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A SUBTRACTIVE MODELLING APPROACH FOR PREDICTING THE RADIATION OF A CYLINDRICAL SHELL IN A WAVEGUIDE

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ABSTRACT

Numerical methods for predicting the underwater radiated noise of submerged structures are of paramount importance for ensuring high performance of systems for naval applications. When it comes to studying submerged structures in a shallow water environment, the influence of the sea surface and seabed cannot be neglected as strong coupling arises between the system and the environment boundaries. In this paper, the radiation from a cylindrical shell submerged in a perfect acoustic waveguide composed of an upper free surface and a rigid floor is investigated using a subtractive modelling approach based on the previously developed CTF (Condensed Transfer Function) and rCTF (reverse Condensed Transfer Function) methods. These methods enable prediction of the behavior of a system from the combination of the condensed transfer functions of the coupled and de-coupled subsystems which the original system is composed of. Following this principle, the radiation of the cylindrical shell in the waveguide can be investigated from the calculation of the condensed transfer functions of the waveguide, of the water volume occupied by the cylindrical shell, and of the uncoupled shell. Results from this approach are compared to a recently developed analytical procedure.

Keywords: *subtractive modelling, numerical methods, substructuring, vibroacoustics, acoustic radiation*

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1. INTRODUCTION

Studying the vibroacoustic behavior of cylindrical shells is of ongoing relevance as they can be used to analyze industrial systems such as underwater vehicles. It is then of paramount necessity to develop computationally efficient tools to model those systems accurately over short simulation times. Classic numerical element-based methods such as the Finite Element Method (FEM) or the Boudary Element Method (BEM) can provide accurate results on complex systems where no analytical formulation is available but they are very time consuming, especially at high frequencies, hence restricting their use to low-frequency analysis. This highlights the necessity of developing numerical methods which can be computationally efficient in a broader frequency range. To overcome frequency limitations of the element-based methods, substructuring methods have been developed over the last decades. They allow studying a complex vibroacoustic system by partitioning it into a number of smaller systems, called subsystems. Those subsystems can be studied separately using different methods (analytical, numerical, or experimental), before reassembling them using frequency response functions. In particular, substructuring methods have a certain history of application in the naval engineering domain, as they have been used to study the vibroacoustic behavior of submerged shells coupled to axisymmetric [1] and non-axisymmetric [2] internal frames, via the development of the Circumferential Admittance Approach (CAA) and the Condensed Transfer Function (CTF) approach, respectively. These methods are based on the coupling of subsystems. However, one can sometimes be interested in decoupling subsystems rather than coupling them. To this end, the reversed Condensed Transfer Function (rCTF) method was subsequently developed and applied to study a partially coated cylindrical shell [3, 4]. In this paper, we are interested in modelling the radiation of a cylindrical shell immersed in an acoustic waveguide comprised of an upper free surface and a lower rigid floor, to simulate shallow water conditions. Compared to free-field conditions where an infinite water domain is considered, a strong coupling arises from the interaction between the shell and the boundaries of the waveguide. An analytical formulation of this problem has been derived recently [5], which gives us a reference result. The objective of this study is to solve this problem using subtractive modelling via the CTF and rCTF methods, and compare the performances of this approach to the analytical resolution.

2. MODEL

We are interested in an infinitely long cylindrical shell of thickness h_s, radius R, Young's modulus E_s, Poisson's ratio ν and density ρ_s, immersed in an acoustic waveguide filled by a heavy fluid of density ρ_f and sound speed c_f as shown in Fig. 1. The waveguide is closed on its upper boundary by a free surface (for which a pressure release boundary condition is considered) and on its lower boundary by a rigid floor. The total depth of the waveguide is H, where the distance from the origin (which is the center of the shell) to the free surface is H_{fs}, and the distance from origin to the rigid floor is H_{sb}.

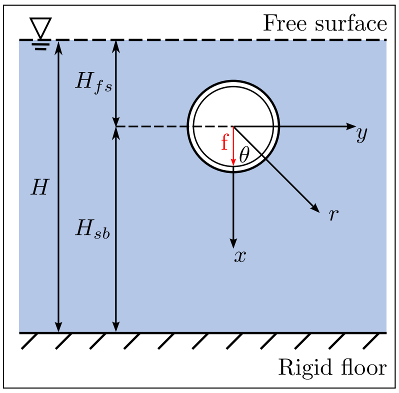

Figure 1. Cylindrical shell in a perfect acoustic waveguide.

The shell is excited by a harmonic line force f along the z direction at an angle θ_0 from the x axis. The system is described by cylindrical coordinates (r, θ, z). As the system is uniform along the z-coordinate, it can be reduced to a 2-D problem where the shell is excited by a mechanical point force at the coordinate $z = 0$.

To study this problem using subtractive modelling, as illustrated in Fig. 2, two steps are necessary. The first step, called the decoupling procedure, consists in removing a disk of fluid (subsystem 2) from a perfect acoustic waveguide (system 1+2). This step can be performed using the rCTF approach [4]. Then, the excited shell (subsystem 3) is added using the CTF method [2] to achieve the recoupling procedure. As such, the disk of fluid removed from the waveguide in the decoupling procedure must correspond to the volume occupied by the shell.

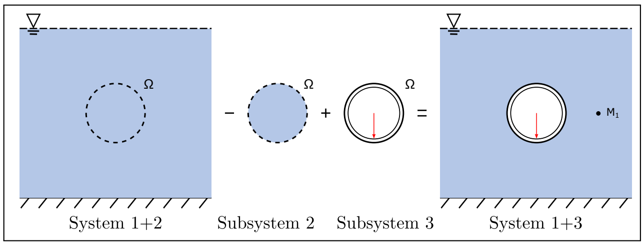

Figure 2. Study of the considered problem using rCTF and CTF methods.

3. PRESENTATION OF THE SUBTRACTIVE MODELLING APPROACH

3.1 Theoretical formulation

Let us consider two systems coupled along a boundary Ω. A set of orthonormal functions, called condensation functions, is defined on Ω: $\{\varphi^\zeta\}_{1\leq\zeta\leq N}$. For each system α, the pressures p_α and normal velocities u_α on Ω can be expressed as a linear combination of the condensation functions

$$\begin{cases} p_\alpha(\theta) \simeq \sum_{\zeta=1}^{N} P_\alpha^\zeta \varphi^\zeta(\theta) \\ u_\alpha(\theta) \simeq \sum_{\zeta=1}^{N} U_\alpha^\zeta \varphi^\zeta(\theta) \end{cases}, \quad \theta \in \Omega \tag{1}$$

where P_α^ζ and U_α^ζ are the unknowns. They can be estimated by defining, for each system taken individually, condensed transfer functions (CTFs) between φ^ζ and φ^ξ ($\xi \in [\![1, N]\!]$, $\zeta \in [\![1, N]\!]$). The CTFs are defined depending on the nature of the subsystem (acoustical or mechanical) and of the decoupling or coupling boundary Ω (tangible or fictitious). As system 1+2 is an acoustical system with a fictitious decoupling boundary, the CTFs are defined by prescribing a velocity jump $\delta u_{1+2} = \varphi^{\xi}$ on Ω, meaning that the CTFs will result in condensed impedances [3,4]

$$Z_{1+2}^{\zeta\xi} = \frac{P_{1+2}^{\zeta}}{\delta U_{1+2}^{\xi}} = \frac{\langle \bar{p}_{1+2}, \varphi^{\zeta} \rangle}{\langle \delta u_{1+2}, \varphi^{\xi} \rangle} = \langle \bar{p}_{1+2}, \varphi^{\zeta} \rangle \tag{2}$$

where $\langle \bullet, \bullet \rangle$ is the scalar product, and $\bar{p}_{1+2}$ corresponds to the resulting pressure on Ω when the system is excited by $\delta u_{1+2} = \varphi^{\xi}$. Subsystem 2 is an acoustical subsystem with a tangible boundary, the CTFs are then defined by applying a prescribed velocity $u_2 = \varphi^{\xi}$ on Ω, meaning that the CTFs will result in condensed impedances

$$Z_{2}^{\zeta\xi} = \frac{P_{2}^{\zeta}}{U_{2}^{\xi}} = \frac{\langle \bar{p}_{2}, \varphi^{\zeta} \rangle}{\langle u_{2}, \varphi^{\xi} \rangle} = \langle \bar{p}_{2}, \varphi^{\zeta} \rangle \tag{3}$$

As for subsystem 3, it is a mechanical subsystem with a tangible boundary, hence the CTFs are defined by applying a mechanical pressure $p_3 = \varphi^{\xi}$ on Ω, meaning that the CTFs will result in condensed admittances

$$Y_{3}^{\zeta\xi} = \frac{U_{3}^{\zeta}}{P_{3}^{\xi}} = \frac{\langle \bar{u}_{3}, \varphi^{\zeta} \rangle}{\langle p_{3}, \varphi^{\xi} \rangle} = \langle \bar{u}_{3}, \varphi^{\zeta} \rangle \tag{4}$$

where $\bar{u}_3$ corresponds to the normal velocity on Ω when the system is excited by $p_3 = \varphi^{\xi}$.

Then, following some developments of the rCTF approach (described in [4]) and CTF approach (described in [2]), we can express the pressure radiated by the shell at point M_1 in the waveguide as

$$p_{1+2}(M_1) = \mathbf{Z_1^{M_1}} \left(\mathbf{I} - \mathbf{Y_3 Z_1} \right)^{-1} \mathbf{\tilde{U}_3} \tag{5}$$

where

- $\mathbf{I}$ is the identity matrix
- $\mathbf{Y_3}$ is the condensed admittance matrix of subsystem 3.
- $\mathbf{\tilde{U}_3}$ is the vector of the free condensed velocities of the shell induced by the radial point force.

$\mathbf{Z_1}$ and $\mathbf{Z_1^{M_1}}$ are quantities related to subsystem 1, obtained from the decoupling process, and which corresponds to the waveguide with a rigid disk occupying the removed water domain. $\mathbf{Z_1}$ is the condensed impedance matrix of subsystem 1 and can be obtained from $\mathbf{Z_2}$ and $\mathbf{Z_{1+2}}$, the condensed impedance matrices of subsystem 2 and system 1+2, respectively

$$\mathbf{Z_1} = \mathbf{Z_2} \left(\mathbf{Z_2} - \mathbf{Z_{1+2}} \right)^{-1} \mathbf{Z_{1+2}} \tag{6}$$

As for $\mathbf{Z_1^{M_1}}$, it corresponds to the vector of the point condensed impedance of subsystem 1 and can be expressed as

$$\mathbf{Z_1^{M_1}} = \left(\mathbf{Z_1} + \mathbf{Z_2} \right) \mathbf{Z_2}^{-1} \mathbf{P_{1+2}^{M_1}} \tag{7}$$

where $\mathbf{P_{1+2}^{M_1}}$ is the vector of the condensed pressures at the surface Ω of system 1+2, induced by a monopole source of unit volume velocity located at point M_1.

3.2 Condensation functions and calculation of the condensed transfer functions

To apply the subtractive modelling approach presented in 3.1, the condensation functions must be defined. In this work, we are interested in two different kinds of condensation functions: gate functions and complex exponential functions.

Using gate functions as condensation functions amounts to dividing Ω into a number of segments. They are then defined on Ω according to their length L_ζ

$$\varphi^{\zeta}(\theta) = \begin{cases} \frac{1}{\sqrt{L_\zeta}} & \text{if} \quad \theta \in [\theta_{\zeta-1}, \theta_\zeta] \\ 0 & \text{elsewhere} \end{cases}, \zeta \in [\![1, N]\!] \tag{8}$$

where $\theta_{\zeta-1}$ and θ_ζ are the boundaries of the segment ζ. In practice, when using gate functions, the condensed transfer function between two segments of the subsystems are computed by exciting a single segment (while keeping all the other rigid), and measuring the response on the given receiving segment.

Concerning the complex exponential functions, the set of $N = 2K + 1$ condensation functions is defined on Ω as

$$\varphi^{\zeta}(\theta) = \frac{1}{\sqrt{2\pi R}} \mathrm{e}^{\mathrm{j}\zeta\theta}, \quad \zeta \in [\![-K, K]\!] \tag{9}$$

For each kind of condensation functions, the number N of condensation functions taken into account for the calculation plays a key role in the convergence of the subtractive modelling method. The number of condensation function follows a wavelength based criterion at the highest considered frequency. For the gate functions, it yields

$$L_\zeta \leq \frac{\lambda_f}{2} \tag{10}$$

where λ_f is the flexural wavelength of the shell. As for the complex exponential functions, the maximum index K is defined as

$$K \geq \frac{L_s}{\lambda_f} - \frac{1}{2} \tag{11}$$

where L_s is the length of the shell.

Once the condensation functions have been defined, the CTFs can be calculated for each subsystem. In the present case, they are computed as follows:

- for $\mathbf{Z_{1+2}}$, an analytical formulation is implemented with the image source method to account for the infinite reflections of acoustic waves off the waveguide boundaries [5].
- for $\mathbf{Z_2}$, an analytical formulation is implemented based on a decomposition of the acoustic pressure on the circumferential orders of the subsystem.
- for $\mathbf{Y_3}$, an analytical formulation is implemented based on the resolution of the Flügge's equations of motion of the shell.

4. APPLICATION TO THE SYSTEM OF INTEREST

4.1 Results

Now that the formulation for studying the system of interest has been presented and the condensation functions have been defined, the equations of section 3.1 can be applied. All the parameters of the calculation (dimensions of the shell, material properties of the shell and of the surrounding fluid) are given in Table 1. The calculation is performed between 5 Hz and 3000 Hz with 5 Hz increments so that the comparison with the results obtained in [5] remains consistent. According to the criteria related to the number of condensation functions (Eq. 10 for the gate functions and Eq. 11 for the complex exponential functions), it is necessary to divide Ω into 110 gates ($N = 110$) for the gate functions and to have a maximum index $K = 55$ ($N = 111$) for the complex exponential functions. The results of the calculations are compared to a reference FEM calculation as it was done in [5], using COMSOL Multiphysics®.

In the following, we will be interested in two quantities:

- the pressure radiated by the shell at a point located at 1 m to the right of the shell (see Fig. 2), for which the expression is given by Eq. 5.

Table 1. Calculation parameters.

Parameter	Notation	Value	Unit
Shell radius	R	1002.5	mm
Shell thickness	h_s	5	mm
Young's modulus	E_s	210	GPa
Poisson's ration	ν_s	0.3	-
Shell density	ρ_s	7850	kg.m^{-3}
Shell loss factor	η_s	0.01	-
Fluid density	ρ_f	1000	kg.m^{-3}
Fluid speed of sound	c_f	1500	m.s^{-1}
Fluid loss factor	η_f	0.001	-

- the mean quadratic velocity of the shell.

The later quantity can be expressed as

$$\langle u^2 \rangle = \frac{1}{2\pi R} \int_0^{2\pi} |u(\theta)|^2 R \mathrm{d}\theta = \frac{1}{2\pi R} \sum_{\zeta} |U^{\zeta}|^2 \tag{12}$$

with

$$U^{\zeta} = \left(\mathbf{I} - \mathbf{Y_3 Z_1}\right)^{-1} \mathbf{V_3} \tag{13}$$

where $\mathbf{V_3}$ is the vector of the point condensed velocities of the uncoupled shell induced by the mechanical excitation.

The results for the mean quadratic velocity of the shell are shown in Fig. 3 for both the gate functions and complex exponential functions, with a zoom in the low frequency range (≤500 Hz). It is observed that there is an excellent agreement between the results for the complex exponentials and those obtained by the FEM (Fig. 3b). There is a slight shift in frequency that becomes apparent at higher frequencies due to the difference in the shell theories used in the two calculations. For the gate functions however (Fig. 3a), the results are not as good as we can observe that the frequency shifts of the rCTF curve are greater than for the complex exponentials. This means that the appareant agreement observed around 1400 Hz or 2500 Hz doesn't actually correspond to the same resonances of the shell for the subtractive modelling and the FEM calculations. Some important discrepancies can also be observed in the very low frequencies (≤60 Hz). The difference between those two cases lies in the nature of the condensation functions, as complex exponentials are continuous on Ω while gate functions are not. This means that the sums in Eq. 1 are not as accurate for the gate functions as they can be for the complex exponentials.

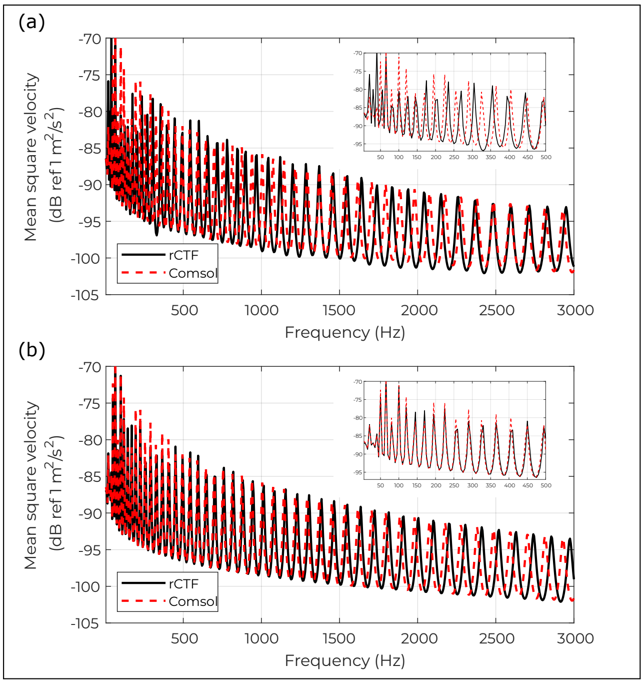

Figure 3. Mean square velocity of the shell using (a) Gate functions (b) Complex exponential functions.

Concerning the radiated pressure, the results for both kinds of condensation functions are showcased in Fig. 4. It is observed that the global trend is globally respected for the gate functions (Fig. 4a), as opposed to the exponential functions (Fig. 4b) for which the calculation fails, especially in the low frequency range ($\leq$1000 Hz). The reason for this fail is still unsure at this point, but one explanation could come from the fact that using complex exponentials as condensation functions amounts to decompose the quantities on the circumferential orders of the systems. It has been observed in [5] that the low circumferential orders have little to no influence on the radial velocity of the shell while they contribute to the radiated pressure. This means that an error of decoupling on the low circumferential orders, which is likely to appear due to the sensitive nature of the rCTF method [4], would affect the radiated pressure but not the shell's vibrations, as it is observed here.

This is not the case for the gate functions as the CTF of a system is not directly linked to a particular circumferential order. For these condensation functions, some unwanted resonances seem to appear, especially in the low frequencies, where some important peaks appear below 200 Hz and between 600 Hz and 900 Hz. Above 1000 Hz, some peaks still appear but they are of much smaller amplitude. To circumvent this issue, a new coupling/decoupling boundary can be defined outside the near field of the shell. This will be investigated in the next section.

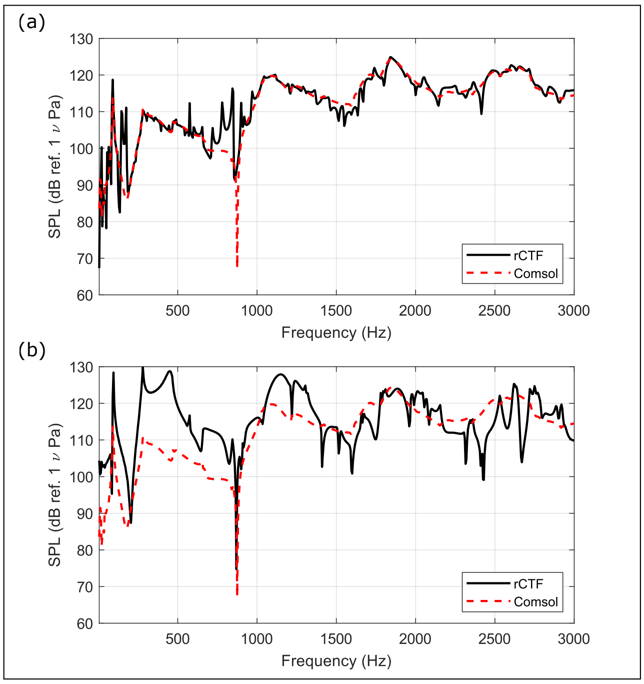

Figure 4. Pressure radiated by the shell using (a) Gate functions (b) Complex exponential functions.

4.2 Partitioning outside the near field of the shell

It has already been observed in previous studies related to substructuring approaches (see for example Ref. [6] for the Patch Transfer Function (PTF) method) that partioning the subsystems at the fluid-structure interface leads to poor convergence of the method for the case of heavy fluid. Indeed, for frequencies well below the critical frequency (which is the case here), the pressure in the near field of the structure varies mainly according to the flexural wavelength of the structure. To circumvent this issue, Maxit et al. [7] propose to shift the coupling boundary inside the fluid domain, at a sufficiently large distance from the structure so it can be considered that the partitioning is no longer located in its near field. Following this suggestion, a new subtractive modelling problem is proposed, illustrated in Fig. 5. Instead of considering the uncoupled cylindrical shell for the recoupling procedure (subsystem 3), the recoupled subsystem is a cylindrical shell coupled to a thin fluid layer for which the thickness is determined according to the procedure described in [7] ($h_f = 0.3$ m). Subsequently, the removed subsystem of the decoupling procedure (subsystem 2) will be a water disk of the same size of the volume occupied by the shell surrounded by the fluid layer.

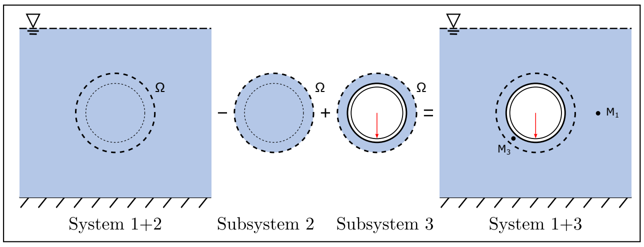

Figure 5. Subtractive modelling when partitioning outside of the near field of the shell.

As a result of this change in the subsystems, a new formulation must be derived. The pressure at point M_1, located outside of Ω, can be expressed as

$$p_{1+2}(M_1) = -\left(\mathbf{I} + \mathbf{Z_1}\mathbf{Z_2}^{-1}\right)\mathbf{P_{1+2}^{M_1}}\left(\mathbf{Z_1} + \mathbf{Z_3}\right)^{-1}\tilde{\mathbf{P}}_\mathbf{3} \tag{14}$$

where

- $\mathbf{Z_3}$ is the condensed impedance matrix of subsystem 3.
- $\mathbf{P_3}$ is the vector of the condensed pressures at the surface Ω of the uncoupled subsystem 3 induced by external mechanical force applied on the shell.

As for the pressure at M_3 initially belonging to subsystem 3

$$p_{1+2}(M_3) = \tilde{p}_3(M_3) + \mathbf{P_3^{M_3}}\left(\mathbf{Z_1} + \mathbf{Z_3}\right)^{-1}\tilde{\mathbf{P}}_\mathbf{3} \tag{15}$$

where

- $\tilde{p}_3(M_3)$ is the pressure radiated at point M_3 of the uncoupled subsystem 3 induced by the external mechanical force applied on the shell.
- $\mathbf{P_3^{M_3}}$ is the vector of the condensed pressures at the surface Ω of the uncoupled subsystem 3, induced by a monopole source of unit volume velocity located at point M_3.

It can be noted that the new coupling interface is purely acoustical, which means that all the CTFs are now condensed impedances. All the new quantities that must be computed (i.e. $\mathbf{Z_3}$, $\mathbf{P_3}$, $\tilde{p}_3(M_3)$ and $\mathbf{P_3^{M_3}}$) are calculated using an analytical wavenumber approach, by the means of a Fourier series decomposition along the circumference of the subsystem. One of the interests of this approach is that the criterion to define the number of condensation functions is now based on the acoustic wavelength inside the fluid domain at the highest considered frequency, instead of the flexural wavelength of the shell. As a result, the calculation is performed using 33 gates for the gate functions and a maximum index $K = 17$ ($N = 35$) for the complex exponential functions.

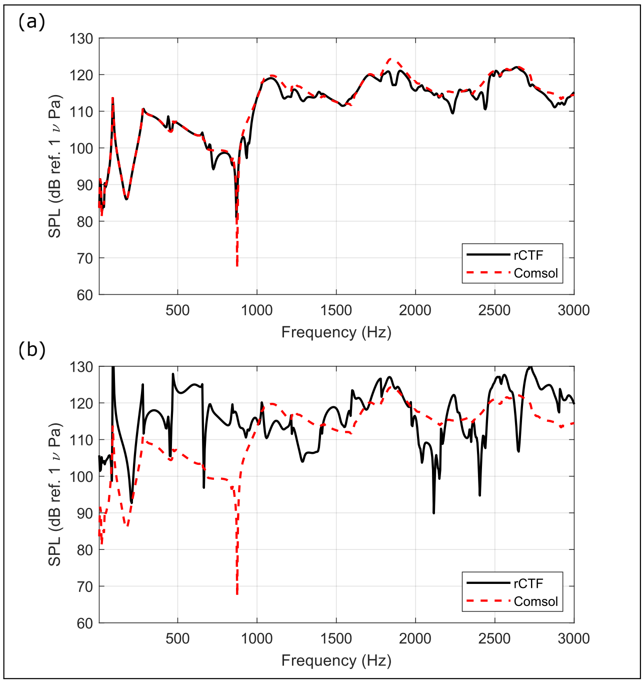

Figure 6. Pressure radiated by the shell when partitioning outside of the near field using (a) Gate functions (b) Complex exponential functions.

The results for this approach are shown in Fig. 6 for the radiated pressure at the same point as that in section 4.1. It can be observed, for the gate functions (Fig.

6a), that there is a clear improvement of the results compared to the previous study in the low frequency range. Indeed, there is an almost perfect match between the results for frequencies below 1000 Hz. The discrepancies observed in this frequency range correspond to common anti-resonances between the waveguide (system 1+2) and the water disk (subsystem 2) arising from the rCTF method as it has already been observed in previous studies [4]. Above 1000 Hz, the results are quite similar to those presented in section 4.1.

Concerning the complex exponentials (Fig. 6b), the subtractive modelling calculation still fails to give good results, as it was observed in section 4.1, and the previously proposed explanation is still valid in this case.

5. CONCLUSION

In this paper, an alternative method has been proposed to address the problem studied in [5], focused on the prediction of the sound radiated by a cylindrical shell in an acoustic waveguide. Based on the previously developed CTF and rCTF method, a subtractive modelling approach has been proposed. It consists of considering the acoustic waveguide, from which a water disk is removed, and to which a cylindrical shell is added. The theoretical formulation for this approach was presented and the numerical results were compared to a reference FEM calculation.

Two kinds of condensation functions have been investigated to study this system. The complex exponential functions have proved to be very accurate to predict the vibrations of the shell, but they failed to describe correctly the radiated pressure due to the influence of the low circumferential orders of the systems which are not correctly calculated by the decoupling process. This problem remains an open investigation which is currently addressed to improve the convergence of the results.

As for the gate functions, it has been shown that the discontinuous nature of these functions, along with the difficuly arrising from partitioning at the vibroacoustic interface of the systems, prevented from correctly capturing the vibrations of the shell. However, the results for the radiated pressure for these condensation functions were quite accurate, especially when the partitioning was performed outside of the near field of the shell. The remaining errors were located at the common resonances and anti-resonances between the waveguide and the water disk, which is an inherent of decoupling procedures such as the rCTF method. However, this issue could be solved by considering a global decoupling approach for the decoupling part, as this method has proven to give more accurate results for the rCTF method [4].

6. ACKNOWLEDGMENTS

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