

*Quantitative Approaches to
High Net Worth Investment*

Edited by Andrew Rudd and Stephen Satchell



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Haymarket House
28-29 Haymarket
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Tel: + 44 (0)20 7484 9700
E-mail: books@incisivemedia.com
Sites: www.riskbooks.com
www.incisivemedia.com

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About the Editors

Andrew Rudd is chairman and CEO of Advisor Software, Inc, which he founded in 1995 to deliver world class analytics to the retail financial services market. He is an expert in asset allocation, modern portfolio theory, risk management and performance measurement. Andrew is also a co-founder and former chairman of Barra, Inc, where he served as CEO from 1984 to 1999. Under his guidance Barra developed risk management technologies, investment portfolio analysis methods, trading technologies and a vision for the modern management of large pools of assets that have subsequently become the professional standard on a global basis. Andrew is the co-author of two industry-leading books on institutional investing: *Modern Portfolio Theory: The Principles of Investment Management* and *Option Pricing*. In addition he has written numerous journal articles and research papers on a wide range of domestic and international investment practices and theories. Andrew received his Bachelor of science degree with honours in mathematics and physics from Sussex University and earned an MSc in operations research, an MBA in finance and international business and a PhD in finance and operations research from the University of California, Berkeley. He was professor of finance and operations research at Cornell University in Ithaca, New York and also serves as a trustee of the University of California, Berkeley, Foundation and a member of the investment committee of the University of Massachusetts Foundation.

Stephen (Steve) Satchell was born in New Zealand but has spent his working life in the UK, a great deal of at the University of Cambridge, where he was the reader in financial econometrics until his retirement. He focuses on both empirical and theoretical aspects of econometrics, finance, risk measurement and utility theory. His very strong econometric techniques knowledge has proved invaluable for his links with financial institutions. Stephen is a professor in the discipline of finance at Sydney University, a Fellow of Trinity College, Cambridge, and a visiting academic at Royal Holloway College, University of London. He holds two PhDs (Cambridge and London), an MComm (UNSW), an MSc (Sydney) and

an MA (Cambridge). He is an Honorary Actuary and senior editor of three journals and speaks frequently at practitioner conferences. He has refereed academic journals and has published extensively on equity return and risk models, style rotation, asset allocation, trading rules, volatility, option prices, exchange rates and property markets. His interest in quant and high net worth grew out of his working life as a financial consultant and academic advisor to a range of more traditional wealth managers, whom he shepherded through the interesting transition period, much of it contextualised by the global financial crisis.

About the Authors

Timothy (Tim) Adler is a managing partner and head of research and technology for Windham Capital Management, LLC. Tim joined Windham in 2004 and is responsible for managing the research for all of Windham's investment solutions and proprietary technologies. He has a Bachelor of science degree from the Royal Melbourne Institute of Technology and a Master of business administration degree from Boston College. Tim is a member of the CFA Institute and the Boston Security Analysts Society.

Nicolas (Nic) Brunetti has been vice president of investment research and development in the global asset allocation group at Fidelity Investments Asset Management since January 2013, responsible for providing research support to Fidelity Personalized Portfolios, including separately managed accounts (SMAs). Nic joined Fidelity as a senior investment manager in 2011. Before this, he was a quantitative analyst and senior vice president at Putnam Investments from 2008 to 2011. He held various roles at Citi from 1999 to 2008. As a member of Citi's US Large Cap Core portfolio management team, Nic conducted the primary research underlying the firm's tax-aware US equity core process for high net worth clients. He has been in the investments industry since 1999. Nic earned his Bachelor of engineering degree from The Cooper Union, his Master of science degree from Cornell University, and his Master of business administration degree from the MIT Sloan School of Management. He is a Chartered Financial Analyst (CFA) charterholder.

Greg B. Davies is head of behavioural investment philosophy at Barclays, leading a team of specialists who help people make better financial decisions. He is an Associate Fellow at the University of Oxford's Saïd Business School, and co-authored the book *Behavioral Investment Management* in 2012. He has authored papers in multiple academic disciplines, and is a frequent media commentator on behavioural finance. Greg holds an MPhil in economics, and a PhD in behavioural decision theory, both from the University of Cambridge. He co-created the "reality opera" *Open Outcry*, turning the

behaviour of a functioning trading floor into a musical performance, which received its première in November 2012.

Dan diBartolomeo is president and founder of Northfield Information Services, Inc, which creates mathematical models of financial markets. He sits on the board of numerous financial industry organisations, including the London Quant Group, the International Association for Quantitative Finance and the Chicago Quantitative Alliance. His publication record includes 30 books, book chapters and research journal articles. In addition, Dan is a visiting professor at the CARISMA centre of Brunel University and has been admitted as an expert witness in litigation matters regarding investment practices and derivatives in both US federal and state courts.

Greg N. Gregoriou, a native of Montreal, obtained his joint PhD in Finance at the University of Quebec at Montreal. He has published 49 books, 60 refereed publications in peer-reviewed journals and 20 book chapters since his arrival at SUNY (Plattsburgh) in August 2003. He is a research associate at the EDHEC business school in Nice, France, and also a research associate at the Endowed Chair of La Caisse de dépôt et placement du Québec (CDPQ) in Portfolio Management at the University of Quebec at Montreal. He is also Senior Advisor to the Ferrell Asset Management Group in Singapore and a lecturer in finance at McGill University, Department of Continuing Studies. Greg is hedge fund editor and editorial board member for the *Journal of Derivatives and Hedge Funds*, as well as editorial board member for the *Journal of Wealth Management*, the *Journal of Risk Management in Financial Institutions*, *Market Integrity*, *IEB International Journal of Finance*, the *Journal of Quantitative Methods for Social Sciences* and the *Brazilian Business Review*. Many of Greg's books have been translated into Chinese and Russian; his co-authored articles have appeared in the *Review of Asset Pricing Studies*, *Journal of Portfolio Management*, *Journal of Futures Markets*, *European Journal of Operational Research*, *Annals of Operations Research*, *Computers and Operations Research*, among others. His interests focus on hedge funds, funds of funds and cumulative translation adjustments.

Robert Jalali is a research officer and a PhD candidate at the University of Cambridge. He lectures on various finance topics for Judge Business School and for the Wharton School's executive education programme. Robert is an active commercial real estate broker in the

US, with over 18 years of industry experience. He has been involved in land acquisition, development, investment and financing activities in residential and commercial markets. He has also acted as a financial consultant to the World Bank within the privatisation and private sector development group.

Robert Jukes is a global strategist for Canaccord Genuity Wealth Management, responsible for providing top-down macroeconomic ideas and strategies. He is a member of the asset allocation committee, and has a particular interest in portfolio construction and risk. He moved to the wealth management division in January 2008, having initially joined as global equity strategist in the institutional equities division in May 2007. Prior to joining the firm, Robert spent 10 years at Credit Suisse, latterly as global equity strategist. He started his career in finance as a bond analyst for Paine Webber after a stint in academia and the civil service as a statistician and an economist. He has had a number of papers published.

Mark Kritzman is president and CEO of Windham Capital Management. He is also a founding partner of State Street Associates, and he teaches a graduate finance course at the Massachusetts Institute of Technology. He has written numerous articles for academic and professional journals and is the author of six books, including *Puzzles of Finance* and *The Portable Financial Analyst*. In 2004, Mark was elected a Batten Fellow at the Darden Graduate School of Business Administration, University of Virginia. Mark has an MBA with distinction from New York University and a CFA designation.

Francois-Serge Lhabitant is the CEO/CIO of Kedge Capital, where he oversees more than US\$6.5 billion of capital invested in hedge fund strategies. He was formerly a member of senior management at Union Bancaire Privée and a director at UBS/Global Asset Management. On the academic side, Francois-Serge is a professor of finance at the EDHEC Business School (France) and a visiting professor at the Hong Kong University of Science and Technology. He holds an engineering degree from the Swiss Federal Institute of Technology, a BSc in economics, an MSc in banking and finance and a PhD in finance from the University of Lausanne. He is the author of several books on risk management, alternative investments (hedge funds) and emerging markets, and has published numerous research and

scientific articles. Francois-Serge is a member of the European Advisory Board of the International Association of Financial Engineers (IAFE) and a member of the investor steering committee at the Alternative Investment Management Association (AIMA). He has been a member of the Scientific Committee of the Autorité des Marchés Financiers, the French financial markets regulatory body.

Antonia Lim is the global head of quantitative research for wealth and investment management at Barclays. She has over a dozen years' experience in wealth management and previously held senior roles at Kleinwort Benson and Dresdner Bank, specialising in asset allocation and security selection. Antonia graduated from the University of Oxford with a Masters in physics and an academic scholarship. She is a CFA charterholder and a director of the not-for-profit organisation London Quant Group.

Colin Lizieri is Grosvenor Professor of real estate finance at the University of Cambridge. Colin has over 30 years' experience in real estate research, consultancy and expert witness work, advises international organisations, central banks, government, professional bodies and private firms on real estate investment and has published widely on real estate finance and investment. His book *Towers of Capital* explores global capital flows and office markets. He has a BA from the University of Oxford and a PhD from the London School of Economics. He is a Fellow of the Royal Institution of Chartered Surveyors, a Fellow of the Royal Geographical Society and Weimer Fellow of the Homer Hoyt Institute.

Anders Petterson is the founder and managing director of ArtTactic Ltd, a London-based art market research and analysis company set up in 2001. ArtTactic has been a pioneer in using crowd-sourcing techniques for gathering and processing intelligence on the art market. Anders previously worked at JP Morgan, where he was responsible for debt capital markets and structured products for banks and corporates. He also worked as an independent research and evaluation consultant for Arts & Business in London between 2002 and 2007, and has been involved in a number of large research and evaluation projects in the cultural sector. Anders lectures on the topic of "art as an asset class" for Cass Business School, Sotheby's Institute of Art, Christie's Education and the Institut d'Etudes Supérieures des

Arts (IESA) in London. He is a board member of the Professional Advisors to the International Art Market (PAIAM).

Keith Quinton has been a portfolio manager at Fidelity Management and Research Company (FMRC), the investment advisor for Fidelity's family of mutual funds since October 2006, responsible for co-managing the Disciplined Equity Fund. Keith was a portfolio manager at Pyramis Global Advisors and its predecessor organisations from 2001 to 2006, where he managed various institutional products. He has been in the industry since 1983, holding the positions of quantitative analyst, senior quantitative analyst, head of US Equities, senior vice president of quantitative equity research and equity strategist. Keith earned his Bachelor of arts degree in Russian and his Master of business administration degree from Dartmouth College, New Hampshire.

Bernd Scherer is the chief investment officer at FTC Capital GmbH in Vienna. Before joining FTC, Bernd was managing director at Morgan Stanley, London, and professor of Finance at EDHEC Business School from January 2010 to September 2012. He was also head of quantitative research and head of portfolio engineering for Deutsche Asset Management in New York. Bernd has published more than 50 papers in academic journals, as well as in practitioner journals. He has authored or edited eight books on quantitative finance, is a reviewer for international finance journals and has written several contributions for the *Financial Times*. He served five years as a board member for the London Quant Group.

Edward Smith is a global strategist for Canaccord Genuity Wealth Management and provides top-down macro-based, multi-asset research for the global investment process and presents to the asset allocation committee. He co-designed, with Robert Jukes, the firm's forward-looking approach to risk management. Edward joined the wealth management division in 2007 from BlackRock, where he had worked with liability-driven investments for institutions after graduating from Oxford. He is a CFA charterholder and has completed a course in quantitative economic methods at Birkbeck.

Nandini Srivastava is a PhD student at the faculty of economics, University of Cambridge. Her research interests include macroeconomics, econometrics and applied financial economics. Her dissertation is based on non-linear dynamic models in the foreign

exchange and the art markets. She has been a referee for the *Journal of Asset Management* and the *Journal of Derivatives and Hedge Funds*.

Susan Thorp is professor of finance and superannuation at the University of Technology, Sydney. Her research focuses on financial econometrics and long-horizon wealth management. Her publications include studies of market integration, retirement savings portfolio management, annuitisation and retirement income streams. She is a member of the Centre for the Study of Choice and the Quantitative Finance Research Centre at UTS, and the National Centre for Econometric Research, QUT. Susan gained her BEc (Hons) from the University of Sydney, joined the economic group at the Reserve Bank of Australia, and later gained her PhD from the University of New South Wales.

Jarrold Wilcox founded Wilcox Investment, Inc, in 2001, serving families. He has been at various times director of international equities, of currency and of research at Panagora Asset Management, director of international investments at Batterymarch Financial Management and chief investment officer at Colonial Management Associates. Earlier, he was an assistant professor at MIT's Sloan School and a consultant at The Boston Consulting Group. Jarrold serves on the advisory board of the *Journal of Portfolio Management*, and is co-author of the CFA Research Institute's *Investment Management for Taxable Private Investors* and of *Financial Advice and Investment Decisions: A Manifesto for Change*.

Oliver Williams has over 15 years' experience in structuring and trading financial derivatives. He joined JP Morgan Swaps Group in 1994 and subsequently moved to Credit Suisse in 2002, before leaving to pursue academic research. Since 2009 he has specialised in design and implementation of computerised systematic trading models across all asset classes. He holds an MA in computer science and management studies, an MPhil in economics and a PhD in financial economics (all from the University of Cambridge), has co-authored several papers and book chapters on investment topics and is co-inventor of a social-networking patent. Oliver is managing director of Scalpel Research Ltd and a partner in Markham Rae LLP.

Introduction

Andrew Rudd; Stephen Satchell

Advisor Software; University of Cambridge

Who are the high net worth investors (HNWIs) and what makes them tick? Our experience is that most people aspire to being acknowledged as "high net worth", except those that actually belong to this category. Truly high net worth individuals tend to shun, if possible, the notoriety that comes with extraordinary wealth, but if they are public figures, such as athletes, entertainment stars or politicians, they typically accept their situation with as little comment as possible. A desire for confidentiality and, perhaps, some fear of being a target play some role in this. Certainly, for those who gained their wealth as a result of inheritance there would appear to be little to be gained from being acknowledged as an HNWI beyond an emotional and egotistical sense of well-being arising from being part of an unusual, distinguishable group.

High net worth investors clearly have greater wealth than those who are not, but how much more? And are there other characteristics, such as location, of this high net worth demographic which distinguish them from others? Most people would accept that billionaires belong to the HNWI category. According to the Wealth-X (2013) report,¹ at the time of writing there is an estimated global population of 2,160 billionaires, an increase of 185 over 2011–12. Out of the 2012–13 global population of billionaires, there were an estimated 480 in the US, 147 in China and 140 in the UK. We believe that having a net worth of US\$30 million or above should be indicative of HNWI status. The Wealth-X report estimates this global population amounts to 187,380, of which 60,280 are in the US, 11,245 in China and 10,515 in the UK. By region, the estimated population is 42,895 in Asia, 53,440 in Europe and 14,750 in Latin America.

In contrast, it is not clear to us that having US\$1 million or more in investible wealth defines, in most peoples' minds, membership of the HNWI club. Although achieving millionaire status is unusual,²

the media in the developed countries of the world tend to emphasise the amassing of staggering wealth and so accumulating a million dollars does not seem to be such an achievement as it once was. However, this is a standard used quite frequently in the industry, presumably to engender positive feelings among would-be clients. The millionaire cut-off is used in the World Wealth Report 2013 (Capgemini and RBC Wealth Management 2013), another survey of the global wealthy population, which estimates a global population of 12 million millionaires, with 3.7 million in North America, 3.7 million in the Asia-Pacific region and 3.4 million in Europe.

Other than the level of wealth, are their investment portfolios different from those of the less wealthy? Do these investors have other common attributes that are different to the less wealthy? Or are the high net worth investors only interesting because they have more wealth? Certainly, we would expect to, and do, see greater exposure in the portfolios of the HNWI to those assets which only they can afford. For example, we do observe in the portfolios of the HNWI real assets such as significant real estate beyond a primary residence, financial assets and products with high minimum investments, such as alternatives, collections of art, jewellery, and "luxury collectibles" such as automobiles, boats or jets.

One other aspect of HNWI behaviour towards their wealth is different to that of the less wealthy: as reported in Capgemini and RBC Wealth Management (2013, p. 20), "HNWI focus on wealth preservation is pronounced globally, but regional differences exist, with HNWIs in some emerging markets focused more on wealth growth". A focus on wealth preservation can only be pursued if the accumulated wealth is not needed for living expenses or other important lifetime goals, suggesting by implication that only the HNWI can afford to attend to the preservation of wealth. A related observation concerns the HNWI attitude towards risk. It is generally observed that many investors become HNWI as a result of concentrated bets, in either an operating business or investment portfolio, but that continuing to maintain such concentrated bets as an HNWI potentially leads to loss of wealth (JP Morgan Private Banking 2011). As a result we observe a preference towards a generally conservative approach to investing and wealth preservation, with this observation being stronger among older HNWIs and those in the upper wealth segments (Capgemini and RBC Wealth Management 2013, p. 23).

This book brings together a wide variety of studies on the investment behaviour of high net worth investors. We have placed an emphasis on quantitative methods, largely because this is an area in which there has been a great deal of innovation. Although the chapters vary considerably in the amount of actual quantitative finance within them, they all rely on an analytical perspective to illustrate their approach and emphasise their results.

We have tried to cover most of what we consider to be the important areas for HNWIs. Contributions to the book cover the spectrum from academic to practical approaches and also discuss some of the related issues that arise in the lives of HNWIs. The two great universals – death and taxes – are covered. We also consider some near-universals such as fraud, luxury goods, divorce and real estate. Other aspects of the human condition, such as dynastic risk and longevity, were beyond the scope of this book.

STRUCTURE OF THE BOOK

in Chapter 2 Jarrod Wilcox looks at the management of discretionary wealth. This is both intellectually interesting and driven by a lifetime of practical experience. Jarrod was one of the first to use this approach, and many subsequent chapters, and work elsewhere, are in debt to his ideas.

In Chapter 3 Dan diBartolomeo introduces a new procedure for asset allocation, called the analytic hierarchy process (AHP). This is a robust procedure which allows non-experts to "formulate multiple decision criteria in the form of questions, with each question having multiple discrete answers". The advantages of such a technique is that it can be used to supplement mean-variance, which is well known to suffer from a lack of robustness. Dan makes the case that AHP provides this robustness. The chapter is further enhanced by a technical appendix with extra information on AHP.

In Chapter 4 Keith Quinton and Nicolas Brunetti address issues of tax alpha. This chapter should be of direct use to HNWIs and their advisors, as it gives practical information on how to improve after-tax returns. As Quinton and Brunetti explain, tax alpha is not about forecasting, it is about understanding tax laws and fund management. They show that the use of quantitative approaches in the context of tax alpha leads to noticeable improvements.

In Chapter 5, Greg Davies and Antonia Lim present an analysis described by the phrase “managing anxiety to improve financial performance”, which should be applicable to HNWIs. This is a most interesting approach that is difficult to summarise in a few phrases, but the thrust of what they are doing is to suggest behavioural perturbations that make the use of conventional financial tools, such as mean-variance, more palatable. They make the case that HNWIs (generally, with some exceptions) face the issue of managing their flows of funds and assets rather than their stocks and thus require more continuous monitoring than the more typical “buy and hold” of the smaller investor. The term “anxiety” covers what the man on the Clapham omnibus might think of as anxiety but actually has a broader meaning. Anxiety is, to quote Davies and Lim, “any short-term behavioural responses to investors’ immediate circumstances, whether good or bad, that might make them uncomfortable enacting, or sticking with, the long-term normative solution”.

Chapter 6 by Robert Dukes and Edward Smith present a fascinating practitioner model to capture regime switches in investing. This has been built for individual investors. In many ways this makes a similar point to Chapter 5 in that it offers private investors alternatives to static investment. The procedures they provide allow for predictable volatility fluctuations to be modelled and incorporated into a choice of assets. As Dukes and Smith themselves note, after 2007 the notion of static fixed weight investment seems not only unappealing but also non-intuitive.

In Chapter 7 Oliver Williams and Anders Petterson address the role of expert opinion in markets for collectibles. The full picture of HNWI behaviour should take account of how money is spent. Spending above essentials (discretionary wealth, as discussed in Chapter 2) is frequently spent on art, stamps, coins, etc, and such investments have the attractive feature that they can be enjoyed by investors, while the assets themselves increase in value. The determination of the price of such assets, however, is a much more complex matter and is often determined by taste-makers, that is, the expert opinion or people of high status. The obvious reason why this is so is the lack of visible earnings accruing to the assets and, in many cases, the absence of reasonable data on which to base values. This chapter looks at how collecting information about taste-makers from auctions and elsewhere can lead to predictable changes in the realised

prices of artworks. The procedures are highly quantitative and can be applied not only to the art market but also to any similar market.

The cost of socially responsible investing is examined by Mark Kritzmman and Timothy Adler in Chapter 8. There are many instances of funds/portfolios owned or sponsored by wealthy individuals that set out to achieve specific goals of a moral nature. The cost of such activity, as Kritzmman and Adler note, is rather hard to quantify, but at the simplest level boils down to adding extra constraints to an optimisation problem. We would expect therefore socially responsible investment to impact negatively upon performance. That it indeed does sometimes, but on other occasions is shown not to do so, is an interesting fact in itself and, to some extent, is the focus of Kritzmman and Adler’s work.

As Chapter 9, we are delighted to include a *Journal of Wealth Management* paper by Greg Gregoriou and Francois-Serge Lhabitant titled “Madoff: A Flock of Red Flags”. This chapter makes the point, very clearly, that there were numerous signals related to the performance of returns and structure of Mr Madoff’s operation that wealth managers should have picked up. The fact that in many cases they did not and were happy to send their clients’ money to be managed says as much about the naivety of certain experienced managers as it does about the deficiencies of their counterparty.

Chapter 10, by Colin Lizieri and Robert Jalali, considers real estate as an asset class and provides a useful overview for those considering commercial real estate for investment purposes. It can be contrasted with earlier chapters that discuss deviations from long-term investment, as real estate is often seen primarily in long-run terms. Lizieri and Jalali present a balanced view of the costs and benefits of real estate investment.

In Chapter 11 Nandini Srivastava and Stephen Satchell investigate the role of luxury goods for high net worth investment and also for high net worth consumption. They consider collectibles but also look at other classes of luxury goods. The analysis used is more in line with microeconomics, and behaviour is defined in terms of utility functions. Using such tools, Srivastava and Satchell provide definitions of utility from luxury goods based on such notions as status and wealth being an end in themselves (often referred to as the “spirit of capitalism” model) rather than a means to increased consumption. The approach is highly quantitative.

Chapter 12, by Susan Thorp and Stephen Satchell, is titled "Modelling Sustainable Spending Plans for Family Offices, Foundations and Trusts". They consider the problem facing HNWIs as one that can be described in terms of an infinitely lived investor. This provides a convenient analytic framework for solving the optimal consumption rate while preserving wealth. They also address issues of family-name extinction.

Chapter 13 by Bernd Scherer investigates marriage, taken, not as the union of two souls, but as a complex multi-period derivative. One of the issues surrounding the possession of a great deal of wealth is the costliness of marital breakdown. Scherer considers marital breakdown as a contingent event and this chapter provides a fascinating analysis of this issue.

Finally, in Chapter 14, Andrew Rudd looks at the asset allocation decisions for HNWIs and expresses the view that the very wealthy tend to be relatively secretive about their financial activities and do not willingly provide data for quants to analyse and use to build models. This provides a whole range of new challenges, not least the observation that frequently multi-generational wealth preservation is probably more important motivation for the high net worth than wealth growth. Interestingly, a wealth-preservation strategy reflected in high cash holdings flies in the face of conventional finance theory, which is almost universal in assuming decreasing absolute risk aversion. Rudd focuses on the practicalities of wealth preservation from a US perspective, which not only should be useful reading for those interested in the subject but leads to a number of results for appropriate asset allocation policy.

As often happens in a book of this kind, certain themes are planned *ex ante* in the construction of the book, such as discretionary wealth, while others emerge *ex post*. In the latter category, the idea of separating short-term planning from long-term planning is one example, and readers will no doubt detect others. We hope we have covered most aspects of the topic but we are aware that history has a great tendency for surprising us and that events will create topicality possibly even as the book goes to press. As an example, one topic not considered explicitly is the optimal choice of residence and its close relative, global tax arbitrage, but Chapters 4 and 10 would certainly help a researcher to prepare to investigate these areas in greater detail. Our text is designed to benefit researchers, investors and their advisors.

- 1 A survey of the global wealthy population.
- 2 For example, roughly coinciding with the top 1% of the population in the US.

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Modelling Sustainable Spending Plans for Family Offices, Foundations and Trusts

Stephen Satchell; Susan Thorp

University of Cambridge; University of Technology, Sydney

A critical question for many high net worth individuals (HNWIs) is how to best adjust spending rates when the investment outlook changes. This chapter sets out optimal spending plans for HNWIs or family offices. Unlike standard approaches, the plans derived here allow for the fact that family offices often decide spending plans and investment strategy separately. The plans also account for the fact that many such HNWIs have high risk tolerance but a preference for steady consumption streams.

Using a sophisticated recursive utility function (Weil 1990; Epstein and Zin 1989), we first model the optimal disbursement rate for a perpetual entity with a predetermined asset allocation. Then, under general assumptions about investment returns, we use properties of stochastic dominance to estimate modifications to the current consumption rate when the investment outlook changes. The capacity of the decision-maker to vary spending through time (elasticity of intertemporal substitution) is key to understanding optimal increases or decreases in consumption.

We begin by setting out the background needed to model optimal consumption paths under conditions of uncertainty, showing how the recursive utility set-up is well adapted to modelling HNWIs' decisions, and briefly foreshadowing the main results. We address adjustments to the set-up needed when planning for a real family with finite survival prospects, and then describe and solve the analytical model for optimal consumption before giving a simple empirical example illustrating the important results. We derive principles of scenario analysis, covering changes in both expected returns

and volatility. Overall, in this chapter we set out a sophisticated but robust approach to planning disbursement rates from trusts or endowments, including how to modify the optimal spending rule for revisions in the investment outlook.

BACKGROUND AND MODELLING FRAMEWORK

Globally, in 2011, there were 11 million individuals with over US\$1 million in investable wealth, amounting to more than US\$42 trillion in assets. More than 3 million HNWI's resided in each of Asia-Pacific, Europe and North America with the highest concentration of wealth in North America (CapGemini and RBC Wealth Management 2012, p. 5). HNWI's often employ skilled professionals in family offices, foundations and trusts to manage their wealth for current and future generations (Martiros and Millay 2006).¹ While concerns for privacy make it difficult to measure the size of the HNWI sector separately, Martiros and Millay infer that it is substantial. The discussion of HNWI plans in this chapter extends naturally to many similar types of organisations, including perpetual charities and foundations. The charity sector is known to be very large: in 2012, the UK Charity Commission reported over 161,000 charities, holding investments in excess of £78 billion with annual spending over £53 billion,² and, for the US, Standard & Poor's Money Market Directories reported over 5,000 endowments and foundations, controlling more than US\$946 billion in assets.³

The wealth management problem for these organisations has some important non-standard features that are addressed here. Specifically, the setting combines the Epstein–Zin–Weil (EZW) recursive utility model (Epstein and Zin 1989; Weil 1990), which allows a separation of tastes for risk from tastes for consumption smoothing, with general assumptions on investment returns, and in an infinite time horizon. The model is particularly suited to HNWI individuals or families who plan investment separately from spending. For example, if assets are concentrated in operating businesses, and/or if investment decisions are delegated to managers, investment goals are likely to be made largely independently of the spending interests of family members. Amit *et al* (2008) reported that 58% of family offices in their survey sample operated family businesses, with 77.5%

of those holding a controlling interest. On average, around one quarter of wealth is tied to operating business in the US and Europe. Such HNWI individuals or families are likely to be more constrained in asset allocation than in spending decisions. Moreover, most family offices have investment management committees. While the family offices surveyed reported that trans-generational wealth management was their overarching purpose, investment goals ranged from aggressive wealth growth to conservative preservation. Non-expected utility models such as EZW give the flexibility needed to match this disjunction in tastes between investment management and spending.

In addition, when investment returns can be treated as independent and identically distributed (iid) the asset allocation and consumption decisions in this model are theoretically separable, reflecting a division between investment and spending policies. So the analysis to follow derives spending plans under alternative scenarios for returns that are conditional on a separately managed portfolio. The framework also allows family office managers and trustees to carry out general scenario analysis without assuming that returns processes are lognormal. HNWI portfolios include private equity, hedge funds and real estate in significant quantities in both Europe and North America (Amit *et al* 2008) and since there is evidence that returns to these asset classes are typically non-normal more general distributional assumptions are needed.⁴ Consequently, the analysis is robust to many of the irregularities of financial returns processes.

The general problem of an entity making spending and investment plans over a finite or infinite horizon, subject to uncertainty, has generated a huge literature. Standard models usually comprise time-additive von Neumann–Morgenstern utility, often with uncertainty generated by lognormal diffusions, with only a few cases where explicit solutions can be derived. Merton's seminal model (Merton 1969) analyses an infinitely lived entity with a constant relative risk aversion utility function.⁵ In the case where all asset returns are lognormally distributed and some regularity conditions on the rate of discounting of future utility are satisfied, the optimal rate of consumption is constant, and optimal wealth is lognormal and bounded below. In Merton's case all calculations are done continuously rather than discretely. Although the key features of the Merton

solution (a constant disbursement rate and strictly positive wealth) are interesting and the solution is relatively easy to compute, it only partly addresses the problem of this chapter.

First, a continuous time framework does not fit the decision-making of a family office or trust, where boards may meet quarterly or less often (Amit *et al* 2008). So this analysis solves for annual spending rates over an infinite horizon. (The survival prospects of families, and how the problem could be modified in the light of variation in survival, are also discussed below.) Second, joint log-normality seems an excessively restrictive assumption for returns, given the asset classes invested in by HNWI, family offices and foundations. Third, as noted above, Amit *et al* (2008) and Martiros and Millay (2006) describe processes of investment management that are delegated to groups of in-house or external managers, so the model below allows asset allocation decisions to be decided separately from disbursement rates. Fourth, beneficiaries and/or family members may want smooth spending paths. Models which apply the usual time-separable expected utility functions limit the scope of analysis by constraining relative risk aversion to be the inverse of the elasticity of intertemporal substitution, so that agents who have low risk aversion must also be willing to transfer consumption across time. However, for HNWI individuals or families, risk aversion and aversion to intertemporal substitution are likely to be conceptually and practically distinct: many family trusts can tolerate considerable uncertainty over returns while aiming for fairly smooth payments to beneficiaries over time.

Recursive or non-expected-utility preferences as proposed by Kreps and Porteus (1978, 1979) allow a partial separation of tastes for risk and intertemporal consumption. Whereas the von Neumann-Morgenstern agent is interested only in the conditional expectation of all future consumption (the timing of the resolution of uncertain outcomes does not matter), the Kreps-Porteus agent also cares how soon uncertainty over consumption will be resolved. If an entity is highly risk averse but willing to redistribute consumption across time, then they prefer an early resolution of uncertainty, but if an entity is tolerant of risk and, relatively speaking, dislikes transferring consumption across time, then later resolution is better. As Weil (1990) points out, this amounts to a trade-off between the safety and stability of utility, where safety is improved by early resolution of

risk and stability by late resolution. The model used here adapts Weil's version of the Kreps-Porteus preferences to the dynamic consumption problem of HNWI, and explores the properties of the model under scenario analysis.⁶

Giovannini and Weil (1989) and Weil (1990) showed that the optimal constant disbursement rate for an HNWI with EZW utility is set by the rule

$$m = 1 - (\delta \varphi^{(1-\rho)/(1-\alpha)})^{1/\rho} \quad (12.1)$$

where m is the proportion of wealth spent each year, δ is a parameter which is a component of time preference,⁷ α is relative risk aversion, $1/\rho$ is the elasticity of intertemporal substitution and φ is the expected value of $\tilde{Z}^{1-\alpha}$, the risk-adjusted return to wealth, where \tilde{Z} is the gross return to the investment portfolio or family business.

Under constant relative risk aversion (CRRA) preferences, the spending rule simplifies to $m = 1 - (\delta \varphi)^{1/\alpha}$. The analysis to follow also gives the conditions for the convergence of the value function for this problem. Given plausible parameter values and historical estimates of investment returns, optimal spending rates might lie between 1% and 3% of wealth per annum in real terms.

Shifting probability mass from the lower to the upper tail of the returns density, and working with mean-preserving spreads, allows for an analytical and numerical calculation of the trade-off between income and substitution effects and the ensuing changes to disbursement rates when the investment returns distribution changes. Further, while superficial intuition might predict that spending out of an endowment will be positively related to an optimistic investment outlook and negatively linked to pessimism, this is true only for a sub-set of preferences and the reverse reaction can be optimal. Estimation of these effects indicates that optimal consumption rates are remarkably sensitive to small changes in beliefs about future returns distributions. The direction of revisions to optimal consumption depends on whether the elasticity of intertemporal substitution is greater than or less than 1, not on tastes for risk, but the size of revisions will be sensitive to relative risk aversion. Stochastic dominance arguments confirm, extend and illustrate analytical results found in Giovannini and Weil (1989), Weil (1990) and Bhamra and Uppal (2006), which demonstrate the pivotal role of the elasticity of intertemporal substitution for consumption paths.

Having set out the reasons for our choice of utility function and approach to returns distributions, we now address the question of the planning period, or expected lifetime of the foundation, family office or trust.

MODELLING FOR PERPETUAL ENTITIES AND FAMILIES

A crucial question for disbursement decisions is the planning horizon of the endowment or trust. Embedded in the planning horizon are questions of intra- and inter-generational equity, so important to the management of family offices and foundations. Two main strands of economic literature have studied the management of wealth over generations. The first relates to perpetual foundations such as university endowments, and the second relates to families.

Perpetual foundations and endowments

Studies of university endowment behaviour look for a disbursement rate rule that satisfies "inter-generational equity" while preserving capital over the long horizon (see, for example, Tobin 1974; Litvack *et al* 1974; Nichols 1974). Most are not interested in deriving optimal portfolio allocations for endowments. Tobin proposed consuming out of permanent foundation income (ie, from the long-run rate of return on assets). However, Woglom (2003) showed that Tobin's definition of inter-generational equity (fixed real consumption across time) implied a zero rate of intertemporal substitution. For agents with CRRA utility functions this means infinite risk aversion, an assumption that is contradicted by endowment investment patterns. Using a deterministic, continuous-time model, Woglom argued that endowments should consume from recurrent capital gains, but he relaxed the inter-generational equity constraint to allow optimal real consumption to vary over time.

University spending and investment were readdressed in later papers by Merton (1990, 2003), who considered optimal consumption and portfolio allocation at the whole university level, rather than the endowment level. When income streams (gifts, bequests, etc) and the costs of university activities co-vary with investment returns, he argued that university portfolio managers can hedge against future cost changes and adjust to non-tradeable income sources by employing replicating strategies. Dybvig (1995, 1999) viewed the inter-generational equity question differently, proposing

that most endowments will want short-run spending certainty while maintaining long-run viability. He argued for a dynamic portfolio insurance strategy where the institution creates a riskless perpetuity matched to the current minimum spending level while maintaining exposure to risky returns.⁸ This strategy is probably too conservative for most HNWIs, and the analysis to follow allows more flexible intertemporal consumption and investment plans.

Families and family survival

Studies of university endowments treat the decision-making entity as a unified whole with one set of preferences. The second line of literature studying family utility maximisation explores intra- and inter-generational preferences. As Xu (2007) notes, the family is a place of both conflict and cooperation: Amit *et al* (2008, p. 10) reported that the average single family office in their survey sample served "13 households, 40 family members and two to three generations". A simplifying approach to inter-generational transfers is to treat the head (altruist or dictator) as deciding consumption among current and future members of the family so that the welfare of the family is indistinguishable from the welfare of the head (Becker 1974, 1981).

For family trusts and family offices, a trust deed or constitution can stand in the place of a family "head", deciding on allocations between beneficiaries. The fact that most HNWI families use formal agreements is evidence that cooperative bargaining (Manser and Brown 1980; McElroy and Horney 1981) and exchange between family members (Cigno 1993, 2006, 2007) are not effective or stable structures. Further, the utility of future generations is often valued, as well as the interests of the current family members. Becker and Tomes (1986) and Becker and Barro (1988) discussed cases where consumption is divided equally among children in each time period and then aggregated. In this case, trusts and foundations have to plan for expected survival rates.

The survival of families has been a question of interest to mathematicians and demographers for hundreds of years but few empirical estimates of family line survival are available (Albertsen 1995). For England and Wales, for example, the average number of live daughters a woman of child-bearing age of a particular cohort can expect to have in her lifetime (gross reproduction rate) rose to 0.96 in 2008 from a low point of 0.80 in 2000.⁹ Since the probability that

Table 12.1 Estimated probability of family survival and expected family size

Number of generations	Single-branch family		Triple-branch family	
	Probability of survival	Expected family size	Probability of survival	Expected family size
0	1.000	1.000	1.000	3.000
1	0.620	0.949	0.945	2.847
2	0.452	0.901	0.836	2.702
3	0.354	0.855	0.731	2.564
4	0.289	0.811	0.641	2.433
5	0.243	0.770	0.566	2.309
10	0.126	0.592	0.333	1.777
25	0.037	0.285	0.105	0.854
50	0.008	0.077	0.024	0.024
150	≈ 0.000	≈ 0.000	≈ 0.000	≈ 0.000

a female line eventually reaches extinction depends on the average number of daughters born to women in the family, the gross reproduction rate of below 1 implies that a UK family traced from an average mother along the female line would eventually become extinct.

Satchell and Thorp (2011) showed how the theory of branching processes and birth statistics can be used to estimate family survival functions. The pattern of family survival depends on overall fertility, the probabilities of particular numbers of births and the number of branches in the original family. Table 12.1 reports the estimated probability of survival and expected family size (in terms of one gender only) using average fertility patterns of mothers born in England and Wales in 1960. The data shows that, while not expected to survive forever, families have non-zero probabilities of survival over several hundred years. The three-branch family in the table is not expected to have a survival probability of less than 50% for six generations. Another interesting implication of the model, explored in Satchell and Thorp, is that optimal spending plans by family foundations and trusts will vary with survival prospects, being optimally higher in years when survival is more likely. For a single-branch family, this result implies a hyperbolic discount function, while for a multiple-branch family the function is non-monotonic.

Modelling in the remainder of this chapter assumes a constant discount factor since including a time-varying discount rate would rule out analytical solutions. While the assumption of constant discounting is a simplification of survival prospects, assuming an infinite horizon is a reasonable approximation to the very long horizon of any multiple-branch family aiming for sustainable inter-generational wealth transfers.

ANALYTICAL APPROACHES TO OPTIMAL DISBURSEMENT RATES

The standard problem for intertemporal utility maximisation is to find the optimal functional form for consumption and the set of asset allocations that will maximise the expected multi-period utility of wealth through time. Here, the decision-maker is infinitely lived but makes annual consumption plans. Proofs for all the propositions that follow appear in Appendix A.

Recursive utility

Giovannini and Weil (1989) and Weil (1990) find the closed-form solution for the optimal consumption path of an infinitely lived entity that maximises a discrete-time recursive utility function.¹⁰ The aggregator function for utility has two arguments; the first represents the value of current consumption and the second represents expected future utility over uncertain future consumption

$$\begin{aligned}
 L_t &= U[C_t, E_t L_{t+1}] \\
 &\equiv \frac{1}{(1-\delta)(1-\alpha)} \\
 &\quad \times \{(1-\delta)C_t^{1-\rho} \\
 &\quad + \delta[1 + (1-\delta)(1-\alpha)E_t L_{t+1}]^{(1-\rho)/(1-\alpha)}\}^{(1-\alpha)/(1-\rho)} - 1
 \end{aligned}
 \tag{12.2}$$

where $\delta \in (0, 1)$, $\alpha > 0$ and $\rho > 0$, and where C_t is consumption in the form of spending by beneficiaries and costs.

Time preference in Equation 12.2 is represented by the aggregator function, so that the derivative of Equation 12.2 with respect to expected future utility can be viewed as a subjective discount factor. If the aggregator is convex with respect to expected future utility, the agent prefers early resolution of uncertainty, or safety over stability. If Equation 12.2 is concave with respect to its second argument, then

the agent prefers a stable certainty equivalent path of future consumption. As Weil points out, δ is the subjective discount factor in the case of certainty and in the linear CRRA case where $\alpha = \rho$. It is straightforward to show that the convexity or concavity of the aggregator function depends on the relative sizes of α and ρ , being convex when $\alpha > \rho$ and concave when $\alpha < \rho$. Convexity implies more rapidly increasing patience and concavity more slowly increasing patience as expected future utility rises. Agents who are more risk tolerant and value smoothness ($\alpha < \rho$) prefer late resolution, and agents who dislike risk but tolerate larger swings in certainty equivalent utility ($\alpha > \rho$) prefer early resolution.

Another way to view the parameters of the model is to recognise that the coefficient of relative risk aversion for timeless gambles is α and the constant elasticity of intertemporal substitution for deterministic consumption paths is ρ . If either parameter approaches unity, then preferences become logarithmic in that dimension, so that we get logarithmic risk preferences when $\alpha \rightarrow 1$ and logarithmic intertemporal substitution preferences when $\rho \rightarrow 1$. Under the special case where $\alpha = \rho$, the utility function represents the preferences of an individual with CRRA and for whom the inverse of the risk aversion parameter is the elasticity of intertemporal substitution.¹¹

Wealth

The HNWI's optimisation problem also depends on the wealth generated by investment income and donations. The amount of money available for investment, I_t , is given by

$$I_t = W_t - C_t \quad (12.3)$$

where W_t is the wealth at time t . If I_t is invested in n assets, buying $N_{i,t}$ shares in the i th asset at a price $P_{i,t}$, then

$$I_t = \sum_{i=1}^n N_{i,t} P_{i,t} \quad (12.4)$$

If we define the random return to the i th asset as the random variable

$$\bar{z}_{i,t} \equiv \frac{\bar{P}_{i,t+1}}{P_{i,t}} \quad (12.5)$$

then the stochastic wealth of the charity at time $t + 1$ is

$$W_{t+1} = I_t \sum_{i=1}^n w_{i,t} \bar{z}_{i,t} \quad (12.6)$$

where $w_{i,t} \equiv N_{i,t} P_{i,t} / I_t$ represents the relative weights of the assets in the portfolio, so that $\sum_{i=1}^n w_{i,t} = 1$ and saving from wealth is fully invested in each period.

The charity consumes C_t by spending on administration and providing funding to beneficiaries. Setting aside questions of portfolio allocation, and assuming for now that no donation income is received, the budget constraint is

$$W_{t+1} = (W_t - C_t) \bar{Z}_t \quad (12.7)$$

where $\bar{Z}_t \equiv \sum_{i=1}^n w_{i,t} \bar{z}_{i,t}$ is the random growth in investments from t to $t + 1$. If $C_t = m_t W_t$, then Equation 12.7 is

$$W_{t+1} = (1 - m_t) W_t \bar{Z}_t \quad (12.8)$$

This gives us a difference equation

$$\begin{aligned} W_t &= W_0 \prod_{i=0}^{t-1} (1 - m_i) \bar{Z}_i \\ &= W_0 \bar{V}_{t-1} \prod_{i=0}^{t-1} (1 - m_i) \end{aligned} \quad (12.9)$$

where \bar{V}_{t-1} is the accumulated value of one unit of wealth invested at $t = 0$ and held until time t ; it is random and assumed to be non-negative.

Proposition 12.1. If \bar{Z}_i is a positive iid random variable and $\bar{Z}_i^{1-\alpha}$ is a well-defined random variable such that $\mathbb{E}(\bar{Z}_i^{1-\alpha}) = \varphi$ exists for $0 < \alpha < \infty$, it follows that $\mathbb{E}(\bar{V}_{t-1}^{1-\alpha}) = \varphi^t$ for all integer $t > 0$.

Income and new contributions

Many HNWIs and family offices rely entirely on investment income after a foundational business has been sold (Amit *et al* 2008). However, it is possible to generalise to the case where new sources of income or new contributions Y_t are received during the time period $t - 1$ to t but invested at the end of the period. Some family foundations may also receive charitable donations, for example. (Income received during the period cannot be invested in this discrete-time framework until the market opens in integer time.) This means that the wealth equation (Equation 12.8) needs to be adjusted to

$$W_t = (1 - m_t) W_t \bar{Z}_{t-1} + Y_{t-1} \quad (12.10)$$

Then

$$W_t = W_0 \bar{V}_{t-1} \prod_{i=0}^{t-1} (1 - m_i) + \sum_{j=0}^{t-1} Y_j \left(\frac{\bar{V}_{t-1}}{\bar{V}_j} \right) \prod_{i=0}^{t-1-j} (1 - m_i) \quad (12.11)$$

where \bar{V}_0 is assumed to equal 1.

It is apparent that no closed-form solution to the optimisation problem described by Equation 12.2 exists for additive income for general distributions. However, using the fact that new sources of income must be positive, they can be expressed as a multiplicative addition to wealth. Define the cumulative growth in income from new sources as a proportion of wealth $\bar{Y}_{t-1} = \prod_{i=0}^{t-1} \bar{y}_i$, and rewrite the wealth constraint as

$$W_t = W_0 \bar{V}_{t-1} \bar{Y}_{t-1} \prod_{i=0}^{t-1} (1 - m_i) \quad (12.12)$$

In this case the new interpretation of the risk-adjusted expected return to wealth, φ , is

$$\mathbb{E}(\bar{Z}_{t-1}^{1-\alpha} \bar{Y}_{t-1}^{1-\alpha}) = \mathbb{E}(\bar{Z}_{t-1}^{1-\alpha}) \mathbb{E}(\bar{Y}_{t-1}^{1-\alpha}) + \text{cov}(\bar{Z}_{t-1}^{1-\alpha}, \bar{Y}_{t-1}^{1-\alpha}) = \varphi \quad (12.13)$$

We now go on to solve the optimisation problem for consumption or the disbursement rate subject to the wealth process set out above.

Optimal consumption path

Giovannini and Weil (1989) and Weil (1990) show that the optimal disbursement rate will be a constant proportion of wealth when the horizon is infinite and the return to invested wealth \bar{Z}_i is positive iid.

Proposition 12.2 (Giovannini and Weil 1989; Weil 1990). The consumption-to-wealth ratio m that maximises aggregated utility (Equation 12.2) for $t = 0, \dots, \infty$, subject to the wealth constraint (Equation 12.9), where \bar{Z}_i is positive iid, is constant and given by

$$m = 1 - (\delta \varphi^{(1-\rho)/(1-\alpha)})^{1/\rho} \quad (12.14)$$

and the optimised value of Equation 12.2 is

$$L(W) = \frac{(\psi W)^{1-\alpha} - 1}{(1-\delta)(1-\alpha)} \quad (12.15)$$

for $\psi = [(1-\delta)m^{-\rho}]^{1/(1-\rho)}$.

For CRRA utility, when $\alpha = \rho$ the optimal disbursement rate simplifies to $m = 1 - (\delta \varphi)^{1/\alpha}$. In the case of logarithmic risk preferences when $\alpha \rightarrow 1$, the consumption-to-wealth ratio is myopic over investment risk even when the investment opportunity set is non-constant, and in the case of logarithmic intertemporal substitution preferences when $\rho \rightarrow 1$ the consumption-to-wealth ratio is constant for all values of α . Thus, in either logarithmic case, the optimal disbursement rate is independent of our assumption about $\mathbb{E}(\bar{V}_{t-1}^{1-\alpha})$. In other words, the consumption rate is independent of asset allocation, although the amount of wealth drawn down is not.¹²

The feasibility and dynamic stability of this plan can be ensured by placing conditions on model parameters. The dynamic spending plan in Equation 12.14 is feasible (satisfying strictly positive wealth and consumption constraints) when the rate of disbursement is positive so that $\delta \varphi^{(1-\rho)/(1-\alpha)} < 1$, or for the CRRA case, when $\delta \varphi < 1$. Since the (gross) return to wealth is always non-negative so that $\varphi \geq 0$, and given that the discount rate $\delta \in (0, 1)$, a consumption-to-wealth ratio strictly less than 1 is sufficient to ensure feasibility. Dynamic stability, such that the expected value of optimised utility is bounded at the infinite horizon, is also satisfied by $\delta \varphi < 1$ in the CRRA case, but the conditions for feasibility and dynamic stability do not always coincide in the non-linear recursive utility case.¹³ Proposition 12.3 sets out a sufficient condition for dynamic stability that utilises the binomial form of the aggregator function (Equation 12.2).

Proposition 12.3. Under Newton's Generalised Binomial Theorem (Graham *et al* 1994), the aggregator function in Equation 12.2 is the sum of a convergent infinite series if $m/(1-m) < 1$ so that the spending rate is less than the saving rate.

The proof of this proposition is set out in Appendix A, and begins by inserting the value function (Equation 12.15) into the aggregator function (Equation 12.2). An inspection of the result shows its similarity to a generalised binomial form. Newton's Generalised Binomial Theorem thus gives the criteria for convergence, which verifies the transversality condition, and it follows that the value function converges to zero in the limit.

The convergence condition $m/(1-m) < 1$ applies where the discounted value of expected future utility (the second argument in

the aggregator function) exceeds the value of current consumption (the first argument in the aggregator function), and amounts to the requirement that the optimal spending rate be less than the saving rate. If the reverse is true and the value of current consumption exceeds discounted expected utility, then the rate of spending needs to exceed the rate of saving to achieve dynamic stability. For most of the empirical applications to follow, where the optimal spending response to moderate changes to the investment outlook is modelled, the spending rate must be less than the saving rate. This condition is equivalent to $m < \frac{1}{2}$, which is not a binding constraint for most conventional parameterisations of the problem.

The conditions for optimal portfolio selection for this problem are well known and we do not repeat them here. Importantly, when returns are iid, portfolio choice is dependent only on tastes for risk, not preferences over intertemporal substitution, and does not depend on expectations of the future consumption path.¹⁴

In the following sections, we take advantage of this separation and treat the portfolio allocation as predetermined (although not necessarily optimal). But, for any given asset allocation, however determined, it is possible to calculate the impact on the ideal disbursement rate caused by changes in the distribution of future returns, risk attitudes and/or portfolio weights. The scenario analysis is given below (see p. 248).

EMPIRICAL ILLUSTRATION

The empirical implications of the explicit solution for the optimal consumption-to-wealth ratio (Equation 12.14) can be illustrated using a simulated sample of portfolio returns. Amit *et al* (2008) report that HNWI and family offices hold capital in public and private equity, hedge funds, real estate, fixed income, commodities and collectibles. The actual holdings of HNWI are not available, so we construct a returns series using an asset allocation similar to the Wellcome Trust in 2005 as a long-lived trust established by an HNWI.¹⁵ Figure 12.1 shows the proportions of total funds invested in each asset class for simulated portfolio returns and these weights are fixed for the whole sample period.¹⁶

The simulated data are monthly real portfolio returns over the period January 1990 to June 2006 (198 observations), based on individual asset class returns from standard indexes and deflated using

Figure 12.1 Asset allocation of simulated portfolio

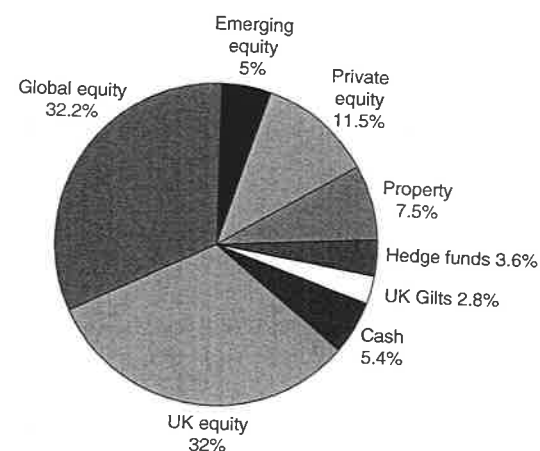


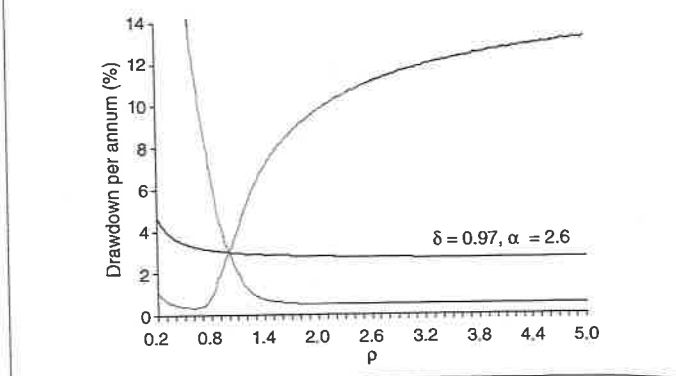
Table 12.2 Summary statistics, real annualised portfolio returns, January 1990–June 2006

Mean (%)	4.75
Standard deviation (%)	13.02
Skewness	-0.69
Kurtosis	3.88
Jarque–Bera	21.87
(p-value)	(0.000)

consumer prices and earnings data. It is reasonable to expect that wages are an important cost for a family office, and deflation using consumer prices alone will overstate real spending power, so inflation is computed as 50% consumer price driven and 50% purely due to wage increases.

The mean (log) real annualised return to this portfolio is 4.75% with volatility of 13% per year. Summary statistics in Table 12.2 show that the (monthly) data is significantly non-normal: negatively skewed and leptokurtic. However, the autocorrelation structure of the de-meaned returns and squared de-meaned returns supports an assumption that real portfolio returns are iid. Ljung–Box *Q* statistics, not reported here, are insignificant to at least 50 lags for the

Figure 12.2 Optimal disbursement rate



de-meaned returns. The squared residuals have one significant autocorrelation at lag 10.

Equation 12.14 is the optimal rate of disbursement rate for an infinitely lived charity under a fixed asset allocation, given time preference parameter δ , consumption smoothing parameter ρ and relative risk aversion α . Another key determinant is the mean of the risk-aversion-scaled portfolio return, $\mathbb{E}(\tilde{Z}_i^{1-\alpha}) = \varphi$. To estimate φ , the monthly portfolio returns were bootstrapped using 120,000 random draws, then summed to get 10,000 annual real (gross) returns and the sample mean was computed

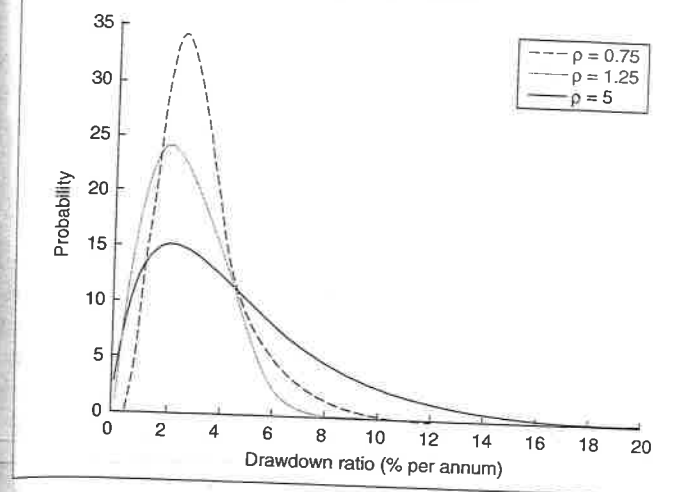
$$\hat{\varphi} = \frac{1}{j} \sum_{i=1}^j \tilde{Z}_i^{1-\alpha} j = 10,000 \quad (12.16)$$

for $\delta = 0.97$ and $\alpha > 0$.

The estimated optimal disbursement rate \hat{m} is shown in Figure 12.2 for values of the intertemporal substitution parameter ρ between 0.2 and 5, and with risk aversion $\alpha = 2.6$. This value for risk aversion is estimated from the portfolio weights of the Wellcome Trust, assuming that the portfolio is optimal.¹⁷ The estimated risk aversion parameter is only indicative, and serves as a reference point for numerical illustrations.

The light grey curves in Figure 12.2 give an approximate 95% error range for the estimate of m . A beta distribution is fitted to 1,000 bootstrapped estimates of $\hat{v}_\alpha := (\delta \hat{\varphi}^{(1-\rho)/(1-\alpha)})^{1/\rho}$ by maximum likelihood, after filtering out values that do not meet the feasibility and

Figure 12.3 Disbursement rate error distribution



boundary conditions. From the estimated beta parameters, quantiles $1 - \hat{v}_{\alpha,0.025} = 1 - F^{-1}(0.025)$ and $1 - \hat{v}_{\alpha,0.975} = 1 - F^{-1}(0.975)$ can be inferred as a guide to the accuracy of \hat{m} . Consistent with the solution for logarithmic intertemporal substitution preferences, the optimal consumption rate, $\hat{m} = (1 - \delta)$, is 3% per annum when $\rho = 1$. As tolerance for consumption transfer through time decreases and ρ increases, the disbursements fall from around 4.7% when $\rho = 0.2$, reaching 2.8% when $\rho = 5$.¹⁸

The error range around \hat{m} widens rapidly as the elasticity of intertemporal substitution (EIS) diverges from 1 in either direction. Figure 12.3 graphs the estimated beta distributions of the optimal disbursement rate at three indicative values of the intertemporal substitution parameter. When the EIS is relatively high, at 1.33 ($\rho = 0.75$), the error distribution is more right-skewed and dis-bursed than when the EIS falls to 0.8 ($\rho = 1.25$), where the distribution is more tightly packed around the 3% logarithmic disbursement rate. However, as the EIS moves away from 1, falling to 0.2 ($\rho = 5$), the probability distribution becomes more right-skewed again, and uncertainty over the optimal spending rate increases. This pattern indicates the increasing importance of the stochastic risk-scaled-returns parameter φ to optimal consumption paths as

the EIS diverges from 1, since at $\rho = 1$ consumption depends only on the discount parameter δ , which is assumed to be known with certainty.

Hence, a moderately risk averse HNWI will spend between 5% and 2% of wealth each year, but the uncertainty surrounding that optimal solution is very large and increasing as the EIS diverges from 1.

SCENARIO ANALYSIS

HNWIs need a way of assessing whether their chosen disbursement rate is robust to changes in beliefs about future returns, an exercise usually called scenario analysis. A natural approach is to set past history as the benchmark and build optimistic or pessimistic outlooks. Alternatively, a range of drawdown rates for a cross-section of foundations or family offices with different beliefs about the returns distribution could be estimated.

In this section we set out a simple procedure to conduct scenario analysis that is not highly dependent on specific assumptions about distributions of returns. The analysis directly connects the desired consumption-to-wealth ratio with stochastic dominance properties of alternative returns distributions.

The influence of the returns distribution on optimal spending rates is via the expectation of risk-scaled portfolio returns, φ . To gauge the optimal spending response to optimistic and pessimistic investment scenarios, consider changes in the expected risk-scaled portfolio return φ , keeping constant tastes for risk, α , and intertemporal substitution, ρ , fixed but varying general distributional characteristics. The change in optimal disbursement rate as φ varies depends on the relative sizes of α and ρ

$$\frac{\partial m}{\partial \varphi} = -\delta^{1/\rho} \frac{(1-\rho)}{(1-\alpha)} \varphi^{(1-\rho)/((1-\alpha)\rho)-1} \quad (12.17)$$

Since α , ρ and φ are positive, the response of the optimal disbursement rate to an increase in φ will be positive when $\rho > 1$ and $\alpha < 1$, and when $\rho < 1$ and $\alpha > 1$. If both α and ρ are greater than 1 or less than 1, then the response of the optimal disbursement rate to an increase in φ will be negative. However, the influence of relative risk aversion on φ itself needs to be accounted for. It turns out that this can be done using the properties of stochastic dominance.

Table 12.3 Revisions to drawdown rates under first-order transformations

Risk aversion	Intertemporal substitution parameter	
	$0 < \rho < 1$	$\rho > 1$
$0 < \alpha < 1$	$\frac{\partial G}{\partial Z} > 0 \Rightarrow \varphi \text{ increases}$	$\frac{\partial G}{\partial Z} > 0 \Rightarrow \varphi \text{ increases}$
	$\frac{\partial m}{\partial \varphi} < 0 \Rightarrow m \text{ decreases}$	$\frac{\partial m}{\partial \varphi} > 0 \Rightarrow m \text{ increases}$
$\alpha > 1$	$\frac{\partial G}{\partial Z} < 0 \Rightarrow \varphi \text{ decreases}$	$\frac{\partial G}{\partial Z} < 0 \Rightarrow \varphi \text{ decreases}$
	$\frac{\partial m}{\partial \varphi} > 0 \Rightarrow m \text{ decreases}$	$\frac{\partial m}{\partial \varphi} < 0 \Rightarrow m \text{ increases}$

First-order stochastic dominance

Proposition 12.4. If Z_i^Δ first-order stochastically dominates Z_i , then φ is increased if $0 < \alpha < 1$ and decreased if $\alpha > 1$.

First-order stochastic dominance (FSD) implies that $E_\Delta[G(Z)] \geq E[G(Z)]$ for $G(\cdot)$ any increasing function. Now apply the result for $\varphi = E(\tilde{Z}^{1-\alpha})$, to see that $G(Z) = Z^{1-\alpha}$ is positive increasing for $0 < \alpha < 1$ (hence, $\varphi_\Delta > \varphi$), and positive decreasing for $\alpha > 1$, so that $\varphi_\Delta < \varphi$, where Δ is an FSD transformation.

Consider now the change in m under an FSD shift $F_\Delta(Z)$, for each of four combinations of values for relative risk aversion and the elasticity of intertemporal substitution. Two effects interact to determine the response of the consumption-to-wealth ratio to FSD transformations of the returns density. The first depends on the properties of the function $G(Z)$, and the second on the sign of the derivative of m with respect to φ given in Equation 12.17. The outcomes are summarised in Table 12.3.

So, regardless of the size of the relative risk aversion parameter, transformations of the returns distribution that are described by first-order stochastic dominance mean a decrease in the consumption-to-wealth ratio whenever $0 < \rho < 1$ and an increase in the consumption-to-wealth ratio when $\rho > 1$. Weil (1990) showed this result for lognormally distributed portfolio returns, but here it is generalised to the case of any well-behaved continuous returns distribution. The former case, $0 < \rho < 1$ fits decision-makers with high elasticities of intertemporal substitution, and the latter case,

$\rho > 1$, agents with low elasticities of intertemporal substitution. For optimistic returns scenarios and where the willingness to transfer consumption over time is high, the substitution effect dominates the income effect and the HNWI reduces spending rates, whereas for HNWI with low elasticities of intertemporal substitution, the income effect dominates the substitution effect, and they increase spending rates. These effects are independent of tastes for risk when returns are iid.

Now consider reshaping the returns distribution to reflect optimistic scenarios for investment that are consistent with FSD transformations. For optimistic outlooks, the aim is to make extremely poor returns unlikely relative to the recent past by shifting tail mass from the left tail to the right tail of the distribution. For an arbitrary positive continuous density, $\text{pdf}(x)$ we consider two points x_1 and x_u and the probabilities

$$P_1 = \int_0^{x_1} \text{pdf}(x) dx, \quad P_u = \int_{x_u}^{\infty} \text{pdf}(x) dx, \quad P_{md} = \int_{x_1}^{x_u} \text{pdf}(x) dx \quad (12.18)$$

and clearly $P_u + P_1 + P_{md} = 1$.

Construct a new density by the following shift

$$P'_u = P_u + \Delta, \quad P'_1 = P_1 - \Delta, \quad P'_{md} = P_{md} \quad (12.19)$$

where $0 < \Delta < \min(P_u, P_1)$ and

$$\text{pdf}'(x) = \begin{cases} \frac{P'_u \text{pdf}(x)}{P_u} & \text{for } x_u < x < \infty \\ \text{pdf}(x) & \text{for } x_1 < x < x_u \\ \frac{P'_1 \text{pdf}(x)}{P_1} & \text{for } 0 < x < x_1 \end{cases}$$

It is easy to check that $\text{pdf}'(x)$ is still a well-defined density, although it is no longer continuous at $x = x_1$ or $x = x_u$. Note that since a continuous density with zero probability mass at any point was assumed, the discontinuities induced by our transformation will not affect the existence of the integrals. Furthermore, the above transformation can be called "optimistic" in that it transfers probability from the lower tail to the upper tail of the density, while a "pessimistic" transformation does the reverse. For an optimistic transformation

$$\int_0^{\infty} \text{pdf}'(x) dx < \int_0^{\infty} \text{pdf}(x) dx$$

which satisfies FSD, and the following corollary holds.

Corollary 12.5. If $G(x)$ is a positive increasing function, then

$$\int_0^{\infty} G(x) \text{pdf}'(x) dx > (<) \int_0^{\infty} G(x) \text{pdf}(x) dx, \quad (12.21)$$

for $\text{pdf}'(x)$ the result of an optimistic (pessimistic) transformation, respectively. The opposite result applies to positive decreasing functions.

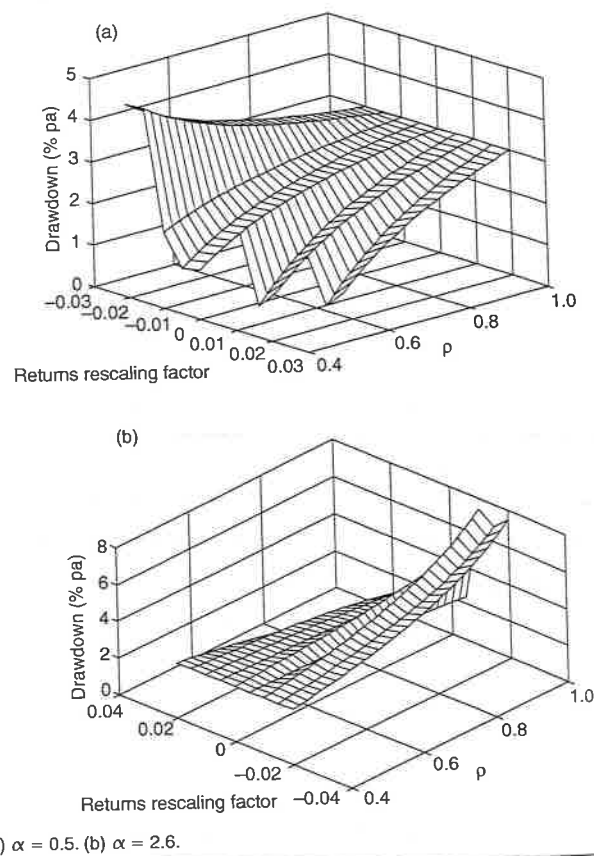
Empirical illustration

Figures 12.4 and 12.5 show the impact on the optimal spending rate of a range of transformations of the distribution of \tilde{Z}_t , the portfolio return. Panels (a) and (b) in Figure 12.4 show graphs for the optimal spending rate when $\alpha = 0.5$ and $\alpha = 2.6$, respectively, and ρ ranges from 0.4 to 1. A positive rescaling of the returns distribution of size, say, 0.02, shifts 2% of the total probability mass from the left tail to the right tail of the distribution and matches an optimistic outlook for investment returns. In the same way, a negative rescaling of 0.02 shifts the same probability mass from the right tail to the left tail, when the investment outlook is bleak. Whenever $\rho = 1$ the optimal spending rate is 3% per year, but, as ρ shrinks, EIS increases and spending falls with optimistic expectations and rises with pessimistic expectations.¹⁹ A very flexible foundation or family office facing better prospects does best by decreasing current spending rates in favour of future consumption, with substitution effects dominating income effects.

For HNWI, foundations or family offices with low elasticities of intertemporal substitution, where $\rho > 1$, optimistic transformations of the portfolio returns distribution increase the optimal disbursement rate, as they enjoy higher income in the current period rather than favouring future consumption. Figure 12.5 graphs changing spending rates as optimism increases and EIS decreases.

Table 12.4 shows specific examples of the numerical scale of changes in disbursement rates. While the size of the EIS relative to 1 determines the direction of revisions to disbursement rates, relative risk aversion influences the scale of the change. When $\rho = 2$ (EIS = 0.5) and $\alpha = 2.6$, for example, optimal spending at the historical average return is 2.8% per year. Reducing the probability of left tail returns by 4 percentage points raises spending by 180 basis points (bp) from 2.8% to 4.6% per year. The same size shift in the direction of pessimism reduces spending by 240bp from 2.8% to

Figure 12.4 Optimal disbursement rate under transformations of the portfolio returns distribution, $0 < \rho \leq 1$

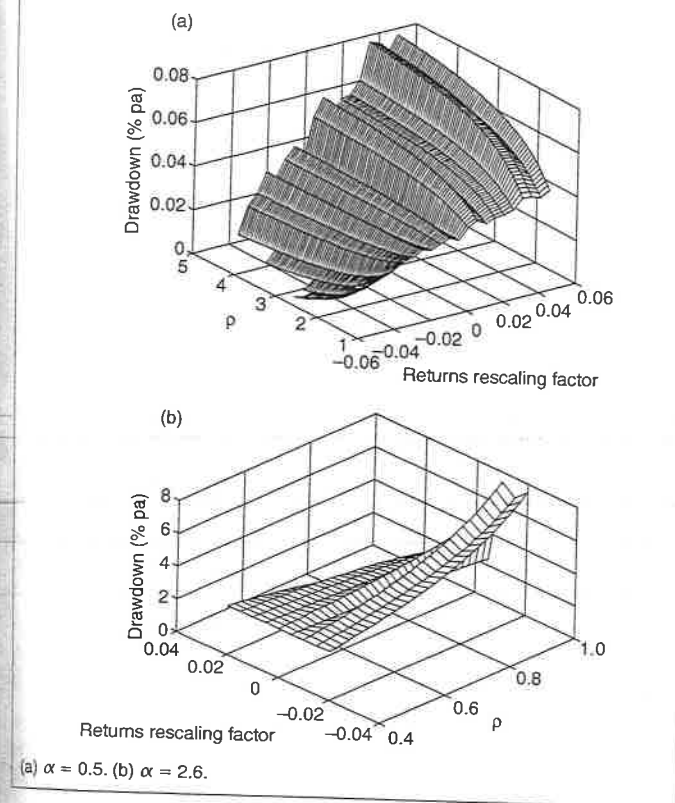


0.4% per year. For lower risk aversion, the revisions to benchmark spending are an increase of 220bp for the optimistic scenario and a decrease of 180bp for the pessimistic scenario.

Second-order stochastic dominance

The first discussion considered first-order transformations of the returns distribution. Now we consider second-order changes.

Figure 12.5 Optimal disbursement rate under transformations of the portfolio returns distribution, $1 \leq \rho \leq 5$



Proposition 12.6. If Z_i^ω second-order stochastically dominates Z_i , then φ is increased if $0 < \alpha < 1$ and decreased if $\alpha > 1$.

Second-order stochastic dominance (SSD) implies that

$$E_\omega[G(Z)] \geq E[G(Z)]$$

for $G(\cdot)$ any increasing, concave function. Applying this result for $\varphi = E(Z^{1-\alpha})$, we see that $G(Z) = Z^{1-\alpha}$ is positive increasing and concave for $0 < \alpha < 1$ (hence, $\varphi_\omega > \varphi$), and positive decreasing and convex for $\alpha > 1$, so that $\varphi_\omega < \varphi$, where ω is an SSD transformation.

Table 12.4 Numerical example of changes to drawdown rates (FSD)

(a) $\rho = 2$, EIS = 0.5			
RRA	Benchmark (%) $\Delta = 0.00$	Optimistic (%) $\Delta = 0.04$	Pessimistic (%) $\Delta = -0.04$
$\alpha = 2.6$	2.8	4.6	0.4
$\alpha = 0.5$	3.5	5.7	0.7

(b) $\rho = 0.8$, EIS = 1.25			
RRA	Benchmark (%) $\Delta = 0.00$	Optimistic (%) $\Delta = 0.04$	Pessimistic (%) $\Delta = -0.04$
$\alpha = 2.6$	3.1	1.9	4.3
$\alpha = 0.5$	2.7	1.9	3.6

The change in m for each of four combinations of α and ρ , given an optimistic transformation of the returns density, is summarised in Table 12.5. If our transformation creates SSD over the original distribution, then the optimal consumption-to-wealth ratio m decreases whenever $0 < \rho < 1$. If $\rho > 1$, SSD implies the opposite effect, where m increases as risk shrinks and decreases as risk rises (for a constant expected return). This result confirms the reasoning in Weil (1990) that responses to mean-preserving spreads of the returns distribution depend only on the value of ρ , but it is shown that this result holds for any second-order stochastic dominance transformation of the returns distribution.

To illustrate the result, consider a mean-preserving spread of the distribution as a special case of SSD. For an arbitrary positive continuous density, $\text{pdf}(x)$, where $x_i = \mu_x + \varepsilon_i$, $\varepsilon_i \sim \text{iid}(0, \sigma_\varepsilon^2)$, construct a mean-preserving spread by the following transformation of x_i

$$x'_i = \mu_x + (1 + \omega)\varepsilon_i, \quad 0 < \omega < \infty \quad (12.22)$$

The mean of both distributions is

$$\mathbb{E}(x'_i) = \mathbb{E}(x_i) = \mu_x \quad (12.23)$$

and, for $0 < \omega < \infty$, the variance of the transformed variable x'_i is greater than the variance of x_i

$$\text{var}(x'_i) = (1 + \omega)^2 \sigma_\varepsilon^2 > \text{var}(x_i), \quad (12.24)$$

Table 12.5 Revisions to drawdown rates under second-order transformations

Risk aversion	Intertemporal substitution parameter	
	$0 < \rho < 1$	$\rho > 1$
$0 < \alpha < 1$	$\frac{\partial G}{\partial Z} > 0, \frac{\partial^2 G}{\partial Z^2} < 0 \Rightarrow \varphi \text{ increases}$ $\frac{\partial m}{\partial \varphi} < 0 \Rightarrow m \text{ decreases}$	$\frac{\partial G}{\partial Z} > 0, \frac{\partial^2 G}{\partial Z^2} < 0 \Rightarrow \varphi \text{ increases}$ $\frac{\partial m}{\partial \varphi} > 0 \Rightarrow m \text{ increases}$
$\alpha > 1$	$\frac{\partial G}{\partial Z} < 0, \frac{\partial^2 G}{\partial Z^2} > 0 \Rightarrow \varphi \text{ decreases}$ $\frac{\partial m}{\partial \varphi} > 0 \Rightarrow m \text{ decreases}$	$\frac{\partial G}{\partial Z} < 0, \frac{\partial^2 G}{\partial Z^2} > 0 \Rightarrow \varphi \text{ decreases}$ $\frac{\partial m}{\partial \varphi} < 0 \Rightarrow m \text{ increases}$

Table 12.6 Numerical example of changes to drawdown rates (SSD)

(a) $\rho = 2$, EIS = 0.5			
RRA	Benchmark (%) $\omega = 0.00$	Optimistic (%) $\omega = -0.5$	Pessimistic (%) $\omega = 0.5$
$\alpha = 2.6$	2.8	3.5	1.3
$\alpha = 0.5$	3.5	3.7	3.4

(b) $\rho = 0.8$, EIS = 1.25			
RRA	Benchmark (%) $\omega = 0.00$	Optimistic (%) $\omega = -0.5$	Pessimistic (%) $\omega = 0.5$
$\alpha = 2.6$	3.1	2.7	3.8
$\alpha = 0.5$	2.7	2.6	2.8

These are sufficient conditions for the second-order stochastic dominance of $\text{pdf}(x)$ over $\text{pdf}'(x)$. The variance of x_i can be shrunk by choosing an optimistic transformation such that $-1 < \omega < 0$, so that the transformed distribution $\text{pdf}'(x)$ dominates the original distribution, $\text{pdf}(x)$.

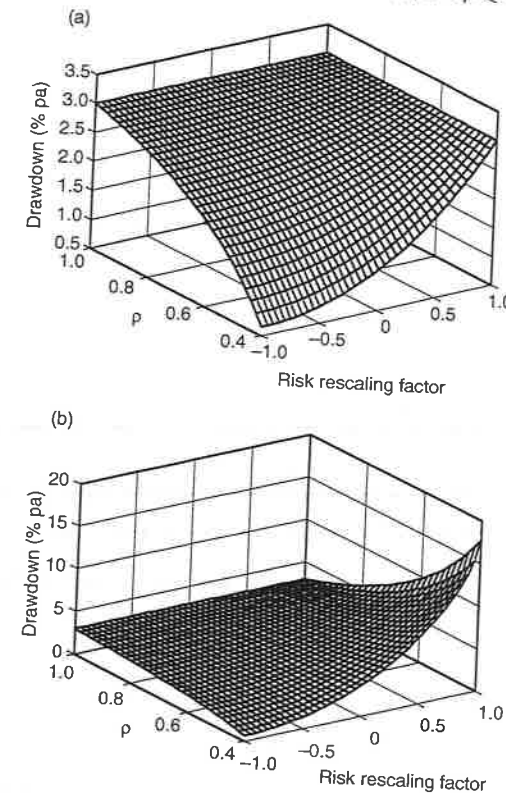
Corollary 12.7. If $G(x)$ is a positive increasing, concave function, then

$$\int_0^\infty G(x) \text{pdf}'(x) dx > (<) \int_0^\infty G(x) \text{pdf}(x) dx \quad (12.25)$$

for $\text{pdf}'(x)$ the result of an optimistic (pessimistic) transformation. The opposite result applies to positive decreasing, convex functions.

Empirical illustration

Figures 12.6 and 12.7 graph the optimal disbursement rate when the variance, but not the mean, of the distribution of \tilde{Z}_i is increased or decreased. In Figure 12.6 the standard deviation is shrunk from its historical value to almost zero (rescaling to -1), or pessimistically raised to twice the historical size (rescaling to 1), while setting $\alpha = 0.5$ or $\alpha = 2.6$, and allowing ρ to range from 0.4 to 1 . For HNWI with low elasticities of intertemporal substitution, when $\rho > 1$, increases in risk lower optimal spending rates, with the effect becoming more dramatic as EIS shrinks. Figure 12.7 graphs these changing spending rates as optimism over volatility increases and EIS decreases.

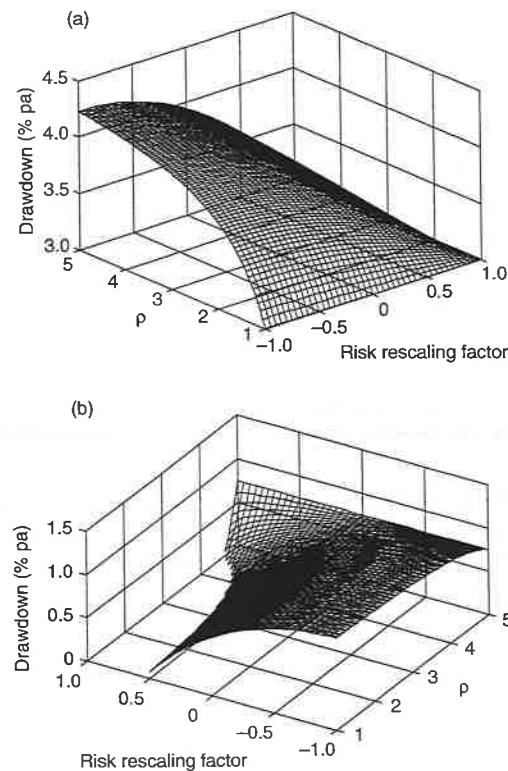
Figure 12.6 Optimal disbursement rate under mean-preserving spread transformations of the portfolio returns distribution, $0 < \rho \leq 1$ 

(a) $\alpha = 0.5$. (b) $\alpha = 2.6$.

Numerically, changes in spread create relatively small revisions to the disbursement rates, as can be seen in Table 12.6. Again the direction of changes depends on the EIS, but the size and original benchmark are also influenced by risk aversion.

For $\rho = 2$ and $\alpha = 0.5$, the optimal spending rate based on historical returns is around 3.5%, and shrinking volatility by 50% causes a small increase in disbursements towards 3.7%, while increasing volatility by 50% decreases disbursements by about the same amount. For $\rho = 2$ and $\alpha = 2.6$, the same changes on the optimistic

Figure 12.7 Optimal disbursement rate under mean-preserving spread transformations of the portfolio returns distribution, $1 \leq \rho \leq 5$



(a) $\alpha = 0.5$. (b) $\alpha = 2.6$.

side raise spending by 70bp and on the pessimistic side decrease spending by 150bp.

Lower current spending as a reaction to improved prospects is not necessarily irrational or irresponsible. On the contrary, such episodes could be evidence for a high level of willingness to transfer disbursements into the future. However, if an HNWI favours smoother consumption, then unwillingness to shift consumption towards the future dominates, and optimal spending rises and falls as the outlook brightens or blackens. Somewhat surprisingly, this is true whatever

the degree of risk aversion. Preferences for early or late resolution of uncertainty do not determine the direction of response. While the benchmark level of spending, m , will be sensitive to both risk aversion and the intertemporal elasticity, whether spending decreases or increases from that level in response to scenario changes depends only on whether the elasticity of intertemporal substitution is less than or greater than 1.

CONCLUSION

In this chapter we built and solved a model of the ideal constant disbursement rate for a foundation or trust. The specific features of management decisions for an HNWI or family office were built in. First, the ideal rate of spending depends on preferences for safety and smoothness in expected consumption, tastes which can be represented in an EZW utility framework. The EZW, or recursive, utility separates risk tolerance from intertemporal consumption preferences, so that if returns to investment are iid, then the asset allocation and consumption decisions are separable, and spending rates can be treated as contingent on a pre-set portfolio. Descriptions of the governance structures of family offices and foundations indicate that investment choices are not always made simultaneously with choice of spending rates, so flexibility between spending and investment planning are critical.

The ideal spending rate depends on investment returns and risk, the risk preferences of the decision-maker and their capacity for transferring consumption from the present to the future, or the elasticity of intertemporal substitution. Using a simulated returns distribution, we derived an ideal spending rate of 3% per annum when the elasticity of intertemporal substitution was $\rho = 1$. As tolerance for consumption transfer through time decreases and ρ increases, the disbursements fall from around 4.7% when $\rho = 0.2$ to 2.8% when $\rho = 5$. However, this result is fixed for stipulated investment returns.

In order to investigate the responsiveness of disbursement policies to changes in the shape of very general returns distributions, we considered scenario analysis. The effects of optimistic and pessimistic transformations of the returns distribution were identified using the properties of stochastic dominance. Analytical results were derived for revisions to expected returns (FSD) and for revisions to diffusion of returns (SSD).

Without assuming a specific functional form for the probability density, the effects on optimal spending due to a transfer of probability mass from the lower to the upper tail (FSD), and vice versa, and the effects of mean-preserving spread (SSD) were derived, incorporating important idiosyncratic features of actual returns distributions. These experiments can represent either revisions to the beliefs of an HNWI, or a cross-section of beliefs about investment opportunities from a sample of such individuals or families.

While the optimal drawdown rate depends on both tastes for risk and the elasticity of intertemporal substitution (EIS), scenario analysis shows that whether optimal spending rates increase or decrease in response to first and second-order dominance changes in returns depends entirely on the EIS. Whenever the EIS is less than 1, income effects dominate substitution effects, and optimistic changes to returns (FSD and/or SSD) raise current spending. The reverse holds when the EIS is greater than 1, and when the EIS is unitary spending rates are immune to revision and depend only on time preference.

Foundations, trusts and family offices have been treated here as always having an interest in future regardless of time horizon. This approach is an approximation for an HNWI or family office for which survival is at least likely into the distant future, even though eventual extinction is inevitable, as discussed above. Moreover, the importance of internal family relationships have been subsumed into an assumption of unitary preferences, which is very likely an over-simplification. Amit *et al* (2008) report that single family offices (SFOs), especially later generation SFOs, commonly perform family education, counselling services and relationship management, emphasising the limitations of the framework used here.

APPENDIX A

Proof of Proposition 12.1 Since \tilde{Z}_i is iid, $\tilde{Z}_i^{1-\alpha}$ is iid and

$$\begin{aligned}\mathbb{E}_0(\tilde{V}_{t-1}^{1-\alpha}) &= \mathbb{E}\left[\left(\prod_{i=0}^{t-1} \tilde{Z}_i\right)^{1-\alpha}\right] = \mathbb{E}\left[\prod_{i=0}^{t-1} \tilde{Z}_i^{1-\alpha}\right] \\ &= \prod_{i=0}^{t-1} \mathbb{E}(\tilde{Z}_i^{1-\alpha}) = \prod_{i=0}^{t-1} \varphi = \varphi^t\end{aligned}$$

□

Proof of Proposition 12.2 Substituting Equation 12.15 into Equation 12.2 and using the expressions for consumption and the wealth constraint (Equation 12.9) gives

$$\begin{aligned}L_t &= \frac{1}{(1-\delta)(1-\alpha)} \\ &\quad \times \{(1-\delta)(m_t W_t)^{1-\rho} \\ &\quad + \delta[\mathbb{E}_t(\psi \tilde{Z}_t (1-m_t) W_t)^{1-\alpha}]^{(1-\rho)/(1-\alpha)}\}^{(1-\alpha)/(1-\rho)} - 1\end{aligned}\quad (12.26)$$

Maximising Equation 12.26 over m_t is the same as maximising over consumption, and gives the first-order condition as a function of ψ

$$\begin{aligned}\frac{\partial L_t}{\partial m_t} &= (1-\rho)(1-\delta)m_t^{-\rho} W_t^{1-\rho} \\ &\quad - (1-\rho)\delta\psi^{1-\rho}[\mathbb{E}_t(\tilde{Z}_t^{1-\alpha})]^{(1-\rho)/(1-\alpha)}(1-m_t)^{-\rho} W_t^{1-\rho} \\ &= 0\end{aligned}\quad (12.27)$$

Rearranging Equation 12.27 gives

$$m_t = \left\{1 + \left[\frac{\delta}{1-\delta}\psi^{1-\rho}([\mathbb{E}_t(\tilde{Z}_t^{1-\alpha})]^{1/(1-\alpha)})^{1-\rho}\right]^{1/\rho}\right\}^{-1}\quad (12.28)$$

If $\psi = [(1-\delta)m_t^{-\rho}]^{1/(1-\rho)}$, then Equation 12.28 becomes

$$m_t = \left\{1 + \left[\frac{\delta}{1-\delta}(1-\delta)m_t^{-\rho}([\mathbb{E}_t(\tilde{Z}_t^{1-\alpha})]^{1/(1-\alpha)})^{1-\rho}\right]^{1/\rho}\right\}^{-1}\quad (12.29)$$

and rearranging confirms that

$$m_t = 1 - [\delta([\mathbb{E}_t(\tilde{Z}_t^{1-\alpha})]^{1/(1-\alpha)})^{1-\rho}]^{1/\rho}\quad (12.30)$$

and if $\mathbb{E}_t(\tilde{Z}_t^{1-\alpha}) = \mathbb{E}(\tilde{Z}_t^{1-\alpha}) = \varphi$ then

$$m_t = m = 1 - [\delta(\varphi^{(1-\rho)/(1-\alpha)})]^{1/\rho}$$

□

Proof of Proposition 12.3 Newton's Generalised Binomial Theorem states that, for any $r \in \mathbb{R}$, if $|a| < 1$, then

$$\sum_{t \geq 0} \binom{r}{t} a^t$$

converges to $(1+a)^r$. This result implies, for $a = x/y$

$$(y+x)^r = \sum_{t \geq 0} \binom{r}{t} y^{r-t} x^t = \sum_{t \geq 0} \binom{r}{t} a^t = (1+a)^r\quad (12.31)$$

Using the aggregator function (Equation 12.2) and substituting the value function (Equation 12.15), we obtain

$$\begin{aligned}
 L_t &= \frac{1}{(1-\delta)(1-\alpha)} \\
 &\quad \times \{(1-\delta)C_t^{1-\rho} \\
 &\quad + \delta[\mathbb{E}_t\{[(1-\delta)m^{-\rho}]^{1/(1-\rho)}W_{t+1}^{1-\rho}\}^{(1-\rho)/(1-\alpha)}\}^{(1-\alpha)/(1-\rho)} \\
 &\quad - 1 \\
 &= \frac{1}{(1-\delta)(1-\alpha)} \\
 &\quad \times \{(1-\delta)C_t^{1-\rho} \\
 &\quad + \delta[(1-\delta)m^{-\rho}][\mathbb{E}_t(W_{t+1}^{1-\rho})^{(1-\rho)/(1-\alpha)}]^{(1-\alpha)/(1-\rho)} - 1 \\
 &= \frac{1}{(1-\delta)(1-\alpha)} \\
 &\quad \times \{(1-\delta)m^{1-\rho}W_t^{1-\rho} \\
 &\quad + \delta(1-\delta)m^{-\rho}(1-m)^{1-\rho}W_t^{1-\rho}\varphi^{(1-\rho)/(1-\alpha)}\}^{(1-\alpha)/(1-\rho)} - 1
 \end{aligned}
 \tag{12.32}$$

and Equation 12.32 will be the convergent sum of the generalised binomial expansion above if

$$\begin{aligned}
 a &= \frac{x}{y} = \frac{(1-\delta)m^{1-\rho}W_t^{1-\rho}}{\delta(1-\delta)m^{-\rho}(1-m)^{1-\rho}W_t^{1-\rho}\varphi^{(1-\rho)/(1-\alpha)}} \\
 &= \frac{m}{1-m} < 1
 \end{aligned}$$

and $r = (1-\alpha)/(1-\rho)$ is a real number. (Note that this condition restricts $\rho \neq 1$.)

The generalised binomial expansion in this case is

$$\sum_{t \geq 0} \binom{r}{t} a^t = \sum_{t \geq 0} \frac{r(r-1)(r-2) \cdots (r-t+1)}{t!} \left(\frac{m}{1-m}\right)^t \tag{12.33}$$

In each period, the first ratio in Equation 12.33 grows by a factor $(r-t+1)/t$, which in the limit as $t \rightarrow \infty$ goes to -1 , so $m/(1-m) < 1$ gives convergence to a bounded value for the summation in Equation 12.33. \square

Proof of Proposition 12.4 Note that if Z_t^Δ first-order stochastically dominates Z_t , then $F_\Delta(Z) \leq F(Z)$, where $F_\Delta(Z)$ and $F(Z)$ are the respective distribution functions. We denote expectations with respect to

Table 12.7 Data sources for each returns series

Asset class	Data	Mnemonic/source	Weight (%)
1 UK equity	FTSE All-Share	FTALLSH(RI)	32.2
2 Global equity	MSCI World ex UK	MSWFUK\$(RI)~U\$, to BPN using BBGBSP(ER)	32.0
3 Overseas equity	MSCI Emerging Markets	MSEMKF\$(RI)~U\$, to BPN using BBGBSP(ER)	5.0
4 UK gilts	FTA	FTBGTTF(RI)~£	2.8
5 Property	IPD	UKIPDRI.F	7.5
6 Hedge funds	CSFB/Tremont hedge fund	CSTHEDG~£	3.6
7 Private equity	UK Trusts Private equity	ITVCAPT(RI)~£	11.5
8 Cash	Three-month CD rate	Bank of England CPI: UKCPHARMF	5.4
9 Inflation	Average of CPI and earnings	Wages: UKWAGES.E	

them by $\mathbb{E}_\Delta(\cdot)$ and $\mathbb{E}(\cdot)$. FSD implies that $\mathbb{E}_\Delta[G(Z)] \geq \mathbb{E}[G(Z)]$ for $G(\cdot)$ any increasing function (see Huang and Litzenberger (1988) for the proof). If $G(Z) = Z^{1-\alpha}$, $0 < \alpha < 1$, then $G(Z)$ is increasing in Z , and hence, under $F_\Delta(Z)$, φ is increased. If $\alpha > 1$, then $G(Z)$ is a decreasing function in Z and under $F_\Delta(Z)$ the reverse happens: φ decreases. \square

Proof of Proposition 12.6 Note that if Z_i^ω second-order stochastic dominates Z_i , then

$$\int_0^Z F_w(s) ds \leq \int_0^Z F(s) ds \quad \text{for all } Z \in (0, \infty)$$

where $F_w(Z)$ and $F(Z)$ are the respective distribution functions. We denote expectations with respect to them by $\mathbb{E}_w(\cdot)$ and $\mathbb{E}(\cdot)$. SSD implies that $\mathbb{E}_w[G(Z)] \geq \mathbb{E}[G(Z)]$ for $G(\cdot)$ any increasing, concave function (see Huang and Litzenberger (1988) for proof). If $G(Z) = Z^{1-\alpha}$, $0 < \alpha < 1$, then $G(Z)$ is increasing and concave in Z , and hence, under $F_w(Z)$, φ is increased. If $\alpha > 1$, we have $G(Z)$ a decreasing and convex function in Z , and under $F_w(Z)$ the reverse happens: φ decreases. \square

APPENDIX B

Table 12.7 lists data sources for each returns series. A consistent series of returns to hedge funds was not available prior to January 1994, so from January 1990 to December 1993 the allocations to UK, global, emerging and private equity were each increased by 0.9% and hedge funds set to zero. Total portfolio return is the weighted sum of log changes in each returns index and the cash rate (expressed on a monthly basis) minus the log change in the inflation rate

$$\dot{p} = \frac{1}{2} \left[\ln \left(\frac{\text{CPI}_t}{\text{CPI}_{t-1}} \right) + \ln \left(\frac{\text{earnings}_t}{\text{earnings}_{t-1}} \right) \right]$$

All the series are from Datastream, apart from the cash rate, which is from the Bank of England database.

¹ There is surprisingly little quantitative research published in the area of family trusts, endowments and foundations. Studies of family offices include Amit *et al* (2008) and references therein, and, from a sociological perspective, Gilding (2005). The literature on the optimal spending and investment plans for university endowments originates with Tobin (1974), Litvack *et al* (1974), Nichols (1974), but also features in Merton (1990) and more recently in Dybvig (1999), Woglom (2003) and Merton (2003). Empirical studies of the structure and investment performance of endowments include Brown (1999), Lerner *et al* (2005), Brown *et al* (2010), Dimmock (2007) and Acharya and Dimson (2007).

- 2 See <http://www.charity-commission.gov.uk/showcharity/registerofcharities/registerhome.aspx>.
- 3 See <http://www.mmdwebaccess.com/SPContent/Endowment>.
- 4 Returns to hedge funds, real estate and private equity can be serially correlated. See Satchell *et al* (2012) for analysis of spending in the EZW model when returns are not iid.
- 5 Merton also addresses the problem for a finite horizon.
- 6 The analysis also links to the large asset pricing and life-cycle literature using the EZW utility function (see, for example, Campbell 1993; Tallarini 2000; Campbell and Viceira 2002; Campbell and Vuolteenaho 2004; Bansal and Yaron 2004; Gomes and Michaelides 2005; Hansen *et al* 2005). Many of these studies either fix the EIS at 1 (or at some calibrated value), or use linearizations of the problem that constrain the EIS to be close to 1, and most assume conditional or unconditional lognormality in the returns process.
- 7 In a time-additive utility model δ would simplify to the rate of time preference, but time preference is generally endogenous in non-expected utility settings. See Backus *et al* (2005, pp. 321–90) for a general discussion of recursive preferences.
- 8 Constant proportion portfolio insurance is the optimal investment strategy of an investor or endowment protecting a fixed minimum level of consumption, a result implicit in Merton (1971) and explicit in Kingston (1989).
- 9 ONS data covers the decade 1998 to 2008.
- 10 Bhamra and Uppal (2006) solve a related problem for a finite horizon with non-constant investment opportunity set.
- 11 Giovannini and Weil (1989) and Campbell (1993) derive and discuss special cases.
- 12 This result has been widely employed in the asset pricing literature to help match up high equity premiums with relatively smooth consumption paths (see, for example, Campbell 1993).
- 13 Smith (1996) derives the feasibility and transversality condition for a related aggregator function in continuous time, but the model here is different in significant ways and Smith's result does not transfer directly.
- 14 Bhamra and Uppal (2006) set out the implicit portfolio optimality condition and the explicit optimal portfolio weights for simple examples of constant and stochastic investment opportunity sets. Explicit analytical results for portfolio choice under stochastic investment are limited to a two-state process for the risky asset.
- 15 The Wellcome Trust was founded by businessman and philanthropist Sir Henry Wellcome and supports biomedical and medical humanities research.
- 16 A consistent series of returns to hedge funds are not available prior to January 1994, so from January 1990 to December 1993 the allocations to UK, global, emerging and private equity were each increased by 0.9%, and hedge funds set to zero. Appendix B lists data sources for each returns series.
- 17 Optimal portfolio weights will satisfy a vector of moment conditions in the risk-scaled portfolio return and returns to individual assets. When returns are iid and the disbursement rate is constant, the conditions are (Bhamra and Uppal 2006, Equation 17)

$$\mathbb{E}[m^{\rho(1-\alpha)/(1-\rho)} Z_t^{-\alpha}(z_h)] = \mathbb{E}[Z_t^{-\alpha}(z_h)] = 0$$

This system of moment conditions and the portfolio returns data described above can be used to estimate α by generalised method of moments. Estimation results are available from the authors on request.

¹⁸ Chen *et al* (2007) estimate parameters of the EZW utility function at the aggregate level and find that the EIS is greater than 1 and risk aversion is in the range 17–60. See also Gruber (2006) for a high estimate for the EIS. Other authors find lower values for both parameters (see, for example, Vissing-Jorgensen and Attanasio 2003; Vissing-Jorgensen 2002; Epstein and Zin 1991).

- 19 The slightly jagged shape of the surface is caused by the bootstrap process: a different set of random draws is made at each combination of ρ and Δ . Edges of the surface are not smooth because the feasibility and boundary conditions are not met for some extreme values of ρ and α .

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Asset Allocation and Divorce Risk

Bernd Scherer

FTC Capital GmbH

In reality the standard marriage contract we sign on the happiest day of our life is a complex structured product that gives each party a set of contingent claims that both are hard to value and have an impact on the optimal asset allocation decisions of both partners. From our analysis we find that divorce law can lead to lower equity allocations than both partners find individually optimal. It can make strategic divorce (gaining access to the spouse's resources not accessible during marriage) attractive and lead to welfare-increasing prenuptial agreements for both partners.

Portfolio advice to private investors as it appears in textbooks excludes many aspects of real life and instead focuses on risk factors stemming from the investment opportunity set alone. Real life is more complex than that. We look at the impact of divorce risk and the importance of understanding the implicit contingent claims in divorce law in order to correctly arrive at an asset allocation that is in the client's best interest. In this contribution we try to answer the question of how divorce laws affect the asset allocation for married couples. Despite the practical relevance of this question, either to provide normative investment advice for married couples or to evaluate the welfare consequences of divorce laws, the literature on this is extremely sparse.

The only papers close to our work are by Love (2008) and Voena (2010). Both derive theoretical models for optimal investment decisions in a marriage. Both models are richer, as they include a larger set of decision variables (investment, consumption and labour supply) and explicitly model a multi-period optimisation problem. However, this richness comes at a cost. In Voena (2010) investment returns are considered deterministic, and Love (2008) ignores contingent claims between partners. This chapter instead focuses on an investment problem with stochastic asset returns and contingent