the general 20D dynamics from the numerical perspective and thereby illustrates what has been shown analytically. Section 8.7 concludes and gives a perspective on future developments.

8.2 Reformulating the structure of the economy

Tables 8.1 to 8.9 (and subsequent accounting presentations) provide a brief survey of the changes we shall make in this chapter with respect to the structure of the 18D core dynamics of the small open economies considered and investigated in Part II. These changes basically concern the financing conditions and the investment behaviour of firms assumed in Part II. We thus do not repeat here many of the structural details of this 18D model of disequilibrium growth, in particular of the real part of the economy, but refer the reader back to Part II for the full details of this model type (with fixed proportions in production).[5] When presenting the new intensive form of the now 20D dynamics we will however attempt to motivate its equations to some extent. These equations concern

- quantity adjustment processes and growth (sales expectations, inventories and the stock of labour besides the capital stock growth and growth of the housing stock of the economy);
- price adjustment processes (wages, prices, inflationary expectations and the price of housing services);

[5] The case of neoclassical smooth input and output substitution is considered in detail in Chiarella *et al.* (1999a,b), see also Chiarella *et al.* (2000).

- asset market dynamics (bond price and exchange rate dynamics, both including capital gain expectation formation);
- policy rules and government debt (the accumulation of short- and long-term bonds, wage and import taxation and a Taylor interest rate policy rule);
- as new laws of motion the dynamics of the debt to capital ratio and the dynamics of interest on loans.

Note that we will ignore value-added taxes on consumption goods in the following and thus set the parameter τ_v of the original 18D dynamics equal to zero for notational simplicity.

8.2.1 *Changes in the financial sector of the economy*

Let us first reformulate the financial part of the economy where all additions made with respect to the general framework presented in Part II are marked by bold letters. Note that we here switch from pure equity financing to pure loan financing as far as the external fund-raising of firms is concerned and that therefore the expected returns of firms are no longer distributed to households (but retained) in this revision of the 18D core model of Part II in order to allow concentration on the effects of debt financing for firms' performance (and also for worker households later on).

Table 8.1 shows that firms now use loans in the place of equities as instrument to finance (part of) their investment expenditures. These loans are supplied by pure asset holders in the gross amount $\dot{\Lambda}_f^b$ following the loan demand of firms. Loans are just an amount of money lent to firms (with a price of unity) and they exhibit a variable rate of interest i_λ which is applied to all loans (old and new ones, Λ_f, $\dot{\Lambda}_f^b$) in a uniform manner so that there is no term structure of interest rates as far as loans are concerned. Furthermore, in order to keep things simple, we assume that a certain fraction δ_λ of the stock of loans Λ_f existing at each moment in time is repaid in this moment of time, and that only net amounts of new debt $\dot{\Lambda}_f = \dot{\Lambda}_f^b - \delta_\lambda \Lambda_f$ need to be considered as far as budget equations and asset accumulation are concerned, in order to ease the presentation of the model in later sections of the chapter. Note that money is not treated as an asset in this chapter, due to specific assumptions made in Part II (where 'money' has been treated as a pure medium of account).

8.2.2 *Changes from the viewpoint of national accounting*

We shall consider in this subsection briefly the production and income accounts, and the accumulation and financial accounts of two of the four agents of our economy, firms and asset holders, whose relationship to each other is changed by the introduction of loans from asset holders to firms (in place of the equities and the dividend payments assumed in the version of the model presented in Part II). These accounts provide basic information on what has been changed compared with the general disequilibrium growth model of Part II.

Table 8.1. *The financial part of the economy (Foreign country data: i_l^*)*

	Short-term Bonds of the Government	Long-term Bonds of the Government	Loans to Firms	Foreign Bonds of the Foreign Government
Workers	$\dot{B}_w$	–	–	–
Asset holders	$\dot{B}_c$	$\dot{B}_1^l$	$\dot{\Lambda}_f = \dot{\Lambda}_f^b - \delta_\lambda \Lambda_f$	$\dot{B}_2^l$
Firms	–	–	$\dot{\Lambda}_f = \dot{\Lambda}_f^b - \delta_\lambda \Lambda_f$	–
Government	$\dot{B}$	$\dot{B}^l$	–	–
Prices	$1\ [i]$	$p_b = 1/i_l$	$1\ [i_\lambda]$	$sp_b^* = s \cdot 1/i_l^*$
Expectations	–	$\pi_b = \hat{p}_b^e$	–	$\epsilon_s = \hat{s}^e$
Stocks	$B = B_w + B_c$	$B^l = B_1^l + B_1^{l*}$	Λ_f	B_2^l
Growth	$\hat{B}, \hat{B}_w, \hat{B}_c$	$\hat{B}^l, \hat{B}_1^l$	$\hat{\Lambda}_f = \dot{\Lambda}_f^b/\Lambda_f - \delta_\lambda$	$\hat{B}_2^l$

Table 8.2. *Production account of firms*

Uses	Resources
Depreciation $p_y \delta_k K$	Consumption $p_y C_w + p_y C_c + p_y G$
Imports (including import taxes) $p_m J^d$	Durables (Housing) $p_y I_h$
Wages (including payroll taxes) $w^b L_f^d$	Exports $p_x X$
Interest on Loans $i_\lambda \Lambda_f$	Gross Fixed Business Investment $p_y I$
Actual Profits + Inventories $\Pi = r^a p_y K + p_y \dot{N}$	Actual Inventory Investment $p_y \dot{N}$
= Intended Profits + Inventories: $r^e p_y K + p_y \mathcal{I}$	Intended Inventory Investment $p_y \mathcal{I}$

We start with the accounts of the sector of firms – shown in Table 8.2 – which organise production Y, employment L_f^d of their workforce L_f^w and gross business fixed investment I and which use (in the present formulation of the model) loans Λ_f and expected retained earnings (plus windfall profits) as financing instruments for their desired net investment. There are import taxes τ_m on imported commodities and payroll taxes τ_p (with respect to hours worked L_f^d in the sector of firms). There are no subsidies and no longer value-added taxes τ_v on the consumption goods produced by firms, for reasons of simplicity. Note that all accounts are expressed in terms of the domestic currency. Note also that our one-good economy assumes that this good can be used for consumption and investment purposes (also for new housing supply).

Firms use up all imports J^d as intermediate goods which thereby become part of the unique homogeneous good Y that is produced for domestic purposes. They have replacement costs with respect to their capital stock, pay import taxes and wages including payroll taxes, and, as a new item, have to pay interest $i_\lambda \Lambda_f$ on their stock of loans Λ_f. Their accounting profit is therefore equal to actual pure profits $r^a p_y K$ (based on actual sales) and notional income gone into actual inventory changes (besides the depreciation fund for capital stock replacement).

Table 8.3. *Income account of firms*

Uses	Resources
Savings S_f^n	Profits Π

Table 8.4. *Accumulation account of firms*

Uses	Resources
Gross Fixed Investment $p_y I$	Depreciation $p_y \delta_k K$
Inventory Investment $p_y \dot{N}$	Savings S_f^n
–	Financial Deficit FD

Note that firms have sales expectations that follow actual sales in an adaptive fashion. They therefore experience (unexpected) windfall profits (or losses) for the financing of their fixed investment when their actual inventory changes are smaller than (larger than) their desired ones. Firms save all the income they receive and spend it on net fixed investment and on inventories of finished goods. The accumulation account is therefore self-explanatory as is the financial account which only repeats our statements made above that the financial deficit of firms is financed by new loans from pure asset holders.

Note also that the amount $\delta_\lambda \Lambda_f$ of existing loans must be repaid to asset holders (and replaced by new loans by assumption on credit market contracts) in each moment of time which means that the sum of all new loans $\dot{\Lambda}_f^b$ must be diminished by this magnitude in order to arrive at the rate of change of the stock of loans $\dot{\Lambda}_f$ to be considered later on. Note also that all goods are now valued at producer prices p_y which do not differ from consumer prices $p_v = (1 + \tau_v) p_y$ in the presently considered model ($\tau_v = 0$). There are also no direct (capital) taxes in the sector of firms, neither on property nor on profits. Note finally that the accumulation account of firms is based on realised magnitudes and thus refers to their intended inventories plans only implicitly. In the production account of the firms shown in Table 8.2, the important (single) change is depicted in bold-face letters.

The income account of firms is formally seen the same as in Part II and shown in Table 8.3.

The change in the accumulation account is also only an implicit one (based on the change in profits) and shown in Table 8.4.

There is finally the financial account of firms (see Table 8.5) where the debt financing of investment is the (single) new element (bold-face letters).

Turning next to the sector of asset holders we know from Part II that investment in housing as well as the supply of housing services has been exclusively allocated to

Table 8.5. *Financial account of firms*

Uses	Resources
Financial Deficit FD	**Loans** $\dot{\Lambda}_f = \dot{\Lambda}_f^b - \delta_\lambda \Lambda_f$

Table 8.6. *Production account of households (asset holders)*

Uses	Resources
Depreciation $p_y \delta_h K_h$	Rent $p_h C_h^w$
Rent Earnings Π_h	

Table 8.7. *Income account of households (asset holders)*

Uses	Resources
Tax Payment $\tau_c r B_c + \tau_c B_1^l$	Interest Payment $r B_c + B_1^l$
Taxes $\tau_c(p_h C_h^w - p_y \delta_h K_h)$	Interest Payment $s(1 - \tau_c^*) B_2^l$
Tax Payment $\tau_c i_\lambda \Lambda_f$	**Interest on Loans** $i_\lambda \Lambda_f$
Consumption $p_y C_c$	Rent Earnings Π_h
Savings S_c^n	

this sector. The production account of asset holders (see Table 8.6) therefore shows the actual sale (not the potential sale) of housing services (set equal to the demand for housing services by assumption) which is divided into replacement costs and actual earnings or profits on the uses side of the production account.

The income account of asset holders (see Table 8.7) comes from various sources: interest payments on short- and long-term domestic bonds and on long-term foreign bonds (net of tax payments which must be paid abroad), interest income on loans to firms and profits from housing rents. All domestic profit income is subject to tax payments at the rate τ_c and after tax income is by definition divided into the consumption of domestic commodities (including houses, but not housing services) and the nominal savings of asset owners.

The accumulation account (see Table 8.8) shows the sources for gross investment of asset holders in the housing sector, namely depreciation and savings, the excess of which (over housing investment) is then invested into financial assets as shown in the financial account. Note here that short-term bonds are fixed-price bonds $p_b = 1$ (which are perfectly liquid), while loans as well as long-term bonds are not perfectly liquid, the latter since they have the variable price $p_b = 1/i_l$ which shows that they are of the

Table 8.8. *Accumulation account of households (asset holders)*

Uses	Resources
Gross Investment $p_y I_h$	Depreciation $p_y \delta_h K_h$
Financial Surplus FS	Savings S_c^n

Table 8.9. *Financial account of households (asset holders)*

Uses	Resources
Short-term Bonds $\dot{B}_c$	Financial Surplus FS
Long-term Bonds $p_b \dot{B}_1^l$	
Foreign Bonds $s p_b^* \dot{B}_2^l$	
Loans $\dot{\Lambda}_f = \dot{\Lambda}_f^b - \delta_\lambda \Lambda_f$	

type of consols or perpetuities (the same holds true for imported foreign bonds, which are of long-term type solely).[6]

The central new topic in the income account of asset holders is of course the interest on the loans supplied to the worker households. The accumulation account is formally seen the same as in Part II. The loans of pure asset holders to worker households is again the single new topic as compared with Part II of the book.

The financial account of asset holders is shown in Table 8.9. There is no taxation of financial wealth (held or transferred) in the household sector and there is also no real property tax. Furthermore, although asset holders will consider expected net rates of return on financial markets in their investment decision, there is no taxation of capital gains on these markets as the model is presently formulated, which descriptively seems realistic.

We do not present the accounts of the worker households here as there is no change in their treatment as compared with Part II). Later on we shall however reinterpret the quantity B_w, their stock of short-term assets, as liabilities to the sector of asset holders, in other words as a negative quantity, and thus will get also debtor–creditor relationships between our two types of households, workers and pure asset holders, in addition to the one between firms and asset holders. We also do not present the foreign

[6] Due to the assumption of a given nominal rate of interest on foreign bonds, these bonds can always be sold at a given international price if this is desired by domestic residents, but they are then of course subject to exchange rate risks. Note that foreign bond purchases by domestic residents are treated as residual in the wealth accumulation decisions of the asset holders of the model of this chapter.

account, the balance of payments, here, as there is also no change in this single account, representing trade in goods, in capital and interest payments.

There are finally the accounts of the fiscal and the monetary authority which are slightly altered through the above additions to the accounts of firms and asset holders. We simply state in this regard that the term $\tau_c i_\lambda \Lambda_f$ has now to be added to the resources side of the income account of the government as the sole change in the activity accounts of the government.

Having presented the model from the ex post point of view as far as its new elements as compared with Part II are concerned we now turn to the structural form of the model and present in the following section its technological and behavioural relationships, various definitions and the budget equations of the agents of the domestic economy, and finally also the laws of motion for quantities, prices and expectations to the extent they are experiencing change by the modifications of the model described above.

8.3 The augmented 18+2D system: investment, debt and price level dynamics

We now start with the presentation of the structural equations of the advanced model of disequilibrium growth, here only with respect to the changes needed for a treatment of the problem of debt deflation. We will compare these changes with the building blocks of the Keen (2000) model step by step.

Module 1 of the model provides definitions of important rates of return r^e, r^a, an expected one based on sales expectations Y^e of firms and the actual one based on the actual sales Y^d of firms. Note that actual production Y exceeds expected sales by planned inventory changes $\mathcal{I}$ and that it differs from actual sales by actual inventory changes. Note also that the currently expected and the actual pure rate of profit (net of depreciation $p_y\delta_k K$ and of interest payments $i_\lambda\Lambda_f$) are both based on actual exports $X = x_y Y$, actual imports $J^d = j_y Y$ and the actual employment $L^d = l_y Y$ of the workforce of the firms. Our choice of notation of production coefficients indicates that we are assuming a technology with fixed input/output coefficients ($Y^p = y^p K$ the potential output of firms) where export supply is in fixed proportion to actual output Y, as is import demand and labour demand (the latter coefficients are however subject to Harrod neutral technological change: $\hat{l}_y = -n_l$). For the details of all the notation as well as further explanations of the equations used in this model the reader is referred to Part II. Note again that the changes made to the model are represented by bold letters in the following.[7]

1. Definitions (Rates of Return and Real Growth):

$$r^e = \frac{p_y Y^e + p_x x_y Y - \boldsymbol{i_\lambda \Lambda_f} - w^b l_y Y - p_m j_y Y - p_y \delta_k K}{p_y K}, \tag{8.1}$$

[7] Note also that there holds as a relationship between actual and expected profits: $r^a p_y K = r^e p_y K + p_y(\mathcal{I} - \dot{N}) = r^e p_y K + p_y(Y^d - Y^e)$.

$$r^a = \frac{p_y Y^d + p_x x_y Y - \boldsymbol{i_\lambda \Lambda_f} - w^b l_y Y - p_m j_y Y - p_y \delta_k K}{p_y K}, \tag{8.2}$$

$$\gamma = n + n_l \quad \text{all given magnitudes.} \tag{8.3}$$

The two rates of profits used in Part II are now defined by subtracting the interest payments of firms to asset holders based on the amount of loans Λ_f they have accumulated over the past. Furthermore trend growth in the world economy is given by the rate γ which is identified with the natural rate of growth n of the domestic working population plus the given rate of Harrod neutral technological progress n_l for reasons of simplicity.

Keen (2000, p. 83) considers a prototype 3D model of classical growth where besides the direct investment of capitalists (who own the firms and who reinvest all of their profit income, based on the pure profits of firms) there is also only pure loan financing of the remaining investment expenditure of the firms. These loans are supplied by financial asset holders (called banks in his paper) which are to be treated explicitly if his approach is to be compared with the one we present in the following. There are no demand constraints on the market for goods in Keen's (2000) paper which implies that his (uniquely determined) measure of the profitability $r = r^e = r^a$ of the firms' activities is based on actual = potential output $Y^p = y^p K$ throughout (with no reference to export or import activities of firms due to the assumption of a closed economy in his case). This gives as pure actual (= expected) rate of profit, used to describe the investment behaviour of firms in his paper, a rate of the type:

$$r = \frac{p_y Y^p - \boldsymbol{i_\lambda \Lambda_f} - w l_y Y - p_y \delta_k K}{p_y K}.$$

These profits accrue directly to the real capital owning households who do not consume, but only invest (and are therefore named firms in Keen (2000) and in the following for simplicity).

Module 2 provides the equations that concern the household sector where two types of households are distinguished, workers and pure asset or wealth owners. Of course, these two types of household are only polar cases in the actual distribution of household types. Nevertheless we believe that it is useful to start from such polar household types before intermediate cases are introduced and modelled. There is no change in the behavioural equations of worker households, as compared with Part II, which are therefore not repeated in the present chapter. In Keen (2000) workers spend what they get: $p_y C_w = wL^d$ (as in the original Goodwin (1967) growth cycle model) and wage income wL^d is not taxed since there is no government sector in the basic form of his model.[8]

The other type of household of our model, the asset owners, desire to consume C_c (goods and houses as supplied by firms through their domestic production Y) at an

[8] We denote by e the rate of employment L^d/L and assume for labour supply natural growth at a given rate $n = \hat{L}$.

amount that is growing exogenously at the rate γ which is therefore independent of their current nominal disposable income Y_c^{Dn}. The consumption decision is thus not treated here as an important decision of these pure asset holders. Their nominal interest and rent income (after taxes) diminished by the nominal value of their consumption $p_y C_c$ is then spent on the purchase of financial assets as well as on investment in housing services supply (for worker households). Note here that our one good representation of the production of domestic commodities entails consumption goods proper and houses so that asset holders buy houses for their consumption as well as for investment purposes.

2. Households (Asset Holders):

$$Y_c^{Dn} = (1-\tau_c)[i_\lambda \Lambda_f + iB_c + B_1^l + p_h C_h^w - p_y \delta_h K_h] + s(1-\tau_c^*)B_2^l, \tag{8.4}$$

$$\hat{C}_c = \gamma, \tag{8.5}$$

$$S_c^n = Y_c^{Dn} - p_v C_c = \dot{B}_c + \frac{\dot{B}_1^l}{i_l} + \frac{s\dot{B}_2^l}{i^{l*}} + \dot{\Lambda}_f + p_y(I_h - \delta_h K_h), \quad \dot{B}_c = \dot{B} - \dot{B}_w. \tag{8.6}$$

Changes in this sector of the economy are again quite small, since we only have to add interest income (from loans) to the income of asset holders (and of course to remove dividend payments as there are no equities in the changed model) and to describe later on how much of their savings goes into new loans $\dot{\Lambda}_f$ to firms, both shown above in bold-face letters. Note that the term $s\dot{B}_2^l/i^{l*}$ adjusts residually to the other changes in the wealth composition of asset holders in this chapter.

Keen (2000) does not consider explicitly the agents that supply credit to firms in the forms of loans (called 'banks' in his chapter). Yet there must be a budget equation for these agents, since their interest income will generally differ from their supply of new loans. This means that something like

$$i_\lambda \Lambda_f = \dot{\Lambda}_f + p_y C_c,$$

must be assumed in his approach since there is no demand constraint for the supply of output in his model, which can only be true if the budget constraints for the three types of agents imply that the demand for goods is always equal to their supply: $Y^p = C_w + C_c + I$. The budget equation just shown together with the one that has been assumed above for workers (workers spend what they get) and below for firms – as in Keen (2000) – indeed just guarantee this type of outcome. Note that the consumption of these 'credit institutions' may become negative in the Keen (2000) model, if they lend more than they get as interest rate income, in which case they must be considered as supplying commodities to the goods market (from their stock of goods).[9] Note finally

[9] Such an occurrence of negative consumption for 'banks' may be considered as problematic and must at least be based on the assumption that their accumulated stock of (durable) goods stays non-negative in time which means this type of agent is considered to build up stocks of finished commodities in certain times from which it sells (in order to provide additional loans) in other times (when this is demanded by firms). Note that nothing of this sort is needed in our general model (as long as firms have positive inventories) where moreover the output of firms is always demand determined.

that debt accumulation in Keen (2000) as well as in the present model does not consider debt repayments explicitly (but does consider only the net effect in this respect).

In the next module 3 of the model we describe the sector of firms, whose planned gross investment demand I is assumed to be always served, just as all consumption. We thus assume for the short run of the model – see Part II for the details – that it is always of a Keynesian nature, so that aggregate goods demand is never rationed, due to the existence of excess capacities, inventories, overtime work and other buffers that exist in this model type as well as in real market economies. There is thus only one regime possible, the Keynesian one, for the short run of the model, while supply side forces – concerning price dynamics – come to surface only in the medium and the long run (Keen's (2000) model by contrast is completely supply side based in its evolution of quantities). Note that we only display the investment relationships of the model, since there is no change in the description of technology, the employment policy of the firms and the like; see Part II for the details.

3. Firms (Investment Behaviour):

$$g_k = \alpha_r^k(r^e - i_\lambda) + \alpha_i^k(i_l - i) + \alpha_u^k(u - \bar{u}) + \gamma + \delta_k, \tag{8.7}$$

$$\dot{\Lambda}_f = p_y(I - \delta_k K) + p_y(\dot{N} - \mathcal{I}) - r^e p_y K = p_y(I - \delta_k K) - r^a p_y K, \tag{8.8}$$

$$\hat{K} = I/K - \delta_k = g_k - \delta_k. \tag{8.9}$$

We assume in the sector of firms, without showing this explicitly here, a fixed proportions technology.[10] The capital stock of firms is used to measure potential output $Y^p = y^p K$ in the following, while all other magnitudes are provided by the Keynesian regime and its demand determined output rate $Y = Y^e + \mathcal{I}$, $\mathcal{I}$ the inventory changes desired by firms. The rate of capacity utilisation $u = Y/Y^p$ is defined on the basis of the above concept of potential output and will receive importance when describing the investment behaviour and the pricing policy of firms. Firms employ a labour force of amount L_f^w which supplies labour effort of amount L_f^d as determined by the present state of sales expectations Y^e (plus voluntary inventory production). This labour force of firms is adjusted in a direction that reduces the excess or deficit in the utilisation of the employed labour force, $L_f^d - L_f^w$, which means that firms intend to return to the normal usage of their labour force thereby. An additional growth term for the employed labour force takes account of the trend growth γ of domestic output, but is diminished by the effect of Harrod neutral technical change, n_l, which when working in isolation would allow to reduce the workforce of the firms.

Explicitly presented above is the formulation of the desired gross rate of capital stock accumulation of firms, $g_k = I/K$, which depends on relative profitability, measured by the deviation of the expected rate of profit, r^e, from the interest rate, i_λ, firms have

[10] Smooth input and output substitution is considered, as in Powell and Murphy (1997), in Chiarella *et al.* (1999a,b), see also Chiarella *et al.* (2000), with respect to the three inputs, labour L_f^d, imports (raw materials) J^d, and capital K, and its two outputs (internationally) non-traded and traded goods, Y, X.

to pay on their debt, on the interest rate spread $i_l - i$, between long- and short-term government debt, representing the tightness of monetary policy and its believed effects on economic activity, on the rate of capacity utilisation u of the capital stock of firms in its deviation from the desired rate of capacity utilisation, $\bar{u}$, which is given exogenously, and on trend growth γ and the rate of depreciation δ_k of business fixed investment. When comparing the rates r^e, i_λ in their investment decision firms decide to increase their investment projects via additional debt if $r^e - i_\lambda > 0$ holds (and vice versa). They do not pay attention here to (expected) inflation and the implied real rate of interest on their loans when making this decision. This would make the considered dynamics much more involved, in particular through the medium-run rates of return then to be used as in Chiarella *et al.* (2000, Part III) in conjunction with the expected medium-run rate of inflation used in the wage-price module of this chapter. We expect that such an extension would add further momentum to the debt effects to be investigated in later sections of this chapter.

The budget equation (8.8) of firms says that firms have to finance net investment and all inventory changes $\dot{N}$ (unintended inventory changes $\mathcal{I}$) by the profits that are based on actual output Y (expected sales Y^e, respectively) or by making new loans. Note here that unintended inventory disinvestment gives rise to windfall profits to firms which are retained and thus used to finance part of the fixed business investment as shown by the above budget equation if $\dot{N} - \mathcal{I} < 0$ holds true. The last equation of the above module of the model finally defines the growth rate of the capital stock which is determined by the net rate of capital accumulation planned by firms (due to the Keynesian nature of the short run of the model).

Keen (2000) assumes as budget equation of firms (owned by capitalists) the following equation

$$\dot{\Lambda}_f = p_y(I - \delta_k K) - r p_y K,$$

where r is the actual pure rate of profit. Firms therefore finance net investment $I - \delta_k K$ by means of pure profits $r p_y K$ (which are always reinvested) and new debt, the latter being determined residually. There are no (unintended) inventory changes, but there is full capacity growth with goods demand always equal to goods supply. Furthermore, he assumes that gross investment is driven by the pure rate of profit (net of interest) $r = r^e$ solely which gives

$$I/K = \alpha^k(r - r_{min}) + \delta_k \quad \text{in place of his equation} \quad I/Y = \alpha^k(r),$$

if we use the notation of our modelling approach and neglect the non-linearity in the investment behaviour considered in Keen (2000). These two equations constitute two of the three laws of motions of his basic model of debt accumulation and wage inflation or deflation.

The next equation describes the change in the public sector of the economy, as compared with Part II, which only concerns tax collection, where taxes on the interest

paid for loans are to be added (and taxes on dividends to be ignored) in the equation describing the tax income of the government.

4. Government (Fiscal Authority):

$$T^n = \tau_w[wL^d + w^u(L - L^w) + w^r\alpha_l L_2] + \tau_p wL^d,$$
$$+ \tau_c[i_\lambda \Lambda_f + iB + B^l + p_h C_h^w - \delta_h K_h] + \tau_m s p_m^* J^d. \quad (8.10)$$

Keen (2000) considers the government sector in a later section of his paper, there based on dynamic government expenditure and taxation rules that differ from the ones underlying the present approach. This module of the model may be used as in Keen (2000) to consider the topic of automatic stabilisers and the like.

There is no change in the description of the dynamics of quantities and prices. We however here present briefly the two Phillips curves (PCs) that describe money wage and price level dynamics since these equations are of course of importance when the problem of debt accumulation is to be approached in a deflationary (and of course also in an inflationary) environment.

5. Wage-Price Adjustment Equations, Expectations:

$$\hat{w}^b = \beta_{w_e}(e - \bar{e}) + \beta_{w_u}(u_f^w - \bar{u}_f^w) + \kappa_w(\hat{p}_y + n_l) + (1 - \kappa_w)(\pi^c + n_l), \quad (8.11)$$
$$\hat{p}_y = \beta_p(u - \bar{u}) + \kappa_p(\hat{w}^b - n_l) + (1 - \kappa_p)\pi^c. \quad (8.12)$$

The two equations just shown describe the dynamics of nominal (gross) wages as dependent on demand pressure terms, here specifically the outside and the inside employment of workers, $e - \bar{e}$, $u_f^w - \bar{u}_f^w$, measured as deviation from Non-Accelerating Inflation Rate of Unemployment (NAIRU) levels, and on cost pressure terms, here the short-term actual and the medium-term expected rate of price inflation, $\hat{p}_y$, π^c, augmented by the rate of productivity growth, n_l. We shall set $\bar{u}_f^w$ equal to 1 in the following which means that each employed worker provides one unit of labour if there is no over- or under-employment within firms. Similarly, price inflation depends on demand pressure in the market for goods, here solely measured by the rate of capacity utilisation, $u - \bar{u}$, in its deviation from the NAIRU rate of capacity utilisation,[11] and on wage cost pressure, diminished by productivity growth. These equations have been explained in their details in Part II and here serve the purpose of indicating how inflationary or deflationary processes (based on demand pressure as well as cost-push terms) are to be integrated into an environment where firms use debt to finance at least part of their investment expenditures.

Keen (2000) considers a money wage PC, $\hat{w} = \beta_w(e)$, based on demand pressure on the (outside) labour market solely, and assumes with respect to the price level p_y that it is a given magnitude ($\equiv 1$ implying of course $\pi^c = 0$). His third law of motion of

[11] The term NAIRU is used in an extended way in this chapter and should be read as Non-Accelerating Inflation Rate of Utilisation. Note here also that a second term in the price PC could be given by the deviation of desired inventory holdings from actual inventory holdings.

the considered growing economy can thus be obtained from the first of the above two PCs by assuming $\beta_{w_u} = 0, \kappa_w = 1$. This gives for the dynamics of the share of wages v in national income of the Keen model:

$$\hat{v} = \beta_{w_e}(e - \bar{e}) - n_l,$$

where we again use a linear form for the time being. In a final section, he briefly considers prices for consumption and capital goods separately, but does not yet represent their dynamics by way of formalised laws of motion.

The module of asset price dynamics of Part II is to be augmented in the present context by just one equation describing the dynamics of the interest rate on loans (while all other adjustment processes in these markets remain as before):

6. Asset Prices, Expectations and Interest Rate Adjustments:

$$\dot{i}_\lambda = \beta_{i_\lambda}(i_l - i_\lambda). \tag{8.13}$$

We here simply assume that the rate of interest on loans follows the rate of interest on long-term government bonds with some time delay, measured by the speed of adjustment β_{i_λ}, similar to the other delayed interest rate parity conditions used in our model. In Keen (2000) the rate of interest i_λ on loans is a given magnitude, so that there is no need there to formulate a law of motion for this interest rate variable. Note that it may take considerable time until the steering of the short-term rate of interest i by the central bank (via a Taylor interest rate policy rule) can actually exercise a significant effect on the interest rate on loans governing the firms' investment decision in the present model.

Summing up the dynamics of the core model in Keen (2000) builds on a money wage dynamics of type $\hat{w} = \beta_w(e - \bar{e})$, $e = L^d/L$ with only labour market demand pressure influences (since the price level is still kept constant), on an investment-demand driven growth path $\alpha^k(r - r_{min})$ that is partly financed by loans (at a given rate of interest) and on the budget equation of firms $\dot{\Lambda}_f = p_y(I - \delta_k K) - rp_yK$ where r is given by $\frac{p_yY^p - i_\lambda\Lambda_f - wl_yY^p - p_y\delta_k K}{p_yK}$. These three dynamic laws operate in a fixed proportions technological environment (exhibiting Harrod neutral technical change) with natural labour force growth, no savings out of wages and no effective demand constraint on the market for goods. We shall reconsider this fundamental approach to debt-financed economic growth in intensive form, and also its implications, in the next sections of the chapter.

We have added to this model type in particular an endogenous determination of the price level and of the rate of interest paid on loans, and also a Keynesian demand constraint. Furthermore, as shown in Chiarella *et al.* (2000), there are detailed descriptions of the behaviour of the fiscal and the monetary authority in our extension of this model, more advanced types of structural relationships for consumption, investment and financial wealth accumulation (still without feedback effect on the real side of the economy due to the lack of wealth effects in consumption) and also a detailed treatment of asset markets and their dynamics with heterogeneous expectations formation on these markets as well as with respect to wage and price formation.

8.4 Intensive form representation of the 20D dynamics

In this section we present our modification of the 18D core model of Part II in intensive form in order to allow for the consideration of debt financing of the investment undertaken by firms and the problem of debt deflation in this model type. To simplify the model slightly we assume in the following that $C_c(0)$ holds initially (and thus for all times) and thus neglect the consumption of asset holders altogether (which does not contribute to the present investigation very much under the assumptions made). We stress that the resulting dynamics on the state variable level are then of dimension twenty, due to the additional laws of motion formulated in the preceding section for the accumulation of debt by firms and for the interest rate paid by them on their loans.

We start with a compact presentation (including brief comments on their contents) of these twenty laws of motion (now including the state variables $\lambda_f = \Lambda_f/(p_yK)$ and i_λ), and will present thereafter the unique interior steady state solution of these dynamics. These laws of motion around the steady state of the dynamics, appropriately grouped together and all in per (value) unit of capital form, and in efficiency units if necessary, are the following ones.

As first group we consider the quantity adjustment mechanisms with respect to the market for goods, concerning sales expectations y^e and actual inventories ν, and for labour, concerning the employment policy of firms, l_f^{we}, and also concerning the evolution of full employment labour intensity l^e (both in efficiency units) and of the stock of housing (everything per unit of the capital stock of firms):

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + (\gamma - (g_k - \delta_k))y^e, \tag{8.14}$$

$$\dot{\nu} = y - y^d - (g_k - \delta_k)\nu, \tag{8.15}$$

$$\dot{l}_f^{we} = \beta_l(l_f^{de} - l_f^{we}) + [\gamma - (g_k - \delta_k)]l_f^{we}, \tag{8.16}$$

$$\hat{l}^e = \gamma - (g_k - \delta_k), \tag{8.17}$$

$$\hat{k}_h = g_h - \delta_h - (g_k - \delta_k). \tag{8.18}$$

The first of these five laws for quantity movements describes the adjustment of sales expectations y^e in view of the observed expectational error $y^d - y^e$ based on currently realised sales y^d, augmented by a term that takes account of the fact that this adjustment is occurring in a growing economy. Next, inventories ν change according to the gap between actual output y and actual sales y^d, again reformulated such that growth of the capital stock, the measurement base for the considered intensive form variables, is taken account of. Employment of firms, l_f^{we}, is changed in order to reduce the discrepancy that currently exists between the actual employment l^{de} of the employed and their normal employment, here measured both by l_f^{we} (everything measured in efficiency units). The growth rate of the factor endowment ratio l^e (in efficiency units) is simply given by the difference between the natural rate of growth (including Harrod neutral technical change) and the growth rate of the capital stock $g_k - \delta_k$. Similarly, the growth rate of the housing stock (per unit of the capital stock of firms) is simply given by the difference of the accumulation rates of the stock of houses and the capital stock.

Next we consider the nominal dynamics in the real sector of the economy which are described by four dynamical laws. Note here that the laws of motion for wages, w^e, net of payroll taxes and in efficiency units, and prices, p_y, are here formulated independently from each other and show that reduced form or across markets PCs (exhibiting only one rate of inflation as remaining cost-push term) are generally not as simple as is often assumed in the literature:[12]

$$\hat{w}^e = \pi^c + \kappa[\beta_{w_e}(l^{we}/l^e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - 1) + \kappa_w \beta_p(y/y^p - \bar{u})], \tag{8.19}$$

$$\hat{p}_y = \pi^c + \kappa[\kappa_p(\beta_{w_e}(l^{we}/l^e - \bar{e}) + \beta_{w_u}(l_f^{de}/l_f^{we} - 1)) + \beta_p(y/y^p - \bar{u})], \tag{8.20}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p}_y - \pi^c) + (1 - \alpha_{\pi^c})(0 - \pi^c)), \tag{8.21}$$

$$\hat{p}_h = \beta_h(\frac{c_h^w}{k_h} - \bar{u}_h) + \kappa_h \hat{p}_y + (1 - \kappa_h)\pi^c. \tag{8.22}$$

As already noted we now use reduced form PCs for wage inflation $\hat{w}^e$ and price inflation $\hat{p}_y$ which both depend on the demand pressures in the markets for labour (external and internal ones: $l^{we}/l^e - \bar{e}, l_f^{de}/l_f^{we} - 1$) as well as for goods, $y/y^p - \bar{u}$. The change of the rate of inflation expected over the medium run, π^c, is determined as a weighted average of adaptively formed expectations and regressive ones (which implies that the steady state rate of inflation is zero in the present model). Finally, the inflation rate for housing services depends on the demand pressure term $\frac{c_h^w}{k_h} - \bar{u}_h$ in the market for these services,[13] and on actual and perceived cost-push expressions, here simply based on a weighted average concerning the inflation rate of domestic output.

There follow below the dynamical laws for long-term bond price dynamics and exchange rate dynamics (including expectations) which basically formulate a somewhat delayed adjustment towards interest rate parity conditions and are supplemented by heterogeneous expectations formation (of partially adaptive and partially perfect type). Note that perfect foresight, concerning the proportion $1-\alpha_s$ of market participants, can be removed from explicit representation as it coincides with the left-hand side of the

[12] Such disentangled laws of motion for nominal prices and wages are obtained from their originally interdependent presentation – see the preceding section – by solving the two linear equations of this section with respect to the variables $\hat{w}^e - \pi_l, \hat{p}_y - \pi_l$. This implies the expressions shown in equations (8.19) and (8.21), which both depend, via demand-pull and cost-push inflation pass through considerations, on our measures of demand pressure on the market for labour *as well as* on the market for goods; and on the expected medium-run inflation π^c in addition, the only cost-push term that is still explicitly shown in the equations (8.19) and (8.21). It is intuitively obvious that the removal of wage or price inflation cost-push pressure, $\hat{w}^e$, $\hat{p}_y$, from price or wage dynamics must imply that both the goods and the labour market demand pressure will be present in the resulting disentangled PCs which thus are in a significant way more general than the ones usually considered in the theoretical or applied literature on price PCs unless one assumes – as some kind of Okun's law – that the two demand pressure variables used are positive multiples of each other. But even then the composed parameters of our reduced form equations (8.19) and (8.21) clearly show the complex way the labour and the goods market are interrelated in these two equations.

[13] Where $\frac{c_h^w}{k_h}$ represents the rate of capacity utilisation demanded on this market and $\bar{u}_h$ its NAIRU level.

corresponding price adjustment equation, giving rise to the fractions in front of these adjustment equations (see Chiarella *et al.* (2000) and here Part II for the details):[14]

$$\hat{p}_b = \frac{\beta_{p_b}}{1 - \beta_{p_b}(1 - \alpha_s)}[(1 - \tau_c)i_l + \alpha_s \pi_{bs} - (1 - \tau_c)i], \quad i_l = 1/p_b, \tag{8.23}$$

$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}), \tag{8.24}$$

$$\hat{s} = \frac{\beta_s}{1 - \beta_s(1 - \alpha_s)}[(1 - \tau_c)i_l^* + \alpha_s \epsilon_s - ((1 - \tau_c)i_l + \pi_b)], \quad i_l = 1/p_b, \tag{8.25}$$

$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{s} - \epsilon_s). \tag{8.26}$$

Note that the literature generally only considers the border case where $\alpha_s = 0$ is used in conjunction with infinite adjustment speeds on the two considered markets. This gives rise to two interest parity conditions coupled with myopic perfect foresight on bond price and exchange rate movements, a situation of knife-edge instability, which is stabilised by means of the so-called jump variable technique.[15]

The next set of dynamical laws concerns the evolution of short- and long-term debt of the government (the relative issuing of which is here subject to fixed proportions $\alpha_b^g, 1 - \alpha_b^g$), its wage and import taxation policy and the interest rate policy of the central bank.

$$\dot{b} = \alpha_b^g[gy^e + rb + b^l - t^w - t^c + w^a] - (\hat{p}_y + g_k - \delta_k)b, \tag{8.27}$$

$$\dot{b}^l = (1 - \alpha_b^g)/p_b[gy^e + rb + b^l - t^w - t^c + w^a] - (\hat{p}_y + g_k - \delta_k)b^l, \tag{8.28}$$

$$\hat{\tau}_w = \alpha_{\tau_{w1}}\left(\frac{\lambda_g}{\bar{\lambda}_g} - 1\right), \quad \lambda_g = \frac{b + p_b b^l}{y^e}, \tag{8.29}$$

$$\hat{\tau}_m = \alpha_{\tau_m}\frac{p_x^* x - (1 + \tau_m)p_m^* j^d}{p_x^* x}, \quad x = x_y y, \; j^d = j_y y, \tag{8.30}$$

$$\dot{i} = -\beta_{i_i}(i - i_l^*) + \beta_{i_p}\hat{p}_y + \beta_{i_u}(y/y^p - \bar{u}). \tag{8.31}$$

Since these laws of motion, with the exception of the interest rate policy rule, are not of central interest in the following analysis we here only briefly state that the first two are immediate consequences of the government budget constraint (based in particular on various sources of tax income), that wage taxation is here adjusted in the direction of a target ratio of government debt, $\bar{\lambda}_g$, and that import taxes are adjusted in a way that ensures a balanced trade account in the steady state (which greatly simplifies the calculation of the steady state of the model). The interest rate policy rule (8.31) is of interest since it could be of help to counteract accelerating debt deflation, by lowering, if still possible, nominal interest rates in situations of depressed output and price deflation.

[14] These laws of motion are made of secondary importance in this chapter as we shall assume in this part of model low adjustment speeds for the time being (which is not realistic). More appropriate treatments are thus left here for future research.

[15] See Blanchard (1981) for an early and typical example of this type of macro-modelling.

This rule states that the central bank attempts to steer the domestic short-term interest rate i towards its international level, i_l^*, but deviates from this general target in view of current price inflation $\hat{p}_y$ and the current state of the business cycle as measured by demand pressure on the market for goods $u - \bar{u}$. Note here that the inflationary target of the central bank and thus steady state inflation is assumed as zero in the present chapter in order to ease the presentation of the state variables of the model.

There remain the two dynamical laws that are new to the model:

$$\dot{\lambda}_f = g_k - \delta_k + y - y^d - \beta_n(\beta_{n^d} y^e - \nu) - \gamma\beta_{n^d} y^e - r^e - (\hat{p}_y + g_k - \delta_k)\lambda_f, \tag{8.32}$$

$$\dot{i}_\lambda = \beta_{i_\lambda}(i_l - i_\lambda). \tag{8.33}$$

Although the dynamical law for absolute debt accumulation considered in the preceding section is by and large a simple one, its representation on the intensive form level is somewhat complicated due to the fact that unintended inventory changes are involved (and expressed in intensive form) besides the rate of capital accumulation $g_k - \delta_k$, and due to the fact that debt is now calculated in per value unit of capital (divided by $p_y K$) which transformed to growth rates gives rise the addition of $-(\hat{p}_y + g_k - \delta_k)\lambda_f$. By contrast, there is no change needed in the law of motion for interest on loans since it only involved state variables of the model right from the start. We shall consider in the next section the evolution of the ratio λ_f in situations of increasing generality, at first only coupled with laws of motion for nominal wage adjustment (and thus given prices) and the evolution of labour intensity in a supply side growth model. Thereafter we include a static simplification of the quantity adjustment processes on the goods market considered above (leading to a demand driven growth model) and add the price level dynamics (8.20) to not only allow for debt accumulation, but also for goods price deflation in situations of depressed rates of capacity utilisation. Finally we will also study the more general case where quantity and price adjustment processes interact (still without much stress on fiscal policies, asset markets, all kinds of expectations, the housing sector and the openness of the economy).

The above twenty laws of motion for the state variables of the model make use in addition of the following supplementary intensive form definitions and abbreviations (which are not explained here in detail, since we only provide the essential features of the modelling approach of Part II of the book):

$$y = y^e + \beta_n(\beta_{n^d} y^e - \nu) + \gamma\beta_{n^d} y^e,$$

$$l_f^{de} = l_y^e y, \quad (l_y^e \text{ the labour coefficient in efficiency units}),$$

$$l_g^{de} = l_g^{we} = \alpha_g g y^e,$$

$$l^{de} = l_f^{de} + l_g^{de},$$

$$l^{we} = l_f^{we} + l_g^{we},$$

$$y_{w1} = w^e \left[l^{de} + \alpha^u(l^e - l^{we}) + \alpha^r \frac{L_2(0)}{L_1(0)} l^e \right] / p_y,$$

$$c_g^w = c_y(1 - \tau_w) y_{w1},$$

$$c_h^w = p_y c_h (1 - \tau_w) y_{w1} / p_h,$$

$$r^e = y^e - \delta_k + (s p_x^*/p_y) x_y y - i_\lambda \lambda_f - ((1 + \tau_p) w^e / p_y) l_f^{de} - ((1 + \tau_m) s p_m^* / p_y) j_y y,$$

$$g_k = \alpha_r^k (r^e - i_\lambda) + \alpha_i^k (i_l - i) + \alpha_u^k (y/y^p - \bar{u}) + \gamma + \delta_k, \quad (i_l = 1/p_b),$$

$$g_h = \alpha_r^h ((1 - \tau_c)((p_h/p_y) c_h^w / k_h - \delta_h) - ((1 - \tau_c) i_l - \pi^c)) + \alpha_i^h (i_l - i)$$
$$+ \alpha_u^h \left(\frac{c_h^w}{k_h} - \bar{u}_h \right) + \gamma + \delta_h, \quad (i_l = 1/p_b),$$

$$y^d = c_g^w + g_k + g_h k_h + g y^e,$$

$$\pi_b = \alpha_s \pi_{bs} + (1 - \alpha_s) \hat{p}_b,$$

$$t^w = \tau_w w^e \left[l^{de} + \alpha^u (l^e - l^{we}) + \alpha^r \frac{L_2(0)}{L_1(0)} l^e \right] / p_y + \tau_p w^e l^{de} / p_y + \tau_m s p_m^* j_y y / p_y,$$

$$t^c = \tau_c \left[i_\lambda \lambda_f + i b + b^l + (p_h/p_y) c_h^w - \delta_h k_h \right],$$

$$w^a = w^e \left[\alpha^u (l^e - l^{we}) + \alpha^r \frac{L_2(0)}{L_1(0)} l^e + (1 + \tau_p) l_g^{de} \right] / p_y.$$

These equations state how output y depends on expected demand and inventories, how employment is determined in the private and the public sector, how disposable wage income y_{w1}^Λ of workers is formed and used for goods and housing services consumption, how the expected rate of pure profits and the rates of accumulation for capital and houses are defined on the intensive form level, how aggregate demand per unit of capital y^d is composed from consumption, investment and government demand and how average bond price expectations are composed from adaptive and perfect expectations. There are finally three expressions for wage and import taxation, property income taxation and transfer payments to and from the government.

Inserting these equations into the above twenty laws of motion gives rise to an explicit system of twenty autonomous non-linear differential equations in the twenty state variables of the model, given by equations (8.14)–(8.33). Note that we have to supply as initial conditions the relative magnitude $\frac{L_2(0)}{L_1(0)}$ in order to get a complete characterisation of the dynamics and that the evolution of price levels is subject to zero-root hysteresis, since it depends on historical conditions due to our assumptions on the interest rate policy rule of the central bank and the accompanying assumption of costless cash transactions (during each trading period) for the four agents of the model; see Part II for details.

We present next the twenty steady state values of the model. All these values should normally have an index 'o' to denote their steady state character. As we have done earlier, to not overload the notation we do not add this index to the following list of steady state values. Note that the steady state values of level magnitudes are all expressed in per unit of capital form and if necessary in efficiency units; see Part II for the details in the case of the 18D core model.

Note also that we have now debt of firms and of the government in the model and that we therefore denote their actual and steady debt to capital ratios by choosing appropriate indexes in both cases: λ_f, λ_g. Note finally that the steady state is parametrically dependent on a given output price level p_y which is not determined by the model (due to the Taylor type interest rate policy pursued by the central bank) and thus can be supplied from the outside in an arbitrary fashion:

$$y^e = \frac{y^p \bar{u}}{1+\gamma\beta_{n^d}} \quad [y = y^p \bar{u}], \tag{8.34}$$

$$\nu = \beta_{n^d} y^e, \tag{8.35}$$

$$l_f^{we} = l_f^{de} = l_y^e y^p \bar{u} \quad [\text{total employment: } l^{we} = l_f^{we} + l_g^{we}, l_g^{we} = \alpha_g g y^e], \tag{8.36}$$

$$l^e = (l_f^{we} + \alpha_g g y^e)/\bar{e}, \tag{8.37}$$

$$k_h = \frac{d_h(y^e(1-g) - (\gamma+\delta_k))}{d_y(i_l^* + \delta_h) + (\gamma+\delta_h)d_h}, \tag{8.38}$$

$$w^e = \frac{\omega^{be} p_y}{1+\tau_p} \quad \left[\omega^{be} = \frac{y^e - \delta_k - i_l^*\lambda_f - i_l^*}{l_f^{we}}\right], \tag{8.39}$$

$$p_y = \text{arbitrary}, \tag{8.40}$$

$$\pi^c = 0, \tag{8.41}$$

$$p_h = p_y(i_l^* + \delta_h)/\bar{u}_h, \tag{8.42}$$

$$p_b = 1/i_l^*, \tag{8.43}$$

$$\pi_{bs} = 0, \tag{8.44}$$

$$s = \frac{s_o - [\tau_w y_{w1} + \tau_p \frac{w^e}{p_y} l^{we}]}{\tau_m p_m^* j_y y/p_y}, \tag{8.45}$$

$$\epsilon_s = 0, \tag{8.46}$$

$$b = \alpha_b^g \bar{\lambda}_g y^e, \tag{8.47}$$

$$b_l = i_l^*(1-\alpha_b^g)\bar{\lambda}_g y^e, \tag{8.48}$$

$$\tau_w = 1 - \frac{p_h \bar{u}_h k_h}{c_h p_y y_{w1}}, \tag{8.49}$$

$$\tau_m = \frac{p_x^* x_y - p_m^* j_y}{p_m^* j_y}, \tag{8.50}$$

$$i = i_l^*, \tag{8.51}$$

$$\lambda_f = \frac{\gamma - i_l^*}{\gamma}, \tag{8.52}$$

$$i_\lambda = i_l^* = [r^e]. \tag{8.53}$$

With respect to the two equations for the wage tax rate τ_w and for the rate of exchange s of the model we have to apply (besides the steady values calculated for of y, l^{we}, and ω^{be}, see the above) the further defining expressions

$$c_h^w = \bar{u}_h k_h,$$

$$t_o^c = \tau_c[i_l^*\lambda_f + i_l^* b + b^l + (p_h/p_y)c_h^w - \delta_h k_h],$$

$$s_o = g y^e + i_l^* b + b^l - t_o^c + \frac{w^e}{p_y}\left[\alpha^u(l^e - l^{we}) + \alpha^r \frac{L_2(0)}{L_1(0)} l^e\right] + (1+\tau_p)\frac{w^e}{p_y}\alpha_g g y^e - \frac{\gamma b}{\alpha_b^g},$$

$$y_{w1} = w^e\left[l^{we} + \alpha^u(l^e - l^{we}) + \alpha^r \frac{L_2(0)}{L_1(0)} l^e\right]/p_y,$$

in order to have a determination of the interior steady state solution that is complete.

Note that the value of the exchange rate s will be indeterminate when we have $\tau_m = 0$ in the steady state and that the above formula for s cannot be applied then. Note furthermore that the parameters of the model have to be chosen such that k_h, τ_w, s are all positive in the steady state.[16] Note finally that the parameter α_s, the proportion of adaptive forecasters, must always be larger than $1 - 1/\beta_x$ for $x = p_b, s, p_e$ in order to satisfy the restrictions established in Chiarella *et al.* (2000) and here in Part II.

Equation (8.34) gives the steady state solution of expected sales y^e per unit of capital K (and also output y per K) and equation (8.35) provides on this basis the steady inventory-capital ratio N/K. Equation (8.36) represents the amount of workforce (per K) employed by firms which in the steady state is equal to the hours worked by this workforce. It also shows total employment (per K) where account is taken of the employment in the government sector in addition. Equation (8.37) represents full employment labour intensity (in the steady state), while the last expression for the quantity side of the model, in equation (8.38), provides the steady value of the housing capital stocks per unit of the capital stock of firms.

Equation (8.39) concerns the nominal wage level (net of payroll taxes and in efficiency units) to be derived from the steady state value for gross real wages ω^{be}, which include payroll taxes, which depends on the amount of interest to be paid on the loans of firms. The steady state value of the price inflation rate expected to hold over the medium run is zero – see equation (8.41) – since the inflationary target of the central bank is zero in the present formulation of the model. This also implies that all nominal magnitudes

[16] There are further simple restrictions on the parameters of the model due to the economic meaning of the variables employed.

(up to nominal wages) have no long-run trend in them and that all expected rates of change – see equations (8.41), (8.44), (8.46) – must be zero in the steady state. Again, in equation (8.40), p_y can be any value due to the assumptions made on monetary policy and money balances. Note that all nominal magnitudes, up to the price for long-term bonds p_b – see equation (8.43) – depend on p_y and thus change proportionally when this price level magnitude is changed. As remaining nominal magnitudes we have the price level p_h for housing rents (in equation (8.42)), to be calculated from the uniform rate of interest i_l^* of the economy in the steady state (provided by the world economy), and the nominal exchange rate, s, in equation (8.45), which is given by a complicated expression to be obtained from the government budget constraint, due to the import taxation rule followed by the government. Note here that the equations for the steady state of the economy are presented in the same order as its laws of motion. They have to be reordered from the mathematical point of view when solved in a recursive fashion.

There follows the steady state value of short-term government debt per unit of capital $b = B/(p_v K)$ as well as the one for long-term domestic bonds, in equations (8.47) and (8.48), which are both simple consequences of the debt adjustment rule of the government and the rigid proportions by which government splits its debt in short- and long-term components. The steady state value of the wage tax rate – see equation (8.50) – is obtained from wage income-spending relationships of worker households, here performed on the basis of the housing services demanded and supplied in the steady state,[17] while the steady value of the import tax rate, in equation (8.50), just balances the trade account (when import taxes are included into it). With respect to the public sector, there is finally the interest rate policy rule of the central bank, which due to its formulation implies that the interest rate on short-term government debt must settle down at the given foreign rate, i_l^*, in the steady state.[18]

Again, the new equations are equations (8.52) and (8.53), where the steady debt to capital ratio of firms is easily obtained from the budget constraint of firms and is positive if and only if the world rate of interest is smaller than the natural rate of growth (including the rate of technical progress) of the domestic economy. Finally, the steady value of the rate of interest on loans, i_λ, is provided which quite obviously must settle down at $i = i_l^*$. This closes the presentation of the interior steady state solution of our 20D dynamical model.

We have used in the preceding section as point of reference for the general 20D model the extended supply side growth cycle dynamics formulated and investigated in Keen (2000) which includes loans to firms and thus debt financing of (part of) their investment expenditures in a very fundamental way. We have thus now at our disposal two polar cases for the discussion of debt accumulation and debt deflation, a very basic classical one where the stress should lie on analytical results and a proper inclusion

[17] Making use of gross steady wage income y_{w1} and the marginal propensity to spend this income on housing services.

[18] The steady value of the short-term rate of interest equals its long-run equivalent as there is no risk or liquidity premium in the 18D version of Part II as well as in the present 20D extension of it.

of deflationary processes – see the next section – and a very detailed Keynesian one where the question should be how it compares numerically with the insights obtained for the smaller models.

Taken together, and based on the linear behavioural assumptions used in our approach to debt and deflation, the equations of the theoretical starting point of the investigation can be represented as 3D dynamical systems in the state variables $v = wL^d/p_y Y^p$, the wage share, $e = L^d/L$, the rate of employment, and $\lambda_f = \Lambda_f/p_y K$, the debt to capital ratio of the firms, as follows:[19]

$$\hat{v} = \beta_w(e - \bar{e}) - n_l, \tag{8.54}$$

$$\hat{e} = \alpha^k(r - r_{min}) - (n + n_l), \tag{8.55}$$

$$\dot{\lambda}_f = \alpha^k(r - r_{min})(1 - \lambda_f) - r, \tag{8.56}$$

where $r = y^p(1 - v) - \delta_k - i_\lambda \lambda_f$ is the actual rate of profit in this supply driven approach to economic growth.[20] As stated, we use a linear PC mechanism and a linear investment function in this representation of the Keen (2000) model and leave the discussion of behavioural non-linearities for future investigations. Note that we have made use of the notation of our general model presented above in order to express the laws of motion of the Keen (2000) core dynamics.

There is not yet a foreign and a government sector in this form of the Goodwin growth cycle model (up to the indication of credit supplying institutions: see our discussion in the preceding section), but only the interaction of firms (capitalists) and worker households. The first two equations of this model would in fact be identical to the original Goodwin (1967) growth cycle approach if debt would not be there in the formulation of the pure profit rate of the model and if $\alpha^k = 1, r_{min} = 0$ would hold, in which case capitalists would just invest all income not going into wages and thus would determine the rate of growth of the employment rate as the difference between capital stock growth $\hat{K} = r$ and effective labour supply growth $n + n_l$. But α^k will here be assumed as larger than 1 – see the next section – which in particular means that investment must be financed to some extent via loans which, of course, then implies the redefinition of the rate of profit of firms as shown above.

The third equation of this model is easily derived from the budget equation of firms[21]

$$\dot{\Lambda}_f = \alpha^k(r - r_{min})K - rK,$$

by making use of the definitional relationship $\dot{\lambda}_f = \dot{\Lambda}_f/K - \hat{K}\lambda_f$, $\lambda_f = \Lambda_f/K$. We stress that the dynamics automatically guarantee that v, e stay positive when they start positive, but that $r = y^p(1-v) - \delta_k - i_\lambda \lambda_f \geq 0, e \leq 1$ need not be fulfilled at all times. Furthermore, we should have $\alpha^k(r - r_{min}) + \delta_k \geq 0$ at all times, since disinvestment

[19] Note again that the price level p_y is kept fixed in the core version of the Keen model (and set equal to one) and that the rate of interest i_λ is also a given magnitude in this model.

[20] Due to the assumption $Y^d \equiv Y \equiv Y^p$ in the Keen (2000) paper.

[21] Note again that this model assumes $p_y = 1$.

can by assumption at most occur at rate δ_k. Note that this last inequality can be used to argue that $r \geq 0$ is not really needed for the viability of the model under the assumed investment behaviour. Referring in addition to overtime work when the labour market is exhausted may finally be used to argue that the constraint $r \geq r_{min} - \delta_k/\alpha^k$ is really the only one that is crucial for a meaningful working of the model.

We shall explore in the next section these growth dynamics (with debt accumulation) with respect to the state variables v, e, λ_f analytically in order to see what we can learn from their properties for the general 20D dynamics. Conversely, these 20D dynamics provides us with the perspective of how to augment the 3D core case by price level dynamics in order to obtain a basic case where debt accumulation and deflation can be investigated in their interaction analytically, then in a 4D situation of supply driven growth.

This basic proper model of debt deflation is augmented in the 20D situation by Rose effects in the wage-price interaction (which say that either wage or price flexibility must be destabilising with respect to the implied real wage adjustments), by Keynes effects (which here are more direct than is usually the case due to the monetary policy rule assumed), by Mundell effects (which state that the interaction between price inflation and expected price deflation must be destabilising if the adaptive component of these expectations is operating with sufficient speed), by Metzler effects (which imply accelerator-type instability of the inventory adjustment mechanism when it operates with sufficient speed) and by cumulative (destabilising) effects in financial markets (if adjustments are fast) due to positive feedback loops between expected changes and resulting actual changes of financial variables in our delayed adjustment processes towards overall interest rate parity (uniform rates of return). All these effects are of course partial in nature and must be studied in their interaction in a full analysis of the 20D model. However, we will only consider in the next section effects that concern the real part of the economy in its interactions with the debt accumulation of firms and thus leave the other markets in the financial sector of the economy for later investigations (by assuming low adjustment speeds in the market for long-term domestic and foreign bonds). These two financial markets are thus very 'tranquil' in the present chapter which concentrates on the effects of credit relationships between households and firms (not households and the government) and the possibilities of the central bank to neutralise the destabilising nature debt deflation by way of its interest rate policy rule.

8.5 Debt effects and debt deflation

In subsection 1 of this section we shall consider the Keen (2000) 3D growth cycle dynamics from the analytical point of view. We then extend these dynamics in subsection 2 by a law of motion for the price level that is a special case of the one used in the 20D case and analyse the features of these 4D dynamics (now including deflation or inflation besides debt accumulation of firms). In Section 8.6 we then approach these 3D and 4D and also the general 20D dynamics from the numerical perspective, with

particular stress on the occurrence of debt deflation. We then provide a brief discussion of another possibility where the combination of high debt and deflation may lead the economy into recessions or depressions, namely the situation of a debtor–creditor relationships within the household sector coupled with marginal propensities to consume that are higher for debtors than for creditors.

8.5.1 3D debt accumulation

Let us first consider the steady state of the dynamics (8.54)–(8.56) presented in the preceding section as the simplest case that allows for debt-financed (cyclical) growth. This steady state is uniquely determined, since no situation on the boundary of the positive orthant (or economic state space) can be steady in these growth dynamics. This unique steady state is given by:

$$e_o = \bar{e} + n_l/\beta_w, \tag{8.57}$$

$$v_o = \frac{y^p - \delta_k - r_o - i_\lambda \lambda_f^o}{y^p} \quad (r_o = r_{min} + (n + n_l)/\alpha^k), \tag{8.58}$$

$$\lambda_f^o = 1 - \frac{r_o}{n + n_l} = \frac{\alpha^k - 1}{\alpha^k} - \frac{r_{min}}{n + n_l}. \tag{8.59}$$

This set of steady state values shows that steady employment increases with the rate of technical progress and decreases with the speed of adjustment of nominal wages. Profitability depends positively on the minimum rate of profit (which separates positive from negative net investment) and on the natural rate of growth, and negatively on the speed of adjustment of investment with respect to changes in the pure rate of profit earned by firms, while just the opposite holds true for the debt to capital ratio in the place of the pure rate of profit.

Note that the rate of pure profits need not coincide with the rate of interest on loans in the steady state as there is no mechanism in the model that would promote their equalisation. We will assume in this subsection that $i_\lambda < n + n_l$ and $\alpha^k > 1$ holds (a necessary condition for a positive debt to capital ratio in the steady state which needs to be coupled with an assumption on the relative size of r_{min} in order to get a positive steady state value for λ_f).[22] Furthermore, the size of output per capital y^p should be such that the steady share of wages v_o is positive (and less than one which is always the case under the assumption just made).

To show that this steady state solution is the only one it suffices to exclude that $e_o = 0$ or $v_o = 0$ can be steady state values of the model. With respect to $e_o = 0$ this is obvious, since the state variable v cannot be steady in this case. With respect to $v_o = 0$ we first note that there is a unique solution of equations (8.58), (8.59) – set equal to zero – with respect to the values of r, λ_f as they are shown above (since $e_o > 0$ holds).

22 Note that the steady debt ratio must always be smaller than one.

Assume now with respect to the parameter y^p of the Keen (2000) model that it satisfies

$$y^p > \delta_k + r_o + i_\lambda \lambda_f^o,$$

which means that there is a meaningful steady state solution $v_o > 0$ to the model.

Since r_o, λ_f^o are uniquely determined there cannot therefore be an additional steady state with $v_o = 0$. We thus know that there is not only a uniquely determined interior steady state solution of the dynamics (8.54)–(8.56), but have shown in addition that there cannot be another steady state solution on the boundary on the positive orthant of the considered three dimensional state space (in contrast to many other systems that involve rates of growth formulation).

Referring again to overtime work (here assumed to come about when the labour market is exhausted),[23] we do not exclude the case $e_o > 1$ from consideration in the following, and do also allow for steady rates of profit r that are larger than $n + n_l$. Note that the Goodwin (1967) growth cycle is obtained if $\alpha^k = 1, r_{min} = 0$ is assumed which gives $\dot{\lambda}_f = r(1-\lambda_f) - r$ which remains zero when we start from a situation of no debt: $\lambda_f(0) = 0$.

Proposition 8.1 *Assume $\alpha^k > 1, 0 < i_\lambda < n + n_l$.*[24] *Then: the steady state (8.57)–(8.59) of the dynamics (8.54)–(8.56) is locally asymptotically stable for all admissible parameter values.*

Proof: Concerning the calculation of the determinant of the Jacobian of the dynamics (8.54)–(8.56) at the steady state we can first of all state that its third row can be reduced to $(0, 0, -(n + n_l))$ by the addition of an appropriate multiple of its second row without changing its size. This immediately implies that this determinant is equal to $-\beta_w v_o \alpha^k y^p e_o (n + n_l)$ and thus negative which provides one of the Routh–Hurwitz conditions for local asymptotic stability. With respect to the sum a_2 of the principal minors of order 2 one furthermore immediately gets the expression $\beta_w v_o \alpha^k y^p e_o < 0$ since two of these minors are equal to zero. Furthermore one has for the entry J_{33} of the Jacobian J (which gives the trace of J) in the considered situation the expression:

$$\begin{aligned} J_{33} &= -(n + n_l) + i_\lambda(1 - \alpha^k) + \alpha^k i_\lambda \lambda_f^o \\ &= -(n + n_l) + i_\lambda - \alpha^k i_\lambda (1 - \lambda_f^o) = -(n + n_l) - i_\lambda \alpha^k r_{min}/(n + n_l). \end{aligned}$$

The trace of J is therefore negative since $i_\lambda < n + n_l$ by assumption and since $\lambda_f^o < 1$ holds. The coefficients $a_1 = -\text{trace } J, a_2, a_3 = -\det J$ of the Routh–Hurwitz polynomial are therefore all positive and thus all support the local asymptotic stability claimed by the above proposition. Finally, we also have $a_1 a_2 - a_3 > 0$, since the expression for $\det J$ is part of the all positive expressions contained in $a_1 a_2$ and thus cannot make the expression $a_1 a_2 - a_3$ less or equal to zero (the latter if $i_\lambda > 0$ holds). □

[23] See the 20D model for a more plausible treatment of overtime work.
[24] In the case $i_\lambda = 0$ we have the Goodwin growth cycle dynamics coupled with an isolated adjustment process in the debt to capital ratio.

We thus have the strong result that a partial debt financing of investment demand turns the centre type dynamics of the original Goodwin (1967) growth cycle (all orbits are closed) into ones that imply convergence to the steady state, at least in a certain neighbourhood of this steady state.

Proposition 8.2 *We consider again the situation $\alpha^k > 1, 0 < i_\lambda < n + n_l$. Assume furthermore as a special case that $\beta_w = 0, n_l = 0$ holds, so that there is no adjustment in the wage share occurring when the other two state variables of (8.54)–(8.56) are changing. Then: for each level of the wage share v satisfying $y^p(1-v) - \delta_k - r_{min} > 0$ there exists a threshold value $\bar{\lambda}_f \geq 0$ of the debt to capital ratio λ_f above which this ratio will increase beyond any bound according to the dynamics (8.54)–(8.56).*

Proof: If the state variable v is stationary by assumption we get that the third law of motion of the dynamics is independent of the remaining two state variables. It is then given by:

$$\dot{\lambda}_f = \alpha^k i_\lambda \lambda_f^2 + [(1 - \alpha^k) i_\lambda - \alpha^k \tilde{r}]\lambda_f + (\alpha^k - 1)\tilde{r} - r_{min},$$

with $\tilde{r}$ being given by $y^p(1 - v) - \delta_k - r_{min} > 0$. The right-hand side of this equation represents a polynomial of degree 2 $p(\lambda_f) = c_o \lambda_f^2 + d_y \lambda_f + d_h$ with $c_o > 0, d_y < 0$. The minimum of this function is at $\lambda_f = -d_y/(2c_o) > 0$ and it exhibits of course only positive values after the larger of its two roots has been passed (if it is real, otherwise all values of $p(\lambda_f)$ are positive even for all $\lambda_f > 0$). Initial values of the debt to capital ratio λ_f which lie to the right of this root therefore imply a purely explosive behaviour of this ratio as long as there is no sufficiently strong counteracting change in the wage share v. □

We have pointed out at the end of the preceding section that, in the minimum, the side condition $r \geq r_{min} - \delta_k/\alpha^k$ should always be fulfilled in order to allow for economically meaningful trajectories (along which gross investment should always stay non-negative). The threshold for an explosive evolution of the debt to capital ratio found to exist in Proposition 8.2 may however still be so large that explosiveness can only occur in a domain where the system is not economically viable. In this case the proposition simply states that the dynamics will not always be globally stable from the purely mathematical point of view, but does not yet prove that critical developments in the debt to capital ratio may also come about at initial situations to which there corresponds an economically meaningful environment. To show that such situations will indeed exist is the aim of the following Proposition 8.3.

Proposition 8.3 *We assume as before $\alpha^k > 1, 0 < i_\lambda < n + n_l$ and as a special case again that $\beta_w = 0, n_l = 0$ holds. Then: for the steady state value of the wage share, v_o,*[25] *the threshold value $\bar{\lambda}_f \geq 0$ of the debt to capital ratio λ_f of Proposition 8.2 implies a rate of profit $\bar{r} \in (0, r_{min})$. The considered dynamics (8.54)–(8.56) therefore*

[25] Note here that the steady state value of e, the rate of employment, is no longer uniquely determined in the considered case.

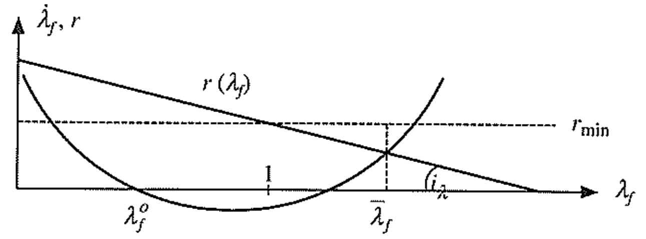


Figure 8.4 Debt and profit curves around the steady state share of wages

become divergent for values of λ_f that lie in an economically meaningful part of the state space.

Proof: We shall show below that the situation depicted in Figure 8.4 holds true under the assumed assumptions.

First, we show that the threshold value $\bar{\lambda}_f$ must be larger than one in the considered situation. To see this it suffices to show that the polynomial considered in Proposition 8.2 is still negative at $\lambda_f = 1$. At $\lambda_f = 1$ we have $\dot{\lambda}_f = -r = y^p(1 - v_o) - \delta_k - i_\lambda$. This gives $\dot{\lambda}_f = -[r_o + (i_\lambda \lambda_f^o - i_\lambda)]$. Inserting the steady state values (8.58), (8.59) into this expression then implies

$$\dot{\lambda}_f = -[r_{min} + \frac{n}{\alpha^k} + i_\lambda(\frac{\alpha^k - 1}{\alpha^k} - \frac{r_{min}}{n} - 1)].$$

Rearranging the shown items then gives

$$\dot{\lambda}_f = -[r_{min}(1 - \frac{i_\lambda}{n}) + \frac{1}{\alpha^k}(n - i_\lambda)] < 0$$

due to the assumption $n > i_\lambda$. From this result there follows immediately that the second root of the considered polynomial $\bar{\lambda}_f$ must be larger than one (while the first coincides with the steady state due to our assumption $v = v_o$).[26]

Let us now calculate the rate of profit at this threshold value $\bar{\lambda}_f$. Since we have $\dot{\lambda}_f = 0$ at this value (yet not a steady rate of employment, but instead a falling one), we get for $\bar{r} = y^p(1 - v_o) - \delta_k - i_\lambda \bar{\lambda}_f$ the expression

$$0 = \alpha^k(\bar{r} - r_{min})(1 - \bar{\lambda}_f) - \bar{r},$$

which in turn gives

$$\bar{r} = \frac{\alpha^k r_{min}(1 - \bar{\lambda}_f)}{\alpha^k(1 - \bar{\lambda}_f) - 1} = \frac{r_{min}}{1 - 1/(\alpha^k(1 - \bar{\lambda}_f))}.$$

Due to the above considerations we know however that the denominator of this expression is larger than one which implies that $\bar{r}$ must lie in the open interval $(0, r_{min})$. □

[26] Note that the smaller root can be negative, meaning that firms are creditors not debtors in the steady state, if $r_{min} > 0$ and if the parameter α^k is sufficiently close to one.

Should a shock throw the economy out of the steady state to a value of λ_f slightly above the threshold value $\bar{\lambda}_f$ it will be caught in a situation where λ_f is monotonically increasing accompanied by a falling rate of employment e until the domain of economically meaningful values for these two state variables is left. We stress that this result is obtained on the basis of a wage share that remains fixed at its steady state value and which therefore neither improves nor worsens the considered situation through its movements in time. This result will also hold true for all adjustments in the wage share that are sufficiently slow. At present it is however not clear whether a strongly falling wage share (based on a high value of the parameter β_w), which significantly improves the profitability of indebted firms, can lead us back to the steady state. This may depend on the size of the implied change in gross investment and its consequences for the change of the debt of firms.

For sufficiently small parameter values β_w we however know that the dynamics will produce explosiveness of the debt to capital ratio λ_f and implosiveness for the rate of employment e beyond threshold values $\bar{\lambda}_f, \bar{r}$. For sufficiently high debt, measured relative to the level of the capital stock, we thus get that debt accumulation feeds itself and will lead to larger and larger debt to capital ratios at least if there is no sufficient support for the pure rate of profit from downward changes in the wage share. Yet, as there is no price deflation, there cannot be a 'perverse' adjustment (a rise) of the wage share in such a situation of depressed profitability and high debt accumulation. Such a problematic situation comes about when there is sluggish or no downward adjustment in the level of nominal wages, but – due to insufficient goods demand, which is not yet a possibility in the considered model of Keen (2000) – downward adjustment in the price level causing increases in the real wage and the wage share. This scenario will be investigated by a suitable 4D simplification of the general 20D model in the next subsection.

8.5.2 4D debt deflation

Let us thus now extend the model (8.54)–(8.56) to include into it in a minimal way the possibility for price level deflation and thus the possibility for the occurrence of debt deflation (high levels of debt combined with declining profitability due to falling output prices). In order to achieve this, we set all parameters of the general 20D model that characterise the fiscal and monetary authority, the foreign sector, the housing sector and the asset markets equal to zero and thus get in particular given rates of interest (no Keynes effect, no cumulative asset market behaviour), with all interest rates equal to the then given rate of interest on loans. We will furthermore ignore the delayed Metzlerian quantity adjustment on the market for goods and assume that firms adjust their labour force with infinite speed which identifies employment l^{de} with the employed workforce l^{we} as now unique expression for the utilisation of the labour force. We assume finally that inflationary expectations remain fixed at their steady state level (no Mundell effect) by setting adjustment coefficients equal to zero there too. This gives rise to the following type of nominal dynamics for wages w^e, prices p_y and debt λ_f

coupled with an investment driven growth path, here represented by the dynamics of full employment labour intensity l^e (measured in efficiency units):[27]

$$\hat{w}^e = \kappa[\beta_{w_e}(l^{de}/l^e - \bar{e}) + \kappa_w\beta_p(y/y^p - \bar{u})], \tag{8.60}$$

$$\hat{p}_y = \kappa[\kappa_p\beta_{w_e}(l^{de}/l^e - \bar{e}) + \beta_p(y/y^p - \bar{u})], \tag{8.61}$$

$$\hat{l}^e = -[\alpha_r^k(r - i_\lambda) + \alpha_u^k(y/y^p - \bar{u})], \tag{8.62}$$

$$\dot{\lambda}_f = [\alpha_r^k(r - i_\lambda) + \alpha_u^k(y/y^p - \bar{u}) + \gamma](1 - \lambda_f) - r - \hat{p}_y\lambda_f. \tag{8.63}$$

Where the Metzlerian feedback mechanism from actually observed aggregate demand to expected demand to planned output and income and back to aggregate demand,

$$y^d = d_y\frac{w^e}{p_y}l^{de} + \alpha_r^k(r - i_\lambda) + \alpha_u^k(y/y^p - \bar{u}) + \gamma + \delta_k \to y^e \to y \to y^d$$

will be simplified and specialised to the following static (and again linearised) relationship:

$$\begin{aligned} y^d = y^e = y &= y\left(\frac{w^e}{p_y}, \lambda_f\right) \\ &= \bar{u}y^p + d_w\left(\frac{w^e}{p_y} - (\frac{w^e}{p_y})_o\right) + d_\lambda(\lambda_f - \lambda_f^o) \qquad (d_w, d_\lambda \le 0), \end{aligned}$$

which will be used in the following as a shortcut for the delayed feedback chain of the general case (and its richer concept of aggregate demand) in order to integrate the effects of price inflation and deflation into the Keen (2000) model as presented and analysed in the preceding subsection.[28] Otherwise the 4D dynamics are just a subdynamics of the general 20D dynamics considered in this chapter. Note that the budget equations of the credit-giving institution (here the pure asset holders) are no longer subject to the problem we observed for the banks of the 3D Keen model. Note furthermore that Goodwin type dynamics are obtained when $i_\lambda, \lambda_f(0), d_w, d_\lambda$ are all zero,[29] while the more general Rose (1967) type of real wage dynamics demands $i_\lambda, d_\lambda = 0$ (with wage flexibility as a stabilising factor and price flexibility destabilising if (as is assumed) $d_w < 0$ holds). Finally, the Fisher debt mechanism is obtained (due to $d_\lambda < 0$) by setting $\beta_{we}, \kappa_w, d_w = 0$. The above goods market representation therefore allows for Rose effects of traditional type (where price flexibility is destabilising) and for Fisher debt effects (where price flexibility should also be destabilising), but it excludes Mundell effects for example (that would also demand the inclusion of inflationary expectations into the above model).

27 $\bar{e}, \bar{u}$ the NAIRU utilisation rates of the labour force and the capital stock.
28 Note that this shortcut of the originally delayed quantity adjustment process of Metzlerian type demands that the steady state value of this function y must be equal to $y^p\bar{u}$ in order to get a steady state solution for this 4D simplification of the 20D dynamics.
29 Also in the further special case where $\alpha_r^k = 1, \alpha_u^k = 0, \gamma = i_\lambda$ holds.

We assume that the propensity to invest dominates the propensity to consume with respect to the impact of real wages $\frac{w^e}{p_y}$ on consumption and investment (the orthodox point of view) and take also from the above feedback chain and its shown shortcut the assumption that output depends negatively on the debt to capital ratio λ_f. The partial derivatives of the function $y(\frac{w^e}{p_y}, \lambda_f)$ are therefore both assumed as negative in the following ($d_w, d_\lambda < 0$). Since l^{de} is strictly proportional to output y, due to the fixed proportions technology assumed, we have that this employment magnitude exhibits the same type of dependence on the real wage and the debt to capital ratio as output y. Finally we of course again have $r = y - \delta_k - \frac{w^e}{p_y}l^{de} - i_\lambda\lambda_f$ for the rate of pure profits r.

The above represents the simplest way to integrate from the perspective of the 20D model the dynamics of the price level into our representation of the Keen (2000) model by abstracting from Metzlerian delayed output adjustment, from the distinction between the inside and the outside employment rate, from inflationary expectations, the housing sector, a fiscal and monetary authority, a foreign sector and from endogenous interest rate determinations.

Let us first calculate the interior steady state of the dynamics (8.60)–(8.63). This steady state is uniquely determined up to the steady level of prices p_y and is characterised by[30]

$$\lambda_f^o = 1 - i_\lambda/\gamma, \tag{8.64}$$

$$y_o = y^p\bar{u}, \tag{8.65}$$

$$l_o^{de} = y_o l_y^e, \tag{8.66}$$

$$l_o^e = l_o^{de}/\bar{e}, \tag{8.67}$$

$$r_o = i_\lambda, \tag{8.68}$$

$$\left(\frac{w^e}{p_y}\right)_o = \frac{y_o - r_o - \delta_k - i_\lambda\lambda_f^o}{l_o^{de}}, \tag{8.69}$$

$$p_y^o = \text{determined by initial conditions}, \tag{8.70}$$

$$w_o^e = p_y^o\left(\frac{w^e}{p_y}\right)_o. \tag{8.71}$$

Due to the new form of the investment function[31]

$$I/K = \alpha_r^k(r - i_\lambda) + \alpha_u^k(y/y^p - \bar{u}) + \gamma + \delta_k$$

we now have a different steady debt to capital ratio which is solely determined by trend growth γ in its deviation from the given rate of interest i_λ on loans. We again assume that $\gamma - i_\lambda > 0$ holds in order to get a positive steady state ratio λ_f. The two NAIRUs

30 We use l_y^e to express employment per unit of output measured in efficiency units (a given magnitude).
31 Which must be non-negative along the relevant trajectories of the dynamics.

on the labour and the goods market, $\bar{e}, \bar{u}$, and our consistency assumption that y is equal to $y^p\bar{u}$ in the steady state imply (on the basis of the given technology) the steady state ratios for actual and full employment labour intensity (in efficiency units), l^{de}, l^e in the usual way. Having determined the rate of profit through the rate of interest on loans implies on this basis a well-defined level of real wages measured in efficiency units, $(\frac{w^e}{p_y})_o$, which is positive if y^p is chosen sufficiently high relative to $\gamma, \delta_k, i_\lambda$ and $\bar{u}$. This real wage level then determines the nominal wage level on the basis of a given price level which is determined through historical (initial) conditions.

Proposition 8.4 *Assume* $0 < i_\lambda < \gamma, d_\lambda = 0$ *and* $\beta_p, \kappa_p = 0$,[32] *implying that the price level is a given magnitude in this special case. Assume furthermore that the investment parameter* α_r^k *is chosen such that* $\alpha_r^k i_\lambda - \gamma > 0$ *holds true. Then: the steady state (8.64)–(8.71) of the dynamics (8.60)–(8.63) is locally asymptotically stable for all other admissible parameter values.*

Proof: Note first of all that the dynamics are now of dimension three by assumption. Concerning the calculation of the determinant of the Jacobian of these reduced dynamics (8.60), (8.62), (8.63), at the steady state, we can first of all state that its third row can be reduced to $(0, 0, -\gamma)$ by the addition of an appropriate linear combination of the first two rows of this determinant without changing its sign. This immediately implies that this determinant can be characterised by the following remaining sign structure:

$$\det J = \begin{vmatrix} - & - & 0 \\ + & 0 & - \\ 0 & 0 & -\gamma \end{vmatrix}$$

and must thus be negative which provides one of the Routh–Hurwitz conditions for local asymptotic stability. With respect to the sum a_2 of the principal minors of order 2 one furthermore gets from the full sign structure of the Jacobian matrix J in the case $d_\lambda = 0$:

$$J = \begin{pmatrix} - & - & 0 \\ + & 0 & - \\ - & 0 & - \end{pmatrix},$$

thus the Jacobian is the sum of two positive and one zero determinant and thus unambiguously positive. Note furthermore that the entry

$$J_{33} = -\gamma + i_\lambda - \alpha_r^k i_\lambda(1 - \lambda_f) = -\gamma - i_\lambda\gamma(\alpha_r^k i_\lambda - \gamma)$$

in the preceding matrix is negative and larger in amount than γ due to the assumption made with respect to the parameter α_r^k. The trace of J is therefore negative, too, since $i_\lambda < \gamma$ by assumption and since $\lambda_f^o < 1$ holds again. The coefficients $a_1 = -\text{trace } J, a_2, a_3 = -\det J$ of the Routh–Hurwitz polynomial are therefore all positive and thus all support the local asymptotic stability claimed by the above proposition. Finally, we also have $a_1a_2 - a_3 > 0$, since the expression for $\det J$ is part of

[32] This implies $\kappa = 1$.

the all positive expressions contained in a_1a_2 and thus cannot make the expression $a_1a_2 - a_3$ less or equal to zero. □

We thus have that the steady state of the reduced dynamics (8.60), (8.62), (8.63) (where there is no adjustment of prices due to the demand pressure on the market for goods) is locally asymptotically if the influence of the debt to capital ratio λ_f on the level of output and employment, both in intensive form, is sufficiently weak. Furthermore, since the determinant of the full 4D dynamics is always zero these dynamics will be convergent with respect to the three state variables w^e, l^e, λ_f also for all speeds of adjustments β_p (and parameters κ_p) chosen sufficiently small, since the eigenvalues of the full dynamics are continuous functions of the parameters of the model.

Proposition 8.5 *Assume now (as was originally the case) that* $d_\lambda < 0$ *holds. Then: the steady state (8.64)–(8.71) of the dynamics (8.60)–(8.63) is not locally asymptotically stable for all price adjustment speeds* β_p *chosen sufficiently large.*

Proof: The interdependent part of the dynamics (8.60)–(8.63) can be reduced to the dynamics of the state variables $\omega^e = \frac{w^e}{p_y}$, the real wage, and again l^e, λ_f, as follows:

$$\hat{\omega}^e = \kappa[(1 - \kappa_p)\beta_{w_e}(l^{de}/l^e - \bar{e}) - (1 - \kappa_w)\beta_p(y/y^p - \bar{u})], \tag{8.72}$$

$$\hat{l}^e = -[\alpha_r^k(r - i_\lambda) + \alpha_u^k(y/y^p - \bar{u})], \tag{8.73}$$

$$\begin{aligned} \dot{\lambda}_f &= [\alpha_r^k(r - i_\lambda) + \alpha_u^k(y/y^p - \bar{u}) + \gamma](1 - \lambda_f) - r \\ &\quad - \kappa[\kappa_p\beta_{w_e}(l^{de}/l^e - \bar{e}) + \beta_p(y/y^p - \bar{u})]\lambda_f. \end{aligned} \tag{8.74}$$

Regarding the terms in the trace of the Jacobian of these dynamics at the steady state that depend on the parameter β_p one obtains

$$\omega_o^e\kappa(1 - \kappa_w)\beta_p(-d_w)/y^p + \kappa\beta_p(-d_\lambda)/y^p,$$

which is based on positive expressions throughout (up to the possibility that either κ_w or κ_p can be equal to one).[33] Therefore the trace of J can always be made positive by choosing the parameter β_p sufficiently large. □

The local stability result for the 3D Keen model is therefore overthrown in the case where relative goods demand is negatively dependent on the debt to capital ratio and where the price level adjusts with respect to demand pressure on the market for goods with sufficient speed. In such a case, we conjecture and will test this assertion numerically, that a process of deflation will continue without end accompanied by higher and higher debt ratios of firms which eventually will lead to zero profitability and bankruptcy.

Proposition 8.6 *Assume again that* $d_\lambda < 0, \beta_p > 0$ *holds. Assume now that nominal wages are completely fixed* $(\beta_w = \kappa_w = 0)$. *Then: the dynamics (8.60)–(8.63) are*

[33] The first expression shows the strength of the destabilising Rose or price level flexibility effect and the second is the Fisher debt effect.

monotonically explosive, implying higher and higher real wages and debt to capital ratios, for initial debt to capital ratios chosen sufficiently high (in particular larger than one) and all real wage levels above their steady state value.

Proof: The real economic dynamics considered in the proof of the preceding proposition can then be reduced to

$$\hat{\omega}^e = -\beta_p(y/y^p - \bar{u}), \tag{8.75}$$

$$\dot{\lambda}_f = [\alpha_r^k(r - i_\lambda) + \alpha_u^k(y/y^p - \bar{u}) + \gamma](1 - \lambda_f) - r - \beta_p(y/y^p - \bar{u})\lambda_f, \tag{8.76}$$

since l^e does no longer feed back on the state variables of these dynamics. Since both ω^e and λ_f are larger than their steady state values, we get from the first law of motion that ω^e must be rising further (due to falling price levels caused by $y < y^p\bar{u}$). Furthermore, since also $r - i_\lambda$, $1 - \lambda_f < 0$ holds we get that $\dot{\lambda}_f$ must be larger than

$$\gamma(1 - \lambda_f) - i_\lambda - \beta_p(y/y^p - \bar{u})\lambda_f > -\gamma\lambda_f - \beta_p(y/y^p - \bar{u})\lambda_f.$$

If therefore $-\beta_p(y/y^p - \bar{u}) > \gamma$ has come about by choosing λ_f sufficiently high we have that $\dot{\lambda}_f > 0$ must be true so that both ω^e and λ_f will be rising which further strengthens the conditions for their monotonic increase. □

We thus get as in Proposition 8.3, but much easier and much more severe (through the occurrence of price deflation), that there will indeed occur situations of now debt *deflation* where profitability falls monotonically and where the debt of firms is increasing beyond any limit, therefore leading to economic collapse sooner or later.

Proposition 8.7 *Assume as always $0 < i_\lambda < \gamma$ and $\alpha_r^k > 1$. Assume furthermore that $\beta_p = 0, \kappa_p = 1$, so that the price level is determined by cost-push considerations solely and hence by a conventional markup equation of the type*

$$p_y = (1 + m)\frac{wL^d}{Y} = (1 + m)wl_y = (1 + m)w^e l_y^e.$$

Assume that the given markup m is such that the implied real wage ω^e (in efficiency units) is equal to its steady state level. Next, assume a given level of nominal wages (measured in efficiency units), which means that $\beta_{we} = 0, \kappa_w = 0$.[34] *Assume finally that the investment parameter α_u^k is chosen such that $\alpha_u^k > y^p(1 - \omega^e l_y^e)\frac{\gamma - i_\lambda}{i_\lambda}$ holds true.*[35] *Then: the steady state (8.64)–(8.71) of the dynamics (8.60)–(8.63), which can then be reduced to adjustments of the debt to capital ratio basically, is locally asymptotically stable for all values of the parameter $d_\lambda < 0$.*

[34] The nominal wage is therefore growing in line with labour productivity.
[35] This inequality is equivalent to the inequality

$$\alpha_u^k > [(\gamma - i_\lambda)^2 + 2(\gamma - i_\lambda) + \delta_k(\gamma/i_\lambda - 1)]/\bar{u}.$$

Proof: In the assumed situation we have $\hat{p}_y = 0$ due to the given level of nominal wages and thus get a single independent law of motion for the debt to capital ratio λ_f:

$$\dot{\lambda}_f = [\alpha_r^k(r(\lambda_f) - i_\lambda) + \alpha_u^k(y(\lambda_f)/y^p - \bar{u}) + \gamma](1 - \lambda_f) - r(\lambda_f).$$

We have to show that the derivative of the right-hand side of this equation is negative at λ_f^o. Note first that $r'(\lambda_f) = y'(\lambda_f)(1 - \omega^e l_y^e) - i_\lambda = d_\lambda(1 - \omega^e l_y^e) - i_\lambda$ holds with a real wage ω^e that stays at this steady state level. Next, the derivative of the $\dot{\lambda}_f$ equation with respect to λ_f evaluated at the steady state is calculated and reads:

$$-\gamma + [\alpha_r^k r'(\lambda_f) + \alpha_u^k y'(\lambda_f)/y^p](1 - \lambda_f) - r'(\lambda_f).$$

This expression can be rearranged as follows:

$$-\gamma + (\alpha_r^k - 1)r'(\lambda_f)(1 - \lambda_f) + \alpha_u^k d_\lambda/y^p(1 - \lambda_f) - r'(\lambda_f)\lambda_f.$$

From this expression we get through further rearrangement

$$-(\gamma - i_\lambda\lambda_f) + (\alpha_r^k - 1)r'(\lambda_f)(1 - \lambda_f) - d_\lambda(-\alpha_u^k/y^p(1 - \lambda_f) + (1 - \omega^e l_y^e)\lambda_f)$$

with $\lambda_f = 1 - i_\lambda/\gamma$, $1 - \lambda_f = i_\lambda/\gamma$. This expression must be negative since $i_\lambda < \gamma$, $\lambda_f < 1$, $\alpha_r^k > 1$, $r' < 0$ and due to

$$\alpha_u^k > y^p(1 - \omega^e l_y^e)\lambda_f/(1 - \lambda_f) = y^p(1 - \omega^e l_y^e)\frac{\gamma - i_\lambda}{i_\lambda}.$$

In a similar way it can also be shown that the above derivative is negative for all $\lambda_f \in (0, \lambda_f^o)$, hence there is convergence to the steady state for all positive debt to capital ratios below the steady state ratio. It is however not possible to provide an easy expression for the upper limit of the basin of attraction of the steady state (which may be less than one). □

We have formulated Proposition 8.7 in view of an intended policy application which however can only be sketched here. Consider the case where the debt to capital ratio λ_f is so large that there are cumulative forces at work (as in Proposition 8.6) which would lead to higher and higher debt and lower and lower profitability in the considered economy. In the case considered in Proposition 8.7 there are three possible ways to break this catastrophic tendency in the evolution of the economy:

- An increase in nominal wages w^e which under the assumptions of Proposition 8.7 causes an immediate increase in the price level p_y and thus an immediate decrease in the ratio λ_f, which (if strong enough) may lead the economy back to the basin of attraction of its steady state.
- A decrease in the rate of interest i_λ on loans which moves the steady state of the economy to a higher sustainable debt to capital ratio.
- A decrease in the sensitivity of output y (through appropriate fiscal policies) with respect to λ_f, meaning a value of the parameter d_λ that is smaller in amount (which may enlarge the basin of attraction of the steady state).

There is therefore scope for economic policy to move the economy out of regions of developing debt deflation into regions where it converges back to the steady state. The details of such possibilities must however be left for future research.

8.6 Numerical simulations: from low to high order dynamics

In this section we provide numerical examples for the propositions on the 3D and 4D dynamics presented in the preceding sections and will also present some simulation runs of the general 20D dynamical system. Part II has discussed the various feedback mechanisms it contains and given some indication of the shape and size of the basins of attraction in the 18D case.[36]

8.6.1 *The 3D dynamics*

We start the numerical analysis of the 3D dynamics (8.54)–(8.56) by stressing again that they are of the Goodwin (1967) growth cycle type (where all orbits are closed curves around the steady state) when one assumes the parameter values: $i_\lambda = 0, r_{min} = 0, \alpha^k = 1$. There are also further cases where the closed orbit structure is obtained as we shall see in the following.

As a first example we now consider the case where there holds: $\alpha^k = 1.5$; $\beta_w = 0.5$; $n = 0.03$; $n_l = 0.03$; $r_{min} = 0$; $\bar{e} = 0.9$; $i_\lambda = 0.05$; $y^p = 0.45$; $\delta_k = 0.1$; and where we exercise a very large shock on the debt to capital ratio, giving it three times the size of its steady state value (from which the dynamics starts). The first thing to notice is that the debt to capital ratio converges back to its steady state value in a time span of approximately fifty years and does so monotonically while the real cycle keeps its basic shape. The result is that the size of this cycle is shock dependent since the disappearance of motion in the debt ratio makes the wage-share employment-rate dynamics again self-contained and thus of the Goodwin (1967) closed orbit type (the size of which depends as in the Goodwin growth cycle model on the history of the economy). If there is strong convergence of λ_f back to its steady state value (and this appears to be the case in many situations even when shocks are large) the involved Goodwin cycle mechanism comes to a rest once the debt ratio comes sufficiently close to its steady state value again. Figure 8.5 provides an example of such dynamics.

In the next 3D example in Figure 8.6 we make use of less sensitive investment behaviour now based on a minimum rate of profit that is larger than zero. In this case we get sluggishly convergent Goodwin-type growth cycle behaviour which we exhibit in the lower graphs of Figure 8.6 for the time interval (200, 260). As in many other convergent cases we have here only a weak reduction in amplitude over time, in particular since debt is relatively small for many reasonable choices of the parameters α^k and r_{min}. We now also observe a basically positive correlation of the employment rate and the debt to capital ratio.

[36] The simulation studies in this and the preceding chapters were performed in Fortran, Gauss or SND. See Chiarella *et al.* (2002) with respect to the latter simulation package.

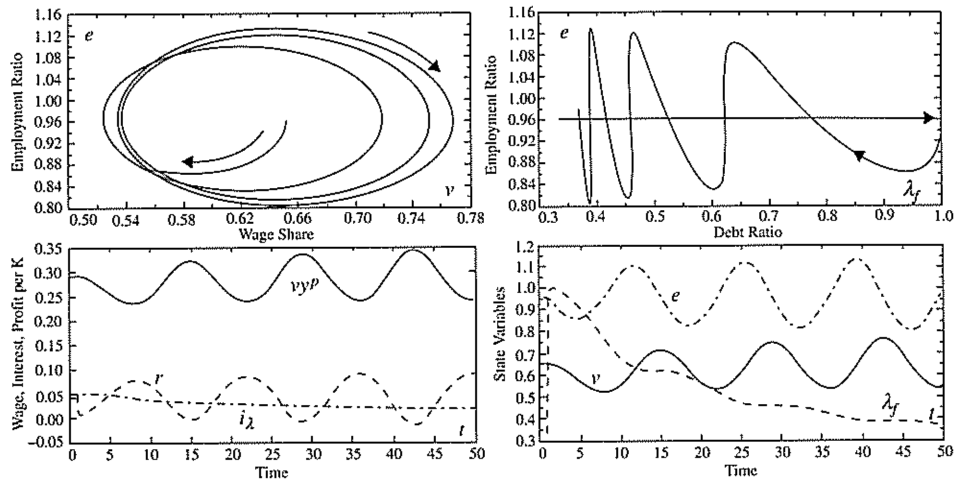


Figure 8.5 Debt convergence and shock-dependent persistent cyclical growth

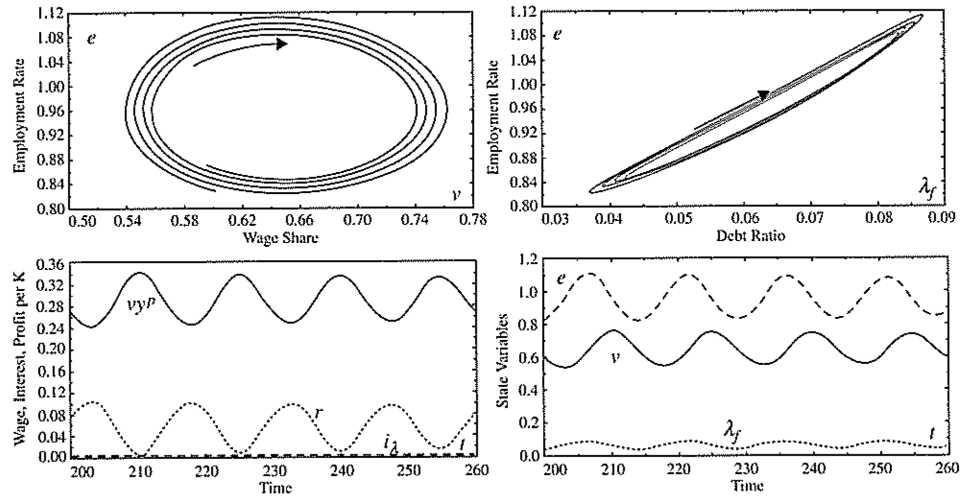


Figure 8.6 Slow convergence through debt-financed investment

The parameters underlying Figure 8.6 are given in Table 8.10 where we see that the change only concerns the assumed investment behaviour in comparison with the situation of Figure 8.5.

We conclude from these and various other simulations that the Goodwin real growth cycle often plays a dominant role in the shaping of the dynamics, while the debt dynamics either die out, lead to Goodwin cycles with slowly declining amplitude, or lead to strongly explosive behaviour (not shown) if the shocks in the debt ratio are made very large. It is therefore time to add the nominal price dynamics to the real growth model which is characterised by firms that finance part of their investment through new debt.

Table 8.10. *Parameter values underlying the simulations of Figure 8.8*

$\alpha^k = 1.3$; $\beta_w = 0.5$; $n = 0.03$; $n_l = 0.03$; $r_{min} = 0.01$; $\bar{e} = 0.9$; $i_\lambda = 0.05$; $y^p = 0.45$; $\delta_k = 0.1$

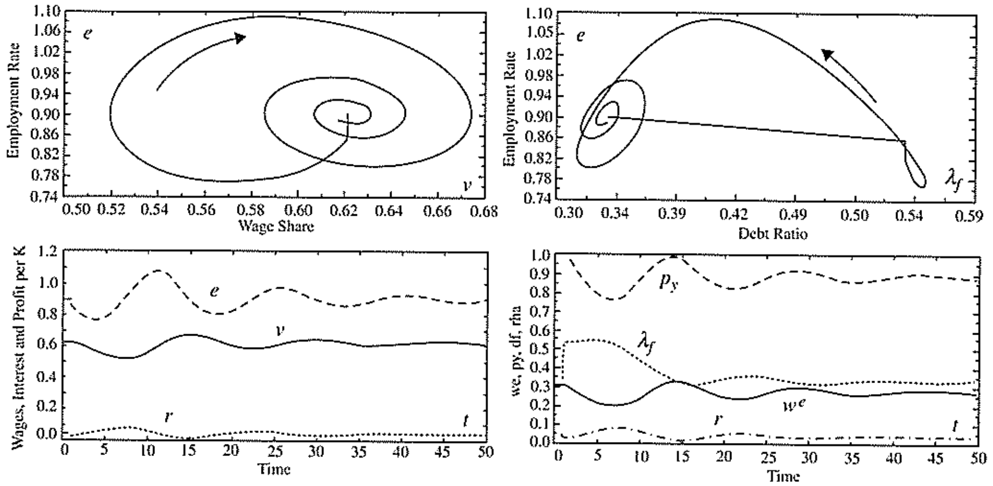


Figure 8.7 Faster convergence through a stabilising Rose effect

8.6.2 *The 4D dynamics*

We consider now the 4D dynamics (8.60)–(8.63) with both wage and price level adjustment.[37] We thus assume now that price adjustments are based on demand pressure as well as a wage cost-push term, and that wage adjustments (expressed in efficiency units) fully incorporate price inflation ($\kappa_w = 1$), a situation in which the real wage dynamics depend only on demand pressure in the market for labour and not on that in the market for goods. There is thus only a stabilising Rose (1967) effect present with respect to real wage adjustment (since $d_w < 0$ holds and since goods market equilibrium is in this situation irrelevant for real wage dynamics). This effect is of course the stronger the larger the parameter β_{we} becomes. Furthermore the debt effect on output is comparably weak here since $d_\lambda = -0.1$, and the steady debt ratio as well as the dynamic one (and thus also interest payments) are small in the present situation, which in sum gives rise here to a fast cyclical adjustment of the employment rate, of the wage share and of the debt to capital ratio to their steady state positions.

Note however that the initial phase of the dynamics (see Figure 8.7) exhibits high (and even rising) debt and falling price levels which however in the current situation create no long-lasting problem for the economy. We expect that this situation will change when the wage adjustment speed is decreased or the price adjustment speed increased and the parameter d_λ made more negative, because of the normal Rose effect

[37] We assume as a starting point the following parameter set: $\alpha_r^k = 1.3$; $\alpha_u^k = 1.3$; $\beta_{we} = 0.3$; $\beta_p = 0.5$; $\kappa_w = 1$; $\kappa_p = 0.5$; $\gamma = 0.06$; $\bar{e} = 0.9$; $\bar{u} = 0.9$; $i_\lambda = 0.04$; $y^p = 0.45$; $\delta_k = 0.1$; $l_y^c = 2$; $d_w = -0.5$; $d_\lambda = -0.1$.

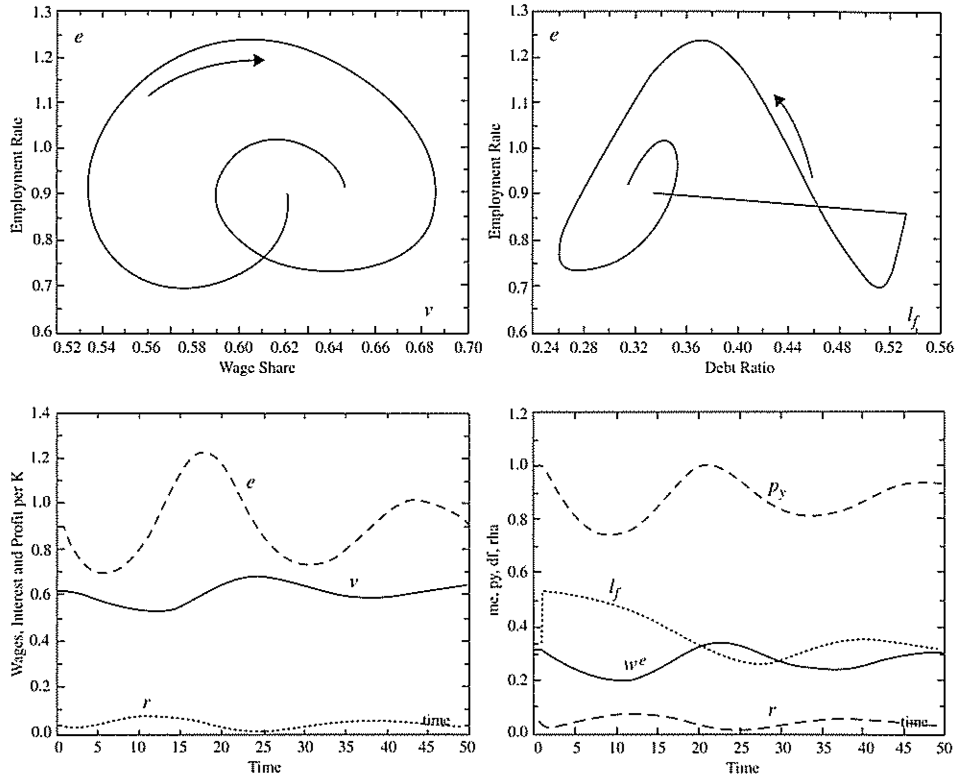


Figure 8.8 Slower convergence through more sluggish wages

with respect to real wage adjustments and a destabilising Fisher debt effect. A partial example for this is shown in Figure 8.8. Yet, even in this figure we have still a rising rate of profit despite high debt and falling prices and thus still a situation where the conflict about income distribution helps to prevent debt deflation from becoming a real threat to the rate of pure profits of firms.

Such a situation is assumed away in Figure 8.9 where we have $\beta_{we} = 0$ coupled with $\kappa_w = 1$, which implies that wages are following prices passively such that the wage share stays constant (furthermore we now also assume $d_\lambda = -0.2$ and $\beta_p = 0.552$).

As Figure 8.9 shows, we have a marked dip in the rate of profit when the sudden increase in the debt ratio occurs (at $t = 1$), which nevertheless slowly reverses thereafter since the debt ratio declines back to its steady value and since deflation no longer causes the dynamics to collapse. Note however that, although the rate of capacity utilisation converges back to its normal rate, the rate of employment does not show a similar tendency as there is no demand pressure effect from the rate of employment on the share of wages.[38] Increasing further the size of the shock in the debt to capital ratio will, however, eventually lead to monotonic divergence and thus to economic breakdown.

[38] See Fair (2000) for an empirical study of wage and price PCs where only demand pressure in the goods market is important.

Table 8.11. *The parameter set for Figure 8.10*

$\alpha_r^k = 1.3;\ \alpha_u^k = 1.3;\ \beta_{we} = 0.1;\ \beta_p = 0.1;\ \kappa_w = 1;\ \kappa_p = 0.5;\ \gamma = 0.06;\ \bar{e} = 0.9;$ $\bar{u} = 0.9;\ i_\lambda = 0.025;\ y^p = 0.45;\ \delta_k = 0.1;\ l_y^e = 2;\ d_w = 0;\ d_\lambda = -0.03$

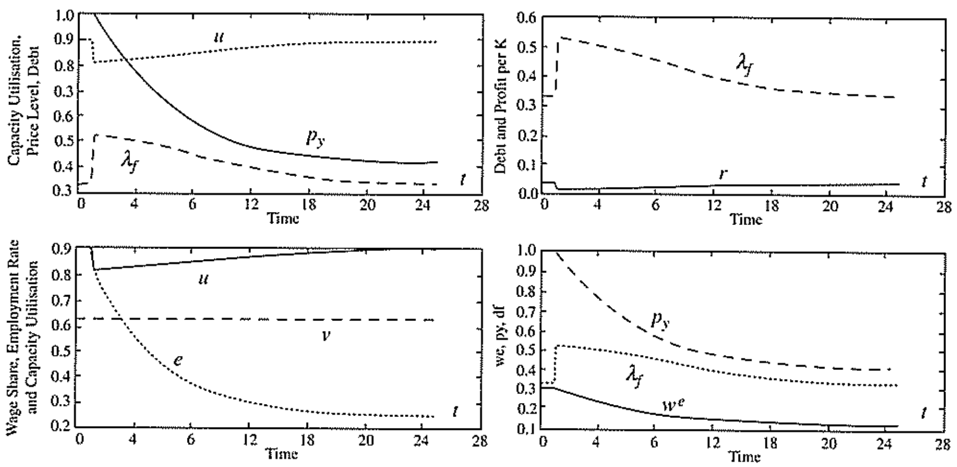


Figure 8.9 Deflation and converging debt

Figure 8.10 is based on the parameter set shown in Table 8.11 so that we have sluggishly reacting price and wage levels, now coupled with a low rate of interest on loans and thus a higher steady state ratio for λ_f. There is little movement in the wage share at first and no real wage effect on output (no Rose effect), but only a small negative effect of increasing debt on y. As we can see the dynamics are explosive in the present case, with at first rapidly rising profitability, due to the decline in debt and in the wage share occurring after the initial increase of debt at $t = 1$. Later on, however, the wage share starts rising, lowering the rate of profit significantly which then leads to increasing debt to capital ratios, falling capacity utilisation and falling prices, and to economic breakdown soon thereafter (although the wage share seems to start declining again).

Clearly, there is debt deflation in the final phase of the time series shown, and the question may therefore be posed whether positive price shocks, placed appropriately in such periods of deflation, can prevent economic collapse, extending its life beyond the 70 years that it here runs before (numerical) breakdown occurs. To obtain some insight into this issue, in Figure 8.11 we have added such positive price shocks (at $t = 58.70$) to the dynamics shown in Figure 8.10 and do indeed observe that these shocks counteract debt deflation for some time, by stopping the occurrence of falling price levels, restoring profitability and lowering the debt to capital ratio, which also leads to higher capacity utilisation due to its negative dependence on debt to capital

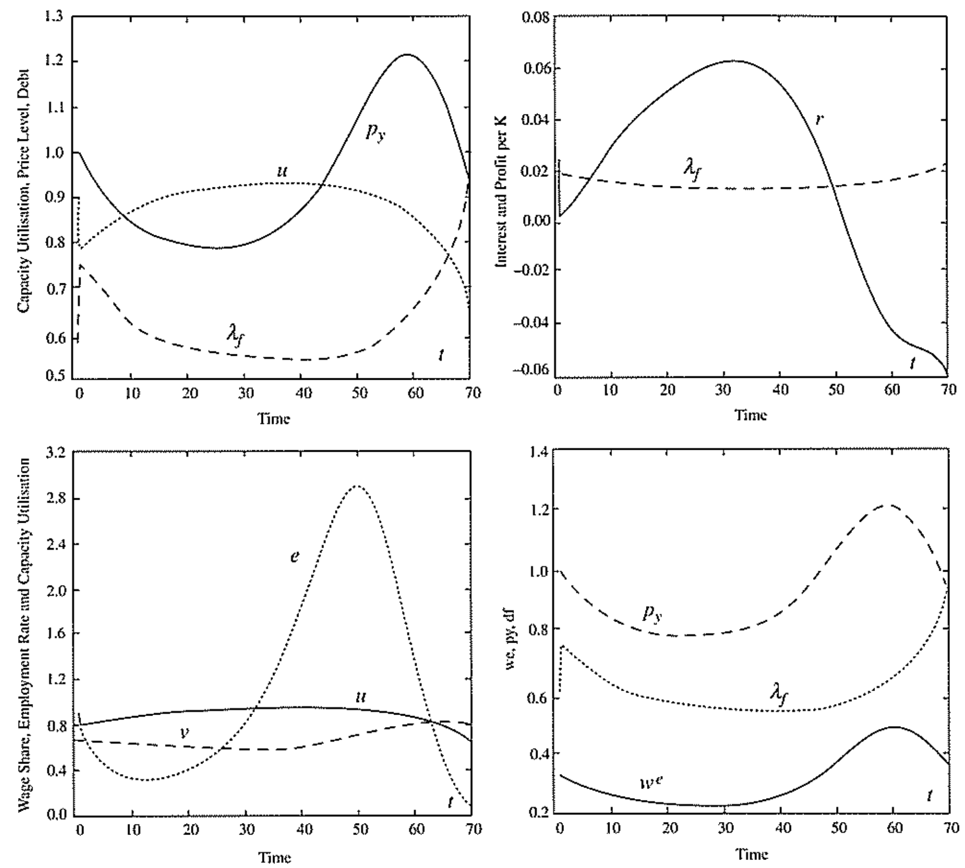


Figure 8.10 Debt deflation in the case of a sluggishly adjusting wage share

ratio λ_f. Note however that employment reacts in an extreme fashion and with long swings (basically due to the sluggish adjustment of nominal wages in the face of a large disequilibrium in the market for labour).

This closes our investigation of basic growth cycle models with debt financing, the possible occurrence of deflation and the role of the wage share in such a situation. Further numerical investigation is provided in Chiarella *et al.* (2001a,b) concerning the Fisher debt effect and the Rose real wage feedback mechanism. We have seen that (with and without profitability increasing adjustments in the wage share) debt will often converge back to its steady state value after debt shocks of considerable size. Undamped fluctuations are however possible and may lead to periods of strong debt deflation where positive price shocks may help to avoid economic collapse. Further increases in price flexibility will, however, lead to strong explosiveness (not shown), in the present model due to the joint working of the Rose real wage and the Fisher debt effects if both of the parameters d_w and d_λ are chosen significantly below zero.

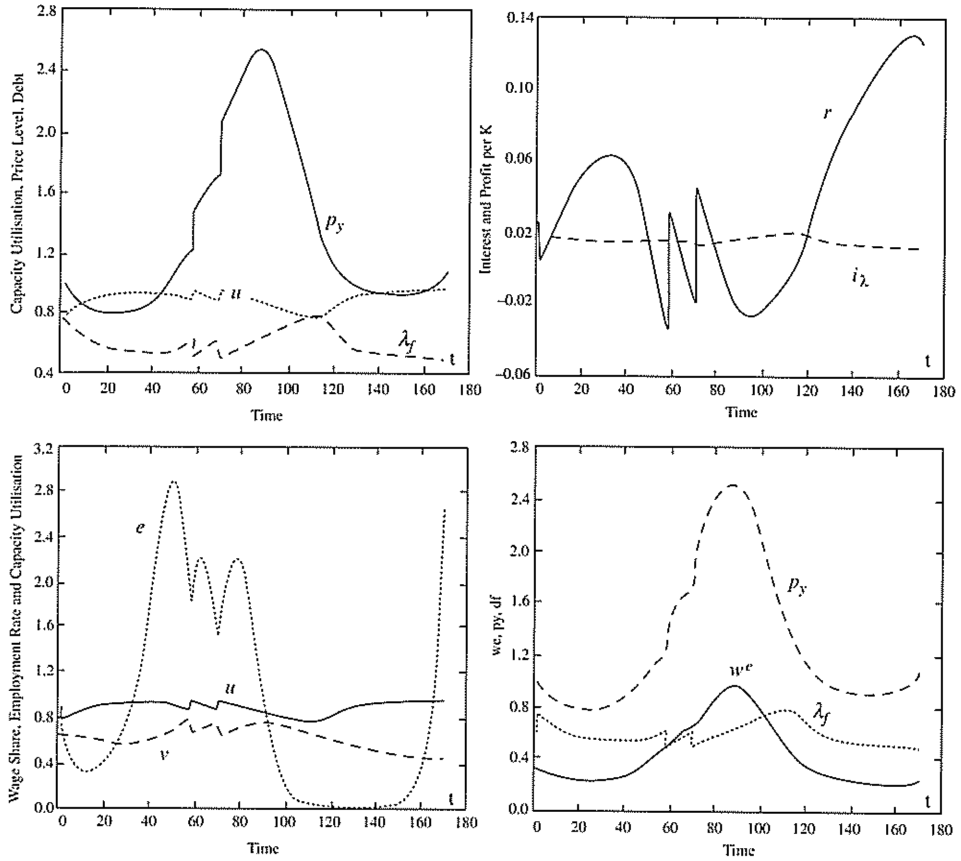


Figure 8.11 Positive price shocks (temporarily) stop debt deflation

8.6.3 *The 20D dynamics*

Now we consider simulations of the intensive form of the 20D dynamics laid out in Section 8.5. Let us first of all stress that debt financing is the least involved in the 20D dynamics in the case where the parameter values $\alpha_r^k = 1$, $\alpha_i^k = \alpha_u^k = 0$, $\beta_{i_\lambda} = 0$ and $i_\lambda(0) = \gamma$ hold. We then have $g_k = r^e + \delta_k$, which implies that only unexpected inventory changes have to be financed by loans (which should not matter very much for the dynamics of the model and thus should not allow debt deflation to play a significant role in this case). Furthermore, the qualitative properties of the original 18D dynamics considered in Part II should not change radically as far as the role of adjustment speeds is concerned if all expected profits are retained and not paid out as dividends, as was assumed in the 18D model (where fixed business investment was financed – in the background via the issuing of new equities). In our numerical simulations we have used the above simplified situation to find cases where the steady state is asymptotically stable (not shown) and from where we could then start, through

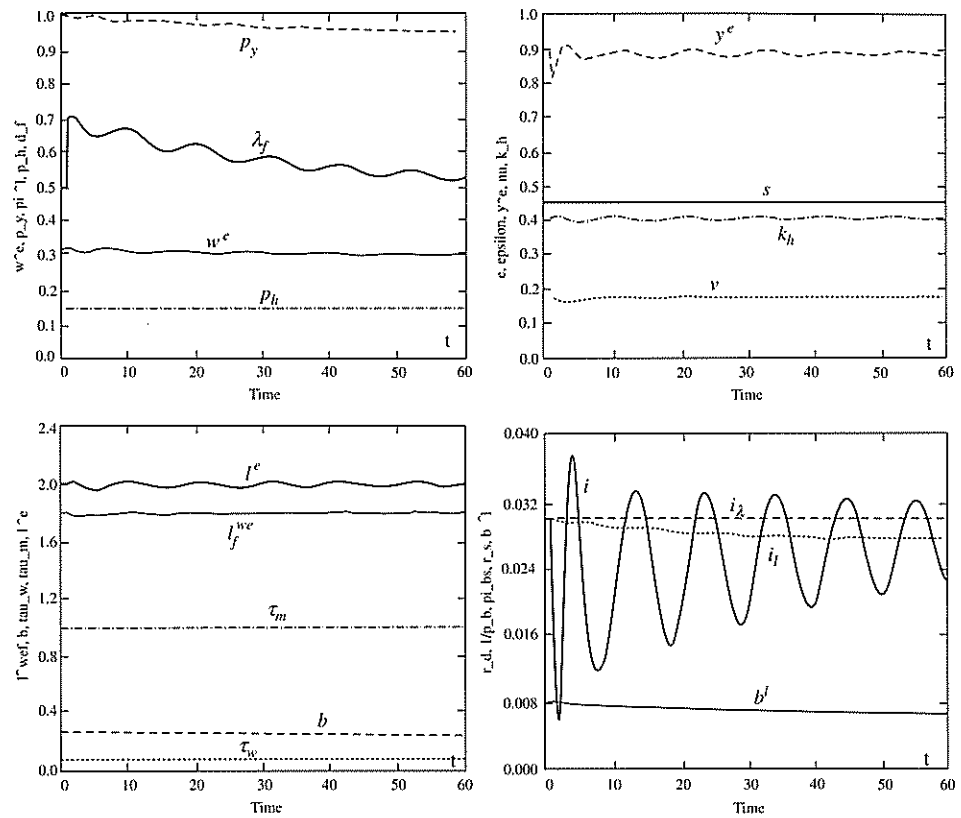


Figure 8.12 Asymptotic stability in the 20D case

parameter modifications, the investigation of destabilising debt deflation in the 20D case.

We first show in Figure 8.12 a case of asymptotic stability of the steady state of the 20D dynamics. The parameter values underlying Figure 8.12 are those specified in Table 8.12, with the exception that $\beta_{i_p} = 0.5$, $\kappa_w = 0$ and $\beta_p = 0.2$. We stress that the steady state is indeed asymptotically stable since also the price level will converge to a given level (and thus not fall forever) in the considered situation. Note that this case already departs from the above reference situation to a considerable degree and that we have assumed that monetary policy works with sufficient strength in order to overcome the instabilities here already present in the private sector of the economy. These destabilising forces again basically derive from the Rose and the Fisher debt effect which in this extended framework can be schematically presented as

$$p_y \downarrow \to \omega^e \uparrow \to y^d \downarrow \to y^e \downarrow \to y \downarrow \to \hat{p}_y \downarrow \quad \text{(the Rose effect)},$$

$$p_y \downarrow \to \lambda_f \uparrow \to r^e \downarrow \to y^d \downarrow \to y^e \downarrow \to y \downarrow \to \hat{p}_y \downarrow \quad \text{(the Fisher effect)}.$$

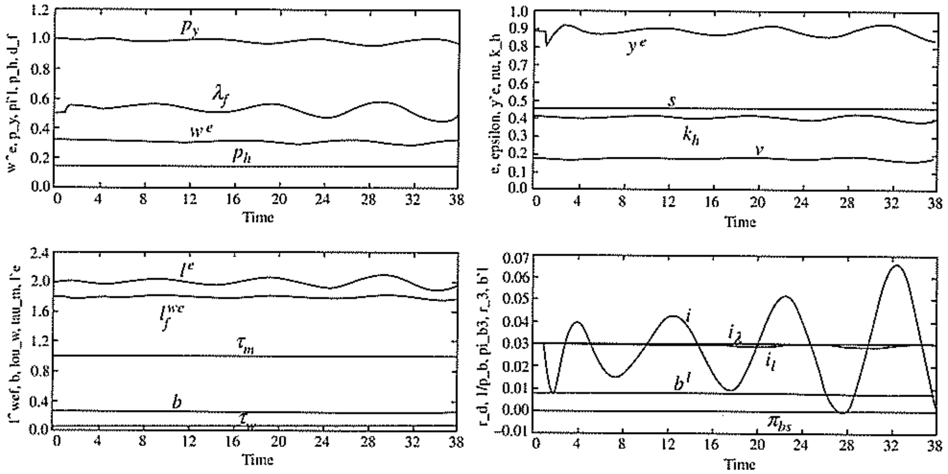


Figure 8.13 Destabilising price flexibility

Note that the partial Rose effect only works in this way if investment reacts more sensitively to real wage changes than consumption, in which case the cost effect of increasing real wages dominates the purchasing power effect they have in this model (as is the case in the following numerical simulations of the 20D dynamics). Note furthermore that asset markets react very sluggishly in the situations considered in this subsection and that the inventory adjustment mechanism exhibits slow inventory adjustments coupled with fast sales expectations which give it (from a partial perspective) the features of a stable dynamic multiplier process. Finally, the Mundell effect of inflationary expectations is also absent, due to the parameter choices made in the following. We consequently concentrate in this subsection on the two effects shown in the above boxes and on the role of the interest rate policy rule as a stabilising instrument in such an environment (because of its close relationship to the Keynes effect in the alternative case of a money-supply policy rule). Note finally that rates of return are equalised in the 20D case, in contrast to the 3D and 4D situations considered in the preceding subsections.

Next, we increase the parameter reflecting price flexibility to $\beta_p = 0.35$ and indeed in Figure 8.13, get a situation where the steady state is no longer attracting. We stress that monetary policy (the stabilising Keynes effect) is needed in order to obtain this only slightly explosive situation. However, the type of monetary policy that is assumed seems to be too weak here to again enforce convergence to the steady state.

Next, in Figure 8.14 we consider a case where there is some sort of isolated debt deflation, over the horizon shown, coupled with declining government debt and corresponding rates of interest. There are however no real effects visible over the horizon shown, which only occur later on when the situation becomes more and more extreme. The parameters specific to this situation are obtained from Table 8.12 but with $\beta_p = 1, \beta_{i_p} = 1, \kappa_w = 1$.

Table 8.12. *The simulation of the 20D dynamics – parameter values for Figures 8.12–8.15 with the exceptions noted in the text*

$\beta_{w_e} = 1$; $\beta_{w_u} = 0$; $\beta_p = 1$; $\beta_{\pi_l} = 0$; $\beta_{p_b} = 0.4$; $\beta_{\pi_{bs}} = 0$;
$\beta_e = 0$; $\beta_\epsilon = 0$; $\beta_n = 0.1$; $\beta_{n_d} = 0.2$; $\beta_{y_e} = 10$; $\beta_h = 0$;
$\beta_l = 0.5$; $\beta_{i_i} = 0.5$; $\beta_{i_p} = 1$; $\beta_{i_u} = 0.5$; $\beta_{r_d} = 0$;
$\alpha_g = 0$; $\alpha_l = 0.5$; $\alpha_b^g = 0.5$; $\alpha_{\pi^c} = 0$; $\alpha_r^h = 0$; $\alpha_u = 0$; $\alpha_i^h = 0$; $\alpha_r = 0$;
$\alpha_u^h = 0$; $\alpha_{\tau_w} = 0$; $\alpha_r^k = 1$; $\alpha_{\tau_m} = 0.5$; $\alpha_i^k = 0.5$; $\alpha_s = 1$; $\alpha_u^k = 0$;
$L_1(0) = 20,000$; $L_2(0) = 5,000$; $\kappa_p = 0$; $\kappa_w = 1$; $\kappa_h = 0$;
$\bar{u} = 0.9$; $\bar{u}_h = 0.9$; $n = 0.03$; $\bar{\bar{e}} = 0.9$; $n_l = 0.03$; $\bar{d} = 0.6$; $p_m^\star = 1$;
$g = 0.33$; $p_x^\star = 1$; $c_l = 0.7$; $r_l^\star = 0.03$; $\delta = 0.1$; $\tau_c = 0.5$;
$\delta_h = 0.1$; $\tau_p = 0.3$; $\gamma = 0.06$; $c_h = 0.1$; $x_{x_y} = 0.2$; $j_y = 0.1$;
$l_y^e = 2$; $y^p = 1$; $p_y(o) = 1$; $i_o = r_l^\star$;

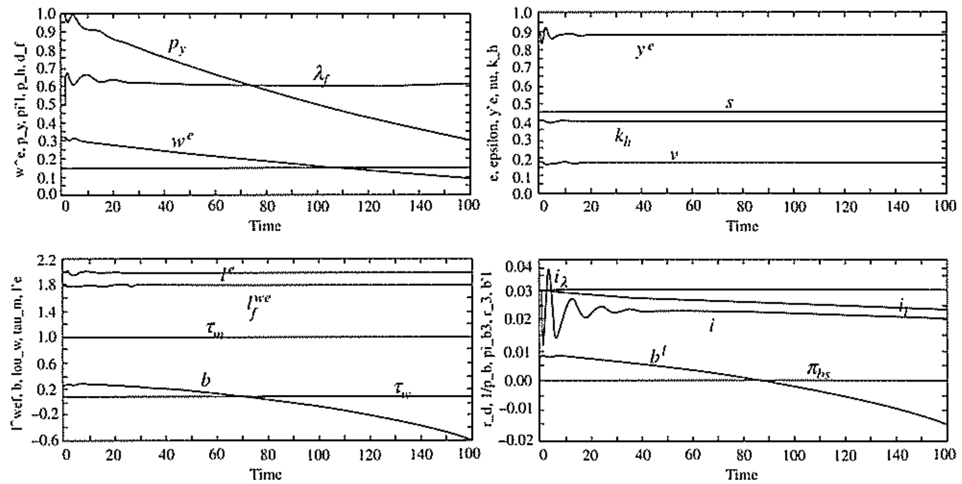


Figure 8.14 Pure debt deflation

The final situation presented in this subsection is given by Figure 8.15 where the deflationary process just considered is interrupted from time to time by positive price shocks which stop the monotonic development shown in Figure 8.14, decrease the real debt of firms and add fluctuations to the real magnitudes also shown in Figure 8.14.

These few numerical examples of the working of the 20D dynamics (still with a simplified choice of parameter values) show that much remains to be done for a proper demonstration of the consequences of debt deflation in a fully specified Keynesian model of monetary growth. Such investigations, which call for more refined numerical tools and more carefully considered parameter choices (in particular with respect to empirically observed parameter sizes), must however be left for future research. In addition, changes need to be made in the specification of the investment behaviour of firms and the way interest on loans is determined in order to extend the here still very

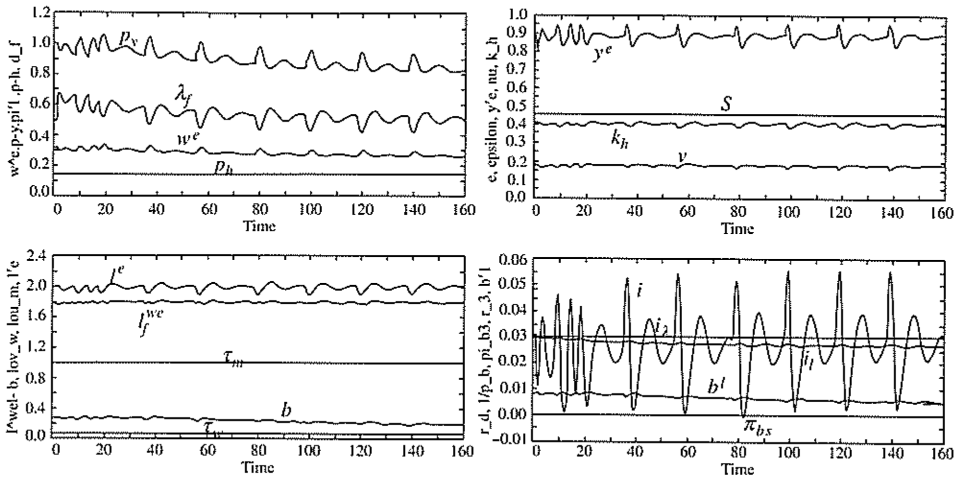


Figure 8.15 Positive price shocks in order to stop debt deflation

basic treatment of debt and deflation to more virulent cases of debt deflation than could be considered here.

8.7 Summary and outlook

In this chapter we have applied the integrated Keynesian 18D dynamics of Part II, with their price and quantity adjustment processes, their growth laws, asset market descriptions and fiscal and monetary policy rules, to the problem of describing and investigating situations where high debt of firms becomes combined with deflationary processes on the goods market, leading to falling profitability when there is no accompanying sufficiently large fall in real wages.[39]

To achieve this we have assumed as modification of Part II that firms use debt (in addition to retained pure profits) in place of equities to finance their investment expenditures (fixed business investment and inventories) and have derived the growth law of the debt to capital ratio from the budget equation of firms. In contrast to the very stylised situation of pure equity financing considered in Part II, where firms basically had no retained earnings, we now have pure profits of firms (over and above their debt service and factor costs) that in their relation to the interest rate on loans determine their investment plans. Although wealth effects on consumption and asset holdings are lacking in both of the considered dynamics we have seen in the present chapter that the level of debt and corresponding interest rate payments influence economic activity via investment behaviour and thus may significantly influence the fluctuating growth patterns to which this model type generally gives rise. Using loans in place of equities

[39] Real wages may even rise in such situations if prices fall faster than nominal wages.

implies that the rate of interest on loans has to be added to the endogenous variables of the model and this has been done in this chapter in the simplest way possible, by assuming that it adjusts to the long-term rate of interest on government bonds with a given time delay. The original 18D dynamical system thereby became a 20D dynamical system that served as point of reference for various types of simpler dynamics that we have considered in this chapter.

The most basic type of debt accumulation in a growing economy was obtained by making use of Keen's (2000) extension of the Goodwin (1967) growth cycle model. In addition to the reinvestment of the pure profits of firms this model also allows for debt financed investment in this supply driven growth context, thereby extending the dynamic interaction of the share of wages with real capital accumulation by the law of motion for the debt to capital ratio which feeds back into the real part of the dynamics via the pure rate of profit that it defines. The wage squeeze of the Goodwin model has thereby been augmented by a certain type of interest rate squeeze mechanism. This basic situation was investigated both analytically and numerically and gave rise to local stability assertions as well as global instabilities, depending on the size of the shock applied to the debt to capital ratio in particular. Integrating debt financing into the Goodwin growth cycle therefore gives rise to a new phenomenon, the occurrence of corridor stability, in this classical model of fluctuating growth. The interest rate squeeze mechanism therefore introduces a different type of behaviour as compared with the classical profit squeeze mechanism of real wage adjustments in view of demand pressure on the labour market. This may be explained by the lack of a PC mechanism as far as the credit market of the model is concerned.

Yet, in this basic approach, debt accumulation occurs without the possibility that firms have to face falling output prices simultaneously, a possibility that is not easily incorporated into a model where there is full capacity growth. In view of the established general 20D model, as a next step we have therefore integrated into the 3D dynamics a demand constraint for firms on the market for goods, reflecting two basic goods market characteristics of the general case. These two characteristics are represented by a negative impact effect of both real wages and real debt per unit of capital on this demand constraint. Using this shortcut to a full description of goods market adjustment processes of the 20D case, we then made use of the price PC of the 20D case in order to add as a fourth law of motion to the 3D dynamics a theory of price inflation based on demand pressure terms and cost-push elements. In this extended 4D model, we could again show asymptotic stability of the steady state for sluggishly adjusting price levels and, by as appropriate choice of parameters for debt deflation, instability for price flexibility chosen sufficiently large. Furthermore, if wages do not fall by a sufficient degree, the possibility of debt deflation could be demonstrated and policies that possibly could stop such an outcome were sketched (again analytically as well as numerically).

The decisive step away from supply side driven capital accumulation to demand side determined growth patterns was however to a certain extent preliminary, as the static shortcut of the dynamic feedback chain leading from expected demand to actual output to aggregate demand and back to expected demand is not an exact representation of the features of this delay driven feedback chain. The full feedback chain must therefore be used eventually if Keynesian growth is formulated, as it should be, with sluggish price as well as quantity adjustment processes. In this respect this chapter has offered however only a range of preliminary numerical illustrations that also downplayed important, but for the current question not central, aspects of the general model, namely activities of the state (the exception being the use of a Taylor type monetary policy rule), asset market behaviour, international aspects and the housing sector. As in the 4D dynamics we therefore concentrated in these examples on Rose type real wage dynamics and Fisher type debt deflation,[40] which both stress the destabilising potential of price flexibility in depressed situations due to its adverse effects on real wages and real debt. With the fully integrated 20D dynamics as a perspective we thus have been able to show how the question of debt deflation may be approached with respect to integrated models of monetary growth of an applicable nature. However we must also admit that much remains to be done in order to develop a deeper understanding of processes of debt deflation, which, as has been argued, are currently an important theme in public discussions on the state of the world economy.

The present chapter, with its general 20D model, has in fact not fully exploited the possibilities for a strong debt deflation mechanism that its 4D simplification may contain. This is due to the fact that debt operates on investment behaviour solely via the budget constraint of firms and not as in the 4D case through a direct adverse effect on effective demand. In addition interest rates were following long-term bond market interest rates with a time delay and thus did not have any direct relationship to the level of loans per unit of capital. The role played by debt accumulation in the 20D model thus resembles more the role the government budget constraint has for economic stability or instability than in fact the situation where debt and falling prices significantly depress investment behaviour and thus economic activity. By reducing pure profits, debt and deflation can however put the evolution of the debt of firms on an explosive path that cannot be counteracted in the way the government can counteract the explosive evolution of its own debt.

We shall return to the above issues and additional ones in future investigations of the general 20D model where more advanced mathematical tools will be used to determine regions of stability with respect to speed of adjustment parameters and boundaries where stability gets lost and basins of attraction; a preliminary investigation of these issues has been carried out by Chiarella *et al.* (2003b). In this way we hope to contribute to the understanding of the adjustment features of structural macroeconometric dynamic models for the USA, Germany, Australia and other countries to the point that the insights developed can actually be applied.

[40] We also had stabilising Keynes as well as destabilising Mundell effects.

9 Bankruptcy of firms, debt default and the performance of banks

The preceding chapters have shown that debt accumulation when combined with price dynamics may give rise to instability. A stylised fact of periods of financial fragility is that over-indebtedness leads to the insolvency of borrowers. Firms go bankrupt and default on loans. The impact of the failure of firms and non-performing loans plays a central role in the theories of financial fragility developed by Minsky and Fisher.[1] Non-performing loans may have a boomerang effect on the financial sector, by undermining the profitability of commercial banks. In this chapter, the preceding models are extended to take into account three aspects of debt over-indebtedness over the business cycle:

1. Bankruptcy of firms
2. Debt default
3. Non-performing loans and banking crises.

Bankruptcies may have ambiguous effects on the business cycle. On the one hand, the market sanctions bad performance by bankruptcy. It eliminates the weakest and most fragile firms and establishes favourable conditions for economic recovery. Similarly, in a Schumpeterian approach, the creative destruction argument points in the same direction. Recessions are productive as they are periods during which new technologies and new organisations are implemented. Likewise, bankruptcy also improves the average output to capital ratio, which paves the way for economic recovery. This is in fact a key element of the so-called reproductive cycle (see Gordon *et al.* (1983)). On the other hand, using a Keynesian line of argument bankruptcies may have a destabilising effect on consumption through unemployment and nominal wages. In addition, in situations of widespread over-indebtedness such as during a currency crisis, a strict enforcement of bankruptcy procedures would eliminate firms that would be profitable in a normal environment. Perverse effects may outweigh positive effects and impede recovery. In such a situation, public intervention might be required to steer the economy out of recession.

Bankruptcies can also generate debt default. Debt default may be seen as a way to solve the debt crisis, as it may reduce the Fisher effect – to the extent that investment

[1] See for instance Fisher (1933) or Minsky (1986).

depends negatively on the debt level.[2] Just as over-indebtedness may reduce firms' access to credit in the downturn and worsen the depression, debt default may ease credit constraints and may enable a faster economic rebound. It may well be seen as a reverse Fisher effect. Neo-institutionalism[3] highlights however the misallocation of resources that may result. Sgard (2002), for instance, notes that debt default requires a redefinition of the microeconomic relationships between borrowers and lenders. If debt default is not sanctioned by bankruptcy, moral hazard may spread and worsen bad credit allocation. Andrieu-Lacu (2006) argues that debt default without the sanction of bankruptcy explains the length and protracted nature of the Japanese crisis.

In turn, debt default generates non-performing loans and may affect the credit supply of the banks. As debt defaults constitute a loss for banks, financial fragility may lead to a banking crisis and may trigger additional unstable mechanisms. The extent to which banks can absorb these losses is likely to explain the depth and protracted nature of financial crises. As a matter of fact, the resilience of the banking sector determines the length of time it takes to resolve a crisis. The cost of banking crises in terms of GDP can actually be quite large – the IMF has estimated around 11 per cent for the twin crises in 1998; see IMF (1998).

In order to take into account the impact of bad debt on banks, commercial banks must be modelled more precisely. In the previously discussed models, households financed firms directly in the absence of a commercial bank. We here model a commercial bank, which collects deposits and supplies loans to firms as well as invests in public bonds. This detailed banking sector is necessary to take into account the role of banks' performance on the business cycle. There are different approaches to modelling commercial banks. Most of the literature on commercial banks focuses on the role of reserve requirements in the transmission of monetary policy. The so-called 'lending channel' disregards the role of bank profits and capital. More recently the so-called 'banks' capital channel' has received more attention and evidence has been found that bank performance affects credit supply (see Van den Heuvel (2002)). Banks' balance sheets and profitability are pro-cyclical and add a further channel to the financial accelerator. During periods of expansion, bank health improves and banks tend to take more risks and to extend credit beyond normal limits. Borio *et al.* (1999) show that bank profitability is strongly pro-cyclical and that risks, interest rate spreads and provisions for bad loans are counter-cyclical. Risks are often underestimated in periods of booms and overestimated in periods of recessions and contribute to a rapid growth of credit in

[2] As shown by the vast literature on the financial accelerator in closed and open economies. See for instance Bernanke *et al.* (1996).

[3] Institutional economics is a field of economics which studies the institution at the basis of capitalist economies. Five groups of institutions are usually considered: the monetary regime, the degree of competition, the labour nexus, the organisation of firms and the state/private sector interactions. A central question is what set of institutions produces an economic path which is stable and viable over time. Another central theme is to understand how institutions evolve over time and what are the key factors driving these changes. Institutional economics has been flourishing in France with the Regulation school Aglietta (1976), Boyer (1986) as well as in the US with the work of Epstein and Gintis (1995). Neo-institutionalism usually refers to a neoclassical interpretation of the aforementioned questions; see North (1990).

the upturn and a fast contraction of credit in the downturn. Gambacorta and Mistrulli (2004) show for a panel of Italian banks over the 1990s that excess capital of banks, as well as maturity mismatch, explain credit supply. Specifying the budget constraint of banks enables one to take into account the impact of bank performance on credit supplied. From this perspective, debt defaults weaken the financial situation of the banks and may lead to a credit crunch and banking crises. The financial accelerator in fact produces a boomerang effect on financial institutions.

Public authorities have two main policy options for supporting banks. Monetary authorities can rely on decentralised policies, which consist of organising the support of failing banks by the banking sector. This procedure belongs to a buy-in principle, which requires little public funding. On the other hand, fiscal and monetary authorities may step in directly. The bailout principle implies large public spending, and may differ slightly depending on the type of public intervention: for example, recapitalisation, fund injections, transfers of non-performing loans to a public entity, and partial or total nationalisation. The bailout principle implies a transfer of losses from the banking sector to the public sector, or inversely a transfer of funds from taxpayers to financial institutions. This model also extends public intervention beyond monetary and fiscal policy to account for bailout procedures.

We first present the main modifications undertaken with respect to the preceding chapter. We then present our artificial economy through the stocks and flows tables as in Godley (1999) and Dos Santos and Zezza (2004), before discussing the main equations of our model with a special focus on the strategy for modelling debt default, bankruptcy and commercial banks. The discrepancies between debt default with and without bankruptcy are discussed in a small three dimensional model. We then perform simulations to identify the impact of bankruptcy and debt default as well as bank performance on the business cycle. We briefly discuss loss socialisation and its impact on credit supply. Note that we also show that the main properties of the models presented in the previous chapters are still at work, especially with respect to the wage-price dynamics, debt deflation as well as with respect to monetary and fiscal policies.

9.1 Debt targeting, debt default and bankruptcy

In this section we provide a description of how the general model of the preceding chapter may be further extended and modified in order to allow for further stabilising or destabilising feedbacks caused by the simultaneous occurrence of high debt and deflation, concerning in particular debt default, the bankruptcy rate of firms and the resilience of the banking system.

The main changes undertaken in this chapter are threefold. First, the equations for capital and debt accumulation are modified to take into account the rate of bankruptcy and debt default. Second, the model considers the case of a commercial bank, which plays the role of a financial intermediary between lenders and borrowers. In this chapter, banks supply credit as in the endogenous theory of money. Modelling a commercial bank requires us to make a choice between the exogenous and the endogenous theories of money, as the causality between deposits and credits differs across these two theories. Third, the investment function of firms is slightly modified to address the question of credit rationing, which constitutes the main transmission channel between bank performance and the real economy. Credit supply depends positively on both the net wealth of borrowers as well as the profitability of banks.

To this end we first reformulate the equation for debt and capital accumulation of Section 8.3 as follows:

1. Firms: Actual Debt and Actual Capital Stock Growth

$$\dot{\Lambda}_f = p_y(I - \delta_k K) - r^a p_y K - \varphi_d(r)\Lambda_f, \quad (\varphi_d'(r) < 0), \tag{9.1}$$

$$\hat{K} = g_k - \delta_k - \varphi_b(r), \quad (\varphi_b'(r) < 0). \tag{9.2}$$

The equations (8.8) and (8.9) for the dynamics of firm debt and capital accumulation are modified to reflect the impact of bankruptcy. Thus in equations (9.1) and (9.2) the default rate φ_d as well as the bankruptcy rate φ_b enter negatively the equations for firm debt and capital accumulation respectively. It is here assumed that they both depend negatively on firm profitability r. The chapter will discuss explicitly the case in which the rates of debt default and bankruptcy differ. Assuming identical rates of bankruptcy and debt default in Equations (9.1) and (9.2) leaves the debt to capital ratio of firms unaffected, implying no feedback effects from the debt dynamic to the real economy. Both rates differ because of a composition effect, which arises as firms that go bankrupt have zero net wealth. The rate of debt default is then greater than the rate of bankruptcy, which improves the net wealth of firms at the macroeconomic level.

Defaults here reduce the debt level of firms in their dependence on the sector of commercial banks (since these firms stop paying interest) and therefore happen as if there were a debt-reducing gift from these banks to firms. The chapter also discusses the case in which debt default does not result from bankruptcy. Past episodes of financial crisis show that bankruptcy procedures are not necessarily enforced strictly. In Japan for instance (see the following chapter), debt default was massive while bankruptcy was relatively rather limited.

The second set of changes concerns commercial banks. In the preceding chapter, households financed firms directly in the absence of financial intermediation. The financial system now consists of a commercial bank that makes profits and has a non-zero net wealth. Debt default affects bank balance sheets and income statements in two ways. First, default reduces the interest payments received every period from borrowers by reducing the value of outstanding loans. Second, debt default is a loss that must be reported by banks and that enters banks' net wealth negatively.

Lastly in equation (9.3) we reformulate the investment function of firms given by (8.7) to address the case of credit rationing. The investment function is augmented by two elements: firms' debt to capital ratio λ_f and banks' profitability r_{nb}. The former captures the idea that credit supply is made on the basis of borrower net wealth as in the financial accelerator literature. Any improvement of the debt to capital ratio of firms loosens credit rationing and fosters investment. On the one hand, this implies a negative feedback of debt on its rate of change. On the other hand, increasing real debt – caused by a falling price level – will reduce investment behaviour and lead via goods demand to further (destabilising) downward pressure on the output price level of firms. The latter captures the idea that credit supply is made on the basis of bank profitability. Credit supply increases with bank performance and pushes investment upward. This transmission channel is potentially destabilising as new loans mechanically improve bank profits up to the point where over-indebtedness produces debt default. The investment function can be seen here as a reduced form equation reflecting firms' heterogeneity with respect to credit rationing as in Duménil and Lévy (1999).

2. Firms: Investment Behaviour

$$g_k = \alpha_r^k(\cdot) + \alpha_i^k(\cdot) + \alpha_u^k(\cdot) - \alpha_\lambda^k(\lambda_f - \lambda_{f0}) + \alpha_{irb}(r_{nb} - r_{nb0}) + \gamma + \delta_k. \tag{9.3}$$

In this chapter we will consider in more detail situations where the direct debt relationship between firms and pure asset holders is replaced by commercial bank intermediation. We will then use lower dimensional dynamical models to shed more light on the role of firm bankruptcy and indebtedness for the stability of the macroeconomy.

9.2 Tabular representations of stocks and flows

Tables 9.1 and 9.2 give a broad view of the economic system considered in this chapter. Table 9.1 displays the balance sheets of the different agents and Table 9.2 displays their income and expenditure.

Our economy is composed of six kinds of agents: workers, asset holders or rentiers, firms, commercial banks, a central bank and a government. With the exception of workers, other agents have assets and/or liabilities. The assets of firms consist of the stock of capital (machines, buildings) resulting from past investments and the stock of inventories. Firms hold no financial assets but have financial liabilities in the form of credit Λ.[4] Banks make use of deposits from households D_c to meet reserves requirements R, to supply credit Λ and to hold short-term public bonds B_b^s, the residual. The latter quantity plays the role of a buffer and is such that banks satisfy their budget constraints. Reserves are held at the central bank, which also creates money, B_{cb}^s through open market operations. Money is seen here as high-powered money. It only includes cash held by households H_c and reserves.[5] The central bank accommodates all demand for money in line with the usual post-Keynesian tradition of endogenous money. Open market operations consist in buying or selling short-term government bonds B^s issued by the government to finance its deficit. Another source of financing is available to the government in the form of long-term bonds $p_b B^l$. Eventually, rentiers are the ultimate lenders to all other agents. They have only financial assets and no liabilities. They hold cash H_c, deposits D_c, as well as short- and long-term bonds B_c^s and $p_b B^l$.

4 Equities are ignored at this stage since they make the portfolio much more complicated.
5 Deposits and short-term bonds are not included in this restrictive definition of money but would fit a broader definition as they are very liquid at near zero cost.

Table 9.1. *Balance sheets*

	Rentiers	Firms	Banks	CB	Government	Total
Capital		$+pK$				$+pK$
Inventories		$+p\mathrm{N}$				$+p\mathrm{N}$
Cash	$+H_c$			$-H_c$		0
Deposits	$+D_c$		$-D_c$			0
Reserves			$+R$	$-R$		0
Loans		$-\Lambda$	$+\Lambda$			0
ST bonds	$+B_c^s$		$+B_b^s$	$+B_{cb}^s$	$-B^s$	0
LT bonds	$+p_b B^l$				$-p_b B^l$	0
Total	W_c	W_f	W_b	0	W_g	$+p(K+\mathrm{N})$

Table 9.2. *Flows of funds*

	Workers	Rentiers	Firms	Banks	CB	Gvt	Total
Consumption	$-pC$		$+pC$				0
Investment			$+pI$				$+pI$
Gvt spending			$+pG$			$-pG$	0
Wages	$+wL^d$		$-wL^d$				0
Taxes	$-pT_w$	$-pT_c$	$-pT_f$			$+pT$	0
Inventory accumulation			$+p\dot{N}$				$+p\dot{N}$
Loan interest			$-i\Lambda$	$+i\Lambda$			0
ST bond interest		$+iB_c^s$		$+iB_b^s$	$+iB_{cb}^s$	$-iB^s$	0
LT bond interest		$+B^l$				$-B^l$	0
Bank transfers		$+\alpha_{rb}r_b$		$-\alpha_{rb}r_b$			0
CB transfers					$-iB_{cb}^s$	$+iB_{cb}^s$	0
Total	0	S_c	S_f	S_b	0	S_g	S_T

Concerning income flows, firms pay wages wL^d to workers and interest rate $i\Lambda$ to banks. They sell goods that are consumed by workers pC and the government pG. The demand for investment goods pI is made by firms to themselves as there is no distinction between firms in charge of producing investment and consumption goods. They also finance inventories by use of profits rather than debt. They are eventually taxed at a rate τ_f. Workers receive wages and consume their entire income net of taxes pT_w. Banks receive interest payments on credit and short-term bonds $i\Lambda + iB_b^s$ while deposits are not remunerated. Banks distribute part of their profits to asset holders. It is assumed that rentiers own commercial banks but that they are not traded on the stock market. The share of non-distributed profits increases banks' own funds. To ensure that the central bank's net wealth is zero, its profits are transferred to fiscal authorities.[6] Government

[6] Introducing the central banks' own funds is left for future research.

income is also composed of taxes raised on households and firms. Spending is made up of demand for goods pG to firms and of interest on public debt $iB^s + B^l$. Eventually, asset holders also receive interest payments on public bonds $iB_c^s + B^l$, and banks' income $i\Lambda + iB_b^s$.

9.3 Commercial banks and pro-cyclical credit supply

This section presents the equations of the model for each type of agent.

9.3.1 *Firms*

The first block of equations for firms is similar to the equations of the model of Section 5.4.3. Firms still have a fixed proportion production function as in Chapter 5. Potential output Y^p is a certain proportion y^p of the stock of capital K as a constant production function is assumed. Labour demand L^d grows together with the level of production. The rate of employment e is the ratio of people employed over the active labour population, and u (the rate of capacity utilisation) is simply the ratio of actual production over potential production.

3. Firms

$$Y^p = y^p K, \tag{9.4}$$

$$L^d = Y/x, \tag{9.5}$$

$$e = L^d/L, \tag{9.6}$$

$$u = Y/Y^p, \tag{9.7}$$

$$\dot{Y^e} = \beta_{ye}(C + I + G - Y^e) + nY^e, \tag{9.8}$$

$$\mathcal{I} = \beta_n(N^d - N) + nN^d, \tag{9.9}$$

$$Y = Y^e + \mathcal{I}, \tag{9.10}$$

$$N^d = \beta_{nd} Y^e, \tag{9.11}$$

$$\dot{N} = Y - Y^d, \tag{9.12}$$

$$r = (pY - wL^d - \delta_k pK - i\Lambda)/(pK), \tag{9.13}$$

$$Y^d = pC + pI + pG. \tag{9.14}$$

The stock-flow principle of our model also requires the specification of inventories explicitly. There are implicit inventories as the discrepancy between production and demand is met either by increasing or decreasing inventories. Disequilibrium on the goods market requires that we distinguish output from aggregate demand and expected production. Firms produce Y, which is the sum of expected production Y^e and expected inventories $\mathcal{I}$. On the basis of aggregate demand Y^d, firms form expectations regarding the level of production. Similarly, firms have a desired stock of inventories N^d which is proportional to expected output. Expected inventories $\mathcal{I}$ adjust to the discrepancy between desired and actual inventories. The actual change in inventories $\dot{N}$ is simply the difference between output and aggregate demand. The profit rate r is the difference between the revenue from selling goods produced and the different costs that firms face (mainly wages, depreciation and interest payments). Aggregate demand consists of consumption, investment and public expenditure.

There are three new elements in the block of equations of firms: bankruptcy of firms, debt default of firms and credit rationing.

$$\varphi_b = \beta_{\varphi_b}(\bar{r} - r) + \delta_\varphi, \tag{9.15}$$

$$\hat{K} = I/K - \delta_k - \varphi_b, \tag{9.16}$$

$$\dot{\Lambda} = p(I - \delta_k K) - (1 - \tau_f)rpK + (Y - Y^d) - \varphi_d \Lambda, \tag{9.17}$$

$$\varphi_d = \varphi_b/\lambda, \tag{9.18}$$

$$\lambda = \Lambda/pK, \tag{9.19}$$

$$I/K = \alpha_{iu}(u - \bar{u}) + \alpha_{i\lambda}(\lambda_0 - \lambda) + \alpha_{ir}(r - r_0) + \alpha_{irb}(r_{nb} - r_{nb0}) + n + \delta. \tag{9.20}$$

There are two ways to model firm bankruptcy, and these are not equivalent in terms of the model properties and in terms of wealth transfers. On the one hand, firms might be homogeneous and the rate of bankruptcy is the same as the rate of default. On the other hand, firms might be heterogeneous. Bankrupt firms are firms that have zero net wealth. In this case, the rate of default is greater than the rate of bankruptcy.

Assuming that firms are homogeneous and that they have a debt to capital ratio of 50 per cent, which also entails a net wealth to capital ratio of 50 per cent. A 40 per cent rate of bankruptcy entails a 40 per cent rate of default and a 40 per cent rate of wealth loss. In such a case, the debt and net wealth to capital ratio is constant and equal to $\frac{1}{2}$. Bankruptcy leaves the debt to capital ratio unaffected and therefore does not affect the Fisher effect. What may change the properties of the model is the fact that the output to capital ratio and the labour to capital ratio both rise. The former increases the rate of utilisation of the productive capacity u. It might have a positive effect on investment and real wages. It may also have a negative effect on employment, which may reduce real wages and aggregate demand.

The assumption that the firms are heterogeneous leads to slightly different results. It may in fact be more realistic to assume that firms that go bankrupt are firms that have a net zero wealth $K_2 = \Lambda_2$ (see Figure 9.1). The rate of bankruptcy and default are now different and bankruptcy modifies the debt to capital ratio. The rate of bankruptcy is $\varphi_b = K_2/(K_1 + K_2)$. The rate of default is $\varphi_d = \Lambda_2/\Lambda = K_2/\Lambda > \varphi_b$. Put differently, $\varphi_d = \frac{K_2}{K}\frac{K}{\Lambda} = \frac{\varphi_b}{\lambda}$.[7] In terms of wealth transfers, bankruptcy reduces the stock of capital while debt default reduces the stock of borrower liabilities. The net effect is an increase of the net wealth to capital ratio. Bankruptcy is here associated with an increase in borrowers' aggregated wealth, and with a transfer of losses to lenders. This specification integrates a form of balance sheet composition effect.

[7] Note that $K = K_1 + K_2$, $\Lambda = \Lambda_1 + \Lambda_2$ and $\lambda = \Lambda/K$.

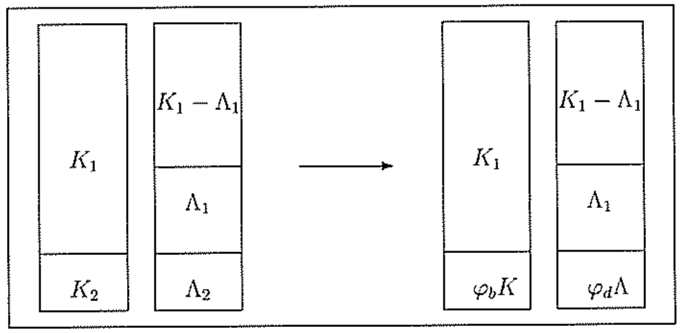


Figure 9.1 Bankruptcy – heterogenous firms

Formalising bankruptcies modifies the block of equations of firms. The main difference concerns the equation for the capital stock and debt change. Bankruptcy is expressed as a negative function of the difference between the steady state and current profit rate $(\bar{r} - r)$. In a downturn, firms go bankrupt only when profits are very low. It is here assumed that closing physical assets are not bought out by existing firms. There is therefore no secondary market for the stock of capital. Physical assets are destroyed at no cost. The bankruptcy function could include a much greater variety of elements, such as the output capacity or the level of indebtedness. We choose to keep the model as simple as possible in order to ease the analysis of the feedback channels. At the equilibrium, the rate of bankruptcy is constant at a rate δ_φ. Given that investment is already a function of profits, these enter twice into the equation of the capital stock and with the same sign. It might therefore be more meaningful to replace the investment and the bankruptcy function by only one equation that would deal with a form of net investment function. Nevertheless, investment and bankruptcy might not be exactly of the same nature, and bankruptcy might not be equivalent to a negative investment. If negative investment or bankruptcy reduces the stock of capital, bankruptcy does not appear directly in the aggregate demand, while investment does. In other words, reducing investment or increasing bankruptcy do not have the same effect on aggregate demand Y^d. The stock of fixed capital grows with investment I and decreases with depreciation δ and bankruptcy φ_b. Bankruptcy affects the economy by reducing the stock of capital. The creative destruction mechanism is captured by the increasing output to capital ratio, while the Keynesian effect is captured by the higher labour to capital ratio.

The rate of debt default φ_d reduces the stock of existing debt Λ. It is equal to the rate of bankruptcy divided by the debt to capital ratio. Given that the rate of debt default is larger than the rate of bankruptcy, firms' net wealth increases with bankruptcy. Debt default is likely to be stabilising through two main mechanisms represented in Figure 9.2. By reducing the stock of debt, firms restore their profitability as debt service decreases.

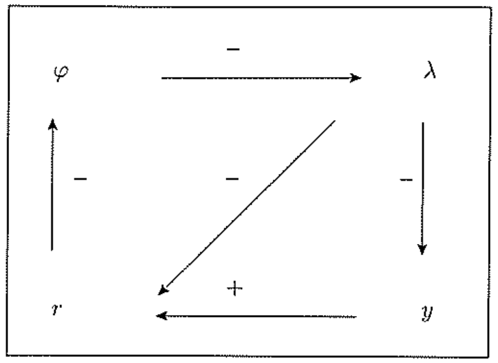


Figure 9.2 Stabilising debt default – intensive form

Lower debt is also likely to reduce the Fisher effect and to restore investment. Default has an opposite effect to debt deflation – it alleviates the debt burden while the Fisher effect magnifies the debt burden. The level of debt depends on retained profits on the one hand and on investment on the other hand. It also decreases with bankruptcy as it generates debt default. As explained previously, to the extent that bankrupt firms have zero net wealth, the rate of default is larger than the rate of bankruptcy. The debt to capital ratio therefore decreases with bankruptcy. In addition, inventories affect the quantity of external funds. As they are financed by profits, change in inventories reduces internal funds by the quantity $Y - Y^d$.

It is necessary to introduce some type of credit rationing into the model if the impact of bank performance on firms' investment is taken into account. Thus we write

$$I^d/K = I^d(u, r),$$
$$I^r/K = I^r(r, \dot{\lambda}^s).$$

Credit rationing is here introduced in a very simple way. The investment function I displayed in this chapter is seen as a reduced form equation for desired investment I^d and restricted investment I^r. Firms might not be able to invest this quantity as they might be rationed by financial institutions. In the case of rationing, investment I^r is equal to internal plus external funds: the quantity of debt supplied by banks and firms' retained earnings $I^r = \dot{\Lambda}^s + (1 - \tau_f)rpK$. Realised investment is therefore the minimum between firms' desired investment and restricted investment $I = min(I^d, I^r)$. When such a *min* function is specified it is no longer possible to derive the stability conditions. We therefore make use of a reduced form equation for investment I, similar to Duménil and Lévy (1999). This equation can be interpreted as reflecting the heterogeneity of firms. The economy is made up of firms which are not rationed and firms which are rationed. It gathers the various elements of desired and restricted investment. Investment

I is still a basic function that includes a Harrodian accelerator u and the profit rate rpK. Credit rationing is also a function of the basic financial accelerator λ, but is also augmented by variable for banks. Note that λ is similar to the net wealth to capital ratio:

$$\left[\frac{pK - \Lambda}{pK} - \left(\frac{pK - \Lambda}{pK}\right)_0\right] = \left[\left(\frac{\Lambda}{pK}\right)_0 - \frac{\Lambda}{pK}\right].$$

Restricted investment is also a positive function of bank profitability net of default losses with a sensitivity α_{irb}. The latter parameter therefore captures the sensitivity of credit supply to bank performance. At the steady state, investment grows at a constant rate equal to the growth rate of the population n and the rate of depreciation of the capital stock δ.

9.3.2 Commercial banks: credit rationing and money creation

This subsection presents a detailed banking sector that tries to overcome some of the usual shortcomings related to the formalisation of financial institutions. Although post-Keynesian theories usually give a central role to financial intermediaries, banks' assets and liabilities are usually not modelled. Here we shall focus in particular on the link between bank performance and credit supply.

The balance sheet of banks is displayed in Table 9.3[8] and is composed as follows. Banks use deposits D_c to grant loans Λ. Part of the deposits, R, must be held as reserves at the central bank for prudential requirements. They are a fixed proportion θ of deposits. Eventually, banks adjust their budget constraint by selling or buying government bonds $B_b^s = D_c + OF - \Lambda - R$. In case deposits are not large enough to finance reserves and loans, banks sell public bonds. There are therefore no idle resources. Financial institutions make the best use of existing resources and invest excess reserves in financial assets. Holding of public bonds acts as a buffer that adjusts to changes in deposits, loans and reserves. In the absence of central bank advances, banks adapt their asset structure to finance investment. As banks' assets are greater than liabilities and as profits are positive, banks have a positive net wealth, called own funds (OF). Own funds increase with banks' profits and decrease with debt default. Debt default must appear twice in the balance sheet. It reduces the value of the existing stock of debt. It also appears as a loss in bank own funds. The stock of debt cannot be greater than the stock of deposits plus own funds minus compulsory reserves as banks have no advances from the central bank $\Lambda \leq D_c + OF - R$. Eventually, profits are made out of interest on loans and public bonds. Both interest rates are the same and deposits are not remunerated for simplicity. This type of banking system behaviour is close to the behaviour of North American banks, and it may be expressed as the set of equations

[8] This table is from Lavoie and Godley (2004).

Table 9.3. *The balance sheet of banks: assets adjustments*

Assets	Liabilities
Λ	D_c
R	OF
B_b	
Assets - Liabilities = 0	

Table 9.4. *Banks' balance sheets: CB advances*

Assets	Liabilities
Λ	D_c
R	A
	OF
Assets - Liabilities = 0	

4. Commercial Banks

$$\dot{R} = \theta \dot{D}_c, \tag{9.21}$$

$$\dot{B}_b^s = \dot{D}_c + \dot{OF} - \dot{\Lambda} - \dot{R}, \tag{9.22}$$

$$r_b p K = i\Lambda + i B_b^s, \tag{9.23}$$

$$\dot{OF} = (1 - \alpha_{rb}) r_b p K - \varphi_d \Lambda. \tag{9.24}$$

An alternative, which takes into account the European banking system would specify central bank advances. Instead of adjusting their assets structure to finance investment, banks would rely on central bank advances, which are defined as $A = \Lambda + R - D_c - OF$ (see Table 9.4).[9] To better understand this specification, it is useful to make an analogy with firms. In the case of the central bank's advances, it is straightforward. The change in bank assets results from bank investment. Investment has an active component in the form of credit supply, and a passive component in the form of compulsory reserves. Banks then raise external funds to meet the need for external finance.

The presentation of financial intermediaries given here is in line with what the reader will find in Lavoie and Godley (2004).

The supply of credit is a function of borrowers' characteristics as specified in the base line model. The quantity of loan supplied depends on firms' net wealth. A cumulative loop takes place, as easing credit supply increases capital accumulation, which improves firms' net wealth in return, as illustrated in Figure 9.3. In addition, we express credit rationing with respect to bank performance as shown in equations (9.25) and (9.26) below.

9 This table is also from Lavoie and Godley (2004).

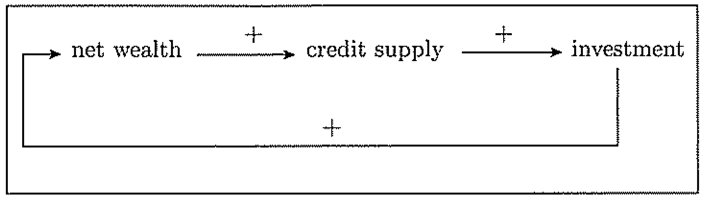


Figure 9.3 Credit rationing

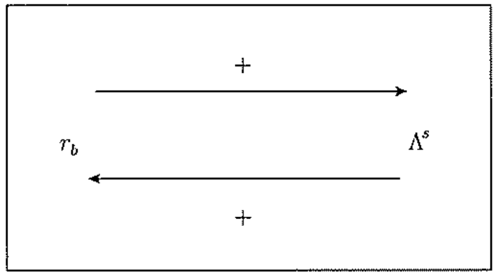


Figure 9.4 Banks' profitability and credit supply

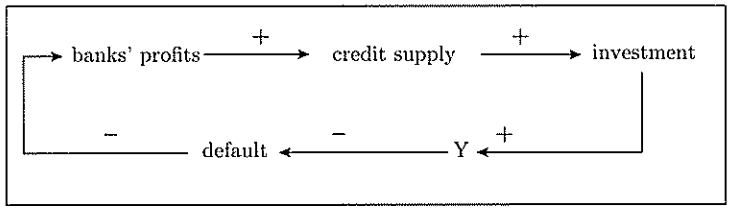


Figure 9.5 Debt default and banks' profits

$$\dot{\Lambda}^s = \Lambda^s(W_f, r_{nb}) \tag{9.25}$$

$$r_{nb} = i_\lambda \Lambda + i B_b - \varphi_d \Lambda \tag{9.26}$$

We argue that banks increase credit when their profitability r_b increases. When bank profits increase, banks are more willing to expand their assets further. In addition, banks may be less selective with respect to borrowers' ability to serve debt commitments (Figure 9.4). Banks' income is in fact strongly pro-cyclical and amplifies the financial accelerator, as shown by Borio *et al.* (1999). Another pro-cyclical mechanism goes through the dynamics of debt default. Debt defaults are losses for banks. Debt defaults reduce the value of banks' assets and their profits. As debt defaults decline in the upturn, firms' profitability increases further and stimulates credit supply. On the contrary, large scale debt defaults in the downturn depress credit supply further, Figure 9.5 illustrates this process.

9.3.3 Asset holders: Blanchard asset market dynamics

We improve the behavioural equations of asset holders by specifying a Tobin type portfolio function to allocate asset holders' savings between the different assets. They allocate their savings to three kinds of financial assets: short-term bonds B_c^s, long-term bonds $p_b B^l$ and money M_c, which includes deposits D_c and cash H_c. Their income Y^c is made up of interest on public short-term bonds, $i B_c^s$, and on long-term bonds, B^l, as well as a share of banks' income $\alpha_{rb}(i\Lambda + i B_b^s)$ to ensure stock-flow consistency. Asset holders are taxed at a rate τ_c and do not consume their income.

The choice between short- and long-term bonds B_c^s, B^l follows a Tobin like portfolio decision while money is a function of asset holder wealth.[10] Bonds are held in a certain proportion of the agent's wealth[11] and this proportion varies with respect to the differential rate of return between the different assets.[12] The return on short-term bonds is the interest rate i set by the central bank, whereas the rate of return on long-term bonds is the inverse of their price $1/p_l$. The long-term bonds considered here yield a fixed income of one money unit (e.g. euro, dollar) per bond. The interest rate or the return on long-term bonds is given by the sum paid as interest B^l over the nominal value of bonds $p_b B^l$. The equations for asset holders are:

5. Asset Holders

$$S_c^n = \dot{B}_c^s + p_b \dot{B}^l + \dot{H}_c + \dot{D}_c, \tag{9.27}$$

$$Y_c^n = (1-\tau_c)(r_b p K + i B_c^s + B^l), \tag{9.28}$$

$$S_c^n = Y_c^n, \tag{9.29}$$

$$B^l = f_l(i, r_b^e) W_c^n / p_b, \tag{9.30}$$

$$B_c^s = f_b(i, r_b^e) W_c^n, \tag{9.31}$$

$$W_c^n = W_c - M_c = B_c^s + p_b B^l, \tag{9.32}$$

$$r_b^e = 1/p_b, \tag{9.33}$$

$$M_c^d = \alpha_m (p_b B^l + B_c^s), \tag{9.34}$$

$$\dot{M}_c = \beta_{mc}(M_c^d - M_c) + (n + \bar{\pi}) M_c, \tag{9.35}$$

$$\dot{H}_c = c \dot{M}_c, \tag{9.36}$$

$$\dot{D}_c = (1-c)\dot{M}_c. \tag{9.37}$$

The bond demand functions satisfy $f_l + f_b = 1$. Assets demands follow the gross substitution principle, which implies that $\partial f_b/\partial i > 0$, $\partial f_b/\partial r_b^e < 0$, $\partial f_l/\partial i < 0$ and $\partial f_l/\partial r_b^e > 0$. The bond price is the clearing variable that ensures the equilibrium of financial markets. If the interest rate is set by the central bank and does not clear the short-term bond market, it is shown that flows consistency holds and ensures equilibrium in the short-term bond market.

[10] Net of money.
[11] Net of money.
[12] Taxes are assumed to be lump sum such that they do not affect rentiers' portfolio allocation.

In addition to equities and public bonds, rentiers' wealth is composed by money M_c. Money demand is excluded from the portfolio decision for simplicity as the assumption of endogenous money does not allow the interest rate to clear the market for short-term bonds. Asset holders have a desired quantity of money holdings M_c^d that is expressed as a proportion of their wealth net of money. The effective change in money demand $\dot{M}_c$ adjusts towards this ratio. The expression for money demand is similar to the Metzler inventory formulation (see Chiarella *et al.* (2005) for instance). Money is held in two forms. In order to have cash and deposits in the model, it is assumed that a proportion c of M_c is held as cash H_c and a proportion $(1-c)$ is held as deposits in banks in line with the basic IS–LM model without a portfolio (Sargent, 1987). Even if the portfolio formulation does not include all assets, the key point is that there is a portfolio allocation between one risky and one safe asset.

9.3.4 Public sector

The public sector gathers both fiscal authorities and a central bank.

6. Fiscal Authorities

$$pT_w = \tau_w w L^d, \tag{9.38}$$

$$pT_c = \tau_c Y_c^n, \tag{9.39}$$

$$pT_f = \tau_f r p K, \tag{9.40}$$

$$pG = \psi p K, \tag{9.41}$$

$$\begin{aligned}\dot{\psi} &= \psi\big(- \beta_{gd}(B^s + p_b B^l - (B_0^s + p_{b0} B_0^l)) \\ &\quad - \beta_{ge}(e - \bar{e}) - \beta_{gg}(\psi - \psi_0)\big),\end{aligned} \tag{9.42}$$

$$\begin{aligned}\dot{B}_{cb}^s + \dot{B}^s &= \alpha_b\big(pG + i B^s + B^l - pT_c - pT_w - pT_f\big), \\ &\text{with } \dot{B}^s = \dot{B}_c^s + \dot{B}_b^s,\end{aligned} \tag{9.43}$$

$$p_b \dot{B}^l = (1-\alpha_b)\big(pG + i B^s + B^l - pT_c - pT_w - pT_f\big). \tag{9.44}$$

The government taxes profits, financial and labour incomes and consumes goods in a proportion ψ of the capital stock. This proportion changes with respect to two elements: the level of public debt in line with some kind of Maastricht criteria and the level of employment in a very Keynesian fashion. The budget deficit is financed by either short- or long-term bonds in the proportions α_b and $(1-\alpha_b)$ respectively. The quantity B^s is now the quantity of short-term bonds available to the public (household and banks). It is the fraction of the deficit financed by short-term bonds minus bonds bought by the central bank through open market operations $\dot{B}^s = \alpha_b(pG + i B^s + B^l - pT_c - pT_w - pT_f) - \dot{B}_{cb}^s$. It is now possible to define bonds held by asset holders as the difference between bonds available to the public minus bonds held by banks $B_c^s = B^s - B_b^s$.

7. Monetary Authorities: Endogenous Money and the Taylor Rule

$$\dot{i} = \alpha_i(i^* - i), \tag{9.45}$$

$$i^* = (i_0 - \bar{\pi}) + \hat{p} + \gamma_{ip}(\hat{p} - \bar{\pi}) + \gamma_{iu}(u - \bar{u}), \tag{9.46}$$

$$\dot{B}^s_{cb} = \dot{H}_c + \dot{R} = \mu \dot{M}_c \text{ with } \mu = \theta(1 - c) + c. \tag{9.47}$$

The assumption of endogenous money is a key pillar of the post-Keynesian research agenda and is seen as one of the arguments against orthodox approaches (Godley, 1999).[13] Surprisingly, post-Keynesian models that explicitly formulate endogenous money rarely specify the role of the central bank or how monetary policy is conducted (Godley, 1999; Tadeu Lima and Meirelles, 2003, for instance), even if the use of a Taylor rule is an implicit recognition of endogenous money. In fact, the supply of money must adjust to the demand of money as the interest rate is fixed. In our case, monetary authorities steer the interest rate towards a target $\alpha_i(i^* - i)$ that depends on the long-term interest rate and on two measures of the business cycle: deviations from the output gap $\gamma_{iu}(u - \bar{u})$ and from the long-term inflation rate $\gamma_{ip}(\hat{p} - \bar{\pi})$ (Taylor, 1993). In this framework, the interest rate policy is strongly counter-cyclical as interest rates are raised when output and inflation are higher than their long-term value, as shown schematically in Figure 9.6.

In an endogenous money framework (Deleplace and Nell, 1996), money creation results from the supply of loans by commercial banks. Meanwhile in the presence of a central bank, their interaction and the path of money creation is more difficult to trace. It must be shown that the demand for high-powered money by economic agents is accommodated by the central bank and equals money injected through open market operations. On the one hand, loan supply generates investments by firms and the distribution of bank profits to asset holders. It generates a demand for high-powered money, as part of asset holders' income is held as cash ($\dot{H}_c$) and part of asset holders' deposits are held as reserves ($\theta(1 - c)\dot{M}_c$) by banks at the central bank. On the other hand, the demand for short-term bonds by private agents changes as asset holders allocate their new revenue through the portfolio and as banks' holdings of bonds are adjusted to finance new credit. In order to fix the interest rate through the Taylor rule, given a change in bond demand, the central bank must implement open market operations.[14] The quantity of bonds bought by the central bank is equal to the total

13 According to Godley, p.394 "It is no exaggeration to say that the endogenous money view is potentially lethal not merely to monetarism, which has now been discredited, but to the neoclassical paradigm itself."

14 In the case where banks rely on central banks' advances to finance credit operations, the role of the central bank in the process of money creation is more easily identified. Banks demand advances to finance the share of loans that is not covered by new deposits. The central bank acts as the bank of commercial banks and prints money for that purpose. It leads us to one of the points of disagreement about endogenous money. Some who argue in favour of a weak hypothesis of endogenous money consider that the central bank can still influence money creation to some degree by restricting the quantity of money advances. Contrarily, those advocating a strong assumption of endogenous money consider that in such a case, commercial banks can still adjust the structure of their financial assets or rely on financial innovations (Palley, 2002).

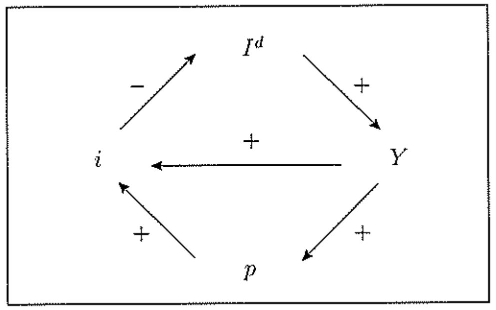


Figure 9.6 Taylor rule

demand of cash $\dot{H}_c + \dot{R}$ as given in equation (9.47). It follows that the amount of high-powered money in the economy is smaller than the amount of money in the broad sense $c\dot{M}_c + \theta(1 - c)\dot{M}_c < \dot{M}$, which gives us the definition of the money multiplier $\mu = \theta(1 - c) + c$.

9.3.5 Workers

Workers' households receive labour income wL^d that is a function of nominal wage and labour demand from which taxes must be subtracted. Workers consume all their income to underline that savings out of wages are small and relatively lower than savings out of profits.[15] The Kaldor neo Pasinetti theorem provides a theoretical rationale for this assumption (Kaldor, 1966) that has strong empirical support (Marglin, 1984). The active labour population grows at a constant rate n. These assumptions lead to the following equations:

8. Wage-price Interaction: the Rose Effect

$$pC = (1 - \tau_w)wL^d, \tag{9.48}$$

$$S_w^n = 0, \tag{9.49}$$

$$\hat{L} = n. \tag{9.50}$$

The so-called Rose effect comes from the wage-price interaction formulated by Rose (1967) and used by Chiarella *et al.* (2003a) to model a Goodwinian conflict over income distribution. In this formulation, wages and prices are adjusting to some measures of labour and goods market disequilibrium. Two Phillips curves (PCs) are specified instead of the usual single one by expressing price changes as a function of the goods market disequilibrium. In this framework, inflation not only results from wage inflation but also from the ability of firms to increase or decrease prices. In other words, the

15 Asset holders' saving rate equals 1.

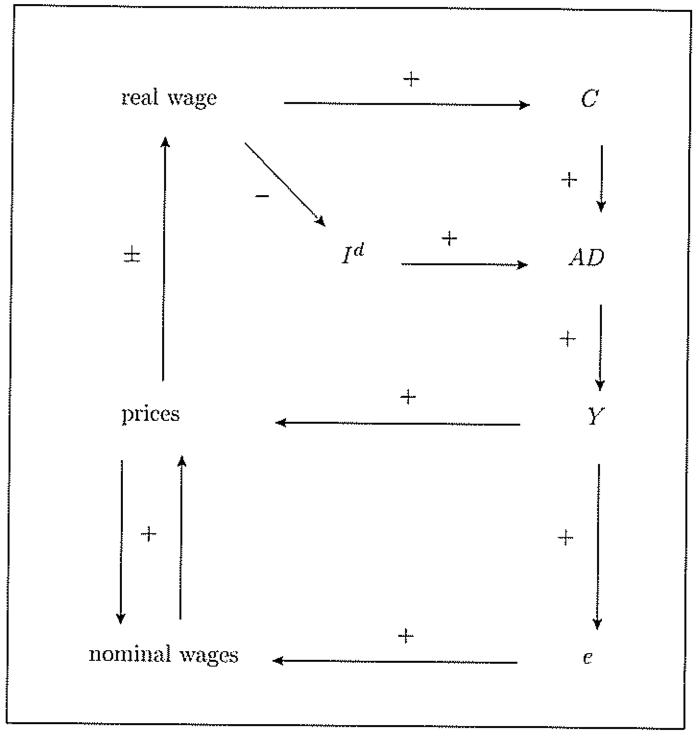


Figure 9.7 Rose effect

markup on wage cost is not fixed. In this respect, it shares some similarities with the conflict approach to inflation by Rowthorn (1977). Thus prices and nominal wages adjust according to

$$\hat{p} = \beta_p(u - \bar{u}) + \kappa_p\hat{w} + (1 - \kappa_p)\bar{\pi}, \tag{9.51}$$

$$\hat{w} = \beta_w(e - \bar{e}) + \kappa_w\hat{p} + (1 - \kappa_w)\bar{\pi}. \tag{9.52}$$

Nominal wages adjust to some measure of the disequilibrium on labour market $\beta_w(e-\bar{e})$ to which is added a cost-push element linked to changes in inflation $\kappa_w\hat{p}$. Prices react to deviation of the rate of capacity utilisation of the capital stock from its steady state value $\beta_p(u - \bar{u})$. And a cost-push element linked to variations of the nominal wage is also added $\kappa_p\hat{w}$. We adopt here a very simple formulation of the wage-price spiral by considering constant expectations $\bar{\pi}$ about price inflation instead of the usual backward and/or forward expectations that we considered in Chapter 6.

Four possible scenarios may arise and these are displayed in Figure 9.7. The relative speeds of adjustment of nominal wages and prices determine the sign of real wage change. Taking for instance the case of a positive shock on output, a faster speed of adjustment of prices entails a reduction of real wage. The overall effect of the real wage adjustment depends on the sensitivity of aggregate demand on the real wage

(Bhaduri and Marglin, 1990).[16] In a wage-led economy, aggregate demand lowers and the disequilibrium on the output market is counterbalanced. Inversely, in a profit-led economy, aggregate demand increases and excess demand worsens. The other two cases take place when nominal wages are more flexible than prices. The positive shock on output raises the real wage. These speeds of adjustment are destabilising in a wage-led economy and stabilising in a profit-led economy.

9.4 Reduced form equations and steady state

The reduced form equations give rise to an eleven-dimensional system of differential equations with respect to real wage, labour population, expected output, inventories, interest rate, expected capital gains, short- and long-term bonds, public spending, money and debt. Thus we write[17]

$$\begin{aligned}
\dot{\omega} &= \omega\kappa\big((1 - \kappa_p)\beta_w(e - \bar{e}) + (\kappa_w - 1)\beta_p(u - \bar{u})\big),\\
\dot{l} &= l\big(n - \delta_\varphi - (g_k - \delta_k - \varphi_b)\big),\\
\dot{y}_e &= \beta_{ye}(y^d - y_e) + (n - \delta_\varphi + \delta_k - g_k + \varphi_b)y_e,\\
\dot{i} &= \beta_{ii}(i_0 - i) + \beta_{ip}(\hat{p} - \bar{\pi}) + \beta_{iu}(u - \bar{u}),\\
\dot{b}^s &= \alpha_g(\psi + ib^s + b^l - t_c - t_w - t_f) - \mu\dot{m}_c - (\hat{p} + g_k - \delta_k - \varphi_b)b^s,\\
\dot{b}^l &= (1 - \alpha_g)(\psi + ib^s + b^l - t_c - t_w - t_f)/p_b - (\hat{p} + g_k - \delta_k - \varphi_b)b^l,\\
\dot{\psi} &= \psi(-\beta_{gd}(b^s + p_b b^l - b_0^s + p_{b0}b_0^l) - \beta_{ge}(e - \bar{e}) - \beta_{gg}(\psi - \psi_0)),\\
\dot{m}_c &= \beta_{mc}(\alpha_m(b_c^s + p_b b^l) - m_c) + (n + \bar{\pi} - \delta_\varphi - \hat{p} - g_k + \delta_k + \varphi_b)m_c,\\
\dot{\lambda} &= g_k - \delta - (1 - \tau_f)r - \varphi_b + (y - y^d) - (\hat{p} + g_k - \delta - \varphi_b)\lambda,\\
\dot{of} &= (1 - \alpha_{rb})r_b - \varphi_b - (\hat{p} + g_k - \delta_k - \varphi_b)\ of.
\end{aligned}$$

Finally some algebraic relationships must be added for investment g_k, the rate of profit r, bankruptcy φ_b, labour demand l^d, taxes t, asset holders' bonds b_c, the money multiplier μ, the employment e and output rate u, the long-term interest rate i_l, bank holding of bonds b_b^s as well as the growth rate of price $\hat{p}$. These are summarised by

$$\begin{aligned}
g_k &= \alpha_{iu}(u - \bar{u}) + \alpha_{i\lambda}(\lambda_0 - \lambda) + \alpha_{ir}(r - \bar{r}) + \alpha_{irb}(r_{nb} - r_{nb0}) + n + \delta_k,\\
r_f &= y - \omega y/x - \delta_k - i\lambda,\\
\kappa &= 1/(1 - \kappa_w\kappa_p),\\
\varphi_b &= \beta_{\varphi_b}(\bar{r} - r) + \delta_\varphi,\\
l^d &= y/x,
\end{aligned}$$

16 Bhaduri and Marglin formulate the dichotomy between a wage-led and a profit-led economy.
17 With regard to the last of these equations, for of, note that OF is defined by equation (9.24) and $of = OF/K$.

$$
\begin{aligned}
t &= t_w + t_c = \tau_w \omega l^d + \tau_c(ib^s + i\lambda + b^l) + \tau_f r,\\
y^d &= (1-\tau_w)\omega l^d + g_k + \psi,\\
b_c^s &= b^s - b_b^s,\\
\mu &= \theta(1-c) + c,\\
e &= l^d/l,\\
u &= y/y^p,\\
i_l &= 1/p_b,\\
r_b &= 1/p_b,\\
r_b &= i_\lambda \Lambda + i B_b,\\
r_{nb} &= 1 - \alpha_{rb} r_b - \varphi_b,\\
b_b^s &= (1-\theta)(1-c)m_c + of - \lambda,\\
\hat{p} &= \kappa\big(k_p \beta_w (e - \bar{e}) + \beta_p(u - \bar{u})\big) + \bar{\pi}.
\end{aligned}
$$

Note that p_b clears the long-term bond market and ensures that $b^{ls} = b^{ld}$. The market clearing values of p_b are the solutions of a polynomial expression of second order.[18]

Setting $\hat{w} = 0$ and $\hat{p} = 0$ give the steady state value for e_0 and u_0, which yield y_0 and l_0. At the steady state, output y_0 equals normal production $y^p\bar{u}$ such that there is no disequilibrium on the goods market. The steady state for labour demand l_0^d is given by the steady state level of production $y^p\bar{u}$ and the proportion between production and employment given by x. Choosing the steady state value of the profit rate $r_0 = \bar{r}$ and setting $\dot{d} = 0$ give the steady state debt. It depends on the steady state of the population growth rate, retained earnings and price growth rate. From the government budget constraint you get the steady state for public bonds b_0^s and b_0^l. The steady state value of the expected capital gain on bonds dynamic is calculated from $\dot{\pi}_{b0} = 0$. Given b_{c0}^s and substituting into $\dot{m}_c = 0$ gives m_{c0}. From the definition of the rate of profit we get i_0 and assuming that the rate of return on bonds is equalised ($i_0 = r_{b0}^e$) gives us p_{b0}.

[18] One may calculate that

$$p_{bi} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (i = 1, 2),$$

where

$$
\begin{aligned}
a &= b^l(\alpha_{b0} - \alpha_{b1}(i - i_0) + \alpha_{b2}(\pi_b - r_{b0}^e) - 1),\\
b &= b_c^s(\alpha_{b0} - \alpha_{b1}(i - i_0) + \alpha_{b2}(\pi_b - r_{b0}^e)) + \alpha_{b2} b^l,\\
c &= b_c^s \alpha_{b1}.
\end{aligned}
$$

There are two solutions and it is assumed that $p_b = \max\{p_{b1}, p_{b2}\}$.

The steady state calculations may be summarised as:

$$
\begin{aligned}
r_0 &= \bar{r},\\
y_0 &= y^p \bar{u},\\
y_{e0} &= y_0/(1 + \beta_{nd}(n - \delta_\varphi)),\\
l_0^d &= y_0/x = y^p\bar{u}/x,\\
l_0 &= l_0^d/\bar{e} = y^p\bar{u}/\bar{e}/x,\\
v_0 &= \beta_{nd} y_{e0},\\
u_0 &= (y_{e0} - n - delta - g)x_0/(y_0(1-\tau_w)),\\
\lambda_0 &= (-\delta - g - y_0 u_0(1-\tau_w)/x_0 + y_0 - (1-\tau_f)r_0 - \delta_\varphi)/(n + \bar{\pi} - \delta_\varphi),\\
i_0 &= (y - u_0 y/x - \delta_k - r_0)/\lambda_0,\\
p_{b0} &= 1/i_{l0},\\
A &= 1 + (1-\theta)(1-c)\alpha_m,\\
b_0^s &= \alpha_b\big(g - \tau_w \omega_0 y_0/x_0 - \tau_c i_0 \lambda_0 - \alpha_r r_{f0}\big)/(n + \bar{\pi} - \delta\varphi),\\
B_0^l &= (1-\alpha_b)\big(g - \tau_w \omega_0 y_0/x_0 - \tau_c i_0 \lambda_0 - \alpha_r r_{f0}\big)/(n + \bar{\pi} - \delta\varphi)p_b,\\
of_0 &= \big((1-\alpha_{rb})i_{\lambda 0}\lambda_0 - \delta_\varphi + \delta_{\varphi_s} + (1-\alpha_{rb})i_0((1-\theta)(1-c)\alpha_m(b_0 + p_{b0}b_0^l)\\
&\quad - \lambda_0)/A\big)/(\bar{\pi} + n - \delta_\varphi - (1-\alpha_{rb})i_0/A),\\
b_{c0} &= b_0 - b_{b0},\\
m_{c0} &= \alpha_m(b_{c0} + p_{b0}b_0^l),\\
\hat{p}_0 &= \bar{\pi},\\
r_{nb0} &= i_0\lambda_0 + i_0 b_{b0} - \delta_\varphi. \qquad (9.53)
\end{aligned}
$$

9.5 Debt default without and with bankruptcy

In this subsection, we raise the issue of debt default with and without bankruptcy. In a capitalist economy in which property rights are strictly enforced debt default results from firm bankruptcy, as in the model just presented. There are nevertheless numerous contemporary examples of debt default without bankruptcy as illustrated by the Japanese and Argentinian cases. Debt default and bankruptcy both reduce indebtedness. Debt default and bankruptcy differ with respect to loss sharing. In the case of bankruptcy, borrowers suffer some losses. In the case of debt default without bankruptcy, all losses are borne by creditors.

Bankruptcy and debt default therefore raise an important institutional issue: the role of property right enforcement in resolving over-indebtedness situations. Neoclassical institutionalism has pointed to the key aspect of the legal framework in various fields of economic theory. With regards to financial fragility, it puts forward that a necessary element to solve periods of financial fragility quickly is a strict enforcement of property rights, through bankruptcy procedures. There are two main issues. First, a virtuous microeconomic framework ensures in the neoclassical line of argument that the proper incentives are given to economic agents. The resulting efficient responses are transmitted to the macroeconomic level. Meanwhile, by focusing too much on the incentives given by the legal framework, mainstream economics minimises the direct and potentially powerful effects of debt default in alleviating the financial burden of firms. The moral hazard argument is over-emphasised. In addition, it may lead to extreme conclusions. Following this line of reasoning, it is better that firms go bankrupt today rather than adopting biased behaviours tomorrow. Put differently, a crisis today is better than a crisis tomorrow. Second, from a Schumpeterian perspective, bankruptcy is the instrument through which the market selects viable firms from the failing ones. In a Darwinist line of argument, bankruptcy eliminates the weakest. It is said that crises are creative. Our macro approach enables us to discuss whether debt default without and with bankruptcy can solve financial fragility. We also discuss the impact of demand regimes and labour and good markets' institutions.

The following sections discuss the respective properties of debt default without and with bankruptcy. For that purpose, the high order model of the previous section is reduced to three dynamic equations in order to better understand the mechanisms at work. In particular, we derive the stability conditions for two cases, debt default without bankruptcy and debt default with bankruptcy. For each situation, we discuss how the type of demand regime, wage-led or profit-led, affects the results. Lastly, in order to better understand the interaction of bankruptcy with the other equations of the model, we derive the stability conditions for the subsystem starting with two dimensions and ending at three dimensions.

9.5.1 *Debt default without bankruptcy*

The following 3D dynamical system consists of three equations for wages, output adjustment and debt:

$$\dot{\omega} = \omega\kappa\big((1-\kappa_p)\beta_w(e-\bar{e}) + (\kappa_w - 1)\beta_p(u-\bar{u})\big),$$
$$\dot{y} = \beta_y(y^d - y) + (n + \delta_k - g_k)y,$$
$$\dot{\lambda} = g_k - \delta_k - (1-\tau_f)r - \varphi_d\lambda - (\hat{p} + g_k - \delta_k)\lambda.$$

Debt default, $\varphi_d\lambda$, enters debt accumulation negatively. The rate of debt default φ_d, is a negative function of profitability. Debt default has a clear counter-cyclical effect on the business cycle as it contributes to the reduction of debt in the downturn and to an increase of debt in the upturn.

The following algebraic quantities, including the rate of default, are required on the right-hand side of the above 3D dynamical system for ω, y and λ:

$$g_k = \alpha_{iu}(u-\bar{u}) + \alpha_{i\lambda}(\lambda_0 - \lambda) + \alpha_{ir}(r - r_0) + n + \delta_k,$$
$$\varphi_d = \beta_{\varphi_d}(\bar{r} - r) + \delta_\varphi,$$

$$r = y - \omega y/x - \delta_k - i\lambda,$$
$$\kappa = 1/(1-\kappa_w\kappa_p),$$
$$l^d = y/x,$$
$$\hat{p} = \kappa\big(\kappa_p\beta_w(e-\bar{e}) + \beta_p(u-\bar{u})\big) + \bar{\pi},$$
$$y^d = (1-\tau_w)\omega l^d + g_k + \psi.$$

Steady states are similar to the steady states of the preceding section with the exception that the rate of bankruptcy no longer appears. The steady state is thus given by

$$r_0 = \bar{r},$$
$$y_0 = y^p\bar{u},$$
$$l_0^d = y^p\bar{u}/x,$$
$$l_0 = y^p\bar{u}/\bar{e}/x,$$
$$u_0 = \bar{u},$$
$$\lambda_0 = (n - (1-\tau_f)r_0)/(n + \bar{\pi} + \varphi_d),$$
$$i_0 = (y - \omega y/x - \delta - r_0)/\lambda_0,$$
$$\hat{p}_0 = \bar{\pi},$$
$$\omega_0 = (y_0 - \delta - g - g_{k0})x/\big((1-\tau_w)y_0\big),$$
$$e_0 = \bar{e},$$
$$g_{k0} = n + \delta.$$

The 2D model As illustrated by Figure 9.2, the introduction of debt default modifies the interaction between debt and output. The stability conditions of the two dimensional model may be summarised as follows:

- Debt accumulation is stabilising when wage and price dynamics are not taken into account.
- Debt default tends to increase the stability of the economy. It reduces the debt to capital ratio, which sustains the profit rate and limits credit rationing.

Proposition 9.1 *Assuming that $\beta_y > y_0$ and β_{φ_d} is large enough such that*

$$\beta_{\varphi_d} > ((1-\tau_f)(1-\omega_0/x_0) - (1-\lambda_0)g_{ky})/\lambda_0(1-\omega_0/x_0),$$

assume furthermore that

$$(1-\tau_w)\omega_0/x_0 + \alpha_{iu}/y^p + \alpha_{ir}(1-\omega_0/x_0) < 1$$

holds. Assume finally that the adjustment speeds of β_w, β_p are all zero.

Then:
The Jacobian of the independent subdynamics:

$$\dot{y} = \beta_y(y^d - y) + (n + \delta_k - g_k)y,$$
$$\dot{\lambda} = g_k - \delta_k - (1 - \tau_f)r - \varphi_d\lambda - (\bar{\pi} + g_k - \delta_k)\lambda,$$

has a stable steady state.

Proof: The entries of the Jacobian matrix are:

$$J_{11} = \beta_y\Big((1 - \tau_w)\omega_0/x_0 + g_{ky} - 1\Big) - y_0 g_{ky} < 0,$$
$$J_{12} = (\beta_y - y_0)g_{k\lambda} < 0,$$
$$J_{21} = (1 - \lambda_0)g_{ky} - (1 - \tau_f)(1 - \omega_0/x_0) + \lambda_0\beta_{\varphi_d}(1 - \omega_0/x_0) > 0,$$
$$J_{22} = (1 - \lambda_0)g_{k\lambda} + (1 - \tau_f)i_0 - \beta_{\varphi_d}i_0\lambda_0 - (\bar{\pi} + \delta_\varphi + n) < 0,$$

with $g_{k\lambda} < 0$ and $g_{ky} > 0$. Thus it follows that trace J < 0 and $\det J > 0$. □

Remark 9.1 As β_{φ_d} enters J_{21} positively and J_{22} negatively, $J_{21} > 0$ and $J_{22} < 0$. In other words, a β_{φ_d} large enough ensures that debt is not cumulative with the output and is not cumulative with itself. Default is unambiguously stabilising at this stage. Output has a positive effect on profitability. In the upturn, default decreases and increases the debt to capital ratio. It limits the increase of firms' net wealth in the boom. By contrast, debt depresses profits and increases debt default, which smoothes the feedback effect of debt on itself.

The 3D model The stability conditions of the three dimensional model may be summarised as follows:

- Debt accumulation is prone to instability if $\alpha_{i\lambda}$ is large and if prices are flexible.
- Debt default seems to have a strong stabilising effect. It smoothes the depression-inducing effects of credit rationing and sustains profitability; and reduces the debt to capital ratio.

We discuss the stability conditions for the case of wage-led aggregate demand and profit-led aggregate demand. Debt default is stabilising in both cases.

9.5.1.1 *The case of a wage-led aggregate demand*

Proposition 9.2 *Assume in addition to what has been assumed in Proposition 9.1 that* $(1 - \tau_w) > \alpha_{ir}$ *holds, that* β_{φ_d} *satisfies* $\beta_{\varphi_d} > (1 - \tau_f - \alpha_{ir}(1 - \lambda_0))/\lambda_0$, *that* β_p *and* κ_p *are larger than* β_w *and* κ_w *and that* β_p *and* κ_p *are not too large such that*

$$g_{ky}(1 - \lambda_0) - (1 - \tau_f)(1 - \omega_0/x_0) > \lambda_0\kappa(\kappa_p\beta_w/x_0 l_0 + \beta_p/y^p).$$

Then:
The Jacobian of the then independent subdynamics

$$\dot{y} = \beta_y(y^d - y) + (n + \delta_k - g_k)y,$$
$$\dot{\lambda} = g_k - \delta_k - (1 - \tau_f)r - \varphi_d\lambda - (\hat{p} + g_k - \delta_k)\lambda,$$
$$\dot{\omega} = \omega\kappa\big((1 - \kappa_p)\beta_w(e - \bar{e}) + (\kappa_w - 1)\beta_p(u - \bar{u})\big),$$

has the properties $\det J < 0$, $trJ < 0$, $J_1 + J_2 + J_3 > 0$, $(-trJ)(J_1 + J_2 + J_3) + \det J > 0$.
Thus the steady state of these reduced dynamics is locally asymptotically stable.

Proof: Some of the entries of the Jacobian are left unchanged. They are $J_{11} < 0$, $J_{12} < 0$ and $J_{22} < 0$. Due to the introduction of price dynamics, the other entries are

$$J_{13} = \beta_y(1 - \tau_w)y_0/x_0 + g_{kw}(\beta_y - y_0) > 0,$$
$$J_{21} = g_{ky}(1 - \lambda_0) - (1 - \tau_f)(1 - \omega_0/x_0) + \lambda_0\beta_\varphi(1 - \omega_0/x_0)$$
$$- \lambda_0\kappa(\kappa_p\beta_w/x_0 l_0 + \beta_p/y^p) > 0,$$
$$J_{23} = g_{kw}(1 - \lambda_0) + (1 - \tau_f)y_0/x_0 - \lambda_0\beta_{\varphi_d}y_0/x_0 < 0,$$
$$J_{31} = \omega\kappa\big((1 - \kappa_p)\beta_w/x_0 l_0 + (\kappa_w - 1)\beta_p/y^p\big) < 0,$$
$$J_{32} = 0,$$
$$J_{33} = 0,$$

with $g_{kw} = -\alpha_{ir}y_0/x_0 < 0$.
The result then follows from standard calculations. □

Remark 9.2 Increasing the dimension of the model modifies some of the entries of the Jacobian matrix. With respect to the 2D model, the entry J_{21} is increased by $-\lambda_0\kappa(\kappa_p\beta_w/x_0 l_0 + \beta_p/y^p)$ which may change the sign of the entry if β_p and κ_p are sufficiently large. In other words, price flexibility may produce a counter-cyclical real debt to capital ratio. Firms' net wealth to capital ratio is pro-cyclical and the financial accelerator is unstable.

The only new effect of debt default appears through J_{23} which is augmented by $-\lambda_0\beta_{\varphi_d}y_0/x_0$. Changes in real wage affect profits and debt default, which feeds back on output through debt. This new element tends to intensify the negative effect of real wage on real debt. It therefore increases stability (see Figure 9.8).

The last condition (that $(\mathrm{tr}J)(J_1 + J_2 + J_3) + \det J > 0$) holds as β_y enters quadratically in $(-\mathrm{tr}J)(J_1 + J_2 + J_3)$ but not in $\det J$. This is so since β_y enters both the trace and J_2 as well as J_3. In addition, β_{φ_d} tends to increase the sign of the four conditions. The last condition is increasing with β_{φ_d}. The trace is decreasing with β_φ, $J_1 + J_2 + J_3$ is increasing with β_φ while $\det J$ is decreasing with β_φ. □

$$\begin{tikzcd} \omega \arrow[r, "-"] & r \arrow[r, "-"] & \varphi \arrow[d, "-"] \\ y \arrow[u, "-"] & & \lambda \arrow[ll, "-"] \end{tikzcd}$$

Figure 9.8 The intensive form dynamics – a stabilising channel of debt default via the effect of real wages on profits

9.5.1.2 The case of a profit-led aggregate demand

Proposition 9.3 *Assume in addition to what has been assumed in Proposition 9.1 that* $(1 - \tau_w - \alpha_{ir}) < 0$ *holds, that* β_{φ_d} *satisfies* $\beta_{\varphi_d} > (1 - \tau_f - \alpha_{ir}(1 - \lambda_0))/\lambda_0$*, that* β_w *and* κ_w *are different from zero and sufficiently larger than* β_p *and* κ_p *and that* β_p *and* κ_p *are not too large such that*

$$g_{ky}(1 - \lambda_0) - (1 - \tau_f)(1 - \omega_0/x_0) > \lambda_0\kappa(\kappa_p\beta_w/x_0 l_0 + \beta_p/y^p).$$

Assume furthermore that τ_w *is large enough and* τ_f *is small enough such that*

$$(1 - \lambda_0)\beta_y(1 - \tau_w) - (\beta_y - y_0)(1 - \tau_f) < 0.$$

Assume finally that $\alpha_{i\lambda}$ *is small enough and* α_{ir} *is large enough such that*

$$-\lambda_0\beta_{\varphi_d}(J_{12}y_0/x_0 - i_0 J_{13}) < 0.$$

Then:
The Jacobian of the independent subdynamics

$$\begin{aligned}
\dot{y} &= \beta_y(y^d - y) + (n + \delta_k - g_k)y, \\
\dot{\lambda} &= g_k - \delta_k - (1 - \tau_f)r - \varphi_d\lambda - (\hat{p} + g_k - \delta_k)\lambda, \\
\dot{\omega} &= \omega\kappa\big((1 - \kappa_p)\beta_w(e - \bar{e}) + (\kappa_w - 1)\beta_p(u - \bar{u})\big),
\end{aligned}$$

has the properties $\det J < 0$, $trJ < 0$, $J_1 + J_2 + J_3 > 0$, $(-trJ)(J_1 + J_2 + J_3) + \det J > 0$. *Thus, the steady state of these reduced dynamics is locally asymptotically stable.*

Proof: The main results of the preceding section still hold. As aggregate demand is profit-led we have $J_{13} < 0$, the real wage must be labour market-led so that $J_{31} > 0$. We require investment to be sensitive to profitability (α_{ir} is large) and nominal wages to be more flexible than prices (β_w and κ_w are relatively larger than β_p and κ_p). As β_p and κ_p are relatively small, they do not make debt counter-cyclical, and so $J_{21} > 0$. In addition, i_0 is sufficiently small such that $J_{22} < 0$ holds. The propensity to consume is lower than 1 so that $J_{11} < 0$; and β_y is still larger than y_0 to achieve $J_{12} < 0$.

Eventually, $J_{32} = J_{33} = 0$, and $J_{23} < 0$. Note that similarly to the wage-led aggregate demand case, β_φ tends to reinforce the sign of the entries J_{2i}.

Although the trace is negative and the sub-determinants are positive, the sign of the determinant is not clear at first glance. Here again, the model is stable if $J_{12}J_{23} - J_{13}J_{22} < 0$. Taxes on profits must be small while taxes on wages must be sufficiently large such that $(1-\lambda_0)\beta_y(1-\tau_w)-(\beta_y-y_0)(1-\tau_f) < 0$ is true. In addition, the interest rate must be sufficiently small such that $(1 - \tau_f)i_0 - n - \bar{\pi} < 0$ holds. It must also be the case that if $\alpha_{i\lambda}$ is small enough and α_{ir} is large enough then $-\lambda_0\beta_\varphi(J_{12}y_0/x_0 - i_0 J_{13}) < 0$ is true. Note that this latter condition may be hard to fulfil as i_0 is assumed to be small. Nevertheless, in such a case the determinant is decreasing with β_{φ_d}. Default stabilises the economy. In other words, if the destabilising Fisher effect $\alpha_{i\lambda}$ is smaller than the stabilising Rose effect α_{ir}, then default increases stability. □

9.5.2 Debt default with bankruptcy

The following 3D dynamic takes into account the case of debt default with bankruptcy:

$$\begin{aligned}
\dot{\omega} &= \omega\kappa\big((1 - \kappa_p)\beta_w(e - \bar{e}) + (\kappa_w - 1)\beta_p(u - \bar{u})\big), \\
\dot{y} &= \beta_y(y^d - y) + (n - \delta\varphi + \delta_k + \varphi_b - g_k)y, \\
\dot{\lambda} &= g_k - \delta_k - (1 - \tau_f)r - \varphi_b - (\hat{p} + g_k - \delta_k - \varphi_b)\lambda.
\end{aligned}$$

The equations are similar to the high order model of Section 9.4 except that inventories do not enter the debt dynamics and the distinction between expected and actual output. The rate of bankruptcy φ_b is a negative function of firms' profitability and reduces the stock of capital. Debt default is a consequence of bankruptcy and can be expressed as a function of the rate of bankruptcy.

The following variables must be introduced into the above equations.

$$\begin{aligned}
g_k &= \alpha_{iu}(u - \bar{u}) + \alpha_{i\lambda}(\lambda_0 - \lambda) + \alpha_{ir}(r - r_0) + n + \delta_k, \\
\varphi_b &= \beta_\varphi(\bar{r} - r) + \delta_\varphi, \\
r &= y - \omega y/x - \delta_k - i\lambda, \\
\kappa &= 1/(1 - \kappa_w\kappa_p), \\
l^d &= y/x, \\
\hat{p} &= \kappa\big(\kappa_p\beta_w(e - \bar{e}) + \beta_p(u - \bar{u})\big) + \bar{\pi}, \\
y^d &= (1 - \tau_w)\omega l^d + g_k + \psi.
\end{aligned}$$

Steady states are left unchanged with the exception that inventories no longer appear. Thus

$$\begin{aligned}
&e_0 = \bar{e}, && u_0 = \bar{u}, && y_0 = y^p\bar{u}, \\
&r_0 = \bar{r}, && g_{k0} = n + \delta_k, && \hat{p}_0 = \bar{\pi}, \\
&l_0^d = y^p\bar{u}/x, && l_0 = y^p\bar{u}/\bar{e}/x,
\end{aligned}$$

$$\lambda_0 = (n - (1-\tau_f)r_0 - \delta\varphi)/(n + \bar{\pi} - \delta\varphi),$$
$$\omega_0 = (y_0 - \delta - g - g_{k0})x/\big((1-\tau_w)y_0\big),$$
$$i_0 = (y - \omega y/x - \delta_k - r_0)/\lambda_0.$$

The 2D model The stability conditions for the two dimensional model may be summarised as follows:

- As seen previously, debt is stabilising in a two dimensional model. Debt accumulation has a strong disciplinary effect.
- Default with bankruptcy increases stability. It combines the stabilising effect of default and bankruptcy. In periods of depression and inversely in periods of expansion, the stock of capital is reduced, which increases the output to capital ratio. The remaining firms are on average more efficient which enables the economy to rebound.

Proposition 9.4 *Assume that $\beta_y > y_0$ and β_φ is large enough such that*

$$\beta_\varphi > ((1-\tau_f)(1-\omega_0/x_0) - (1-\lambda_0)g_{ky})/(1-\lambda_0)(1-\omega_0/x_0).$$

Assume furthermore that

$$(1-\tau_w)\omega_O/x_0 + \alpha_{iu}/y^p + \alpha_{ir}(1-\omega_0/x_0) < 1.$$

Assume finally that the adjustment speeds of β_w, β_p are all zero.
Then:
The Jacobian of the independent subdynamics:

$$\dot{y} = \beta_y(y^d - y) + (n - \delta\varphi + \delta_k + \varphi_b - g_k)y,$$
$$\dot{\lambda} = g_k - \delta_k - (1-\tau_f)r - \varphi_b - (\bar{\pi} + g_k - \delta_k - \varphi_b)\lambda.$$

has a stable steady state.

Proof: The entries of the Jacobian matrix are:

$$J_{11} = \beta_y\Big((1-\tau_w)\omega_0/x_0 + g_{ky} - 1\Big) - y_0 g_{ky} - y_0\beta_\varphi/y^p - \beta_\varphi(1-\omega_0/x_0) < 0,$$
$$J_{12} = (\beta_y - y_0)g_{k\lambda} + \beta_\varphi y_0 i_0 < 0,$$
$$J_{21} = (1-\lambda_0)g_{ky} - (1-\tau_f)(1-\omega_0/x_0) + \beta_\varphi(1-\lambda_0)(1-\omega_0/x_0) > 0,$$
$$J_{22} = (1-\lambda_0)g_{k\lambda} + (1-\tau_f)i_0 - \beta_\varphi i_0(1-\lambda_0) - (\bar{\pi} + n - \delta\varphi) < 0,$$

with $g_{k\lambda} < 0$ and $g_{ky} > 0$. The result readily follows as trace J < 0 and det J > 0. □

Remark 9.3 With respect to default without bankruptcy, default appears in the entries J_{21} and J_{22} with the same sign but with a weight $1-\lambda_0$ that reflects the balance sheet composition at the sectoral level.

Bankruptcy enters negatively in J_{11} and positively in J_{12}. The effect of bankruptcy on output is stabilising as J_{11} is decreasing with β_φ. Conversely, change in output with

respect to debt tends to increase with β_φ. The entries J_{12} change sign only for very large and unrealistic values of β_φ.

Default increases the pro-cyclical tendency of debt with respect to output. It also increases the negative feedback effect of debt on itself. We do not need to assume that τ_f is large enough to ensure $J_{21} > 0$ and $J_{22} < 0$ as these conditions are fulfilled if β_φ is larger than $(1-\tau_f)/(1-\lambda_0)$ and smaller than $-g_{k\lambda}(\beta_y - y_0)/y_0 i_0$.

The 3D model The stability conditions for the full model may be summarised as follows:

- Debt accumulation is prone to instability when the wage-price dynamic is incorporated.
- When default results from bankruptcy, a wage-led real sector is stabilised by a higher output to capital ratio and lower debt level.
- With respect to a profit-led real sector, default resulting from bankruptcy may trigger an unstable feedback channel between wage flexibility, output and debt. Wage flexibility which is stabilising in a profit-led demand regime has some perverse effects when combined with debt. In periods of depression, real wage decreases, which reduces bankruptcies. In turn, the output to capital ratio decreases further. In other words, the real wage dynamics weaken the destructive creation effect. In addition, lower bankruptcies increase debt, which depresses output further. Nominal wage flexibility also weakens the positive effect of debt default.

9.5.2.1 The case of a wage-led aggregate demand

Proposition 9.5 *Assume in addition to what has been assumed in Proposition 9.1 that $(1-\tau_w) > \alpha_{ir} - y_0\beta_\varphi/\beta_y$ holds, that β_φ is sufficiently large that $\beta_\varphi > (1-\tau_f - \alpha_{ir}(1-\lambda_0))/(1-\lambda_0)$ holds, that β_p and κ_p are larger than β_w and κ_w and that β_p and κ_p are not so large such that*

$$g_{ky}(1-\lambda_0) - (1-\tau_f)(1-\omega_0/x_0) + (1-\lambda_0)\beta_\varphi(1-\omega_0 x_0) > \lambda_0\kappa(\kappa_p\beta_w/x_0 l_0 + \beta_p/y^p).$$

Then:
The Jacobian of the independent subdynamics:

$$\dot{y} = \beta_y(y^d - y) + (n - \delta\varphi + \delta_k + \varphi_b - g_k)y,$$
$$\dot{\lambda} = g_k - \delta_k - (1-\tau_f)r - \varphi_b - (\hat{p} + g_k - \delta_k - \varphi_b)\lambda,$$
$$\dot{\omega} = \omega\kappa\big((1-\kappa_p)\beta_w(e-\bar{e}) + (\kappa_w - 1)\beta_p(u-\bar{u})\big),$$

has a stable steady state.

Proof: The entries J_{11}, J_{12} and J_{22} are left unchanged. The other new or modified entries are

$$\begin{aligned}
J_{13} &= \beta_y(1-\tau_w)y_0/x_0 + (\beta_y - y_0)g_{kw} + y_0\beta_\varphi y_0/x_0 > 0,\\
J_{21} &= (1-\lambda_0)g_{ky} - (1-\tau_f)(1-\omega_0/x_0) + \beta_\varphi(1-\lambda)(1-\omega_0/x_0),\\
&\quad - \lambda_0\kappa(\kappa_p\beta_w/x_0 l_0 + \beta_p/y^p) > 0,\\
J_{23} &= g_{kw}(1-\lambda_0) + (1-\tau_f)y_0/x_0 - \beta_\varphi(1-\lambda_0)y_0/x_0 < 0,\\
J_{31} &= \omega\kappa\big((1-\kappa_p)\beta_w/x_0 l_0 + (\kappa_w - 1)\beta_p/y^p\big) < 0,\\
J_{32} &= 0,\\
J_{33} &= 0.
\end{aligned}$$

It is straightforward to calculate that trace J < 0, det J $=< 0$, $J_1 + J_2 + J_3 > 0$, and $-\mathrm{tr}J(J_1+J_2+J_3) + \det J > 0$. The result then follows from application of the Routh–Hurwitz conditions. □

Remark 9.4 J_{11} and J_{12} are left unchanged with respect to the 2D case. In a wage-led real sector, default with bankruptcy increases J_{13} by $y_0\beta_\varphi y_0/x_0$. An increase of the real wage reduces profit and increases bankruptcies, which in turn further raises output. This effect is stabilising.

J_{21} is increased by $-\lambda_0\kappa(\kappa_p\beta_w/x_0 l_0 + \beta_p/y^p)$ as prices affect real debt. As underlined in the baseline model, large price flexibility may lead debt to be counter-cyclical with $J_{21} < 0$. J_{22} is left unchanged. It is decreasing with bankruptcy. Debt is unlikely to feed back on itself when default is allowed. Eventually, J_{23} decreases with bankruptcies. Higher real wage increases bankruptcies and default, which in turn reduces the debt to capital ratio. These feedback channels are displayed in Figure 9.9.

In line with the preceding model, a wage-led aggregate demand is stabilised by a goods market-led real wage. Prices are more flexible than nominal wages if β_p and κ_p are greater than β_w and κ_w.

In the case of debt accumulation with a wage-led real sector, default with bankruptcy increases the stability of the economy. The positive feedback channel between real wage, bankruptcy, debt and output is represented in Figure 9.9.

9.5.2.2 *The case of a profit-led aggregate demand*

Lemma 9.1 *Assume in addition to what has been assumed in Proposition 9.1 that* $\alpha_{ir} > (1-\tau_w) + y_0\beta_\varphi/\beta_y$ *holds, that* β_φ *is sufficiently large that* $\beta_\varphi > (1-\tau_f - \alpha_{ir}(1-\lambda_0))/(1-\lambda_0)$ *holds, that* β_w *and* κ_w *are different from zero and sufficiently larger than* β_p *and* κ_p *are not too large such that these hold*

$$g_{ky}(1-\lambda_0) - (1-\tau_f)(1-\omega_0/x_0) + (1-\lambda_0)\beta_\varphi(1-\omega_0/x_0) > \lambda_0\kappa(\kappa_p\beta_w/x_0 l_0 + \beta_p/y^p).$$

In addition assume that τ_w *is large enough and* τ_f *is small enough such that*

$$(1-\lambda_0)\beta_y(1-\tau_w) - (\beta_y - y_0)(1-\tau_f) < 0.$$

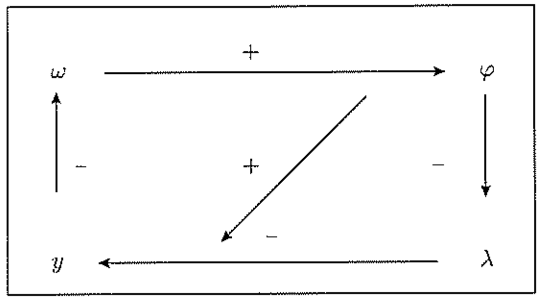


Figure 9.9 The intensive form dynamics – a stabilising channel of debt default via the effect of goods market-led real wage

Finally assume that $\alpha_{i\lambda}$ *is small enough,* α_{ir} *is large enough and* β_φ *is not so large such that*

$$-\lambda_0\beta_\varphi(J_{12}y_0/x_0 - i_0 J_{13}) < 0.$$

Then:

The Jacobian of the independent subdynamics:

$$\begin{aligned}
\dot y &= \beta_y(y^d - y) + (n - \delta\varphi + \delta_k + \varphi_b - g_k)y,\\
\dot\lambda &= g_k - \delta_k - (1-\tau_f)r - \varphi_b - (\hat p + g_k - \delta_k - \varphi_b)\lambda,\\
\dot\omega &= \omega\kappa\big((1-\kappa_p)\beta_w(e-\bar e) + (\kappa_w - 1)\beta_p(u - \bar u)\big)
\end{aligned}$$

has a stable steady state.

Proof: The following conditions hold: trace J < 0, det J $=< 0$, $J_1 + J_2 + J_3 > 0$ and $-\mathrm{tr}J(J_1+J_2+J_3) + \det J > 0$. The result then follows from application of the Routh–Hurwitz conditions. □

Remark 9.5 With respect to a wage-led real sector, the entry J_{13} is negative in a profit-led real sector. The real wage tends to increase the output to capital ratio through bankruptcy, so reducing the degree to which the real sector is profit-led. J_{13} is negative if $\alpha_{ir} > (1-\tau_w) + y_0\beta_\varphi/\beta_y$. A significant sensitivity of bankruptcy to the profit rate may turn a profit-led aggregate demand into a wage-led one. The entries J_{21}, J_{22} and J_{23} are similar to the wage-led case.

As seen previously, a profit-led aggregate demand is stabilised by a labour market-led real wage. The entry J_{31} is positive if β_w and κ_w are larger than β_p and κ_p. Although the stability conditions trace J < 0, $J_1 + J_2 + J_3 > 0$ and $-\mathrm{tr}J(J_1+J_2+J_3) + \det J > 0$ are straightforward, the sign of the determinant is less clear. In the preceding sections, a destabilising feedback channel between real wage, debt and output was identified. We argued that nominal wage flexibility may lead to a debt-deflation spiral, and that its positive effect on profit may not be large enough to restore stability (det J < 0 if $J_{12}J_{23} - J_{13}J_{22} < 0$). We argue that bankruptcy generates an additional destabilising feedback channel with nominal wage flexibility. In other words, nominal wage flexibility tends

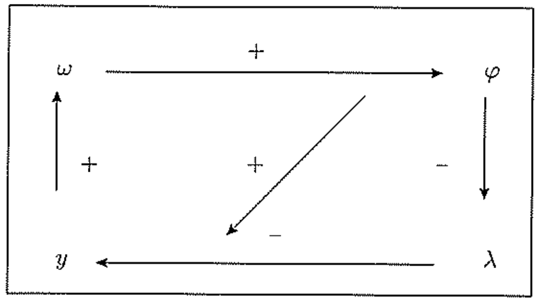


Figure 9.10 Destabilising channels of bankruptcy with a profit-led AD

to reduce the positive effects of bankruptcy and default. The quantity J_{13} is increased by $y_0\beta_\varphi y_0/x_0$. An increase of the real wage reduces profit, increases bankruptcies and output, which in turns raises the real wage further. Nominal wage flexibility reduces the stabilising interaction between output and the real wage through bankruptcy. In addition, J_{23} decreases with bankruptcies. Higher real wage increases bankruptcies and debt default, which reduces the debt to capital ratio. In turn, output increases and the real wage is pushed upward. In the case of debt accumulation with a profit-led real sector, default with bankruptcy decreases the stability of the economy. The unstable feedback channels between real wage, bankruptcy, debt and output are displayed in Figure 9.10.

9.6 Simulations: baseline scenarios

In this section, we seek to gauge by use of simulations the impact of debt default, bankruptcy and bank performance on the business cycle. We also consider the effect of the resulting supply of credit on the economy as a whole.

9.6.1 Debt default and bankruptcy

In Figure 9.11(a) and 9.11(b), debt default takes place without bankruptcy and is stabilising. Credit rationing is moderate $\alpha_{i\lambda} = 0.35$, aggregate demand is profit-led $\alpha_{ir} = 1.9$ and the real wage is labour market-led $\beta_w = 0.5$, $\beta_p = 0.05$, $\kappa_w = 0.75$ and $\kappa_p = 0.5$. In such a case, default is highly stabilising. Increasing the parameter of debt default from 0.1 to 0.5 accelerates the convergence of the economy. To the extent that the baseline finance-led model with a profit-led aggregate demand is very unstable, default smoothes out some of the destabilising feedbacks. An important channel is the profit rate channel. Eigenvalues diagrams clearly show that default is stabilising, as the maximum real part of the eigenvalues is decreasing in β_φ and β_{φ_d}.

Bankruptcy has ambiguous effects when aggregate demand is profit-led as can be seen in Figures 9.11(c) and 9.11(d). The sensitivity of aggregate demand to profit is large, $\alpha_{ir} = 1.9$, $\tau_w = 0.325$ and the real wage is labour market-led with $\beta_w = 1$ and

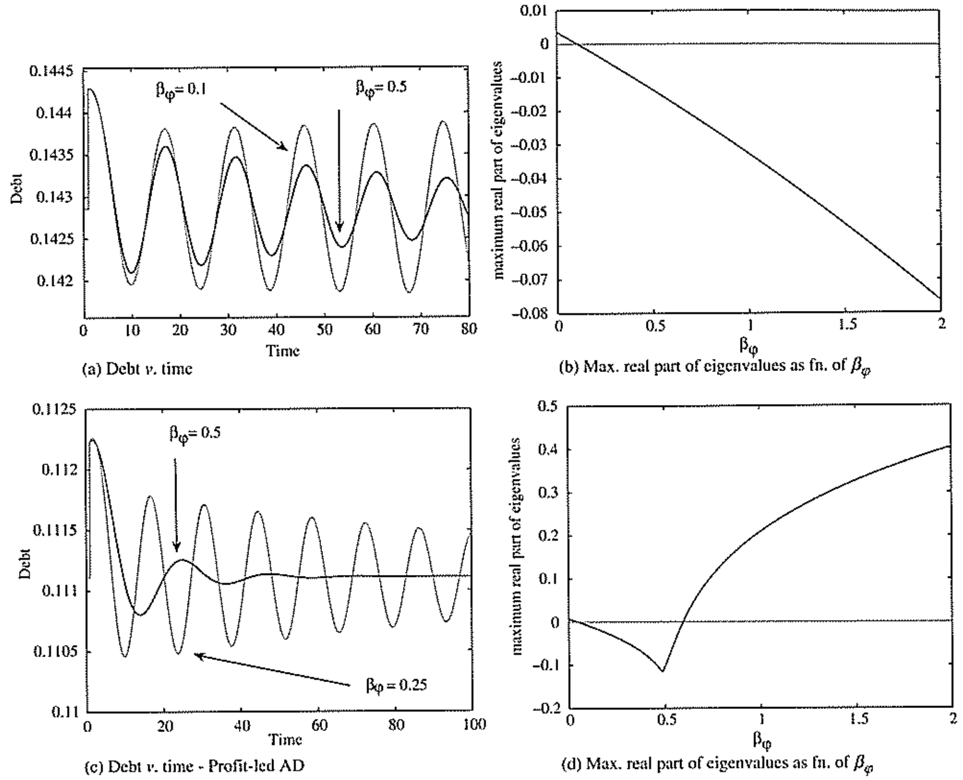
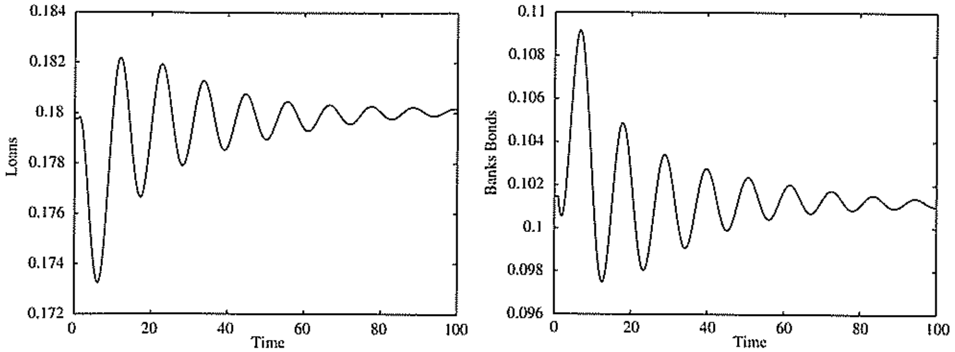


Figure 9.11 Debt default and bankruptcy – the 3D model

$\beta_p = 0.1$. With a moderate sensitivity of bankruptcy to profitability, bankruptcy is stabilising. Increasing β_φ from 0.25 to 0.5 stabilises the business cycle. As the weakest firms are eliminated and as debt default reduces the Fisher effect, the economy converges faster. By contrast, the maximum real part of the eigenvalues graph (Figure 9.11(d)) shows that after a certain point, a large β_φ destabilises the economy.

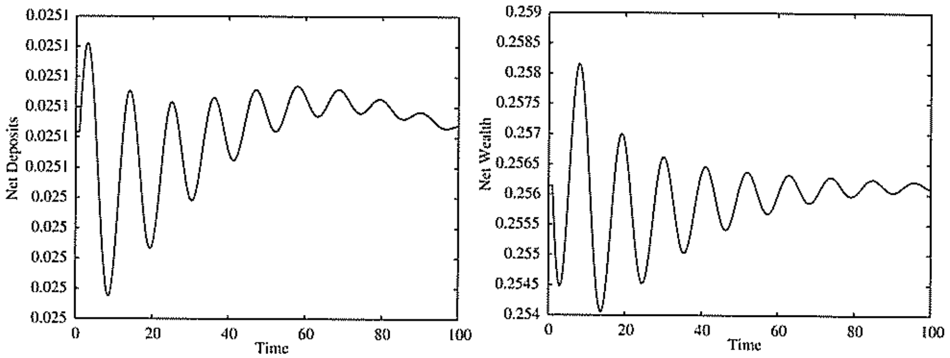
9.6.2 Banks' budget constraint

Figures 9.12 and 9.13 display banks' balance sheet. The main parameters involve a wage-led aggregate demand ($\alpha_{ir} = 0.1, \tau_w = 0.1$) and goods market-led real wage dynamics ($\beta_w = 0.05, \beta_w = 0.4$, $\kappa_p = \kappa_w = 0.4$). The monetary policy is active and weights inflation and the output gap similarly with $\alpha_{iu} = \alpha_{ip} = 0.4$. The fiscal policy is counter-cyclical ($\beta_{ge} = 0.2$) and there is no credit rationing with respect to the borrower or lender characteristics ($\alpha_{i\lambda} = \alpha_{irb} = 0$). A 1 per cent positive shock to output generates damped business cycle oscillations of approximately fifteen years. There are two main results. First, the banks' balance sheet sums to zero at every point in time. Second, when credit decreases, banks will hold excess reserves. Instead of having unused cash, they buy public bonds. And given the change in deposits, net wealth



(a) Loans v. time (b) Bank bonds v. time

Figure 9.12 The balance sheet of banks - loans and bank bonds



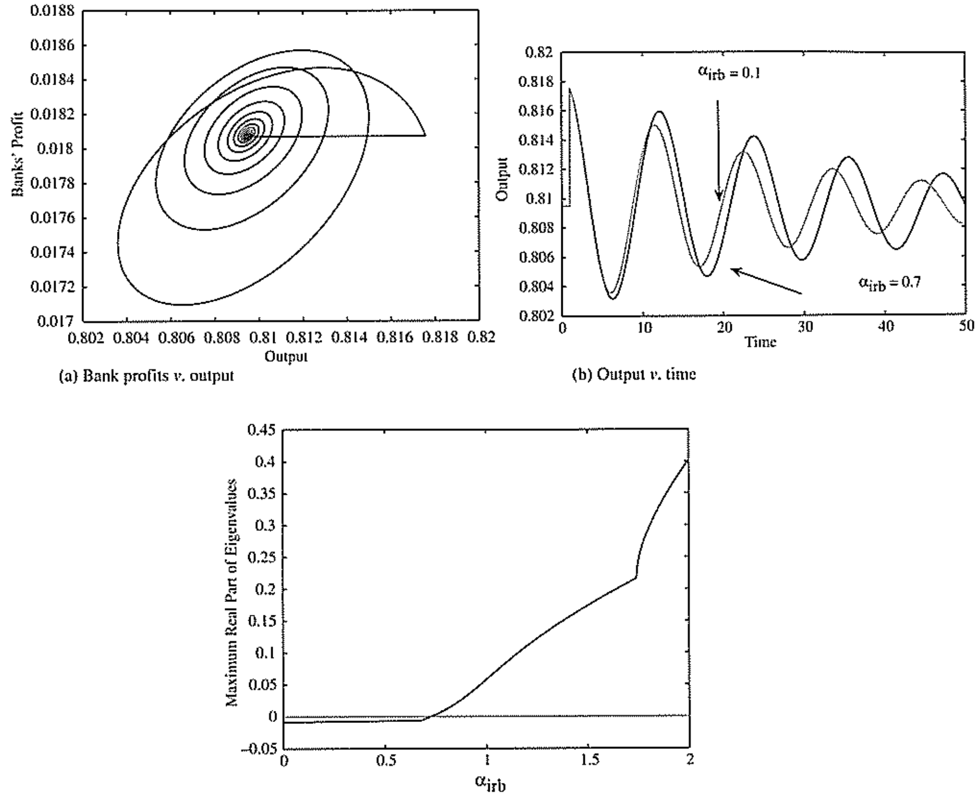
(a) Net deposits v. time (b) Net wealth v. time

Figure 9.13 The balance sheet of banks – net deposits and net wealth

decreases. The evolution of banks' net wealth is roughly in line with the dynamics of credit supply.

9.6.3 Pro-cyclical profits and credit supply

Figure 9.14 illustrates the case of credit rationing with respect to bank profits. In particular, we check whether the profitability of firms and therefore own funds are pro-cyclical. We then enquire whether the resulting credit supply is pro-cyclical as well. With respect to the parameters of the preceding simulations, we increase the sensitivity of credit supply with respect to bank profits from 0 to 0.1. In the boom, firms' desired investment increases and their need for external funds also increases. Credit expands and bank assets and profitability improve. As bank credit supply is now a function of bank income, they ease credit rationing. A cumulative loop is unleashed in which higher credit supply leads to better profitability of banks and larger credit supply. Note that this simulation confirms that bank profitability is pro-cyclical, which implies an expansion of the balance sheet of banks; see Figure 9.14(a). This bank based version of the financial accelerator brings instability. Increasing α_{irb} from 0.1 to 0.7 generates wider



(a) Bank profits v. output (b) Output v. time

(c) max. real part of eigenvalues as fn. of α_{irb}

Figure 9.14 Banks' pro-cyclical profitability and credit supply

oscillations of the output as shown in Figure 9.14(b). Figure 9.14(c) displays the effect of a high sensitivity of credit supply to bank profitability on economic stability. The maximum real part of the eigenvalues are increasing with α_{irb}, so clearly pro-cyclical credit supply is destabilising.

9.6.4 Debt default and credit crunch

Figure 9.15 shows the impact of debt default on the credit supply of banks. In Figure 9.15(a) debt default takes place but credit rationing is independent of bank profitability and it is assumed here that the parameter for credit rationing (α_{irb}) is 0. The maximum real part of the eigenvalues decreases slightly with β_{φ}. As shown in the second model, bankruptcy may have some positive effects through debt default. In addition, active monetary and fiscal policies are likely to smoothe the negative effect of bankruptcy on unemployment. Now, increasing the sensitivity of credit supply to bank profitability reverses the effect of default. In the upturn, banks' profits and credit supply both increase. This effect is amplified by the reduction of debt default. As output increases, firms' general profitability improves. Meanwhile, in the downturn, debt default contributes to reducing credit supply further by reducing bank assets and

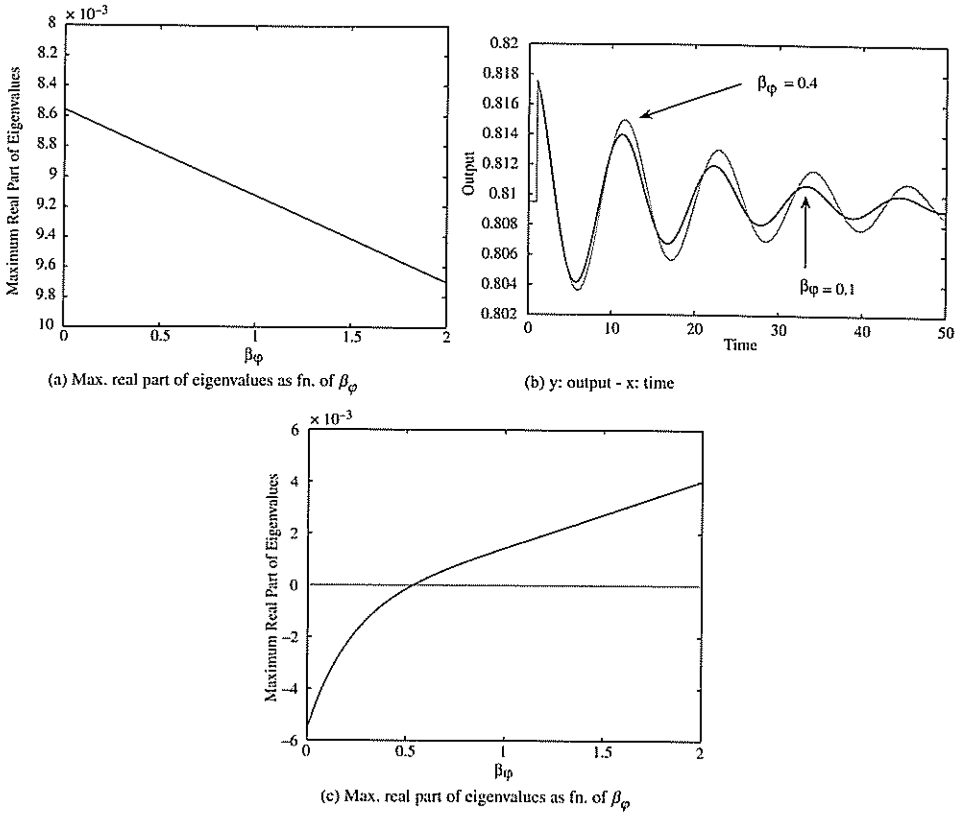


Figure 9.15 Debt default and credit crunch

income (in addition to the deterioration brought by the depressed economic environment). Increasing the speeds of adjustment of bankruptcy to profitability from 0.1 to 0.4 increases output fluctuations. In the same line of argument, the maximum real part of the eigenvalues is increasing with β_φ as shown in Figure 9.15(c) where now the parameter for credit rationing (α_{irb}) is 0.7. Put differently, financial instability is enhanced when the banking sector's financial health is affected by debt default. In such a case, a banking crises may unfold and require public intervention.

9.6.5 Bank bailouts and loss socialisation

Large defaults deteriorate the own funds of banks and generate a credit crunch. To avoid a systemic crisis and to relax credit rationing, the government can bail out banks. There are different ways to support banks. In this framework, banks transfer part of the non-performing loans to the government, so a proportion $\varphi_s \Lambda$ is socialised in this way. This transfer allows banks to sustain net equity and increases government debt, with $\varphi_s < \varphi_d$. Such a transfer mechanism requires an adjustment to the budget constraints of banks and government. The change in own funds of banks still decreases with debt default, but it is sustained by loss socialisation $\varphi_s \Lambda$. The share of default socialised in

this way appears positively in the government budget constraint. It slightly modifies the steady state for government debt, as well as for own funds of banks. The latter increase with loss socialisation, while at the same time government net wealth decreases.[19]

In extensive form the dynamics of the banks become:[20]

$$\dot{OF} = (1 - \alpha_{rb}) r_b p K - \varphi_d \Lambda + \varphi_s \Lambda, \tag{9.54}$$

$$\varphi_s = \beta_{\varphi s}(\varphi_b - \delta_\varphi) + \delta_{\varphi s}, \tag{9.55}$$

$$\dot{B}^s_{cb} + \dot{B}^s = \alpha_b\big(pG + iB^s + B^l + \varphi_s \Lambda - pT_c - pT_w - pT_f\big), \tag{9.56}$$

$$p_b \dot{B}^l = (1 - \alpha_b)\big(pG + iB^s + B^l + \varphi_s \Lambda - pT_c - pT_w - pT_f\big). \tag{9.57}$$

The resulting effect is ambivalent. On the one hand, own funds of banks are sustained and the economy is stabilised. Increasing $\beta_{\varphi s}$ from 0.1 to 0.5 increases loss socialisation. For a given default, the own funds of banks decrease less. Credit rationing is smoothed and the recession is limited. The business cycle fluctuations are narrower as we see in Figure 9.16(b). Meanwhile, the government finances this spending by issuing additional public bonds. If public debt increases too much, public debt starts accumulating and brings instability. Loss socialisation contributes to the solution of the private debt problem, but is likely to lead to a public debt crisis. This effect appears through the maximum part of eigenvalues that turns positive for a large socialisation of bad debt, as shown in Figure 9.16(c). It is actually well known that public intervention has a direct positive effect but may have indirect and unexpected perverse effects.

9.7 Simulations: extended studies

In this section we reproduce some of the findings of the previous small size models (in particular of Chapter 8), using wage-led and profit-led aggregate demand. It turns out that the finance-led regime is still destabilising through credit rationing and may lead to debt-deflation spirals, whereas fiscal and monetary policies stabilise the business cycle.

9.7.1 Wage-led aggregate demand

The wage share plays an important role in the simulation of Figure 9.17 as the first difference of output with respect to real wages ($\dot{y}/\dot{w}$) is equal to 0.1236. Taxes on

[19] Steady state for banks' own funds and public debt is modified as follows:

$$\begin{aligned}
of_0 = {} & \big((1-\alpha_{rb})i_{\lambda 0}\lambda_0 - \delta_\varphi + \delta_{\varphi_s} + (1-\alpha_{rb})i_0((1-\theta)(1-c)\alpha_m(b_0 + p_{b0}b^l_0) - \lambda_0)/A\big) \\
& /(\bar{\pi} + n - \delta_\varphi - (1-\alpha_{rb})i_0/A), \\
A = {} & 1 + (1-\theta)(1-c)\alpha_m, \\
b^s_0 = {} & \alpha_b\big(g + \delta_{\varphi s} - \tau_w \omega_0 y_0/x_0 - \tau_c i_0 \lambda_0 - \alpha_r r_{f0}\big)/(n + \bar{\pi} - \delta\varphi), \\
B^l_0 = {} & (1-\alpha_b)\big(g + \delta_{\varphi s} - \tau_w \omega_0 y_0/x_0 - \tau_c i_0 \lambda_0 - \alpha_r r_{f0}\big)/(n + \bar{\pi} - \delta\varphi) p_b.
\end{aligned}$$

[20] Note that Equation (9.54) is a modification of Equation (9.24), Equation (9.55) is new, and Equations (9.56) and (9.57) are modifications of Equations (9.43) and (9.44) respectively.

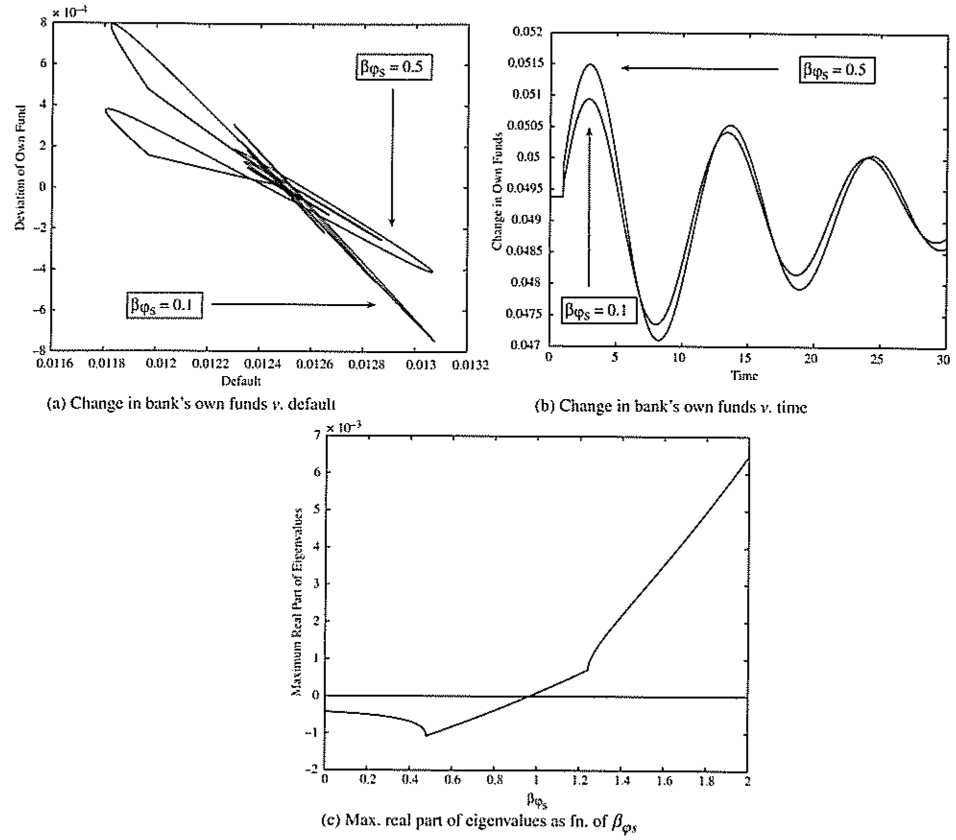


Figure 9.16 Bank bailout

wages amount (τ_w) 'only' to 50 per cent of real income, while investment strongly depends on the Harrodian multiplier with $\alpha_{iu} = 0.8$ and less on profitability since $\tau_f = 0.4$. Note also that the Fisher effect is kept relatively small at $\alpha_\lambda = 0.1$. The Rose effect is stabilising. Nominal wages are quite flexible ($\beta_w = 0.4$ and $\kappa_w = 0.4$) but prices are even more flexible ($\beta_p = 0.6$ and $\kappa_p = 0.75$). The Taylor rule is weak as the interest rate does not react much to the output gap or to inflation given that $\beta_{iu} = \beta_{ip} = 0.1$. Monetary authorities are more concerned with keeping the interest rate at its equilibrium value $\beta_{ii} = 0.4$. Public spending tends to be counter-cyclical as the debt target is almost zero ($\beta_{g1} = 0.001$) and the employment level is important ($\beta_{g2} = 0.4$). In this set-up, public intervention might be more effective through fiscal rather than monetary policy. The destabilising effects of long-term bond dynamics we kept small by making two assumptions: public spending is mainly financed through short-term debt ($\alpha_b = 0.85$), and there is a weak feedback effect between bond price and capital gain expectations ($\beta_{\pi_b^e} = 0.1$).

The outcome of a 5 per cent shock on output is a business cycle of approximately twelve years that converges towards the steady state at a somewhat slow speed. The

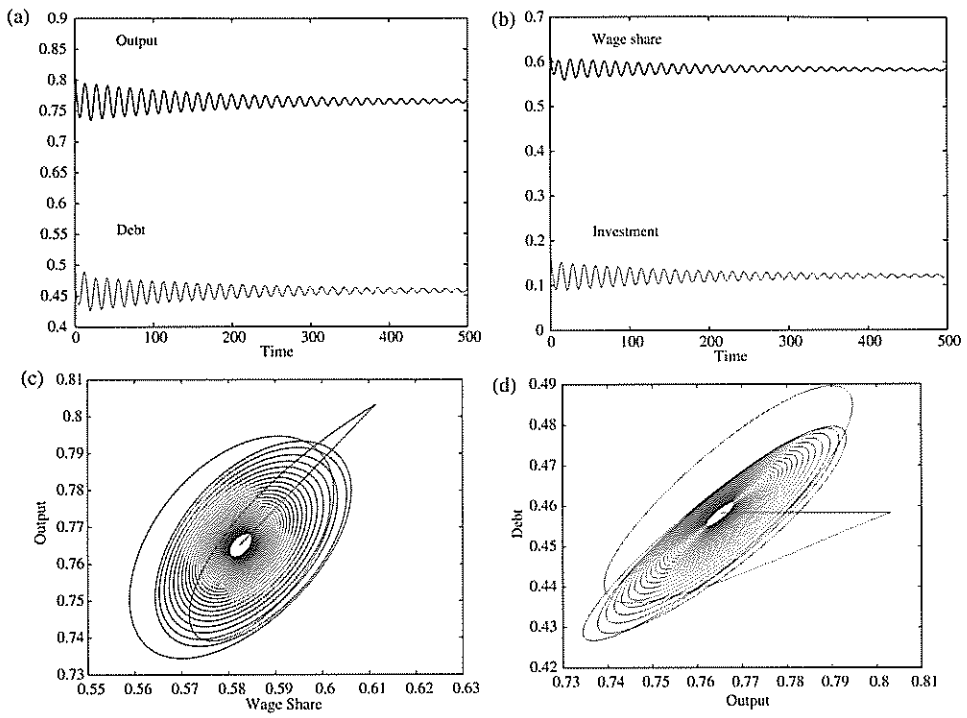


Figure 9.17 Business cycle – wage-led – stability

wage share is pro-cyclical, as is investment, due to its dependence on the output gap even if with a lag. As a result of high investment and high wage share in the upturn, indebtedness is also pro-cyclical. It entails that the Fisher effect is rather small when prices are slightly negative over the first cycles. The main stabilising element of the model is the wage-price dynamics that generate a counter-cyclical real wage. As prices react faster than nominal wages to the disequilibrium on the labour and goods markets, the real wage slows down in the upturn so smoothing aggregate demand and stabilising the economy. In line with theoretical intuition an increase in the price flexibility parameter β_p increases the stabilising influence of the Rose effect. Figure 9.18 displays all of the effects just discussed; note that β_p has been increased from 0.6 to 0.9. The economy converges much faster to its steady state compared with Figure 9.17.

Figure 9.19 shows the maximum real part of eigenvalue diagrams for the wage-price dynamics and confirms the properties of the wage-price dynamics with a wage-led aggregate demand. Fast adjustment of prices is stabilising as the maximum real part of the eigenvalues are negative for large value of β_p and κ_p. On the contrary, the maximum real part of the eigenvalues turns positive for a large coefficient of nominal wage β_w

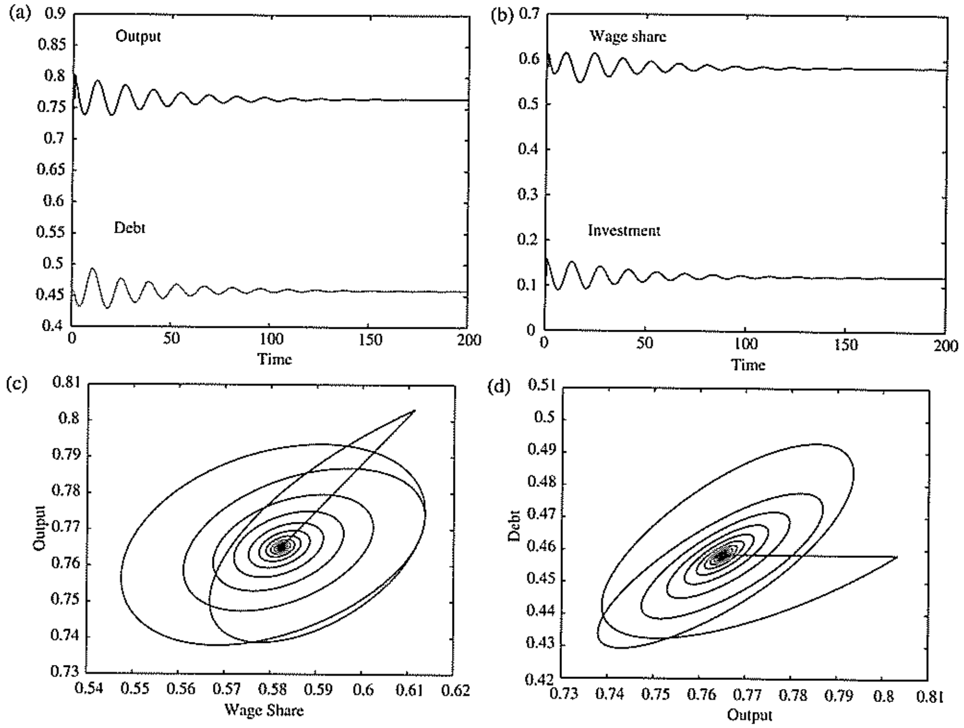


Figure 9.18 Business cycle – wage-led – stabilising higher price flexibility

and κ_w. More precisely, in a wage-led aggregate demand, the Rose effect is stabilising if the derivation of the change in real wage with respect to output is negative.

9.7.2 *Profit-led aggregate demand*

The mirror case of wage-led aggregate demand is profit-led aggregate demand where higher wages have a depressing effect on output. Increasing the coefficient for the sensitivity of investment to profit from 0.4 to 0.6 shifts the slope of $\frac{\dot{y}}{\omega}$ to -0.1234. Other parameters that do not affect the IS curve were also changed. The output gap sensitivity of investment α_{iu} is reduced from 0.8 to 0.4. In addition, given that aggregate demand is profit-led, nominal wages are made more flexible than prices ($\beta_p = 0.3$, $\beta_w = 0.6$, $\kappa_p = 0.7$ and $\kappa_w = 0.6$). In such a way, the real wage adjusts to dampen economic cycles.

The wage share has now an inverse relationship with the output while profit rate is positively related to output as shown in Figures 9.20(a) and (b). Decreasing β_p from 0.3 to 0.1 makes the Rose effect even more stabilising. As illustrated in Figures 9.20(c) and (d) the economy reaches the point of rest much faster. The sign of $\frac{\partial \dot{\omega}}{\partial y}$

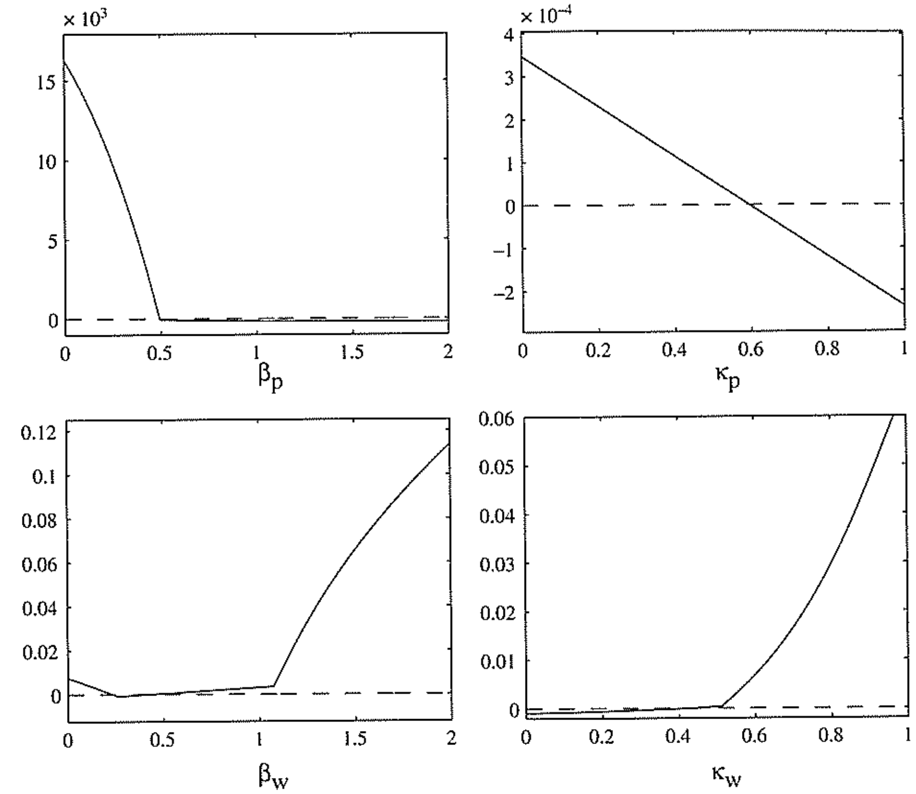


Figure 9.19 Maximum real parts of eigenvalues – wage-led – Rose effect

increases from 0.0346 to 0.0739, confirming that the Rose effect is stabilising in this case.

9.7.3 *Debt deflation*

In this subsection, we try to reproduce a debt-deflation spiral that is finance-led both in the cases of a wage-led and a profit-led aggregate demand set-up. It appears that debt deflation is more likely to occur in a profit-led demand regime as in this framework price flexibility has unambiguously destabilising effects contrary to wage-led aggregate demand. In the former case, price flexibility is destabilising in two ways, through the Rose and through the Fisher effect. In the latter case, price flexibility is stabilising through the Rose effect and destabilising through the Fisher effect.

Figure 9.21 illustrates the case of debt deflation. It is based on the parameters of Figure 9.17 except that the parameter $\alpha_{i\lambda}$ for the Fisher effect in the investment function is increased from 0.1 to 0.2 while the parameter α_{iu} for the Harrodian component of investment is decreased from 0.8 to 0.4. Due to the strong effect of $\alpha_{i\lambda}$ the speed of adjustment of price is slightly reduced from 0.6 to 0.5. If faster price adjustment

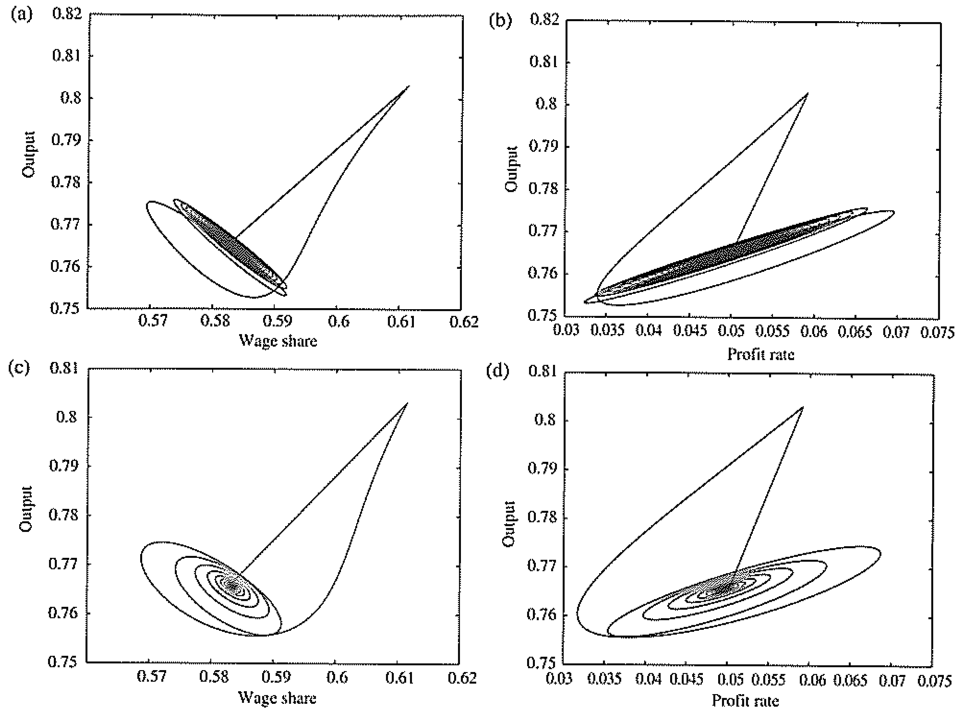


Figure 9.20 Business cycle – profit-led – stability

is stabilising through the Rose effect, then it is destabilising through the Fisher effect. Eventually, the interest rate is made more sensitive to the output gap by setting $\beta_{iu} = 0.2$ to counteract price deflation. As a result of the stronger Fisher effect, debt increases when price growth rate is negative. The cycle is exploding as illustrated in Figure 9.21(b). Figure 9.21(d) shows that the Fisher effect may be quite strong. For values of $\alpha_{i\lambda}$ greater than 0.2 the system loses stability. Contrary to Figures 9.19(a) and (d), Figures 9.21(b) and (c) show the maximum real part of the eigenvalues positive for large values of β_p and κ_p. Even if strong the Fisher effect may not necessarily lead to explosiveness. Price deflation may be contained within reasonable bounds as illustrated in Figures 9.21(e) and (f). A more stable Rose effect (which occurs when β_w and κ_w are reduced from 0.4 to 0.3 and 0.325 respectively) together with a 20 per cent shock to debt generates damped fluctuations where price deflation does not exceed 2 per cent.

Figure 9.22 illustrates the case of debt deflation in a situation of profit-led aggregate demand. The parameters are identical to Figure 9.20 except that the sensitivity of investment to indebtedness ($\alpha_{i\lambda}$) is increased from 0.1 to 0.115. This slight change generates debt deflation of a moderate magnitude. Price deflation does not exceed 1.5 per cent and fluctuations are converging slowly to the equilibrium. In Figures 9.22(c) and (d) the maximum real parts of eigenvalues with respect to β_p and

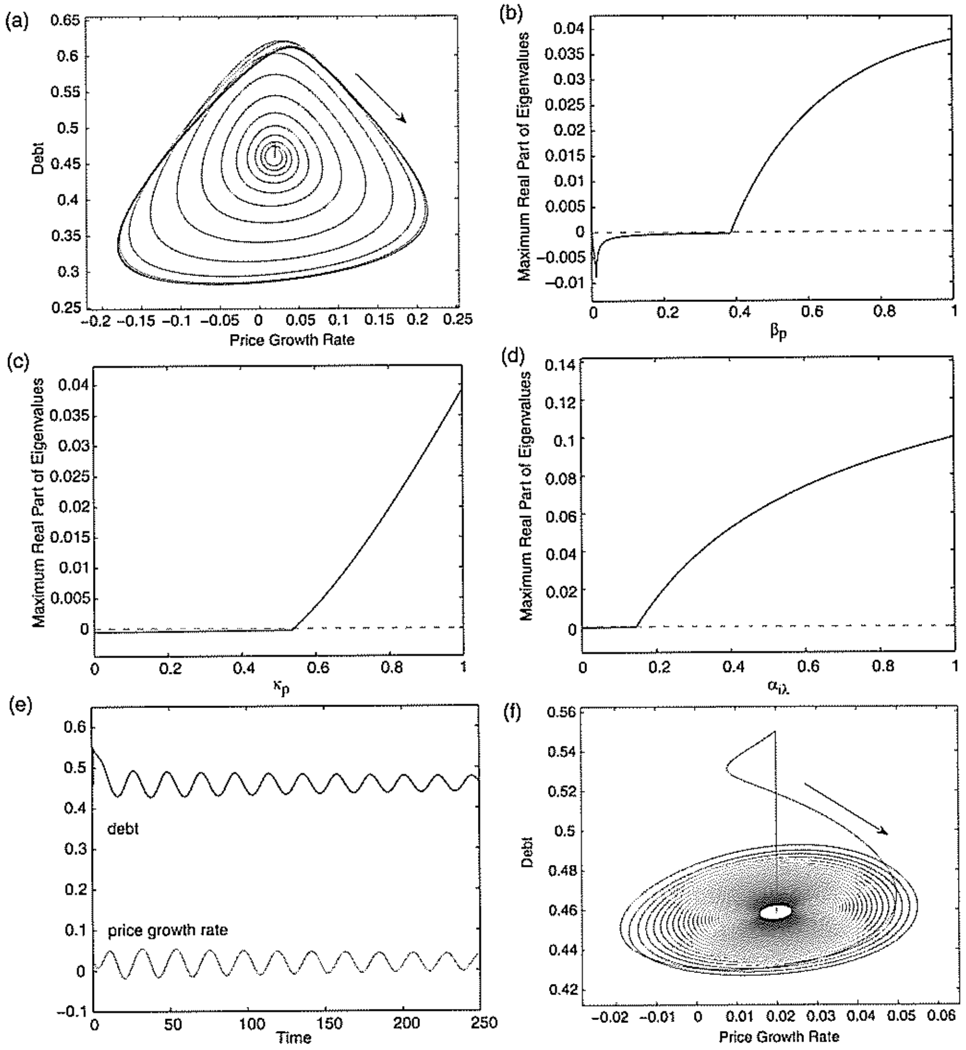


Figure 9.21 Debt deflation – wage-led AD

κ_p are always positive, underlining that price flexibility may very quickly become destabilising.

9.7.4 Interest rate policy rules

The conclusion of the first part of the model (Sections 9.7.1, 9.7.2 and 9.7.3) is that the private sector is prone to instability when the economy is finance-led and when nominal wages and prices are free to adjust to disequilibrium on the goods and labour markets. Public interventions might therefore play this stabilising role through monetary and

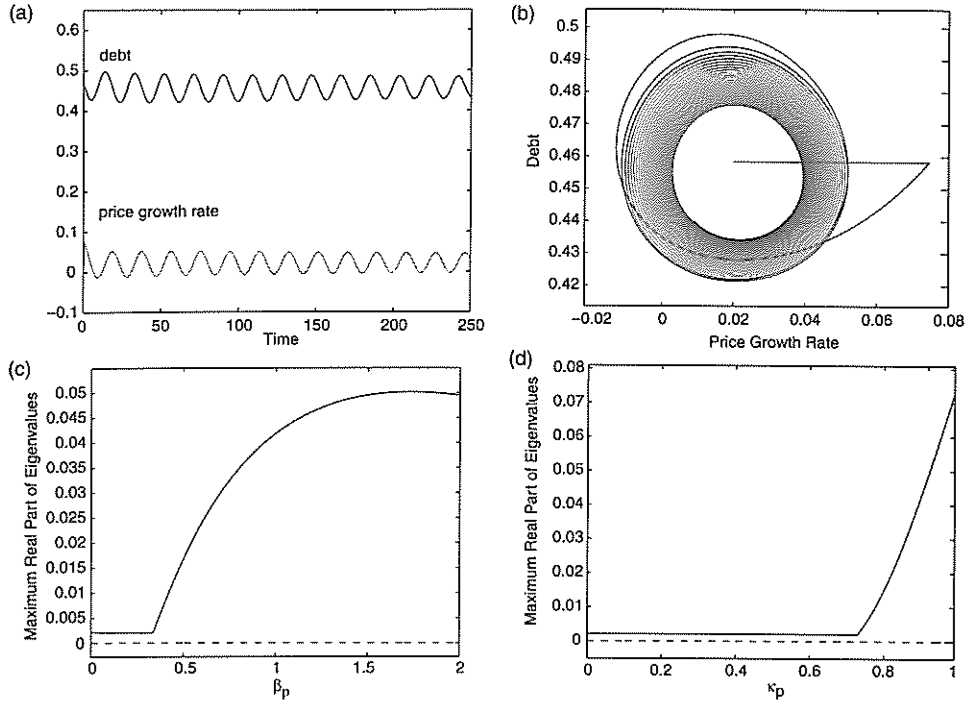


Figure 9.22 Debt deflation – profit-led AD

fiscal policy. Figure 9.23 assesses the effect of monetary policy on the stability of the model based on the parameters of the coefficients of Figure 9.17. Increasing the speed of adjustment of the interest rate with respect to the output gap, β_{iu}, from 0.1 to 0.4 has a positive effect. The economy converges much faster. Eigenvalue diagrams give additional evidence that monetary policy may be beneficial. It tells in addition that the speed of convergence of the interest to its steady state level β_{ii} must not be too large, here greater than 0.8. The reaction to the output gap, β_{iu}, must be kept within a certain corridor. This corridor is quite large in the present case, and is stabilising for values between 0.05 and 1.6. Eventually, the sensitivity of the interest rate to inflation is stabilising if the speed of adjustment is greater than 0.05. This efficiency of the interest rate policy might be linked with the strong destabilising forces generated by the output, price and debt dynamics through the Fisher effect. Chiarella *et al.* (2003a) found slightly different results. In a model without debt they argue that the Taylor rule is stabilising for speed of adjustments that belongs to a small corridor. Faster and slower speed of adjustment brings instability, indicating that the monetary authorities should be neither be too quick nor too slow to react.

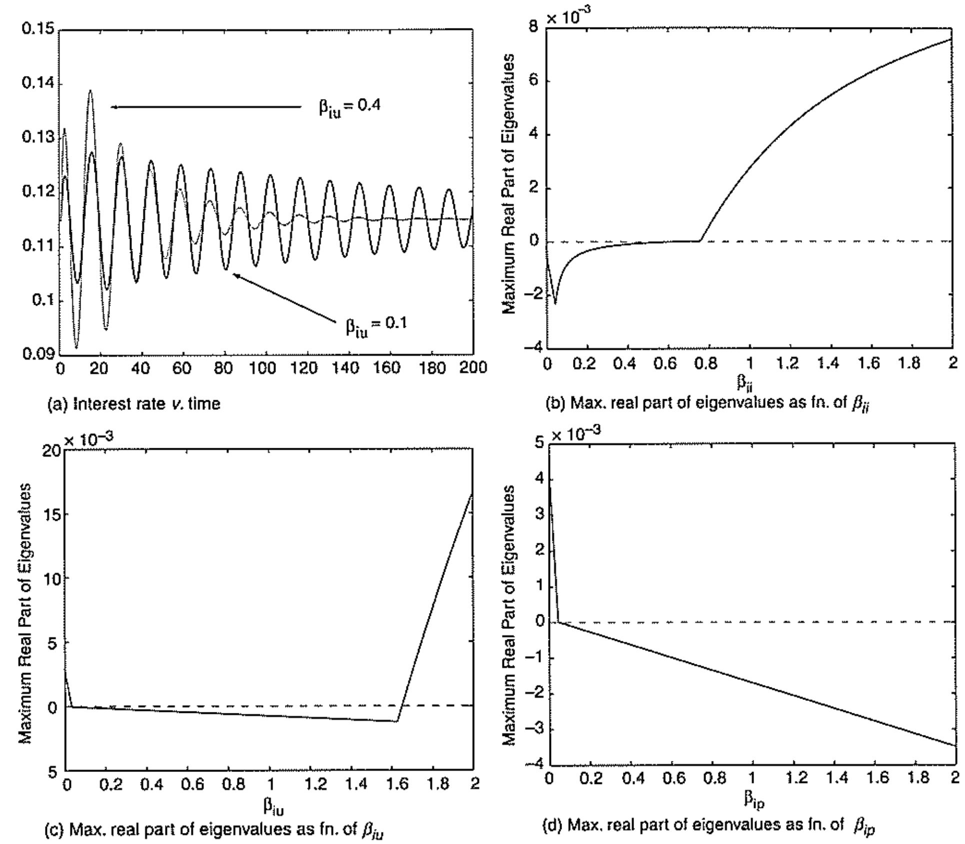


Figure 9.23 Taylor rule

9.7.5 Fiscal policy

Figure 9.24 illustrates the effects of the fiscal policy on the stability of the model. Parameters are very similar to the coefficients of Figure 9.17. Few changes are applied: α_{iu} is decreased to 0.6, β_p is increased to 0.9 while the interest rate reacts slightly faster to inflation and to the output gap ($\beta_{ip} = \beta_{iu} = 0.2$). With the speed of adjustment of public spending to employment, β_{ge}, equal to 0.4, the economy fluctuates on a fifteen- to seventeen-year basis and converges towards the equilibrium. Increasing β_{ge} to 0.5 has a stabilising effect on the business cycle which converges faster. The maximum real parts of eigenvalues confirms the stabilising effects of a counter-cyclical fiscal policy. Eigenvalues are all negative between 0.2 and 1.4. Very slow or very fast speeds of adjustment are not stabilising; on the contrary, the economy loses stability when public spending reacts to the level of public debt. Eigenvalues are all positive for any speed of reaction. This illustration of some kind of Maastricht criteria underlines the destabilising effect of pro-cyclical policy when aggregate demand is fully taken into account.

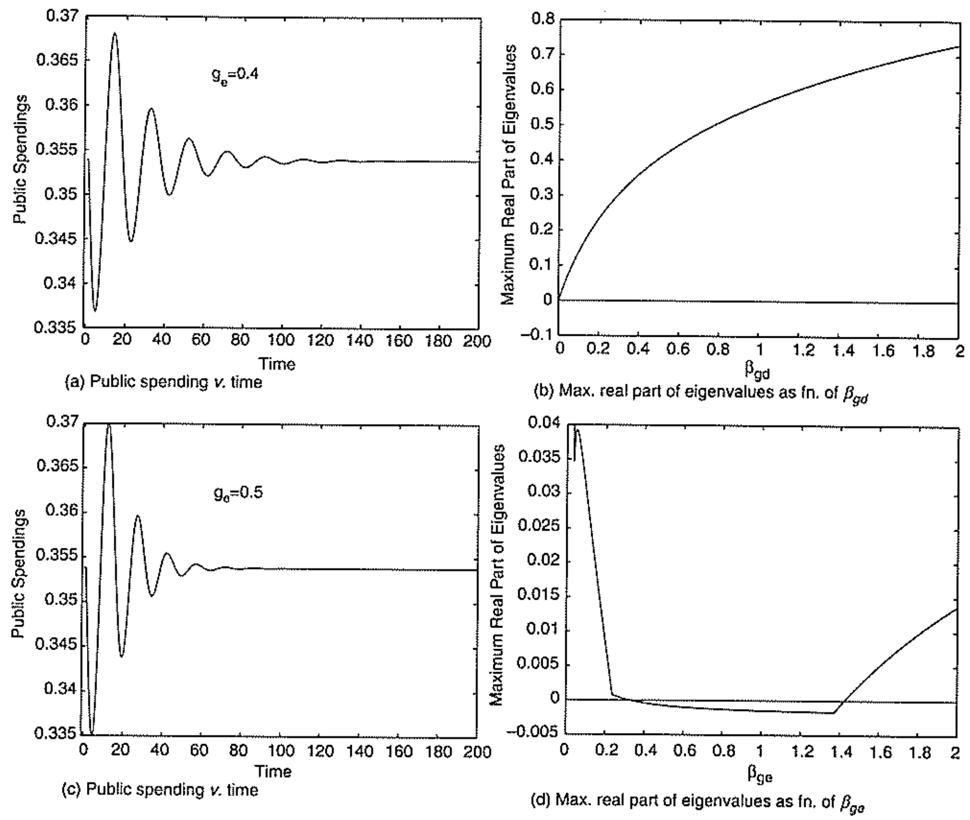


Figure 9.24 Fiscal policy

9.8 Conclusions

In the present period of financial instability, the fragility of banks has received increasing attention. The behaviour of banks is at the heart of the credit relationship. Bank performance alters the quantity of credit supplied and influences the accumulation of debt. In addition, there is a growing concern that debt defaults disrupt the credit relationship through their effect on the financial health of banks. Banking fragility from the asset side becomes very likely to the extent that over-indebtedness brings large scale debt defaults. In the case of a large credit loss, the government plays a central role in stabilising financial institutions. To avoid systemic risk, the socialisation of losses is necessary. The transfer of loss to public entities is a key instrument in the restoration of normal credit supply. A recession, in contrast, may be far worse.

In this section, we have tried to model some of the main mechanisms associated with the financial situation of banks and credit behaviour. There are three main aspects. First, it was shown that banks' profits are pro-cyclical and that their profits push up credit supply. In this sense, the behaviour of banks can cause economic oscillations to exhibit larger amplitude. This aspect also highlights an additional channel to the

financial accelerator. Second, the over-indebtedness of borrowers reacts on the financial statements of banks and is an additional source of credit instability. By reducing banks' net equity in the downturn, credit disruption can worsen the recession. When financial institutions are brought into the picture, debt defaults become destabilising. This effect alters the conclusion of the preceding model. Debt defaults and bankruptcies are stabilising if they do not too much affect the financial system. These mechanisms are potentially strong and raise the question of the attitude of government to banks' financial distress. Third, bank bailouts by public authorities contribute to the restoration of the credit relationship. In line with the literature on banking crisis management, socialisation of losses is necessary when bank losses become very large. Meanwhile, the substitution of private debt by public debt carries the risk that a private debt crisis is transferred into a public sector one. It is in fact likely that loss socialisation triggers a self-accumulation of public debt.

10 Japan's institutional configuration and its financial crisis

In 1990, very few economists predicted that the stock market crash in Tokyo would trigger more than a decade of economic recession and stagnation. Unlike most developed economies, Japan had remained a dynamic economy over the 1970s and 1980s, experiencing neither stagflation nor rising unemployment even though economic growth had slowed. From 1973 to 1990, Japan grew at a rate of 3.9 per cent compared with the average of 2.5 per cent in Organization for Economic Co-operation and Development (OECD) countries and maintained close to full employment, with a 2 per cent unemployment rate.

The Japanese institutional setting[1] at the heart of this sustained period of growth was based on: i) a wages policy based on life employment and progressive income to ensure the support and the involvement of employees in the achievement of competitiveness; ii) an accommodating financial system that adjusted its profitability objectives to firms' performances and that formed tight links with borrowers; iii) governmental coordination of private sector strategies and expectations (Boyer, 2004).

Nevertheless, the 1980s were characterised by a growing disequilibrium linked to credit expansion and financial bubbles. The belief that Japanese organisations were able to dominate key industries led to a large expansion of credit. In reaction to these profit perspectives, financial assets, in particular real estate, attracted new investors and led to a large asset price inflation. The evolution of two factors modified the Japanese institutional setting. First, the increasing pressure on employees made their commitment to firm objectives ever more difficult. Second, the liberalisation of financial markets destabilised the main banking system and made the coordination of agents by public entities more hazardous. Increasing competition among banks led to a larger supply of credit and as a consequence to new sets of activities, including asset speculation, with far less monitoring.

With the burst of the real estate bubble, the large debt level pushed many borrowers into insolvency. The resulting non-performing loans degraded the health of financial institutions and prompted a deep and long lasting banking crisis (Hoshi and Kashyap, 1999; Calomiris and Mason, 2003). The length of the stagnation was subject to many

1 See the definition of institutional economics in footnote 3 in Chapter 9.

interpretations (Wilson, 2000). Some stressed the weakening of aggregate demand resulting from lower investment and consumption. Others highlighted the macroeconomic mistakes of monetary and fiscal authorities (Kuttner and Posen, 2001). The liquidity trap, the asset bubble and price deflation, and the recession of 1998 have been interpreted as resulting from monetary mistakes (Bernanke, 2000; Nakaso, 2001), inadequate tax cuts and fiscal stimulus. Neo-institutionalists argued that the weak enforcement of bankruptcy laws led to the slow resolution of non-performing loans (Andrieu-Lacu, 2006), and that procedures put in place to resolve the banking crisis in fact delayed its management and prevented speedier reaction to it.

The main elements of the Japanese crisis consisted of debt and price dynamics, the effect of default on the banking sector, as well as the reaction of monetary and fiscal authorities. The theoretical models developed in Part III of this book provide a good framework to interpret the Japanese crisis as they combine these different elements and allow assessment of their respective effects.

In this chapter, we examine the three main elements that shape the behaviour of the private sector and the effect of their interaction in terms of stability. We first use estimation techniques to explore whether aggregate demand in Japan has been wage- or profit-led. While household consumption has been a key pillar of accumulation in Japan, firms' profitability has also been very important, and it is not clear which effect has dominated.

We then ask whether the real wage was labour market- or goods market-led. Again, the Japanese case is complex. Although mass lay-offs are not a key adjustment variable in Japan, the particularities of Japanese wage bargaining, and increasing job flexibility in the 1990s do not necessarily imply a relatively more rigid nominal wage.

Finally, we examine whether there has been a financial accelerator effect. While the main banking system involves close links between firms and banks, the progressive financial market liberalisation may have altered these relationships and favoured credit rationing.

Further, we discuss whether the private sector was shaped by unstable forces that may explain the crisis of the 1990s. The absence of institutional complementarity[2] may be an explanation for the Japanese crisis. We then assess what has been the overall effect of government intervention. As some have argued that the length of the crisis can be largely explained by policy mistakes, it is of interest to ask whether the speed of adjustment of the central bank interest rate falls within our theoretical stability corridors. In particular, it is challenging to ask why debt deflation was kept within a limited range and did not give rise to an increasing debt level. There are two possible answers. First, private sector instability was moderate which implied moderate debt deflation. Second, public intervention limited the extent of price deflation.

2 A main result of institutional economics is actually to show that there is no single model of capitalist economies, but that different capitalist regimes are likely to coexist. To describe this variety of organisation, institutional economics uses the concept of institutional complementarity. Institutional complementarity can be defined following two principles. A first one is a Pareto-efficiency criterion – institutions must be shaped such that all economic agents are better off. A second definition considers that a set of institutions is complementary, if it contains destabilising trajectories but maintains business cycles within viable magnitudes.

Given the lack of data for non-performing loans, we estimate the parameters of the following 5D model, which has equations for real wage, debt, output, the active population to capital ratio and a Taylor interest rate rule:

$$\dot{\omega} = \omega\kappa\big((1-k_p)\beta_w(e-\bar{e}) + (k_w-1)\beta_p(u-\bar{u})\big), \tag{10.1}$$

$$\dot{\lambda} = g_k - \delta - (1-\tau_f)r - (\hat{p} + g_k - \delta)\lambda, \tag{10.2}$$

$$\dot{y} = \beta_y(y^d - y) + (n + \delta_k - g_k)y, \tag{10.3}$$

$$\dot{l}^e = l^e\big(n - (g_k - \delta_k)\big), \tag{10.4}$$

$$\dot{i} = \beta_{ii}(i_0 - i) + \beta_{ip}(\hat{p} - \bar{\pi}) + \beta_{iu}(u - \bar{u}). \tag{10.5}$$

While small size models could be estimated from Japanese data, the lack of quarterly data concerning non-performing loans and the management of the banking crisis prevent such an exercise. We are nevertheless able to discuss two main aspects. First, as noted in the previous model, debt default is a possible way to resolve the debt problem. Nevertheless, its effect largely depends on whether it results from the bankruptcy of firms and how the real sector behaves. Second, debt default has potentially stabilising effects if it does not affect the banking sector on a large scale. The deep banking crisis in Japan suggests the opposite. Nevertheless, the various government interventions to tackle this issue potentially offset these deleterious effects.

10.1 A stable profit-led real sector

In this section, we estimate a version of the model restricted to the real side variables: namely, wages, prices, output and employment. One of the aims is to understand whether instability is purely related to real mechanisms. Price deflation might well result from real effect. In such a case, the Japanese financial disruptions may just be the result of a real crisis but not the triggering factor. An important source of instability in the real sector could be a lack of complementarity between income distribution and the determination of real wages. In fact, a profit-led economy with rigid nominal wages is prone to crisis.

The system of equations (10.1)–(10.5) is reformulated in discrete time in order to be estimated for the Japanese case. First, the active population to capital ratio is a definition and therefore cannot be estimated. Second, we decompose the real wage equation into two separate equations, for nominal wages and prices respectively. Third, the impact of firms' debt to capital ratio on capacity u is not considered here to abstract from financial variables. This leaves us with the following system of four equations:[3]

$$\hat{w}_t = \beta_{we}(e_{t-1} - e_0) + k_{wp}\hat{p}_t, \tag{10.6}$$

$$\hat{p}_t = \beta_{pu}(u_{t-1} - u_0) + k_{pw}\hat{w}_t, \tag{10.7}$$

[3] See Flaschel *et al.* (2009) for full details of how the system (10.6)–(10.9) is derived.

$$\hat{u}_t = \alpha_u(\hat{u}_{t-1} - \hat{u}_0) + \alpha_v(\hat{v}_{t-1} - \hat{v}_0), \tag{10.8}$$

$$\hat{e}_t = \alpha_{eu1}\hat{u}_{t-1} + \alpha_{eu2}\hat{u}_{t-2} + \alpha_{eu3}\hat{u}_{t-3}. \tag{10.9}$$

We rely on Generalized Method of Moments (GMM) type estimations for the coefficients. It is well known that GMM is suited for dealing with the issue of endogenous variables. As instruments, we use the past value of the explanatory variables up to three lags for production capacities. We make use of quarterly data over the period 1980–2004; a period that comprises the boom and bust aspect of the Japanese crisis. Data sources are described in the Appendix in Section 10.8. Of interest is the information published by the Ministry of Finance (MOF) related to firms' financial statements. This enables us to test directly for the effect of financial variables on economic activity.

In Section 10.2 production capacities are illustrated in Figure 10.1(a).[4,5] The 'boom and bust' aspect of the Japanese economy appears clearly. The 1980s are characterised by an increasing trend in economic activity and wider oscillations. Production capacities peak at 1.15 in 1991. The 1990s was a period of lower production capacity tending to follow a downward trend. From 1991 to 1993, it dropped to 0.95 and reached its lowest point in 2002 at 0.87. The recovery between 1993 and 1997 also appears clearly, as production capacities are restored to 1.07 in 1997 before the second depression of the 1990s starts in 1998. Production capacities drop to 0.87 in 2002 before the economy starts to recover and most financial disruptions end.[6]

The employment rate is closely linked with the business cycle (Figure 10.1(b)). The 1980s are characterised by near full employment. Employment deteriorates until 1987 to reach 97 per cent but it is only a 1 per cent drop over seven years. And the boom of the late 1980s raises employment back to its pre-1980s level. With the deterioration of economic activity, employment worsens. It decreases at first very slowly but it then decreases steadily. The fall starts in 1992 only and unemployment reaches 5.5 per cent in 2002. The economic rebound over the mid-1990s hardly appears in the data. The rapid deterioration of employment over the second half of the 1990s might be the result of the progressive deregulation of the labour market and the rise of unconventional employment.[7]

Wage change[8] follows the level of employment but has a clear negative trend (Figure 10.1(c)). Over the 1980s, periods of economic expansion generated accelerating wage inflation, e.g. between 1987 and 1991, but inflation is still cooling down over that period. The economic crisis increased (the rate of) wage disinflation. Wage inflation slowed three times over the 1990s. At the beginning of the crisis it drops from 0.04 to 0.02; during the years 1997–8 it drops from 0.02 to just below 0; and in 2000–1

[4] They are normalised to 1 in 2,000, which explains why they are above 1 for the period of the 1980s.
[5] The data sources used for the figures and tables of this chapter are listed in the Appendix in Section 10.8.
[6] The problems of the banking system are almost all resolved at this date.
[7] See Lechevalier (2003) for a detailed presentation of the debate about the changing nature of the Japanese labour market.
[8] The graph represents the variation of the wage index in logarithm.

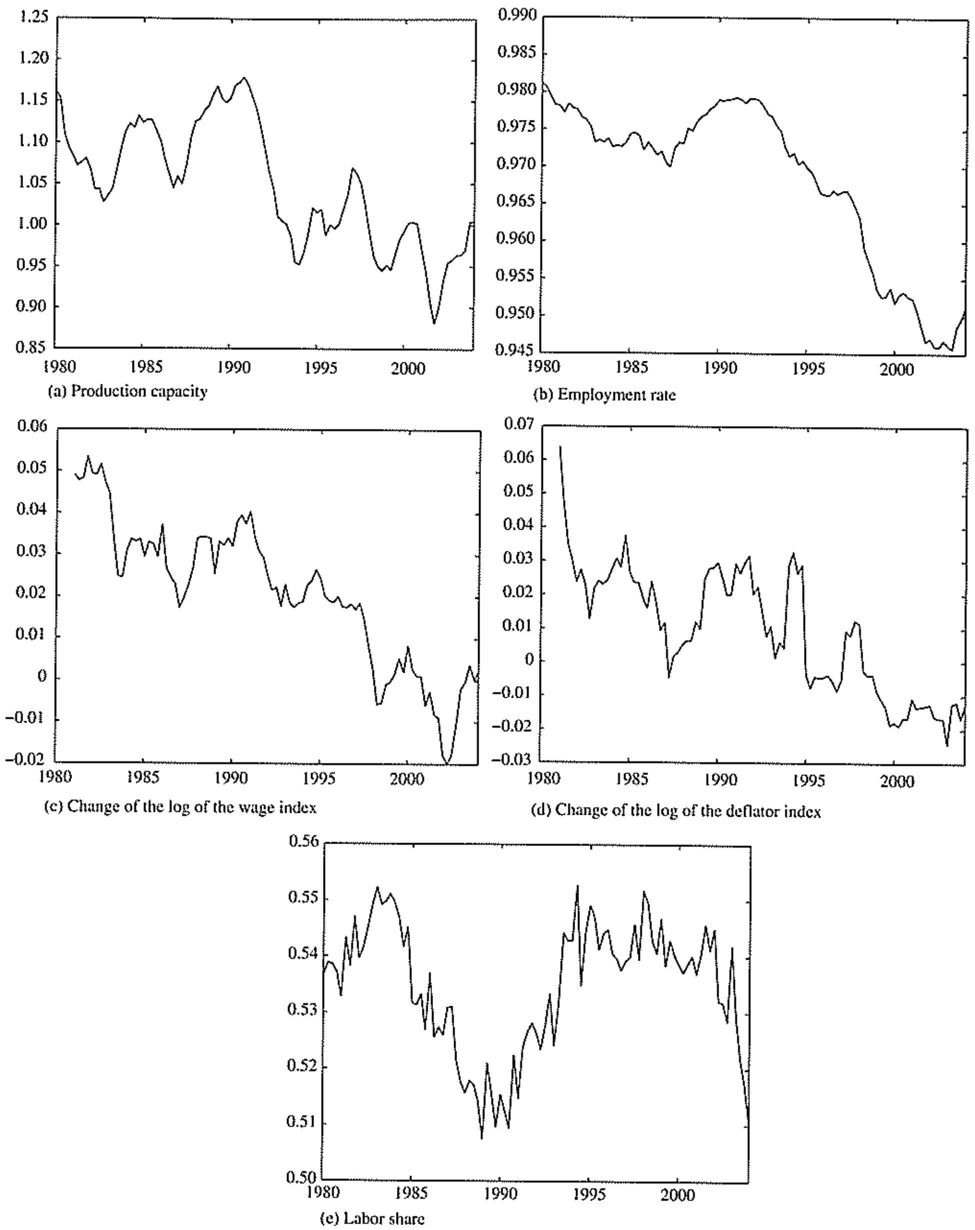


Figure 10.1 Japan – the main economic indicators

it drops from 0 to –0.02. In 1997 and 2001, wage inflation turns negative and wages decrease. It is surprising that nominal wages drop: this has been a rare occurrence in developed economies since the Second World War.

Similarly to wages, prices are also pro-cyclical. The change of the GDP deflator follows a downward trend, which underlines the strong disinflationary forces at work over that period. Inflation accelerates in the late 1980s up to 0.04 in 1991, before

Table 10.1. *Phillips–Perron unit test results for labour shave data*

Variable	Adj. Test Stat	Prob	Variable	Adj. Test Stat	Prob
u	–3.153	0.10	$d(u)$	–5.187	0.00
e	–1.330	0.87	$d(e)$	–7.663	0.00
$\ln(w)$	–0.123	0.99	$d\ln(w)$	–20.063	0.00
$\ln(p)$	0.371	0.99	$d\ln(p)$	– 9.696	0.00
v	–1.830	0.68	$d(v)$	–14.908	0.00

decreasing quickly in the 1990s. Price inflation is zero over the period 1994–6, before turning negative from 1998 onward (Figure 10.1(d)). Compared to the change in wages, disinflation seems to be much more gradual.[9] There is no abrupt decrease of inflation. On the contrary there are two peaks, in 1994 and in 1998.

The labour share appears in Figure 10.1(e). It fluctuates at around 53 per cent of GDP. It drops during the late 1980s and early 1990s from 55 per cent in 1984 to 51 per cent in 1990. The labour share then recovers in the first phase of the crisis, up to 55 per cent in 1994. From 1995, it follows a decreasing trend especially in the late 1990s. The labour share seems to be negatively linked with the business cycle. It actually decreases in the late 1980s during the boom years, while it increases in the early 1990s when the crisis starts. The same trend prevails after the early 1990s. The labour share decreases over the 1994–7 rebound, while it increases at the beginning of the deflation.

Given that we deal with times series, we must check for stationarity to avoid any problem related to spurious regressions. We carry out Phillips–Perron unit root tests for each series in order to account, not only for residual autocorrelation as is done by the standard Augmented Dickey-Fuller (ADF) tests, but also for possible residual heteroscedasticity when testing for stationarity. The Phillips – Perron test specifications and results are shown in Table 10.1. As often with linear economic series, variables are non-stationary. Nevertheless, taking the first difference is sufficient to provide stationarity.[10] An alternative possibility to make the series stationary is to divide each series by the stock of capital. Although the outcome is very much in line with our theoretical models, which express variables over the stock of capital in the reduced form equations, questions arise as a result of the implicit assumption that such a procedure entails – in particular dividing each series by the capital stock assumes that each series can be explained by the stock of capital with a unit root coefficient. In such a case, cointegration procedures are probably more suited than short-term estimation techniques such as GMM employed below. To correct for seasonality, we applied either the X12 procedure or we integrated quarterly dummies in the regressions.

The main results of this first estimation are given in Table 10.2 and indicate that wages are more flexible than prices in Japan.[11] Wages are more sensitive to the employment

9 The graph represents the variation of the deflator index in logarithms.
10 We write $d()$ to indicate first differences of the variable in the brackets.
11 The number of stars gives the level of significance of the coefficient: *** is 1 percent, ** is 5 percent and * is 10 percent.

Table 10.2. *Estimations results: the real model*

$\hat{w}$	β_{we}	κ_{wp}	*cst.*			R^2	DW
	0.363***	0.1	0.008***			0.79	1.77
$\hat{p}$	β_{pu}	κ_{pw}	*cst.*			R^2	DW
	0.024**	0.459**	−0.001			0.32	2.17
$\hat{u}$	α_u	α_v	*cst.*			R^2	DW
	−0.067***	−0.276**	−0.002			0.94	0.84
$\hat{e}$	α_{eu1}	α_{eu2}	α_{eu3}	α_{eu4}	*cst.*	R^2	DW
	0.015***	−0.025**	0.029	−0.020	−0.003**	0.20	1.86

rate $\beta_{we} = 0.363$ than prices to capacity production $\beta_{pu} = 0.024$. Nevertheless, at this stage wages do not take into account inflation as κ_{wp} is non-significant while prices are very much sensitive to inflation costs $\kappa_{pw} = 0.459$. Put differently, nominal wages are largely determined by demand pressure while prices are largely determined by cost-push elements. Nonetheless, wages in Japan are more flexible than prices for the period 1980–2004. The Harrodian accelerator is negative and small, $\alpha_u = -0.067$, which implies stability of the output on itself. Finally, the labour share appears with a negative sign $\alpha_v = -0.267$ and is significant at 5 per cent. This suggests that the economy is profit-led and confirms the graphical analysis. Overall, the real sector is stable as aggregate demand is profit-led and the real wage is labour market-led. The real sector can be represented by a 2D model for output and wage dynamics (equations (10.1) and (10.3)) that we reproduce here:

$$\dot{\omega} = \omega\kappa\big((1-k_p)\beta_w(e-\bar{e}) - (1-k_w)\beta_p(u-\bar{u})\big),$$
$$\dot{y} = \beta_y(y^d - y) + (n - (g_k - \delta))y.$$

The estimation gives us the following signs of the Jacobian matrix, which imply stability:

$$J = \begin{vmatrix} - & - \\ + & 0 \end{vmatrix}$$

10.2 Pro-cyclical financial markets

A key aspect of our theoretical work is to integrate the effect of financial variables on the output dynamics. In particular, we make use of a measure of the net wealth of firms. Given that the theoretical model uniformly points to the destabilising effect of firms' net wealth on investment, any significant empirical evidence should identify a destabilising mechanism. Figure 10.2 displays a measure of firms' wealth. It is made up of assets minus liabilities divided by fixed assets. Data are taken from the MOF which publishes quarterly data on firms' financial statements. Firms' net wealth has a clear 'boom and bust' aspect over the 1980s and early 1990s. It increases continuously over the 1980s to reach 1.03 in 1990. It then drops to 0.88 in 1993 before rebounding to 0.97 in 1997.

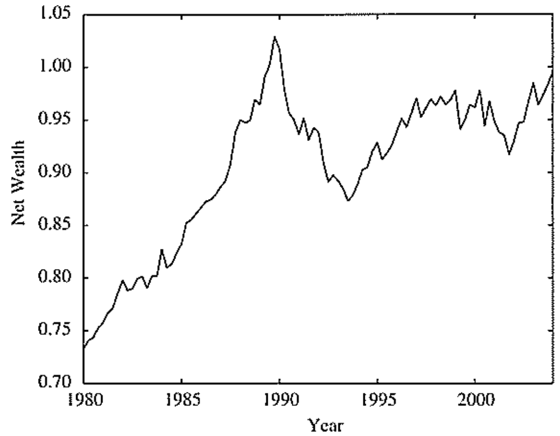


Figure 10.2 Japan - an indicator of firms' wealth (assets minus liabilities divided by fixed assets)

Nevertheless, the deflation of the late 1990s does not appear very clearly. Estimating the effect of banks' net wealth on investment requires inclusion of a measure of firms' assets and liabilities in the equation (10.8) for capacity u. Furthermore, firms' budget constraints are not directly estimated, as it is an accounting identity. The new IS curve to be estimated is now

$$\hat{u}_t = \alpha_u(u_{t-1} - u_0) + \alpha_v(v_{t-1} - v_0) + \alpha_{nw}(\lambda_{t-1} - \lambda_0). \qquad (10.10)$$

Even if normalised by the stock of capital, firms' net wealth is non-stationary and would require differencing to become stationary (Table 10.3). Nevertheless, given that firms' net wealth enters capacity production as a function of the discrepancy between its value and its steady state value,[12] the resulting variable is very likely to be stationary. Concerning the real parameters, the estimated coefficients (see Table 10.4) are mostly the same as previously found. The real wage is still labour market-led, β_{we} increased slightly from 0.363 to 0.409 and κ_{wp} is still not significant. The sensitivity of prices to the output gap is slightly larger too, at 0.039 compared with 0.024 previously. The major change concerns the sensitivity of prices to wage costs which dropped from 0.459 to 0.15. The output gap is still negatively correlated to itself with $\alpha_u = -0.082$ and aggregate demand is still profit-led as the labour share is negatively correlated to output, $\alpha_v = -0.201$. The parameter for firms' net wealth is positive and significant, $\alpha_{nw} = 0.065$. There is evidence that firms' balance sheets matter for the output. Although the parameter may seem rather small, the theoretical models underline that the economy may lose stability for a very small value of α_{nw}. Given that the effect of firms' net wealth is unambiguously destabilising in the model, the firms' financial accelerator has contributed to instability.

[12] The average of firms' steady state over the sample.

Table 10.3. *Phillips–Perron unit test results for interest rate data*

Variable	Adj. Test Stat	Prob	Variable	Adj. Test Stat	Prob
NW	−2.068	0.55	NW	−9.895	0.00

Table 10.4. *Estimations results: with credit rationing*

$\hat{w}$	β_{we}	κ_{wp}	*cst.*			R^2	DW
	0.409***	0.20	0.009***			0.760	1.76
$\hat{p}$	β_{pu}	κ_{pw}	*cst.*			R^2	DW
	0.039***	0.15*	0.001			0.31	2.12
$\hat{u}$	α_u	α_v	α_{nw}	*cst.*		R^2	DW
	−0.082***	−0.201*	0.065**	0.001		0.94	0.83
$\hat{e}$	α_{eu1}	α_{eu2}	α_{eu3}	α_{eu4}	cst.	R^2	DW
	0.022***	−0.023***	0.014***	−0.009**	−0.003***	0.237	1.94

10.3 Less than optimal fiscal and monetary policies

The Japanese government used a vast array of macroeconomic tools to try to lift Japan out of recession over the 1990s. The effectiveness of such tools has often been questioned. Public spending was considered inefficient, politically motivated and slowly implemented while monetary policy was considered inadequate, especially during the liquidity trap of the late 1990s. Our theoretical model underlines the fact that for fiscal and monetary policy, public intervention is efficient only to the extent that it is implemented at the right speed. A reaction that is either too slow or too rapid adds to the disequilibrium. We first estimate a Taylor rule and then discuss the various fiscal packages implemented by different governments in an attempt to stimulate the economy.

The question of monetary policy is a difficult question in the Japanese case. Although economists usually refer to the Taylor rule to assess whether monetary policy is too loose or too tight, the question arises whether this is still the required framework for the Japanese case. In the late 1990s, Japan's interest rate reached a zero bound and the central bank was not able to reduce the interest rate further to fight price deflation. Some have argued that evaluating monetary policy by use of a Taylor rule in such a case could give a negative interest rate. Others have focused on the alternative tools or channels that monetary policy should mobilise to fight deflation, such as the exchange rate (Eggertsson and Woodford, 2003, 2004, for instance).

For the estimation exercise, monetary policy is best illustrated by the call rate in Figure 10.3. The central bank interest rate decreased continuously until the late 1980s, and may have contributed to real estate speculation. Conversely, the interest rate increases in the late 1980s probably contributed to triggering the crisis. The interest rate more than doubled from 3.5 per cent to 8 per cent between 1987 and 1991.

Table 10.5. *Phillips–Perron unit root test results on the interest rate*

Variable	Adj. Test Stat	Prob
i	−3.611	0.007

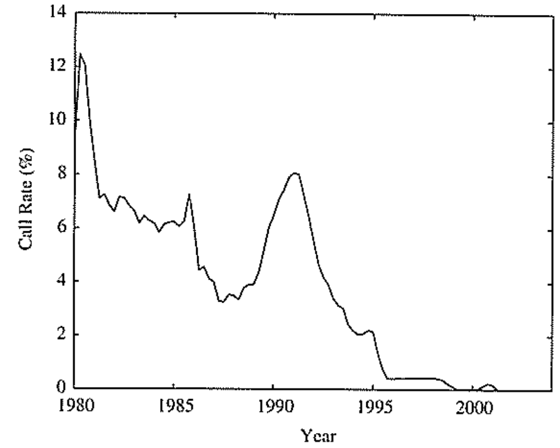


Figure 10.3 The call rate in Japan: 1980–2004

Once the crisis erupted, monetary authorities continuously decreased the interest rate to support the economy. Nonetheless, from 1999 monetary policy faced the zero bound interest rate and the so-called liquidity trap.

The unit roots test performed on the interest rate data shows that the series is stationary and does not need to be differenced (see Table 10.5). Introducing the interest rate estimation into the system does not modify the main results found previously. We have estimated the interest rate rule Equation (10.5) in the form

$$i_t = \beta_{ii} i_{t-1} + \beta_{ip} \hat{p}_t + \beta_{iu}(u_{t-1} - u_0). \qquad (10.11)$$

The speeds of adjustment of nominal wages are still larger than prices, even though nominal wage sensitivity to employment is smaller at $\beta_{we} = 0.275$ (see Table 10.6).[13] The cost-push element in nominal wages is now positive and significant at $\kappa_{wp} = 0.207$ (again see Table 10.6), while it was not significant in the previous estimations. Price flexibility to the output gap is still relatively small $\beta_{pu} = 0.018$ while the cost-push element is large, being $\kappa_{pw} = 0.654$. Aggregate demand is still profit-led $\alpha_{uv} = -0.135$ which is consistent with the real wage. The sensitivity of output to itself and the effect

[13] Table 10.6 comes from the estimation of equations (10.6), (10.7), (10.9), (10.10) and (10.11).

Table 10.6. *Estimations results: with government policy*

$\hat{w}$	β_{we}	k_{wp}	$cst.$			R^2	DW
	0.275***	0.207***	0.008***			0.805	1.84
$\hat{p}$	β_{pu}	k_{pw}	$cst.$			R^2	DW
	0.018***	0.654***	−0.003***			0.311	1.99
$\hat{u}$	α_u	α_v	α_{nw}	$cst.$		R^2	DW
	−0.067***	−0.135*	0.100***	−0.003***		0.944	0.82
$\hat{e}$	α_{eu1}	α_{eu2}	α_{eu3}	α_{eu4}	cst.	R^2	DW
	0.053***	−0.081***	0.052***	−0.021***	−0.004***	0.376	2.57
i	β_{ii}	β_{ip}	β_{iu}			R^2	DW
	0.887***	0.074***	0.040***			0.981	1.13

of firms' balance sheets are left relatively unaffected. α_u and α_{λ_f} increased respectively from −0.082 to −0.067 and from 0.065 to 0.1. Up to this point, we still have a stable real sector with an unstable impact of financial variables. In this context, the Taylor rule appears to give mixed results. The interest rate is sensitive to both inflation and the output gap with a larger weight on the price rather than on the output component. Meanwhile, coefficients seem rather small, $\beta_{ip} = 0.074$ and $\beta_{iu} = 0.040$. Both coefficients are too small with respect to the existing empirical evidence of the Taylor rule. This might well be due to the liquidity trap as the interest rate was rigid downward to price and output dynamics. More generally, the numerous studies that test the Taylor rule in Japan do not give clear results. As underlined by Kuttner and Posen (2004), existing studies find opposite results, monetary policy being either too loose or too tight. In particular, they show that results depend largely on the measure of the output gap, on the trend estimate of the output gap by making use of real-time versus post-estimate data, and on the type of Taylor rule (backward or forward) estimated. All existing studies nevertheless show that the interest rate is more sensitive to prices than to the output gap in line with our own conclusion. With respect to the theoretical models, these coefficients lie on the boundary of the stability corridor. Faster reactions by the central bank would increase stability. A key point is that monetary policy did not provoke instability, although it lacked efficiency.

With respect to fiscal policy, we do not estimate the determinants of government policy (is it pro-cyclical or counter-cyclical?) as it is more difficult to estimate than a Taylor rule and as the size of the sample limits the number of equations that can be estimated. At the same time, it is well known that the government adopted numerous fiscal packages to sustain economic activity (OECD, 2002). The total amounted to 136 trillion yen, which is 27 per cent of GDP. This number is surprisingly much larger than the cost of the banking crisis. The vast majority was devoted to public works (25 per cent of total package), in an attempt to alleviate the loss of real estate and construction companies. Large real estate projects stimulated the construction sector and sustained land prices at the heart of the banking crisis. Another significant proportion

of spending financed tax cuts (12 per cent of the total package). These various tax cuts aimed primarily at alleviating the fiscal burden of high income households. A temporary income cut was passed in 1994 for a three-year period. At the same time, the tax base was enlarged. It was balanced by a 2 per cent point increase (up to 5 per cent) of the value-added tax which falls more heavily on small incomes as they are mostly spent on consumption goods. The income tax was repealed in 1997 but was replaced by a permanent income tax for corporations and top incomes in 1998 (Kuttner and Posen (2001)). In Japan, taxes fall mostly on salaried urban workers rather than on small business owners and rural residents.

If the figures tend to show that spending increased in line with the various crises, the attitude of the government has been slightly more complex. In fact, fiscal authorities hesitated between sustaining aggregate demand and maintaining the deficit within reasonable limits. This explains why fiscal expansions followed fiscal contractions and why tax cuts followed tax increases. Announced projects were not always implemented as the central administration delegated the job to local government, but did not allocate the corresponding funds. The packages of August 1992 and April 1993 for instance amounted to 24 trillion yen, while originally targeting double this amount. Another example is the fiscal structural reform law in 1997 that aimed at restoring fiscal balance but which worsened the deflationary trends and was postponed (Kuttner and Posen (2001)).

10.4 Debt default without bankruptcy

A key feature of the Japanese crisis is that default was massive but bankruptcy was rare. Put differently, the transfer of losses from borrowers to lenders took place on a very large scale. We have seen in the theoretical sections of the previous chapter that debt default without bankruptcy has, to some extent, a stabilising effect when aggregate demand is profit-led. Contrary to some neo-institutionalist claims, the violations of property rights in Japan were probably not at the heart of the long-lasting recession.

Figure 10.4 shows the rate and the number of firm bankruptcies from 1975 onwards. The rate of bankruptcy decreases in the boom to reach its lowest point in 1990 at 0.05 per cent. It then increases during each recession, between 1991 and 1994, between 1997 and 1999, and between 2001 and 2002. Nevertheless, it is almost always contained at a low rate of between 0.1 per cent and 0.15 per cent. The number of bankrupt firms never exceeded 20,000 cases. Decomposition by types of industry tells us that all sectors incurred higher bankruptcies during the crisis, but that this was more concentrated within the construction sector as well as within the wholesale and retail trade sectors. The number of bankruptcies was thus relatively small with respect to the depth of the Japanese crisis.

Conversely, debt default was very large during the Japanese crisis, as can be seen from Figure 10.5. The Financial Supervisory Agency (FSA), which was set up to tackle the issue of bad debt in the early 1990s, has proposed two official measures of non-performing loans: the *risk management loans* and the *loans disclosed under the financial*

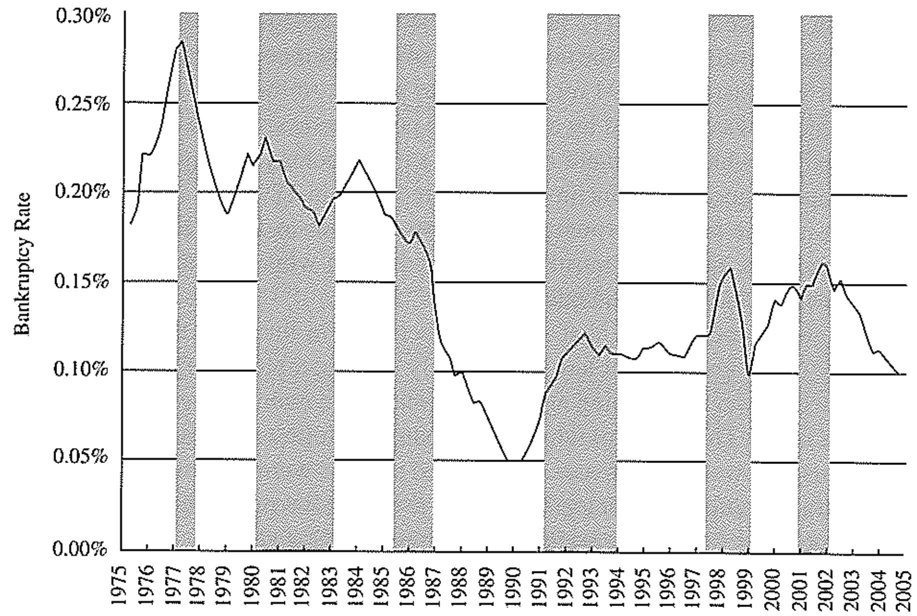


Figure 10.4 Firms' bankruptcy, reproduced from Kageyama and Harada (2007). The shaded zones indicate the recessionary periods.

reconstruction law. While their breakdowns differ slightly, their results are very similar: outstanding loans were 34.8 and 35.3 trillion yen respectively at the end of March 2003. Figure 10.5(a) gives an estimate of the stock of bad debt in Japanese financial institutions between 1993 and 2006 under the *risk management loans* measure.[14] Financial institutions hid the bad debt problem until 1995. From March 1995 to March 1996, the quantity of bad debt doubled, from 15,000 billion yen to 30,000 billion yen as financial institutions started to write them off. As a percentage of GDP, the stock of bad debt amounted to 6 per cent of GDP in March 1996. Bad debt levels then stayed constant over the mid-1990s and early 2000s despite the fact that they had been progressively written off. In fact, borrowers defaulted until 2002 and financial institutions encountered difficulties in absorbing non-performing loans. A peak was reached in March 2002, when the stock of bad debt amounted to 8.5 per cent of GDP. It then decreased quickly. In March 2006, it reached 2.5 per cent, the level prevailing in 1993. The cumulative loss for the disposal of non-performing loans reached 100,000 billion yen in 2006. This is equal to 20 per cent of GDP (Figure 10.5(b)). Banks also built provisions for non-performing loan losses. In terms of the stock of bad debt, the size increased from 30 per cent in 1993 to 60 per cent in March 1998. At the 1998 peak, provisions aggregated to 18,000 billion yen.

[14] Numbers are dated to March of each year.

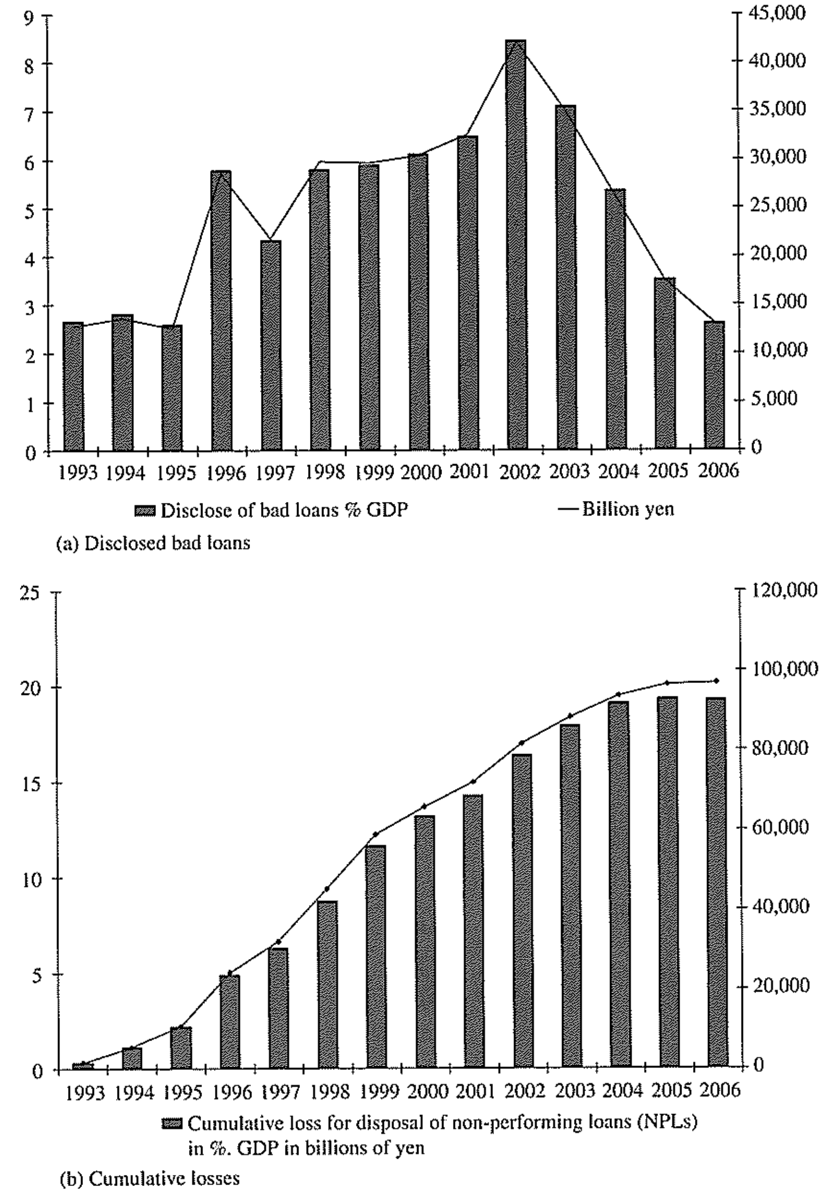


Figure 10.5 Loss related to default. Source: FSA

10.5 Bad debt and banking crises

At the same time, although default alleviated borrowers' debt burden, the banking system was deeply affected by these large losses. The resulting destabilising effect on banks overturned the stabilising effects of default on non-financial firms. In line with the theoretical model, default becomes unambiguously destabilising if it deeply affects the banking sector and leads to credit rationing and a credit crunch.

Losses for NPLs[15] have affected banks' financial health negatively for more than a decade. Although lending margins and gross profits improved over the 1990s, loan losses generated negative profit before capital gains over almost the entire period of the Japanese stagnation. Banks' profitability reached its lowest points in 1995 (–7 trillion yen), 1997 (–7.9 trillion yen) and 1998 (–8.3 trillion yen) when loan losses were the largest (13.3, 13.5 and 13.5 trillion yen respectively). In 2002, profitability improved along the gradual decrease of debt default but was still negative (–1 trillion yen). As a result, many banks went bankrupt in Japan and the number of financial institutions fell dramatically. The large disruption to credit relations that the banking crisis generated is a key element to understanding the length of the Japanese recession.

The degree to which the banking system was affected by NPLs is controversial. According to Fukao (2002), the disclosed figures understate the real situation as a result of the rules of accounting. The bad loans of banks should be double in 2002 – 71 trillion yen, instead of 42 trillion yen. In addition, loan loss and therefore provisions, are underestimated as losses are assessed with respect to a one-year time frame, while automatic debt roll-over should imply a three-year time frame. The estimation of non-performing loans according to banks' self-assessment is actually far greater (see Figure 10.6). In the bank self-assessment procedure, the definition of credits that fall into the category *need special attention* is much wider. Included in this definition are loans that *need attention* while the *financial reconstruction law* considers only loans that *need special attention*. Figure 10.6 displays the difference between the *financial reconstruction law* and banks' self-assessment of NPLs. For each column of Figure 10.6(a), the first three elements equal the estimate by the *financial reconstruction law* while the fourth element is the discrepancy between FSA assessment and banks' assessment. The difference is large, for instance in September 2001 the banks' estimate of NPLs was between three and four times bigger than the official figures. NPLs reach 20 per cent of GDP under this definition. At the same time, the discrepancy concerns the less risky loans. It does not matter so much for the measure of banks' losses (see Inaba *et al.* (2004) for details).

10.6 Delayed and weak government response

The banking crisis has been actively combatted by the Japanese monetary authorities in order to limit credit disruption. As underlined in the theoretical model of Section 9.6.5, an appropriate answer by the government to the banking crisis may cancel most of the perverse effects linked to NPLs. Meanwhile, despite the effort of the monetary authorities, the length of the banking crisis can to some extent be attributed to a slow and inappropriate response.

In Japan, the management of the banking crisis can be divided into two main time periods. Until 1997, the resolution of failing institutions followed the usual procedure,

[15] NPL = non-performing loans.

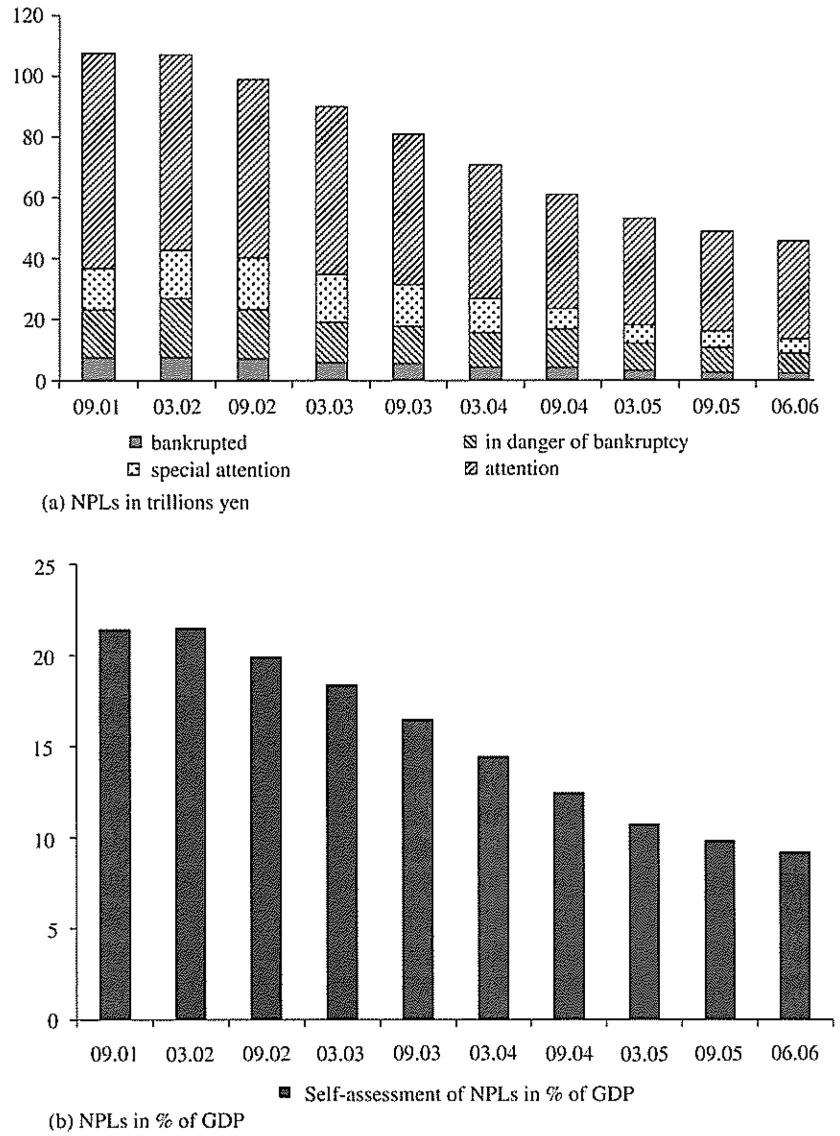


Figure 10.6 Banks' self-assessment of NPLs. Source: FSA

based on a buy-in principle. The first cases of distressed financial institutions were relatively small and specialised in the real estate market. Monetary authorities organised the distribution of loss between shareholders and creditors. The government assumed a small share of the losses in the Jusen case: 685 billion yen in the form of loss compensation. From 1997, although the government did not want to directly support financial institutions, the worsening of the banking crisis forced it to transfer public funds to financial institutions and to shift to buy-out procedures. A possible explanation of the length of the banking crisis lies in the reluctance of the Japanese government in the early 1990s to tackle the issue of bad debt on a large scale, as this would have required buying out banks.

10.6.1 The early response: buy-in of failing banks

In Japan, until 1997 banking failures were traditionally dealt with by the private sector. They took place under Merger and Acquisition assisted by public authorities or under Purchase and Assumption. The so-called 'cohort procedure' follows a buy-in principle and corresponds to a distribution of losses within financial institutions. This procedure is a direct consequence of the high degree of stability of the financial system since the Second World War. Financial institutions were very sound and there was no major bankruptcy until the 1990s. Furthermore, monetary authorities were concerned with avoiding any buy-out of the financial sector. They tried to avoid any incentives that would lead to moral hazard. In addition, the population strongly disapproved of such measures, as the banking sector had historically larger privileges than other sectors of the economy. Losses were distributed between shareholders, creditors and assuming institutions. Financial institutions purchased assets and assumed liabilities of the failed institutions or merged with them. The rescuing institutions agreed on the share of losses and public spending was not massively mobilised: at most limited to partial recapitalisation and by the transfer of assets to the Deposit Insurance Corporation (DIC). This type of procedure addressed the case of rather small bankruptcies and relied on the solidarity of the main financial actors.

A good illustration of the buy-in procedure is the famous Jusen case. Along with credit cooperatives, the Jusen companies were the first bankrupt financial institutions of the early 1990s. The Jusen companies were non-bank financial institutions founded by larger banks to grant housing loans to households. There were eight Jusen,[16] and the oldest was founded in 1971. The two banks (Long Term Credit Bank of Japan and Nippon Credit Bank)[17] that would be privatised in 1997 were in fact the mother banks of two Jusen companies: Daiichi Jutaku Kinyu and Nippon Housing Loan respectively. These institutions were created as larger banks had little interest in supplying small and complicated credits to households. Jusen could not accept deposits but received funds from mother banks. Nevertheless, the deregulation of the banking system in the early 1980s lowered profit margins of banks and the real estate bubble made real estate credit more worthwhile. Banks therefore started to compete with Jusen; which in turn were driven away from housing credit towards real estate and construction companies. Jusen loan portfolios were composed of 95.6 per cent housing loans in 1980, while

[16] Nippon Jutaku Kinyu, Jutaku Loan Service, Juso, Sogojukin, Daiichi Jutaku Kinyu, Chigin Seiho Jutaku Loan, Nippon Housing Loan and Kyodou Jutaku Loan.

[17] LTCB and NCB.

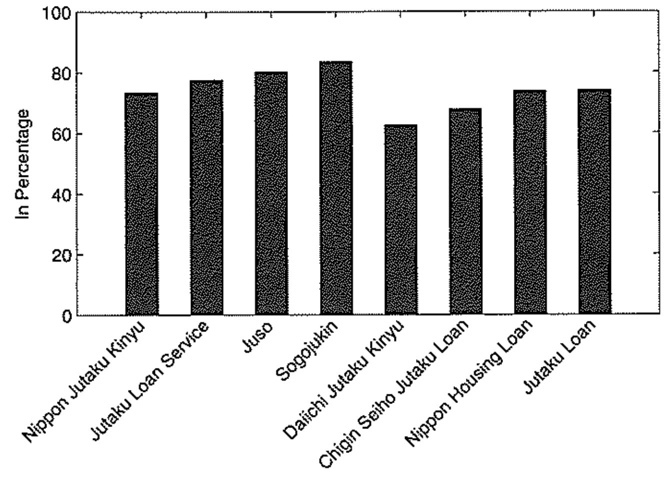


Figure 10.7 Bad assets of the Jusen companies in June 1995, reproduced from Kataoka (1997)

loans to companies amounted to 78.4 per cent by 1991. The major creditors of Jusen were agricultural cooperatives that benefited from special legislation.

In the late 1980s, the real estate market crash reduced Jusen assets and increased non-performing loans. The fall in real estate was aggravated by the credit crunch that followed as real estate assets were used as collateral for new loans. The real estate crash placed borrowers into insolvency. In addition, mother banks tended to transfer non-performing loans to their Jusen companies in the early 1990s. As a result, in 1995 the Jusen were plagued by non-performing assets up to 60–80 per cent of their total assets, as shown in Figure 10.7. The rapid increase of loan losses reduced their capital structure and pushed them into insolvency.

In December 1995, the MOF proposed a resolution plan that distributed losses between the different creditors of the Jusen according to Table 10.7. The main element was that assets that were likely to be recovered were sold to the Housing Recollection Company (HRCC), which tried to sell the collateral real estate assets at the best price. Losses linked to non-recoverable assets were divided between the different creditors. Out of the non-performing loans, bad assets of Category Four[18] were associated with a certain loss. These losses amounted to 6.3 trillion yen, and were borne by originating banks for 3.5 trillion yen, by other banks for 1.7 trillion yen and by agricultural cooperatives for 530 billion yen. Eventually, the government had to pay 680 billion yen. The small price paid by agricultural cooperatives gave rise to intense debate, as they were deeply involved in the Jusen. The cooperatives argued that banks were the effective

[18] Jusen assets are classified into four categories
1) asset tier 1; normal asset
2) asset tier 2; collectable, but not very soon
3) asset tier 3; difficult to collect
4) asset tier 4; a loss.

Table 10.7. *Jusen Resolution Corporation in December 1995 in billion yen. Source: MOF*

7 Jusen		First write-off		Sell HRCC		Capital subscription	
Normal assets	3,490	Mother banks	3,500	Normal assets	3,490	Mother banks	2,400
Bad asset 2	2,050	Other banks	1,700	Bad asset 2	2,050	Other banks	2,100
Bad asset 3	1,240	Agri-coops	530	Bad asset 3	1,240	Agri-coops	2,300
Bad asset 4	6,270	Government	680				
Deficit	140						
Total	13,190	Total	6,410	Total	6,780	Total	6,800

shareholders of the Jusen and that they should bear most of the losses. They considered themselves as simple creditors who should be served first. In 1994, their funding represented 42.5 per cent of total funds. NPLs of the other two categories were supposed to be at least partially recovered and transferred to the HRCC. The latter was financed by financial institutions at very low interest rates. Mother banks contributed 2.4 trillion yen, other banks 2.1 trillion yen and agricultural cooperatives 2.3 trillion yen. As the banking crisis deepened in the mid-1990s and came to involve a larger number of more significant institutions, the participation of public authorities became necessary, as existing banks were not financially strong enough to bear the costs associated with buy-in procedures.

10.6.2 The ineluctable buy-out of failing banks

Although monetary authorities were reluctant to buy out the financial sector, the depth of the crisis forced them to do so. It was not until 1997–8 that monetary authorities accepted such a response to the banking crisis. This delayed response is probably a major cause of the long-lasting banking crisis. Before that time, there were no adequate institutions to buy out financial institutions as the DIC could not raise large funds. In addition, the population was opposed to such use of public funds. In autumn 1997, the Bank of Japan (BOJ) refused to bring liquidity support to Sanyo Securities. The BOJ argued that its role was to help banking institutions, and not securities houses such as Sanyo. In addition, the amount of default was rather limited. Nevertheless, the shock triggered by the Sanyo default destabilised the interbank market and forced the BOJ to inject liquidity through the purchase of bonds. Three weeks later, the difficulties of Yamaichi Securities were this time answered by the BOJ with liquidity support. Meanwhile, the BOJ had no regulatory framework in which to act, as securities houses fell outside the scope of the DIC. It provided outstanding loans of 325 billion yen, that would eventually not be recovered. Consequently, a new law was passed in 1998 to enable the DIC to address the financial fragility of all financial institutions and raise larger funds.

These two examples highlight that 'lender of last resort' (LLR) activities evolved over the banking crisis to address new situations. The different types of LLR activities

conducted by the BOJ, the MOF and the DIC may be classified into five categories (Nakaso, 2001).

1. **Emergency liquidity assistance to a failed deposit-taking institution.** This was the most frequent type of LLR. It provided funds to failed deposit-taking institutions to avoid disruptions of their activities. Assets and liabilities were transferred to the DIC, which covered its losses.
2. **Provision of liquidity to interbank markets.** The interbank liquidity crisis after the Sanyo default forced the government to inject liquidity through the purchase of eligible bills. It injected 22 trillion yen, which were fully paid back.
3. **Emergency liquidity assistance to a failed non-bank financial institution.** With the failure of Yamaichi, monetary authorities started to support non-banks in order to avoid systemic risks. The absence of an institutional framework raised the issue of who bears the losses.
4. **Provision of risk capital to a financial institution.** In the early stage of the crisis, systemic risk arose with the solvency of some banks. To ease the takeover by the assuming institutions, capital was injected into the failing institutions. Until 1998, there was no clear framework for capital injection, and so these turned out very costly.
5. **Emergency assistance to a temporarily illiquid institution.** This type of lending is very close to the theoretical definition of LLR activities. In Japan however no lending of this kind took place. A possible explanation is that this type of instrument is used as a crisis prevention measure rather than a crisis management measure.

The LLR types 2 and 5 are the typical situation and are usually neutral in terms of public funds, as loans are paid back quickly and on time. The other three types of LLR activities were implemented on a practical basis, in order to address the development of the banking crisis. The absence of pre-determined rules made the issues and cost of the crisis more unpredictable.

The DIC was the institution that was in charge of implementing LLR activities and transferring funds to financial institutions. It used three procedures, through which we are able to measure the cost of the banking crisis in terms of public funds: first, capital injections to weak banks to help them to meet their capital requirements; second, financial assistance in the form of monetary grant; and third, the purchase of non-performing assets.

Financial assistance was granted between 1992 and 1997. Over that period, the buy-in principle was dominant but the government injected funds to support either the failing or the assuming institution through two mechanisms: it purchased non-performing assets from failed institutions; and provided a monetary grant to assuming financial institutions. With the worsening of the crisis, financial assistance mobilised larger funds. In terms of resources, grants became much larger than asset purchases, amounting to 18.6 trillion yen between 1992 and 2002, mainly concentrated between

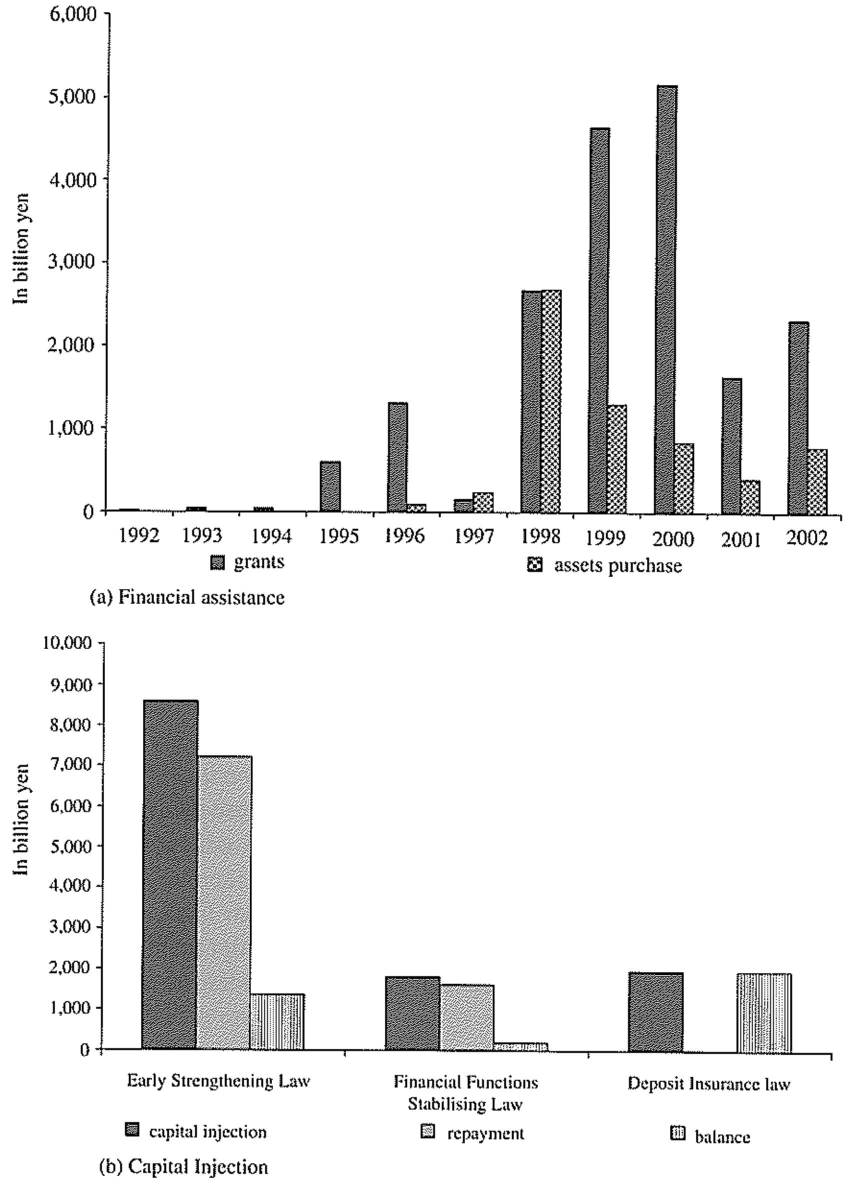


Figure 10.8 Financial assistance and capital injections. Source: FSA

1998 and 2000. Asset purchases were three times smaller, at around 6.3 trillion. Overall, financial assistance totalled almost 25 trillion yen for 180 cases, illustrated in Figure 10.8(a).

Another procedure was capital injections which are summarised in Figure 10.8(b). This took place from 1998 onwards, and was mainly related to buy-out procedures. As sound financial institutions were too weak to support failed institutions, the government

adopted a more direct procedure called the open bank. This was mainly a massive injection of public funds to recapitalise banks. Capital injections took the form of purchase of preferred shares or subordinated bonds/loans. They were both very similar, as preferred shares can be viewed as the lowest-possible grade bond. They also paid a high return as they were very risky in the case of bankruptcy. Preferred shares were characterised by a fixed dividend paid on after-tax profit, while subordinated bonds/loans yielded a high interest rate. Subordinated debt was the last to be paid back among bond holders. Preferred shares are very risky too, as shareholders are paid back after creditors. In case of bankruptcy, they are very likely to entail a full loss. Finally, subordinated bonds/loans may be perpetual in the sense that there is no fixed date of repayment and it happens whenever the issuer so wishes. There were mainly subordinated bonds up to 1999, with preferred shares after this date. These injections were implemented through a series of new laws. They were made under the:

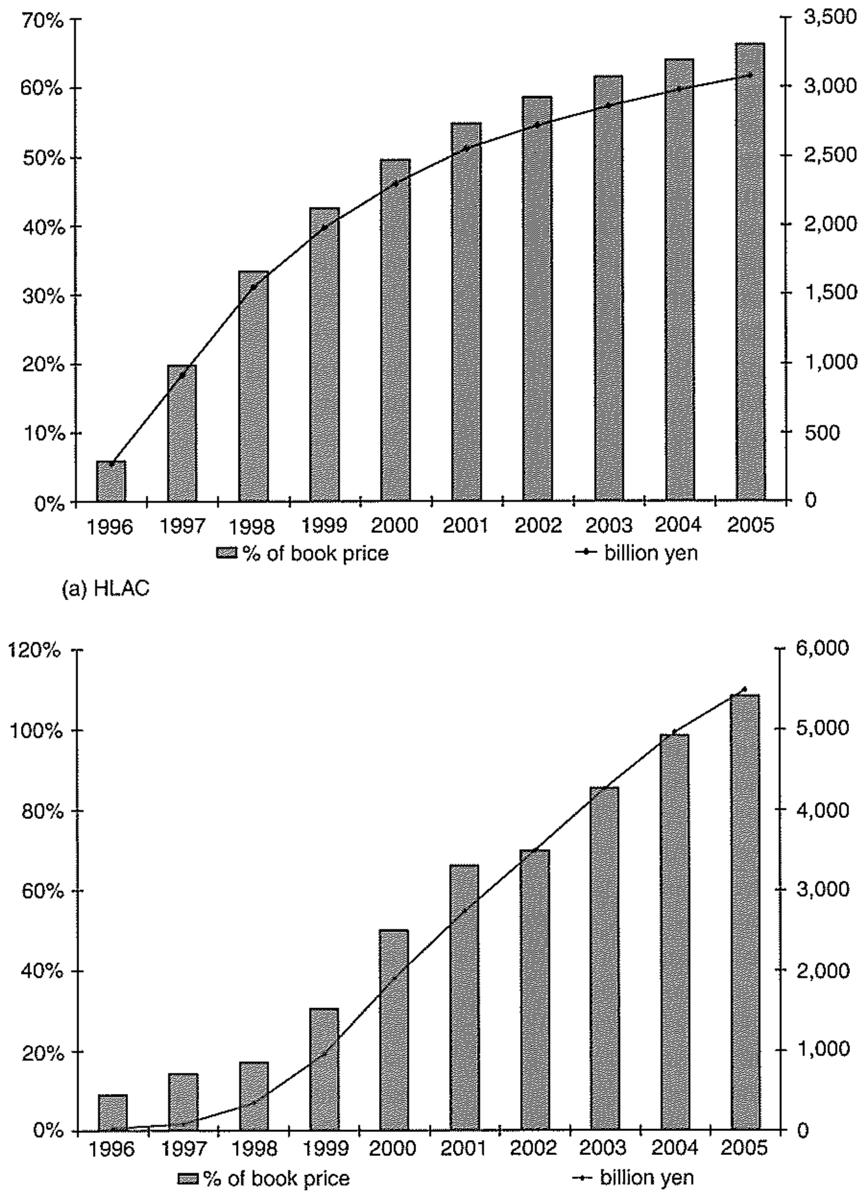
1. Financial Functions Stabilising Law in 1998;
2. Early Strengthening Law in 1999;
3. Deposit Insurance Law in 2003;
4. Financial Reorganisation Promotion Law in 2003;
5. Financial Function Strengthening Law in 2006.

The most important was the Early Strengthening Law that involved 8.6 trillion yen. Overall, capital injections aggregated to 12.4 trillion yen. Most of the injections were paid back after some time. The balances of each law are respectively 1,371, 190, 1,957, 6 and 40.5 billion yen. While they required large public funds, after a few years capital injections were mostly paid back.

The socialisation of losses also took the form of purchasing assets from financial institutions. This happened between 1999 and March 2005 through the Financial Revitalisation Law and was implemented by the Resolution and Collection Corporation (RCC).[19] Sound financial institutions transferred their risky loans to a public entity. The RCC resulted from the merger of two recollection agencies: the Housing Loan Administration Corporation (HLAC) in charge mainly of real estate assets, especially from the Jusen companies; and the Resolution and Collection Bank (RCB). Out of the total of asset transfers (from sound and failed financial institutions) the RCC had contrasting results depending on the nature of assets. HLAC and RCB both had similar amounts of claims, 4,653 and 5,071 billion yen respectively. At the same time, HLAC was only able to collect 66 per cent of claims while the RCB managed to get back more than 100 per cent. Collection by HLAC was made more complicated given the importance of real estate assets in its portfolio and the duration of depressed real estate prices, see Figure 10.9.

Nationalisation also took place over this period. Goods assets were transferred to a bridge bank affiliated to the DIC. Non-performing assets were transferred to the NCC, responsible for their recycling. Shareholders were sanctioned and it was the most costly

19 The number of financial institutions is not counted for duplication of purchases.



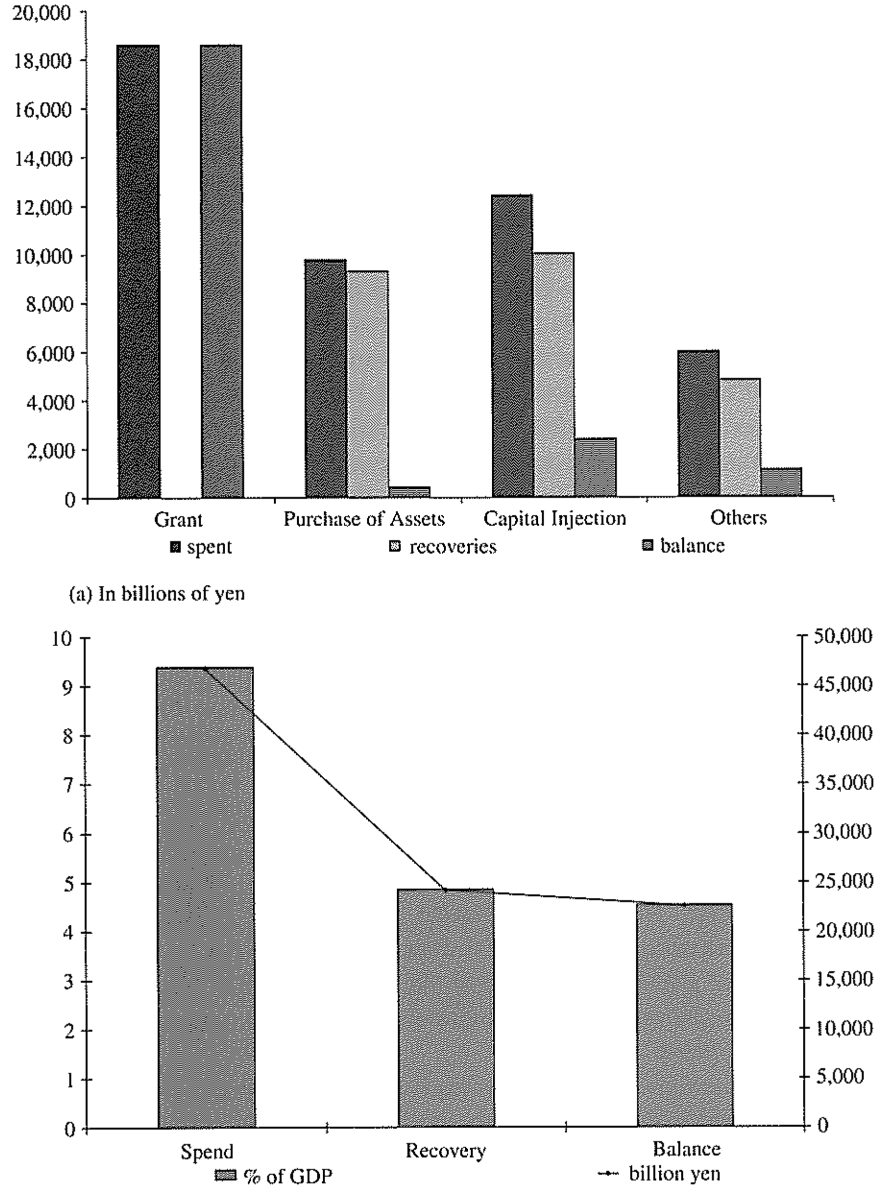
(a) HLAC

(b) RCB

Figure 10.9 Asset purchases – Japan. Source: FSA

procedure in terms of public funds. There were two nationalisations: the Long Term Credit Bank (LTCB) in 1997 and the Nippon Credit Bank in 1998. They were privatised in 1999 and 2000 respectively.

Tax preferences were also mobilised to subsidise banks. From 1998, tax authorities enabled banks to integrate potential losses on assets in their fiscal deduction. In the



(a) In billions of yen

(b) In % of GDP

Figure 10.10 Transfers to the financial system – Japan. Source: FSA

case where bank income was so small that they were not submitted to taxation, this procedure enabled them to capitalise these tax reductions for the future. The time frame of this measure, about five years in 1998, was extended to seven years and became a disguised subsidy. Another fiscal procedure involved the use of related companies by banks to clean up their balance sheet. Before 1998, banks did not have to consolidate subsidiaries and affiliates with less than 50 per cent and 20 per cent stake respectively in their reports. They therefore transferred NPLs above market value to these structures (see Kanaya and David (2000)).

Overall, banking rescues by public authorities involved large transfers of funds towards failed financial institutions. Total transfers amounted to more than 9 per cent of GDP, which is roughly half of the cost of non-performing loans; Figure 10.10 displays the various transfers. Out of the 47 trillion yen, the government recovered 24 trillion and was left with a balance of 23 trillion which corresponds to 4.5 per cent of GDP (see figure 10.10b). Most of the cost was linked to monetary grants provided under financial assistance. Out of the 24 trillion yen recovered, 9 trillion yen were due to the sales of financial assets and were therefore not paid back by financial institutions (see figure 10.10b). The net transfers towards financial institutions amounted therefore to 32 trillion yen, or 6.4 per cent of GDP. To summarise, public action avoided systemic risk and contributed to the reduction of the stock of non-performing assets that destabilised banks' balance sheets. This success came at the cost of large transfers of wealth towards the financial sector. In addition, the difficulties of targeting solvent but illiquid, rather than insolvent, institutions combined with the systemic risk related to the failure of large institutions led authorities to rescue failed institutions.

10.7 Conclusions

In this chapter, we have explored how the Japanese institutional configuration can be characterised and whether it can be used to explain part of the financial crisis of the 1990s. We proceeded step by step: from a real model to a financial model with government intervention.

First, the real sector was characterised by a profit-led aggregate demand and a labour market-led real wage. The feedback channel between these two elements produces attracting forces and ensures that the crisis does not originate in the real sector. Second, there is evidence of a financial accelerator that has pro-cyclical effects and may be a source of instability. Nevertheless, its magnitude is not large enough to explain the deep economic crisis in Japan.

Third, legal institutions in Japan are such that borrowers were able to default without being sanctioned by bankruptcy. This special case has, to some extent, stabilising effects when aggregate demand is profit-led. It is therefore inaccurate to blame the Japanese bankruptcy procedure for the length of the Japanese crisis. Fourth, debt default affected negatively banks' financial health and gave rise to a long-lasting banking crisis. The resulting credit disruptions were largely responsible for the deterioration of economic activity. This underlines the fact that the key dimension to be taken into account to explain Japan's 'lost decade' is the effect of debt default on the banking system.

Further, the fifth result underscores the sluggish and inadequate government response to the crisis. Monetary policy was implemented slowly, and may have been more stabilising if more active. The banking crisis lasted for more than ten years, to a large extent because the government was reluctant to use public spending to solve the bad debt problem in the early 1990s. Monetary authorities preferred to rely on market-based solutions, which – although costless for the state – are not necessarily suited to solve widespread debt problems. Contrary to the moral hazard argument, it was actually the attempt by the government to avoid any moral hazard that resulted in the government response to the bad debt problem being so slow. Put differently, an economy based on banks' credit and intermediation should avoid financial deregulation that may disrupt the credit links between firms and borrowers. In addition, given the importance of the credit relationship, the management of the banking crisis should be fast and public funds should be mobilised if necessary.

From a distributive perspective, we have shown that the costs of over-indebtedness were mainly supported by creditors rather than borrowers. Creditors supported losses on credit equivalent to 20 per cent of GDP, while bankruptcy of the borrowers reached not even 0.25 per cent of the total number of firms at the worst point of the crisis. The overall transfers of public funds to the banking sector aggregated to 10 per cent of GDP, which is half of the financial sector losses. While part of the funds were paid back, the net transfers to financial institutions totalled 6.4 per cent of GDP. With respect to real transfers, the expansionary fiscal packages implemented by the government to sustain economic activities were far larger, at up to 27 per cent of GDP. This suggests that once the crisis became serious, large scale government intervention was unavoidable.

10.8 Appendix: data sources

Table 10.8. *Data sources for Japan*

Indicator	Source
GDP deflator	IMF
Monthly earnings	IMF
GDP	IMF
Index of capacity utilisation	METI
Compensation of employee	CAO
Number of employed	Labour force survey
Labour force	Labour force survey
Call rate	BOJ
Government spending	CAO
Net wealth, fixed assets, and liabilities	MOF

CAO (Central Application Office); IMF (International Monetary Fund); METI (Ministry of Economy, Trade and Industry)

11 Housing investment cycles, workers' debt and debt default

11.1 Introduction

A proper modelling of the housing sector in a structural macroeconomic and/or macroeconometric model needs to consider housing investment, the purchase of houses or housing services and the evolution of the prices charged for them. The main focus in the applied literature on this issue has often been the subsector of office space, but of course the sector of privately owned houses or private rental is also a very large and important sector of the macroeconomy.

What is particularly interesting from the macrodynamic point of view in this type of literature is that there are concepts and issues in the literature on the housing sector that are closely related to important topics of standard macrodynamic theorising. There is the concept of the natural vacancy rate, or of a Non-Accelerating Inflation Rate of Unemployment (NAIRU) in the housing sector, as discussed by Hendershott *et al.* (2002), the concept of overbuilt markets, see Hendershott (1996), and of persistent cycles in the housing sector that in our view bear close resemblance to what is happening in the unemployment-inflation dynamics in the interaction of the labour market with the market for goods and the wage-price spiral.

Due to the size of the housing sector it is therefore of great interest from the macrodynamic point of view to not only study this sector with its interaction of space demand and supply, rental and housing prices and the rates of return they imply and finally the investment behaviour in this sector, but to also consider its interaction with the rest of the macroeconomy where (at least) two real cycle generators may be at work leading to coupled oscillators and maybe to complex business cycle fluctuations. In the present chapter we want to lay foundations for such an investigation and to show that models of such type can even be handled from the theoretical perspective. Ultimately, though, numerical and empirical investigations will be needed, which however are beyond the scope of this chapter as far as empirical issues are concerned; see though the quoted literature for investigations of this issue.

In this chapter[1] we apply the general framework introduced in Part II[2] to the special issue of housing investment cycles, the supply and demand for housing services as part

[1] This chapter is based on Chiarella and Flaschel (2003).
[2] See also Chiarella *et al.* (2000, Part III) in this regard.

of the dynamics, and the price dynamics this implies in the housing sector. Rents in this sector then in turn determine rates of return for housing investment and interact with this investment in the generation of damped, persistent or even explosive cycles thus generated in the housing sector of the economy.

The necessary ingredients for this analysis have by and large already been provided in earlier chapters, but are here modified to a certain extent in order to allow a more specific analysis of the topics mentioned above. The following section will present in this regard the details we need for the analysis of the dynamics originating in the housing sector and that interact with the general business cycle of the model. However we will not repeat the general framework in all of its details, but simply refer the reader to Part II. The reader is therefore referred to earlier chapters for the full models on the extensive form level, the intensive form level and the many subdynamics to which this model type can give rise, as well as investigations of their interaction in the integrated 18D core dynamics of this approach. This overall framework has been motivated by an attempt to understand the basic dynamic feedback mechanisms of the macroeconomy, and their interaction, in large scale macroeconometric models such as that of Powell and Murphy (1997).

The new focus that the current chapter brings is on debt relationships in the household sector, composed of indebted worker households and pure asset holders as creditors. Worker households purchase houses as durable consumption goods (in addition to housing services by part of them) through credit from the asset holders and are thus now characterised by negative bond holdings in place of positive bond holdings in the chapters of Part II. They indeed have – in the aggregate – a marginal propensity to consume out of their disposable income that is larger than 1 (when consumption of non-durables and durables are taken together) and they finance the excess of consumption over their disposable income by new debt and thus credit from the asset holding part of the household sector. In the steady state we will have a constant debt to capital ratio and thus debt of workers growing at a constant rate over time.

Our interest however is to study endogenous fluctuations around this steady growth path and to investigate in addition the possibility that price and wage dynamics may be such that processes of debt deflation are generated, here not as in Chiarella *et al.* (2001a,b) and Chapter 7 with respect to firms and their indebtedness due to past investment behaviour, but rather within the household sector and its debtor-creditor relationship.[3]

In Section 11.2 we present the new components of the model on the extensive form level, in addition to the details from our earlier work that concern the housing sector. Section 11.3 then derives the laws of motion of the general 19D dynamics, which – with special focus on the housing sector – are simplified thereafter to give rise to the core 9D dynamics to be investigated in the remainder of the chapter. Section 11.4 considers 2D to 5D subcases of the integrated 6D real subdynamics of the 9D dynamics and

[3] We add here that the analysis in Chiarella *et al.* (2001a,b) may indeed be applicable to the sector of office space, while the present chapter focuses on private space.

formulates and proves a number of propositions on these subdynamics. The full 9D dynamics are investigated from the numerical perspective in Section 11.5. Section 11.6 concludes.

11.2 Debt relationships in the household sector

In this section we reformulate the general model of disequilibrium growth introduced and investigated in Part II with respect to assets supplied and demanded by the two types of households of our general model, workers and asset holders, who are assumed to make up the household sector.[4]

We will assume that worker households rent housing services and also buy new houses, and that they finance the resulting excess of their consumption over their disposable income via credit (bonds of the fixed-price/variable interest variety of the model of Part II). Such bonds are supplied by the other type of household of the model, the pure asset holders.

Firms produce for domestic purposes a unique good that can be used as a consumption good proper by the two types of households, as a business fixed investment good, as an investment good providing housing services to the workers by asset holders, for the purpose of government consumption and now also for representing the direct demand for houses by both asset holders and workers. These alterations of the original 18D core dynamics of Part II will increase the dimension of these dynamics by one, since debt accumulation of workers will now feed back into the rest of the dynamics due to their consumption habits. We will represent the resulting dynamical system in compact form in the following subsections.

11.2.1 Worker households

We consider the behavioural equations of worker households first, but only to the extent they are changed by the existence of a debtor-creditor relationship between our two types of agents in the household sector. In order to derive the new characteristics of this module of the model let us first present these equations in the form that they were used in the original approach of Part II of the book:

Households (Workers – original formulation):

$$\begin{aligned} Y_w^{Dn} &= (1-\tau_w)[wL^d + w^u(L-L^w) + w^r\alpha_l L_2] + (1-\tau_c)iB_w \\ &= Y_{w1}^{Dn} + (1-\tau_c)iB_w, \\ p_yC_g^w &= c_yY_{w1}^{Dn}, \\ p_hC_h^w &= c_hY_{w1}^{Dn}, \\ S_w^n &= Y_w^{Dn} - p_yC_g^w - p_hC_h^w = \dot{B}_w. \end{aligned}$$

[4] If households are to be considered as heterogeneous in a macromodel then this should be the fundamental distinction in a model with labour supply and asset markets and with only two household types. Such polar types of households, which still appear in a very stylised way in the models of this book, are in our view much more relevant for macroeconomic model building than the distinction between workers and pensioners made in the Overlapping Generations (OLG) type of models.

In these equations, the expression Y_w^{Dn} denotes total nominal disposable income of workers after taxes at the rate τ_w as far as their labour income $wL^d + w^u(L-L^w) + w^r\alpha_l L_2$ is concerned,[5] and after taxes at the rate τ_c with respect to their interest income iB_w, on the stock B_w of short-term bonds accumulated by workers.

We assumed in the original approach of Part II that workers save and thus hold and accumulate bonds in the amounts B_w and $\dot{B}_w$ respectively, but there they reinvested all of their interest income into bond accumulation, which thus did not feed back into the income term Y_{w1}^{Dn} that determined their nominal consumption of goods $p_yC_g^w$ and of housing services $p_hC_h^w$ (with marginal propensities to consume $c_y + c_h < 1$). These assumptions helped to simplify considerably the dynamics of that earlier study since the bond accumulation of workers did not influence aggregate demand and goods market behaviour in this case. Nominal savings S_w^n of workers was invested into short-term bonds solely, since money was not a financial asset in the model of Part II.

In the case of negative savings and thus debt accumulation (a negative B_w will be denoted by the positive expression Λ_w in the following), things are however not so easily disentangled. Since debt is in fact entered into in order to increase the consumption of workers (not only to rent houses as has so far been assumed, but also to buy houses as durable consumption goods), it follows that asset owners thereby become debtors to asset holders, just as the government. Interest payments[6] must then appear in the income expression to be used for determining the consumption demand of workers since these payments reduce the possibility of workers to spend more than they earn.[7] Assuming such a situation leads us to the following reformulation of the above representation of workers' behaviour, based on the augmented form of short-term loans, $\dot{B}_c = \dot{\Lambda}_g + \dot{\Lambda}_w$, supplied by the asset holders to the government as well as to worker households.

Households (Workers – new formulation):

$$\begin{aligned} Y_w^{Dn} &= (1-\tau_w)[wL^d + w^u(L-L^w) + w^r\alpha_l L_2] - (1-\tau_c)i\Lambda_w \\ &= Y_{w1}^{Dn} - (1-\tau_c)i\Lambda_w, && (11.1) \\ p_yC_g^w &= c_yY_w^{Dn}, && (11.2) \\ p_hC_h^w &= c_hY_w^{Dn}, && (11.3) \\ \dot{\Lambda}_w &= p_yC_g^w + p_hC_h^w - Y_w^{Dn}. && (11.4) \end{aligned}$$

[5] Labour income here consists of wage income, unemployment benefits and pension payments, which are all subject to tax payments here at the uniform wage tax rate τ_w. Note however that the model would not be changed very much if differential wage tax rates are allowed for, an observation which also applies to the consumption propensities shown, which at present are the same for employed, unemployed and retired workers.

[6] For simplicity we assume that these are at the short-term rate i in place of i_λ.

[7] Note here that interest payments are deducted before worker households decide on their consumption patterns. In the case where propensities to consume are applied to total wage income (after taxes) the dynamics of the capital to debt ratio to be considered later on do not feed back into the rest of the dynamics.

We assume in this chapter that $c_y + c_h > 1$ holds, so that worker households always consume more than they earn (after the deduction of interest payments).[8] Such an assumption for worker households amounts to assuming that there is no intertemporal budget constraint in the usual sense of the word for this type of household, just as for the government sector. In both cases we will have a given debt to capital ratio in the steady state meaning that part of expenditure is always financed by issuing new debt, which then grows (just as the stock of debt) with the given real growth rate of the world economy. Such an approach is admissible in a descriptively oriented disequilibrium growth model of monetary growth, in particular if it is understood that this model type (and its steady state solution) is to be applied to particular periods of the evolution of actual market economies. Assuming no debt of the government and of workers in the steady state by choosing the parameters of the model appropriately clearly is too limited an approach from the descriptive point of view.

We also note here that debt accumulation, and even more so debt deflation, is still of a fairly simple type. A study of the equations in the above module of the model clearly shows that workers' consumption demand depends negatively on their debt (due to their interest payments and their marginal propensities to consume being in sum larger than one) and thus on the debt to capital ratio $\lambda_w = \Lambda_w/(p_y K)$ to be considered later on. This means that aggregate demand depends negatively on the ratio λ_w and will thus shrink when this ratio is increasing, which happens in particular when there is goods price deflation (taken in isolation).

Such deflation therefore decreases aggregate demand, which via the Metzlerian goods market adjustment process leads to still lower economic activity and from there to further falling goods prices and so on. In this way a deflationary spiral may be established, which drives the economy into ever more depressed situations. The resulting downward spiral in prices and wages and in economic activity depends however on the precise way wages, goods prices and rental prices are falling and what happens to other components of aggregate demand. It should be noted that we have continued to ignore other feedback channels, in particular the ones that concern the effects of falling goods and rental prices.

One can interpret the above description of the behaviour of worker households also in the following way. Assume that workers accumulate debt basically due to their purchase of houses, which can be made explicit if it is assumed that c_y is split into c_y^g and c_y^h, where the first parameter describes the propensity to consume consumption goods proper and the second parameter denotes that portion of goods consumption that goes into purchase of houses. The equation that describes the evolution of the stock of houses K_h^w owned by workers is then given by

$$\dot{K}_h^w = c_y^h Y_w^{Dn}/p_y - \delta_h K_h^w \quad \text{or} \quad \hat{k}_h^w = c_y^h y_w^D/k_h^w - \delta_h - \hat{K},$$

[8] Note that we have assumed here for reasons of symmetry that interest paid on debt leads to tax reduction at the rate τ_c, which however is a detail of the model which is of secondary importance.

where δ_h, y_w^D and k_h^w denote the depreciation rate of houses, the real disposable income of workers (after interest deduction) per unit of capital K and K_h^w/K, respectively.

The steady state value of the stock of houses owned by workers per unit of capital is thus given by $k_h^w = c_y^h y_w^D/(\gamma + \delta_h)$, where γ denotes the steady state rate of growth of the economy. We shall show in Section 11.4 that the steady state value of the debt to capital ratio $\lambda_w = \Lambda_w/(p_y K)$ is given by $(c_y + c_h - 1)y_w^D/\gamma$. Assume now that Λ_w can be considered as the housing mortgage that can and will be bequeathed to the next generation if the side condition $\Lambda_w \le p_y K_h^w$ is fulfilled (since the mortgage is then less than the reproduction value of the stock of houses owned by worker households). In the steady state this side condition amounts to

$$\lambda_w = \frac{c_y + c_h - 1}{\gamma} y_w^D \le \frac{c_y^h}{\gamma + \delta_h} y_w^D,$$

which is fulfilled if

$$c_y + c_h - 1 \le c_y^h \quad i.e., \quad c_y^g + c_h - 1 \le 0.$$

The above thus gives a lower bound on the propensity to consume c_y^h such that the above side condition is fulfilled, at least along the steady state solution of the dynamics.

Note also that the dynamics of the model is based on a Keynesian determination of the short run throughout so that demand is always satisfied in this model type. Situations where this is not the case are analysed in detail in Chiarella *et al.* (2000) and do not lead to important changes in the behaviour of the models described there. Due to the two consumption functions just presented we have that Keynesian goods market features now depend on the stock of debt of workers (through the interest payments that they imply). The debt to capital ratio of workers will therefore now influence the dynamics of the real part of the model in contrast to the situation considered in Part II where workers accumulated a positive stock of short-term bonds.

11.2.2 Pure asset holder households

Next, we consider the other type of households of our model, the pure asset holders who are assumed to consume C_c (goods and houses as supplied by firms through their domestic production Y) at an amount that is growing exogenously at the rate γ, which is thus in particular independent of their current nominal disposable income Y_c^{Dn}. The consumption decision is thus not an important decision for pure asset holders. Their nominal income diminished by the nominal value of their consumption $p_v C_c$ is then spent on the purchase of financial assets, three types of bonds (short-term domestic, long-term domestic and foreign) and equities, as well as on investment in housing supply (for rent to part of the worker households). Note here that the one good view of the production of the domestic good entails consumption goods proper and houses so that asset holders buy houses for their consumption as well as for investment purposes.

<u>Households (Asset Holders):</u>

$$Y_c^{Dn} = (1-\tau_c)[r^e p_y K + iB_c + B_1^l + p_h C_h^w - p_y \delta_h K_h] + s(1-\tau_c^*)B_2^l, \quad (11.5)$$

$$\hat{C}_c = \gamma, \quad (11.6)$$

$$S_c^n = Y_c^{Dn} - p_y C_c \quad (11.7)$$

$$= \dot{B}_c + \dot{B}_1^l/i_l + s\dot{B}_2^l/i_l^* + p_e \dot{E} + p_y(I_h - \delta_h K_h) \quad (\dot{B}_c = \dot{\Lambda}_g + \dot{\Lambda}_w),$$

$$C_h^s = K_h, \quad (11.8)$$

$$r_h = (p_h C_h^w - \delta_h p_y K_h)/(p_y K_h), \quad (11.9)$$

$$g_h = I_h/K_h$$

$$= \alpha_r^h((1-\tau_c)r_h - i^r) + \alpha_i^h(i_l - i) + \alpha_u^h\left(\frac{C_h^w}{C_h^s} - \bar{u}_h\right) + \gamma + \delta_h, \quad (11.10)$$

$$\hat{p}_h = \beta_{p_h}(\frac{C_h^w}{C_h^s} - \bar{u}_h) + \kappa_h \hat{p}_y + (1-\kappa_h)\pi^c, \quad (11.11)$$

$$\hat{K}_h = I_h/K_h - \delta_h. \quad (11.12)$$

This part of the model is basically the same as the one considered in Part II, with the interpretational exception that asset holders in addition now lend to worker households. Equation (11.5) defines the disposable income of asset holders which consists of the dividend payments of firms (who distribute their whole expected profit to equity holders), interest on government bonds (short-term bonds and consols), $iB_c + B_1^l$, to the extent they are held by domestic residents, rents for housing services net of depreciation, and interest payments on foreign bonds held by domestic households (after foreign taxation and expressed in domestic currency by means of the exchange rate s).

Private savings of asset holders S_c^n thus also concern short- and long-term bonds (domestic and foreign with respect to the latter), equities and net housing investment (equation (11.8)). The supply of housing services C_h^s (in equation (11.8)) is assumed to be proportional to that part of the existing stock of houses K_h devoted to the supply of such services (here the factor of proportionality is set to unity for simplicity). We assume for simplicity that there is no resale market for houses as there are for the financial assets of the model, which however is a feature of the model that should be removed in further extensions. Note again that the production of houses is part of the production activities of firms and thus part of the homogeneous supply of the domestic (non-traded) output.

The return on housing services is given by equation (11.9). The demand for housing services C_h^w has already been defined in the preceding module. We assume that housing demand is always satisfied and we can guarantee this in general (up to certain extreme fluctuations in the demand for housing services) by assuming that there is a given benchmark rate of capacity utilisation $\bar{u}_h$ of the housing service supply beyond which there is additional pressure on the price p_h of housing services and also increased effort

to invest into housing supply (which may be of such extent that rationing on the market for housing services is prevented).[9] We have assumed in the workforce sector that the demand for housing services is growing (apart from short-term deviations) with the trend rate γ (underlying the steady state of the model). This implies that housing services per household grow with trend rate $\gamma - n$, where n is the natural rate of growth of the workforce. Therefore, over the growth horizon of the considered economy, we have that worker households consume more and more housing services (measured by square metres per housing unit for example).[10] Equation (11.10) describes the rate of gross investment in housing of asset holders, which depends on the profit rate r_h in the housing sector compared with the required rate of return, which is measured in reference to government consols by $i^r = i_l - \pi^c$ (via Tobin's q as the relative profitability measure). It depends furthermore on the interest spread $i_l - i$ as a measure for the tightness of monetary policy and its perceived (or only believed) effects on the level of economic activity and employment, on the actual rate of capacity utilisation of housing services in its deviation from the natural rate of occupancy (representing short-run demand pressure), $\frac{C_h^w}{C_h^s} - \bar{u}_h$, on the trend rate of growth γ and on the rate of depreciation δ_h in the housing sector.

In equation (11.11) the rate of inflation of the rental price of housing services, $\hat{p}_h$, depends (as does investment) on the rate of capacity utilisation in the housing sector (the demand pressure component) and on a weighted average formed by the actual rate of inflation of producer prices in the production of the domestic good and on the level of this inflation that is expected over the medium term as a medium-term average (the rate π^c), whose law of motion will be provided later on (this weighted average represents the cost-push component in this dynamical equation for the price of housing services). Finally in equation (11.12) actual gross investment plans are always realised and thus determine the rate of growth of the housing stock by deducting the rate of depreciation.

Summing up we can state that the consumption decisions of asset owners are basically driven by exogenous habits[11] (which are independent of their income and wealth position) and that their investment decision in housing service supply precedes the other (financial) asset accumulation decisions. These latter decisions are in the present framework governed by supply side forces based on the new issuing of bonds by the domestic government and of equities by firms. Furthermore, their choice of accumulation (or decumulation) of foreign long-term bonds is here determined as the residual to all these flows in or out of short- and long-term domestic debt and the flow of new equities issued by firms and is thus determined as the last step in the savings decision of asset holders. The essential decisions in this module of the model are therefore the housing investment decision and the pricing rule for housing services. Due to the

[9] See for example Shilling *et al.* (1987) and Rosen and Smith (1983) for such NAIRU approaches to the market for housing services.

[10] Such a construction is needed for the discussion of steady states of the economy and of course is only applicable over certain periods of time in the evolution of market economies.

[11] Here in a way that allows for a fixed parameter representation in the intensive form of the model.

assumptions made on the consumption of asset holders we do not need to consider the asset accumulation of these agents explicitly in the dynamical investigations that follow.

11.2.3 Wage, price and interest rate adjustment processes

Finally, we present the wage, price and interest rate dynamics of the model that are important for the integrated core 5D dynamics of the real part of the model to be investigated subsequently in this chapter. This type of dynamics has started to receive growing attention in recent studies with an empirical orientation. We stress however that we do not yet pay attention to consumer price indices and the role of import prices in the formation of the money wage and the price level Phillips curves (PCs), respectively, see however Chiarella *et al.* (1999a,b) for such additions to the model. This module is the same as the one employed in Chiarella and Flaschel (1999a), which in sum means that the basic change in this chapter with respect to these earlier integrated models of monetary growth concerns solely the budget restriction and the consumption behaviour of worker households.

Wage-Price-Interest Adjustment Equations:

$$\hat{w} = \beta_w(e - \bar{e}) + \kappa_w(\hat{p}_y + n_l) + (1 - \kappa_w)(\pi^c + n_l), \tag{11.13}$$

$$\hat{p}_y = \beta_p(u - \bar{u}) + \kappa_p(\hat{w} - n_l) + (1 - \kappa_p)\pi^c, \tag{11.14}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p}_y - \pi^c) + (1 - \alpha_{\pi^c})(\bar{\pi} - \pi^c)), \tag{11.15}$$

$$\dot{i} = -\beta_{i_i}(i - i_l^*) + \beta_{i_p}(\hat{p}_y - \bar{\pi}) + \beta_{i_u}(u - \bar{u}). \tag{11.16}$$

In equation (11.13) wage inflation $\hat{w}$ responds in the traditional PC manner to the state of the demand pressure in the labour market as measured by the deviations of the rate of employment e from its NAIRU level $\bar{e}$ and there is also the usual accelerator term of price inflation, which is here measured as a weighted average of actual price inflation based on short-term perfect foresight (plus the actual rate n_l of productivity growth) and expected medium-term price inflation (also augmented by the given rate of productivity growth). The law of motion (11.14) for goods prices p_y of the domestic commodity is formulated in a similar way, as a second type of PC. We use the demand pressure measure $u - \bar{u}$, the deviation of actual capacity utilisation of firms from its norm, as the demand pressure cause of price inflation. The cost-push term in the price inflation equation is given as a weighted average of current wage inflation and the one expected for the medium run (both made less severe in their influence on price inflation by the existence of a positive growth rate of labour productivity).

In equation (11.15) expected medium-term inflation π^c in turn is based on a weighted average of two expectations mechanisms, an adaptive one with weight α_π and a forward-looking one with weight $1 - \alpha_\pi$. Forward-looking expectations are here simply based on the inflation target of the central bank $\bar{\pi}$, in the usual way of a regressive scheme of expectations revision. Inflationary expectations are thus following a weighted average of actual inflation and the target rate of the monetary authority. We assume $\bar{\pi} = 0$ in the following and thus will have no inflation in the steady state of the model. Furthermore we will also not consider the destabilising role of inflationary expectations (the so-called Mundell effect) and thus will set $\beta_{\pi^c} = 0$ for reasons of simplicity, in order to concentrate on destabilising real debt and real wage adjustments.

We can see from the above description that only the inflation rate of the domestic good matters in the wage-price module of our economy. Housing, through its rental price (and its rate of change $\hat{p}_h$), is thus ignored in this description of the wage-price interaction. All of this simplifies the feedback structure of the model, but should give way to a domestic price index of the form $p_c = p_y^a p_h^{1-a}$ and its rate of change in the wage equation in future reformulations of the model, see Chiarella *et al.* (1999a,b) in this regard.

The interest rate adjustment rule, equation (11.16), of the monetary authority adjusts the short-term interest rate i towards the given rate of interest in the world economy, but also pays attention to the deviation of the actual rate of price inflation from the targeted one, implying a rising i if the actual rate is above the target (and vice versa). Finally, interest rates are more easily increased if there is demand pressure on the market for goods than in the opposite situation.

We do not go here into the other modules of the model as formulated in Part II, since they by and large will not matter very much in our subsequent investigation of housing investment cycles, consumer debt and wage deflation. These modules concern the sector of firms with its fixed proportions technology (including exports and imports) and an investment behaviour that is similar to the one assumed for asset holders with respect to the housing stock, the government sector whose fiscal policies do not matter here (due to assumptions to be made in the next section), but which makes use of a Taylor type interest rate policy rule (as shown above), asset markets that adjust towards a general prevalence of interest rate parity conditions and Metzlerian adjustment of inventories and sales expectations of firms that generally do not correctly perceive aggregate demand on the market for domestic goods. These equations will be summarised in compact form on the level of intensive or state variables in the next section.

11.3 Intensive form derivation of a simplified 9D dynamics

In this section we present the modification of the 18D core model, which was investigated in Part II from various numerical perspectives, in order to focus now on a detailed consideration of the possibility of housing cycles and the debt financing of the investment undertaken by workers into housing purchases as already contained, but not yet considered in detail, in this original approach to disequilibrium growth dynamics (where interest payments in the sector of worker households did not yet have an impact on their consumption behaviour). To simplify the model slightly we assume throughout the following that $C_c(0) = 0$ holds initially (and thus for all times) and thus neglect the consumption of asset holders altogether (which does not contribute very much to the present investigation under the assumptions to be made). We stress that the resulting dynamics on the state variable level are no longer of dimension 18 as in Part II, but now of dimension 19, since the law of motion of workers' debt (formerly workers' bond accumulation) now feeds back into the market for goods due to the dependence of workers' goods demand on the interest payments for their loans. The present model thus not only reinterprets the worker households' bond accumulation as debt accumulation and adjusts their behavioural equations to this new interpretation, but it also adds a feedback chain to the dynamics that were formerly missing and on which the possibility of the occurrence of debt deflation is now based.[12]

We start with a compact presentation (including brief comments on their contents) of the nineteen laws of motion of the full model analysed in this chapter, and will present thereafter a 9D core case (with a unique interior steady state solution) of these dynamics to be used in the analysis that follows. These laws of motion around the steady state of the dynamics, appropriately grouped and all in per unit of capital terms, are given by the following set of differential equations. As first group we consider here the quantity adjustment mechanisms with respect to the market for goods, concerning sales expectations y^e and actual inventories ν, and for the market for labour, concerning the employment policy of firms, l^{we}, and also concerning the evolution of full employment labour intensity l^e (both in efficiency units) and of the stock of housing (everything per unit of the capital stock of firms):[13]

$$\begin{aligned}
\dot{y}^e &= \beta_{y^e}(y^d - y^e) + (\gamma - (g_k - \delta_k))y^e, \\
\dot{\nu} &= y - y^d - (g_k - \delta_k)\nu, \\
\dot{l}^{we} &= \beta_l(l^{de} - l^{we}) + [\gamma - (g_k - \delta_k)]l^{we}, \\
\hat{l}^e &= \gamma - (g_k - \delta_k), \\
\hat{k}_h &= g_h - \delta_h - (g_k - \delta_k).
\end{aligned}$$

The first of these five laws for quantity dynamics describes the adjustment of sales expectations y^e in view of the observed expectational error $y^d - y^e$ based on currently realised sales y^d, augmented by a term that takes account of the fact that this adjustment is occurring in a growing economy and is expressed in intensive form. Next, inventories ν (per unit of capital) change according to the gap between actual output y and actual sales y^d, again reformulated such that growth of the capital stock, the measurement base for the considered intensive form variables, is taken into account. Employment of firms, l^{we}, changes so as to reduce the discrepancy that exists in each moment of time between the actual employment l^{de} of the employed and their normal employment, here measured by l^{we} (everything in efficiency units due to the assumed form of technical change). The growth rate of the factor endowment ratio $l^e = L^e/K$ (in efficiency units) is simply given by the difference between the natural rate of growth (including Harrod neutral technical change) and the growth rate of the capital stock $g_k - \delta_k$. Similarly,

[12] See Chiarella *et al.* (2001a,b) for similar, but possibly much more severe situations of debt deflation, there with respect to credit relationships between asset holders and firms.

[13] We denote by γ the given trend growth rate in the world economy, which is used as a trend term in the investment equations that apply to the domestic economy.

the growth rate of the housing stock (per unit of the capital stock of firms) is given by the difference of the corresponding net accumulation rates. We will assume in the analytical treatment of the model that employment of firms adjusts with infinite speed ($\beta_l = \infty$) to the actual employment of their labour force so that there is no over- or under employment of this labour force. There thus remain only four laws of motion of quantities of which the first two will in addition be replaced by a static relationship in the further evaluation of the model.

Next we consider the nominal dynamics in the real sector of the economy, which are described by four dynamical laws. Note here that the laws of motion for wages, w^e, in efficiency units, and prices, p_y, are now formulated independently from each other[14] and show that reduced form PCs (exhibiting only price inflation) are generally not as simple as is often assumed in the literature:[15]

$$\begin{aligned}
\hat{w}^e &= \pi^c + \kappa[\beta_w(l^{de}/l^e - \bar{e}) + \kappa_w\beta_p(y/y^p - \bar{u})], \\
\hat{p}_y &= \pi^c + \kappa[\kappa_p\beta_w(l^{we}/l^e - \bar{e}) + \beta_p(y/y^p - \bar{u})], \\
\dot{\pi}^c &= \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p}_y - \pi^c) + (1 - \alpha_{\pi^c})(0 - \pi^c)), \\
\hat{p}_h &= \beta_h\left(\frac{c_h^w}{k_h} - \bar{u}_h\right) + \kappa_h\hat{p}_y + (1 - \kappa_h)\pi^c.
\end{aligned}$$

As already noted we now use reduced form PCs for wage inflation $\hat{w}^e$ and price inflation $\hat{p}_y$, which both depend on the demand pressures in the markets for labour as well as for goods, $y/y^p - \bar{u}$. The change of the rate of inflation expected over the medium run, π^c, is determined as a weighted average of adaptively formed expectations and regressive ones (which realise that the steady state rate of inflation is zero in the present model). Finally, the inflation rate for housing services depends on the demand pressure term $\left(\frac{c_h^w}{k_h} - \bar{u}_h\right)$ in the market for these services,[16] and on actual and perceived cost-push expressions, here simply based on a weighted average concerning the inflation rate of domestic output. We shall assume in the following that $\beta_{\pi^c} = 0$ holds and thus we will not consider the role of inflationary expectations (which would add extra instability to the model if the price level is adjusting with high speed and if β_{π^c} is chosen sufficiently large, the so-called Mundell effect of Keynesian type models that include a wage-price sector).

[14] We do not consider in the present simplified form of the model payroll taxes, and value-added taxes, which helps to simplify the notation.

[15] Such disentangled laws of motion for nominal prices and wages are obtained from their originally interdependent presentation – see the preceding section – by solving the two linear equations of this section with respect to the variables $\hat{w}^e - \pi^c$ and $\hat{p}_y - \pi^c$ that implies the expressions shown below, which make use of both of our measures of demand pressure on the market both for labour and for the goods market (and on expected medium-run inflation). It is intuitively obvious that the removal of wage or price inflation cost-push pressure ($\hat{w}^e$ and $\hat{p}_y$) from the original form of the price or wage dynamics must imply that both the goods and the labour market expressions will be present in the resulting disentangled PCs which thus are in a significant way more general than the ones usually considered in the theoretical or applied literature on price PCs (unless one assumes – as some sort of Okun's law – that all demand pressure variables used are positive multiples of each other).

[16] Where $\frac{c_h^w}{k_h}$ represents the rate of capacity utilisation on this market and $\bar{u}_h$ the corresponding NAIRU level.

Next follow the dynamical laws for long-term bond price dynamics and exchange rate dynamics (including expectations) which basically formulate a somewhat delayed adjustment towards interest rate parity conditions and are supplemented by heterogeneous expectations formation (of partially adaptive and partially perfect type). Note that perfect foresight, concerning the proportion $1-\alpha_s$ of market participants, can be removed from explicit representation as it coincides with the left-hand side of the corresponding price adjustment equation, giving rise to the fractions in front of these adjustment equations; see Part II for details):

$$
\begin{aligned}
\hat{p}_b &= \frac{\beta_{p_b}}{1-\beta_{p_b}(1-\alpha_s)}[(1-\tau_c)i_l + \alpha_s \pi_{bs} - (1-\tau_c)i], \\
\dot{\pi}_{bs} &= \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}), \\
\hat{s} &= \frac{\beta_s}{1-\beta_s(1-\alpha_s)}[(1-\tau_c)i_l^* + \alpha_s \epsilon_s - ((1-\tau_c)i_l + \pi_b)], \\
\dot{\epsilon}_s &= \beta_{\epsilon_s}(\hat{s} - \epsilon_s).
\end{aligned}
$$

These laws of motion are not made use of in the following since we assume in this chapter that the required rate of return i^r used in the description of investment of firms and asset holders is a given magnitude, measured by the world rate of interest i_l^* and since we will also assume that the measure for the tightness of monetary policy, $i_l - i$, is not involved in the formation of these investment plans (by setting the corresponding coefficients of the investment functions of the preceding section equal to zero). A further assumption needed to avoid any further discussion of these laws of motion will be provided when the next block of laws of motion, concerning the government sector, is considered.

Note with respect to the above equations that the literature generally only considers the border case where $\alpha_s = 0$ is used in conjunction with infinite adjustment speeds on the two considered markets. This gives rise to two interest parity conditions which, when coupled with myopic perfect foresight on bond price and exchange rate movements, give rise to a situation of knife edge instability – which is then stabilised by the adherents of the Rational Expectations School through the application of the jump variable technique to those variables they consider as non-predetermined.

The next set of dynamical laws concerns the evolution of short- and long-term debt of the government (the issuing of which is here governed by the fixed proportions $\alpha_b^g, 1-\alpha_b^g$), its wage and import taxation policy and the interest rate policy of the central bank. Note here that we continue to use the letter b to denote government debt per unit of capital and that its short-term debt, b_g, must now be indexed by g since there is also the debt of worker households (which we here denote by λ_w in order to stress their importance for the present investigation):[17]

[17] The expressions t^a, t^c, w^a represent tax payments out of wages and profits and transfer payments of the government that will be of no importance in the core 9D dynamics that is the focus of this chapter.

$$
\begin{aligned}
\dot{b}_g &= \alpha_b^g[gy^e + ib_g + b^l - t^a - t^c + w^a] - (\hat{p}_y + g_k - \delta_k)b_g, \\
\dot{b}^l &= (1-\alpha_b^g)[gy^e + ib_g + b^l - t^a - t^c + w^a]/p_b - (\hat{p}_y + g_k - \delta_k)b^l, \\
\hat{\tau}_w &= \alpha_{\tau_{w1}}\left(\frac{\lambda_g}{\bar{\lambda}_g} - 1\right), \quad \lambda_g = \frac{b_g + p_b b^l}{y^e}, \\
\hat{\tau}_m &= \alpha_{\tau_m}\frac{p_x^* x - (1+\tau_m)p_m^* j^d}{p_x^* x}, \quad (x = x_y y,\ j^d = j_y y), \\
\dot{i} &= -\beta_{i_i}(i - i_l^*) + \beta_{i_p}(\hat{p}_y - \bar{\pi}) + \beta_{i_u}(y/y^p - \bar{u}), \quad \bar{\pi} = 0.
\end{aligned}
$$

Since these laws of motion, up to the interest rate policy rule, are also suppressed by appropriate assumptions in the analysis that follows we here only briefly state that the first two are immediate consequences of the government budget constraint (based in particular on various sources of tax income, now diminished by subsidies that concern the interest payments of worker households), that wage taxation is here adjusted in the direction of a target ratio of government debt, $\bar{\lambda}_g$, and that import taxes are adjusted in a way that ensures a balanced trade account in the steady state (which greatly simplifies the calculation of the steady state of the model). The interest rate policy rule $\dot{i}$ is of interest however since it could be used in counteracting accelerating debt (wage) deflation, by lowering nominal interest rates in situations of depressed activity levels and price deflation. This rule has already been explained in the preceding section. We assume in the following that the wage tax rate is not adjusted at all ($\alpha_{\tau_{w1}} = 0$) and set equal to the steady state value of the general 19D model and that the import taxes are adjusted with infinite speed ($\alpha_{\tau_m} = \infty$). These two assumptions imply in the reduced formulation of the model given below that the evolution of government debt does not feed back into the core dynamics of the model and that the exchange rate does not matter for them (also due to the assumption of given world market prices for both import and export commodities).

As the nineteenth law of motion, which is not new to the model but is now interacting with its core dynamics due to the feedback on the spending behaviour of workers' households, we finally have

$$
\dot{\lambda}_w = (c_y + c_h - 1)y_w^D - (\hat{p}_y + g_k - \delta_k)\lambda_w,
$$

which determines the evolution of the debt to capital ratio of worker indebtedness to the other type of households, namely the asset holders. This law of motion, together with the possibility of housing cycles due to the investment in housing and the rate of return that characterises the housing sector and the possibility of price deflation, will be the focus of interest of this chapter. Note here that the debt to capital ratio λ_w influences its rate of change negatively as far as the term based on the disposable income of workers is concerned, since this income depends negatively on this ratio and since the sum of workers' marginal propensities to spend has been assumed to be larger than one. However, due to this situation, we also have that aggregate demand, economic activity and thus goods price inflation depend negatively on λ_w, which introduces a positive dependence between the rate of change of this ratio and its level. This is indeed the partial debt deflation mechanism of the model we have already described in the preceding section. Note here that we do not yet have credit rationing in the model which would establish a further channel by which aggregate demand may be reduced in deflationary episodes.

Summarising we can thus state that we will basically consider the following subdynamics of the general 19D dynamics in the next section and will do this by making use of further simplifications of these dynamics that allow for the possibility of an analytical treatment:

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + (\gamma - (g_k - \delta_k))y^e, \tag{11.17}$$

$$\dot{\nu} = y - y^d - (g_k - \delta_k)\nu, \tag{11.18}$$

$$\hat{l}^e = \gamma - (g_k - \delta_k), \tag{11.19}$$

$$\hat{k}_h = g_h - \delta_h - (g_k - \delta_k), \tag{11.20}$$

$$\hat{\omega}^e = \kappa[(1-\kappa_p)\beta_w(l^{de}/l^e - \bar{e}) - (1-\kappa_w)\beta_p(y/y^p - \bar{u})], \tag{11.21}$$

$$\hat{p}_y = \kappa[\kappa_p\beta_w(l^{de}/l^e - \bar{e}) + \beta_p(y/y^p - \bar{u})], \tag{11.22}$$

$$\hat{p}_h = \beta_h\left(\frac{c_h^w}{k_h} - \bar{u}_h\right) + \kappa_h\hat{p}_y, \tag{11.23}$$

$$\dot{\lambda}_w = (c_y + c_h - 1)y_w^D - (\hat{p}_y + g_k - \delta_k)\lambda_w, \tag{11.24}$$

$$\dot{i} = -\beta_{i_i}(i - i_l^*) + \beta_{i_p}\hat{p}_y + \beta_{i_u}(y/y^p - \bar{u}). \tag{11.25}$$

Note in these laws of motion we use the real wage $\omega^e = w^e/p_y$ in the place of the nominal wage. These laws of motion make use of the following supplementary intensive form definitions and abbreviations (which are not explained here in detail since we only provide the new features of the modelling approach of Part II):[18]

$y = y^e + \beta_n(\beta_{n^d}y^e - \nu) + \gamma\beta_{n^d}y^e$ (output per unit of capital),

$l^{de} = l_y^e y, \quad l_y^e$ (the labour coefficient in efficiency units),

$y_w^D = (1-\tau_w)\omega^e l^{de} - (1-\tau_c)i\lambda_w$ (real disposable income of workers),

$c_g^w = c_y y_w^D$ (goods consumption of workers – including the purchase of houses),

$c_h^w = p_y c_h y_w^D / p_h$ (housing services consumption of workers),

$r^e = y^e - \delta_k - \omega^e l^{de}$ (the expected real rate of profits of firms),

$g_k = \alpha_r^k(1-\tau_c)(r^e - i_l^*) + \alpha_u^k(y/y^p - \bar{u}) + \gamma + \delta_k$ (gross investment of firms),

$r_h = (p_h/p_y)c_h^w/k_h - \delta_h$ (real rate of profit for housing investment),

[18] Note that output y differs from expected sales y^e due to voluntary inventory investments of firms.

$g_h = \alpha_r^h(1-\tau_c)(r_h - i_l^*) + \alpha_u^h\left(\frac{c_h^w}{k_h} - \bar{u}_h\right) + \gamma + \delta_h$ (gross investment in housing),

$y^d = c_g^w + g_k + g_h k_h + g y^e$ (aggregate demand – including government demand gy^e),

$\tau_w = 1 - (y_{wo}^D + (1-\tau_c)i_l^*\lambda_{wo})/(\omega_o^e l_o^{de})$ (the taxation rule).

Insertion of these equations into the above nine laws of motion gives an explicit system of nine autonomous non-linear differential equations in the nine state variables of the model shown in equations (11.17)–(11.25). Note that we have removed pension payments and unemployment benefits from the above presentation of the model. Note also that the evolution of price levels is subject to zero-root hysteresis, since these depend on historical conditions due to our assumption on the interest rate policy rule of the central bank and the accompanying assumption of costless cash balances (during each trading period) for the four agents of the model; see Chiarella and Flaschel (1999a) for details.

We present next the nine steady state values of the model (including further definitional equations that are needed for their full determination). All of these values should have an index 'o' (denoting their steady state character); in order not to overload the notation we do not add this index to the following list of steady state values. Note that the steady state values of level magnitudes are all expressed in per unit of capital form and if necessary in efficiency units; see Chiarella *et al.* (2003b) for the details in the case of the 18D core model. Note also that we have now debt of workers and of the government in the steady state of the model and that we therefore denote their actual and steady state debt to capital ratios by choosing appropriate indices λ_w, λ_g in both cases.[19] Note finally that the steady state is parametrically dependent on a given output price level p_y which is not determined by the model (due to the Taylor type interest rate policy pursued by the central bank) and thus can be supplied from the outside in an arbitrary fashion:[20]

$$y^e = \frac{y}{1+\gamma\beta_{n^d}} \quad (y = y^p\bar{u}), \tag{11.26}$$

$$\nu = \beta_{n^d}y^e, \tag{11.27}$$

$$l^e = l^{de}/\bar{e} \quad (l^{de} = l_y^e y), \tag{11.28}$$

$$k_h = \frac{(1-g)y^e - (\gamma+\delta_k)}{c_y(i_l^* + \delta_h)/c_h + \gamma + \delta_h} \quad (c_h^w = \bar{u}_h k_h), \tag{11.29}$$

$$p_h/p_y = (i_l^* + \delta_h)/\bar{u}_h, \tag{11.30}$$

[19] Here λ_g is the aggregate debt of the government sector.

[20] The inflation target of the central bank, $\bar{\pi}$, is a zero rate of inflation here (which is not true for actual central bank behaviour in general). This implies in the present model that steady state inflation will be zero, too, which in turn implies that the levels of nominal magnitudes are fixed in the steady state (in efficiency units solely as far as nominal wages are concerned).

$$y_w^D = (p_h/p_y)k_h\bar{u}_h/c_h, \tag{11.31}$$

$$y_{w1}^D = y_w^D + (1-\tau_c)i_l^*\lambda_w, \tag{11.32}$$

$$\tau_w = 1 - y_{w1}^D/(\omega^e l^{de}), \tag{11.33}$$

$$\omega^e = \frac{y^e - \delta_k - i_l^*}{l^{de}} \quad (w^e = \omega^e p_y), \tag{11.34}$$

$$p_y = \text{determined by initial conditions}, \tag{11.35}$$

$$p_h = p_y(i_l^* + \delta_h)/\bar{u}_h, \tag{11.36}$$

$$i = i_l^*, \tag{11.37}$$

$$\lambda_w = \frac{c_y + c_h - 1}{\gamma} y_w^D. \tag{11.38}$$

Note that λ_w is positive in the steady state due to our assumption that $c_y + c_h > 1$, so that workers' debt grows in line with the capital stock in the steady state (as do workers' interest payments). Note also again that the steady state is inflation-free due to our assumption about monetary policy and that nominal wages rise with labour productivity in the steady state.

Equation (11.26) gives the steady state solution of expected sales y^e per unit of capital K (and also of output y per K) as determined by the desired utilisation rate of firms and the inventory policy they have to adopt due to demand growth in the steady state. Equation (11.27) provides the steady inventory-capital ratio N/K, which says that inventories (to be produced in addition to actual sales) must grow at the same speed as the capital stock. Equation (11.28) represents (in efficiency units) the amount of workforce (per K) employed by firms in the steady state as well as full employment labour intensity which is larger than actual labour intensity (in efficiency units) due to the assumed NAIRU rate of employment, $\bar{e} < 1$. The last expression for the quantity side of the model – in equation (11.29) – provides the steady value of the housing capital stock per unit of the capital stock of firms, which is obtained on the basis of the calculation of the income magnitudes shown and the debt to capital ratio of worker households to be determined below. Equation (11.34) concerns the wage level (in efficiency units), real and nominal, to be derived from the steady state value for the rate of profit which is given by the world rate of interest i_l^*. Note that all nominal magnitudes (up to nominal wages) exhibit no long-run trend and that the steady price level of output prices p_y is not determined by the model.

As remaining nominal magnitude we have the price level p_h for housing rents (in equation (11.36)), to be calculated from the uniform rate of interest i_l^* of the economy in the steady state (which also characterises the rate of return in the housing sector). There follows the steady value of the debt to capital ratio λ_w of workers, the only debt ratio to be considered in the following due to the assumption of a given wage tax rate. With respect to the public sector, there is finally the interest rate policy rule of the central bank, which due to its formulation implies that the interest rate on short-term

government debt must also settle down at the given foreign rate, i_l^*, in the steady state. Again, the new equation is equation (11.37), where the steady state debt to capital ratio of workers is easily obtained from their budget constraint of workers and is positive if and only if $c_y + c_h > 1$ holds true. This closes the presentation of the interior steady state solution of our reduced 9D dynamical model. We note that the debt to capital ratio of workers rises with $c_y + c_h$, y_{w1}^D and τ_c and falls with γ and i_l^*.

11.4 2D, 3D and 5D subcases of integrated 6D real subdynamics

Using an approach similar to that of Chapter 6 of Part II of the book we simplify the 9D dynamics of the preceding section further, by assuming in place of the Metzlerian delayed feedback chain that is based on a goods market disequilibrium adjustment process of the type

$$y^d = c_y y_w^D + \alpha_r^k(r^e - i_l^*) + \alpha_u^k(y/y^p - \bar{u}) + \gamma + \delta_k \to y^e \to y \to y^d,$$

a simplified static and linearised[21] equilibrium relationship of the kind

$$y^d = y^e = y = y(\omega^e, \lambda_w) = \bar{u}y^p + d_w(\omega^e - \omega_o^e) + d_\lambda(\lambda_w - \lambda_w^o), \qquad d_w, d_\lambda \le 0.$$

This relationship between output, real wages and real debt will be used in the following as a shortcut for the delayed feedback chain of the general case (and its richer concept of aggregate demand and its determinants) in order to study the effects of wage and price inflation and deflation on debt and real wages in a significantly simplified version of the 9D model. Note that this shortcut of the originally delayed quantity adjustment process of Metzlerian type requires that the steady state value of this function y must be equal to $y^p\bar{u}$ in order to get a steady state solution for this 7D simplification of the 9D dynamics. Note also that we concentrate in this presentation of goods market equilibrium on the effects of real wage increases and debt ratio increases which both are assumed to have a (non)-negative influence on goods market behaviour, that is in the case of real wages that the resulting decrease in investment demand outweighs the implied increase in consumption.

Let us furthermore assume that $\kappa_h = 1$ holds, so that the cost-push term in the dynamics of rental prices is given solely by the current rate of inflation on the market for goods. This assumption allows us to reduce the dynamics to a consideration of relative prices only, namely the real wage (as before) and the real rental price. On the basis of this assumption and the above short cut for goods market dynamics the dynamical system to be investigated in the following reads

$$\hat{l}^e = \gamma - (g_k - \delta_k), \tag{11.39}$$

$$\hat{k}_h = g_h - \delta_h - (g_k - \delta_k), \tag{11.40}$$

$$\hat{\omega}^e = \kappa[(1-\kappa_p)\beta_w(l^{de}/l^e - \bar{e}) - (1-\kappa_w)\beta_p(y/y^p - \bar{u})], \tag{11.41}$$

21 Around the interior steady state of the model, given by ω_o^e, λ_w^o.

$$\hat{q}_h = \beta_h \left(\frac{c_h^w}{k_h} - \bar{u}_h \right) \quad (q_h = p_h/p_y), \tag{11.42}$$

$$\dot{\lambda}_w = (c_y + c_h - 1) y_w^D - (\kappa[\kappa_p \beta_w (l^{de}/l^e - \bar{e}) + \beta_p (y/y^p - \bar{u})] + g_k - \delta_k)\lambda_w, \tag{11.43}$$

$$\dot{i} = -\beta_{i_i}(i - i_l^*) + \beta_{i_p}\kappa[\kappa_p \beta_w (l^{de}/l^e - \bar{e}) + \beta_p (y/y^p - \bar{u})] + \beta_{i_u}(y/y^p - \bar{u}), \tag{11.44}$$

now with the static relationships

$$y = \bar{u} y^p + d_w(\omega^e - \omega_o^e) + d_\lambda(\lambda_w - \lambda_w^o) \quad (d_w, d_\lambda \le 0),$$
$$l_f^{de} = l_y^e y,$$
$$y_w^D = (1 - \tau_w)\omega^e l^{de} - (1 - \tau_c) i \lambda_w,$$
$$c_h^w = c_h (1 - \tau_w) y_w^D / q_h,$$
$$r = y - \delta_k - \omega^e l^{de},$$
$$g_k = \alpha_r^k (1 - \tau_c)(r - i_l^*) + \alpha_u^k (y/y^p - \bar{u}) + \gamma + \delta_k,$$
$$r_h = q_h c_h^w / k_h - \delta_h = c_h (1 - \tau_w) y_w^D / k_h - \delta_h,$$
$$g_h = \alpha_r^h (1 - \tau_c)(r_h - i_l^*) + \alpha_u^h \left(\frac{c_h^w}{k_h} - \bar{u}_h \right) + \gamma + \delta_h,$$
$$\tau_w = 1 - y_{w1}^D / (\omega^e l^{de}) \quad (y_{w1}^D = y_w^D + (1 - \tau_c) i_l^* \lambda_w).$$

Neglecting the interest rate policy (11.44) of the monetary authority for the moment (by setting the corresponding adjustment parameters equal to ∞, 0 and 0, respectively which implies $i = i_l^*$) we have that interest payments of workers are based on a given rate of interest. The resulting 5D dynamics are then based on the growth laws for full employment labour intensity, housing capital per unit of capital, real wages and real rental prices and finally as the financial variable the debt to capital ratio of worker households.

The underlying interior steady state solution of these 5D dynamics (and also of the 6D dynamics) is characterised by

$$y = y^p \bar{u},$$
$$l^{de} = l_y^e y,$$
$$l^e = l^{de}/\bar{e},$$
$$r = r_h = i_l^*,$$
$$\omega^e = \frac{y - \delta_k - i_l^*}{l^{de}},$$
$$q_h = (i_l^* + \delta_h)/\bar{u}_h,$$

$$\lambda_w = \frac{c_y + c_h - 1}{\gamma + (c_y + c_h - 1)(1 - \tau_c) i_l^*} y_{w1}^D, \quad y_{w1}^D = (1 - \tau_w)\omega^e l^{de},$$
$$c_h^w = c_h y_w^D / q_h, \quad y_w^D = y_{w1}^D - (1 - \tau_c) i_l^* \lambda_w,$$
$$k_h = c_h^w / \bar{u}_h,$$
$$\tau_w = 1 - y_{w1}^D / (\omega^e l^{de}).$$

The above 5D dynamical system can be subdivided further into a natural real growth cycle model of the Goodwin (1967), Rose (1967) type concerning the interaction of capital accumulation and income distribution and into a 3D dynamical system where we can study the interaction of the growth of the housing stock (for rental purposes) with real rental price adjustments and the debt to capital ratio of workers used in particular to finance their investment into their own stock of houses.

In order to obtain the first set of subdynamics we have to assume in addition to the assumptions already made that $d_\lambda = 0$ holds true, so that there is no debt effect with respect to the state of the goods market used in the following dynamical subsystem for describing real wage and investment dynamics. The resulting dynamics in the full employment labour intensity l^e and the real wage ω^e are basically of Goodwin (1967) growth cycle type augmented by Rose (1967) type effects of the real wage on its rate of change (as we shall see in detail below), namely

$$\hat{l}^e = -(\alpha_r^k (1 - \tau_c)(r - i_l^*) + \alpha_u^k (y/y^p - \bar{u})), \tag{11.45}$$

$$\hat{\omega}^e = \kappa[(1 - \kappa_p)\beta_w (l^{de}/l^e - \bar{e}) - (1 - \kappa_w)\beta_p (y/y^p - \bar{u})]. \tag{11.46}$$

We now have as remaining static relationships

$$y = \bar{u} y^p + d_w(\omega^e - \omega_o^e) \quad (d_w < 0),$$
$$(l^{de} = l_y^e y),$$
$$r = y - \delta_k - \omega^e l^{de}.$$

The assumption $d_w < 0$ represents what we call a negative Rose effect since it implies that wage flexibility is stabilising and price flexibility destabilising just as in the original contribution of Rose (1967).

The steady state of these 2D dynamics is characterised by

$$y = y^p \bar{u},$$
$$l^{de} = l_y^e,$$
$$l^e = l^{de}/\bar{e},$$
$$r = i_l^*,$$
$$\omega^e = \frac{y - \delta_k - i_l^*}{l^{de}}.$$

Assuming on the other hand no fluctuations in the capital stock of firms, $\hat{K} = g_k - \delta_k = \gamma$, and in the real wage of workers, $\omega^e = \omega_o^e$, by contrast gives rise to interacting dynamics between the stock of houses, $k_h = K - h/\mathrm{K}$, offered for rent on the market for housing services, the real rental price of housing services, $q_h = p_h/p_y$, and the real debt to capital ratio $\lambda_w = \Lambda_w/(p_y K)$:[22]

$$\hat{k}_h = \alpha_r^h(1-\tau_c)(r_h - i_l^*) + \alpha_u^h(c_h^w/k_h - \bar{u}_h), \tag{11.47}$$

$$\hat{q}_h = \beta_h\left(\frac{c_h^w}{k_h} - \bar{u}_h\right), \tag{11.48}$$

$$\dot{\lambda}_w = (c_y + c_h - 1)y_w^D - (\kappa[\kappa_p\beta_w(l^{de}/l_o^e - \bar{e}) + \beta_p(y/y^p - \bar{u})] + \gamma)\lambda_w, \tag{11.49}$$

here with the static relationships:

$$\begin{aligned}
y &= \bar{u}y^p + d_\lambda(\lambda_w - \lambda_w^o) \quad (d_\lambda < 0),\\
l^{de} &= l_y^e y,\\
y_w^D &= (1-\tau_w)\omega_o^e l^{de} - (1-\tau_c)i_l^*\lambda_w,\\
c_h^w &= c_h y_w^D/q_h,\\
r_h &= q_h c_h^w/k_h - \delta_h,
\end{aligned}$$

which reduces further to a 2D system where the evolution of debt does not matter if $c_h^w = const$ is assumed.

The interior steady state of these 3D dynamics is characterised by

$$\begin{aligned}
y &= y^p\bar{u},\\
l^{de} &= l_y^e y,\\
r_h &= i_l^*,\\
q_h &= (i_l^* + \delta_h)/\bar{u}_h,\\
\lambda_w &= \frac{c_y + c_h - 1}{\gamma + (c_y + c_h - 1)(1-\tau_c)i_l^*}y_{w1}^D, \quad y_{w1}^D = (1-\tau_w)\omega^e l^{de},\\
c_h^w &= c_h y_w^D/q_h, \quad y_w^D = y_{w1}^D - (1-\tau_c)i_l^*\lambda_w,\\
k_h &= c_h^w/\bar{u}_h.
\end{aligned}$$

Note that the steady state values can be obtained from the above dynamics in this order and that these calculations in particular imply that the excess demand situations underlying the β_w, β_p terms in the dynamics (11.47)–(11.49) are both zero in the steady state which in particular again implies that the price level is stationary in the steady

[22] Note here that $\hat{K} = \gamma$ implies $l^e = l_o^e$ if we start from the steady state and if shocks only occur in the state variables of the presently considered dynamics.

state. Note also that this implies that the steady state value of λ_w is uniquely determined, as is claimed above.

We stress that the study of these partial subdynamics is not to be considered as being completely specified from the economic point of view, but should be viewed as an approach, eventually leading back to the fully specified 9D dynamics, that generates propositions of the 'as if' variety, in the case of the above 3D dynamics, as if the debt effect on output can be considered as the one that dominates the outcome of the interaction of aggregate demand, sales expectations and output decisions of firms on the market for goods (as formulated for the full 9D dynamics).

With respect to these latter dynamics we now have as first propositions:

Proposition 11.1 *Assume that c_h^w is fixed at its steady state value ($\hat{C}_h^w = \gamma$). Then: the steady state of the dynamics (11.47), (11.48) is globally asymptotically stable for all positive starting values for k_h, q_h, so that all trajectories in the positive orthant of $\Re^2$ converge to the steady state values of k_h, q_h shown above.*

Proof: Concerning the Jacobian or linear part of the growth dynamics (11.47), (11.48), and paying no attention to the fact that we have growth rates in the place of time derivatives on the left-hand side, we get at all positive tuples (k_h, q_h) the qualitative expression

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} - & + \\ - & 0 \end{pmatrix}.$$

We thus in particular have trace $J < 0$, det $J > 0$ and $J_{12}J_{21} \neq 0$. As shown in Flaschel (1984) these three conditions imply the assertion, due to a particular application of Olech's theorem on global asymptotic stability. □

We observe with respect to Proposition 11.1 and its proof that these dynamics would be of the Goodwin (1967) centre type were there not the negative (stabilising) influence of the state variable k_h on its own evolution.

The next proposition adds the influence of the debt to capital ratio to the dynamics just considered (via the effect this ratio has on the output of firms) and it of course adds also the budget law that determines the evolution of real debt per unit of capital. Contrary to what one might expect, we here find that these additional aspects do not endanger the stability result just obtained, if price adjustment is sufficiently sluggish, which due to the increased dimension of the dynamics can now however only be shown in an appropriately chosen neighborhood of the steady state. Yet, the included debt effects will be destabilising if the adjustments caused by goods and labour market disequilibrium in the wage-price module of the model become sufficiently pronounced.

Proposition 11.2 *The steady state of the dynamics (11.47)–(11.49) is locally asymptotically stable, if the parameters β_p and κ_p (or β_w) are chosen sufficiently small (such that $J_{33} < 0$ holds; see the proof). Conversely, this steady state will be unstable if the parameters β_p or β_w, the latter for $\kappa_p > 0$, are chosen sufficiently large (such that $J_{33} > 0$ holds).*

Proof: Due to the continuity of eigenvalues with respect to parameter changes we only need to consider the assertion of local asymptotic stability in the case where $\beta_p = \kappa_p = 0$ holds. The Routh–Hurwitz theorem then states that all eigenvalues of the considered Jacobian will have negative real parts if the Routh–Hurwitz coefficients fulfil $a_1 = -\text{trace}\, J > 0$, $a_2 = J_1 + J_2 + J_3 > 0$, $a_3 = -\det J > 0$ and finally $a_1 a_2 - a_3 > 0$, a situation which, as just stated, is not changed if small variations of the parameters β_p and κ_p away from zero are allowed for. Note here that the coefficient a_2 represents the sum of the principal minors of order 2 of the Jacobian J.

It is easy to show that the trace $J_{11}+J_{22}+J_{33}$ of the Jacobian J must be negative in the assumed situation, since all auto-feedbacks of the system (11.47)–(11.49) are negative, thus all three coefficients making up the trace are negative here. Concerning J_1, J_2 and J_3, whose indices refer to the row and column not considered in these subdeterminants, one also gets immediately (from what has just been shown for the trace) that both J_1 and J_2 must be positive, since J_{31} and J_{32} are both zero so that only multiplication with respect to elements from the diagonal of J is involved here. With respect to J_3 one gets furthermore that the dynamical law for k_h can be reduced to

$$\hat{k}_h = \alpha_r^h (1 - \tau_c) q_h,$$

without changing the sign of J_3, by making use of the linear dependencies that exist with respect to the second dynamical law (for q_h) that is involved in the calculation of J_3. From this we see qualitatively that

$$J_3 = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} 0 & + \\ - & - \end{pmatrix} > 0,$$

as was claimed above.

Calculating $\det J$ one can use a similar linear dependency in addition in order to arrive at

$$\det J = \begin{vmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{vmatrix} = \begin{vmatrix} 0 & + & 0 \\ - & - & - \\ 0 & 0 & - \end{vmatrix}.$$

This not only shows that $\det J$ must be negative, but also that $-\det J$ must be equal to or smaller than $J_1(-J_{33})$ which finally gives that also $a_1 a_2 - a_3 > 0$ must hold true, since $a_1 a_3$ is based on positive expressions throughout.

Concerning the second assertion, on instability, one simply has to note that the third law of motion implies (with positive parameters β_p, κ_p and β_w) for the entry J_{33} of J at the steady state:

$$J_{33} = (c_y + c_h - 1)((1 - \tau_w)\omega_o^e l_y^e d_\lambda - (1 - \tau_c) i_l^*) - \gamma$$
$$+ \kappa[\kappa_p \beta_w l_y^e (-d_\lambda)/l_o^e + \beta_p (-d_\lambda)/y^p]\lambda_w.$$

This immediately shows that trace J can be made as positive as is desired by choosing either β_p or β_w (the latter for $\kappa_p > 0$) sufficiently large, since $-d_\lambda > 0$ and $\lambda_{w^o} > 0$ hold.

Note finally that in the present formulation of the dynamics (11.47)–(11.49) we always have that the third law of motion is independent of the other ones, so that J_{31} and J_{32} are always zero (which simplifies the above stability arguments further). The benchmark for asymptotic stability therefore is the situation where $J_{33} < 0$ holds true and instability in the present situation is therefore solely due to the law of motion for the debt to capital ratio λ_w. □

In view of this last observation on the (in)stability of the model we should however stress that we have approached this proposition and its proof from a slightly more general perspective than was really necessary, in order to indicate how it can be applied to more general situations than considered above. Assume for example that the marginal propensities to consume c_y and c_h both depend positively on the relative price for housing services q_h such that real expenditure on housing services c_h^w depends negatively on q_h (but, as assumed, not nominal expenditures on these services). Assume also that domestic output y depends positively on q_h. Proposition 11.2 basically also holds true in such an augmented situation, since trace J stays negative, since linear dependencies again imply that J_1, J_2 and J_3 are all positive and since $\det J$ can in this way be reduced to the form

$$\det J = \begin{vmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{vmatrix} = \begin{vmatrix} 0 & + & 0 \\ - & 0 & 0 \\ 0 & 0 & - \end{vmatrix},$$

which again shows that this determinant is negative and dominated by the positive expressions in $a_1 a_2$. The situation of Proposition 2 therefore can be generalised to cases where the third law of motion is no longer independent of the other two differential equations.

Proposition 11.3 *The steady state of the dynamics (11.47)–(11.49), if locally asymptotically stable, is never globally asymptotically stable, but will be explosive in the debt to capital ratio if this ratio is chosen sufficiently large.*

Proof: We know in the assumed situation that $J_{33} < 0$ holds true at the steady state. Considering the right-hand side of equation (11.49) it is, however, obvious from the preceding proof that there must be a second root of this equation; where $\dot{\lambda}_w = 0$ holds and where $J_{33} > 0$ is true. This follows from the fact that the right-hand side of this equation is a polynomial of order 2 in the state variable λ_w with a positive coefficient in front of the λ_w^2 term. To the right of this root, the debt to capital ratio will increase beyond any bound, since $\dot{\lambda}_w > 0$ is then given for all points in time. □

Let us now consider the other subdynamics (11.45)–(11.46) of the 5D system (11.39)–(11.43) where it is assumed that the rate of interest on the debt of workers is a given magnitude ($= i_l^*$) and not subject to policy considerations by the central bank. Neglecting again the growth rate formulation of these dynamics, the Jacobian of the right-hand side of this system reads, for all points in the state space,

$$J = \begin{pmatrix} 0 & -\alpha_r^k(1-\tau_c)r_{\omega^e} - \alpha_u^k d_w/y^p \\ -\kappa(1-\kappa_p)\beta_w l^{de}(l^e)^{-2} & \kappa(1-\kappa_p)\beta_w l_y^e d_w/l^e + \kappa(1-\kappa_w)\beta_p(-d_w)/y^p \end{pmatrix},$$

with r_{ω^e} given by $d_w(1-\omega^e l_y^e) - l_y^e y < 0$.

Proposition 11.4 *Assume that $\beta_p = 0$ (or $\kappa_w = 1$) holds. The interior steady state of the dynamical system (11.45) and (11.46) is globally asymptotically stable for all positive starting values l^e and ω^e, so that all trajectories in the positive orthant of $\Re^2$ converge to this steady state in the current situation.*

Proof: Concerning the Jacobian J just calculated we get in this case for all points of $\Re^2$ the qualitative expression

$$J = \begin{pmatrix} 0 & + \\ - & - \end{pmatrix}.$$

We thus in particular have trace $J < 0$, det $J > 0$ and $J_{12}J_{21} \neq 0$ and so obtain the asserted global asymptotic stability as in Proposition 11.1 by an appropriate application of Olech's theorem on global asymptotic stability. □

This method of proof cannot be applied in the case $\beta_p > 0$ since we then have opposing signs in the element J_{22} of the trace of J with respect to the β_w and β_p expressions. Trace J may therefore change its sign (for large l^e for example) in the considered state space, although it may be negative at the steady state and thus imply local asymptotic stability, but not global asymptotic stability. In view of this we define a critical value for the parameter β_p (in the case $\kappa_w < 1$, for the steady state) by the expression

$$\beta_p^H = \frac{1-\kappa_p}{1-\kappa_w}\frac{\beta_w l_y^e/l^e}{y^p}.$$

With respect to this value we then get:

Proposition 11.5 *1. The interior steady state of the dynamical system (11.45), (11.46) is locally asymptotically stable for $\beta_p < \beta_p^H$.*

2. It is unstable for $\beta_p > \beta_p^H$.

3. At β_p^H there occurs a Hopf bifurcation, where the steady state loses its stability (in general) by way of the death of an unstable limit cycle or the birth of a stable limit cycle as this parameter value is crossed from below.

Proof: We have $J_{33} = 0$ at the bifurcation point and < 0 (> 0) to the left (to the right) of it, which proves the first two assertions since det $J > 0$. The third assertion is a standard one in the case where det $J > 0$ holds throughout at the steady state. □

We observe that assertion 3 also holds with respect to Proposition 11.2 in a similar and more trivial way (although the resulting dynamical system is formally seen to be of dimension three). In sum we therefore have the result that increasing price flexibility

may be dangerous for asymptotic stability for two reasons, applying in two different subdynamics of the 5D dynamics of this section; due to its adverse effects on the debt to capital ratio (a Fisher debt effect) and due to its adverse effect on real wage adjustment (a Rose effect). We expect of course that these two destabilising mechanisms are jointly present in the integrated 5D dynamics and thus do not overthrow economic intuition when brought together in a higher dimensional environment.

This is easily shown for the dynamical system (11.39)–(11.43) since the Jacobian J of this growth rate system at the steady state reads with respect to the elements that depend on the parameter β_p:

$$J = \begin{pmatrix} -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- \\ -- & -- & \kappa(1-\kappa_w)\beta_p(-d_w)/y^p & -- & \kappa(1-\kappa_w)\beta_p(-d_\lambda)/y^p \\ -- & -- & -- & -- & -- \\ -- & -- & \kappa\beta_p(-d_w)\lambda_w/y^p & -- & \kappa\beta_p(-d_\lambda)\lambda_w/y^p \end{pmatrix} \tag{11.50}$$

Note that this expression only applies to the steady state of the dynamics and that we have used in this respect in particular that inflation is zero in the steady state. Obviously the trace expressions and the instability arguments based on them in the case of the disentangled 2D and 3D dynamics considered earlier apply again, showing that the trace of J can be made positive if the parameter β_p is chosen sufficiently large. Note however that the point where trace J becomes zero, and positive thereafter, is now not given by a simple expression.

Proposition 11.6 *1. The interior steady state of the 5D dynamical system (11.39)–(11.43) is locally asymptotically stable if it is assumed that y_w^D depends positively on the real wage ω_e, if the parameters β_p, β_w and β_h are sufficiently small, and if κ_p is sufficiently close to 1.*

2. Asymptotic stability gets lost by way of a Hopf bifurcation, at least in the case where $\kappa_p < 1$ holds (no stabilising real wage based Rose effect), if the parameter β_w is sufficiently large.[23]

3. Increasing the parameter β_p leads from a negative to a positive determinant of the Jacobian of the considered dynamics at the steady state, so the loss of stability need not occur via a Hopf bifurcation as the parameter β_p is increased, since real parts of eigenvalues may now become positive by a movement along the real line.

We observe that the assumption that y_w^D depends positively on the real wage is a plausible one since it means that labour demand, which depends negatively on the real wage, is not so sensitive in this respect that the wage sum is in fact decreased by an increase in the real wage. The mathematical condition underlying this assumption is that output elasticity with respect to real wages (in absolute terms) is less than 1 which is true at the steady state if the condition $(-d_w)\omega^e < \bar{u}y^p$ holds.

[23] Loss of stability is not obvious for the parameter β_h, but is of the same type if it occurs.

Proof: 1. Let us first consider the case where $\beta_p = 0$, $\beta_h = 0$ and $\kappa_p = 1$ hold and where therefore ω^e and q_h stay fixed at their steady state values. The remaining 3D system in the state variables l^e, k_h, λ_w (in this order) then gives rise to a Jacobian J at its interior steady state which is of the form

$$J = \begin{pmatrix} 0 & 0 & + \\ 0 & - & - \\ - & 0 & - \end{pmatrix},$$

if the parameter β_w is chosen sufficiently small that $J_{33} < 0$ holds. It is again easy to show that the Routh–Hurwitz conditions are fulfilled in such a case, in the same way as they were shown to hold in Proposition 2.

Let us next investigate the case where $\beta_p = 0$, $\beta_h > 0$ and $\kappa_p = 1$ holds so that the resulting dynamics therefore have become of dimension four (with q_h as the fourth state variable). It is then again easy to show that the determinant of the enlarged Jacobian can be reduced to the form (if the assumption on β_w is again made)

$$\det J = \begin{vmatrix} 0 & 0 & + & 0 \\ 0 & - & - & 0 \\ - & 0 & - & 0 \\ 0 & - & - & - \end{vmatrix}.$$

This determinant is therefore positive (since the upper 3×3 minor has been shown to be negative). Parameter values β_h sufficiently close to zero therefore imply that the real parts of the three eigenvalues which were negative (in the case $\beta_h = 0$) must stay negative also for small positive β_h which implies that the fourth eigenvalue will move from zero to a negative value in order to have a positive determinant of the Jacobian of the 4D system.

We now move in the same way from $\beta_p = 0$ and $\kappa_p = 1$ to values of these parameters sufficiently close to this situation. This then gives a 5D system whose fifth eigenvalue is no longer zero by necessity. We show again that the determinant of the Jacobian of this 5D system is negative and thus get in the same way as in the preceding step that the fifth eigenvalue must change from zero to a negative value in order to fulfil the condition on the determinant just stated. Therefore if the parameter changes are again that small that the negative real parts of the first four eigenvalues remain negative we get in sum that all real parts of the eigenvalues of the Jacobian of the full 5D dynamics must be negative. So the interior steady state is in fact locally asymptotically stable under the stated conditions (the proof has in fact shown that there are at least three real eigenvalues in such a situation).

It remains to show that the determinant of the 5D Jacobian is indeed negative under the stated conditions. To this end we first of all observe that the right-hand side equations

of the dynamical system (11.39)–(11.43) can be reduced to the following expressions (in the case $\beta_p = 0$):

$$\begin{aligned} &(\hat{k}_h)\ldots && +q_h, \\ &(\hat{q}_h)\ldots && -k_h, \\ &(\hat{l}^e)\ldots && -g_k, \\ &(\hat{\omega}^e)\ldots && +y/l^e, \\ &(\dot{\lambda}_w)\ldots && +y_w^D, \end{aligned}$$

without change in the sign of the determinant. Hence we obtain the following sign structure

$$\det J = \begin{vmatrix} 0 & + & 0 & 0 & 0 \\ - & 0 & 0 & 0 & 0 \\ ? & ? & 0 & + & + \\ ? & ? & - & - & - \\ ? & ? & 0 & + & - \end{vmatrix} < 0,$$

which gives the desired result.

2. Assertion 2 is easy to show in the case $\kappa_p = 1$ since we then have that the parameter β_w is only present in the fifth law of motion and there with a positive effect on the trace of J via

$$J_{55} = \kappa \beta_w l_y^e (-d_\lambda)/l_o^e \lambda_w,$$

which means that the trace of J can be made positive if β_w is chosen sufficiently large. Note that things are more difficult in the case $\kappa_p < 1$ since we then have a stabilising Rose effect of wage flexibility, which counteracts the destabilising debt deflation effect of wage-price inflation (of the case $\kappa_p = 1$) just considered.

3. In order to prove this assertion we have to calculate that part of the considered determinant of the 5D system which depends on the parameter β_p. We again only show the items that are relevant for this calculation (where $\beta_p > 0$ now holds):

$$\det J(\beta_p) = \begin{vmatrix} 0 & + & 0 & 0 & 0 \\ - & 0 & 0 & + & - \\ 0 & 0 & 0 & + & + \\ ? & ? & ? & +\beta_p & +\beta_p \\ ? & ? & + & ? & ? \end{vmatrix}.$$

Here, $\det J(\beta_p)$ denotes only that part of the determinant which in fact depends on the parameter β_p. Inspecting the original 5D dynamics one of course notes that the parameter β_p also appears in its fifth law of motion, but that it can be removed from the row of the corresponding Jacobian with respect to the calculation of their signs by means of an appropriate multiple of the fourth row without change in the qualitative structure of the remaining terms. Furthermore, it can be shown that β_p can be removed from the fourth column of the above determinant (by means of $(-d_w/d_\lambda)$ times the fifth column) without changing the positive sign in J_{34}.[24] This implies as remaining terms for the considered determinant in its dependence on the parameter β_p:

$$\det J(\beta_p) = \begin{vmatrix} 0 & + & 0 & 0 & 0 \\ - & 0 & 0 & + & - \\ 0 & 0 & 0 & + & + \\ ? & ? & ? & + & +\beta_p \\ ? & ? & + & ? & ? \end{vmatrix}.$$

We therefore get that the linear function $\det J(\beta_p)$ is upward sloping. Since we know already that $\det J(\beta_p)$ is negative for $\beta_p = 0$ we thus have the result that there is a unique value for β_p where $\det J$ must be zero (and that it is positive thereafter). □

Let us finally consider the full 6D dynamical system (11.39)–(11.44) of this section and investigate to what extent monetary policy (11.44) can contribute to the stability of the 5D dynamics of the private sector. Due to the peculiar role of debt in the considered dynamics we however obtain a negative result in this regard:

Proposition 11.7 *1. The interior steady state of the 6D dynamical system (11.39)–(11.44), which in the 5D case was locally asymptotically stable for $\partial y_w^D/\partial\omega^e > 0$, $\beta_p, \beta_w, \beta_h$ sufficiently small and κ_p sufficiently close to 1, becomes unstable (for all parameter choices) if the interest rate policy rule is switched on by choosing a positive value for either β_{i_p} or β_{i_u} (the other parameters in this feedback policy still being zero).*

2. Asymptotic stability is regained in the situation considered in assertion 1, if either β_{i_p} or β_{i_u} is negative and sufficiently small (the other remaining at zero).

3. In the situation considered in assertion 2, stability gets lost (in general) by way of a Hopf bifurcation, if the parameter β_p is made sufficiently large.

Proof: 1. The 6D dynamical system (11.39)–(11.44) can now be reduced to the following form if attention is only paid to the calculation of the sign of the determinant of the Jacobian at the steady state and if the case $\beta_{i_u} > 0$ is considered for example (the case $\beta_{i_p} > 0$ may be proved in the same way):

$$\begin{aligned} (\hat{l}^e)\ldots &\quad -g_k, \\ (\hat{k}_h)\ldots &\quad +g_h, \\ (\hat{\omega}^e)\ldots &\quad +\beta_w l^{de}/l^e, \\ (\hat{q}_h)\ldots &\quad +\beta_h\left(\frac{c_h^w}{k_h} - \bar{u}_h\right), \end{aligned}$$

[24] See (11.50) and note in this regard that r_{ω^e} is given by $d_w(1-\omega^e l_y^e) - l_y^e y$ which implies that J_{34} is larger than J_{35} as was claimed above.

$$\begin{aligned} (\dot{\lambda}_w)\ldots &\quad +(c_y + c_h - 1)y_w^D - (g_k - \delta_k)\lambda_w, \\ (\dot{i})\ldots &\quad +\beta_{i_u}(y/y^p - \bar{u}). \end{aligned}$$

With the same objective in mind this situation can be reduced further to:

$$\begin{aligned} (\hat{l}^e)\ldots &\quad +\omega^e, \\ (\hat{k}_h)\ldots &\quad +q_h, \\ (\hat{\omega}^e)\ldots &\quad +1/l^e, \\ (\hat{q}_h)\ldots &\quad +\beta_h\left(\frac{1}{k_h}\right), \\ (\dot{\lambda}_w)\ldots &\quad -i, \\ (\dot{i})\ldots &\quad -\lambda_w. \end{aligned}$$

It follows that the sign of $\det J$ must be negative, which turns around one of the necessary and sufficient Routh–Hurwitz conditions for local asymptotic stability.
2. In the case assumed by assertion 2 we get, in the place of the just shown result, that

$$\begin{aligned} (\hat{l}^e)\ldots &\quad +\omega^e, \\ (\hat{k}_h)\ldots &\quad +q_h, \\ (\hat{\omega}^e)\ldots &\quad +1/l^e, \\ (\hat{q}_h)\ldots &\quad +\beta_h\left(\frac{1}{k_h}\right), \\ (\dot{\lambda}_w)\ldots &\quad -i, \\ (\dot{i})\ldots &\quad +\lambda_w, \end{aligned}$$

and thus $\det J > 0$ in this case. Continuity of eigenvalues with respect to parameter changes then again ensures that the stability result shown for the 5D case is preserved by such an addition to the interest rate policy rule.
3. Since $\det J$ is unambiguously positive in the situation considered by assertion 2 we immediately obtain the assertion from the fact that the trace of J is an upward sloping linear function of the parameter β_p, due to the destabilising Rose effect and the destabilising Fisher effect as far as price level flexibility is concerned and due to the fact that the β_p term in the interest rate policy rule does not concern the trace of the matrix J. Note that we do not prove the (not very restrictive) speed condition of the Hopf bifurcation theorem here (which in the present case is very difficult to obtain), but only assume that it will be fulfilled in nearly all conceivable situations. □

Note that the seemingly perverse result of assertion 2 is not really implausible if one notes the following characteristic of the dynamics under consideration. A policy of decreasing nominal interest rates in the situation of a depressed economy (or a deflationary one) in order to push economic activity back to normal activity does not work well in the present context, since this tends to increase disposable income of workers and thus their consumption and indebtedness, which by assumption leads to a further decline in the output of firms and thus does not necessarily have the consequences intended by this monetary policy (inducing further interest rate reductions). Monetary policy of this type therefore can only be expected to work if interest rate reductions speed up economic activity. Such a situation is however only present in the general 19D model of this chapter, where investment behaviour responds positively to a chain of interest rate reductions in general. This to some extent shows that the 6D dynamical system investigated in this section must be embedded in not only the general 9D situation where sluggish quantity adjustments of Metzlerian type make the feedback chains on the market for goods less fast and more involved and where nominal price adjustments matter, but must allow for the case where long-term interest rates respond to short-term ones and thus lead to responses of investment behaviour in view of the adjustments that occur in the financial markets. Such a task can however at present only be undertaken numerically, some examples of which are discussed in the next section.

11.5 Numerical investigation of housing cycles and debt deflation

In this section we briefly present some numerical illustrations of the investment cycles that are implied by the model of this chapter and the processes of debt accumulation and debt deflation to which it can give rise. These numerical illustrations provide a first impression of the dynamics that the model is capable of generating and only serve the purpose of illustration. Detailed numerical simulations should take a closer look on the various feedback channels that characterise the dynamical models of this chapter.[25] These illustrations must therefore be continued and considered in more depth in future studies of this model type, where also more refined debt deflation mechanisms than the still simple one of this chapter should be integrated.

In Figure 11.1 we show a case where damped oscillations are generated by the 9D dynamics in the case of a positive rental price shock, here still in the presence of a peg of the nominal rate of interest. We see that capacity utilisation rates in the goods and the labour market are basically fluctuating in line with each other, while the capacity utilisation rate of space is first leading and later on lagging behind these two measures of the business cycle (which then also become weaker in their positive correlation). These rates are all decreasing initially, since we had a positive rental price shock, which not only reduces the demand for housing services but also other consumption demand and thus economic activity.

We have a less than normal return in the housing sector soon after the positive price shock in this sector due to a significant decrease in the demand for housing services,

[25] The parameters underlying the numerical illustrations are (up to the changes discussed within the figures themselves): $\beta_w = 0$; $\beta_p = 0$; $\beta_n = 0.3$; $\beta_{n^d} = 0.1$; $\beta_{y^e} = 1$; $\beta_h = 0.2$; $\beta_{i_i} = 0$; $\beta_{i_p} = 0$; $\beta_{i_u} = 0$; $\alpha_r^h = 0.25$; $\alpha_u^h = 0.25$; $\alpha_r^k = 0$; $\alpha_u^k = 0$; $\kappa_p = 0$; $\kappa_w = 0$; $\bar{u} = 0.92$; $l_y = 2$; $\bar{U}_h = 0.9$; $\bar{e} = 0.95$; $c_y = 0.7$; $i_l^* = 0.08$; $\delta_k = 0.1$; $\tau_c = 0.5$; $\delta_h = 0.1$; $g = 0.1$; $\gamma = 0.06$; $c_h = 0.4$; $y^p = 1$; $\kappa_h = 0$; $p_o = 5$.

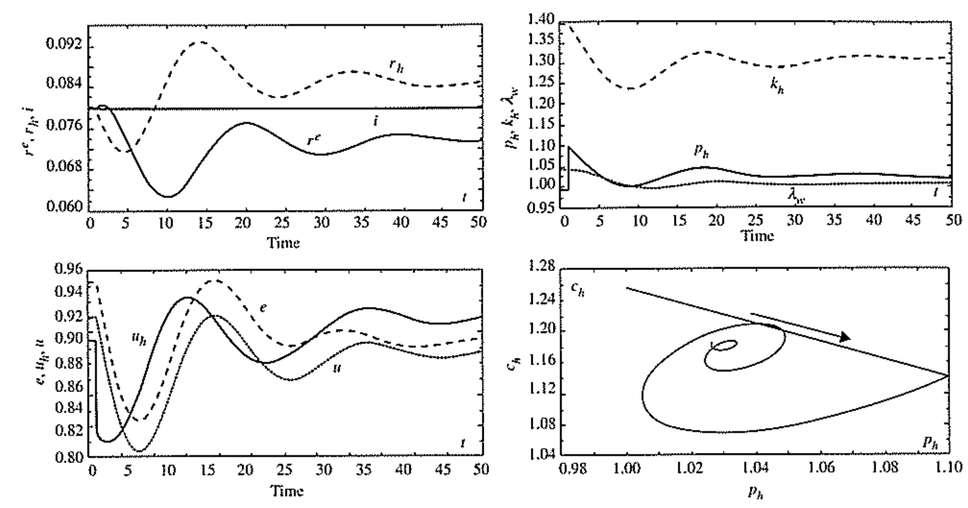


Figure 11.1 Damped fluctuations in the supply of housing services and rental prices

see the figure bottom right, which is accompanied by reduced capital formation in this sector relative to the goods-producing industry. This holds over a long-run horizon of fifty years, over which the demand for housing services does not return to its initial level again (although capacity utilisation in the housing sector does reach high levels in between). The opposite holds true, in particular with respect to the rate of return of the goods manufacturing sector. Bottom right we finally see a mild cyclical evolution with respect to occupied rental space and rental prices. We stress that the considered situation is still an extreme one, since neither wages nor goods prices respond to demand pressure on their respective markets so far, which allows for zero roots and thus path dependence and asymmetries in the time series that are shown. The considered situation is indeed a very sluggish one with respect to cycle lengths, since the economy does not yet respond to certain demand pressures to a sufficient degree.

In Figure 11.2 we have increased the adjustment speed of goods prices (away from its zero level to 0.2) which – due to an adverse real wage or Rose effect – destabilises the economy leading to higher volatility in all variables just discussed. This also removes the path-dependency from the shown time series, allows for basically symmetric fluctuations of utilisation rates around their steady state levels, with the rate of capacity utilisation of space now always leading the other two measures of the business cycle.

On average, profitability in the housing sector still remains depressed, while the opposite seems to hold in goods manufacturing, implying that the capital stock underlying the supply of housing services is still shrinking relative to the one in goods manufacturing. The variable that is subject to a positive shock is now the debt to capital ratio of worker households which leads to an immediate decline in their demand, in particular for housing services, a recession in all markets of the economy and the start of the business cycle from the resulting decrease in economic activity. There is now

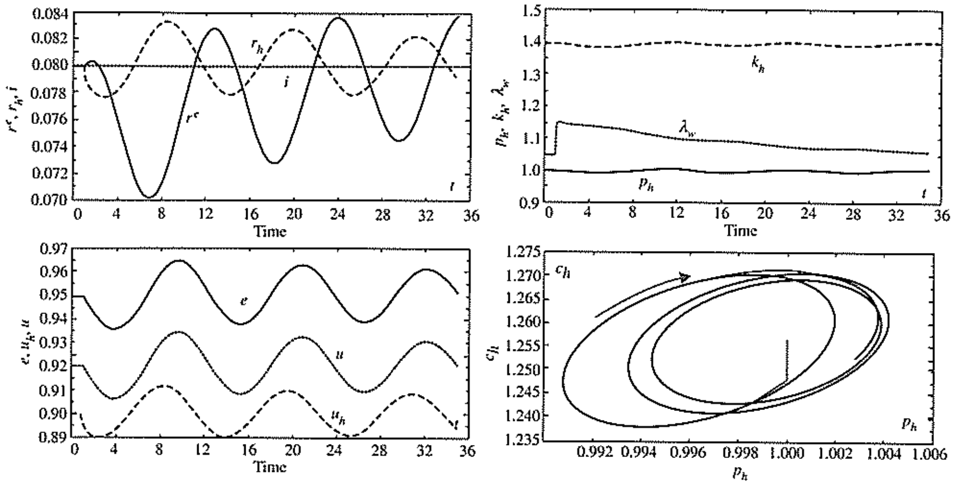


Figure 11.2 More volatile fluctuations through flexible goods-price level adjustments

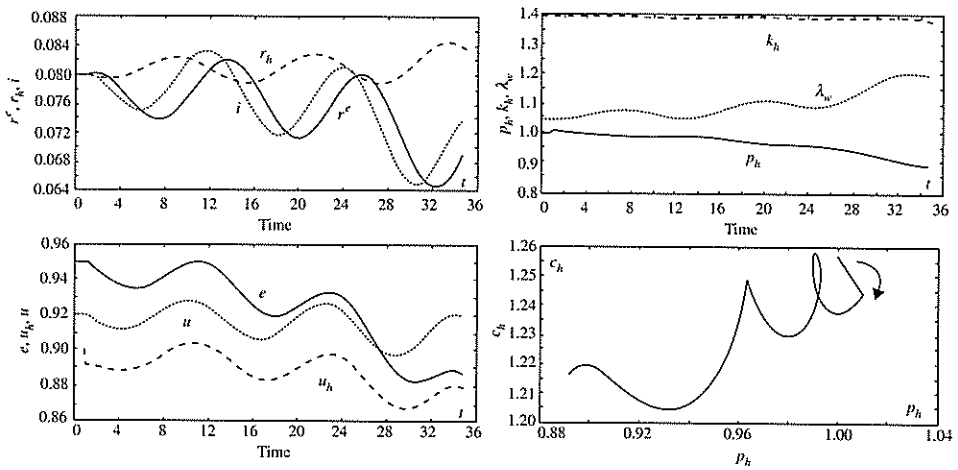


Figure 11.3 Implosive fluctuations and debt deflation

significant overshooting and a nearly persistent cycle in the interaction between rented space and rental prices and a pronounced negative correlation in the evolution of the rates of return in housing services and manufacturing.

Next, in Figure 11.3, we allow for much stronger price adjustments, and now also adjustment of wages with respect to demand pressure on the labour market, and return to the case of a positive shock in rental prices. We now indeed get price deflation with respect to all three price levels of the model. We also allow for an active interest rate policy of the central bank which here follows economic activity closely and is thus

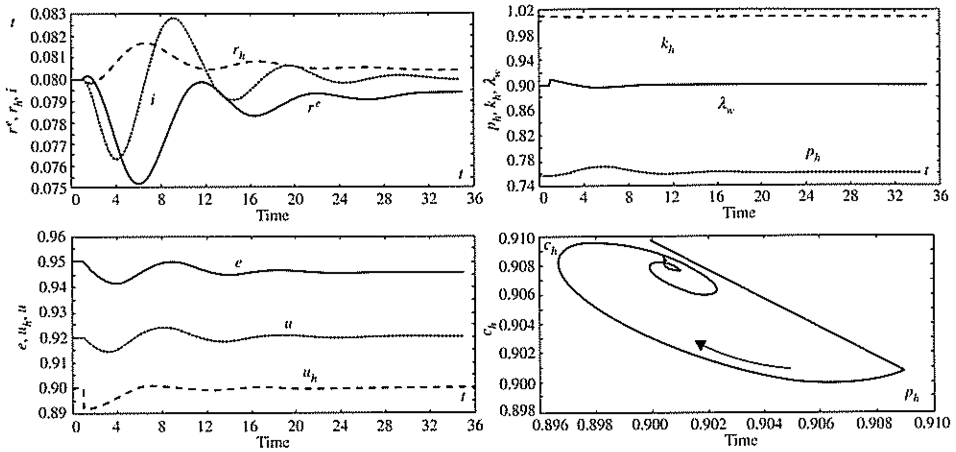


Figure 11.4 Damped fluctuations based on absolute downward wage rigidity

meant to be counter cyclical. The negative correlation between the rates of return in the provision of goods and space is still there and now there is a positive correlation between our three measures of economic activity, which in addition exhibit a significant downward trend. This is the novel thing in this cyclically fairly explosive situation accompanied by the significant upward trend in workers' debt to capital ratio and the shown downward trend in space rental prices as well as occupied space.

The explosive fluctuations of the preceding figure can however be removed and turned into damped oscillations when wages, although remaining flexible in the upward direction, are made downwardly rigid by an appropriate non-linearity in the money–wage PC.[26] This is shown in Figure 11.4 where the trends in the debt to capital ratio and the rental prices are removed by this downward rigidity in nominal wages (an important cost-pressure term in the evolution of space and goods prices). This asymmetric rigidity therefore helps to overcome the deflationary forces indicated in the preceding figure. Yet, due to the lack of a downward adjustment in the money wage we have no longer a uniquely determined NAIRU level on the labour market and need not have a situation in which the rate of employment recovers to its original steady state level (which is here still determined exogenously).

Figure 11.5 finally shows what indeed can happen in the economy if in particular this downward rigidity of money wages is removed to a larger degree. The shown situation of a strong process of debt deflation and increasing depression must however be considered in much more detail than is possible here. In this chapter we primarily attempted to supplement other work by the authors on the occurrence of debt deflation forces in the sector of firms by here considering debtor-creditor relationships in the

26 See Chiarella *et al.* (2000) for a detailed discussion of this type of downward rigidity in the money–wage PC.

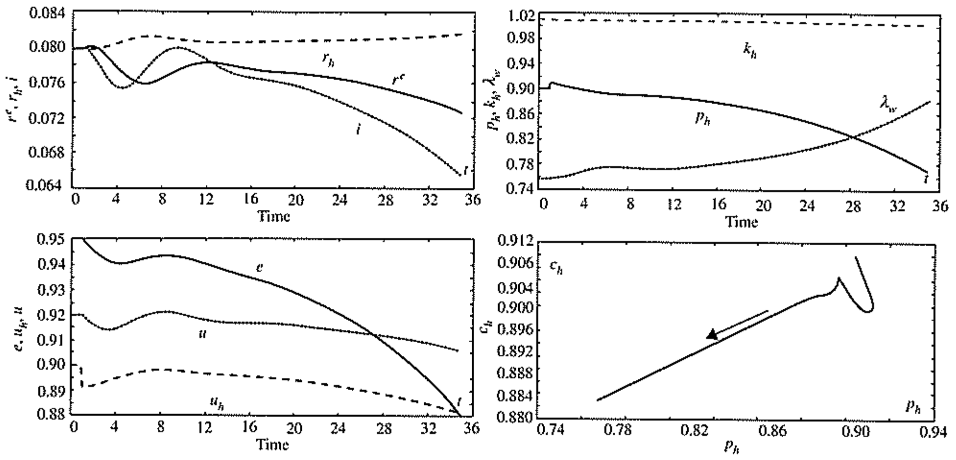


Figure 11.5 Monotonic debt deflation instead of cyclical recovery due to downward wage adjustment

household sector solely and the dynamics that this particular relationship may imply for the long-run evolution of the macroeconomy.

This closes the numerical investigation of the model, here still without worker households' debt default and bankruptcy as far as their past investment in houses (not the purchase of housing services) is concerned. In the next section we will add such debt default and bankruptcy to the housing market, comparable with what we considered for firms in Chapter 7 (to a certain degree). This is still done by adding certain leakages to the model, but not yet by considering housing booms and busts in the form of bubbles in the resale market for housing investment (which does not yet exist in this model type).

Note here also that we did not treat office space cycles explicitly in this chapter, but leave this for future investigation (see the introduction of this chapter for notes on the respective literature). Note also that the (p_h, c_h) cycle shown in Figure 11.4 bottom right is just another type of Goodwin (1967) cycle, in particular since c_h and the utilisation rate of housing space u_h are strictly positively correlated. Note finally that a similar, but not so strict correlation holds true with respect to the price levels p_h, p_y.

11.6 Debt default and bankruptcy in the private housing market

As an addition to the model of this chapter in this section we provide a description of how this general model may be extended and modified in order to allow for further stabilising or destabilising feedbacks caused by the simultaneous occurrence of high debt and deflation, here concerning in particular debt default and the bankruptcy rate of housing owned by worker households.

For this purpose we first reformulate the housing investment behaviour and the financing of this investment of the workers in the household sector:

Worker Households: Housing Investment Behaviour

$$\dot{K}_h^w = c_y^h Y_w^{Dn}/p_y - \delta_h K_h^w - \varphi_b(i) K_h^w, \quad \varphi_b'(i) > 0, \varphi_d \Lambda_w = \varphi_b p_y K_h^w,$$

see Section 11.2.1

$$\dot{\Lambda}_w = p_y C_g^w + p_h C_h^w - (1-\tau_w)[wL^d + w^u(L - L^w) + w^r \alpha_l L_2] - (1-\tau_c) i \Lambda_w - \varphi_d(i)\Lambda_w.$$

We simply add in the first equation the situation of bankruptcy of some worker households as far as their holding of houses is concerned (as an additional leakage effect for the stock of houses K_h^w they are holding). Moreover, there will be in such situations debt default as far as these households are concerned (concerning Λ_w) which is here assumed to be of the extent: $\varphi_d \Lambda_w = \varphi_b p_y K_h^w$. We assume that the default rate φ_d as well as the bankruptcy rate φ_b depend (among other things) positively on the interest rate i, since φ_d is assumed to depend on this rate. We do not consider in this section a loan rate that differs from the short-term interest rate.

Defaults here just reduce the debt level of workers in their dependence on the sector of pure asset holders (since these workers stop paying interest) and are therefore happening as if there is a debt-reducing gift from these households to the worker households, in this extreme form doing no direct harm to the working of the economy as long as asset holder households do not react or are not forced to react to this situation (by credit rationing or – similar to commercial banks – by getting into liquidity difficulties). This is assumed to hold true in the income and savings statements of these households shown below, where part of the savings are no longer net savings, but mere replacement of the debt that has gone into default.

Pure Asset Holders: Debt Default, Income and Savings

$$Y_c^{Dn} = (1-\tau_c)[i_\lambda \Lambda_f + i B_c + B_1^l + p_h C_h^w - p_y \delta_h K_h] + s(1-\tau_c^*) B_2^l,$$

$$S_c^n = Y_c^{Dn} - p_v C_c = \dot{B}_c + \frac{\dot{B}_1^l}{i_l} + \frac{s\dot{B}_2^l}{i^{l*}} + p_y(I_h - \delta_h K_h) + \varphi_d(i)\Lambda_w + \dot{\Lambda}_w.$$

On the intensive form level we have now changes in two of the differential equations of the full model, yet one which does not feed back into its dynamics, changes that are simpler to add than the ones in the case of indebted firms. In fact we only show in the following two laws of motion the additions by which they are to be augmented.

The Extended Dynamics of the Workers' Housing Capital and their Indebtedness

$$\hat{k}_h^w = c_y^h y_w^D / k_h^w - \delta_h - (g_k - \delta_k) - \varphi_b(i), \quad \varphi_b = \varphi_d(i)\lambda_w / k_h^w \quad \text{or}$$

$$\dot{k}_h^w = c_y^h y_w^D - \delta_h k_h^w - (g_k - \delta_k) k_h^w - \varphi_d(i)\lambda_w \quad \text{and}$$

$$\hat{\lambda}_w = (c_y + c_h - 1) y_w^D / \lambda_w - \hat{p}_y + g_k - \delta_k - \varphi_d(i), \quad \varphi_d(i) = a_i(i - i_l^*).$$

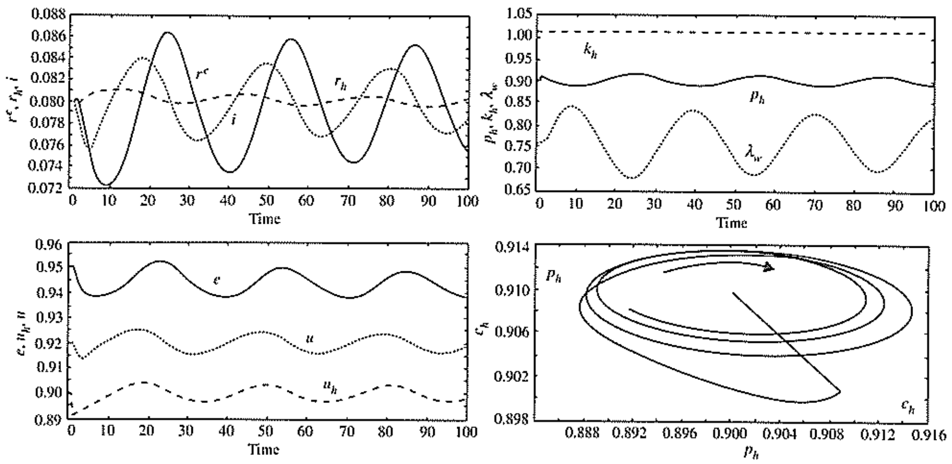


Figure 11.6 Increasing amplitude due to increasing interest rate effect on the default rate of worker households

Note that we do not yet allow that workers' consumption habits change when they go bankrupt with respect to housing capital. The first law of motion is therefore of no importance for the overall stability of the model, since it only describes the housing stock of workers, which may influence their well-being, but – by assumption – does not change their behaviour (since wealth effects are completely disregarded in this model type).

Default and bankruptcy do not have much impact in general and do indeed stabilise the dynamics for small values of the parameter a_i. This result is however not a general one, since larger choices of the parameter a_i may again increase the volatility of the implied trajectories. This is shown (in relation to Figure 11.4) in Figure 11.6 for the value $a_i = 4.5$.

The above high value for the parameter a_i (characterising the function φ_d) may appear as implausible, but may be chosen sufficiently smaller if further indirect effects of the default rate φ_d are taken into account, for example:

- a negative effect of the default rate φ_d, represented by a term $-\alpha_1(i - i_l^*)$, on the price inflation rate $\hat{p}_y$ which in our one good model represents the evolution of prices of all physical commodities (including house consumption and ordinary capital goods);
- a positive effect of the default rate φ_d, represented by a term $\alpha_2(i - i_l^*)$, on the investment rate g_h of pure asset holders into their housing capital stock and their provision of housing services;
- a negative effect of the rate φ_d on the propensity of workers to consume (purchase) houses; and
- the addition of a markup factor on the short-term rate of interest – as far as credit supplied to worker households that own houses is concerned – a markup that depends on the default rate φ_d.

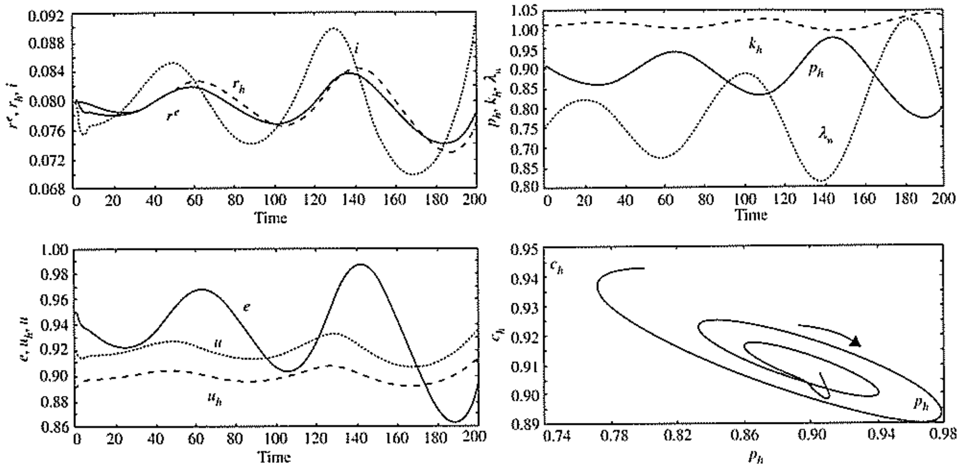


Figure 11.7 Increasing instability due to price level dependency on the default rate of worker households

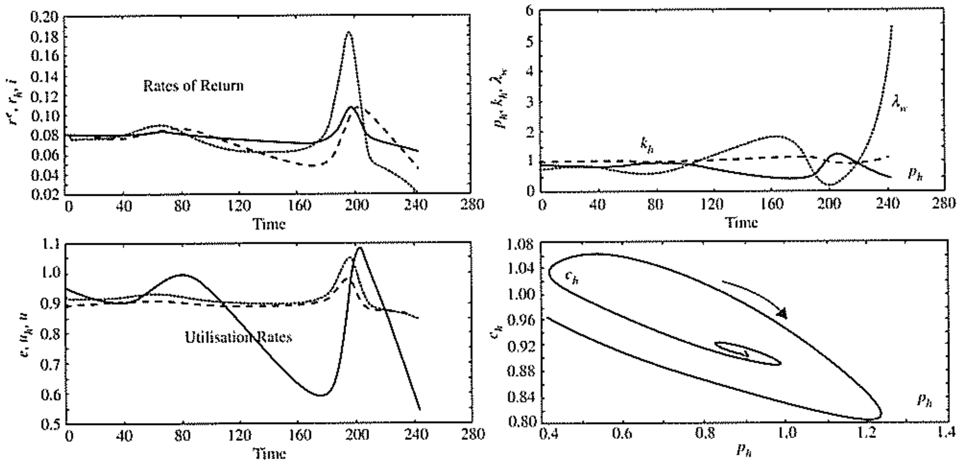


Figure 11.8 Economic breakdown through default dependent price deflation

In the first case, the combination of $a_i = 1$ and $\alpha_1 = 0.35$ is already sufficient to generate the explosive trajectories shown in Figure 11.7 – starting from the simulation shown in Figure 11.6. Introducing negative feedback of default on the price levels p_y, p_h therefore makes the economy subject to increased volatility in its activity levels.

A further slight increase of this parameter to $\alpha_1 = 0.37$ then ultimately produces a breakdown of the economy as shown in Figure 11.8 (if this outcome is not stopped by other means or policy actions).

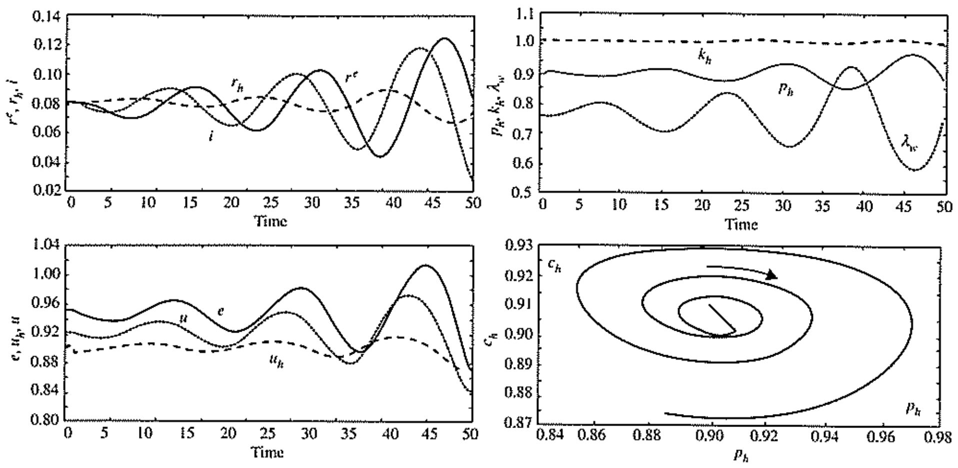


Figure 11.9 Increasing instability due to additional investment in the supply of housing services due to increases in the housing default rate of workers

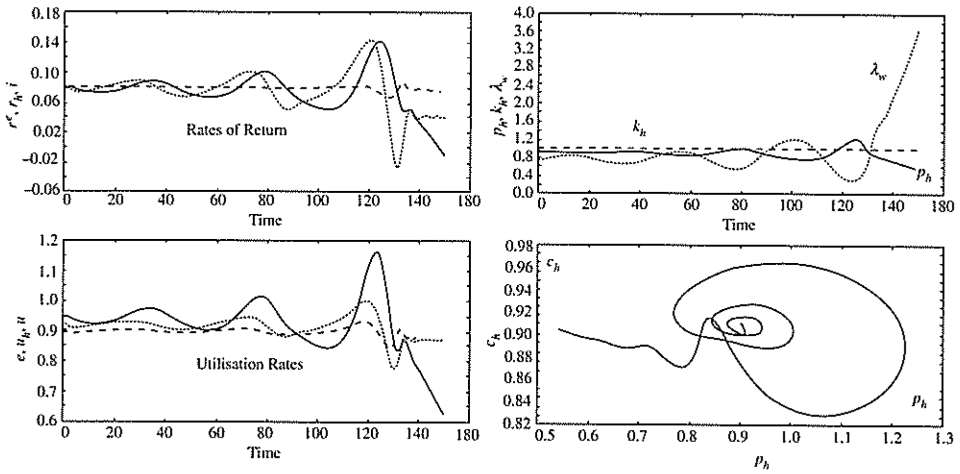


Figure 11.10 Economic breakdown through default dependent price deflation

In the second case the combination of $a_i = 1$ and $\alpha_2 = 0.35$ is sufficient to generate the explosive trajectories (again starting from the simulation shown in Figure 11.6) shown in Figure 11.9.

Finally, choosing both of the positive parameter values, $\alpha_1 = 0.25, \alpha_2 = 0.25$, again produces a breakdown of the economy as shown in Figure 11.10.

We refer the reader back to Part I of the book for a discussion of the current subprime crisis in the US and, spreading out from there, to parts of the world economy. This current debt and liquidity crisis is however much more multi-faceted than what could be included into the type of structural macroeconometric model which we extensively

discussed in Part II of the book. In contrast to Chapters 8 and 9 we did not model commercial banks as intermediaries between pure asset holders and worker households (and thus also not the process of disintermediation). Furthermore, booms and busts in housing prices were here still coupled in a one-to-one fashion with what was happening on the other goods markets of the economy. Nevertheless inflationary and deflationary busts could be shown to be characteristic for the trajectories generated by the 9D subdynamics of this chapter, which however deserves much more investigation than could be done in this final chapter.

11.7 Conclusions

In this chapter we have reconsidered a general disequilibrium model, with an applied orientation and exhibiting a detailed modelling of the private housing sector, which we have developed in Part II, starting from the Murphy model for the Australian economy discussed in Powell and Murphy (1997). This modelling approach is complete with respect to budget equations and stock-flow interactions and can be reduced to a somewhat simplified 18D core model, the dynamics of which were intensively studied in Part II. In the present chapter we have modified this type of model towards the explicit consideration of debtor and creditor households, thus extending the dynamics of the core model by one dimension to 19D by the addition of the dynamics of the debt to capital ratio of the indebted worker households. The subdynamics of these 19D dynamics were investigated theoretically and illustrated numerically. The basic findings were that there is convergence to the balanced growth path of the model for sluggish disequilibrium adjustment processes, that persistent investment cycles in the housing sector can be generated for certain higher adjustment speeds by way of Hopf bifurcations in particular, and that processes of debt deflation may trigger monotonic depressions that become more and more severe when the real debt of debtor households is systematically increased by deflationary spirals in the manufacturing sector in particular.

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