

# Towards Reliability and Scalability in Feature Based Simultaneous Localization and Mapping

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Submitted in fulfillment of the requirement  
for the degree of Doctor of Philosophy

2014



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## Certificate

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## Acknowledgements

Having come to the end of my PhD candidature, it has been a wonderful experience working with some of the best and brightest at the Centre for Autonomous Systems, UTS. Four years has passed by very quickly with much learning and research in the exciting field of Robotics. The road has been filled with many challenges, from late nights spent coding to traveling overseas for competition and conferences.

I have been very privileged to work with Associate Professor Shoudong Huang, Dr Alen Alempijevic and Professor Gamini Dissanayake, who have not only been inspirational but role models to me. I would also like to thank the post docs, Liang Zhao, Zhang Wang, Jack Wang, Gavin Paul and the students, Minjie Liu, Lei Shi, Andrew To, Kasra Khosoussi, Mohammad Norouzi, and Freek De Bruijn.

Finally, I would like to thank my friends and family for their unending support.

# Abstract

*Simultaneous Localization and Mapping (SLAM) has always been an attractive topic in the vibrant field of robotics. Feature based representations of the problem can be seen as one of the most common definitions. In recent years, many SLAM researchers have realized some limitations of filtering based methods and started to focus more on optimization based SLAM techniques. However, this raises several questions surrounding convergence reliability and, similar to filtering, algorithm scalability.*

*In SLAM, sensor noise and non-linearity often causes the problem to become difficult. Converging towards the global minimum in a non-linear least squares formulation is by no means easy. Typically, one would need to start from a good initial estimate, preferably already inside the basin of attraction of the global minimum. In this thesis, we introduce a technique called Iterative Re-Weighted Least Squares bootstrapping to achieve a good initial estimate even when the noise is exceptionally large.*

*As a robot continues to traverse through its environment the complexity of SLAM tends to scale badly with the cumulative nature of graph nodes and edges. To solve large SLAM problems within a reasonable time scale one must also take into consideration elements of accuracy and consistency. In this thesis, we propose two alternative algorithms to handle complexity, Sparse Map Joining and Pose Graph Representation. Both of which contain unique advantages for handling the diverse scenarios within SLAM.*

*A series of quantitative analyses are performed on a number of challenging datasets, both real and simulated. In addition to this we perform a comprehensive case study on a specific type of feature based SLAM problem, RGB-D SLAM. This demonstrates how our technique is capable of avoiding inaccuracies and failure scenarios that is otherwise common in other RGB-D SLAM algorithms.*



# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
1.0.1	SLAM Applications	3
1.0.2	Brief History of SLAM	4
1.1	Motivation	5
1.2	Contributions	6
1.3	Publications	7
1.4	Thesis Outline	8
<b>2</b>	<b>Preliminaries</b>	<b>9</b>
2.1	Extended Kalman Filtering for SLAM	9
2.2	Graph Based SLAM	10
2.2.1	Odometry and Observation Information	11
2.2.2	Linear Least Squares	13
2.2.3	Non-Linear Least Squares	15
2.2.4	Weighted Non-Linear Least Squares	19
2.2.5	Least Squares for SLAM	20
2.3	Front-End and Back-End	21
2.4	Evaluating a SLAM algorithm	22
2.4.1	Chi Squared ( $\chi^2$ )	22
2.4.2	Expected Value of $\chi^2$ / Normalized $\chi^2$	23
2.4.3	$\chi^2$ Ratio	24
2.4.4	Normalized Estimation Error NEES	24
2.5	Related Works	26

---

2.5.1	Improving Reliability . . . . .	26
2.5.2	Overcoming Computational Complexity . . . . .	28
2.6	Summary . . . . .	30
<b>3</b>	<b>Reliable Optimization</b>	<b>31</b>
3.1	Introduction . . . . .	31
3.2	A General Framework for Reliable Optimization . . . . .	32
3.2.1	Defining a Sequence . . . . .	34
3.2.2	Iterative Re-weighted Least Squares . . . . .	35
3.2.3	Formulation for IRLS . . . . .	36
3.2.4	M-Estimators . . . . .	38
3.2.5	Generalized Influence Function . . . . .	40
3.2.6	Initial Influence . . . . .	42
3.2.7	Stopping Condition for IRLS . . . . .	42
3.2.8	Summary of IRLS algorithm . . . . .	43
3.3	Evaluation Criteria . . . . .	43
3.3.1	Benchmark Solution . . . . .	43
3.3.2	Noise Conditions . . . . .	43
3.3.3	Monte Carlo Evaluation . . . . .	44
3.3.4	Success Rate . . . . .	44
3.4	Experiment and Results . . . . .	45
3.4.1	IRLS for Feature Based Graphs . . . . .	45
3.4.2	IRLS on Pose Graphs . . . . .	46
3.4.3	Resulting Maps . . . . .	48
3.5	Discussion . . . . .	50
3.5.1	Importance of Noise Correlation . . . . .	50
3.5.2	Computation Time . . . . .	50
3.6	Summary . . . . .	51

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<b>4</b>	<b>Sparse Map Joining</b>	<b>52</b>
4.1	Introduction . . . . .	52
4.2	Sparse Map Joining . . . . .	53
4.2.1	Building Local Maps . . . . .	53
4.2.2	Marginalization . . . . .	54
4.2.3	Fusing Local Maps . . . . .	55
4.2.4	SMJ Algorithm . . . . .	58
4.3	3D Sparse Map Joining . . . . .	59
4.3.1	Standard 3D Range and Bearing . . . . .	59
4.3.2	SMJ for Bundle Adjustment(BA) . . . . .	60
4.3.3	Dual-Observation Model Joining . . . . .	61
4.4	Evaluation . . . . .	63
4.4.1	Consistency using NEES . . . . .	63
4.4.2	Accuracy using $\chi^2$ Ratio . . . . .	67
4.4.3	Computation Time . . . . .	69
4.4.4	Resulting Maps . . . . .	71
4.4.5	Real Datasets . . . . .	72
4.5	Discussion . . . . .	74
4.6	Summary . . . . .	76
<b>5</b>	<b>Pose Graph Representation</b>	<b>77</b>
5.1	Introduction . . . . .	77
5.2	Pose Graphs . . . . .	78
5.3	Pose Graph Representation of Feature Based SLAM . . . . .	79
5.3.1	Obtaining Relative Pose . . . . .	79
5.3.2	Information Reuse . . . . .	81
5.3.3	Algorithm . . . . .	86
5.4	Evaluation . . . . .	87
5.4.1	Consistency using NEES . . . . .	87
5.4.2	Accuracy using $\chi^2$ Ratio . . . . .	89
5.4.3	Resulting Maps . . . . .	91
5.4.4	Computation Time . . . . .	91



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5.4.5	Real Datasets . . . . .	94
5.5	Discussion . . . . .	95
5.5.1	Further Improving Efficiency . . . . .	95
5.5.2	Euler Angle Parameterization . . . . .	96
5.5.3	Outliers in Feature Observations . . . . .	96
5.6	Summary . . . . .	99
<b>6</b>	<b>Case Study: RGB-D SLAM</b>	<b>100</b>
6.1	Introduction . . . . .	100
6.1.1	Related Work . . . . .	101
6.1.2	Motivation . . . . .	102
6.1.3	Chapter Overview . . . . .	102
6.2	RGB-D Cameras . . . . .	103
6.3	Handling the RGB-D SLAM Front-End . . . . .	106
6.3.1	Feature Selection . . . . .	106
6.3.2	Feature Matching . . . . .	107
6.3.3	Iterative Closest Point (ICP) . . . . .	110
6.3.4	RGB Visual Odometry . . . . .	111
6.3.5	Initializing a New Pose . . . . .	111
6.3.6	Initializing a New Feature . . . . .	111
6.3.7	Loop Closing . . . . .	112
6.4	RGB-D SLAM . . . . .	114
6.4.1	Flow Chart of RGB-D SLAM . . . . .	115
6.4.2	Experiments and Results . . . . .	116
6.5	Robust RGB-D SLAM . . . . .	121
6.5.1	Local Map Building and Joining . . . . .	121
6.5.2	Local Map Switching . . . . .	121
6.5.3	Flow Chart of Robust RGB-D SLAM . . . . .	124
6.5.4	Experiment . . . . .	125
6.6	Discussion . . . . .	129
6.6.1	RGB-D SLAM . . . . .	129
6.6.2	Robust RGB-D SLAM . . . . .	129

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6.7	Summary	131
<b>7</b>	<b>Conclusion and Future Work</b>	<b>132</b>
7.1	Summary of Contributions	133
7.1.1	Reliable Optimization	133
7.1.2	Sparse Map Joining	133
7.1.3	Pose Graph Representation	134
7.1.4	Case Study	134
7.2	Future Work	135
7.2.1	Improving the Reliability	135
7.2.2	Optimal Splitting Strategy	135
7.2.3	Finding the Optimal Subset of Key Poses	136
7.2.4	Issues in RGB-D SLAM	136
	<b>Appendix</b>	<b>138</b>
A	Simulated Datasets	138
B	Reliable Optimization	141
C	Sparse Map Joining	142
C.1	Batch Optimization (BO)	142
C.2	Sequential Optimization (SO)	143
C.3	Divide & Conquer Optimization (DCO)	144
C.4	Sparse Map Joining Algorithm	145
D	Pose Graph Representation	146
E	Schur Complement	148
F	Transforming between Rotation Matrix and Euler Angles	149
	<b>Bibliography</b>	<b>150</b>

# List of Tables

3.1	Generalized function evaluation . . . . .	41
3.2	Pose/feature IRLS evaluation . . . . .	45
3.3	Pose only (Manhattan3500) IRLS evaluation . . . . .	47
3.4	Pose only (City10000) IRLS evaluation . . . . .	47
3.5	IRLS computation time evaluation . . . . .	50
4.1	Noise in simulated datasets . . . . .	63
4.2	Summary of simulated datasets . . . . .	63
4.3	SMJ 2D NEES evaluation . . . . .	65
4.4	SMJ 3D NEES evaluation . . . . .	66
4.5	SMJ 2D $\chi^2$ Ratio evaluation . . . . .	68
4.6	SMJ 3D $\chi^2$ Ratio evaluation . . . . .	68
4.7	SMJ computation time evaluation . . . . .	69
4.8	SMJ real data evaluation . . . . .	72
5.1	Summary of simulated datasets . . . . .	87
5.2	PGR 2D NEES evaluation . . . . .	88
5.3	PGR 3D NEES evaluation . . . . .	88
5.4	PGR 2D $\chi^2$ Ratio evaluation . . . . .	90
5.5	PGR 3D $\chi^2$ Ratio evaluation . . . . .	90
5.6	PGR computation time evaluation . . . . .	92
5.7	PGR real data evaluation . . . . .	94
5.8	Horn vs. Least Squares . . . . .	95

6.1	Feature extraction methods . . . . .	106
6.2	Summary of simulated datasets . . . . .	117
6.3	RGB-D SLAM results . . . . .	117
6.4	RE-RANSAC failure modes . . . . .	122

# List of Figures

1.1	Home Robots . . . . .	3
1.2	Search and Rescue Robot . . . . .	4
2.1	Dynamic Bayesian Network . . . . .	10
2.2	Gauss-Newton vs Gradient Descent . . . . .	16
2.3	Front-End Back-End . . . . .	21
3.1	Graduated non-convexity . . . . .	33
3.2	Basic Sequence . . . . .	34
3.3	Weighted sequence . . . . .	36
3.4	Influence functions . . . . .	38
3.5	Weight functions . . . . .	39
3.6	Generalized function . . . . .	41
3.7	Pose feature result . . . . .	48
3.8	Pose graph (Manhattan3500) result . . . . .	48
3.9	Pose graph (City10000) result . . . . .	49
4.1	Local map marginalization . . . . .	55
4.2	Map joining . . . . .	56
4.3	Batch Optimization . . . . .	57
4.4	Sequential Optimization . . . . .	57
4.5	Divide & Conquer Optimization . . . . .	58
4.6	Dual-observation model joining . . . . .	62
4.7	SMJ simulation results . . . . .	71

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4.8	SMJ real data results . . . . .	73
5.1	Pose/feature to pose graph . . . . .	79
5.2	Compute relative pose . . . . .	80
5.3	Single observation method . . . . .	82
5.4	Multi observation method . . . . .	83
5.5	Ignoring information reuse . . . . .	84
5.6	PGR simulation results . . . . .	91
5.7	Information matrix sparsity . . . . .	93
5.8	PGR real data results . . . . .	94
5.9	Outliers in the data . . . . .	97
5.10	Robust PGR results . . . . .	98
6.1	RGB-D Cameras . . . . .	103
6.2	RGB-D camera model . . . . .	105
6.3	FABMAP loop closing . . . . .	113
6.4	Flow chart for RGB-D SLAM . . . . .	115
6.5	Ground truth comparison . . . . .	119
6.6	Point cloud overlay . . . . .	120
6.7	Flow chart for RGB-D SLAM . . . . .	124
6.8	Visual odometry initial estimate . . . . .	126
6.9	Images at point of switch . . . . .	126
6.10	Optimized graph . . . . .	127
6.11	Point cloud overlay . . . . .	128
6.12	Point cloud overlay vs. architectural floor plan . . . . .	128
1	Circle trajectory . . . . .	138
2	Loop trajectory . . . . .	139
3	Manhattan features trajectory . . . . .	139
4	Sphere features trajectory . . . . .	140

# Nomenclature

## Formatting Style

$\hat{x}$	Measured
$\bar{x}$	Estimated
$\tilde{x}$	Actual

## Subscript

$m$	features index	$i, j$	pose index
$t$	time index		

## Superscript

$\mathcal{M}$	marginalized	$\mathcal{P}$	pose set
$\mathcal{F}$	feature set	$\mathcal{O}$	odometry set
$\mathcal{K}$	pose subset	$\mathcal{L}$	local map
$\mathcal{G}$	global map	$\mathcal{S}$	sensor

**Notations**

$\sim \mathcal{N}$	normally distributed	$\operatorname{argmin}_X$	minimizer
$[\cdot]$	vector elements	$\ \cdot\ $	Euclidean norm
$X^*$	the optimum	$X^{(0)}$	initial estimate

**Variables**

$\mathcal{G}$	undirected graph	$\mathcal{V}$	graph vertices
$\mathcal{E}$	graph edges	$E$	essential matrix
$Z$	measurement vector	$X$	state vector
$\Sigma$	covariance matrix	$\Omega, \Lambda$	information matrix
$\chi^2$	chi squared value	$\nu$	degree of freedom
$\mathcal{P}$	optimisation problem	$w$	weight scalar
$F$	fundamental matrix	$S$	scale
$K$	calibration matrix	$T$	transformation matrix

**Functions**

$g()$	pose to pose	$h()$	pose to feature
$b()$	generalized model function	$\rho()$	m-estimator
$Rot()$	rotation matrix	$Proj()$	projection matrix
$Horn()$	Horn's method	$\psi()$	influence function



# Abbreviations

SLAM	Simultaneous Localisation and Mapping
ML	Maximum Likelihood
GN	Gauss-Newton
GD	Gradient Decent
PDL	Powell's Dog-Leg
LM	Levenberg-Marquardt
STD	Standard Deviation
SGD	Stochastic Gradient Descent
SBA	Sparse Bundle Adjustment
iSAM	Incremental Smoothing and Mapping
$g^2o$	General Graph Optimization
ParallaxBA	Parallax Angle Bundle Adjustment
Alg	Algorithm
RMSE	Root Mean Squared Error
NEES	Normalized Estimation Error Squared

IRLS	Iterative Re-weighted Least Squares
GT	Ground Truth
IMU	Inertial Measurement Unit
RPE	Relative Pose Error
ATE	Absolute Trajectory Error
SIFT	Scale Invariant Feature Transform
SURF	Speeded-Up Robust Features
I-SLSJF	Iterated Sparse Local Submap Joining Filter
EIF	Extended Information Filter
EKF	Extended Kalman Filter
SMJ	Sparse Map Joining
BO	Batch Optimization
SO	Sequential Optimization
DCO	Divide and Conquer Optimization
LAGO	Linear Approximation for Graph Optimization
TORO	Tree based netwORk Optimizer
MO	Multi Observation method
SO	Single Observation method
PGR	Pose Graph Representation
MAP	Maximum a Posteriori

DBN	Dynamic Bayesian Network
ICP	Iterative Closest Point
RE-RANSAC	Re-projection Error RANdom SAMpling Consensus
EM-RANSAC	Essential Matrix RANdom SAMpling Consensus
VO	Visual Odometry
FABMAP	Fast Appearance Based Mapping
IR	Infrared
M-Estimator	Maximum likelihood-type Estimator
GPS	Global Positioning System