Heliyon 9 (2023) e22486

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

New trading strategy in investment and a new anomaly: A study of the hedge funds from emerging and developed markets

Wing-Keung Wong [a,b,c], Tsun Se Cheong [c,d], David Chui [c], Zhihui Lv [e,*], João Paulo Vieito [f]

[a] Department of Finance, Fintech Center, and Big Data Research Center, Asia University, Taiwan
[b] Department of Medical Research, China Medical University Hospital, Taiwan
[c] Department of Economics and Finance, The Hang Seng University of Hong Kong, Hong Kong
[d] Australia-China Relations Institute, University of Technology Sydney, Australia
[e] School of Mathematics and Statistics, Guangdong University of Foreign Studies, China
[f] Polythecnic Institute of Viana do Castelo, Portugal

A R T I C L E I N F O

JEL classification:
C4
D81
G11
I10

Keywords:
Hedge fund strategies
Emerging and developed markets
Relative value
Equity hedge
T-Bill
Portfolio optimization
Stochastic dominance

A B S T R A C T

This paper introduces a new trading strategy in investment: including the asset (Asset A) with the highest mean, the asset (Asset B) that stochastically dominates many other assets, and the asset (Asset C) with the smallest standard deviation in their portfolio to form portfolios in the efficient frontier for emerging and developed markets that could get higher expected utility and/or expected arbitrage opportunities. To test whether our proposed new trading strategy performs better, we set a few conjectures including the conjectures that investors should include any one, two, or three of Assets A, B, and C from emerging and developed markets. We test whether the conjectures hold by employing both mean-variance and stochastic dominance (SD) approaches to examine the performance of the portfolio formed by using hedge funds from emerging and developed markets with and without Assets A, B, and C, the naïve 1/N portfolio, and all other assets studied in our paper. We find that most of the portfolios with assets A, B, and C++ stochastically dominate the corresponding portfolio without any one, two, or all three of the A, B, and C strategies and dominate most, if not all, of the individual assets and the naïve 1/N portfolio in the emerging and developed markets, implying the existence of expected arbitrage opportunities in either emerging or developed markets and the market is inefficient. In addition, in this paper, we set a *conjecture* that combinations of portfolios with no arbitrage opportunity could generate portfolios that could have expected arbitrage opportunity. Our findings conclude that the conjecture holds and we claim that this phenomenon is a new anomaly in the financial market and our paper discovers a new anomaly in the financial market that expected arbitrage opportunity could be generated.

We also conduct an out-of-sample analysis to check whether our proposed approach will work well in the out-of-sample period. Our findings also confirm our proposed new trading strategy to include Assets A, B, and C in the portfolio is the best strategy among all the other strategies used in our paper and gets the highest expected wealth and the highest expected utility for the emerging and developed markets. Our findings contribute to the literature on the emerging and developed markets of hedge funds and the reliability of alternative risk frameworks in the

* Corresponding author.
E-mail addresses: wong@asia.edu.tw (W.-K. Wong), jamescheong@hsu.edu.hk (T.S. Cheong), davidchui@hsu.edu.hk (D. Chui), 202110004@oamail.gdufs.edu.cn (Z. Lv), joaovieito@esce.ipvc.pt (J.P. Vieito).

https://doi.org/10.1016/j.heliyon.2023.e22486
Received 14 June 2023; Received in revised form 11 November 2023; Accepted 14 November 2023
Available online 26 November 2023

evaluation. Our findings also provide practical experience to academics, fund managers, and investors on how to choose assets in their portfolio to get significantly higher expected utility in emerging and developed markets.

1. Introduction

A group of literature studies trading strategies [1–9]. Recently, Lv et al. [10,11] concluded that including both the asset with the highest mean and the asset with the smallest variance could be a better choice than without including both two assets or either in one's investment even though the mean-variance rule concludes that choosing both two assets or either is indifferent from choosing some other assets. According to this finding, we introduced a new trading strategy in investment: to include the asset (Asset A) with the highest mean, the asset (Asset B) that stochastically dominates many other assets, and the asset (Asset C) with the smallest standard deviation in their portfolio to form the portfolio in the efficient frontier for emerging and developed markets.

On the other hand, Lv et al. [10,11] only found that including both the asset with the highest mean and the asset with the smallest variance could generate portfolios that enable investors to obtain higher expected utility but not expected wealth. Is it possible that combinations of portfolios with no arbitrage opportunity could generate portfolios that could have expected arbitrage opportunity? As far as we know, theories in the literature [12–18], including the theory of stochastic dominance (SD), only find that combinations of portfolios with no arbitrage opportunity could generate portfolios that could enable investors to obtain higher expected utility but could not generate portfolio that have expected arbitrage opportunity. In this paper, we conjecture that it is possible that combinations of portfolios with no arbitrage opportunity could generate portfolios that could generate portfolios that could have expected arbitrage opportunity.

To test whether our beliefs hold, we set a few conjectures including the conjecture that among all the assets investors want to invest, investors should include any one, two, or three of Assets A, B, and C from emerging and developed markets. We test whether our conjecture holds, we employ both the mean-variance rule and stochastic dominance approach to examine the performance of the portfolios formed by different hedging funds from emerging and developed markets, including the portfolio with Assets A, B, and C and other portfolios, including the portfolio without Assets A, B, and C, the naïve 1/N portfolio, and all other assets in the emerging and developed markets studied in our paper.

We find that most of the portfolios with assets A, B, and C stochastically dominate the corresponding portfolio without any one, two, or all three of the A, B, and C strategies and dominate most, if not all, of the individual assets and the naïve 1/N portfolio in the emerging and developed markets, including some first-, second, and high-order dominance relationships. This implies that there are expected arbitrage opportunities in the hedge fund market and the market of the hedge fund is inefficient. Our findings also confirm that our proposed trading strategy to include all Assets A, B, and C in the portfolio is the best in the sense that investors could get the highest expected wealth and/or the highest expected utility than any individual asset, the naïve 1/N portfolio, and any portfolio that include any one or two assets A, B, and C in the portfolio or not include any of A, B, and C assets in the portfolio for the emerging and developed markets. We also conduct an out-of-sample analysis to check whether our proposed approach will work well in the out-of-sample period. Our findings contribute to the advancement of knowledge on the outcomes of hedge fund strategies and the reliability of alternative risk frameworks in the evaluation. Our findings also provide practical experience to academics, fund managers, and investors on how to choose assets in their portfolio to get significantly higher expected utility in emerging and developed markets.

We note that stochastic dominance analysis is a very good approach to evaluate the performance between any pair of assets because it has been formally proven that regardless of any distribution, if one asset statistically dominates another asset, then any risk-averse investor will have a higher expected utility of the dominating asset to the dominated asset [19]. Nevertheless, there are many other risk measures, for example, the mean-variance rule, Sharpe ratio, Sortino ratio, Rachev ratio, Information ratio, Jensen's Alpha, VaR, C-VaR, Omega ratio, Farinelli and Tibiletti ratio, Kappa ratio, etc. in measuring the performances of the assets. These measures are considered to be good measures if the comparison by using these measures is the same as the comparison by using the stochastic dominance approach. Nonetheless, the comparison of using some of these measures is found to be equivalent to that of the stochastic dominance approach only under some conditions [15,20–23] and it is easy to find examples to show that one asset, say, Asset A, is preferred to another asset, say, Asset B, or two assets are indifferent by using one measure that is not stochastic dominance approach but risk-averse investors could have a higher expected utility for Asset B to Asset A. Readers may read such examples in Refs. [17–19]. Thus, the stochastic dominance approach is superior to any of these measures and our paper uses the stochastic dominance approach in the comparison.

To the best of our knowledge, this paper is the first paper in the literature to propose above trading strategies. Our research enriches the body of knowledge concerning hedge funds in both emerging and developed markets and the reliability of alternative risk frameworks. Furthermore, our findings also provide practical insights for academics, fund managers, and investors seeking to optimize assets in their portfolio to get significantly higher expected utility in both emerging and developed markets. In addition, our paper also conducts an out-of-sample analysis to check whether our proposed approach will work well in the out-of-sample period.

The remaining part of the paper is organized as follows. Section 2 provides a literature review of the related literature. Section 3 presents the data and the theoretical foundation. Section 4 discusses the empirical results. Sections 5 and 6 discuss the results of the robustness test and preferences for different types of investors. Section 7 draws inferences from the novel empirical findings. Section 8 concludes the study.

2. Literature review

Over the past three decades, hedge funds, with less regulatory rigidity and more trading flexibility, have attracted a great deal of institutional and wealthy individual investors and have rapidly grown into important investment vehicles, either for investment in themselves or as parts of a portfolio. This comprehensive literature review systematically explores academic and practitioner studies that assess hedge fund performance, role within portfolios, and behavior during crisis periods.

Numerous studies have focused on hedge fund performance, yielding various findings. Bali et al. [24] demonstrate that hedge funds' timing ability improves their future performance, survival probability, and capital attraction. Brown et al. [25] show that hedge funds tend to decrease volatility, following a good performance, and increase volatility, following a bad performance. Ackermann et al. [26] demonstrate a strong positive correlation between performance rate and risk-adjusted return and Liang [27] obtain similar results. Brown et al. [28] highlighted the positive risk-adjusted returns of offshore hedge funds. Sialm et al. [29] found that while funds of funds benefit from local advantages but at the same time, create market segmentation that can destabilize the underlying hedge funds.

Agarwal and Naik [30] employed option-based and buy-and-hold strategies to analyze hedge fund risk exposures. Titman and Tiu [31] provided insights into the behavior of well-informed hedge funds, emphasizing lower factor risk exposure and improved Sharpe ratios, information ratios, and alphas. Fung and Hsieh [32] identified seven asset-based style factors explaining 80 % of monthly return variations in hedge funds. Jagannathan et al. [33] observed significant performance persistence in superior funds. Aiken et al. [34] and Aragon et al. [35] revealed that hedge funds with lower funding risk provide higher returns, essentially due to the increased exposure to equity-mispricing anomalies. Heuson et al. [36] shed light on the importance of skewness in hedge fund returns as an indicator of managerial skill in avoiding large drawdowns.

In addition, Agarwal and Naik [37] point out that different hedge fund strategies depend upon different markets. Agarwal and Naik [38] find that a mix of alternative and passive-indexing investments provide better risk-return tradeoffs. Fung and Hsieh [39] show that the global macro fund is positively correlated with U.S. stocks across various equity-market environments, and it underperforms equities in up markets and outperforms equities in down markets. Sialm [29] also describes that funds of hedge funds overweigh their investments in hedge funds located in the same geographical areas and that funds with a stronger local bias exhibit superior performance.

Hedge funds are often used by investors as a risk reduction tool to decrease portfolio volatility and create more stable return patterns. Do et al. [40] shows that Australian hedge funds included as a part of a portfolio were able to improve returns, irrespective of strategy. Lhabitant and Learned [41] emphasized the role of hedge funds in eliminating specific portfolio risks, although diversification of some hedge fund strategies might seem highly attractive in mean-variance terms but becomes less when considering the skewness and kurtosis. Moreover, Bollen et al. [42] describe that hedge fund allocations successfully reduced portfolio volatility in during specific subperiods but failed to improve the Sharpe ratio, highlighting the nuanced trade-offs associated with hedge fund inclusion. Otruba et al. [43] further demonstrate that hedge funds enhance the risk-return profile of portfolios while reducing the probability of underfunding. Amin and Kat [44] demonstrate that hedge funds improve the portfolios' risk-return profile, when mixed with the S&P 500, but fail to do so as a standalone investment.

In addition, some literature care more on constructing portfolio allocation for hedge funds. Cvitanic et al. [45] create optimal hedge fund strategies, noting that increased risk reduces hedge fund allocations, with low-beta hedge funds serving as strong alternatives to risk-free assets. Morton et al. [46] use utility functions with an expected regret component to construct portfolio allocation for hedge funds. Giamouridis and Vrontos [47] adopt static and dynamic models of time-varying covariances/correlations to construct portfolio allocations and discover that portfolios based on dynamic models achieve higher out-of-sample returns, lower risk, and lower tail risk.

A group of literature has explored hedge fund behavior during crises and financial distress. Liang and Kat [48] show that hedge funds were strongly affected by economic crises, while their role as triggers of crises remains uncertain. Schaub and Schmid [49] highlighted the challenges faced by hedge funds with illiquid portfolios, resulting in lower returns and alphas. Capocci and Hübner [50] and Stoforos et al. [51] show that hedge funds suffered from losses and struggled to achieve higher absolute returns during crises. It also highlights the benefits of relative value arbitrage during the Asian crisis. Under conditions of high volatility and financial stress, Jordão and De Moura [52] show Brazilian hedge funds struggled to generate significant alpha values.

Several studies have analyzed the impact of COVID-19 pandemic on hedge funds. Ben Khelife et al. [53] provided strong evidence of the pandemic's effects on fund managers' performance, except for the Managed Futures and the Relative Value funds strategies, while Berglund et al. [54] show that other strategies appear to have been less impacted. Sung et al. [55] argued that hedge funds reduce investments during market regime changes, such as shifts from normal to crisis situations.

Various methodologies are employed to study the hedge fund performance, ranging from the classical CAPM model [56] to the multi-factor models [57,58] to the panel model [59], two-stage peer group benchmarking approach [60], and performance measures for hedge fund returns based on stochastic discount factors.

Stochastic dominance is widely used in comparing the performance of different assets and different portfolios in different efficient frontiers, offering valuable insights into their relative performance. For example, Fong et al. [61] and Chan et al. [18] used stochastic dominance to assess the performance between the traditional stocks and the internet stocks. Hoang et al. [62] find that stock portfolios including gold stochastically dominate those without gold quoted in Paris in the diversification of French portfolios, while Hoang et al. [63] find that risk-averters prefer portfolios with gold in the efficient frontier, regardless of Shari'ah-compliant or conventional stocks. On the other hand, Hoang et al. [64] find that risk averters prefer not to include gold quoted on the Shanghai Gold Exchange in their stock-bond portfolios, contrasting with DeMiguel et al. [65]. Bouri et al. [66] find that investors prefer to invest in portfolios with wine in the efficient frontier to gain higher expected utility. Furthermore, Venkataraman and Rao [67] derive the statistic that could test for third-order stochastic dominance, develop a program to determine the efficient funds, and find the existence of the second and third orders for some funds over other funds.

Recently, a growing body of literature employ stochastic dominance theory to evaluate hedge fund performance. Wong et al. [68] were pioneers in employing stochastic dominance (SD) techniques to evaluate hedge fund portfolios, concluding that the SD approach serves as a superior filter in hedge fund selection compared to traditional methods. Molyboga et al. [69] argued that the standard tests for persistence in hedge fund ignore performance reporting delays and advocated using a group of tests incorporating large-scale simulations framework and stochastic dominance methodology, mirroring practices in institutional investment. Canepa et al. [70] discovered that top-performing hedge funds follow a distinct strategy than mediocre-performing hedge funds by accepting risk factors that anticipate the troubling economic conditions prevailing after 2006.

To bridge gaps in existing literature, this paper introduces a novel trading strategy for investment. We apply portfolio optimization, mean-variance rule, and stochastic dominance theory to examine this new trading strategy, assess hedge fund performance, and conduct an out-of-sample analysis to analyze the impact of COVID-19. Our findings provide valuable insights for academics, fund managers, and investors, offering guidance on asset selection for higher expected utility in both emerging and developed markets, while making a significant contribution to the literature in this field.

3. Data, methodology, and conjectures

3.1. Data

Obtained from a large database of Hedge Fund Research (HFR),[1] the hedge fund data used in this paper are the monthly hedge fund indexes from December 1989 to March 2020. Due to some missing data on several strategies, in this paper, we only consider hedge funds from the following six HFR types of strategies from emerging and developed markets: equity hedge, event-driven, fund of funds, macro index, relative value, in which the equity hedge indices contain Equity Hedge (Total) index and Equity Market Neutral Index, event-driven indexes consist of Event-Driven (Total) Index, Distressed Index, and Merger Arbitrage Index, fund of funds indexes contain Fund of Funds Composite Index, Conservative Index, and Strategic Index, macro index only contains Marco (Total) Index, relative value indexes contain Relative Value (Total) Index and Fixed Income-Convertible Arbitrage Index and regional index contains only Emerging Markets (Total) Index. Also, we use Fund Weighted Composite Index to represent the overall performance of all the hedge fund indexes, and 6-M TB[2] (six-month Treasury bill) to represent the risk-free asset. Furthermore, all price indexes are converted into returns, for example, since the data are originally provided in an annualized rate form, the returns of hedge fund indexes are the first difference of the natural logarithm of the price index and the bond yields are divided by 1200 to get the monthly yields. For simplicity, we use the first or first two capital initial letters of the index to denote the return of the index. Thus, we use EH, RV, EMN, ED, D, and TB to represent Equity Hedge (Total) Index, Relative Value (Total) Index, Equity Market Neutral Index, Event-Driven (Total) Index, Distressed Index, and six-month Treasury bill, respectively, and all other abbreviations can be obtained similarly.

To gain a better understanding of the movements of different hedge fund indices, we exhibit in Fig. 1 the historical performances of the indices for different hedge fund strategies from December 1989 to March 2020. From the figure, we find that in general, the indices of different hedge fund strategies exhibit an upward trend during the entire period we studied except during the periods of the global financial crisis (in 1998, 2000, 2008) and COVID-19 in 2020 that there exist serious downtrends for each index except the Equity Market Neutral Index, which performs relatively moderate stable during the crises. In addition, Fig. 1 shows that the Equity Hedge (Total) Index (the series above nearly all other series in the entire period.) outperforms all the other indices in general.

3.2. Methodology

Under uncertainty conditions, two well-known approaches can be used to compare the performance of different series, which, in turn, help investors in their asset selection. One approach is the mean-variance rule [71] and the other is the stochastic dominance (SD) approach [72]. The former provides a quick review of the preferences of the indices while the latter gives a proper reveal of the references based on information from the entire distribution of each index. In addition, it is well known that portfolio optimization could provide a better return by choosing a portfolio from the efficient frontier. Thus, in this paper, we will apply portfolio optimization (PO), mean-variance (MV) rule, and stochastic dominance (SD) approach to determine which assets investors should include in their portfolios.

3.2.1. Portfolio optimization (PO) approach

We let $\mathbf{r} = (r_1, \ldots, r_p)^T$ be the returns of assets, $S = (s_1, \ldots, s_p)^T$ with mean $\mathbf{u} = (u_1, \ldots, u_p)^T$ and covariance $\Sigma = (\sigma_{ij})$. Investors could apply the portfolio optimization (PO) theory to obtain the return of the optimal portfolio [71]:

$$R = \max \omega^T u$$

subject to $\omega^T 1 \leq 1$ and $\omega^T \Sigma \omega \leq \sigma_0^2$ where $1 = (1, \ldots, 1)^T$, σ_0^2 is the given level of risk, and $\omega = (\omega_1, \ldots, \omega_p)^T$ with $\omega_i (i = 1, \ldots p)$ be the fraction of the capital invested in asset s_i in which ω could be (non) negative when a short sale is (not) allowed. Readers may refer to

[1] www.hedgefundresearch.com.

[2] https://fred.stlouisfed.org.

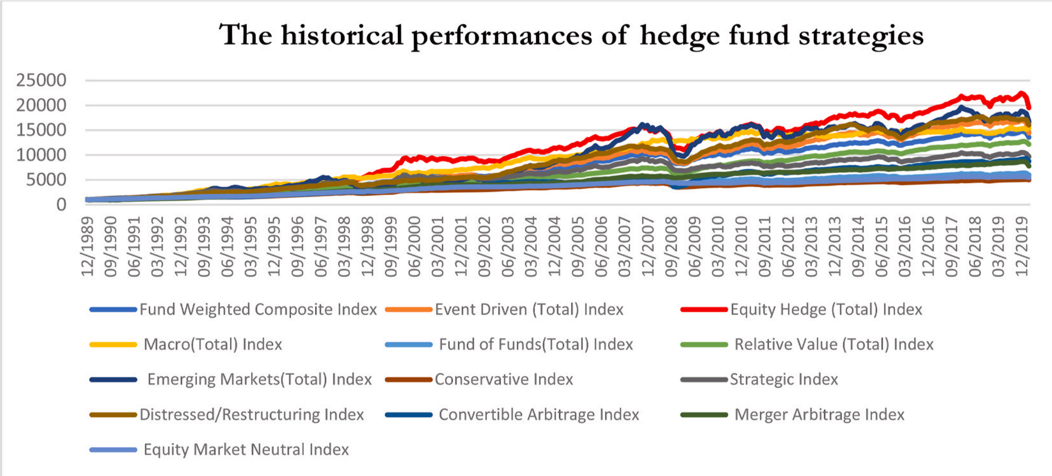


Fig. 1. The historical performances of hedge fund strategies.

Refs. [73,74] to get better estimation approaches to estimate the return of the optimal portfolio.

3.2.2. *Mean-variance (MV) rules*

The mean-variance (MV) rule suggests that investors prefer asset X to asset Y if

$$u_X > u_Y \text{ and } u_X > u_Y$$

Readers may refer to Ref. [19] therein for more details. And the preferences of the MV rule and the SD theory are equivalent under some conditions [4,75,76]. Thus, the MV rule is a good criterion in asset selection.

3.2.3. *Stochastic dominance (SD) approach*

The stochastic dominance (SD) approach is one of the most effective tools for asset selection because ranking assets by the SD rule has been proven to be equivalent to the expected-utility maximization and it does not have any restriction on the distributions of assets and the forms of utility, only require the utility to be increasing concave [19].

We let X and Y be returns of assets or portfolios with cumulative distribution functions (CDFs), F and G, probability density functions (PDFs), f and g, and means, $u_X$andu_Y, respectively, and for any integer j, we use the following notations throughout the paper: $M_j(x) = \int_a^x M_{j-1}(t)dt$ with $M_0 = m$ for $m = f, g$; $M = F, G$.Thus, X dominates Y by FASD (SASD, TASD) denoted by $X \succ_1 Y$ ($X \succ_2 Y$, $X \succ_3 Y$) if $F_1(x) \leq G_1(x)$ ($F_2(x) \leq G_2(x)$, $F_3(x) \leq G_3(x)$) for all x, where FSD, SSD, TSD denote first-, second-, and third-order SD, respectively, For $X \succ_3 Y$, it requires $u_X > u_Y$. In addition, for any integer n, U_n is a set of utility functions for the n^{th}-order investors such that $U_n = \{u : (-1)^{i+1}u^{(i)} \geq 0, i = 1, \ldots, n\}$, where $u^{(i)}$ is the i^{th} derivative of the utility function u. Wong and Ma [76] and others extend the theory to non-derivative utility. We note that in this paper, we assume all investors are risk averse, and thus, the n^{th}-order investors are the n^{th}-order risk averters.

While there are several stochastic dominance tests [77–80], we apply the SD test developed by Davidson and Duclos [81] and modified by Bai et al. [82] to investigate the preference for investors in this paper because Lean et al. [83] and others and show that SD test developed by Davidson and Duclos [81] has good power and decent size.

Assuming that $\{f_i\}$ and $\{g_i\}$ are observations drawn from the returns X and Y of assets or portfolios with CDFs F and G, respectively, we use the following SD test:

$$\mathrm{T_j(x)} = \frac{\widehat{\mathrm{F}}_\mathrm{j}(\mathrm{x}) - \widehat{\mathrm{G}}_\mathrm{j}(\mathrm{x})}{\sqrt{\widehat{\mathrm{V}}_\mathrm{j}(\mathrm{x})}} \; j = 1, 2, 3 \, .$$

where

$$\widehat{V}_j(x) = \widehat{V}_{F_j}(x) + \widehat{V}_{G_j}(x) - \widehat{V}_{FG_j}(x); \; M_j(x) = \frac{1}{N_m(j-1)!}\sum_{i=1}^{N_m}(x - m_i)_+^{j-1};$$

$$\widehat{V}_{M_i}(x) = \frac{1}{N_m}\left[\frac{1}{N_m((j-1)!)^2}\sum_{i=1}^{N_m}(x - m_i)_+^{2(j-1)} - \widehat{M}_j(x)^2\right], M = F, G; m = f, g;$$

$$\widehat{V}_{FM_i}(x) = \frac{1}{N_m}\left[\frac{1}{N_m((j-1)!)^2}\sum_{i=1}^{N_m}(x-m_i)_+^{(j-1)}(x-g_i)_+^{(j-1)} - \widehat{F}_j(x)\widehat{G}_j(x)\right].$$

to test the following hypotheses:

$H_0 : F_j \equiv G_j$ against three alternatives $H_1 : F_j \not\equiv G_j, H_{1l} : F_j \succ_j G_j, H_{1r} : F_j \prec_j G_j$;

We note that $T_j(x)$ will follow N (0,1) asymptotically if H_0 holds. Readers may refer to Davidson and Duclos [81] and Bai et al. [82, 84] for more information on the test. We also note that in this paper, we use the SD test proposed by Davidson and Duclos [81] and Bai et al. [82,84] but not the test proposed by Kouaissah [85] because we compare one prospect (a portfolio formed by using our approach) with another prospect (a corresponding portfolio did not form by using our approach) but do not compare the dominance of each component of one prospect with the other prospect.

3.3. Conjectures

In this paper, we propose a new trading strategy in investment. We believe that among all the assets investors want to consider to be included, he could get higher expected utility in emerging and developed markets if they include the following assets in the portfolios:

Asset A: an asset with the highest mean,

Asset B: an asset that stochastically dominates most of the other assets, and.

Asset C: an asset with the smallest standard deviation.

We note that we do not choose the asset with the highest Sharpe ratio in the portfolio because we have chosen Asset A with the highest mean and Asset C with the smallest standard deviation, and thus, the portfolio formed by using our strategy could have a high Sharpe ratio. Nonetheless, it could be a good extension of our paper to include the asset with the highest Sharpe ratio in the portfolio and compare it with including both Assets A and C in the portfolio, to see which approach has better performance.

Thus, we first set the following conjectures:

Conjecture 1A: investors could get higher expected utility if they include Asset A to form a portfolio in the efficient frontier of emerging and developed markets;

Conjecture 1B: investors could get higher expected utility if they include Asset B to form a portfolio in the efficient frontier of emerging and developed markets; and.

Conjecture 1C: investors could get higher expected utility if they include Asset C to form a portfolio in the efficient frontier of emerging and developed markets.

We then set the following conjectures:

Conjecture 2A: investors could get higher expected utility if they include both Assets A and B to form a portfolio in the efficient frontier of emerging and developed markets;

Conjecture 2B: investors could get higher expected utility if they include Assets A and C to form a portfolio in the efficient frontier of emerging and developed markets; and.

Conjecture 2C: investors could get higher expected utility if they include Assets B and C to form a portfolio in the efficient frontier of emerging and developed markets.

In addition, we set the following conjecture:

Conjecture 3: investors could get higher expected utility if they include Assets A, B, and C to form a portfolio in the efficient frontier of emerging and developed markets.

We believe that the best strategy for investors is to include the asset with the highest mean, the asset that dominates most of the other assets, the naive diversified portfolio was formed by using the 1/N portfolio strategy and the asset with the smallest standard deviation to form a portfolio in the efficient frontier than all other strategies we stated in the above conjectures. Thus, we set the following conjectures:

Conjecture 4A: investors could get higher expected utility if they choose the portfolios from the efficient frontier formed by Assets A, B, and C and other assets of emerging and developed markets than the portfolios from the efficient frontier formed by Assets A and B, and other assets but without Asset C;

Conjecture 4B: investors could get higher expected utility if they choose the portfolios from the efficient frontier formed by Assets A, B, and C and other assets of emerging and developed markets than the portfolios from the efficient frontier formed by Assets A and C, and other assets but without Asset B;

Conjecture 4C: investors could get higher expected utility if they choose the portfolios from the efficient frontier formed by Assets A, B, and C and other assets of emerging and developed markets than the portfolios from the efficient frontier formed by Assets B and C, and other assets but without Asset A;

Conjecture 4D: investors could get higher expected utility if they choose the portfolios from the efficient frontier formed by Assets A, B, and C and other assets of emerging and developed markets than the portfolios from the efficient frontier formed by Assets A and other assets but without Assets B and C;

Conjecture 4E: investors could get higher expected utility if they choose the portfolios from the efficient frontier formed by Assets A, B, and C and other assets of emerging and developed markets than the portfolios from the efficient frontier formed by Assets B and other assets but without Assets A and C; and.

Conjecture 4F: investors could get higher expected utility if they choose the portfolios from the efficient frontier formed by Assets A, B, and C and other assets of emerging and developed markets than the portfolios from the efficient frontier formed by Assets C and other assets but without Assets A and B.

DeMiguel et al. [65] show that the naive diversified portfolio was formed by using the 1/N portfolio strategy. In this paper, we also test whether the portfolios we proposed in this paper outperform the naive 1/N portfolio, we set the following conjecture:

Conjecture 5: investors could get higher expected utility if they include any asset we proposed in this paper, including Asset A with the highest mean, Asset B which stochastically dominates most of the other assets, and Asset C, with the smallest standard deviation to form a portfolio in the efficient frontier of emerging and developed markets than the 1/N portfolio strategy, any individual asset in the portfolio, and the portfolios without the assets we proposed in this paper.

As far as we know, literature only found that combinations of portfolios with no arbitrage opportunity could generate portfolios that could enable investors to obtain higher expected utility but could not generate portfolio that have expected arbitrage opportunity. To extend the literature, we set the following conjecture in this paper:

Conjecture 6: it is possible that combinations of portfolios with no arbitrage opportunity could generate portfolios that could have expected arbitrage opportunity.

4. Empirical findings

In this paper, we will apply portfolio optimization (PO), mean-variance (MV) rule, and stochastic dominance (SD) approach to determine which assets investors should include in their portfolios. We first examine the performances among indices obtained by using different hedge fund strategies and examine the performance of the 6-M T-bill. We recall that we use the first or first two capital initial letters of the index to denote the return of the index. Thus, we use EH, RV, EMN, ED, D, and TB to represent the Equity Hedge (Total) Index, Relative Value (Total) Index, Equity Market Neutral Index, Event-Driven (Total) Index, Distressed Index, and six-month Treasury bill, respectively, and all other abbreviations can be obtained similarly. Readers may refer to Section 3.1 for the full names of all the indices.

4.1. Comparison among different indices

We first conduct an analysis to compare the performances among indices obtained by using different hedge fund strategies and examine the performance of the 6-M T-bill. To do so, we first exhibit in Table 1 some basic descriptive statistics for the returns of indices obtained by using different hedge fund strategies and the return of the 6-M T-bill.

Table 1 exhibits some basic descriptive statistics including mean, standard deviation (Stdev), skewness, excess kurtosis (Kurtosis), and Jarque-Bera (J-B) test for the returns of indices for different hedge fund strategies and the return of 6-M TB. The results show that the means of all the returns are significantly positive at a 1 % level. The estimates of mean and standard deviation indicate that, among all the returns, the Equity Hedge (EH) Total Index has the highest return of 0.0082 and the Emerging Markets (EM) Total Index has the highest risk of 0.0383, while 6-M TB has the smallest return of 0.0022 and the lowest risk of 0.0018. This information can be used to obtain the preference among different hedge fund strategies and T-bills by using the MV rule. Since the results are consistent with but not as good as the preference by using the SD rule, we skip discussing the results.

From the table, the estimates of the skewness for all returns are significantly negative at a 1 % level except for 6-M TB, Marco Total Index, and Equity Market Neutral (EMN) Index in which both 6-M TB and the Marco (M) Total Index are significantly positive at a 1 % level and the EMN Index is not significantly different from zero, implying all returns except 6-M TB, the M and EMN Indices are significantly skewed to the left, both 6-M TB and M Index are significantly skewed to the right, and the distribution of the return for the EMN Index is symmetric. The estimates of kurtosis for all returns are strongly significantly positive at a 1 % level except 6-M TB, implying that all returns except 6-M TB are significantly fat-tailed, while 6-M TB is significantly thin-tailed. In addition, the estimates of both skewness and kurtosis and the Jarque–Bera (J-B) test imply that all the returns are rejected to follow normal distributions.

As displayed in Fig. 1., the EH Index (at the top) outperforms all other strategy indices based on the historical performance of the prices.[3] For this reason, one may believe that the EH Index is the best choice for investment. But is this the case? From Table 1, we find that, YES, the mean of EH Index is the highest among all the assets, but its standard deviation is not the smallest. On the other hand, the standard deviation of 6-M TB is the smallest among all the assets, but the mean of 6-M TB is not the highest. Therefore, based on the mean-variance rule, one cannot conclude that either the EH Index or 6-M TB is the best choice for investment. Based on the mean-variance rule, both EH Index and 6-M TB are indifferent from most of the other assets in our study. To examine the performance of different hedge funds and determine which hedge fund should be included in the portfolios to get higher expected utility, we turn to apply the SD approach to examine the performance of the returns of indices for different hedge fund strategies and the return of 6-M TB and exhibit the results in Table 2.

Table 2 shows that the indices that dominate most of the other indices are Relative Value (RV) Total Index and 6-M TB, both stochastically dominate 9 other indices, and no other index stochastically dominates them. To be precise, the RV Index first-order stochastically dominates the C and EMN Indices, second-order stochastically dominates the FWC, ED, EH, M, FFC, EM, and S Indices, dominating 9 indices totally, and all the other indices that are not stochastically dominated by the RV Index are indifferent from the RV Index, including the D, FICA, MA, and 6-M TB. On the other hand, the 6-M TB third-order stochastically dominates the FWC, ED, EH, M, FFC, EM, C, FICA, MA, and S Indices, dominating 9 indices totally, and all the other indices that are not stochastically

[3] We note that one should include other measures like the Sharpe ratio in the selection.

Table 1
Descriptive statistics for the returns of different indices and the T-bill.

Index	Mean	Stdev	Skewness	Kurtosis	J-B
FWC	0.0072***	0.0191	−0.7455***	2.8927***	163.3403***
ED	0.0074***	0.0200	−1.8940***	9.0091***	1464.609***
EH	**0.0082*****	0.0254	−0.4882***	2.4033***	104.0866***
M	0.0075***	0.0200	0.5865***	1.3555***	49.7712***
FFC	0.0052***	0.0157	−0.6653***	4.3408***	317.3491***
RV	0.0069***	0.0122	−2.0408***	13.1786***	2916.7214***
EM	0.0078***	0.0383	−1.0874***	5.3861***	518.5106***
C	0.0044***	0.0106	−1.6617***	8.2156***	1204.8192***
D	0.0076***	0.0185	−1.0467***	4.5799***	389.9211***
FICA	0.0060***	0.0181	−3.6321***	35.5929***	20196.5139***
MA	0.0057***	0.0121	−2.8993***	17.2847***	5090.4466***
S	0.0063***	0.0232	−0.6297***	4.2484***	302.3575***
EMN	0.0047***	0.0088	−0.2218	1.7853***	52.6528***
TB	0.0022***	**0.0018**	0.2950**	−1.1731***	25.7671***

Note: This table reports the summary statistics including mean, standard deviation (Stdev), skewness, excess kurtosis, and Jarque-Bera (J-B) test. The symbols *, **, and *** denote the significance at 10 %, 5 %, and 1 % levels, respectively. The full names of all the indices are defined in Section 3.1. The values highlighted in boldface are the largest return and the smallest variance.

Table 2
SD results for the returns of indices for different hedge fund strategies and T-Bill.

	ED	EH	M	FFC	EM	RV	C	D	FICA	MA	S	EMN	TB	No. dominate	No. dominated
FWC	=	=	=	=	$\succ_{2,3}$	$\prec_{2,3}$	$\prec_3$	=	=	$\prec_{2,3}$	=	$\prec_3$	$\prec_3$	1	5
ED		=	=	=	$\succ_{2,3}$	$\prec_{2,3}$	$\prec_3$	=	=	$\prec_{2,3}$	=	$\prec_3$	$\prec_3$	1	5
EH			$\prec_{2,3}$	$\prec_{2,3}$	$\succ_{2,3}$	$\prec_{2,3}$	$\prec_3$	=	=	$\prec_{2,3}$	=	$\prec_3$	$\prec_3$	1	7
M				=	$\succ_{2,3}$	$\prec_{2,3}$	=	=	=	$\prec_{2,3}$	$\succ_{2,3}$	$\prec_{2,3}$	=	3	3
FFC					$\succ_{2,3}$	$\prec_{2,3}$	=	=	=	$\prec_{2,3}$	=	$\prec_{2,3}$	$\prec_3$	2	4
EM						$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_3$	0	13
RV							$\succ_{1,2,3}$	=	=	=	$\succ_{2,3}$	$\succ_{1,2,3}$	=	9	0
C								$\prec_{2,3}$	=	=	$\succ_{2,3}$	=	$\prec_3$	5	3
D									=	=	=	$\prec_3$	=	2	1
FICA										=	=	=	$\prec_3$	1	1
MA											$\succ_{2,3}$	=	$\prec_3$	7	1
S												$\prec_{2,3}$	$\prec_3$	1	6
EMN													=	8	1
6-MTB														9	0

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the first row by the i-order SD and Y = Z means that Y and Z do not dominate each other. The significance level is 5 %. "No. dominate" ("No. dominated") is the number of assets in the first column that dominate (being dominated by) the assets in other columns. The full names of all the indices are defined in Section 3.1.

dominated by the 6-M TB are indifferent from the 6-M TB, including the M, RV, D, and EMN. Though both the RV Index and the 6-M TB stochastically dominate 9 other indices and are indifferent from 4 other indices, we consider the RV Index the most dominant index and the 6-M TB the second dominant index because the RV Index first- or second-order dominates 9 other indices but the 6-M TB only third-order dominates 9 other indices. Readers may read Table 2 and notice that the third and fourth most dominant indices are the EMN and MA indices. We skip a detailed discussion on the comparison.

From Table 1, we notice that the Equity Hedge (EH) Total Index has the highest return of 0.0082 and the 6-M TB has the lowest risk of 0.0018. In addition, Fig. 1 shows that the EH Index is on top of all the other indices for most of the period. One may believe that the most dominant index should be the EH Index, not the RV Index. We note that though the EH Index has the highest mean but its mean value is not significantly higher than other indices while its variance could be significantly higher than other indices, and thus, it turns out that the EH Index is stochastically dominated by 7 other indices, and thus, it is definitely not a favorable index among all the indices if one uses the SD rule. Nevertheless, though the EH Index is not a favorable index if one uses the SD rule, it is a favorable index if one uses the mean as the measure of judgment while the 6-M TB is not the most favorable index among all the indices if one uses the SD rule but it is a favorable index if one uses the variance as the measure of judgment. Thus, in this paper, we consider that there are three most favorable indices: the RV Index (according to the SD rule), the EH Index (according to the mean rule), and the 6-M TB (accordingly to the variance rule) and we hypothesize that one should include the RV and EH Indices and the 6-M TB in their formation of a portfolio to get higher expected utility. We first test whether Conjecture 1A holds in this situation:

Conjecture 1A: investors could get higher expected utility if they include the EH Index (the index with the highest mean) to form a portfolio in the efficient frontier.

4.2. Comparison among portfolios with and without index with the highest mean

In this section, to examine whether Conjecture 1A could hold, we apply the SD approach to compare the performance between portfolios with and without the EH Index, the index with the highest mean. Before we make a comparison, we exhibit in Fig. 2 the efficient frontiers obtained by using the portfolio optimization method for the portfolios with and without the EH Index in situations in which a short sale is allowed and not allowed.

Fig. 2 shows that the two frontiers almost coincide except the portions at the starting with lower risk and lower return and the ending point with higher risk and higher return than the frontier of the portfolio with the EH Index is on top of that without the EH Index, especially in the right panel in Fig. 2 in which a short sell is not allowed. To be more precise, the frontiers with the EH Index are much higher than that without the EH Index when the risk is greater than 0.015.

To evaluate the performance between portfolios with and without the EH Index when a short sale is allowed or not allowed, we draw 15 portfolios along each efficient frontier. To do so, for each frontier, we divide the return interval into 14 small equal intervals so that there are 15 returns (points). For each given mean return, we get the corresponding point along the efficient frontier as the corresponding portfolio. Following this approach, we obtain 15 portfolios along the efficient frontier with or without the EH Index. We order the 15 points in each efficient frontier. We then employ the SD approach to compare the first point of the portfolio with the EH Index with the first point of the portfolio without the EH Index and so on when a short sale is allowed or not allowed and exhibit the results in Table 3 of Wong et al. [86].

The table shows that except for the 15th pair portfolio when a short sale is not allowed, there exists no stochastic dominance between the portfolios with the EH Index and the corresponding portfolios without the EH Index, regardless of whether a short sale is allowed or not. While the 15th portfolio with the EH Index second-order stochastically dominates the 15th portfolio without the EH Index when a short sale is not allowed. Based on the SD results from the table, we can conclude that investors could get higher expected utility if they include the EH Index (the index with the highest mean) to form a portfolio in the efficient frontier, inferring that Conjecture 1A could hold only in a small range but at least none of the portfolios without the EH Index stochastically dominates any of the corresponding portfolios with the EH Index. One may wonder why including the EH Index, the index getting the largest mean only increase their expected utility only in a small range, not in a larger range. The reason is that the standard deviation of the EH Index is so large that the EH Index stochastically dominates one index and is stochastically dominated by 7 other indices. Thus, the EH Index is one of the least favorable indices (it is the second least favorable index according to Table 2), and hence, it is the advantage of forming a portfolio that includes one of the least favorable indices, that is the EH Index, in this case, is not worse off.

4.3. Comparison among portfolios with and without the most dominant index

We turn to test whether Conjecture 1B holds that investors could get higher expected utility if they include the RV Index whose stochastically dominates most of the other assets to form a portfolio in the efficient frontier. To do so, we follow the approach we stated in Section 4.2 to compare 15 portfolios with and without the RV Index from their efficient frontiers to evaluate their performance when a short sale is allowed or not allowed. We first exhibit in the left and right panels of Fig. 3 the efficient frontiers between portfolios with and without the RV Index when a short sale is allowed or not allowed, respectively, and display their corresponding SD results in Table 4 of Wong et al. [86].

Fig. 3 shows that the frontier of portfolios with the RV Index is on top of that without the RV Index for nearly the entire range, regardless of whether a short sale is allowed or not allowed, according to the theory of efficient frontier, the portfolios with the RV

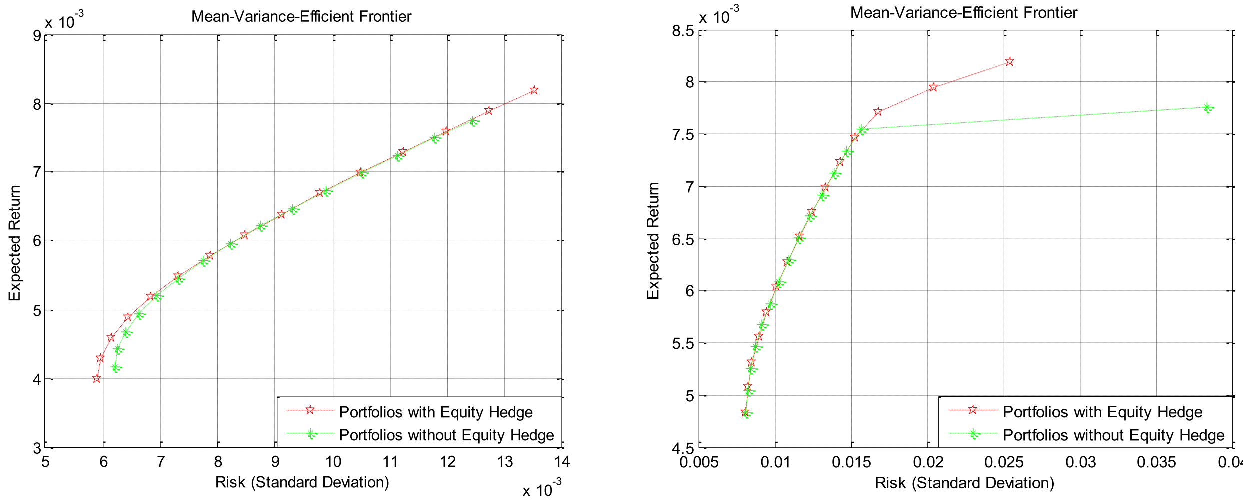


Fig. 2. Efficient Frontiers between portfolios with and without Equity Hedge.
Note: The left and right panels are the Mean-Variance Efficient frontiers when a short sale is allowed or not allowed, respectively.

Table 3
SD analysis between portfolios with and without 6-M TB.

Portfolios with TB	Long	Short	Portfolios without TB
TB1	$\succ_3$	=	TB1′
TB2	$\succ_3$	=	TB2′
TB3	$\succ_3$	=	TB3′
TB4	$\succ_3$	=	TB4′
TB5	$\succ_3$	=	TB5′
TB6	=	$\succ_3$	TB6′
TB7	=	$\succ_{2,3}$	TB7′
TB8	=	=	TB8′
TB9	=	=	TB9′
TB10	=	=	TB10′
TB11	=	=	TB11′
TB12	=	=	TB12′
TB13	=	=	TB13′
TB14	=	=	TB14′
TB15	=	=	TB15′

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the last column by the i-order SD and Y = Z means that Y and Z do not dominate each other. TBn (TBn′) stands for the portfolio with (without) 6-month T-bill for $n = 1, \ldots, 15$. The significance level is 5 %.

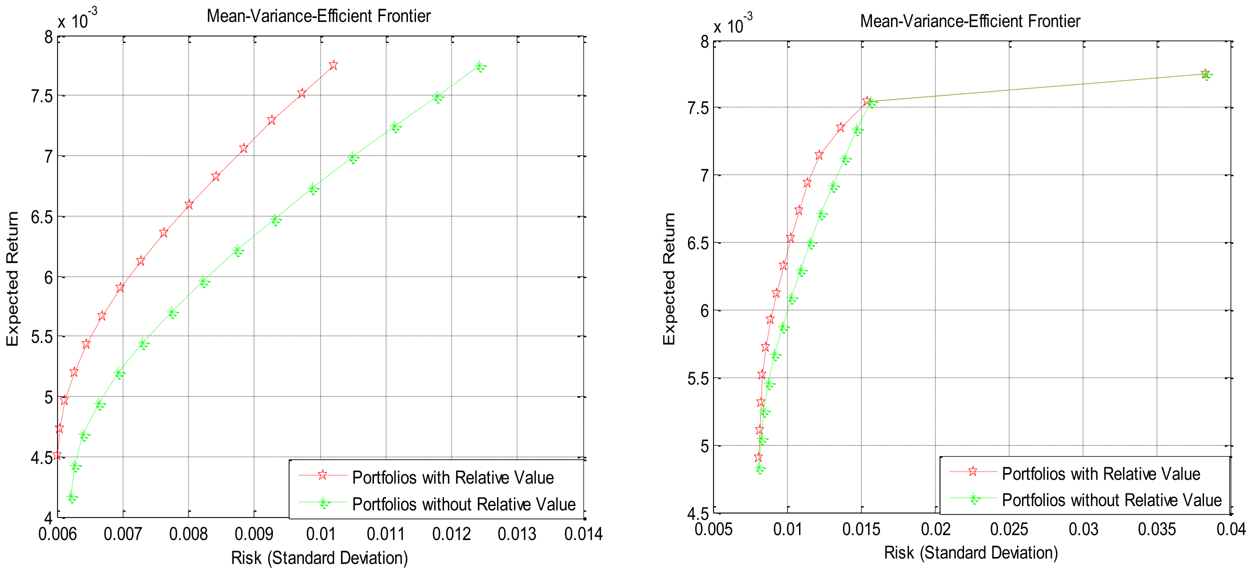


Fig. 3. Efficient Frontiers between portfolios with and without Relative Value.
Note: The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed or not allowed, respectively.

Index in the efficient frontier are preferred to those without the RV Index. However, the table shows that there does not exist any stochastic dominance between any of the portfolios with and without the RV Index, regardless of whether a short sale is allowed or not. Thus, based on the SD results in the table, we CANNOT conclude that investors could get higher expected utility if they include the RV Index (the most dominant index) to form a portfolio in the efficient frontier. Our findings show that, even though the efficient frontier seems to get improvement in both return and risk by including the RV Index in the portfolios, the improvement is not significant enough to get the stochastic dominance test to be significant, and thus, there does not exist any stochastic dominance between any of the portfolios with and without the RV Index and we conclude that Conjecture 1B may NOT hold.

4.4. Comparison among portfolios with and without index with the smallest variance

We turn to test whether Conjecture 1C holds that investors could get higher expected utility if they include the 6-M TB with the smallest standard deviation to form a portfolio in the efficient frontier. To do so, we follow the approach we stated in Section 4.2 to compare 15 portfolios with and without the 6-M TB from their efficient frontiers to evaluate their performance when a short sale is

Table 4
SD analysis between portfolios with and without EH and 6-M TB.

Portfolios with both EH and TB	Long	Short	Portfolios without both EH and TB
ET1	$\succ_3$	=	ET1′
ET2	$\succ_3$	=	ET2′
ET3	$\succ_3$	=	ET3′
ET4	$\succ_3$	=	ET4′
ET5	$\succ_3$	$\succ_3$	ET5′
ET6	=	$\succ_3$	ET6′
ET7	=	=	ET7′
ET8	=	=	ET8′
ET9	=	=	ET9′
ET10	=	=	ET10′
ET11	=	=	ET11′
ET12	=	=	ET12′
ET13	=	=	ET13′
ET14	=	=	ET14′
ET15	$\succ_{2,3}$	=	ET15′

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the last column by the i-order SD and Y = Z means that Y and Z do not dominate each other. ETn (ETn′) stands for the portfolio with (without) equity hedge index and 6-month T-bill for n = 1, …, 15. The significance level is 5 %.

allowed or not allowed. We first exhibit in the left and right panels of Fig. 4 the efficient frontiers between portfolios with and without the 6-M TB when a short sale is allowed or not allowed, respectively, and display their corresponding SD results in Table 3.

Fig. 4 shows that the frontier of portfolios with the 6-M TB is on top of that without the 6-M TB in the beginning and ending periods while the two frontiers are close to each other in between, regardless of whether when a short sale is allowed or not allowed, accordingly to the theory of efficient frontier, the portfolios with the 6-M TB in the efficient frontier are preferred to those without the 6-M TB. To be more precise, the frontiers with the 6-M TB are higher than that without the 6-M TB when the risk is smaller than 0.008 and greater than 0.01 when a short sale is allowed and when the risk is smaller than 0.008 when a short sale is not allowed. Table 3 shows that the first 5 portfolios with the 6-M TB, that is, TB1-TB5 third-order stochastically dominate the corresponding first 5 portfolios without the 6-M TB, that is, TB1′-TB5′ and there exists no stochastic dominance between any other pairs of the portfolios when a short sale is not allowed; on the other hand, the 6th portfolio with the 6-M TB, that is, TB6 third-order stochastically dominates the corresponding 6th portfolio without the 6-M TB, that is, TB6′, and the 7th portfolio with the 6-M TB, that is, TB7 second-order stochastically dominates the 7th portfolio without the 6-M TB, that is, TB7′, and there exists no stochastic dominance between any other pairs of the portfolios when a short sale is allowed. Our findings do conclude that Conjecture 1C holds marginally that investors

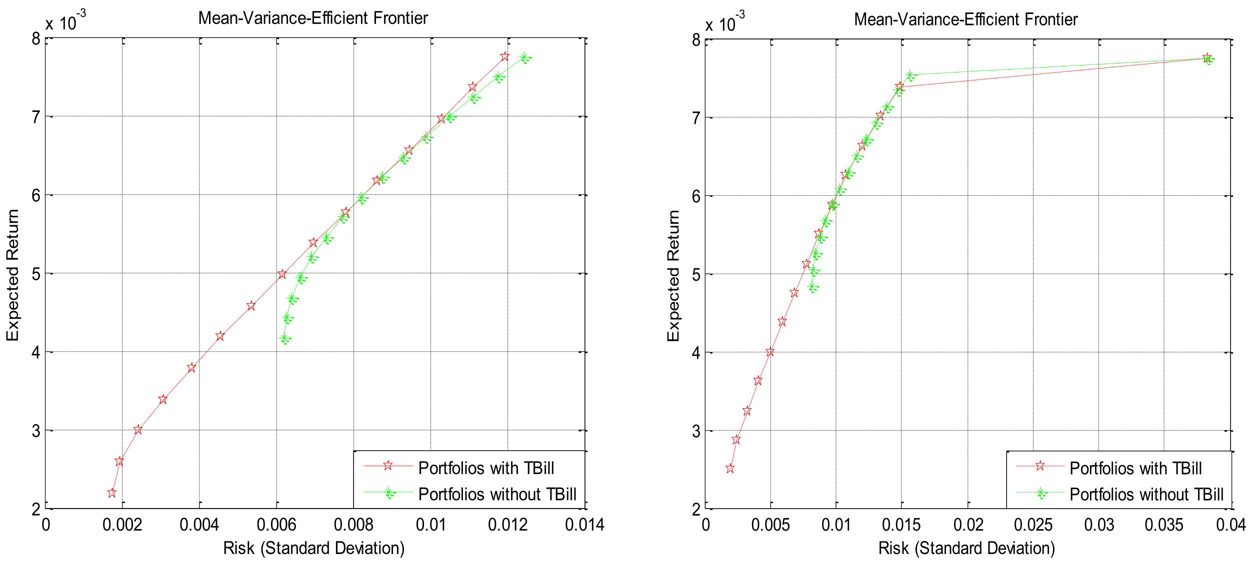


Fig. 4. Efficient Frontiers between portfolios with and without 6-M TB.
Note: the left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

could get marginally higher expected utility if they include the 6-M TB with the smallest standard deviation to form a portfolio in the efficient frontier.

In this paper, we conjecture that the asset with the highest mean; that is, the EH Index in our study, the asset that stochastically dominates most of the other assets; the RV Index in our study, and the asset with the smallest standard deviation; that is, the 6-M TB in our study, should be included in forming their portfolio to get much higher expected utility, and thus, we set Conjectures 1A, 1B, and 1C.

Nevertheless, our findings show that most of the portfolios formed by using one of the EH and RV Indices and the 6-M TB do not stochastically dominate the corresponding portfolio without one of the EH and RV Indices and the 6-M TB, except some of the portfolios with the 6-M TB marginally dominate the corresponding portfolios without the 6-M TB and one of the portfolios with the EH Index, that is EH15, second-order stochastically dominates the corresponding portfolio without the EH Index, that is EH15' when a short sale is not allowed, implying that Conjecture 1B does not hold and both Conjectures 1A and 1C hold marginally. So, do our conjecture that including the asset with the highest mean, the asset that stochastically dominates most of the other assets, and the asset with the smallest standard deviation will get much higher expected utility holds? In order to study the issue, we turn to compare the performance between portfolios with and without any two of the EH and RV Indices and the 6-M TB as shown in the next section.

4.5. Comparison among portfolios with and without the EH and RV indices

We turn to examine whether Conjecture 2A holds that investors could get higher expected utility if they include both the EH Index (with the highest mean) and the RV Index (whose stochastically dominates most of the other assets) to form a portfolio in the efficient frontier. To do so, we follow the approach we stated in Section 4.2 to compare 15 portfolios with and without the EH and RV Indices from their efficient frontiers to evaluate their performance when a short sale is allowed or not allowed. We first exhibit in the left and right panels of Fig. 5 the efficient frontiers between portfolios with and without both the EH and RV Indices when a short sale is allowed or not allowed, respectively, and display their corresponding SD results in Table 6 of Wong et al. [86].

Fig. 5 shows that the frontier of portfolios with both the EH and RV Indices is on top of that without the EH and RV Indices for nearly the entire range, regardless of whether when a short sale is allowed or not allowed. Table 6 of Wong et al. [86] shows that the relationships for all pairs of assets are "equal" except RE15 $\succ_{2,3}$RE15$'$, in the long position, and thus, we do not display them in this paper to save space. Thus, accordingly to the theory of efficient frontier, the portfolios with both the EH and RV Indices in the efficient frontier are preferred to those without the EH and RV Indices. Nonetheless, the table shows that except for the 15th pair portfolio (RE15) when a short sale is not allowed, there is no stochastic dominance between any portfolio with both the EH and RV Indices and the corresponding portfolios without the EH and RV Indices, regardless of whether a short sale is allowed or not. The only exception case is that the 15th portfolio (RE15) with both the EH and RV Indices second-order stochastically dominates the 15th portfolio (RE15$'$) without the EH and RV Indices when a short sale is not allowed. Based on the SD results from the table, we can conclude that Conjecture 2A holds only for RE15 when a short sale is not allowed but does not hold for any other cases that investors could get higher expected utility if they compare RE15 with RE15' when a short sale is not allowed but does not hold for any other cases.

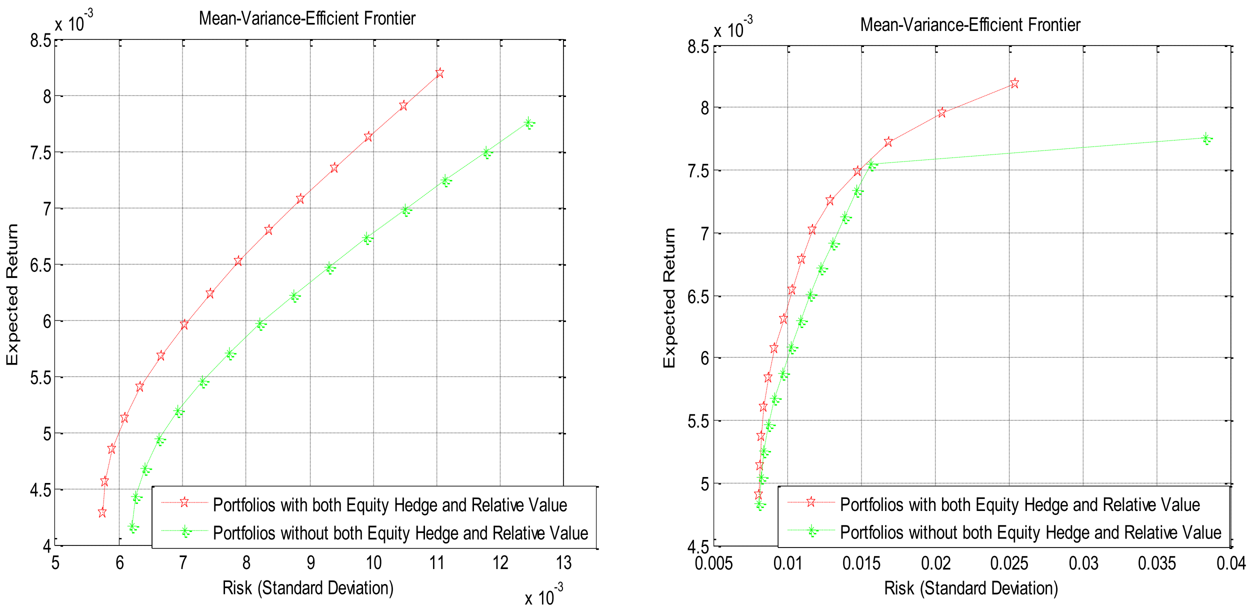


Fig. 5. Efficient Frontiers between portfolios with and without Equity Hedge and Relative Value.
Note: The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

Table 5
SD analysis between portfolios with and without RV and 6-M TB.

Portfolios with both RV and TB	Long	Short	Portfolios without both RV and TB
RT1	$\succ_3$	=	RT1$'$
RT2	$\succ_3$	=	RT2$'$
RT3	$\succ_3$	=	RT3$'$
RT4	$\succ_3$	=	RT4$'$
RT5	$\succ_3$	=	RT5$'$
RT6	$\succ_3$	$\succ_3$	RT6$'$
RT7	$\succ_{2,3}$	$\succ_3$	RT7$'$
RT8	=	$\succ_{2,3}$	RT8$'$
RT9	=	$\succ_{2,3}$	RT9$'$
RT10	=	$\succ_{2,3}$	RT10$'$
RT11	=	$\succ_{2,3}$	RT11$'$
RT12	=	$\succ_{2,3}$	RT12$'$
RT13	=	=	RT13$'$
RT14	=	=	RT14$'$
RT15	=	=	RT15$'$

Note: $Y \succ_i Z$ $(Y \prec_i Z)$ means that Y in the first column significantly dominates (is significantly dominated by) Z in the last column by the i-order SD and Y = Z means that Y and Z do not dominate each other. RTn (RTn$'$) stands for the portfolio with (without) relative value index, and 6-month T-bill for $n = 1, \ldots, 15$. The significance level is 5 %.

Table 6
SD analysis between portfolios with EH, RV, and 6-M TB and portfolios without EH, RV, and 6-M TB.

Portfolios with EH, RV, and TB	Long	Short	Portfolios without EH, RV, and TB
ERT 1	$\succ_3$	=	ERT1$'$
ERT 2	$\succ_3$	=	ERT2$'$
ERT 3	$\succ_3$	=	ERT3$'$
ERT 4	$\succ_3$	=	ERT4$'$
ERT 5	$\succ_3$	$\succ_3$	ERT5$'$
ERT 6	$\succ_{2,3}$	$\succ_3$	ERT6$'$
ERT 7	$\succ_{2,3}$	$\succ_{2,3}$	ERT7$'$
ERT 8	$\succ_{2,3}$	$\succ_{2,3}$	ERT8$'$
ERT 9	$\succ_3$	$\succ_{2,3}$	ERT9$'$
ERT 10	=	$\succ_{2,3}$	ERT10$'$
ERT 11	=	$\succ_{2,3}$	ERT11$'$
ERT12	=	=	ERT12$'$
ERT 13	=	=	ERT13$'$
ERT 14	=	=	ERT14$'$
ERT 15	$\succ_{2,3}$	=	ERT15$'$

Note: $Y \succ_i Z$ $(Y \prec_i Z)$ means that Y in the first column significantly dominates (is significantly dominated by) Z in the last column by the i-order SD and Y = Z means that Y and Z do not dominate each other. ERTn (ERTn$'$) stands for the portfolio with (without) equity hedge index, relative value index, and 6-month T-bill for $n = 1, \ldots, 15$. The significance level is 5 %.

4.5.1. *Comparison among portfolios with and without the EH index and the 6-M TB*

We turn to examine whether Conjecture 2B holds that investors could get higher expected utility if they include both the EH Index (with the highest mean) and 6-M TB (with the smallest standard deviation) to form a portfolio in the efficient frontier. To do so, we follow the approach we stated in Section 4.2 to compare 15 portfolios with and without the EH Index and 6-M TB from their efficient frontiers to evaluate their performance when a short sale is allowed or not allowed. We first exhibit in the left and right panels of Fig. 6 the efficient frontiers between portfolios with and without the EH Index and 6-M TB when a short sale is allowed or not allowed, respectively, and display their corresponding SD results in Table 4.

Fig. 6 shows that the frontier of portfolios with the EH Index and the 6-M TB is on top of that without the EH Index and the 6-M TB in the beginning and ending periods while the two frontiers are close to each other in between, regardless of whether when a short sale is allowed or not allowed. According to the theory of efficient frontier, the portfolios with the EH Index and the 6-M TB in the efficient frontier are preferred to those without the EH Index and the 6-M TB. To be more precise, the frontiers with the EH Index and the 6-M TB are higher than that without the 6-M TB when the risk is smaller than around 0.008 and greater than 0.01 when a short sale is allowed and when the risk is smaller than 0.008 and greater than 0.015 when a short sale is not allowed. Table 4 shows that the first 5 portfolios

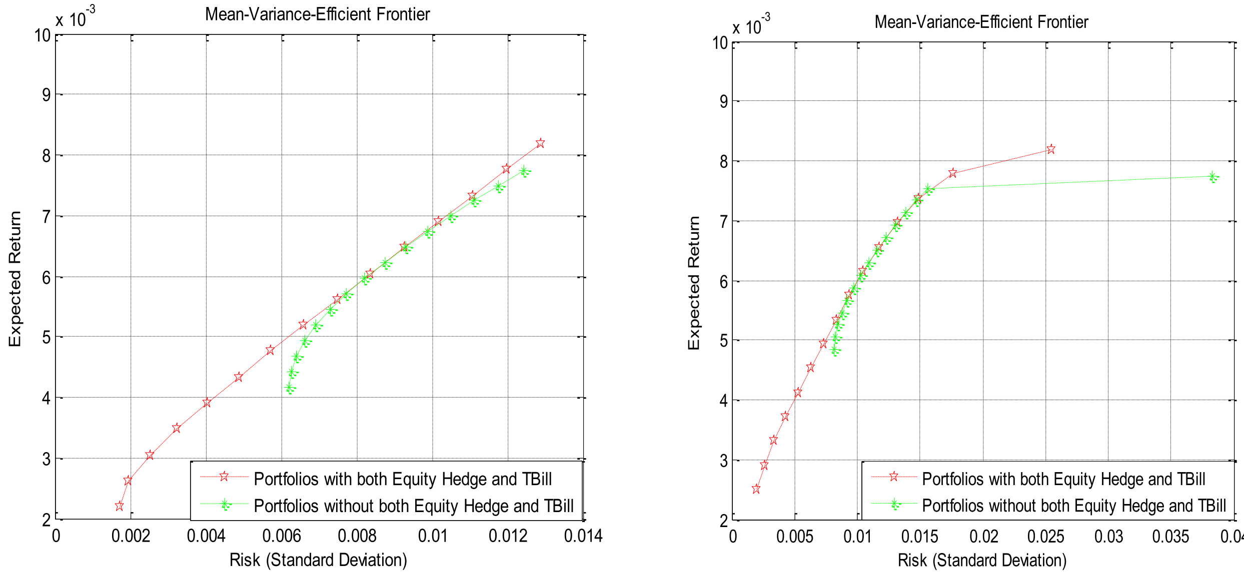


Fig. 6. Efficient Frontiers between portfolios with and without Equity Hedge and 6-M TB.
Note: The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

with the EH Index and the 6-M TB, that is, ET1- ET5 third-order stochastically dominate the corresponding first 5 portfolios without the EH Index and the 6-M TB, that is, ET1′-ET5′, ET15 second-order stochastically dominates ET15′, and there exists no stochastic dominance between any other pairs of the portfolios when a short sale is not allowed. On the other hand, ET5 and ET6 third-order stochastically dominate ET5′ and ET6′ and there exists no stochastic dominance between any other pairs of the portfolios when a short sale is allowed. Our findings do conclude that Conjecture 2B holds marginally that investors could get marginally higher expected utility if they include the EH Index with the highest mean and 6-M TB with the smallest standard deviation to form a portfolio in the efficient frontier**.**

4.6. Comparison among portfolios with and without the RV index and the 6-M TB

We turn to test whether Conjecture 2C holds that investors could get higher expected utility if they include the RV Index whose stochastically dominates most of the other assets and 6-M TB with the smallest standard deviation to form a portfolio in the efficient frontier. To do so, we follow the approach we stated in Section 4.2 to compare 15 portfolios with and without the RV Index and 6-M TB from their efficient frontiers to evaluate their performance when a short sale is allowed or not allowed. We first exhibit in the left and

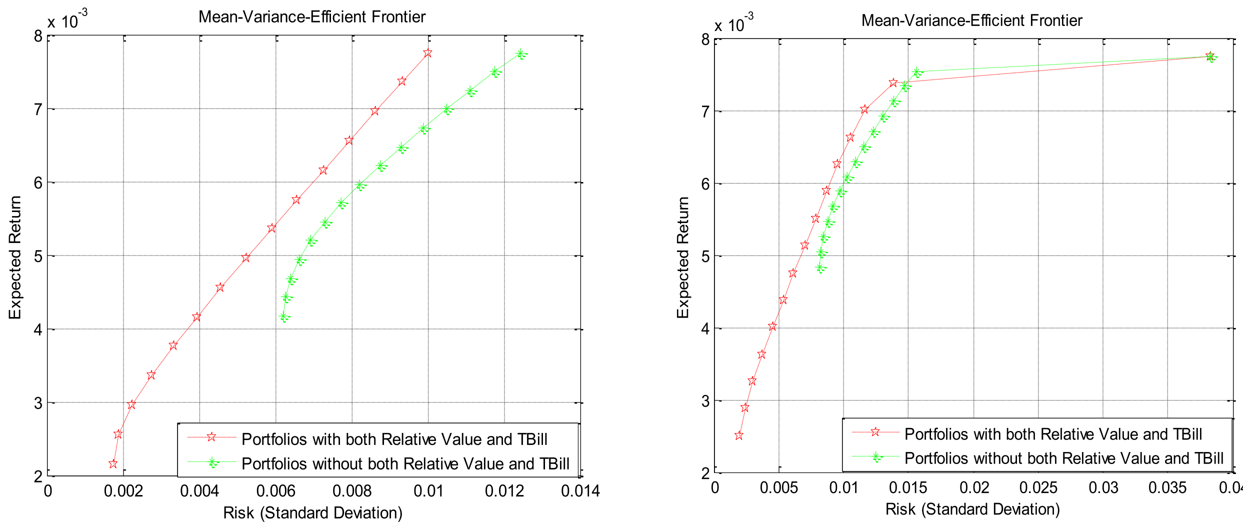


Fig. 7. Efficient Frontiers between portfolios with and without Relative Value and 6-M TB.
Note: The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

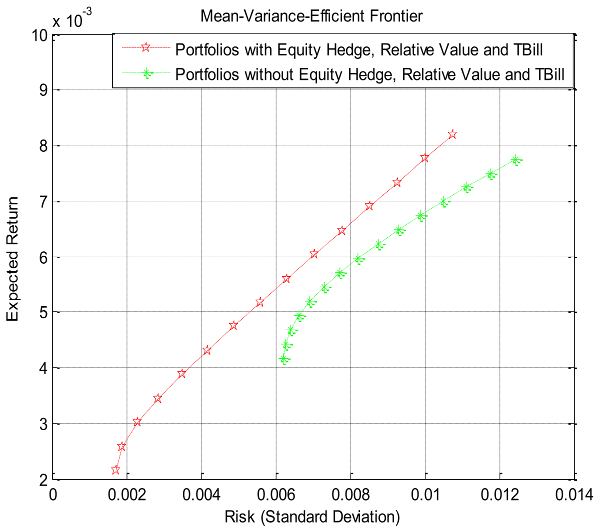
right panels of Fig. 7 the efficient frontiers between portfolios with and without the RV Index and 6-M TB when a short sale is allowed or not allowed, respectively, and display their corresponding SD results in Table 5.

Fig. 7 shows that the frontier of portfolios with the RV Index and 6-M TB is on top of that without the RV Index and 6-M TB for nearly the entire range, regardless of whether when a short sale is allowed or not allowed, accordingly to the theory of efficient frontier, the portfolios with the RV Index and 6-M TB in the efficient frontier are preferred to those without the RV Index and 6-M TB. Table 5 shows that the first 6 portfolios with the RV Index and 6-M TB, that is, RT1- RT6 third-order stochastically dominate the corresponding first 6 portfolios without the RV Index and 6-M TB, that is, $RT1'$-$RT6'$, RT7 second-order stochastically dominates $RT7'$, and there is no stochastic dominance between any other pairs of the portfolios when a short sale is not allowed. On the other hand, RT6 and RT7 third-order stochastically dominates $RT6'$ and $RT7'$, RT8-RT12 s-order stochastically dominates $RT8' - RT12'$ and there is no stochastic dominance between any other pairs of the portfolios when a short sale is allowed. Since there is much second-order stochastic dominance, the findings not only conclude that Conjecture 2C holds marginally but also conclude that Conjecture 2C holds significantly and that investors could get significantly higher expected utility if they include both the RV Index and 6-M TB to form a portfolio in the efficient frontier.

4.7. Comparison among portfolios with and without index with the EH and RV indices and the 6-M TB

We now test whether Conjecture 3 hold that investors could get significantly higher expected utility if they include the EH Index with the highest mean, the RV Index that stochastically dominates most of the other assets and 6-M TB with the smallest standard deviation to form a portfolio in the efficient frontier than all the other cases we studied previously in this paper. To do so, we follow the approach we stated in Section 4.2 to compare 15 portfolios with and without both the EH and RV Indices and 6-M TB from their efficient frontiers to evaluate their performance when a short sale is allowed or not allowed. We first exhibit in the left and right panels of Fig. 8 the efficient frontiers between portfolios with and without the EH and RV Indices and 6-M TB when a short sale is allowed or not allowed, respectively, and display their corresponding SD results in Table 6.

Fig. 8 shows that the frontier of portfolios with the EH and RV Indices and 6-M TB is on top of that without the EH and RV Indices and 6-M TB for nearly the entire range, regardless of whether when a short sale is allowed or not allowed, accordingly to the theory of efficient frontier, the portfolios with the EH and RV Indices and 6-M TB in the efficient frontier are preferred to those without the EH and RV Indices and 6-M TB. Table 6 shows that the first 5 portfolios with the EH and RV Indices and 6-M TB, that is, ERT1- ERT5 third-order stochastically dominate the corresponding first 5 portfolios without the EH and RV Indices and 6-M TB, that is, $ERT1'$-$ERT5'$, ERT9 third-order stochastically dominates $ERT9'$, ERT6- ERT8 and ERT15 second-order stochastically dominate $ERT6'$-$ERT8'$ and $ERT15'$, and is no stochastic dominance between any other pairs of the portfolios when a short sale is not allowed. On the other hand, ERT5 and ERT6 third-order stochastically dominate $ERT5'$ and $ERT6'$, ERT7- ERT11 second-order stochastically dominate $ERT7'$-$ERT11'$ and there is no stochastic dominance between any other pairs of the portfolios when a short sale is allowed. Our findings do conclude that Conjecture 3 holds not only significantly that investors not only get significantly higher expected utility if they include the EH and RV Indices and 6-M TB to form a portfolio in the efficient frontier, but also it is significantly higher than all other cases we studied previously because there are much more second-order stochastic dominances than all other cases we studied previously. Thus, we believe that the best strategy for investors is to include the asset with the highest mean, the asset dominates most of the other assets and the asset with the smallest standard deviation to form a portfolio in the efficient frontier.



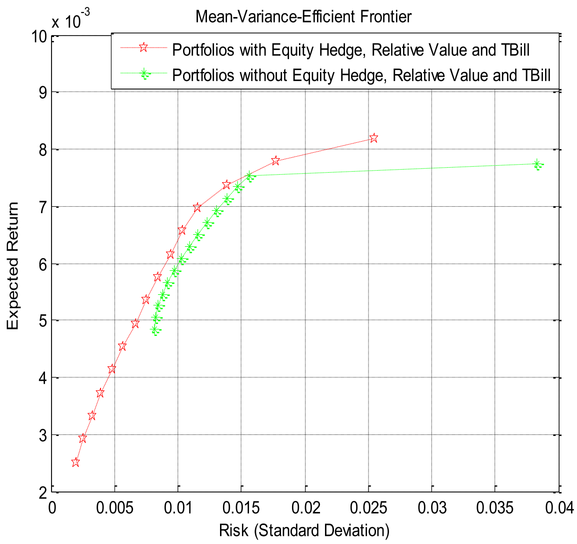


Fig. 8. Efficient Frontiers between portfolios with and without the EH and RV Indices and the **6-M TB.**
Note: The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

To see whether our belief will come true, we make Conjectures 4A to 4F. To save space, we only test Conjectures 4A to 4C as discussed in the following subsections.

4.8. Comparison among portfolios with the EH and RV indices and 6-M TB and those with the EH and RV indices but without 6-M TB

We now test whether Conjecture 4A holds that investors could get significantly higher expected utility if they include the EH Index with the highest mean, the RV Index that stochastically dominates most of the other assets and 6-M TB with the smallest standard deviation to form a portfolio in the efficient frontier than the portfolios from the efficient frontier formed by the EH and RV Indices and other assets but without 6-M TB. To do so, we follow the approach we stated in Section 4.2 to compare 15 portfolios with both the EH and RV Indices and 6-M TB from their efficient frontiers with the portfolios with both the EH and RV Indices to evaluate their performance when a short sale is allowed or not allowed. We display their corresponding SD results in Table 7.

Table 7 shows that the first 6 portfolios with the EH and RV Indices and the 6-M TB, that is, ERT1- ERT6 third-order stochastically dominate the corresponding first 6 portfolios with the EH and RV Indices but without 6-M TB, that is, ER1- ER6, and there is no stochastic dominance between any other pairs of the portfolios when a short sale is not allowed. On the other hand, ERT6 and ERT7 third-order stochastically dominate ER6 and ER7 and there is no stochastic dominance between any other pairs of the portfolios when a short sale is allowed. Our findings do conclude that Conjecture 4A holds that investors could get higher expected utility if they include the EH and RV Indices and 6-M TB to form a portfolio in the efficient frontier than the portfolios from the efficient frontier formed by the EH and RV Indices and other assets but without 6-M TB.

4.9. Comparison among portfolios with the EH and RV indices and 6-M TB and those with the EH index and 6-M TB but without the RV index

We now test whether Conjecture 4B holds that investors could get significantly higher expected utility if they include the EH Index with the highest mean, the RV Index whose stochastically dominates most of the other assets and 6-M TB with the smallest standard deviation to form a portfolio in the efficient frontier than the portfolios from the efficient frontier formed by the EH Index, 6-M TB, and other assets but without the RV Index. To do so, we follow the approach we stated in Section 4.2 to compare 15 portfolios with both the EH and RV Indices and 6-M TB from their efficient frontiers with the portfolios with both the EH Index and 6-M TB to evaluate their performance when a short sale is allowed or not allowed. We display their corresponding SD results in Table 11 of Wong et al. [86]. We note that Table 11 of Wong et al. [86] shows that the relationships for all pairs of assets are "equal", and thus, we do not display them in this paper to save space.

The table shows that there is no stochastic dominance between any of the (ERT) portfolios with the EH and RV Indices and 6-M TB and the (ET) portfolios with the EH Index and 6-M TB but without the RV Index, regardless of whether a short sale is allowed or not. Thus, based on the SD results in Table 11 of Wong et al. [86], we CANNOT conclude that investors could get higher expected utility if they choose the ERT portfolios than the ET portfolios, inferring that Conjecture 4B may NOT hold. To conduct deeper analysis, we count (ERT1, ERT2) how many portfolios the ERT portfolios dominate, (ERT1a, ERT2a) how many portfolios the ERT portfolios are dominated, (ET1, ET2) how many portfolios the ET portfolios dominate, and (ET1a, ET2a) how many portfolios the ET portfolios are dominated for long-only and Short-allowed strategies, respectively. By comparing ERTi with ETi and comparing ERTia with ETia for i = 1,2, we will know whether ERT or ET is better. To do so, we exhibit in Table 8(9) to show ERT1 (ERT2) and ERT1a (ERT2a) and exhibit in Table 10(11) to show ET1 (ET2) and ET1a (ET2a) for long-only and Short-allowed strategies, respectively. From Tables 8 and

Table 7
SD analysis between portfolios with the EH and RValue Indices and 6-M TB and portfolios with the EH and RV Indices but without 6-M TB.

Portfolios with EH, RV, and 6-M TB	Long	Short	Portfolios with EH and RV
ERT1	$\succ_3$	=	ER1
ERT2	$\succ_3$	=	ER2
ERT3	$\succ_3$	=	ER3
ERT4	$\succ_3$	=	ER4
ERT5	$\succ_3$	=	ER5
ERT6	$\succ_3$	$\succ_3$	ER6
ERT7	=	$\succ_3$	ER7
ERT8	=	=	ER8
ERT9	=	=	ER9
ERT10	=	=	ER10
ERT11	=	=	ER11
ERT12	=	=	ER12
ERT13	=	=	ER1 3
ERT14	=	=	ER14
ERT15	=	=	ER15

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the last column by the i-order SD and Y = Z means that Y and Z do not dominate each other. ERTn stands for the portfolio with equity hedge index, relative value index, and 6-month T-bill and ERn stands for the portfolio with both equity hedge index and relative value index for $n = 1, \ldots, 15$. The significance level is 5 %.

Table 8
SD analysis between ERT portfolios, the individual assets, and the naïve 1/N portfolio under Long-only strategy.

	FWC	ED	EH	M	FFC	RV	EM	C	DIS	FICA	MA	S	EMN	TB	Pn	No. dominate/dominated
ERT1	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	12/0
ERT2	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	12/0
ERT3	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	12/0
ERT4	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	=	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_3$	13/0
ERT5	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_3$	14/0
ERT6	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	14/0
ERT7	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\prec_3$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	=	$\succ_{2,3}$	13/1
ERT8	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	=	=	$\succ_{2,3}$	10/0
ERT9	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	=	=	$\succ_{2,3}$	10/0
ERT10	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	=	=	$\succ_{2,3}$	10/0
ERT11	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_3$	=	$\succ_{2,3}$	11/0
ERT12	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_3$	=	=	11/0
ERT13	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_3$	=	=	=	$\succ_{2,3}$	$\succ_3$	=	=	8/0
ERT14	=	=	$\succ_{2,3}$	=	=	=	$\succ_{2,3}$	$\succ_3$	=	=	=	$\succ_{2,3}$	$\prec_3$	=	=	4/1
ERT15	=	=	=	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\succ_{2,3}$	$\prec_3$	=	=	$\prec_{2,3}$	=	$\prec_3$	$\prec_3$	$\prec_{2,3}$	1/8
Pn	=	=	$\succ_{2,3}$	=	=	=	$\succ_{2,3}$	=	=	=	=	$\succ_{2,3}$	$\prec_{2,3}$	$\prec_3$		3/2
Total	13/0	13/0	14/0	13/1	12/1	3/2	15/0	14/1	12/0	3/0	7/1	14/0	10/2	0/1	11/1	144/9

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the first row by the i-order SD and Y = Z means that Y and Z do not dominate each other. ERTn stands for the portfolio with equity hedge index, relative value index, and 6-M TB for $n = 1,\ldots,15$. The significance level is 5 %. The full names of all the indices are defined in Section 3.1. Total only counts the total number of dominate and the total number of dominated for ERT1 to ERT15, it does not include the total number of dominate and the total number of dominated for Pn.

Table 9
SD analysis between ERT portfolios and individual assets and the naïve 1/N portfolio under Short-allowed strategy.

	FWC	ED	EH	M	FFC	RV	EM	C	DIS	FICA	MA	S	EMN	TB	Pn	No. dominate/dominated
ERT1	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	=	$\prec_3$	$\succ_3$	10/1
ERT2	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	13/0
ERT3	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_{2,3}$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	13/0
ERT4	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	=	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_3$	14/0
ERT5	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_3$	15/0
ERT6	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_3$	14/0
ERT7	$\succ_{2,3}$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	14/0
ERT8	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	14/0
ERT9	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	14/0
ERT10	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	14/0
ERT11	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	14/0
ERT12	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	14/0
ERT13	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	13/0
ERT14	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	13/0
ERT15	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	$\succ_{2,3}$	13/0
Pn	=	=	$\succ_{2,3}$	=	=	=	$\succ_{2,3}$	=	=	=	=	$\succ_{2,3}$	$\prec_{2,3}$	$\prec_3$		3/2
Total	15/0	15/0	15/0	14/0	15/0	9/0	15/0	15/0	15/0	11/0	15/0	15/0	14/0	4/1	15/0	187/0

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the first row by the i-order SD and Y = Z means that Y and Z do not dominate each other. ERTn stands for the portfolio with equity hedge index, relative value index, and 6-M TB for any $n = 1, \ldots, 15$. The significance level is 5 %. The full names of all the indices are defined in Section 3.1. Total only counts the total number of dominate and the total number of dominated for ERT1 to ERT15, it does not include the total number of dominate and the total number of dominated for Pn.

Table 10
SD analysis between portfolios with Equity Hedge and 6-M TB and the individual assets and the naïve 1/N portfolio under Long-only strategy.

	FWC	ED	EH	M	FFC	RV	EM	C	DIS	FICA	MA	S	EMN	TB	Pn	No. dominate/dominated
ET1	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$\prec_3$	$\succ_3$	12/1
ET2	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	12/0
ET3	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	12/0
ET4	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$=$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_3$	12/0
ET5	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_3$	13/0
ET6	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\prec_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$=$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	12/1
ET7	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\prec_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$=$	$=$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	10/1
ET8	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	9/0
ET9	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	9/0
ET10	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	9/0
ET11	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	$\succ_3$	$=$	$=$	10/0
ET12	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$=$	$=$	$\succ_{2,3}$	$\succ_3$	$=$	$=$	9/0
ET13	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$=$	$=$	$=$	$\succ_{2,3}$	$\succ_3$	$=$	$=$	$=$	$\succ_{2,3}$	$\prec_3$	$=$	$=$	6/1
ET14	$=$	$=$	$\succ_{2,3}$	$=$	$=$	$\prec_3$	$\succ_{2,3}$	$\succ_3$	$=$	$=$	$=$	$\succ_{2,3}$	$\prec_3$	$=$	$=$	4/2
ET15	$=$	$=$	$=$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\succ_{2,3}$	$\prec_3$	$=$	$=$	$\prec_{2,3}$	$=$	$\prec_3$	$\prec_3$	$\prec_{2,3}$	1/8
Total	13/0	13/0	14/0	12/1	11/1	1/4	15/0	11/1	12/0	1/1	5/1	14/1	8/3	0/2		130/15

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the first row by the i-order SD and $\mathrm{Y} = \mathrm{Z}$ means that Y and Z do not dominate each other. ETn stands for the portfolio with both equity hedge index and 6-M TB for any $\mathrm{n} = 1, \ldots, 15$. The significance level is 5 %. The full names of all the indices are defined in Section 3.1.

Table 11
SD analysis between portfolios with Equity Hedge and 6-M TB and the individual assets and the naïve 1/N portfolio under Short-allowed strategy.

	FWC	ED	EH	M	FFC	RV	EM	C	DIS	FICA	MA	S	EMN	TB	Pn	No. dominate/dominated
ET1	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	=	$\succ_3$	$\succ_3$	=	=	$\succ_3$	$\succ_3$	=	=	$\succ_3$	9/0
ET2	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	13/0
ET3	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_{2,3}$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	=	=	$\succ_3$	11/0
ET4	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	=	$\succ_3$	$\succ_{2,3}$	$\succ_3$	=	$\succ_3$	12/0
ET5	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_3$	14/0
ET6	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_3$	14/0
ET7	$\succ_{2,3}$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	14/0
ET8	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	14/0
ET9	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{1,,2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	13/0
ET10	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	11/0
ET11	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	11/0
ET12	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,,2,3}$	=	$\succ_{2,3}$	$\succ_{1,,2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	11/0
ET13	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,,2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	11/0
ET14	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_{1,,2,3}$	=	=	10/0
ET15	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	=	=	12/0
Total	15/0	15/0	15/0	14/0	15/0	4/0	15/0	15/0	14/0	6/0	11/0	15/0	12/0	1/0		167/9

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the first row by the i-order SD and Y = Z means that Y and Z do not dominate each other. ETn stands for the portfolio with both equity hedge index and 6-M TB for any $n = 1, \ldots, 15$. The significance level is 5 %. The full names of all the indices are defined in Section 3.1. Total only counts the total number of dominate and the total number of dominated for ERV1 to ERV15, it does not include the total number of dominate and the total number of dominated for Pn.

Table 12
SD analysis between portfolios with Relative Value and 6-M TB and the individual assets and the naïve 1/N portfolio under Long-only strategy.

	FWC	ED	EH	M	FFC	RV	EM	C	DIS	FICA	MA	S	EMN	TB	Pn	No. dominate/dominated
RT1	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	12/0
RT2	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	12/0
RT3	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	=	$\succ_3$	$\succ_3$	$\succ_3$	=	$\succ_3$	12/0
RT4	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	=	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_3$	13/0
RT5	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_3$	14/0
RT6	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	14/0
RT7	$\succ_{2,3}$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	13/0
RT8	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\prec_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	=	$\succ_{2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	11/1
RT9	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	=	=	$\succ_{2,3}$	10/0
RT10	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	=	$\succ_{2,3}$	=	=	$\succ_{2,3}$	=	=	$\succ_{2,3}$	9/0
RT11	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	=	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_3$	=	$\succ_{2,3}$	10/0
RT12	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	=	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_3$	=	$\succ_{2,3}$	10/0
RT13	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	$\succ_{2,3}$	=	$\succ_{2,3}$	=	=	$\succ_{2,3}$	$\succ_3$	=	=	9/0
RT14	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	=	=	$\succ_{2,3}$	=	=	=	=	$\succ_{2,3}$	$\succ_3$	=	=	7/0
RT15	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	=	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_3$	$\prec_{2,3}$	0/14
Total	14/1	14/1	14/1	13/1	13/1	4/2	14/0	9/1	13/1	2/1	8/1	14/1	11/1	0/1		144/14

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the first row by the i-order SD and Y = Z means that Y and Z do not dominate each other. RTn stands for the portfolio with both relative value index and 6-M TB for any $n = 1, \ldots, 15$. The significance level is 5 %. The full names of all the indices are defined in Section 3.1. Total only counts the total number of dominate and the total number of dominated for ERV1 to ERV15, it does not include the total number of dominate and the total number of dominated for Pn.

Table 13
SD analysis between portfolios with Relative Value and 6-M TB and the individual assets and the naïve 1/N portfolio under Short-allowed strategy.

	FWC	ED	EH	M	FFC	RV	EM	C	DIS	FICA	MA	S	EMN	TB	Pn	No. dominate/dominated
RT1	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$=$	$=$	$\succ_3$	$\succ_3$	$=$	$\prec_3$	$\succ_3$	9/1
RT2	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	13/0
RT3	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	13/0
RT4	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$=$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	13/0
RT5	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_3$	$\succ_3$	14/0
RT6	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_3$	14/0
RT7	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_3$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	14/0
RT8	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	14/0
RT9	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	14/0
RT10	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$=$	$\succ_{2,3}$	14/0
RT11	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$=$	$\succ_{2,3}$	14/0
RT12	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$=$	$\succ_{2,3}$	14/0
RT13	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_3$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	14/0
RT14	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$=$	$\succ_{2,3}$	13/0
RT15	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$=$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	$=$	$\succ_{1,2,3}$	13/0
Total	15/0	15/0	15/0	14/0	15/0	8/0	15/0	15/0	14/0	11/1	15/0	15/0	14/0	4/1		185/1

Note: $Y \succ_i Z$ ($Y \prec_i Z$) means that Y in the first column significantly dominates (is significantly dominated by) Z in the first row by the i-order SD and Y = Z means that Y and Z do not dominate each other. RTn stands for the portfolio with both relative value and 6-M TB for any $n = 1, \ldots, 15$. The significance level is 5 %. The full names of all the indices are defined in Section 3.1.

10, we find that (ERT1, ET1) = (144, 130) and (ERT1a, ET1a) = (9, 15) and from Tables 9 and 11, we find that (ERT2, ET2) = (187, 167) and (ERT2a, ET2a) = (0, 9). This means that the ERT portfolios dominate 144 and 187 portfolios while the ET portfolios dominate only 130 and 167 assets/portfolios and, on the other hand, the ERT portfolios are dominated by only 9 and 0 assets/portfolios while the ET portfolios are dominated by 15 and 9 portfolios for long-only and Short-allowed strategies, respectively. This shows that the ERT portfolios dominate much more assets/portfolios than the ET portfolios and the ERT portfolios are dominated by much less or even 0 assets/portfolios than the ET portfolios for both long-only and Short-allowed strategies, respectively, which, in turn, conclude that **Conjecture 4B DOES hold.**

4.10. Comparison among portfolios with the EH and RV indices and 6-M TB and those with the RV index and 6-M TB but without the EH index

We now test whether Conjecture 4C holds that investors could get significantly higher expected utility if investors could invest in an (ERT) portfolio in the efficient frontier formed by using the EH Index with the highest mean, the RV Index whose stochastically dominates most of the other assets and 6-M TB with the smallest standard deviation than the (RT) portfolios from the efficient frontier formed by the RV Index, 6-M TB, and other assets but without the EH Index. To do so, we follow the approach we stated in Section 4.2 to compare 15 ERT portfolios and the corresponding RT portfolios to evaluate their performance when a short sale is allowed or not allowed. We display their corresponding SD results in Table 12 of Wong et al. [86].

The table shows that there is no stochastic dominance between any ERT portfolio and its corresponding RT portfolio, regardless of whether a short sale is allowed or not, except the 15th pair portfolio in which the ERT15 portfolio second-order stochastically dominates the RT15 portfolio when a short sale is not allowed. Thus, based on the SD results from Table 12 of Wong et al. [86], we can conclude that Conjecture 4C holds only for the 15th pair of the ERT portfolio when a short sale is not allowed, implying that Conjecture 4C only holds marginally. To conduct deeper analysis, we count (ERT1, ERT2) how many portfolios the ERT portfolios dominate, (ERT1a, ERT2a) how many portfolios the ERT portfolios are dominated, (RT1, RT2) how many portfolios the RT portfolios dominate, and (RT1a, RT2a) how many portfolios the RT portfolios are dominated for long-only and Short-allowed strategies, respectively. By comparing ERTi with RTi and comparing ERTia with RTia for i = 1,2, we will know whether ERT or RT is better. To do so, we exhibit in Table 8(9) to show ERT1 (ERT2) and ERT1a (ERT2a) and exhibit in Table 12(13) to show RT1 (RT2) and RT1a (RT2a) for long-only and short-allowed strategies, respectively. From Tables 8 and 12, we find that (ERT1, RT1) = (144, 144) and (ERT1a, RT1a) = (9, 14) and from Tables 9 and 13, we find that (ERT2, RT2) = (187, 185) and (ERT2a, RT2a) = (0, 1). This means that there exists no dominance between ERT portfolios and RT portfolios for long-only strategies and the ERT portfolios dominate 187 portfolios while the RT portfolios dominate 185 portfolios and, on the other hand, the ERT portfolios are dominated by only 9 and 0 portfolios while the RT portfolios are dominated by 14 and 1 portfolios for long-only and Short-allowed strategies, respectively. Thus, the above findings show that not only ERT15 portfolio outperforms the RT15 portfolio when a short sale is not allowed but also the ERT portfolios dominate much more portfolios than the RT portfolios, but also find that the ERT portfolios dominate more assets/portfolios and are dominated by much fewer assets/portfolios than the RT portfolios for both long-only and Short-allowed strategies, respectively, which, in turn, conclude that **Conjecture 4C DOES hold.**

4.11. Comparison among portfolios with the EH and RV indices and 6-M TB and Individual Funds or the naïve 1/N portfolio

Our analysis confirms that Conjecture 3 and Conjectures 4A to 4F hold. To save space, we skip the discussion for the results of Conjectures 4D to 4F. We skip our discussion to compare the (ERT) portfolios with any Individual Fund because the results are obvious when one looks into the results from Tables 8 and 9 Now, we turn to test whether Conjecture 5 holds that investors could get higher expected utility if investors could invest in an (ERT) portfolio in the efficient frontier formed by using the EH Index with the highest mean, the RV Index whose stochastically dominates most of the other assets and 6-M TB with the smallest standard deviation than the 1/N portfolio strategy. To do so, we exhibit in the second last column of Tables 8 and 9 whether the ERT portfolio stochastically dominates or is dominated by the 1/N portfolio strategy for long-only and Short-allowed strategies, respectively. Table 8 shows that the first 5 ERT portfolios; that is, ERT1-ERT5, third-order stochastically dominate the naïve 1/N portfolio, Pn, ERT6- ERT11 second-order stochastically dominate Pn, and ERT15 is second-order stochastically dominated by Pn, and there is no stochastic dominance between any other pairs of the portfolios when a short sale is not allowed; with 11 ERT portfolios stochastically dominated Pn and one ERT portfolio, that is, ERT15, is stochastically dominated by Pn for long-only strategy and, on the other hand, Table 9 shows that all ERT portfolio; that is, ERT1-ERT15, second- or third-order stochastically dominate the naïve 1/N portfolio Pn without exception for short-allowed strategy, inferring that **Conjecture 5 DOES hold strongly.**

5. Out-of-sample analysis

We follow the approaches used by Bruni et al. [87] and others to conduct the out-of-sample analysis and present the results in this Section. To do so, we choose the 363-month period from December 1989 to March 2020 as the in-sample (holding) period and choose the following-up 12-month period from April 2020 to March 2021 as the out-of-sample (testing) period. We note that the out-of-sample period happens to fall into the COVID-19 pandemic period entirely. To simplify our analysis, we only conduct the out-of-sample analysis by using the results from Table 6. We first obtain the weights of portfolios for portfolios with (without) the EH and RV Indices and 6-M TB from the portfolios in the holding period. We then use the same weights to form the portfolios with (without) the EH and RV Indices and 6-M TB from the portfolios in the testing period and denote them by ERTi (ERTi') for i = 1 to 15 and for Pn and

Pn'. We then follow Bruni et al. [87] and others to apply five common measures for portfolio optimization, including the Sharpe Ratio, Sortino Ratio, Rachev Ratio, Information Ratio, and Jensen's Alpha to compare the performances of the portfolios with and without the EH and RV Indices and 6-M TB when using both long-only and short-allowed approaches in the out-of-sample period from April 2020 to March 2021 and present the results in Tables 14 and 15, respectively. We note that we obtain the results for the entire period including both the holding and testing periods (from December 1989 to March 2021) and for the holding period (from December 1989 to March 2020). We have reported these results in Wong et al. [86]. Readers could read Wong et al. [86] to know the results. In this paper, to save space, we only report the results for the testing period from April 2020 to March 2021.

From Table 14, we find that out of 15 ERT portfolios, there are 12 portfolios using our suggested strategy to include the EH and RV Indices and 6-M TB having higher Sharpe ratios, and 8 portfolios using our suggested strategy having higher Jensen's Alpha than the corresponding portfolios without our suggested strategy; however, there are fewer portfolios using our suggested strategy have higher values in Sortino Ratio, Rachev Ratio, and Information Ratio than the corresponding portfolios without our suggested strategy under the Long-only strategy. From Table 15, we find that out of 15 ERT portfolios, there are fewer portfolios using our suggested strategy having higher values in Sharpe Ratio, Sortino Ratio, Rachev Ratio, Information Ratio, and Jensen's Alpha than the corresponding portfolios without our suggested strategy under the Short-allowed strategy. Based on the results of Tables 14 and 15, one may conclude that the portfolios using our suggested strategy, in general, do not outperform the corresponding portfolios without our suggested strategy. Nevertheless, the results could be due to the fact that the out-of-sample period happens to fall into the COVID-19 pandemic period entirely so the distributions of the portfolios have changed completely in the COVID-19 pandemic period when using the Sharpe Ratio, Sortino Ratio, Rachev Ratio, Information Ratio, and Jensen's Alpha.

6. Inferences and discussions

We draw inferences based on our results regarding arbitrage opportunity, market efficiency, and our conjectures. We first discuss the inference on the arbitrage opportunity.

Table 14
Measures for portfolios with the EH and RV Indices and 6-M TB under Long-only strategy for the testing period (April 2020 to March 2021) .

	Sharpe	Sortino	Rachev	Info. R	Jensen's
ERT1	**1.5616**	**Inf**	−60.0607	**−1.3714**	**0.0001**
ERT1′	0.8644	4.9868	**3.8837**	−1.4612	−0.0030
ERT2	**1.4361**	**16.5284**	**9.1684**	**−1.3683**	**0.0001**
ERT2′	0.8532	5.6788	4.7353	−1.4633	−0.0033
ERT3	**1.2915**	**10.3511**	6.1336	**−1.3790**	**−0.0000**
ERT3′	0.8957	7.1473	**6.7545**	−1.4383	−0.0032
ERT4	**1.2478**	**9.6394**	5.9063	**−1.3875**	**−0.0002**
ERT4′	0.9653	8.9276	**8.2348**	−1.3955	−0.0028
ERT5	**1.2231**	9.2374	5.7850	−1.3941	**−0.0003**
ERT5′	1.0357	**11.2268**	**9.6259**	**−1.3319**	−0.0023
ERT6	**1.1997**	8.4711	5.3962	−1.3959	**−0.0005**
ERT6′	1.0978	**14.2066**	**11.2700**	**−1.2426**	−0.0018
ERT7	**1.1893**	8.3004	5.3416	−1.3907	**−0.0006**
ERT7′	1.1534	**18.3134**	**13.4939**	**−1.1252**	−0.0014
ERT8	**1.1885**	8.4702	5.491001	−1.3723	**−0.0007**
ERT8′	1.2021	**23.9533**	**16.4641**	**−0.9793**	−0.0009
ERT9	1.1864	8.2840	5.3749	−1.3293	−0.0008
ERT9′	**1.2442**	**31.3019**	**20.5799**	**−0.8088**	**−0.0004**
ERT10	1.2064	8.1281	5.1285	−1.2508	−0.0007
ERT10′	**1.2802**	**40.8845**	**26.6569**	**−0.6226**	**0.0001**
ERT11	**1.3169**	8.4010	5.0350	−1.0895	0.0002
ERT11′	1.3109	**56.5518**	**36.4634**	**−0.4317**	**0.0006**
ERT12	**1.3598**	7.1777	4.2471	−0.8934	0.0009
ERT12′	1.3361	**72.6362**	**46.4062**	**−0.2531**	**0.0011**
ERT13	**1.3688**	18.3949	12.9113	−0.2071	0.0012
ERT13′	1.3491	**32.9201**	**21.0221**	**−0.1465**	**0.0015**
ERT14	1.2706	11.8184	7.7706	**0.7979**	−0.0015
ERT14′	**1.3435**	**18.3303**	**11.7391**	−0.0746	**0.0020**
ERT15	**1.1853**	**9.0415**	**6.4204**	**0.9695**	−0.0046
ERT15′	1.1363	6.0597	3.7106	0.5947	**−0.0023**
Pn	**1.3799**	26.5758	16.7397		
Pn′	1.3798	**35.3222**	**22.0791**		

Note: ERTn (ERTn′) stands for the portfolio with (without) equity hedge index, relative value index, and 6-month T-bill for $n = 1,\ldots,15$.and Pn (P_n') is the naïve 1/N portfolio with (without) equity hedge index, relative value index, and 6-month T-bill.

Table 15
Measures for portfolios with the EH and RV Indices and 6-M TB under the short-allowed strategy for the testing period (April 2020 to March 2021) .

	Sharpe	Sortino	Rachev	Info. R	Jensen's
ERT1	−0.5599	−0.6727	0.2446	−1.3609	**0.0003**
ERT1′	**0.8974**	**8.5707**	**7.4981**	**−1.1791**	0.0001
ERT2	0.7608	3.3986	3.5321	−1.3602	**0.0003**
ERT2′	**0.9031**	**9.0220**	**8.1169**	**−1.1542**	0.0001
ERT3	0.8591	4.5112	4.5955	−1.3562	**0.0003**
ERT3′	**0.9042**	**9.0451**	**8.7891**	**−1.1252**	0.0002
ERT4	0.8478	4.4663	4.3707	−1.3483	**0.0003**
ERT4′	**0.9018**	**8.7360**	**9.5222**	**−1.0922**	0.0002
ERT5	0.8399	4.4141	4.2054	−1.3359	**0.0003**
ERT5′	**0.8968**	**8.2518**	**8.7947**	**−1.0558**	0.0002
ERT6	0.8348	4.3776	4.1176	−1.3182	0.00025
ERT6′	**0.8899**	**7.7176**	**7.8303**	**−1.0163**	**0.00026**
ERT7	0.8313	4.3518	4.0631	−1.2947	0.00026
ERT7′	**0.8818**	**7.2028**	**7.1013**	**−0.9743**	**0.0003**
ERT8	0.8287	4.3329	4.0261	−1.2648	0.00026
ERT8′	**0.8730**	**6.7362**	**6.5309**	**−0.9304**	**0.00033**
ERT9	0.8268	4.3185	3.9992	−1.2283	0.0003
ERT9′	**0.8638**	**6.3251**	**6.0723**	**−0.8850**	**0.0004**
ERT10	0.8254	4.3072	3.9788	−1.1850	0.0003
ERT10′	**0.8545**	**5.9668**	**5.6957**	**−0.8389**	**0.0004**
ERT11	0.8242	4.2981	3.9628	−1.1351	0.0003
ERT11′	**0.8451**	**5.6556**	**5.3808**	**−0.7925**	**0.0004**
ERT12	0.8232	4.2906	3.9500	−1.0792	0.0003
ERT12′	**0.8359**	**5.3848**	**5.2585**	**−0.7464**	**0.0005**
ERT13	0.8224	4.2844	3.9394	−1.0181	0.0003
ERT13′	**0.8269**	**5.1483**	**5.2191**	**−0.7008**	**0.0005**
ERT14	**0.8217**	4.2791	3.9305	−0.9527	0.0003
ERT14′	0.8182	**4.9406**	**5.1848**	**−0.6561**	**0.0005**
ERT15	**0.8211**	4.2745	3.9230	−0.8842	0.0003
ERT15′	0.8098	**4.7574**	**5.1548**	**−0.6126**	**0.0006**
Pn	**1.3799**	26.5758	16.7397		
Pn′	1.3798	**35.3222**	**22.0791**		

Note: ERTn (ERTn′) stands for the portfolio with (without) equity hedge index, relative value index, and 6-month T-bill for $n = 1, \ldots, 15$.and Pn (P_n') is the naïve 1/N portfolio with (without) equity hedge index, relative value index, and 6-month T-bill.

6.1. *Arbitrage opportunity*

In this section, we examine whether *Conjecture 6* set in this paper holds that combinations of portfolios with no arbitrage opportunity could generate portfolios that could have expected arbitrage opportunity. To do so, we first look into Table 2 and find that there is no SD relationship or there exist second- or third-order SD among all the assets being examined in this paper. We then look into Tables 3–8, and other tables, and there is no SD relationship or there exist second- or third-order SD among all the portfolios being examined in the related sections of our paper. Nevertheless, we do find some first-order SD relationships in Tables 9, 11 and 13 between different portfolios/assets. For example, from the SD results in Tables 8, 9, 11 and 13, our findings show that there exist several pairings of the first-order stochastic dominance between portfolios along the efficient frontiers and the individual assets (e.g. the C, FICA, and EMN Indices), especially when a short sale is allowed. For example, the 10th −15th portfolios, that is, ERT10 – ERT15, in the efficient frontier formed by the EH and RV Indices and the 6-M TB, first-order stochastically dominate the C Index and ERT11 – ERT15 first-order stochastically dominate the FICA Index, when a short sale is allowed. In addition, the 9th −12th portfolios, that is, ET9 – ET12, in the efficient frontier formed by the EH Index, 6-M TB, and other assets but without the RV Index, first-order stochastically dominate the C Index, and the 10th −15th portfolios, that is, RT10 – RT15, in the efficient frontier formed by the RV Index, 6-M TB, and other assets but without the EH Index, first-order stochastically dominate the C Index, when a short sale is allowed.

Arbitrage opportunities will appear if there exists any first-order stochastic dominance relationship [88,89]. Investors will, therefore, increase their wealth and expected utilities if they sell the dominated asset and hold long the dominant asset. When a first-order stochastic dominance exists, investors may only increase their expected wealth, rather than increase their wealth [21]. Thus, our findings imply that there are arbitrage opportunities in either emerging or developed markets for investors between portfolios along the efficient frontiers and the individual assets of hedge fund strategies including the C, FICA, and EMN Indices when a short sale is allowed and investors will get higher expected wealth as well as a higher expected utility if they sell the dominated asset and hold long the dominant asset.

We note that in the finance literature, there are only very few FSD relationships discovered in the finance literature. For example, Tsang et al. [90] did find some FSD relationships in the rental yield of the Hong Kong property market. However, since the rental yield cannot be hedged, the FSD relationships in the rental yield of the Hong Kong property market may be able to last after discovery.

Now, our paper finds that there are several FSD relationships among different hedging fund portfolios and other portfolios/individual assets and our findings conclude that *Conjecture 6* set in this paper holds that combinations of portfolios with no arbitrage opportunity could generate portfolios that could have expected arbitrage opportunity. We note that so far as we know, literature only found that combinations of portfolios with no arbitrage opportunity could only generate portfolios that could enable investors to obtain higher expected utility but not higher expected wealth [10,61–63,66,91–95]. To be more specific, our findings indicate that using hedge fund strategies can create expected arbitrage opportunities in either emerging or developed markets and gain very good profit for investors. We claim that the phenomenon that combinations of portfolios with no arbitrage opportunity could generate portfolios that could have expected arbitrage opportunity is a new anomaly in the financial market and our paper discovers a new anomaly in the financial market that expected arbitrage opportunity could be generated.

The questions raised then will be whether the FSD relationships among different hedging fund portfolios and other portfolios/individual assets can last. If investors could access such mathematical information timely, they will sell the dominated asset and hold long the dominant asset to push up the price of the dominant asset and push down the price of the dominated asset so that the arbitrage opportunity would have disappeared [96,97]. Whether the comments suggested by Bernard and Seyhun [96], Larsen and Resnick [97], and others will hold for the arbitrage opportunity discovered in our paper for the hedging funds and individual assets will cease after our discovery, we will leave it to future study on the issue.

In addition, it's worth considering why the portfolios formed by using proposed trading strategy first-order stochastic dominance other portfolios/assets in our study period? one plausible explanation is that investors might not have had access to this critical information during the period. If they could, many investors would probably have bought long on the portfolios formed by using proposed trading strategy and short sold on other dominanced portfolios/assets in which case the FSD could have disappeared [96,97]. The second question to ponder is why does several pairings FSD relationship exist in so many pairs of portfolios/assets? One plausible explanation is that investors may lack the knowledge and tools required to detect FSD because developing this index requires some advanced tools like the ones used in this paper. That is the reason for explaining that several pairings FSD relationship still exists in the market.

6.2. Market efficiency

To examine whether the market is efficient is one of the most important issues in finance. For example, Fama [98] comments that technical analysis rejects market efficiency because it attempts to make a profit by exploiting currently available information. In this section, we examine whether the market of hedging funds is efficient. As discussed in Section 5.1, there exist first-order stochastic dominance relationships among the hedging funds, implying the existence of expected arbitrage opportunities in the market, which, in turn, infers that the market of hedging funds is inefficient. Another evidence for market inefficiency is the existence of a higher (than one) order of SD among the funds. Falk and Levy [89], Clark and Kassimatis [99], and others suggest that even without FSD, the market is inefficient if there are higher-order SD relationships so that investors can increase expected utility but not expected wealth by switching from holding the dominated asset to holding the dominant asset. Our empirical results show that there exist many pairs of higher-order SD. For example, the first two portfolios, that is, RT1-RT2, in the efficient frontier formed by with Relative Value and 6-M TB and other assets but without the EH Index third-order stochastically dominate the EM Index and the 3rd −14th portfolios, that is, ET3 – ET14, in the efficient frontier formed by the RV Index, 6-M TB, and other assets but without the EH Index, second-order stochastically dominate the EM Index, regardless of whether when a short sale is allowed or is not allowed. Thus, there are many higher-order SD relationships in the hedge fund market, implying that investors can increase expected utility by selling the dominated funds to holding the dominant fund, which, in turn, implies that the market of the hedge fund is inefficient.

6.3. Conjectures

The issue of how to improve expected utility is a hot topic for decades. For example, Markowitz [71] proposed using the mean-variance rule to form a portfolio can improve expected utility and expected wealth. It is common to have the following beliefs: A) some academics and practitioners, for example, Bouri et al. [66] believe that including the asset with the highest mean is a good choice, B) some academics, for example, Hoang et al. [94] believe that including the asset that stochastically dominates most of the other assets to their portfolios is a good choice, and C) some, for example, Kirchler et al. [100] believe that including the assets with the smallest standard deviation is a good choice, and we believe that A, B, and C are not the best choice but D) inclusion of A, B, and C is the best (in the sense that investors could get the highest expected wealth and/or the highest expected utility) as what we have conjectured in Conjecture 3. To examine whether our conjecture holds, we use the hedging fund data from 1990 to 2010 to compare the performance of A, B, C (as stated in Conjectures 1A, 1B, and 1C), D (as stated in Conjecture 3), and any 2 combinations of a, b, and c (as stated in Conjectures 2A, 2B, and 2C) and in this sample, we find that A is the Equity Hedge (EH) Total Index with the highest mean, B is the Relative Value (RV) Total Index that stochastically dominates most of the other assets to their portfolios, C is the 6-month Treasury Bill (TB) with the smallest standard deviation, and thus, we call "inclusion of A, B, and C" be the ERT portfolio; that is, the portfolios along the efficient frontier including the EH and RV Indices and 6-M TB. Nevertheless, our findings show that most of the portfolios formed by using one of the A, B, and C strategies as mentioned above do not stochastically dominate the corresponding portfolio without one of the A, B, and C strategies, except some of the portfolios in C that marginally dominate the corresponding portfolios without C and one of the portfolios with A second-order stochastically dominates the corresponding portfolio without A when a short sale is not allowed, implying that Conjecture 1B does not hold and both Conjectures 1A and 1C only hold marginally. So could we conclude our conjecture that including the asset with the highest mean, the asset that stochastically dominates most of the other assets, and the asset with the smallest standard deviation will get much higher expected utility does not hold?

To study whether our conjecture holds, we turn to compare the performance between portfolios with and without any two of A, B, and C strategies. Using this comparison, we find that some of the portfolios formed by using two of the A, B, and C strategies stochastically dominate the corresponding portfolio without two of the A, B, and C strategies, except the portfolios formed by the A and B strategies, implying that Conjecture 2A does not hold and both Conjectures 2B and 2C hold marginally. This finding only supports our conjecture marginally.

To study whether our conjecture holds, we turn to compare the performance using all A, B, and C strategies together. By using all A, B, and C strategies together, we find that most of the portfolios (that is, the ERT portfolios) stochastically dominate the corresponding portfolio without any one or any two or all three of the A, B, and C strategies, implying that Conjecture 3 and Conjectures 4A to 4F all hold strongly, that investors could get higher expected utility than if they include Assets A, B, and C together to form a portfolio in the efficient frontier than using any one or any two of Assets A, B, and C. In addition, we find that most of ERT portfolios stochastically dominate all the individual assets and the naïve 1/N portfolio, implying that Conjecture 5 hold strongly that investors could get higher expected utility if they include all A, B, and C strategies together than the 1/N portfolio strategy and any individual asset.

7. Concluding remarks

In this paper, we introduce a new trading strategy in investment and examine the performance of different hedge funds from emerging and developed markets and determine which hedge fund(s) should be included in the portfolios to get higher expected utility and/or expected arbitrage opportunities over the period from December 1989 through March 2020 using both portfolio optimization and stochastic dominance approaches. According to the knowledge of mean-variance rule and stochastic dominance approach, we select the three particular strategies, including the asset, say, Asset A (that is, the Equity Hedge (EH) Total Index), with the highest mean, the asset, say, Asset B (that is, the Relative Value (RV) Total Index), that stochastically dominates most of the other assets, and the asset, say, Asset C (that is, 6-M T-bill), with the smallest standard deviation, and we believe that including these assets could have a greater opportunity to obtain higher expected utility and/or expected arbitrage opportunities. To test our belief, we set up 14 conjectures totally, including the conjecture that among all the assets investors want to invest, to examine whether investors should include Assets A, B, and/or C in their formation of the portfolio in the efficient frontier of emerging and developed markets could get higher expected utility.

To check whether some or all of our conjectures hold by examining the performance of the portfolios formed by different hedging funds from emerging and developed markets along the efficient frontier, we compare not only the performance between portfolios with and without any one, two, and all three of A, B, and C strategies, but also the performance between portfolios with all three of A, B, and C strategies and portfolios with any one or two of A, B, and C strategies when a short sale is allowed or is not allowed. We find that most of the portfolios with assets A, B, and C stochastically dominate the corresponding portfolio without any one, two, or all three of the A, B, and C strategies, implying that Conjecture 3 and Conjectures 4A to 4F hold strongly, that investors could get higher expected utility and/or expected arbitrage opportunities than if they include Assets A, B, and C together to form a portfolio in the efficient frontier of emerging and developed markets than using any one, or two of Assets A, B, and C and without Assets A, B, and C.

In addition, we compare the performance between portfolios with all three of A, B, and C strategies and the individual assets and the naïve 1/N portfolio of emerging and developed markets. We find that most of the portfolios with assets A, B, and C stochastically dominate all the individual assets and the naïve 1/N portfolio, implying that Conjecture 5 hold strongly that investors could get higher expected utility and/or expected arbitrage opportunities if they include all A, B, and C strategies together than the 1/N portfolio strategy and any individual asset.

Our findings support that the existence of expected arbitrage opportunities in the market of hedge funds and in the period we studied, that there are expected arbitrage opportunities in the market of hedge funds. For example. there are expected arbitrage opportunities for investors between portfolios along the efficient frontiers and the individual assets of hedge fund strategies, including the C, FICA, and EMN Indices when a short sale is allowed and investors will not only get higher expected utility but also get higher expected wealth if they sell the dominated asset and hold long the dominant asset. We also find many higher-order SD relationships in the hedge fund market, for example, the first 5 portfolios with the EH and RV Indices and 6-M TB third-order stochastically dominate the corresponding first 5 portfolios without the EH and RV Indices and 6-M TB, implying that investors can increase expected utility by selling the dominated funds to holding the dominant fund, which, in turn, imply that the market of the hedge fund is inefficient.

From our findings, we conclude that our proposed new trading strategy to include Assets A, B, and C in the portfolio is the best strategy among all the other strategies used in our paper in the sense that investors could get the highest expected wealth and/or the highest expected utility than inclusion of any one or two assets A, B, and C in the portfolio for the emerging and developed markets.

In addition, our findings conclude that *Conjecture 6* set in this paper holds that combinations of portfolios with no arbitrage opportunity could generate portfolios that could have expected arbitrage opportunity and we claim that the phenomenon that combinations of portfolios with no arbitrage opportunity could generate portfolios that could have expected arbitrage opportunity is a new anomaly in the financial market and our paper discovers a new anomaly in the financial market that expected arbitrage opportunity could be generated.

Our findings contribute to the advancement of knowledge on the outcomes of hedge fund strategies and the reliability of alternative risk frameworks in the evaluation. Our findings also provide practical experience to academics, fund managers, and investors on how to choose assets in their portfolio to get significantly higher expected utility. The limitation of our study is that our findings are just empirical findings from real data. We do not have any theory to support our claim. Thus, further study of our work includes development of new finance theory to support our claim. As far as we know, this is the first paper to discover that the best portfolio (in term of the mean-variance and stochastic dominance approaches) is to include the asset with the highest mean, the asset that stochastically dominates many other assets, and the asset with the smallest standard deviation by using some hedge funds. Extension of our study could test whether our conjecture still holds for more hedge funds or other funds [68,101] or to other assets [102], for example, energy [103,104], wine [66], futures [92,95,105], warrants [91], iShares [93], gold [62,63,94], internet stocks [61], and many others.

We note that there are many trading strategies [1,2,7–9]. Recently, Lv et al. [10,11] find that using both efficient frontier graphs and stochastic dominance analysis could find portfolios that do not lie on the efficient frontier could have higher expected utility than the corresponding portfolios in the efficient frontier. We note that stochastic dominance analysis is a very good approach to evaluate the performance between any pair of assets because it has been formally proven that regardless of any distribution, if one asset statistically dominates another asset, then any risk-averse investor will have a higher expected utility of the dominating asset to the dominated asset [19]. Nevertheless, there are many other risk measures, for example, the mean-variance rule, Sharpe ratio, Sortino ratio, Rachev ratio, Information ratio, Jensen's Alpha, VaR, C-VaR, Omega ratio, Farinelli and Tibiletti ratio, Kappa ratio, etc. in measuring the performances of the assets. These measures are considered to be good measures because the comparison by using these measures is the same as the comparison by using the stochastic dominance approach. Nonetheless, the comparison of using some, if not all, of these measures is found to be equivalent to that of the stochastic dominance approach only under some conditions [23] and it is easy to find examples to show that one asset, say, Asset A, is preferred to another asset, say, Asset B, or two assets are indifferent by using one measure that is not stochastic dominance approach but risk-averse investors could have a higher expected utility for Asset B to Asset A [19]. Thus, the stochastic dominance approach is superior to any of these measures and our paper uses the stochastic dominance approach in the comparison.

Future research can explore the preference for other types of investors [62,63,94]. It's important to highlight that our hypothesis is not confined solely to portfolios including the Equity Hedge Total Index, Relative Value Total Index, and 6-Month T-bills. We believe that our conjecture could hold for most, if not all, combinations of assets. In this paper, we only use the Equity Hedge Total Index, the Relative Value Total Index and 6-M T-bills as an illustration to support that our conjecture could hold. Future research could investigate cases in which our conjecture does not hold.

Data availability statement

Data included in article/supp. material/referenced in article. Or data will be made available on request.

Funding

This work was supported by Guangdong Basic and Applied Basic Research Foundation (2022A1515110084), the Guangzhou Municipal Science and Technology Bureau (202201010246), Guangdong Provincial Department of Education (2021KQNCX020), Research Grants Council (RGC) of Hong Kong (project number 12500915), and Ministry of Science and Technology (MOST, Project Numbers 106-2410-H-468-002 and 107-2410-H-468-002-MY3).

CRediT authorship contribution statement

Wing-Keung Wong: Writing – review & editing, Writing – original draft, Methodology, Conceptualization. **Tsun Se Cheong:** Writing – review & editing, Conceptualization. **David Chui:** Writing – review & editing, Validation, Methodology. **Zhihui Lv:** Writing – review & editing, Writing – original draft, Methodology, Data curation, Conceptualization. **João Paulo Vieito:** Writing – review & editing, Validation, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

The first author would like to thank Robert B. Miller and Howard E. Thompson for their continuous guidance and encouragement. This research has been supported by Asia University, China Medical University Hospital, The Hang Seng University of Hong Kong, University of Technology Sydney, Guangdong University of Foreign Studies, Polythecnic Institute of Viana do Castelo, Research Grants Council (RGC) of Hong Kong (project number 12500915), and Ministry of Science and Technology (MOST, Project Numbers 106-2410-H-468-002 and 107-2410-H-468-002-MY3), Taiwan. This work was also supported by Guangdong Basic and Applied Basic Research Foundation (2022A1515110084), the Guangzhou Municipal Science and Technology Bureau (202201010246), Guangdong Provincial Department of Education (2021KQNCX020).

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.heliyon.2023.e22486.

References

[1] J. Conrad, G. Kaul, An anatomy of trading strategies, Rev. Financ. Stud. 11 (3) (1998) 489–519, https://doi.org/10.1093/rfs/11.3.489.
[2] W. Fung, D.A. Hsieh, Empirical characteristics of dynamic trading strategies: the case of hedge funds, Rev. Financ. Stud. 10 (1997) 275–302, https://doi.org/10.1093/rfs/10.2.275.
[3] W.K. Wong, B.K. Chew, D. Sikorski, Can the forecasts generated from E/P ratio and bond yield be used to beat stock markets? Multinatl. Finance J. 5 (1) (2001) 59–86, https://doi.org/10.17578/5-1-3.
[4] W.K. Wong, M. Manzur, B.K. Chew, How rewarding is technical analysis? Evidence from Singapore stock market, Appl. Financ. Econ. 13 (7) (2003) 543–551, https://doi.org/10.1080/0960310022000020906.
[5] R. Pardo, The Evaluation and Optimization of Trading Strategies, John Wiley & Sons, 2011, https://doi.org/10.1002/9781119196969.
[6] C.R. Harvey, Y. Liu, Evaluating trading strategies, J. Portfolio Manag. 40 (5) (2014) 108–118, https://doi.org/10.3905/jpm.2014.40.5.108.
[7] R. Lu, V.T. Hoang, W.K. Wong, Do lump-sum investing strategies really outperform dollar-cost averaging strategies? Stud. Econ. Finance 38 (3) (2021) 675–691, https://doi.org/10.1108/SEF-04-2018-0107.
[8] R. Lu, J.J. Wang, W.K. Wong, Investment based on size, value, momentum and income measures: a study in the Taiwan stock market, Annals of Financial Economics (2022), 2250027, https://doi.org/10.1142/S2010495222500270.
[9] R. Lu, C.C. Yang, W.K. Wong, Time diversification: perspectives from the economic index of riskiness, Annals of Financial Economics 13 (3) (2018), 1850011, https://doi.org/10.1142/S2010495218500112.
[10] Z.H. Lv, A.M.Y. Chu, W.K. Wong, T.C. Chiang, The maximum-return-and-minimum-volatility effect: evidence from choosing risky and riskless assets to form a portfolio, Risk Manag. 23 (4) (2021) 97–122, https://doi.org/10.1057/s41283-021-00069-4.
[11] Z.H. Lv, C.-K. Tsang, N.F. Wagner, W.K. Wong, What is an optimal allocation in Hong Kong stock, real estate, and money markets: an individual asset, efficient frontier portfolios, or a naïve portfolio? Is this a new financial anomaly? Emerg. Mark. Finance Trade 59 (5) (2023) 1554–1571, https://doi.org/10.1080/1540496X.2022.2136941.
[12] J. Hadar, W.R. Russell, Stochastic dominance and diversification, J. Econ. Theor. 3 (3) (1971) 288–305, https://doi.org/10.1016/0022-0531(71)90024-X.
[13] L. Tesfatsion, Stochastic dominance and the maximization of expected utility, Rev. Econ. Stud. 43 (2) (1976) 301–315, https://doi.org/10.2307/2297326.
[14] C.K. Li, W.K. Wong, Extension of stochastic dominance theory to random variables, RAIRO-Operations Research-Recherche Opérationnelle 33 (4) (1999) 509–524, https://doi.org/10.1051/ro:1999100.
[15] W.K. Wong, Stochastic dominance and mean–variance measures of profit and loss for business planning and investment, Eur. J. Oper. Res. 182 (2) (2007) 829–843, https://doi.org/10.1016/j.ejor.2006.09.032.
[16] X. Guo, W.K. Wong, Multivariate stochastic dominance for risk averters and risk seekers, Oper. Res. 50 (3) (2016) 575–586, https://doi.org/10.1051/ro/2016026.
[17] R.H. Chan, S.C. Chow, X. Guo, W.K. Wong, Central moments, stochastic dominance, moment rule, and diversification with an application, Chaos, Solit. Fractals 161 (2022), 112251, https://doi.org/10.1016/j.chaos.2022.112251.
[18] R.H. Chan, E. Clark, X. Guo, W.K. Wong, New development on the third order stochastic dominance for risk-averse and risk-seeking investors with application in risk management, Risk Manag. 22 (2020) 108–132, https://doi.org/10.1057/s41283-019-00057-9.
[19] H. Levy, Stochastic Dominance: Investment Decision Making under Uncertainty, Springer, 2015, https://doi.org/10.1007/978-3-319-21708-6.
[20] C. Ma, W.K. Wong, Stochastic dominance and risk measure: a decision-theoretic foundation for VaR and C-VaR, Eur. J. Oper. Res. 207 (2) (2010) 927–935, https://doi.org/10.1016/j.ejor.2010.05.043.
[21] X. Guo, X.J. Jiang, W.K. Wong, Stochastic dominance and omega ratio: measures to examine market efficiency, Arbitrage Opportunity, and Anomaly, Economies 5 (4) (2017) 38, https://doi.org/10.3390/economies5040038.
[22] X. Guo, C.Z. Niu, W.K. Wong, Farinelli and Tibiletti ratio and stochastic dominance, Risk Manag. 21 (3) (2019) 201–213, https://doi.org/10.1057/s41283-019-00050-2.
[23] C.Z. Niu, W.K. Wong, Q.F. Xu, Kappa ratios and (Higher-Order) stochastic dominance, Risk Manag. 19 (3) (2017) 245–253, https://doi.org/10.1057/s41283-017-0020-1.
[24] T.G. Bali, S.J. Brown, M.O. Caglayan, U. Celiker, Does industry timing ability of hedge funds predict their future performance, survival, and fund flows? J. Financ. Quant. Anal. 56 (6) (2021) 2136–2169, https://doi.org/10.1017/S0022109020000794.
[25] S.J. Brown, W.N. Goetzmann, R.G. Ibbotson, Offshore hedge funds: survival and performance 1989– 1995, J. Bus. 72 (1999) 91–118, https://doi.org/10.3386/w5909.
[26] C. Ackermann, R. McEnally, D. Ravenscraft, The performance of hedge funds: risk, return, and incentives, J. Finance 54 (3) (1999) 833–874, https://doi.org/10.1111/0022-1082.00129.
[27] B. Liang, On the performance of hedge funds, Financ. Anal. J. 55 (1999) 72–85, https://doi.org/10.2469/faj.v55.n4.2287.
[28] S.J. Brown, W.N. Goetzmann, J. Park, Careers and survival: competition and risk in the hedge fund and CTA industry, J. Finance 56 (5) (2001) 1869–1886, https://doi.org/10.1111/0022-1082.00392.
[29] C. Sialm, Z. Sun, L. Zheng, Home bias and local contagion: evidence from funds of hedge funds, Rev. Financ. Stud. 33 (10) (2020) 4771–4810, https://doi.org/10.1093/rfs/hhz138.
[30] V. Agarwal, N.Y. Naik, Risks and portfolio decisions involving hedge funds, Rev. Financ. Stud. 17 (1) (2004) 63–98, https://doi.org/10.1093/rfs/hhg044.
[31] S. Titman, C. Tiu, Do the best hedge funds hedge? Rev. Financ. Stud. 24 (1) (2010) 123–168, https://doi.org/10.1093/rfs/hhq105.
[32] W. Fung, D.A. Hsieh, Hedge fund benchmarks: a risk based approach, Financ. Anal. J. 60 (5) (2004) 65–68, https://doi.org/10.2469/faj.v60.n5.2657.
[33] R. Jagannathan, A. Malakhov, D. Novikov, Do hot hands exist among hedge fund managers? An empirical evaluation, J. Finance 65 (1) (2010) 217–255, https://doi.org/10.1111/j.1540-6261.2009.01528.x.
[34] A.L. Aiken, C.P. Clifford, J.A. Ellis, Q. Huang, Funding liquidity risk and the dynamics of hedge fund lockups, J. Financ. Quant. Anal. 56 (4) (2021) 1321–1349, https://doi.org/10.1017/S0022109020000393.
[35] G. Aragon, V. Nanda, H. Zhao, Investor protection and capital fragility: evidence from hedge funds around the world, Rev. Financ. Stud. 34 (3) (2021) 1368–1407, https://doi.org/10.1093/rfs/hhaa051.
[36] A.J. Heuson, M.C. Hutchinson, A. Kumar, Predicting hedge fund performance when fund returns are skewed, Financ. Manag. 49 (4) (2020) 877–896, https://doi.org/10.1111/fima.12304.
[37] V. Agarwal, N. Naik, Multi-period performance persistence analysis of hedge funds, J. Financ. Quant. Anal. 35 (3) (2000) 327–342, https://doi.org/10.2307/2676207.
[38] V. Agarwal, N.Y. Naik, On taking the “alternative” route: the risks, rewards, and performance persistence of hedge funds, J. Altern. Investments 2 (2000) 6–23, https://doi.org/10.3905/jai.2000.318973.
[39] W. Fung, D.A. Hsieh, A primer on hedge funds, J. Empir. Finance 6 (3) (1999) 309–331, https://doi.org/10.1016/S0927-5398(99)00006-7.

[40] V. Do, R. Faff, J. Wickramanayake, An empirical analysis of hedge fund performance: the case of Australian hedge funds industry, J. Multinatl. Financ. Manag. 15 (2005) 377–393, https://doi.org/10.1016/j.mulfin.2005.04.006.
[41] F.S. Lhabitant, M. Learned, Hedge fund diversification: how much is enough? J. Altern. Investments (2002) 23–49, https://doi.org/10.3905/jai.2002.319062. Winter.
[42] N.P. Bollen, J. Joenväärä, M. Kauppila, Hedge fund performance: end of an era? Financ. Anal. J. 77 (3) (2021) 109–132, https://doi.org/10.1080/0015198X.2021.1921564.
[43] S. Otruba, C. Quesada, S. Scholz, The added value of hedge funds in an asset-liability framework, J. Asset Manag. 6 (6) (2006) 433–444, https://doi.org/10.1057/palgrave.jam.2240193.
[44] G. Amin, H. Kat, Hedge fund performance 1990–2000: do the "money machines" really add value? J. Financ. Quant. Anal. 38 (2003) 251–274, https://doi.org/10.2307/4126750.
[45] J. Cvitanic, A. Lazrak, L. Martellini, F. Zapatero, Optimal allocation to hedge funds: an empirical analysis, Quant. Finance 3 (2003) 1–12, https://doi.org/10.1088/1469-7688/3/1/303.
[46] D.P. Morton, E. Popova, I. Popova, Efficient fund of hedge funds construction under downside risk measures, J. Bank. Finance 30 (2) (2006) 503–518, https://doi.org/10.1016/j.jbankfin.2005.04.016.
[47] D. Giamouridis, I.D. Vrontos, Hedge fund portfolio construction: a comparison of static and dynamic approaches, J. Bank. Finance 31 (1) (2007) 199–217, https://doi.org/10.1016/j.jbankfin.2006.01.002.
[48] B. Liang, H.M. Kat, Hedge fund performance: 1990–1999, Financ. Anal. J. 57 (2001) 11–18, https://doi.org/10.2469/faj.v57.n1.2415.
[49] N. Schaub, M. Schmid, Hedge fund liquidity and performance: evidence from the financial crisis, J. Bank. Finance 37 (2013) 671–692, https://doi.org/10.1016/j.jbankfin.2012.09.019.
[50] D. Capocci, G. Hübner, Analysis of hedge fund performance, J. Empir. Finance 11 (2004) 55–89, https://doi.org/10.1016/j.jempfin.2002.12.002.
[51] C.E. Stoforos, S. Degiannakis, T.B. Palaskas, Hedge fund returns under crisis scenarios: a holistic approach, Res. Int. Bus. Finance 42 (2017) 1196–1207, https://doi.org/10.1016/j.ribaf.2017.07.056.
[52] G.A. Jordão, M.L. De Moura, Performance analysis of Brazilian hedge funds, J. Multinatl. Financ. Manag. 21 (3) (2011) 165–176, https://doi.org/10.1016/j.mulfin.2011.02.002.
[53] S. Ben Khelife, C. Urom, K. Guesmi, R. Benkraiem, American hedge funds industry, market timing and COVID-19 crisis, J. Asset Manag. (2022) 1–10, https://doi.org/10.1057/s41260-022-00266-0.
[54] A. Berglund, M. Guidolin, M. Pedio, Monetary policy after the crisis: a threat to hedge funds' alphas? J. Asset Manag. 21 (3) (2020) 219–238, https://doi.org/10.1057/s41260-020-00160-7.
[55] S. Sung, D. Chun, H. Cho, D. Ryu, Hedge fund market runs during financial crises, Economic Research-Ekonomska Istraživanja 34 (1) (2021) 266–291, https://doi.org/10.1080/1331677X.2020.1782245.
[56] W.F. Sharpe, Capital asset prices: a theory of market equilibrium under conditions of risk, J. Finance 19 (1964) 425–442, https://doi.org/10.1111/j.1540-6261.1964.tb02865.x.
[57] E.F. Fama, A.K.R. French, Multifactor explanations of asset pricing anomalies, J. Finance 51 (1996) 55–84, https://doi.org/10.1111/j.1540-6261.1996.tb05202.x.
[58] M.M. Carhart, On persistence in mutual fund performance, J. Finance 52 (1997) 57–82, https://doi.org/10.1111/j.1540-6261.1997.tb03808.x.
[59] R. Steri, M. Giorgino, D. Viviani, The Italian hedge funds industry: an empirical analysis of performance and persistence, J. Multinatl. Financ. Manag. 19 (2009) 75–91, https://doi.org/10.1016/j.mulfin.2008.07.001.
[60] M. Wilkens, J. Yao, P.J. Oehler, N. Jeyasreedharan, Evaluating the performance of hedge funds using two-stage peer group benchmarks, J. Asset Manag. 16 (4) (2015) 272–291, https://doi.org/10.1057/jam.2015.3.
[61] W.M. Fong, H.H. Lean, W.K. Wong, Stochastic dominance and behavior towards risk: the market for internet stocks, J. Econ. Behav. Organ. 68 (1) (2008) 194–208, https://doi.org/10.1016/j.jebo.2008.03.013.
[62] T.H.V. Hoang, H.H. Lean, W.K. Wong, Is gold good for portfolio diversification? A stochastic dominance analysis of the Paris stock exchange, Int. Rev. Financ. Anal. 42 (2015) 98–108, https://doi.org/10.1016/j.irfa.2014.11.020.
[63] V.T.H. Hoang, Z.Z. Zhu, A.E. Khamlichi, W.K. Wong, Does the Shari'ah screening impact the gold-stock nexus? A sectorial analysis, Resour. Pol. 61 (2019) 617–626, https://doi.org/10.1016/j.resourpol.2018.10.002.
[64] V.T.H. Hoang, Z.Z. Zhu, B. Xiao, W.K. Wong, The seasonality of gold prices in China: does the risk-aversion level matter? Account. Finance 60 (3) (2020) 2617–2664, https://doi.org/10.1111/acfi.12396.
[65] V. DeMiguel, L. Garlappi, R. Uppal, Optimal versus naive diversification: how inefficient is the 1/N portfolio strategy, Rev. Financ. Stud. 22 (5) (2009) 1915–1953, https://doi.org/10.1093/rfs/hhm075.
[66] E. Bouri, R. Gupta, W.K. Wong, Z.Z. Zhu, Is wine a good choice for investment? Pac. Basin Finance J. 51 (2018) 171–183, https://doi.org/10.1016/j.pacfin.2018.07.002.
[67] S.V. Venkataraman, S.V.D.N. Rao, Stochastic dominance algorithms with application to mutual fund performance evaluation, Int. J. Finance Econ. 28 (1) (2023) 681–698, https://doi.org/10.1002/ijfe.2444.
[68] W.K. Wong, K.F. Phoon, H.H. Lean, Stochastic dominance analysis of Asian hedge funds, Pac. Basin Finance J. 16 (3) (2008) 204–223, https://doi.org/10.1016/j.pacfin.2007.07.001.
[69] M. Molyboga, S. Baek, J.F. Bilson, Assessing hedge fund performance with institutional constraints: evidence from CTA funds, J. Asset Manag. 18 (7) (2017) 547–565, https://doi.org/10.1057/s41260-017-0053-8.
[70] A. Canepa, M.D.L.O. González, F.S. Skinner, Hedge fund strategies: a non-parametric analysis, Int. Rev. Financ. Anal. 67 (2020), 101436, https://doi.org/10.1016/j.irfa.2019.101436.
[71] H.M. Markowitz, Portfolio selection, J. Finance 7 (1) (1952) 77–91, https://doi.org/10.1111/j.1540-6261.1952.tb01525.x.
[72] G. Hanoch, H. Levy, The efficiency analysis of choices involving risk, Rev. Econ. Stud. 36 (3) (1969) 335–346, https://doi.org/10.2307/2296431.
[73] Z.D. Bai, H.X. Liu, W.K. Wong, Enhancement of the applicability of Markowitz's portfolio optimization by utilizing random matrix theory, Math. Finance 19 (4) (2009) 639–667, https://doi.org/10.1111/j.1467-9965.2009.00383.x.
[74] P.L. Leung, H.Y. Ng, W.K. Wong, An improved estimation to make markowitz's portfolio optimization theory users friendly and estimation accurate with application on the US stock market investment, Eur. J. Oper. Res. 222 (1) (2012) 85–95, https://doi.org/10.1016/j.ejor.2012.04.003.
[75] W.K. Wong, Stochastic dominance theory for location-scale family, J. Appl. Math. Decis. Sci. (2006) 1–10, https://doi.org/10.1155/JAMDS/2006/82049.
[76] W.K. Wong, C.H. Ma, Preferences over location-scale family, Econ. Theor. 37 (1) (2008) 119–146, https://doi.org/10.1007/s00199-007-0254-3.
[77] G.F. Barrett, S.G. Donald, Consistent tests for stochastic dominance, Econometrica 71 (1) (2003) 71–104, https://doi.org/10.1111/1468-0262.00390.
[78] O. Linton, E. Maasoumi, Y.J. Whang, Consistent testing for stochastic dominance under general sampling schemes, Rev. Econ. Stud. 72 (3) (2005) 735–765, https://doi.org/10.1111/j.1467-937X.2005.00350.x.
[79] P. Ng, W.K. Wong, Z.J. Xiao, Stochastic dominance via quantile regression with applications to investigate arbitrage opportunity and market efficiency, Eur. J. Oper. Res. 261 (2) (2017) 666–678, https://doi.org/10.1016/j.ejor.2017.02.047.
[80] S.C. Chow, M.R. Valenzuela, W.K. Wong, New tests for richness and poorness: a stochastic dominance analysis of income distributions in Hong Kong, Asia Pac. J. Oper. Res. 39 (4) (2021), 2040025, https://doi.org/10.1142/S0217595920400254.
[81] R. Davidson, J.Y. Duclos, Statistical inference for stochastic dominance and for the measurement of poverty and inequality, Econometrica 68 (6) (2000) 1435–1464, https://doi.org/10.1111/1468-0262.00167.
[82] Z.D. Bai, H. Li, M. McAleer, W.K. Wong, Stochastic dominance statistics for risk averters and risk seekers: an analysis of stock preferences for USA and China, Quant. Finance 15 (5) (2015) 889–900, https://doi.org/10.1080/14697688.2014.943273.

[83] H.H. Lean, W.K. Wong, X.B. Zhang, The sizes and powers of some stochastic dominance tests: a Monte Carlo study for correlated and heteroskedastic distributions, Math. Comput. Simulat. 79 (2008) 30–48, https://doi.org/10.1016/j.matcom.2007.09.002.
[84] Z.D. Bai, H. Li, H.X. Liu, W.K. Wong, Test statistics for prospect and Markowitz stochastic dominances with applications, Econom. J. 14 (2) (2011) 278–303, https://doi.org/10.1111/j.1368-423X.2011.00348.x.
[85] N. Kouaissah, Using multivariate stochastic dominance to enhance portfolio selection and warn of financial crises, Q. Rev. Econ. Finance 80 (2021) 480–493, https://doi.org/10.1016/j.qref.2021.03.015.
[86] W.K. Wong, T.S. Cheong, D. Chui, Z.H. Lv, J.P. Vieito, New Trading Strategy in Investment: A Study of the Hedge Funds from Emerging and Developed Markets, Social Science Research Network Working Paper Series, 2022, 4278494. http://ssrn.com/abstract=4278494.
[87] R. Bruni, F. Cesarone, A. Scozzari, F. Tardella, On exact and approximate stochastic dominance strategies for portfolio selection, Eur. J. Oper. Res. 259 (1) (2017) 322–329, https://doi.org/10.1016/j.ejor.2016.10.006.
[88] R. Jarrow, The relationship between arbitrage and first order stochastic dominance, J. Finance 41 (4) (1986) 915–921, https://doi.org/10.1111/j.1540-6261.1986.tb04556.x.
[89] H. Falk, H. Levy, Market reaction to quarterly earnings' announcements: a stochastic dominance-based test of market efficiency, Manag. Sci. 35 (4) (1989) 425–446, https://doi.org/10.1287/mnsc.35.4.425.
[90] C.K. Tsang, W.K. Wong, I. Horowitz, Arbitrage opportunities, efficiency, and the role of risk preferences in the Hong Kong property market, Stud. Econ. Finance 33 (4) (2016) 735–754, https://doi.org/10.1108/SEF-03-2015-0079.
[91] C.Y. Chan, C. de Peretti, Z. Qiao, W.K. Wong, Empirical test of the efficiency of the UK covered warrants market: stochastic dominance and likelihood ratio test approach, J. Empir. Finance 19 (1) (2012) 162–174, https://doi.org/10.1016/j.jempfin.2011.09.001.
[92] E.A. Clark, Z. Qiao, W.K. Wong, Theories of risk: testing investor behaviour on the Taiwan stock and stock index futures markets, Econ. Inq. 54 (2) (2016) 907–924, https://doi.org/10.1111/ecin.12288.
[93] D. Gasbarro, W.K. Wong, J.K. Zumwalt, Stochastic dominance analysis of iShares, Eur. J. Finance 13 (2007) 89–101, https://doi.org/10.1080/13518470601025243.
[94] V.T.H. Hoang, W.K. Wong, Z.Z. Zhu, Is gold different for risk-averse and risk-seeking investors? An empirical analysis of the Shanghai Gold Exchange, Econ. Modell. 50 (2015) 200–211, https://doi.org/10.1016/j.econmod.2015.06.021.
[95] Z. Qiao, E. Clark, W.K. Wong, Investors' preference towards risk: evidence from the Taiwan stock and stock index futures markets, Account. Finance 54 (1) (2012) 251–274, https://doi.org/10.1111/j.1467-629X.2012.00508.x.
[96] V. Bernard, H.N. Seyhun, Does post-earnings-announcement drift in stock prices reflect a market inefficiency? A stochastic dominance approach, Rev. Quant. Finance Account. 9 (1) (1997) 17–34, https://doi.org/10.1023/A:1008245709673.
[97] G.A. Larsen, B.G. Resnick, A performance comparison between cross-sectional stochastic dominance and traditional event study methodologies, Rev. Quant. Finance Account. 12 (2) (1999) 103–113, https://doi.org/10.1023/A:1008376819903.
[98] E.F. Fama, Efficient capital markets: a review of theory and empirical work, J. Finance 25 (2) (1970) 383–417, https://doi.org/10.2307/2325486.
[99] E. Clark, K. Kassimatis, An empirical analysis of marginal conditional stochastic dominance, J. Bank. Finance 36 (4) (2012) 1144–1151, https://doi.org/10.1016/j.jbankfin.2011.11.006.
[100] M. Kirchler, F. Lindner, U. Weitzel, Rankings and risk taking in the finance industry, J. Finance 73 (5) (2018) 2271–2302, https://doi.org/10.1111/jofi.12701.
[101] H.H. Lean, K.F. Phoon, W.K. Wong, Stochastic dominance analysis of CTA funds, Rev. Quant. Finance Account. 40 (1) (2012) 155–170, https://doi.org/10.1007/s11156-012-0284-1.
[102] Z. Qiao, W.K. Wong, Which is a better investment choice in the Hong Kong residential property market: a big or small property? Appl. Econ. 47 (16) (2015) 1670–1685, https://doi.org/10.1080/00036846.2014.1000534.
[103] H.H. Lean, M. McAleer, W.K. Wong, Market efficiency of oil spot and futures: a mean-variance and stochastic dominance approach, Energy Econ. 32 (2010) 979–986, https://doi.org/10.1016/j.eneco.2010.05.001.
[104] M. Alghalith, X. Guo, C. Niu, W.K. Wong, Input demand under joint energy and output prices uncertainties, Asia Pac. J. Oper. Res. 34 (4) (2017), 1750018, https://doi.org/10.1142/S021759591750018X.
[105] H.H. Lean, M. McAleer, W.K. Wong, Preferences of risk-averse and risk-seeking investors for oil spot and futures before, during and after the Global Financial Crisis, Int. Rev. Econ. Finance 40 (2015) 204–216, https://doi.org/10.1016/j.iref.2015.02.019.