Parallel Packet Transmission Based on OFDM

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Abstract—**This paper proposes a parallel packet transmission (PPT) scheme based on orthogonal frequency division multiplexing (OFDM). The principle of the PPT scheme is to divide a packet into a number of smaller parallel packets, and transmit each smaller packet over an individual subcarrier of the OFDM symbols instead of spreading the data bits in a packet across a number of different subcarriers. It is proved theoretically that the proposed PPT scheme has higher average throughput than the conventional serial packet transmission without precoding. Furthermore, simulation results show that the OFDM system with PPT outperforms the precoded OFDM system with minimum mean squared error equalization in both uncoded and coded cases in terms of average throughput. The PPT scheme provides an alternative and simpler means to combat frequency-selective fading.**

Keywords—**Orthogonal frequency division multiplexing (OFDM), frequency-selective fading, throughput, parallel packet transmission.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1–2] is a key technology used in many existing wireless communication systems [3–6] and also the leading technological standard for future generation wireless radio access networks [7]. With OFDM, the data symbols are modulated onto orthogonal subcarriers via the inverse fast Fourier transform (IFFT), and recovered using the fast Fourier transform (FFT) followed by simple frequency-domain equalization. Unfortunately, OFDM suffers from some disadvantages, one of which is the poor frequency diversity. Since OFDM converts a frequency-selective fading channel into a set of parallel flat fading channels, its diversity performance, measured by the average bit error rate (BER), can only achieve diversity order one [19].

To improve the diversity performance, channel coding has been traditionally applied to the information data bits before data symbol constellation mapping and subcarrier modulation [8–9]. Recently, linear precoding or block spreading for OFDM has been proposed to introduce data symbol correlation among subcarriers so that the frequency diversity performance can be improved [10–17]. The principle of precoding or block spreading is to produce different linear combinations of the input data symbols through a unitary matrix and then modulate the precoded or block spread data symbols onto different subcarriers. Thus, if a subcarrier experiences a deep fade after transmitting over a frequency-selective fading channel, the data symbols can still be recovered from other received subcarriers.

We observe that the diversity performance is normally measured by the average BER over the channel fading variables. However, a more practical system performance measure should be the system throughput which is reflected in the frame error rate (FER) or packet error rate (PER) especially for wireless data transmissions via packet switching. Although the average BER can be improved by using various diversity techniques, the PER and hence the system throughput may not be improved necessarily. In other words, a lower average BER does not necessarily translate to a higher system throughput. Since a packet has to be discarded at the receiver whether it has one or many erroneous bits, an alternative strategy for system design is to allow more error bits to appear in fewer packets to be discarded, instead of spreading the errors across a large number of packets. Admittedly, employing error-correction coding could remove errors in a packet, but it increases redundancy and therefore may reduce the system throughput.

In this paper, the channel capacity of a precoded OFDM system over frequency-selective fading channel is revisited. Then the channel capacity of a precoded OFDM system with the practical minimum mean squared error (MMSE) equalization is derived. It is revealed that the flat fading channel offers higher channel capacity than a frequency-selective fading channel for the precoded OFDM system. This implies that, to achieve higher system throughput, the greater channel capacity of the flat fading channel should be better exploited as an alternative to employing diversity techniques. Based on this observation, we propose a parallel packet transmission (PPT) scheme based on OFDM to improve the system throughput by transmitting data packets independently over parallel subcarriers without resorting to the complicated diversity techniques.

The rest of the paper is organized as follows. In Section II, the precoded OFDM system and frequency-selective fading channel models are presented. In Section III, the ergodic channel capacity of the precoded OFDM system using MMSE equalization is analyzed. Section IV describes the PPT scheme and proves that it provides higher throughput than serial packet transmission without precoding. Section V provides the simulation results to confirm that the OFDM system with PPT outperforms the precoded OFDM system in both uncoded and coded cases in terms of average throughput. Finally, conclusions are drawn in Section VI.

II. SYSTEM AND CHANNEL MODELS

Let's begin with a precoded OFDM transmitter with N subcarriers as shown in Fig. 1 (a). The precoding group size is also chosen as N for simplicity. To transmit a data packet of kMN bits, a block of MN data symbols $X[i]$, $i = 0, 1, \ldots, MN-1$, is first generated after a k bit constellation mapping, and then converted into N parallel data streams via the serial-to-parallel conversion (S/P). Each data stream has M data symbols. Taking one data symbol from each parallel data stream to form a vector of dimension N, the N parallel data streams can be further expressed as a series of M vectors $\mathbf{X}_0$, $\mathbf{X}_1$, …, $\mathbf{X}_{M-1}$, where $\mathbf{X}_m = (X[mN], X[mN+1], \ldots, X[mN+N-1])^T$, $m = 0, 1, \ldots, M-1$, and $(\cdot)^T$ denotes vector transposition. Before IFFT, each data vector $\mathbf{X}_m$ is multiplied by an $N \times N$ unitary matrix $\mathbf{U}$ satisfying the property $\mathbf{U}\mathbf{U}' = \mathbf{U}'\mathbf{U} = \mathbf{I}$ to produce the precoded data vector $\mathbf{Y}_m = \mathbf{U}\mathbf{X}_m$, where $(\cdot)'$ denotes the matrix transposition and complex-conjugation and $\mathbf{I}$ is the identity matrix of order N. After performing the N-point IFFT, each $\mathbf{Y}_m$ is converted into a time-domain vector $\mathbf{y}_m$, which finally forms an OFDM symbol after parallel-to-serial conversion (P/S) and cyclic prefix (CP) insertion or zero-padded (ZP) suffix appending. There are total M consecutive OFDM symbols composed for this data packet transmission, and an acknowledgement from the receiver via a feedback channel is necessary before the next data packet can be sent. We refer to this conventional packet transmission scheme as the serial packet transmission (SPT).

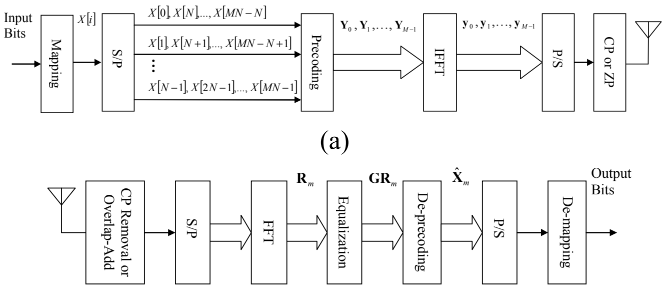

Fig. 1. Precoded OFDM system for serial packet transmission: (a) transmitter and (b) receiver.

After transmitting over a frequency-selective fading channel described by the discrete channel impulse response $h[i]$, $i = 0, 1, \ldots, L-1$, the OFDM symbols arrive at the receiver. By removing the CP or performing an overlap-add operation and transforming the received time-domain signal into the frequency-domain via FFT for each OFDM symbol, the received baseband signal can be expressed as

$$\mathbf{R}_m = \mathbf{H}\mathbf{U}\mathbf{X}_m + \mathbf{V}_m, \quad m = 0, 1, \cdots, M-1, \tag{1}$$

where

$$\mathbf{H} = diag(H[0], H[1], \ldots, H[N-1]) \tag{2}$$

is an $N \times N$ diagonal matrix with the channel discrete frequency response $H[n]$ (the N-point discrete Fourier transform of $h[i]$) as the diagonal elements and $\mathbf{V}_m$ is a zero-mean Gaussian noise vector with covariance matrix $E\{\mathbf{V}_m\mathbf{V}_m'\} = \sigma_V^2\mathbf{I}$, where $E\{\}$ denotes ensemble average. It is also assumed that the data symbols in $\mathbf{X}_m$ are mutually independent with average signal power σ_X^2 so that $E\{\mathbf{X}_m\mathbf{X}_m'\} = \sigma_X^2\mathbf{I}$.

To recover the transmitted data vector $\mathbf{X}_m$, equalization and detection must be performed on the received frequency-domain signal $\mathbf{R}_m$. Due to the complexity of the optimal maximum likelihood (ML) equalization, the MMSE equalization is usually applied in practice, since it can simply use a one-tap equalizer for each subcarrier in the frequency-domain. The MMSE solution of (1) can be expressed as

$$\hat{\mathbf{X}}_m = \left((\mathbf{H}\mathbf{U})'\mathbf{H}\mathbf{U} + \frac{1}{\gamma_{in}}\mathbf{I} \right)^{-1} (\mathbf{H}\mathbf{U})'\mathbf{R}_m = \mathbf{U}'\mathbf{G}\mathbf{R}_m \tag{3}$$

where $\mathbf{G} = \left(\mathbf{H}'\mathbf{H} + \frac{1}{\gamma_{in}}\mathbf{I} \right)^{-1}\mathbf{H}' = diag(G[0], G[1], \ldots, G[N-1])$, in which

$$G[n] = \frac{H^*[n]}{|H[n]|^2 + \frac{1}{\gamma_{in}}}, \quad n = 0, 1, \cdots, N-1, \tag{4}$$

is the one-tap equalizer coefficient for subcarrier n, and $\gamma_{in} = \frac{\sigma_X^2}{\sigma_V^2}$ is the input signal-to-noise ratio (SNR) before equalization. The receiver block diagram is shown in Fig. 1(b). Repeating the equalization and detection process for $m = 0, 1, \ldots, M-1$, all the transmitted data bits are retrieved.

Regarding the frequency-selective fading channel, we assume a normalized tapped delay line model and a full multipath diversity of order L. That is, all channel tap coefficients $h[i]$, $i = 0, 1, \ldots, L-1$, are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero-mean and variance $1/L$. Consequently, $H[n]$ are complex Gaussian variables with zero-mean and unit variance.

III. CAPACITY OF PRECODED OFDM

According to (1), the normalized channel capacity (bits/second/Hz) for a given realization of the channel frequency response can be expressed as

$$C = \frac{1}{N}\log_2 \det\left(\mathbf{I} + \frac{1}{\sigma_V^2}\mathbf{H}\mathbf{U}E\{\mathbf{X}_m\mathbf{X}_m'\}\mathbf{U}'\mathbf{H}' \right)$$
$$= \frac{1}{N}\sum_{n=0}^{N-1}\log_2\left(1 + \gamma_{in}|H[n]|^2\right). \tag{5}$$

The normalized ergodic channel capacity can be evaluated by performing ensemble expectation on $|H[n]|^2$ which are chi-square-distributed with two degrees of freedom and probability density function (pdf) $e^{-\rho}$, i.e.,

$$\overline{C} = \frac{1}{N}\sum_{n=0}^{N-1} E\left\{\log_2\left(1+\gamma_{in}|H[n]|^2\right)\right\} = \int_0^\infty \log_2(1+\gamma_{in}\rho)e^{-\rho}d\rho$$
$$= \frac{\gamma_{in}}{\ln 2}J_1(\gamma_{in}) \quad (6)$$

where $J_1(\gamma_{in}) = \int_0^\infty \frac{e^{-\rho}}{1+\gamma_{in}\rho}d\rho$.

Note that $\overline{C}$ is the same as the normalized ergodic channel capacity for any flat fading channel. This means that applying precoding in a frequency-selective channel does not increase the channel capacity any further beyond what a flat fading channel could have.

After the MMSE equalization is applied for the detection of the precoded OFDM signal, the MMSE is obtained as $\varepsilon_{\min}^2 = E\left\{\left(\hat{\mathbf{X}}_m - \mathbf{X}_m\right)'\left(\hat{\mathbf{X}}_m - \mathbf{X}_m\right)\right\} = \sigma_X^2 trace\left\{\left(\gamma_{in}(\mathbf{HU})'\mathbf{HU}+\mathbf{I}\right)^{-1}\right\}$ $= \sigma_X^2 \sum_{n=0}^{N-1}\frac{1}{\gamma_{in}|H[n]|^2+1}$ and the output SNR is

$$\gamma = \frac{N\sigma_X^2 - \varepsilon_{\min}^2}{\varepsilon_{\min}^2} = \frac{1}{\frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{\gamma_{in}|H[n]|^2+1}} - 1. \quad (7)$$

Therefore, the normalized ergodic channel capacity becomes

$$\overline{C}^{mmse}(N,L) = E\{\log_2(1+\gamma)\}$$
$$= E\left\{\log_2 \frac{1}{\frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{\gamma_{in}|H[n]|^2+1}}\right\} \quad (8)$$

which is a function of the parameters N and L. To show the maximum potential system throughput for the MMSE equalization, we can perform the following asymptotical analysis. First, letting $N \to \infty$, $L \to \infty$, and assuming that $|H[n]|^2$ is ergodic, the average $\frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{\gamma_{in}|H[n]|^2+1}$ in (8) can be replaced by the ensemble average $\int_0^\infty \frac{e^{-\rho}}{\gamma_{in}\rho+1}d\rho$ which is the same as $J_1(\gamma_{in})$. Then, the asymptotical normalized ergodic channel capacity becomes

$$\overline{C}^{mmse}(\infty,\infty) = \log_2\frac{1}{J_1(\gamma_{in})}. \quad (9)$$

Fig. 2 shows the comparison between $\overline{C}$, evaluated using (6), and $\overline{C}^{mmse}(\infty,\infty)$, evaluated using (9), from which we see that the channel capacity of an OFDM system under flat fading is actually higher than that of the precoded OFDM system using the practical MMSE equalization under frequency-selective fading. For comparison, the theoretical Shannon limit $\log_2(1+\gamma_{in})$ is also shown.

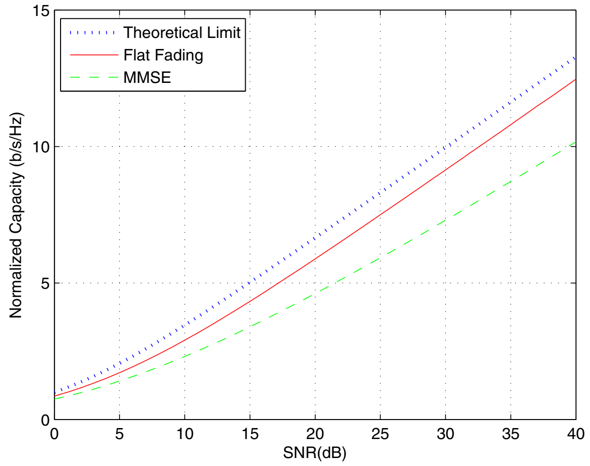

Fig. 2. Comparison between the normalized ergodic channel capacity of OFDM under flat fading and that of the precoded OFDM using MMSE equalization (asymptotical) under frequency-selective fading.

IV. Parallel Packet Transmission

Recall that through OFDM a frequency-selective fading channel can be converted into a number of parallel flat fading sub-channels. To make use of the advantage of the flat fading characteristics, a packet can be transmitted on a single subcarrier rather than across different subcarriers. Provided that there are N subcarriers in the OFDM symbols, N packets can be transmitted independently in parallel. In this way, data symbols in the same packet are no longer modulated on different subcarriers so that no frequency diversity technique is necessary. This transmission scheme is thus called parallel packet transmission, which can be described using the block diagram shown in Fig. 3 (a).

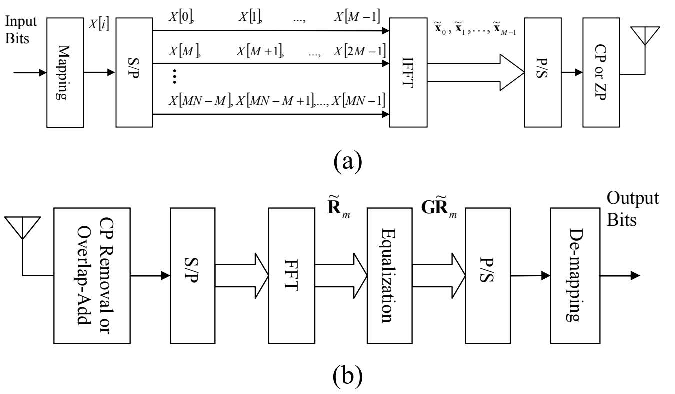

Fig. 3. Parallel packet transmission system based on OFDM: (a) transmitter and (b) receiver.

To transmit packets in parallel, the data symbols $X[i]$,

$i = 0,1,...,MN-1$, form an original serial packet are first divided into N groups $\{X[0], X[1], \ldots, X[M-1]\}$, $\{X[M], X[M+1], \ldots, X[2M-1]\}$, …, and $\{X[MN-M], X[MN-M+1], \ldots, X[MN-1]\}$ via S/P, each of which forms an independent packet and is assigned to only one subcarrier. To produce an OFDM symbol, N data symbols, each of which comes from a different packet, are grouped as a vector, so that a total of M vectors, $\tilde{\mathbf{X}}_0$, $\tilde{\mathbf{X}}_1$, …, $\tilde{\mathbf{X}}_{M-1}$, can be formed, where $\tilde{\mathbf{X}}_m = (X[m], X[m+M], \ldots, X[m+(N-1)M])^T$, $m = 0, 1, \ldots, M-1$. After performing the N-point IFFT, each $\tilde{\mathbf{X}}_m$ is converted into a time-domain vector $\tilde{\mathbf{x}}_m$, which finally forms an OFDM symbol after P/S and CP insertion or ZP appending. There are M consecutive such OFDM symbols composed for N independent packets. At the receiver, an OFDM symbol is processed conventionally as shown in Fig. 3(b), where the MMSE equalization is the same as the one in serial packet transmission, i.e., the same $\mathbf{G}$ matrix is applied to the received signal vector $\tilde{\mathbf{R}}_m$ to obtain an MMSE estimate of $\tilde{\mathbf{X}}_m$. After all the M OFDM symbols are processed, the recovered data bits need to be re-arranged into N parallel packets. Each packet will be parsed and acknowledged independently via a feedback channel. If a packet is checked in error, it will be re-transmitted in the next parallel packet transmission cycle.

To facilitate data decoding and packet error checking, each parallel packet should include a header and a frame check sum (FCS) fields, which will add more overhead to the parallel packet transmission as compared with the serial packet transmission in order to transmit the same number of data bits. However, as long as the data field is sufficiently longer than the header and FCS in a parallel packet, the impact of the additional overhead will be negligible.

To prove that the parallel packet transmission outperforms the serial packet transmission in terms of the achievable system throughput, let's assume a 2^k-ary Gray-coded QAM mapping for the OFDM system with BER $P_e(n)$ for data bits modulated on subcarrier n, $n = 0, 1, \ldots, N-1$. For serial packet transmission without precoding, the data bits in a packet are transmitted on all subcarriers so that the average BER for the packet is $\frac{1}{N}\sum_{n=0}^{N-1} P_e(n)$ and the packet error rate is $BLR_s = 1 - \left(1 - \frac{1}{N}\sum_{n=0}^{N-1} P_e(n)\right)^{B_s}$ where B_s is the number of bits in the serial packet. The throughput is thus given by

$$\begin{aligned} T_s &= k(1 - BLR_s) \\ &= k\left(1 - \frac{1}{N}\sum_{n=0}^{N-1} P_e(n)\right)^{B_s}. \end{aligned} \quad (10)$$

For parallel packet transmission, the packet error rate for the packet transmitted on subcarrier n will be $BLR_p(n) = 1 - (1 - P_e(n))^{B_p}$ where B_p is the number of bits in a parallel packet, and the throughput will be $T_p(n) = k(1 - BLR_p(n)) = k(1 - P_e(n))^{B_p}$. The average throughput for the N parallel packets is thus given by

$$T_p = \frac{1}{N}\sum_{n=0}^{N-1} k(1 - P_e(n))^{B_p}. \quad (11)$$

To prove that $T_p \geq T_s$, we first assume that $B_p = B_s$. According to Jensen's Inequality [20], we have $E[f(X)] \geq f(E[X])$ for a random variable X and any convex function $f(X)$. By defining $f(X) = k(1-X)^{B_s}$ where X takes on values $x_n = P_e(n)$, $n = 0,1,\cdots N-1$, and $E[X] = \frac{1}{N}\sum_{n=0}^{N-1} x_n$, we see that $f(X)$ is a convex function since $1 - x_n \geq 0$ and $f''(x_n) = B_s(B_s - 1)(1 - x_n)^{B_s - 2} \geq 0$. Thus

$$\frac{1}{N}\sum_{n=0}^{N-1} k(1 - P_e(n))^{B_s} \geq k\left(1 - \frac{1}{N}\sum_{n=0}^{N-1} P_e(n)\right)^{B_s}. \quad (12)$$

To transmit the same amount of data bits as that in a serial packet, B_p ($= B_s/N$) is actually much smaller than B_s, which results in even higher T_p. Therefore, the throughput of parallel packet transmission is always higher than that of serial packet transmission.

V. SIMULATION RESULTS

To confirm the advantage of the proposed parallel packet transmission based on OFDM over serial packet transmission, bit-accurate simulation is performed in this section. Both uncoded and coded OFDM systems are simulated. For the coded systems, a convolutional code with coding rate R =½ and constraint length K =7 is used. The generator polynomials are g_0 =133 and g_1 =171 in octal form. Other system parameters are: N = 64, L =16, B_s =8192, and B_p =8192 and 128 respectively. The constellation mappings used are QPSK, 16QAM, and 64QAM respectively, which correspond to the maximum possible normalized throughput 2, 4, and 6 bits/second/Hz respectively. At a given SNR, a packet is transmitted 10000 times with different channel realizations in order to calculate the average throughput and BER.

Fig. 4 and Fig. 5 show the normalized throughput and BER versus input SNR respectively for uncoded OFDM systems. We see that, for serial packet transmission (denoted as SPT), the throughput is very poor without precoding (denoted as w/o PC) and is improved significantly with precoding (denoted as w PC). Parallel packet transmission (denoted as PPT) improves the throughput significantly even though the BER may be worse than that of the preceoded OFDM with serial packet transmission.

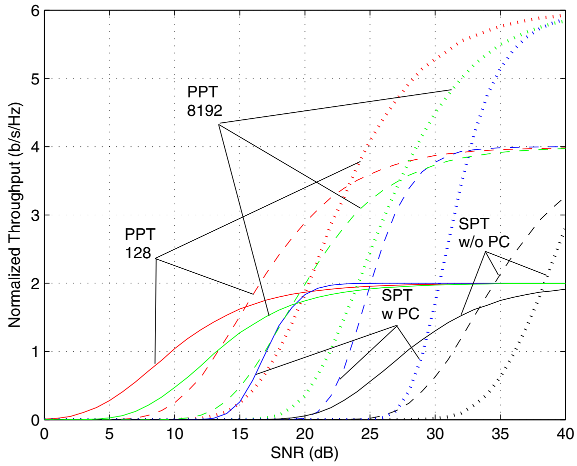

Fig. 4. Simulated throughputs for uncoded OFDM systems (solid lines for QPSK, dashed lines for 16QAM, and dotted lines for 64QAM).

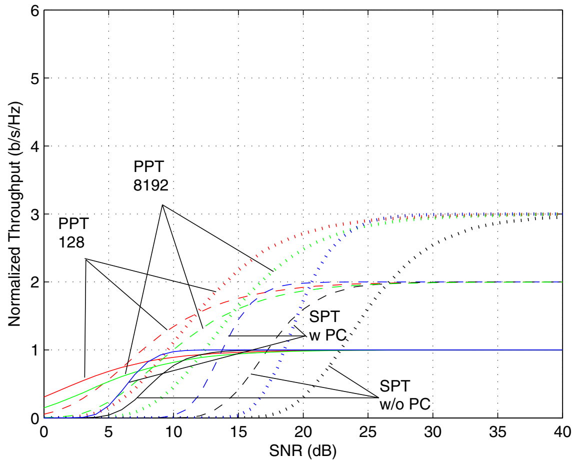

Fig. 6. Simulated throughputs for coded OFDM systems (solid lines for QPSK, dashed lines for 16QAM, and dotted lines for 64QAM).

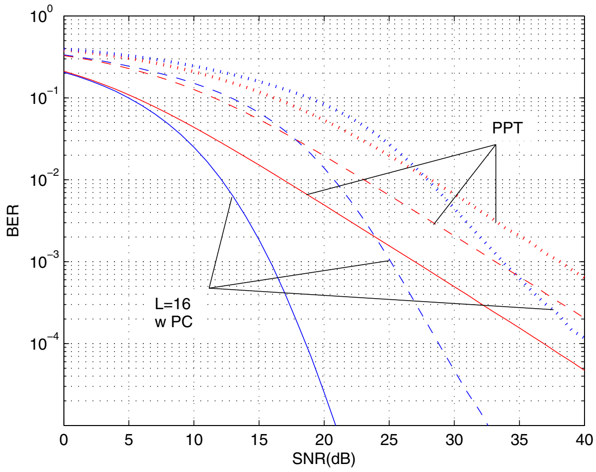

Fig. 5. Simulated BERs for uncoded OFDM systems (solid lines for QPSK, dashed lines for 16QAM, and dotted lines for 64QAM).

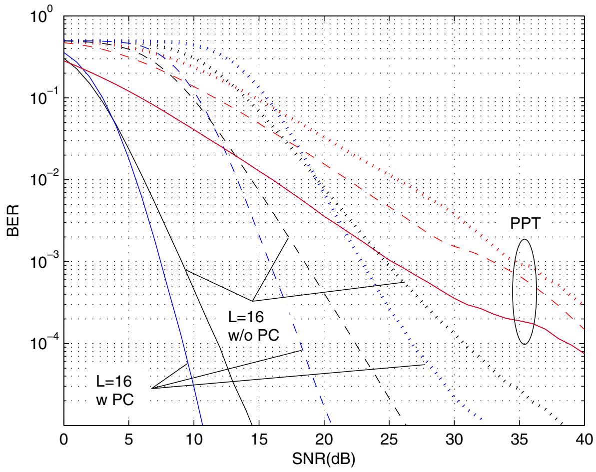

Fig. 7. Simulated BERs for coded OFDM systems (solid lines for QPSK, dashed lines for 16QAM, and dotted lines for 64QAM).

Fig. 6 and Fig. 7 show the simulated throughputs and BER for coded systems. It can be observed that, when channel coding is used, the throughput difference between systems with and without PC is not as significant as that for uncoded systems. This means that channel coding is effective to improve frequency diversity and the combination of channel coding with precoding offers further diversity improvement. However, the parallel packet transmission still outperforms the serial packet transmission in terms of throughput though the BER performance is improved significantly in frequency-selective fading channel after channel coding.

The above simulation results clearly show that the parallel packet transmission can ultimately offer better system performance in terms of throughput than serial packet transmission with or without precoding for both uncoded and coded OFDM systems even though the average BER may be inferior.

VI. Conclusions

We have shown that an OFDM system operating in flat fading channel can offer higher channel capacity than the OFDM system with precoding and MMSE equalization in frequency-selective fading channel, and a lower BER does not necessarily translate to greater throughput. Based on this observation, an OFDM system with parallel packet transmission is proposed which can significantly outperform the conventional OFDM system incorporating complicated diversity techniques such as precoding and MMSE equalization. Although frequency diversity techniques have been proven to be effective in combating frequency-selective fading, they may not be necessary. The parallel packet transmission technique provides an alternative and simpler way to achieved better system performance in frequency-selective fading channels.

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