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Structural Choice Modelling: Theory and Applications to Combining Choice Experiments

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Abstract

We propose and describe a comprehensive theoretical framework that integrates choice models and structural equation models. Referred to as "structural choice modelling," the framework easily combines data from separate but related choice experiments. We describe the mathematical properties of the new framework, including goodness-of-fit and identification and we illustrate how to apply the framework with three empirical examples. The examples demonstrate new ways to evaluate choice processes and new ways to test substantive theory using choice experiments. We show how to combine choice experiments within the same model where there is a common research question, yet the designs and nature of the experiments differ. The seemingly simple notion of combining two or more choice tasks for the same people offers considerable potential to develop and test theory, as illustrated with the new framework.

Keywords: Choice Models, Discrete Choice Experiments, Latent Variables, Structural Equation Models

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1 Introduction

Random Utility Theory (RUT) underpins the choice models used in a wide array of academic and practical situations to model choice processes (e.g., Luce 1959; McFadden 1974; McFadden 1980; Ben-Akiva and Lerman 1985; McFadden 2001). Recently, there has been interest in extending choice models by including ideas and methods from structural equation models (SEMs). For example, Elrod and Keane (1988, 1995, 1997), Walker (2001), Ashok et al. (2002), and Morikawa et al. (2002) show how to combine covariates with factor analytics to create latent variables that form part of the model specification in explaining discrete choices. The observed variables in SEMs reflect variation in underlying latent variables, known as theoretical constructs, in the measurement sub-model. Regression equations and correlations link the latent variables, in the structural sub-model (Jöreskog 1970; Jöreskog 1973; Bollen 1989; Jöreskog and Sörbom 1996). By including latent variables in this way, one can use SEMs to evaluate and test substantive theory. Because SEMs provide a general method, notation and language to evaluate substantive theory, they have become a powerful and commonly used modelling approach in many social science disciplines.

This paper describes an alternate, more general mathematical framework for modelling choice data using latent variables. Referred to as "structural choice modelling," combining data generated from separate but related choice experiments is a particular strength of the framework. Latent variables play a key role. Like SEMs, the latent variables represent the constructs and the links represent relationships between constructs as implied by substantive theory. For example, the latent variables in a structural choice model might represent preferences for the objects studied, including higher-order preferences which capture unobserved sources of heterogeneity. Like SEMs, the fit of the models and competing models allows a researcher to evaluate and test substantive theory. The primary purpose of the paper is to describe the mathematics and notation of structural choice modelling. A secondary purpose is to show how separate but related choice experiments (including stated preference and revealed preference tasks) can be easily combined within the new framework and the benefits of so doing; for example, using latent variables to represent choice processes that are linked by structural models across experiments. Specifying relationships between latent variables within a single choice experiment is possible. However, the emphasis here is on representing choice processes as latent variables and exploring links among different choice tasks/processes.

The general method we propose and describe can model several choice experiments simultaneously, but we confine our discussion and examples to cases involving only two related choice tasks. The three examples are as follows. 1) A study of individuals' hypothetical choices of strategies for reducing carbon emissions combined with a second experiment involving choices of refrigerator attributes which differed by carbon reduction and emissions. We develop and specify a model that describes how individuals' preferences for generic carbon reduction strategies reflect their preferences for refrigerator attributes. The example demonstrates the evaluation of constructs which would be difficult to include as attributes simultaneously within a single DCE. We refer to the latent variables as 'constructs' to emphasise the operationalisation of a component of the substantive theory. The five carbon mitigation strategies available to consumers become the constructs. The constructs are operationalised, each separately, by aggregating different combinations of the more tangible attribute levels. In this case, the constructs are weighted aggregations of covariates representing the levels of refrigerator attributes. The constructs therefore correspond to concepts in the substantive theory (i.e., preferences for refrigerator attributes rather than simply attribute levels). 2) Example 2 specifies a choice model for two revealed preference data sets for the same product category (wine) but in different years. A structural choice model captures the state dependences and develops insight into the stability of individuals' preferences over time. The example introduces the ability to use choice processes in a classic before/after and treatment/control experimental design and in the evaluation of cause and effect. 3) Example 3 examines the impact of the perception profiles of airline brands on the selection of specific trips, again based on choice experiments. The results show that individuals' perceptions have a strong impact and that the nature of the impact varies from brand to brand.

All three empirical examples use the same general framework for specifying, estimating and evaluating structural choice models; each example specifies and evaluates models that link two choice tasks/processes. Taken together, the three examples illustrate the potential of structural choice modelling, and its ability to produce new and different insights about choice behaviour(s) and associated empirical questions.

Next, we review the relationships, in Section 2, between structural models, including structural equation modelling, and discrete choice experiments and then, in Section 3, between latent variables and random utility theory. The mathematics and matrix notation of our approach is developed in Section 4 for a single choice task and then is expanded in Section 5 to two choice tasks. The three examples are presented in Section 6. In Section 7 the literature is revisited and the extensions and new interpretations available through our approach are discussed.

2 Structural Modelling and Discrete Choice Experiments

A particular motivation for structural choice modelling is to better represent the structure of choice processes, especially choice experiments. Because of their consistency with RUT and their ability to handle many real and hypothetical choice situations, discrete choice experiments (DCEs) have seen wide adoption in many fields (e.g., Louviere and Woodworth 1983; Louviere et al. 2000). However, the literature does not fully exploit the potential for DCEs to test substantive theory in a way analogous to SEMs. In SEMs, the latent variables or theoretical constructs and the systematic set of relationships among the variables specifies the substantive theory. The considerable prevalence of SEMs in many substantive literatures is because of this ability to test and empirically evaluate competing substantive theories while not requiring that the constructs within the theories be manifest. DCEs also use latent variables – most notably, utilities are not manifest – and in the literature there is use of constructs through factor analytic choice models for attributes and the characteristics of individuals. We extend the approach to include structural models for specifying relationships among latent preferences for the attributes studied in DCEs.

A further strength of SEMs is that each construct can have its own separate measurement model, increasing the capacity to evaluate competing structural models. We enhance the ability of DCEs in this area by linking two DCEs with different choice tasks but applied to the same individuals; this increases the capacity to separately measure constructs with greater differences. The outcomes from these structural models and linked DCEs are an enhanced capacity for DCEs to represent and empirically evaluate substantive theories. We demonstrate this with the three examples.

The examples show how the use of different choice tasks applied to the same individuals can increase the capacity to test substantive theory. In all three examples the latent variables are constructs reflecting attributes or groups of attributes within a choice task. The partworths for the levels of the attributes/features create the measurement models for the constructs. The variation in the choice tasks means that seemingly similar attributes in both experiments represent quite different constructs. The structural model between and within the two choice tasks then represents the substantive theory linking the constructs. Through using two choice tasks the measurement models for each construct have greater validity. In all three examples the full set of constructs could not be effectively included in just a single choice process. The use of two or more choice tasks with the same individuals, different choice tasks and a linked structural model is better suited to representing and evaluating substantive theory. The examples are included to demonstrate the process and outcomes of specifying constructs and structural models. To fully analyse each example, with the development of substantive theory, the evaluation of the appropriate competing models and a discussion of technical aspects such as estimation and identification would take a complete paper in itself and is a topic for further research.

3 The Foundations of Choice: Random Utility Theory

McFadden (1974, 2001) advanced the expanding theory of choice. He considered Extreme Value Type I errors that led to conditional logit models (CLMs). As is well known in the choice models literature, CLMs separate the utility for each alternative into systematic and random components; the systematic component typically is specified as a fixed coefficient, generalized regression function of attributes of alternatives and covariates associated with individual choosers. More recent work acknowledges that the parameters of the generalized regression functions may vary across the individuals in some systematic way, reflecting an underlying (continuous) distribution of preferences (e.g., Train 2003 and 2009). One also can specify models that assume individuals differ in preferences and error variances and/or that error terms and/or preference parameters can covary, such as the Generalised (G-) MNL model of Fiebig et al. (2010).

Specifically, distributions of preferences across individuals are modelled using the “mixed” logit model (e.g., Ben-Akiva et al. 1997; McFadden and Train 2000; Dube et al. 2002). In this model the utility for individual d and alternative i at choice occasion t is

$$u_{d,i,t} = \eta \; x_{d,i,t} + e_{d,i,t} \qquad (1)$$

where $x_{d,i,t}$ is the vector of observed covariates, η is a vector of utility weights assumed distributed over individuals with means that are not necessarily zero; $e_{d,i,t}$ is an independently and identically distributed “idiosyncratic” error. Over the population of individuals the vector η is assumed to be a multivariate random variable, and the variance-covariance matrix Σ is assumed diagonal. For each individual, choices are independent, but the mixing effect of η across individuals implies that each individual will have unique and enduring preferences over repeated choices (Fiebig et al. 2010).

A vector of latent variables ξ has been introduced as a factor analytic representation, with $\eta = \gamma\xi$ where γ is a matrix of factor weights and the covariates in x can be attributes of alternatives (e.g., Elrod 1988; Elrod and Keane 1995; Keane 1997; Walker 2001) or characteristics of individuals (e.g. Walker 2001; Ashok et al. 2002;

Morikawa et al. 2002; Temme et al. 2008; Bolduc and Daziano 2010; Yáñez et al. 2010; Hess and Stathopoulos 2011). More generally $\eta = \gamma\xi + \varepsilon$; hence, Σ is not diagonal even if the variance-covariance matrix of the vector of latent variables ξ is diagonal. In a manner analogous with SEMs below we further extend the factor analytic approach to allow links between latent variables in the vector ξ of the form $\xi = \beta\xi + \delta$ where β is a matrix of regression parameters.

There is a clear need for factor analytics to combine attributes to form latent variables representing the main components of utility, which can be observed by noting that in some DCEs there are obvious a priori patterns in attributes. For example, if the attribute levels are ordinal, one might specify them as having different "weights" on a single random coefficient, as in many factor analytic models, instead of using separate random coefficients. One also can consider specifying meta-attributes that span a DCE; for example, individuals who prefer a specific level of one attribute, such as leather upholstery, may prefer a specific level of another attribute, such as a sound system with tweeters and subwoofer. Such correlated preference patterns are not the same as interactions of these attributes; for example, the utility of a left shoe is likely to be higher if it is combined with a matching right shoe. In contrast, a correlation effect can occur without two attributes interacting, such as individuals who prefer expensive cars also preferring expensive wines despite the two products not co-occurring. Below we extend factor analytics to specify formal links initially between factors within a DCE, and also between factors in different DCEs.

Walker, Ashok and Morikawa also used latent variable models to estimate the effects of individual difference characteristics in DCEs. Elrod, Keane and Walker used factor analytic models to capture relationships among the attributes and/or to include latent attributes like "seat comfort" as constructs in models. The three examples we introduce illustrate not only how to do this in our proposed approach, but we also show how to extend these models to link attributes across multiple DCEs and different choice tasks, as we now discuss.

4 Model for One Choice Experiment

4.1 Matrix Form

For one DCE, let there be a total of k alternatives over all participants (individuals) and choice sets. According to RUT, alternative i has a utility u_i comprising of a systematic component, v_i, and an error term, e_i, which may have a GEV distribution.

$$u_i = v_i + e_i \tag{2}$$

For simplicity the subscripts for the individual, the choice set and alternative within the choice set are omitted, but reintroduced during model estimation.

The systematic components, v, are specified to be linear combinations of the n covariates, x. The regression parameters, η, in the combination are random coefficients. Traditionally, in random coefficient RUT models these parameters are denoted as β, but we use η to reflect traditions in latent variable models (SEMs).

$$v_i = \eta_1 x_{i,1} + \cdots + \eta_n x_{i,n} \tag{3}$$

In the general model we propose the n random coefficients are specified to be linear functions of m latent variables ξ and a random component ε where

$$\eta_j = \gamma_{j,1}\xi_1 + \cdots + \gamma_{j,m}\xi_m + \varepsilon_j \tag{4}$$

Additionally, each latent variable ξ is a function of the other latent variables and a second random component δ where

$$\xi_j = \beta_{j,1}\xi_1 + \cdots + \beta_{j,m}\xi_m + \delta_j \tag{5}$$

The random components ε and δ typically are specified to have normal distributions but alternative distributions can be assumed.

Equations (4) and (5) can be interpreted as specifying patterns in the random coefficients η for the covariates x. The two equations allow structure in the correlations between the random coefficients based on a small and parsimonious number of latent variables ξ which are the dimensions of utility. Behaviourally, this will appear as individuals showing enduring preferences for particular levels of particular attributes and combinations. There will be a consistency in choice – any one individual's next choice might be well predicted based on his/her past choices – but different individuals will be consistent in different ways. The equations specify the structure to generate data with such consistency.

The model is specified with the following matrix notation for Equation (3) to Equation (5), as below:

$$\begin{bmatrix} v_1 \\ \vdots \\ v_k \end{bmatrix} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & \ddots & \vdots \\ x_{k,1} & \cdots & x_{k,n} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_n \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} \eta_1 \\ \vdots \\ \eta_n \end{bmatrix} = \begin{bmatrix} \gamma_{1,1} & \cdots & \gamma_{1,m} \\ \vdots & \ddots & \vdots \\ \gamma_{n,1} & \cdots & \gamma_{n,m} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_m \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} \xi_1 \\ \vdots \\ \xi_m \end{bmatrix} = \begin{bmatrix} \beta_{1,1} & \cdots & \beta_{1,m} \\ \vdots & \ddots & \vdots \\ \beta_{m,1} & \cdots & \beta_{m,m} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_m \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_m \end{bmatrix} \tag{8}$$

Define row vectors and matrices as follows. Let **V** be the 1 x k vector of deterministic components of utility $\{v_i\}$, **X** be the k x n matrix of observed covariates $\{x_i\}$ where x_i is a 1 x n vector, **H** be the 1 x n vector $\{\eta_j\}$, **Γ** be the n x m matrix of regression parameters $\{\gamma_{ji}\}$, Ξ be the 1 x m vector $\{\xi_i\}$, **B** be the m x m matrix of regression parameters $\{\beta_{ji}\}$, **E** be the 1 x n vector of random components $\{\varepsilon_j\}$ and **Δ** be the 1 x m vector of random components $\{\delta_j\}$. Then Equation (6) to Equation (8) are:

$$\mathbf{V}' = \mathbf{X}\mathbf{H}' \tag{9}$$

$$\mathbf{H}' = \mathbf{\Gamma\Xi}' + \mathbf{E}' \tag{10}$$

$$\mathbf{\Xi}' = \mathbf{B\Xi}' + \mathbf{\Delta}' \tag{11}$$

The matrix notation has several useful properties, the first of which is that Equation (11) can be solved.

$$\mathbf{\Xi}' = [\mathbf{I} - \mathbf{B}]^{-1}\mathbf{\Delta}' \tag{12}$$

Substituting into Equation (10) and Equation (9) gives:

$$\mathbf{H}' = \mathbf{E}' + \mathbf{\Gamma}[\mathbf{I} - \mathbf{B}]^{-1}\mathbf{\Delta}' \tag{13}$$

$$\mathbf{V}' = \mathbf{XE}' + \mathbf{X\Gamma}[\mathbf{I} - \mathbf{B}]^{-1}\mathbf{\Delta}' \tag{14}$$

The $\mathbf{XE}'$ component of Equation (14) is a traditional random coefficient model where $\mathbf{X}$ specifies the covariates and $\mathbf{E}$ the random parameters. The additional component $\mathbf{X\Gamma}[\mathbf{I} - \mathbf{B}]^{-1}\mathbf{\Delta}'$ incorporates the latent variable methods of SEMs.

4.2 Parameters

The variables and parameters are summarised in Table 1. The model has nine matrices of parameters. There are two matrices of regression parameters:

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{1,1} & \cdots & \gamma_{1,m} \\ \vdots & \ddots & \vdots \\ \gamma_{n,1} & \cdots & \gamma_{n,m} \end{bmatrix} \tag{15}$$

Table 1. Notation for Structural Choice Model

Symbol	Greek Name	Description
Random Variables		
η	Eta	Random coefficients (weights) for the covariates
ξ	Ksi	Latent variables
ε	Epsilon	Random components associated with η-variables
δ	Delta	Random components associated with ξ-variables
Parameters		
γ	Gamma	Coefficients of the regressions of η-variables on ξ-variables
β	Beta	Coefficients of the regressions of ξ-variables on ξ-variables
ϕ	Phi	Correlations among random components
μ	Mu	Means of random components
σ	Sigma	Standard deviations of random components

and

$$\mathbf{B}=\begin{bmatrix}\beta_{1,1} & \cdots & \beta_{1,m}\\ \vdots & \ddots & \vdots\\ \beta_{m,1} & \cdots & \beta_{m,m}\end{bmatrix} \tag{16}$$

The random components ε and δ have seven matrices of distribution parameters where ε has a matrix of means

$$E[\mathbf{E}]=\begin{bmatrix}\mu_{\varepsilon,1} & \cdots & \mu_{\varepsilon,n}\end{bmatrix} \tag{17}$$

standard deviations

$$Var(\mathbf{E})=\begin{bmatrix}\sigma^2_{\varepsilon,1} & \cdots & \sigma^2_{\varepsilon,n}\end{bmatrix} \tag{18}$$

(symmetric) correlations

$$Cor(\mathbf{E})=\begin{bmatrix}1 & \cdots & \phi_{\varepsilon,1,n}\\ \vdots & \ddots & \vdots\\ \phi_{\varepsilon,n,1} & \cdots & 1\end{bmatrix} \tag{19}$$

and δ has a matrix of means

$$E[\mathbf{\Delta}]=\begin{bmatrix}\mu_{\delta,1} & \cdots & \mu_{\delta,m}\end{bmatrix} \tag{20}$$

standard deviations

$$Var(\mathbf{\Delta})=\begin{bmatrix}\sigma^2_{\delta,1} & \cdots & \sigma^2_{\delta,m}\end{bmatrix} \tag{21}$$

(symmetric) correlations

$$Cor(\mathbf{\Delta})=\begin{bmatrix}1 & \cdots & \phi_{\delta,1,m}\\ \vdots & \ddots & \vdots\\ \phi_{\delta,m,1} & \cdots & 1\end{bmatrix} \tag{22}$$

and ε and δ have correlations

$$Cor(\mathbf{E},\mathbf{\Delta})=\begin{bmatrix}\phi_{\varepsilon\delta,1,1} & \cdots & \phi_{\varepsilon\delta,1,m}\\ \vdots & \ddots & \vdots\\ \phi_{\varepsilon\delta,n,1} & \cdots & \phi_{\varepsilon\delta,n,m}\end{bmatrix} \tag{23}$$

The nine matrices in Equation (15) to Equation (23) accommodate a larger number of potential parameters. The researcher specifies which parameters are (i) fixed to zero,

(ii) fixed to some other value, or (iii) free to be estimated. The actual number of parameters to be estimated may be quite manageable.

4.3 Properties

Using the same matrix notation it is possible to specify means and covariances that can be used to create R^2 goodness-of-fit statistics for some components of the model.

Write Equation (13) as

$$\mathbf{H}' = \begin{bmatrix} \mathbf{I} & \boldsymbol{\Gamma}[\mathbf{I}-\mathbf{B}]^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{E}' \\ \boldsymbol{\Delta}' \end{bmatrix} \tag{24}$$

The means for the random coefficients for each of the observed covariates in X are;

$$E[\mathbf{H}'] = \begin{bmatrix} \mathbf{I} & \boldsymbol{\Gamma}[\mathbf{I}-\mathbf{B}]^{-1} \end{bmatrix} E\begin{bmatrix} \mathbf{E}' \\ \boldsymbol{\Delta}' \end{bmatrix} \quad \text{where} \quad E\begin{bmatrix} \mathbf{E}' \\ \boldsymbol{\Delta}' \end{bmatrix} = \begin{bmatrix} \mu'_\varepsilon \\ \mu'_\delta \end{bmatrix} \tag{25}$$

which is given by Equation (17) and Equation (20).

The variance-covariance matrix for the random coefficients of the observed covariates in **X** in the discussion of Equation (1) was referred to as Σ. From Equation (24) the matrix now has the general form.

$$\Sigma = Cov([\mathbf{H}'\mathbf{H}]) = \begin{bmatrix} \mathbf{I} & \boldsymbol{\Gamma}[\mathbf{I}-\mathbf{B}]^{-1} \end{bmatrix} Cov\left(\begin{bmatrix} \mathbf{E}' \\ \boldsymbol{\Delta}' \end{bmatrix} \begin{bmatrix} \mathbf{E} & \boldsymbol{\Delta} \end{bmatrix} \right) \begin{bmatrix} \mathbf{I} & \boldsymbol{\Gamma}[\mathbf{I}-\mathbf{B}]^{-1} \end{bmatrix}' \tag{26}$$

where $Cov\left(\begin{bmatrix} \mathbf{E}' \\ \boldsymbol{\Delta}' \end{bmatrix} \begin{bmatrix} \mathbf{E} & \boldsymbol{\Delta} \end{bmatrix} \right)$ is given by the means, variances and correlations of the random components **E** and **Δ** from Equation (17) to Equation (23).

Traditional random coefficient models allowed **Σ** in Equation (26) to be diagonal or fully parameterised with all-correlations free and estimated from data. The diagonal approach is restrictive as it disallows correlations and a priori knowledge about relationships between attribute levels, but full parameterisation often is over-modelled and under-identified. The general modelling approach developed above allows for a correlated multivariate distribution for the random coefficients in the model but imposes a structure on **Σ** that can be parsimonious, identifiable and reflective of a priori knowledge or proposed theory. Thus, it lies between the two extremes of a diagonal and all-correlations forms for **Σ**.

Also, from Equation (12) the mean and variance-covariance matrix of the latent variables **Ξ** are

$$E[\boldsymbol{\Xi}'] = (\mathbf{I}-\mathbf{B})^{-1} E[\boldsymbol{\Delta}'] \tag{27}$$

$$Cov(\boldsymbol{\Xi}'\boldsymbol{\Xi}) = (\mathbf{I}-\mathbf{B})^{-1} Cov(\boldsymbol{\Delta}'\boldsymbol{\Delta}) \left((\mathbf{I}-\mathbf{B})^{-1} \right)' \tag{28}$$

The model specifies two sets of regressions as in Equation (10) for **H** and Equation (11) for **Ξ**. One can calculate R^2 goodness-of-fit statistics to measure the proportion of variation in the dependent variable explained by the model for each regression.

From Equation (10) **H** has random component **E** and explained component $\mathbf{\Gamma\Xi'}$. The diagonal of the matrix $Cov(\mathbf{E'E})$ gives the unexplained variances of **H** and the diagonal of the Σ matrix in Equation (26) gives the total variances. The ratios of these two variances, subtracted from one, gives the proportion of the variance of **H** explained by **Ξ** (i.e., the R^2 goodness-of –fit for **H**).

From Equation (11) **Ξ** has random component **Δ** and an explained component $\mathbf{B\Xi'}$. The diagonal of the matrix $Cov(\mathbf{\Delta'\Delta})$ gives the unexplained variances of **Ξ** and the diagonal of the matrix in Equation (28) gives the total variance. The ratios of these two variances, subtracted from one, gives the proportion of the variance of each **Ξ** which is explained by the other **Ξ** (i.e., the R^2 goodness-of –fit for **Ξ)**.

4.4 Estimation

Individual *d* is presented with a group of choice sets G_d. From choice set $C_{d,i}$ in the group G_d individual *d* selects one alternative $a_{d,i}$. Then the joint probability of the all the choices by a randomly selected individual *d* is:

$$\Pr\{a_{d,1}, a_{d,2}, \ldots \mid C_{d,1}, C_{d,2}, \ldots\} = \int_{\mathbf{E}}\int_{\mathbf{\Delta}} \left(\prod_{C_{d,i} \in G_d} \frac{\exp\left(v_{a_{d,i}}(\mathbf{E},\mathbf{\Delta})\right)}{\sum_{c \in C_{d,i}} \exp\left(v_c(\mathbf{E},\mathbf{\Delta})\right)} \right) f(\mathbf{E},\mathbf{\Delta}) d\mathbf{E} d\mathbf{\Delta} \quad (29)$$

The model has nine parameter matrices as described in Equation (15) to Equation (23). The systematic component of utility, *v*, is a function of the parameters in Equation (15) and Equation (16). The joint probability density function *f* of the random components **E** and **Δ** has the parameters in Equation (17) to Equation (23). In the log likelihood function, *LL*, these parameters are collectively referred to as **θ**. Taking the natural log of Equation (29) and summing over the sample of individuals *d* gives:

$$LL(\boldsymbol{\theta}) = \sum_d \ln\left(\Pr\{a_{d,1}, a_{d,2}, \ldots \mid C_{d,1}, C_{d,2}, \ldots\}\right) \quad (30)$$

We optimize the log likelihood using simulated maximum likelihood usually with 1,000 and up to 10,000 draws. Developmental "proof-of-concept" software estimates the parameters and evaluates the model. The reliable but sometimes slow Nelder-Mead optimizer requires a vector of free parameters whereas the model has nine matrices combining the fixed and free parameters. The programming "trick" is to create two macros, one converting the matrices to the vector of free parameters, and the other doing the reverse.

For ease of presentation and estimation it is convenient for Equation (29) and Equation (30) to assume the logit functional form; that is, it is assumed that the error term in Equation (2) has a Gumbel distribution (McFadden 1974; McFadden 1980; Ben-Akiva and Lerman 1985; McFadden 2001). While, neither the Gumbel distribution, nor the associated logit link function, are necessary assumptions for the model, or the more general modelling approach, described above it is a necessary assumption for the estimation methods used here. Further research is proposed investigating alternate functions and estimation processes (Bolduc and Daziano 2010).

4.5 Identification

Identification issues for latent variable models can be decomposed into a) coding, b) poor model specification, c) scale and d) parsimony. A lack of identification occurs when one or more parameters cannot be uniquely and unambiguously estimated from the data. If identification problems occur, the general approach we propose may produce parameters estimates, but some will be meaningless. If two specifications have the same log likelihood value but one has fewer parameters or smaller standard errors then it is better identified. Furthermore, if standard errors cannot be calculated because the Hessian matrix is singular, a model clearly has identification problems. Key identification issues associated with our proposed approach are as follows:

- There is a coding issue that reflects the mathematical property of random utility maximization models that a constant can be added to the utilities for all alternatives in a choice set and the estimates of the choice probabilities will be unaffected. For a detailed discussion see Ben-Akiva and Lerman (1985) and Walker (2001).

- There is an issue of naive specification of latent variables that arises when a model lacks face validity. The modelling approach we propose is so general that one can specify a latent variable that has no affect on utilities, choice probabilities and/or the data. The parameters of such a latent variable will be meaningless, and removing it from the model specification will leave the model fit unchanged (as assessed by the log likelihood).

- In the SEM literature it is well-known that the scale of the latent variables are not fully identified and can be arbitrarily fixed without loss of generalizability or reducing model fits (Bollen 1989). Latent variables in DCEs have a similar property. There is some confounding between the parameters associated with the latent variables Ξ, the mathematical properties of which are known and will be the focus of further research. As an initial approach to modelling a DCE one should fix the means to zero and the variances to one for the random components $\mathbf{\Delta}$**,** of the latent variables.

- In random utility maximization models care must be taken as the scale of the systematic component of utility v, as in Equation (2), is confounded with the scale of the idiosyncratic error e (Fiebig et al. 2010). This issue assumes greater relevance for models linking two or more DCEs as each has different omitted covariates (i.e., some covariates in one DCE are omitted in the other). Thus, the scale of the systematic utility component will differ between the two DCEs. This impacts the scale of the parameters. However, goodness-of-fit measures, and in particular the R^2 values are unaffected when there is a common scale over all individuals. The mathematical properties, with proofs, of the differences in scale between the two DCEs are known and will be the focus of further research.

- The nine parameter matrices accommodate a large number of potential parameters all of which will not be simultaneously identified. However, for any one substantive theory many of the parameters will not be required and can be fixed to zero improving the identification of the remaining free parameters. In particular a model which introduces correlations between the η through specifying a structure using ξ rather than through specifying correlations directly between the ε can have fewer free parameters and can be better identified. A similar parsimony argument applies to the number of random components with variances other than zero (i.e., the number of ε and δ where the variance is not fixed to zero). In the traditional random coefficient model this is n. Models which are better identified and better fit both the data and the substantive theory can be generated by having fewer ξ than η (i.e. $m < n$) and fixing the variances of ε to zero. This creates parsimony by reducing the number of random components.

5 Combining Choice Experiments

5.1 Data

The first step in modelling two choice tasks/processes (say, DCEs) is to combine the data sets. Figure 1 graphically represents the task. The data for each DCE has the following classes of variables (i) individual ID number, (ii) choice set number, (iii) choice(s) and (iv) covariates. The data sets are merged such that the individual ID numbers and choice set numbers are in the same columns (i.e. variables) in the data file but the covariates are not. Covariates for DCE 2 are treated as separate variables from those for DCE 1. The covariates for the two DCEs are denoted as $\mathbf{X}_{1,1}$ and $\mathbf{X}_{2,2}$ (see Equation (33)). The individual ID numbers match in the two data sets. The same individual has the same ID number in each set. However, the choice set numbers must differ. For example in DCE 1 choice set numbers might be 101, 102 etc., and in DCE 2 they might be 201, 202, etc.

5.2 Partitioned Matrix Form

The matrix notation developed above is now partitioned to reflect the two DCEs. In Equation (6) and Equation (9) there are k alternatives. Let there be k_1 alternatives for DCE 1 and k_2 alternatives for DCE 2, where $k_1 + k_2 = k$. Let $\mathbf{V}_1 = \{v_1, \cdots, v_{k_1}\}$ be the vector of systematic components of utility for DCE 1 and $\mathbf{V}_2 = \{v_{k_1+1}, \cdots, v_k\}$ be the equivalent vector for DCE 2, where $\mathbf{V} = [\mathbf{V}_1 \quad \mathbf{V}_2]$. In Equation (6) and Equation (9) there also are n random coefficients for the covariates. Let there be n_1 random coefficients for DCE 1 and n_2 random coefficients for DCE 2, where $n_1+n_2 = n$. Let $\mathbf{H}_1 = \{\eta_1, \cdots, \eta_{n_1}\}$ be the vector of random coefficient for DCE 1 and $\mathbf{H}_2 = \{\eta_{n_1+1}, \cdots, \eta_n\}$ be the equivalent vector for DCE 2, where $\mathbf{H} = [\mathbf{H}_1 \quad \mathbf{H}_2]$.

Let the matrices of covariates, x, be such that $\mathbf{V}_1' = \mathbf{X}_{1,1}\mathbf{H}_1'$ and $\mathbf{V}_2' = \mathbf{X}_{2,2}\mathbf{H}_2'$. This is equivalent to partitioning the covariate matrix such that

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{1,1} & \mathbf{X}_{1,2} \\ \mathbf{X}_{2,1} & \mathbf{X}_{2,2} \end{bmatrix} \tag{31}$$

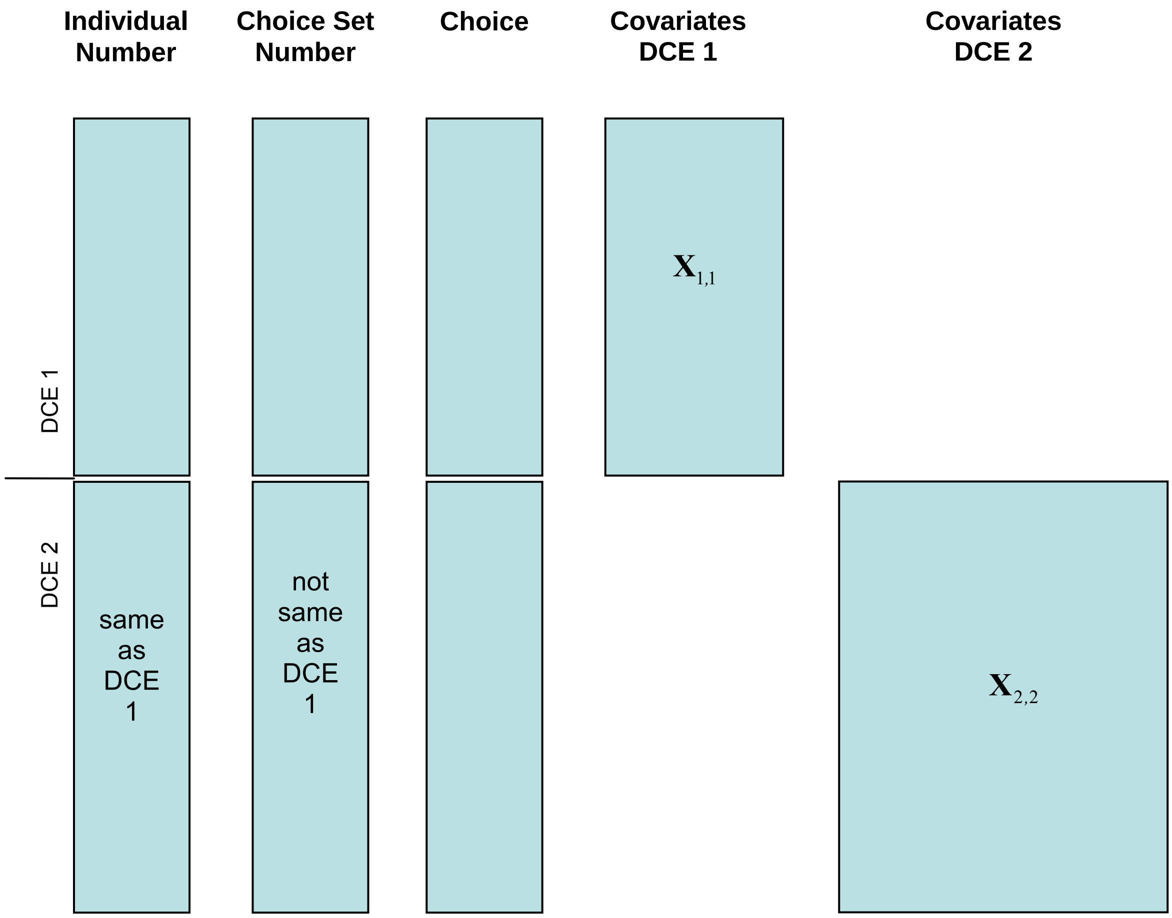

Figure 1. Combined Data

and

$$\begin{bmatrix} \mathbf{V}_1{}' \\ \mathbf{V}_2{}' \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{1,1} & \mathbf{X}_{1,2} \\ \mathbf{X}_{2,1} & \mathbf{X}_{2,2} \end{bmatrix} \begin{bmatrix} \mathbf{H}_1{}' \\ \mathbf{H}_2{}' \end{bmatrix} \tag{32}$$

but where $\mathbf{X}_{1,2} = \{0\}$ and $\mathbf{X}_{2,1} = \{0\}$. Thus, Equation (31) can be written as:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{1,1} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_{2,2} \end{bmatrix} \tag{33}$$

which is shown in Figure 1, and

$$\begin{bmatrix} \mathbf{V}_1{}' \\ \mathbf{V}_2{}' \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{1,1} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_{2,2} \end{bmatrix} \begin{bmatrix} \mathbf{H}_1{}' \\ \mathbf{H}_2{}' \end{bmatrix} \tag{34}$$

In Equation (7) and Equation (10) there are m latent variables. Let there be m_1 latent variables for DCE 1 and m_2 latent variables for DCE 2, where $m_1 + m_2 = m$. Let $\boldsymbol{\Xi}_1 = \{\xi_1, \cdots, \xi_{m_1}\}$ be the vector of latent variables for DCE 1 and $\boldsymbol{\Xi}_2 = \{\xi_{m_1+1}, \cdots, \xi_m\}$ be the equivalent vector for DCE 2, where $\boldsymbol{\Xi} = [\boldsymbol{\Xi}_1 \quad \boldsymbol{\Xi}_2]$.

Let the matrices of γ, the regression parameters, be such that $\mathbf{H}_1' = \boldsymbol{\Gamma}_{1,1}\boldsymbol{\Xi}_1' + \mathbf{E}_1'$ and $\mathbf{H}_2' = \boldsymbol{\Gamma}_{2,2}\boldsymbol{\Xi}_2' + \mathbf{E}_2'$. This is equivalent to partitioning the gamma matrix of regression coefficient such that

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_{1,1} & \boldsymbol{\Gamma}_{1,2} \\ \boldsymbol{\Gamma}_{2,1} & \boldsymbol{\Gamma}_{2,2} \end{bmatrix} \tag{35}$$

and

$$\begin{bmatrix} \mathbf{H}_1' \\ \mathbf{H}_2' \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Gamma}_{1,1} & \boldsymbol{\Gamma}_{1,2} \\ \boldsymbol{\Gamma}_{2,1} & \boldsymbol{\Gamma}_{2,2} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Xi}_1' \\ \boldsymbol{\Xi}_2' \end{bmatrix} + \begin{bmatrix} \mathbf{E}_1' \\ \mathbf{E}_2' \end{bmatrix} \tag{36}$$

but where generally $\boldsymbol{\Gamma}_{1,2} = \{0\}$ and $\boldsymbol{\Gamma}_{2,1} = \{0\}$ in which case, Equation (36) can be written as:

$$\begin{bmatrix} \mathbf{H}_1' \\ \mathbf{H}_2' \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Gamma}_{1,1} & 0 \\ 0 & \boldsymbol{\Gamma}_{2,2} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Xi}_1' \\ \boldsymbol{\Xi}_2' \end{bmatrix} + \begin{bmatrix} \mathbf{E}_1' \\ \mathbf{E}_2' \end{bmatrix} \tag{37}$$

Finally, the beta matrix of regression coefficients is partitioned, and Equation (11) becomes:

$$\begin{bmatrix} \boldsymbol{\Xi}_1' \\ \boldsymbol{\Xi}_2' \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{1,1} & \mathbf{B}_{1,2} \\ \mathbf{B}_{2,1} & \mathbf{B}_{2,2} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Xi}_1' \\ \boldsymbol{\Xi}_2' \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Delta}_1' \\ \boldsymbol{\Delta}_2' \end{bmatrix} \tag{38}$$

where $\mathbf{B}_{1,1}$ is the structural model for DCE 1, $\mathbf{B}_{2,2}$ is the structural model for DCE 2.

5.3 Linked Models for Two Choice Experiments

The term "link" is used here for all aspects of the model that connect the two choice tasks. There are two mechanisms for links. First, the latent variables for one DCE can be functions of the latent variables in the other DCE. The beta matrix for these functions is partitioned in Equation (38) where $\mathbf{B}_{1,2}$ and $\mathbf{B}_{2,1}$ link the two experiments. Secondly, the correlation matrices, as in Equation (19), Equation (22) and Equation (23), are partitioned. Any random components in one DCE can be correlated with any random components in the other DCE. Thus, models can be linked via regressions between latent variables or via correlations between random components.

Regressions have direction, from the explanatory to the dependent variable, whereas correlations do not. Which to use will be determined by the substantive theory and the nature of the two choice tasks. Theories regarding antecedents, temporal sequences and causation should be operationalised as regressions. Also, regressions link the latent variables, are easier to interpret, more parsimonious and better estimated.

If a model has no links, this implies that separate models can be fitted to each data set. In particular, a fixed coefficient model has no links; as does a random coefficient model without correlations. Not surprisingly, therefore, fixed and random coefficient models for the combined data are equivalent to fitting the same models to each data set separately.

5.4 Estimation

In the combined data individual d is presented with two groups of choice sets $G_{d,1}$ from DCE 1 and $G_{d,2}$ from DCE 2. From choice set $C_{d,i,1}$ in the group $G_{d,1}$ individual d selects one alternative $a_{d,i,1}$. Similarly, from choice set $C_{d,i,2}$ in the group $G_{d,2}$ individual d selects one alternative $a_{d,i,2}$. Then the joint probability of the all the choices by a randomly selected individual d is:

$$\Pr\{a_{d,1,1},a_{d,2,1},\ldots,a_{d,1,2},a_{d,2,2},\ldots \mid C_{d,1,1},C_{d,2,1},\ldots,C_{d,1,2},C_{d,2,2},\ldots\} \qquad (39)$$

$$= \int_{\mathrm{E}_1,\mathrm{E}_2}\int_{\Delta_1,\Delta_2}\left(\prod_{C_{d,i,1}\in G_{d,1}}\frac{\exp\left(v_{a_{d,i,1}}(\mathrm{E}_1,\mathrm{E}_2,\Delta_1,\Delta_2)\right)}{\sum_{c\in C_{d,i,1}}\exp\left(v_c(\mathrm{E}_1,\mathrm{E}_2,\Delta_1,\Delta_2)\right)}\right)\left(\prod_{C_{d,i,2}\in G_{d,2}}\frac{\exp\left(v_{a_{d,i,2}}(\mathrm{E}_1,\mathrm{E}_2,\Delta_1,\Delta_2)\right)}{\sum_{c\in C_{d,i,2}}\exp\left(v_c(\mathrm{E}_1,\mathrm{E}_2,\Delta_1,\Delta_2)\right)}\right)$$

$$\times f(\mathrm{E}_1,\Delta_1,\mathrm{E}_2,\Delta_2)\,d\mathrm{E}_1\,d\mathrm{E}_2\,d\Delta_1\,d\Delta_2$$

Taking the natural log of Equation (39) and summing over the sample of individuals d gives:

$$LL(\boldsymbol{\theta}) = \sum_d \ln\left(\Pr\{a_{d,1,1},a_{d,2,1},\ldots,a_{d,1,2},a_{d,2,2},\ldots \mid C_{d,1,1},C_{d,2,1},\ldots,C_{d,1,2},C_{d,2,2},\ldots\}\right) \qquad (40)$$

We have now described the general approach for combining choice tasks/processes including the notation, algebra, statistics, properties and the engine of the method. Our argument has been mathematical but now can be expanded by considering the implications. Three examples are given below which show practical outcomes and more. When SEM was first developed it was recognised as useful and elegant new mathematics but eventually it changed the epistemology of the social sciences through concepts such as "theoretical constructs, measurement models and structural models." It brought quantitative research into new roles in empirically developing social science theory (Bollen 1989). Choice modelling and DCEs too have had a major but different impact on the development of theory and on epistemology particularly in economics and transport (McFadden 2001). The examples below show that combining choice tasks brings new roles to choice modelling, particularly in developing and evaluating theory in the social sciences.

6 Empirical Examples

We now consider three empirical examples to illustrate our proposed modelling approach. In each case two choice tasks (typically, DCEs) are completed by the same individuals. Each example links two choice processes, but designs, choice tasks, outcomes and research objectives differ significantly. Further, the three examples introduce eleven model specifications; however, each specification is nested within one general overarching model (and developmental proof-of-concept software) that depends on the nine parameter matrices.

6.1 Example 1: Consumers' Stated Preferences for Refrigerators and Carbon Reduction Strategies

The substantive motivation is reducing carbon emissions embedded in consumer products. The model captures the extent to which individuals' generic beliefs about reduction strategies are associated with specific product choices that embed these strategies. The substantive research question asks if aspects of individuals' general views of carbon emissions influence their own purchase behaviour; that is, which attitudes, if any, are correlated with behaviour? Over the two choice experiments the choice task varies, first to record an assessment of perceptions of the efficacy of general mitigation strategies and second to record a hypothetical product purchase between alternatives with varying carbon emission attributes. In both experiments specific strategies appear as attributes such as operating efficiency, moderation, recycling, renewable energy, and carbon offsets. In the first choice experiment the constructs are the general assessment of the value of each strategy. In the second choice experiment, the constructs are the preferences for the product attributes/features when the strategies appear as an alternative in a hypothetical purchase situation. The structural model specifies the former constructs as linked to the latter to test the consistency of individuals' preferences in separate but related choice situations.

Specifically, in DCE 1 individuals evaluated sets of generic strategies to change consumer behaviour and chose those strategies they thought would be most effective. In DCE 2 they chose refrigerator features that differed in their carbon emission properties. The study involved a random sample of 1,204 individuals recruited from a major online panel in 2010 in Australia. The design for DCE 1 involved using a balanced incomplete block design to create choice sets that each displayed four carbon reduction strategies. This is known in the literature as best-worst scaling type I (Finn and Louviere 1992; Marley 2009). The design for DCE2 used an orthogonal main effects plan and the presentation of hypothetical profiles per best-worst scaling type II (known as the "profile" case); individuals were shown refrigerators that differed on nine carbon-related dimensions and asked to choose the best and worst attribute level from each profile. The DCEs resulted in 15,652 total choice tasks.

Parameter estimates for two fixed coefficients models are in Table 2 and 3. Renewable fuels are chosen as the most effective generic strategy in DCE 1; it also had a significant influence on preferences for refrigerator attribute levels in DCE2. Now we illustrate a way to link the two DCEs via the correlations between similar constructs in the two DCEs. We begin by fitting a fixed and random coefficient models to the combined DCEs. Next, because the levels of each attribute in DCE 2 are ordinal, we specify a factor structure that captures this, called the “Ordinal Model” (Figure 2). Finally, links are included by allowing the latent variables ξ_1 to ξ_5 in DCE 1 to correlate with ξ_6 to ξ_{14} in DCE 2, forming the “Linked Correlated” Model (Figure 3). The substantive theory suggests relationships between the assessment for one strategy in DCE 1 and its equivalent attribute in DCE 2; each attributes in DCE 2 is correlated with one strategy in DCE 1. While there are potentially 5 × 9 = 45 correlations all are fixed to zero except the nine correlations as in Table 5.

Table 2. Example 1, DCE 1, Generic Strategies – Fixed Model Partworths

Attribute	**Level**	**Covariate**	**Parameter**	**Estimate**	**Se**	**t**
Strategy	Efficiency	X_1	Mu	1.71	0.060	28.6
	Moderation	X_2	Mu	0.79	0.066	12.1
	Renewables	X_3	Mu	1.67	0.060	27.7
	Offsets	X_4	Mu	0		
	Recycling	X_5	Mu	1.51	0.061	24.8

Table 3. Example 1, DCE 2, Refrigerator Attributes – Fixed Model Partworths

Attribute	**Level**	**Covariate**	**Parameter**	**Estimate**	**Se**	**t**
Refrigerant	Hydrofluro	X_6	Mu	2.17	0.209	10.3
	Hydrocarbon	X_7	Mu	2.87	0.207	13.9
Defrost	Sensor	X_8	Mu	2.51	0.208	12.1
Control	Timer	X_9	Mu	3.01	0.206	14.6
Compressor	35%	X_{10}	Mu	2.39	0.213	11.2
Efficiency	45%	X_{11}	Mu	2.48	0.213	11.6
	55%	X_{12}	Mu	2.92	0.210	13.9
	65%	X_{13}	Mu	3.34	0.208	16.0
Configuration	Top Freezer	X_{14}	Mu	2.28	0.209	10.9
	Bottom Freezer	X_{15}	Mu	2.10	0.210	10.0
Running	1 degree	X_{16}	Mu	2.01	0.219	9.2
Temperature	3 degrees	X_{17}	Mu	2.08	0.218	9.5
	5 degrees	X_{18}	Mu	1.80	0.222	8.1
	7 degrees	X_{19}	Mu	1.48	0.229	6.5
Refrigerator	240 liters	X_{20}	Mu	2.06	0.217	9.5
Size	340 liters	X_{21}	Mu	2.24	0.216	10.4
	500 liters	X_{22}	Mu	2.71	0.212	12.8
	720 liters	X_{23}	Mu	2.74	0.211	12.9
Renewable	0%	X_{24}	Mu	1.47	0.227	6.5
Fuel	30%	X_{25}	Mu	1.88	0.220	8.6
	65%	X_{26}	Mu	2.78	0.211	13.2
	100%	X_{27}	Mu	3.63	0.208	17.5
Offsets	0%	X_{28}	Mu	0		
	3%	X_{29}	Mu	0.93	0.246	3.8
	5%	X_{30}	Mu	1.19	0.238	5.0
	7%	X_{31}	Mu	1.51	0.229	6.6
Recyclable	65%	X_{32}	Mu	2.89	0.210	13.8
Content	70%	X_{33}	Mu	3.17	0.209	15.2
	75%	X_{34}	Mu	3.40	0.208	16.3
	100%	X_{35}	Mu	3.55	0.208	17.1

Table 4. Example 1, Log-likelihood values for the Carbon Models

Model	Reference	Number of Parameters	Log Likelihood
Fixed		33	-26847
Random		68	-23913
Ordinal	Figure 2	68	-22725
Linked Correlated	Figure 3	77	-21653

Table 4 shows the Linked Correlated Model fits the data better ($P < 0.001$ in a test of significance compared to the Factor Model as the constrained model). The correlations between the two DCEs are given in Table 5. The model suggests that individuals who prefer specific generic strategies also take these strategies into consideration when making a product purchase. Other models with other correlations can be specified to more fully explore the data and substantive theory. However, the example is sufficient to illustrate the process for linking the two DCEs. DCE 2 is a somewhat more traditional product attribute/feature experiment focusing on choice of particular labelled alternatives. DCE 1 seeks to measure an underlying construct called "perceived strategy effectiveness"; hence, DCE 1 is an example of using a DCE as a choice-based measurement instrument to measure attitudes, beliefs, values, etc. That is, DCE1 measures what voters think policy makers should do and/or what should be emphasised politically. DCE 2 estimates (measures) the likely impacts when the same individuals choose refrigerator attributes. The combined model shows the extent to which individual's preferences as a voter correlate with their preferences as a consumer.

6.2 Example 2: State Dependence in Consumers' Revealed Preferences for Wine

We now consider two revealed preference data sets that involve choices of wines in Italy and model longitudinal and state dependence patterns. The model is analogous to the classic application of SEMs by Jöreskog and Sörbom to sociological panel data investigating 'feelings of alienation' (1996). Our substantive research question asks to what extent is the heterogeneity and segmentation in the purchases of wine the same over the two time periods. Do those individuals with a high preference for traditional quality in period one also one have a similar preference in period two? Note that the data generation process for Example 2 is revealed preference data, not stated preference experiments. The choice task across the two waves is the same; that is, wine purchases. The constructs studied in both purchase occasions are the attributes of the wine. The structural choice modelling framework easily accommodates an assessment of individuals' attribute preferences at both time periods. Further, the structural model links the equivalent attributes over the two time periods and hence evaluates the changes in the latent heterogeneity for the attributes.

Models were specified for two data sets for two different years (2007, 2008); however, the individuals in both datasets are the same, and the product category (wine purchases in Italy) is the same. Three attributes are observed: a) price (high and low), b) format (750 ml bottle and larger pack sizes such as bag-in-box) and c) denomination (DOC/DOCG – the highest levels of traditional regional quality and product control in Italy; GI – a more general type of regional/geographic identification; and table wine – no formal regional identification). Thus, there are seven attribute levels that describe

Table 5. Example 1, Latent Variables in DCE 1 Correlate with Latent Variables in DCE 2. Nine correlations (all with p<0.000) were free and estimated from the data. All other correlations were fixed to zero.

	Efficiency ξ_1	**Moderation** ξ_2	**Renewables** ξ_3	**Offsets** ξ_4	**Recycling** ξ_5
Refrigerant ξ_6	0				
Defrost Control ξ_7	0.7				
Compressor Efficiency ξ_8	0.3				
Configuration ξ_9	0.6				
Running Temperature ξ_{10}		0.98			
Refrigerator Size ξ_{10}		0.2			
Renewable Fuel ξ_{12}			0.4		
Offsets ξ_{13}				0.99	
Recyclable Content ξ_{14}					0.2

Systematic component of utility

$$
\begin{array}{lll}
v = & (\mu_1 + \gamma_{1,1}\xi_1)x_1 + \cdots + (\mu_5 + \gamma_{5,5}\xi_5)x_5 & \text{DCE 1} \\
+ & (\mu_6 + \gamma_{6,6}\xi_6)x_6 + (\mu_7 + \gamma_{7,6}\xi_6)x_7 & \text{DCE 2} \\
+ & (\mu_8 + \gamma_{8,7}\xi_7)x_8 + (\mu_9 + \gamma_{9,7}\xi_7)x_9 & \text{DCE 2} \\
+ & (\mu_{10} + \gamma_{10,8}\xi_8)x_{10} + (\mu_{11} + \gamma_{11,8}\xi_8)x_{11} + (\mu_{12} + \gamma_{12,8}\xi_8)x_{12} + (\mu_{13} + \gamma_{13,8}\xi_8)x_{13} & \text{DCE 2} \\
\vdots & \vdots & \vdots \\
+ & (\mu_{32} + \gamma_{32,14}\xi_{14})x_{32} + (\mu_{33} + \gamma_{33,14}\xi_{14})x_{33} + (\mu_{34} + \gamma_{34,14}\xi_{14})x_{34} + (\mu_{35} + \gamma_{35,14}\xi_{14})x_{35} & \text{DCE 2}
\end{array}
$$

Structural Model

$$
\begin{array}{ccc}
\xi_1 & = & \delta_1 \\
\vdots & \vdots & \vdots \\
\xi_{14} & = & \delta_{14}
\end{array}
$$

δ_1 to δ_{14} are random components with independent normal distributions, zero means and standard deviations equal to one.

Figure 2. Example 1 Ordinal Model.
A factor analytic model is applied to the levels for the renewable attribute in DCE 2.

the three attributes that are effects-coded: one for price, one format and two for denomination. These four covariates are observed in both Time 1 (2007) and Time 2 (2008). The sample size was 693; each individual made 10 purchases in 2007 and 10 in 2008 from a choice set that had eight alternatives.

Parameter estimates for a fixed effects model are in Table 6. Next, a longitudinal model was specified by modifying a random coefficient model. Specifically, the eight random coefficients were specified as latent variables; and each latent variable for Time 2 (2008) was specified as a linear function of its equivalent in Time 1 (2007) as shown in Figure 4. Thus, each random coefficient for 2008 was influenced by its equivalent 2007 coefficient; the heterogeneity in 2008 was partly caused by the heterogeneity in 2007. The model is an analogue of the i) Keane's (1997) state-dependence RUT model and ii) the Jöreskog and Sörbom (1996) SEM alienation example documented in their manual.

The fit of three competing models, fixed, random and longitudinal, are in Table 7. The longitudinal model fits significantly better ($p< 0.001$ in a test of significance compared to the Random Model as the constrained model). The Fixed and Random models describe the change over the two years with aggregate results that do not reveal at a disaggregate level if individual individuals who exhibited higher preferences in Time 1 (2007) continued to exhibit higher preference in Time 2 (2008). Table 7 shows the longitudinal model fits the data much better, reflecting consistency over time (i.e., some individual preferences tended to not change from one year to the next). A useful additional outcome for the Linked Longitudinal model is the R^2 for each of the regression equations (price 17 percent, format 33 percent, DOC/DOCG 25 percent and GI 12 percent). For all four covariates the heterogeneity in 2008 was in part associated with that in 2007, with the highest consistency associated with format and DOC/DOCG, and much less consistency for price and GI. The latter may reflect changes in Italy, changes in individuals, changes in the wine retailing, etc.

6.3 Example 3: Consumers' Stated Preferences for Airline Features and Brand Choices

This example shows how the perceptions of brands can have an influence on the selection of products. The example uses stated preference experiments to investigate the qualities of airlines and their impact on brand selection. Again, over two choice experiments, the choice task varies; the first records preferences for the perceived qualities of airlines (e.g., consistency, credibility, quality, risk and cost) and the second experiment records hypothetical purchases. The constructs in the first choice

Systematic component of utility

As in Figure 2

Structural Model

As in Figure 2

δ_1 to δ_{14} are random components with normal distributions, zero means, standard deviations equal to one and correlations as detailed in Table 5.

Figure 3. Example 1 Linked Correlated Model.
The attributes in the two DCEs are correlated.

Table 6. Example 2, DCE 1 and 2, Fixed Model Partworths

DCE	Attribute	Level	Covariate	Parameter	Estimate	Se	T
1 2007	Price		X_1	Mu	-0.65	0.015	-44.4
1 2007	Format		X_2	Mu	0.02	0.012	1.9
1 2007	Denomination	DOCG	X_3	Mu	-0.37	0.020	-18.4
1 2007		GI	X_4	Mu	-0.19	0.019	-10.0
2 2008	Price		X_5	Mu	-0.53	0.014	-38.8
2 2008	Format		X_6	Mu	0.06	0.012	5.0
2 2008	Denomination	DOCG	X_7	Mu	-0.17	0.018	-9.3
2 2008		GI	X_8	Mu	-0.19	0.018	-10.5

Table 7. Example 2, Log-likelihood values for each of the Longitudinal Models

Model		Number of Parameters	Log Likelihood
Fixed		8	-31512
Random		16	-23557
Linked Longitudinal	Figure 4	20	-22372

Systematic component of utility

$$v = (\mu_1 + \gamma_{1,1}\xi_1)x_1 + \cdots + (\mu_4 + \gamma_{4,4}\xi_4)x_4 \qquad \text{DCE 1 (2007)}$$
$$+ (\mu_5 + \gamma_{5,5}\xi_5)x_5 + \cdots + (\mu_8 + \gamma_{8,8}\xi_8)x_8 \qquad \text{DCE 2 (2008)}$$

Structural Model

DCE 1 (2007)

$$\xi_1 = \delta_1$$
$$\xi_2 = \delta_2$$
$$\xi_3 = \delta_3$$
$$\xi_4 = \delta_4$$

The latent variables in DCE 2 (2008) have as antecedents the equivalent latent variables in DCE 1 (2007)

$$\xi_5 = \beta_{5,1}\xi_1 + \delta_5$$
$$\xi_6 = \beta_{6,2}\xi_2 + \delta_6$$
$$\xi_7 = \beta_{7,3}\xi_3 + \delta_7$$
$$\xi_8 = \beta_{8,4}\xi_4 + \delta_8$$

δ_1 to δ_8, are random components with independent normal distributions, zero means and standard deviations equal to one.

Figure 4. Example 2 Linked Longitudinal Model

experiment are the perceived strengths of the qualities of the brands. In the second choice experiment, the constructs are individuals' preferences for the brands studied. Substantive theory indicates that the perceived qualities for specific brands will be antecedents to purchase, which is the basis for a structural choice model linking the DCEs. Thus, by linking to two experiments it is possible to determine the contribution of the perceptions in DCE 1 to the selection of brands in DCE 2. The study involved a random sample of 200 individuals recruited from a major online panel in Australia where the four brands, Qantas, Virgin Blue, Jetstar and Tiger Airways, were operating nationally. Both DCEs used a BIBD each with sixteen choice sets of set size four. The DCEs resulted in 6400 total choice tasks.

Parameter estimates for the two fixed coefficient models with effects coding are in Table 8 and Table 9. The perception profile (DCE 1) of Qantas is preferred to the other airlines except in the area of risk. Investment was the most preferred perception. In the Trip selection (DCE 2) Qantas was the preferred brand. We now link the two experiments. We begin by fitting the fixed and random coefficients models. Next, because the same brands are in each of the six attributes in DCE 1, we specify a factor structure for each brand called the "Factor Model" (Figure 5). Further, a link was included for each brand between latent variable for the factor in DCE 1 and the choice of brand in DCE 2 called the "Linked Brand" Model (Figure 6).

Table 10 shows the Linked Brand Model fits the data better ($P<0.001$ in a test of significance compared to the Factor Model as the constrained model). The average R^2 for the three regression equations linking the two DCEs is 62 percent. Because of the use of effects coding, with Qantas as the reference level, each of the regression equations models the difference from Qantas. The individual R^2 values are Virgin Blue 60 percent, Jetstar 88 percent and Tiger Airlines 37 percent. Finally, the factor loadings in DCE 1 for the Linked Brand Model are given in Table 11. Those individuals with a preference in trip selection (DCE 2) for Virgin Blue over Qantas favoured the investment and risk perceptions (DCE 1) of Virgin Blue. Similarly those with a preference for Jetstar favoured its risk and cost perceptions and those with a preference for Tiger favoured its risk perception. Risk was the most consistent influence. A substantive conclusion is that individuals' general perceptions and preferences explain variance in their actual brand choices.

Table 8. Example 3, DCE 1, Perceptions of Airlines – Fixed Model Partworths

Attribute	**Level**	**Covariate**	**Parameter**	**Estimate**	**se**	**t**
Investment	Like Qantas		Mu	0.34		
	Like Virgin Blue	X_1	Mu	0.18	0.030	5.9
	Like Jetstar	X_2	Mu	-0.03	0.027	-1.1
	Like Tiger Airways	X_3	Mu	-0.48	0.029	-16.8
Consistency	Like Qantas		Mu	0.18		
	Like Virgin Blue	X_4	Mu	0.01	0.027	0.5
	Like Jetstar	X_5	Mu	0.01	0.027	-7.4
	Like Tiger Airways	X_6	Mu	-0.20	0.025	-0.3
Credibility	Like Qantas		Mu	0.25		
	Like Virgin Blue	X_7	Mu	-0.01	0.025	-9.4
	Like Jetstar	X_8	Mu	-0.01	0.026	3.6
	Like Tiger Airways	X_9	Mu	-0.23	0.026	-2.1
Quality	Like Qantas		Mu	0.20		
	Like Virgin Blue	X_{10}	Mu	0.09	0.026	4.9
	Like Jetstar	X_{11}	Mu	-0.06	0.025	0.5
	Like Tiger Airways	X_{12}	Mu	-0.24	0.025	-9.7

Risk	Like Qantas		Mu	0.10		
	Like Virgin Blue	X_{13}	Mu	0.13	0.026	-1.0
	Like Jetstar	X_{14}	Mu	0.01	0.026	-7.0
	Like Tiger Airways	X_{15}	Mu	-0.24	0.028	9.6
Cost	Like Qantas		Mu	0.12		
	Like Virgin Blue	X_{16}	Mu	0.08	0.028	-23.5
	Like Jetstar	X_{17}	Mu	-0.03	0.028	15.8
	Like Tiger Airways	X_{18}	Mu	-0.18	0.027	-10.0

Table 9. Example 3, DCE 2, Selection of a Trip – Fixed Model Partworths

Attribute	**Level**	**Covariate**	**Parameter**	**Estimate**	**se**	**t**
Brand	Qantas		Mu	0.38		
	Virgin Blue	X_{19}	Mu	0.27	0.015	-16.1
	Jetstar	X_{20}	Mu	0.02	0.015	-8.8
	Tiger Airways	X_{21}	Mu	-0.67	0.015	-11.6
Fare	$450		Mu	1.12		
	$550	X_{22}	Mu	0.45	0.026	-1.0
	$650	X_{23}	Mu	-0.27	0.026	-7.0
	$750	X_{24}	Mu	-1.29	0.028	9.6
Time	5 hours		Mu	0.25		
	7 hours	X_{25}	Mu	-0.25	0.028	-23.5
Change	No		Mu	0.13		
	Yes	X_{26}	Mu	-0.13	0.027	-10.0
In-flight Food	No		Mu	0.17		
	Yes	X_{27}	Mu	-0.17	0.015	-16.1
In-flight Alcohol	No		Mu	0.06		
	Yes	X_{28}	Mu	-0.06	0.015	-11.6
Number of Stops	None		Mu	0.08		
	1 Stop	X_{29}	Mu	-0.08	0.014	-5.5

Table 10. Example 3, Log-likelihood values for the Airline Models

Model Specification	**Number of free parameters**	**Log Likelihood**
Fixed	29	-13253
Random	58	-11930
Factor	58	-11757
Linked Brand	61	-11687

Table 11. Example 3, Factor Loadings in DCE 1 for each Brand in the Linked Brand Model

	Virgin Blue v Qantas	**Jetstar v Qantas**	**Tiger Airways v Qantas**
Investment	.56	.16	.25
Consistency	.23	.15	.15
Credibility	.21	.16	.12
Quality	.15	.16	.12
Risk	.49	.31	.37
Cost	.31	.31	.17

Systematic component of utility

$$\begin{aligned} v \; = \; & (\mu_1 + \gamma_{1,1}\xi_1)x_1 + (\mu_2 + \gamma_{2,2}\xi_2)x_2 + (\mu_5 + \gamma_{3,3}\xi_3)x_3 && \text{DCE 1} \\ \vdots \;\; & \vdots && \vdots \\ + \; & (\mu_{16} + \gamma_{16,1}\xi_1)x_{16} + (\mu_{17} + \gamma_{17,2}\xi_2)x_{17} + (\mu_{18} + \gamma_{18,3}\xi_3)x_{18} && \text{DCE 1} \\ + \; & (\mu_{19} + \gamma_{19,4}\xi_4)x_{19} + (\mu_{20} + \gamma_{20,5}\xi_5)x_{20} + (\mu_{21} + \gamma_{21,6}\xi_6)x_{21} && \text{DCE 2} \\ + \; & (\mu_{22} + \gamma_{22,7}\xi_7)x_{22} + \cdots + (\mu_{29} + \gamma_{29,14}\xi_{14})x_{29} && \text{DCE 2} \end{aligned}$$

Structural Model

$$\begin{aligned} \xi_1 &= \delta_1 \\ \vdots \;\; & \vdots \;\;\; \vdots \\ \xi_{14} &= \delta_{14} \end{aligned}$$

δ_1 to δ_{14} are random components with independent normal distributions, zero means and standard deviations equal to one.

Figure 5. Example 3 Factor Model.
A factor analytic model is applied to each brands in DCE 1.

7 Discussion

Each empirical example that we presented involved two different choice tasks representing different choice processes. In Example 1, one DCE measured generic preferences for carbon mitigation strategies, and the second focused on refrigerator choices. In Example 2, both choice processes similarly focused on preferences, but for two different time periods. That is, circumstances may change across time with unobserved consequences that can be estimated and evaluated. Consider another possibility for Example 2 in which the sample is randomly divided into a treatment group, exposed to some intervention, and a control group, without the intervention. One then could use the proposed modelling approach to examine the causal impact of the treatment on mean preference, preference heterogeneity, and choice consistency. In Example 3 one DCE asked about perception profiles for brands and the other about preference for specific trips. Traditionally, the various constructs in a DCE are introduced as attributes and levels. In these examples the choice tasks themselves have been used to further specify and evaluate the roles of constructs. This is a new development for DCEs.

The examples differ in research questions, designs, numbers of attributes, levels and latent variables; however, all can be analysed with one general model. We considered 11 different model specifications, but all are nested under the general model that we proposed. Each was fully described by specifying the elements of the nine parameter matrices represented by Equation (15) to Equation (23); the elements of these matrices can be free to be estimated or fixed to zero or some other value. A single general model has several potential advantages: 1) various properties developed above including goodness of fit and identification are applicable to an entire family of model specifications, not limited to specific examples; 2) one general estimation software program with standardized interfaces can be used to estimate all the models; and 3) the notation, which can be extended to include path diagrams, provides a way to communicate research hypotheses, substantive theory and associated model specifications in a standardized way, which should lead to clearer model descriptions

Systematic component of utility

As in Figure 5

Structural Model

DCE 1 (Perceptions)

$$\xi_1 = \delta_1$$
$$\xi_2 = \delta_2$$
$$\xi_3 = \delta_3$$

The latent variables ξ_1, ξ_2 and ξ_3 representing brands in DCE 1 (Perceptions) are antecedents to the equivalent latent variables ξ_4, ξ_5 and ξ_6 in DCE 2 (Trip)

$$\begin{aligned}
\xi_4 &= \beta_{4,1}\xi_1 + \delta_4 \\
\xi_5 &= \beta_{5,2}\xi_2 + \delta_5 \\
\xi_6 &= \beta_{6,3}\xi_3 + \delta_6 \\
\xi_7 &= \delta_7 \\
&\vdots \\
\xi_{14} &= \delta_{14}
\end{aligned}$$

δ_1 to δ_{14} are random components with independent normal distributions, zero means and standard deviations equal to one.

Figure 6. Example 3 Linked Brand Model.
Each brand in DCE 1 is linked to the same brand in DCE 2.

and simpler comparisons between empirical applications.

When one combines data from different choice tasks/processes and proposes using latent variables, this gives rise to several important identification and scale issues. Similar to common practices in SEMs, one can resolve some identification issues by constraining the latent variables through fixing some parameters to specific values without loss of generality or model fit. Additionally, our discussion of the examples emphasized goodness-of-fit measures, such as log likelihoods and R^2 which are invariant to scale issues.

The examples demonstrate differences between the latent variable modelling approach we proposed and similar methods suggested by Walker, Ashok, and Morikawa. They introduced latent variables as factors on indicator variables, and

Table 12. Contribution of the existing literature and the general method for analysing two DCEs

Domain	Latent variables and factors in one DCE	Structural model linking latent variables in one DCE	Structural model linking two DCEs
Row 1 - Characteristics of the individual	Walker, Ashok and Morikawa and general method	Walker, Ashok, and Morikawa and general method	General method
Row 2 - Attributes of the alternatives	Walker, Elrod, Keane and general method	General method	General method

viewed the latent variable as a characteristic of an individual. Their approach could be extended to include DCE attributes, as we showed in our examples. Indeed, Example 1 is more similar to their work because DCE 1 measures a subjective or "attitudinal" variable. This was achieved by using a DCE instead of the multi-item scales that they used as indicator variables. Indeed, we could easily use our proposed modelling approach to analyse their data. We briefly summarize and compare our approach with theirs in Table 12. Their approach, analysing the characteristics of the individuals, is represented by the domain of row 1. While our work has paralleled their work we have focussed more on the domain of row 2, analysing attributes. We have moved through the columns of row 2 developing structural models for two DCEs.

The approach we propose extends the work of others in pooling data and in particular RP and SP (Hensher et al. 1999; Louviere et al. 1999) the benefits of which are to better specify and model the random components of utility (Ben-Akiva et al. 2002; Louviere et al. 2002). As with the work of Ben-Akiva and Morikawa (1990) and Morikawa et al. (2002) we develop a joint likelihood function which can be used to estimate switching matrices and choice probabilities of one experiment conditional on the responses to the other. However, our approach using a single model allows for flexibility in specifying how latent variables, and theory, link the preferences in each of the data sets and facilitates the evaluation of competing models.

SEM benefits have been constrained in prior work. Traditionally, data for SEMs have been obtained from rating scales, the limitations of which (particularly cross-culturally), are well-documented (Lee et al. 2007). DCEs yield different and potentially better quality data. The use of choice-based measures leads to less ambiguity regarding the way(s) in which individuals interpret measurement instruments. This reduces the impact of the individual's perception of the instrument and allowing better comparisons between individuals and cultures. More recently, SEMs have been expanded to model categorical variables (but not with choice sets), not just rating scales; DCEs also have been expanded to model latent variables that are characteristics of the individual. Both advances allow some analysis of choices but do not permit one to specify or evaluate structural models for attributes for data collected with choice-based measurement instruments and DCEs. The modelling approach that we proposed and illustrated closes this technical gap. The capacity to model latent variables and structures that exists in SEMs now can be applied to the attributes in DCEs.

Though a non-standard view, RUT models can be seen as exploring the multidimensional nature of utility (Rose et al. 2009). The trade-off by decision makers between alternatives transforms the utilities back to one dimension. The individual selects the alternative from the choice set for which this utility is highest. The factor analytic models of Elrod, Keane and Walker identify higher order dimensions. The latent class models of Kamakura and Russell (1989) are usually interpreted as measuring the probability of each individual being associated with each class but alternatively the classes can be seen as factors in multidimensional utility and the class probabilities as the weights for the transformation to the final one dimension of utility. Viewed in this manner, there is a strong analogy with SEM and its use of confirmatory factors for constructs, as in the measurement models. However, SEM has two additional properties for evaluating multiple dimensions (Jöreskog 1970; Jöreskog 1973; Bollen 1989; Jöreskog and Sörbom 1996). Firstly, it allows regressions to specify the links between the constructs in the structural model. Secondly, identification and accuracy is improved in SEMs by allowing separate measurement models for each construct. In every DCE care is taken to clearly describe the choice task to the individual. It is recognised that changing the description will generate different results which may or may not be useful. We exploit this property by deliberately changing the choice task over the DCEs or other choice tasks/processes. The choice tasks are selected to reflect the constructs and dimensions of utility.

SEMs and choice models both represent long-standing paradigms. Key references for theory and practical applications in both literatures have citation counts in the thousands. Differences in theoretical understanding, practical traditions, audiences and areas of application embedded in each paradigm go beyond the mathematics; they are artificial and are a consequence of method differences. We believe that it is time for a general theory and practice that can integrate both, which should allow these two paradigms to communicate and interact. Naturally, integration requires a fundamental overarching mathematics, which we proposed and illustrated in this paper.

8 Conclusion

Traditionally, choice experiments evaluate substantive theory and answer research questions by manipulating alternatives and their attributes, with the manipulations manifest as different choice alternatives. Using structural choice modelling, the paper shows that one can easily model a variety of relations and links between two or more choice tasks/processes. We believe that our empirical examples show the potential for significant enhancements in evaluating and understanding substantive theory, competing models, the nature of choice, causation and the multidimensional structure of utility. The ability to combine two or more choice tasks and model them based on a substantive theory of how they should be linked hopefully will be seen as a new way to test theory and obtain potentially deeper insights into individual decision making and choice.

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References

Ashok, K., W. R., Dillon and S., Yuan, 2002. Extending discrete choice models to incorporate attitudinal and other latent variables. *Journal of Marketing Research,* 39(February), 31-46.

Ben-Akiva, M. and S. R., Lerman, 1985. *Discrete Choice Analysis: Theory and Application to Travel Demand*, Boston, The MIT Press.

Ben-Akiva, M., D. McFadden, M. Abe, U. Böckenholt, D. Bolduc, D. Gopinath, T. Morikawa, V. Ramaswamy, V. Rao, D. Revelt, D., Steinberg, 1997. Modelling methods for discrete choice analysis. *Marketing Letters*, 8(3) 273-286.

Ben-Akiva, M., D., McFadden, K., Train, J., Walker, C., Bhat, M., Bierlaire, D., Bolduc, Z., Boersch-Supan, D., Brownstone, D. S., Bunch, A., Daly, A. De Palma, D., Gopinath, A., Karlstrom, and M. A., Munizaga, 2002. Hybrid choice models: Progress and challenges. *Marketing Letters*, 13(3), 163-175.

Ben-Akiva, M. and T., Morikawa, 1990. Estimation of switching models from revealed preference and stated intentions. *Transportation Research Part A*, 24(6) 485-495.

Bolduc, D. and R., Alvarez-Daziano, 2010. On estimation of hybrid choice models, in S. Hess and A. Daly (Eds.), *Choice Modelling: The State-of-the-Art and the State-of-Practice*, Emerald, England, 2010.

Bollen, K. A., 1989. *Structural Equations with Latent Variables,* New York, John Wiley & Sons.

Dube, J-P., P., Chintagunta, A., Petrin, B., Bronenberg, R., Goettler, P. B., Seetharaman, K., Sudhir, R., Thomadsen, Y., Zhao, 2002. Structural applications of the discrete choice model. *Marketing Letters*, 13(3), 207-220.

Elrod, T., 1988. Choice map: Inferring a product-market map from panel data. *Marketing Science* 7(1 Winter), 21-40.

Elrod, T. and M. P., Keane., 1995. A factor-analytic probit model for representing the market structure in panel data. *Journal of Marketing Research*, 32, 1-16.

Fiebig, D. G., M. P., Keane, J., Louviere, and N., Wasi, 2010. A generalized multinomial logit model: Accounting for scale and coefficient heterogeneity. *Marketing Science*, 29(3 May June), 393-421.

Finn, A. and J. J., Louviere, 1992. Determining the appropriate response to evidence of public concern: The case of food safety. *Journal of Public Policy and Marketing,* 11(2), 12-25.

Hensher, D., J., Louviere and J., Swait, 1999. Combining sources of preference data. *Journal of Econometrics,* 89, 197-221.

Hess, S. and A. Stathopoulos, 2011. Linking response quality to survey engagement: a combined random scale and latent variable approach. Leads, Institute for Transport Studies, University of Leeds.

Jöreskog, K. G., 1970. A general method for analysis of covariance structures. *Biometrika*, 57, 239-251.

Jöreskog, K. G., 1973. A general method for estimating a linear structural equation system. *Structural equation models in the social sciences*, A. S., Goldberger and O. D., Duncan. New York, Seminar Press, 85-112.

Jöreskog, K. G. and D., Sörbom, 1996. *LISREL 8: User's Reference Guide,* Lincolnwood, IL, Scientific Software International.

Kamakura, W. A. and G. J., Russell, 1989. A probabilistic choice model for market segmentation and elasticity structure. *Journal of Marketing Research*, 36(November), 379-90.

Keane, M. P., 1997. Modeling heterogeneity and state dependence in consumer choice behaviour, *Journal of Business and Economic Statistics*, 15(3 July), 310-327.

Lee, J. A., G. N., Soutar, and J. J., Louviere, 2007. Measuring values using best-worst scaling: The LOV example. *Psychology and Marketing*, 24(12), 1043-1058.

Louviere, J., D. Street, R. Carson, A. Ainslie, J. R. Deshazo, R. Cameron, D., Hensher, R., Kohn, and A., Marley, 2002. Dissecting the random component of utility. *Marketing Letters,* 13(3) 177-193.

Louviere, J. J., D. A., Hensher and J. D., Swait, 2000. *Stated Choice Methods: Analysis and Application*, Cambridge, Cambridge University Press.

Louviere, J. J., R. J., Meyer, D.S., Bunch, R., Carson, B., Dellarert, W. M., Hanemann, D., Hensher, and J., Irwin, 1999. Combining sources of preference data for modeling complex decision processes. *Marketing Letters*, 10(3), 205-217.

Louviere, J. J. and G. G., Woodworth, 1983. Design and analysis of simulated consumer choice or allocation experiments: an approach based on aggregate data. *Journal of Marketing Research*, 20, 350-67.

Luce, R. D., 1959. *Individual Choice Behavior; A Theoretical Analysis,* New York, John Wiley & Sons.

Marley, A. A. J., 2009. The Best-Worst Method for the Study of Preferences: Theory and Application. Working paper, Department of Psychology, University of Victoria, Canada.

McFadden, D., 1974. Conditional logit analysis of qualitative choice behaviour, *Frontiers in Econometrics*, P. Zarembka. New York, NY, Academic Press: 105-142.

McFadden, D., 1980. Econometric models for probabilistic choice among products. *Journal of Business*, 53(3, Pt2), S13-S29.

McFadden, D., 2001. Disaggregate behavioral travel demand's RUM Side: A 30-year retrospective, *The Leading Edge of Travel Behavior Research*, D., Hensher. Oxford, Pergamon Press.

McFadden, D. and K., Train, 2000. Mixed MNL models for discrete response. *Journal of Applied Econometrics*, 15, 447-470.

Morikawa, T., M., Ben-Akiva, and D., McFadden, 2002. Discrete choice models incorporating revealed preference and psychometric data, *Econometric Models in Marketing,* 16, 29-55.

Rose, J. M., D. A., Hensher, S., Caussade, J. de D., Ortúzar, and R.-C. Jou, 2009. Identifying differences in willingness to pay due to dimensionality in stated choice experiments: a cross country analysis. *Journal of Transport Geography,* 17(1), 21-29.

Temme, D., M., Paulssen and T., Dannewald, 2008. Incorporating latent variables into discrete choice models - A simultaneous estimation approach using SEM software, *BuR - Business Research*, 1(2), 230-237.

Train, K., 2003. *Discrete Choice Methods with Simulation* New York, Cambridge University Press.

Train, K., 2009. *Discrete Choice Methods with Simulation* New York, Ed2, Cambridge University Press.

Walker, J. L., 2001. *Extended Discrete Choice Models: Integrated Framework, Flexible Error Structures, and Latent Variables* Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, PhD.

Yáñez, M., S., Raveau and J. de D., Ortúzar, 2010. Inclusion of latent variables in mixed logit models: Modelling and forecasting. *Transportation Research Part A,* 44(9), 744-753.

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