Joint Transmitter and Receiver I/Q Imbalance Estimation in Presence of Carrier Frequency Offset

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***Abstract*—This paper proposes a simple frequency domain joint transmitter and receiver I/Q imbalance estimation method which exploits the phase rotation introduced by carrier frequency offset. Using two frequency domain training sequences inserted in each transmission frame, the transmitter and receiver I/Q imbalances can be jointly estimated over multiple frames. The transmitter I/Q imbalance parameter can be fed back to the transmitter for I/Q imbalance pre-compensation, whereas the receiver I/Q imbalance can be compensated locally followed by conventional frequency domain equalization. Numerical simulation results show that the image rejection ratios for both transmitter and receiver after I/Q imbalance compensation can be improved to over 50 dB which is necessary for multichannel systems with high order modulation and wide transmission bandwidth.**

***Index Terms*—I/Q imbalance, image rejection, carrier frequency offset, spectral efficiency, and multichannel system.**

I. INTRODUCTION

As the wireless networks evolve toward the 5th generation, employing higher order modulation and performing multiple channel aggregation for wireless transmission have become the industry trends [1,2]. Since practical low cost microwave point-to-point systems usually use direct-conversion with in-phase(I)/quadrature(Q) modulation architecture for up and down conversions and channelization, any variation between respective components in the I and Q branches will cause I/Q imbalance. As a result, only about 20 dB image rejection can be achieved in practice, which prevents higher order modulation from being employed. For example, the 1024-quadrature amplitude modulation (1024QAM) requires over 30 dB signal-to-noise ratio (SNR) and hence the image rejection should be at least 40 dB.

I/Q imbalance compensation is commonly implemented in the receiver digital signal processing chain. However, many applications require that both transmitter (Tx) and receiver (Rx) I/Q imbalances be estimated and compensated separately. For example, in a multichannel microwave backhaul, I/Q imbalance at Tx must be pre-compensated deliberately to cancel inter-channel interference so that stringent transmit mask can be satisfied. In addition, since the bandwidth is wider for multichannel systems (about 200 MHz as compared with 7 to 55 MHz for single channel systems), frequency dependency of the I/Q imbalances should be taken into consideration.

Separating Tx and Rx I/Q imbalances is quite challenging since the received signal only sees the overall effect resulted from both Tx and Rx I/Q imbalances. In general, one cannot detect two unknowns from one observation unless there are other independent variables creating variations of the observation. Fortunately, such a variable which causes variation in the received signal does exist in practical low cost systems. This variable is the carrier frequency offset (CFO). It introduces phase rotation to the received signal and can be exploited to separate the Tx and Rx I/Q imbalances.

There are a number of techniques proposed in the literature to jointly estimate Tx and Rx I/Q imbalances as well as CFO [3-9]. However, most of them deal with the problem of compensation for these impairments at receiver rather than pre-compensation at transmitter [3-5]. Frequency domain approach has also been proposed but only frequency-independent I/Q imbalance is considered [6]. Joint estimation of CFO together with other impairments generally has very high computational complexity [7-8].

In this paper, we propose a novel frequency domain approach to jointly estimate frequency-dependent Tx and Rx I/Q imbalances and compensate for them at Tx and Rx separately. We first derive a complete frequency domain received signal model which involves both Tx and Rx I/Q imbalances as well as CFO. Based on this model, we show that, by differently weighting the signal and its image components according to the phase rotation caused by CFO and averaging out the weighted signal and image components over multiple training periods, the Tx and Rx I/Q imbalances can be effectively separated. As a microwave backhaul is usually bi-directional, the estimated Tx I/Q imbalance can be easily fed back for pre-compensation. Channel and CFO estimation then becomes much simpler after I/Q imbalance compensation. We also propose very simple frequency domain training sequences rather than the conventional Golay complementary sequences [9] for joint Tx and Rx I/Q imbalance estimation.

The rest of this paper is organized as follows. In Section II, the complete received signal model, including I/Q imbalances, channel frequency response and CFO, and the transmission frame structure are presented. In Section III, the proposed scheme for joint Tx and Rx I/Q imbalance estimation is described in detail. Simulation

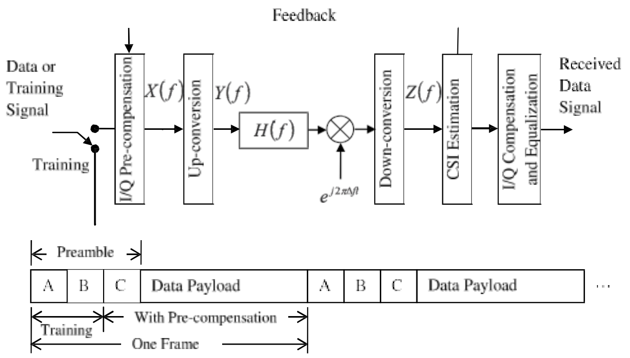

Fig. 1. System model and frame structure.

method and results are provided in Section IV to verify the performance of the proposed scheme. Finally, conclusions are drawn in Section V.

II. Frequency Domain Received Signal Model

Considering the transmission channel and other impairment such as the CFO, the complete system model is shown in Fig. 1 where Δf denotes the CFO. We assume that the link is bi-directional, so that channel state information (CSI) can be fed back to the transmitter via a return channel. We also transmit two segments of training signals denoted as A and B during which the pre-compensation at transmitter is disabled, followed by a segment C and data payload with pre-compensation switched on. Segments A, B, and C constitute the preamble, where A and B are used for I/Q imbalance estimation and C for channel and CFO estimation. Frames with the same structure are transmitted continuously.

The Tx and Rx I/Q imbalances are introduced by the imperfection in the up and down conversion modules. Let the frequency domain baseband signal to the up-converter be $X(f)$. The lowpass equivalent up-converted signal can be expressed as

$$Y(f) = u_t(f) X(f) + v_t(f) X^*(-f) \tag{1}$$

where $u_t(f)$ and $v_t(f)$ are the frequency dependent gains of the baseband signal and its image, respectively. $u_t(f)$ can be regarded as part of the transmission channel frequency response, whereas $v_t(f)$ reflects the self-interference caused by the I/Q imbalance.

To remove the self-interference, a pre-distorted signal $X(f) - \beta_t(f) X^*(-f)$ can be applied to the up-converter to compensate for the Tx I/Q imbalance. Replacing the input signal in (1) with the pre-distorted signal, we see that, if the parameter $\beta_t(f)$ is selected as $\beta_t(f) = \frac{v_t(f)}{u_t(f)}$, the self-interference from the signal's image can be completely removed.

Without transmitter pre-compensation during the training period, the received signal in the frequency domain after down-conversion can be expressed as

$$\begin{aligned} Z(f) =& u_r(f) H(f) e^{j\varphi} Y(f) \\ &+ v_r(f) H^*(-f) e^{-j\varphi} Y^*(-f) + W(f) \end{aligned} \tag{2}$$

where $H(f)$ is the channel frequency response, $e^{j\varphi}$ is a phase factor representing a phase rotation $\varphi = 2\pi\Delta f t$ caused by the CFO, $W(f)$ is the additive white Gaussian noise (AWGN) in the frequency domain, $u_r(f)$ and $v_r(f)$ are the frequency dependent gains of the received signal and its image, respectively. Again, $u_r(f)$ can be regarded as part of the transmission channel frequency response, whereas $v_r(f)$ reflects the self-interference caused by the I/Q imbalance. We assume that the CFO is small enough such that the phase rotation is approximately constant during the training period.

Similarly, the Rx I/Q imbalance can be compensated by performing the cancellation operation $Z(f) - \beta_r(f) Z^*(-f)$ after the down-conversion. If the parameter $\beta_r(f)$ is selected as $\beta_r(f) = \frac{v_r(f)}{u_r^*(-f)}$, the self-interference from the received signal's image can be completely removed.

Substituting (1) into (2), we have

$$Z(f) = u(f) X(f) + v(f) X^*(-f) + W(f) \tag{3}$$

where

$$\begin{aligned} u(f) =& u_r(f) H(f) e^{j\varphi} u_t(f) \\ &+ v_r(f) H^*(-f) e^{-j\varphi} v_t^*(-f) \end{aligned} \tag{4}$$

and

$$\begin{aligned} v(f) =& u_r(f) H(f) e^{j\varphi} v_t(f) \\ &+ v_r(f) H^*(-f) e^{-j\varphi} u_t^*(-f) \end{aligned} \tag{5}$$

are the frequency-dependent gains of the signal and image components, respectively. Note that $v(f)$ represents the combined system level I/Q imbalance including the effects of all practical impairments as well as channel frequency response.

The received signal model shown in (3) incorporates both Tx and Rx I/Q imbalances. From (4) and (5) we see that for a constant channel without CFO the combined I/Q imbalance can be compensated for as a whole at Rx. If channel varies and/or CFO exists, the combined system level I/Q imbalance will also vary. Hence, it is difficult to distinguish between Tx and Rx I/Q imbalances. In many applications, particularly for multichannel systems, stringent transmit mask for each channel must be satisfied at Tx and thus Tx I/Q imbalance should be estimated and pre-compensated at Tx.

III. Joint Tx and Rx I/Q Imbalance Estimation

Exploiting the presence of CFO, we now propose a simple and novel method to jointly estimate Tx and Rx imbalances. It consists of the following steps.

A. Estimation of $u(f)$ and $v(f)$

Specially designed frequency domain training signals are required for this estimation. We propose to use two training signals $A(f)$ and $B(f)$ transmitted in the two segments as shown in the frame structure, satisfying the condition

$$B(f) = \begin{cases} A(f), & \text{for } f > 0 \\ -A(f), & \text{for } f < 0 \end{cases} \tag{6}$$

such that $A(f) A(-f) + B(f) B(-f) = 0$, for $f \neq 0$.

From (3) we obtain the received signals in the two segments as

$$Z_A(f) = u(f) A(f) + v(f) A^*(-f) + W_A(f) \tag{7}$$

and

$$Z_B(f) = u(f) B(f) + v(f) B^*(-f) + W_B(f) \quad (8)$$

where $W_A(f)$ and $W_B(f)$ are independent AWGNs.

The zero-forcing estimate of $u(f)$ is then obtained as

$$u(f) \approx \frac{Z_A(f) A^*(f) + Z_B(f) B^*(f)}{|A(f)|^2 + |B(f)|^2}, \text{ for } f \neq 0. \quad (9)$$

Similarly, $v(f)$ can be estimated as

$$v(f) \approx \frac{Z_A(f) A(-f) + Z_B(f) B(-f)}{|A(-f)|^2 + |B(-f)|^2}, \text{ for } f \neq 0. \quad (10)$$

The noise components will introduce estimation errors, which can be alleviated by taking expectation over multiple independent estimates.

B. Estimation of $\beta_t(f)$ and $\beta_r(f)$

From (4) and (5), and utilizing the fact that $E\left[e^{-j2\varphi}\right] = 0$ where $E[\cdot]$ denotes expectation operation, $\beta_t(f)$ and $\beta_r(f)$ can be expressed as

$$\beta_t(f) = \frac{v_t(f)}{u_t(f)} = \frac{E\left[e^{-j\varphi} v(f)\right]}{E\left[e^{-j\varphi} u(f)\right]} \quad (11)$$

and

$$\beta_r(f) = \frac{v_r(f)}{u_r^*(-f)} = \frac{E\left[e^{j\varphi} v(f)\right]}{E\left[e^{j\varphi} u^*(-f)\right]} \quad (12)$$

respectively, and further estimated using (7) and (8) as well as the estimated phase factor $e^{j\varphi}$ (see Section IV).

This step is critical to separate the transmitter and receiver I/Q imbalances. Without CFO, the separation would be very difficult.

C. Compensation and Equalization

The estimated parameter $\beta_t(f)$ can be fed back to the transmitter as part of the CSI for I/Q pre-compensation, and $\beta_r(f)$ can be used for I/Q imbalance compensation at receiver. After I/Q imbalance compensation, the received signal can be expressed as

$$\begin{aligned} R(f) =& \left[u_r(f) - \beta_r(f) v_r^*(-f)\right] H(f) e^{j\varphi} \\ &\cdot \left[u_t(f) - \beta_t^*(-f) v_t(f)\right] X(f) + W(f) \\ =& e^{j\varphi} \widehat{H}(f) X(f) + W(f) \end{aligned} \quad (13)$$

where $\widehat{H}(f)$ is the overall channel frequency response. Conventional CFO and channel estimation can be applied by using segment C in the preamble, followed by channel equalization to recover the transmitted data signal.

IV. Numerical Simulation Results

A high speed wideband orthogonal frequency division multiplexing (OFDM) system is simulated to evaluate the performance of the proposed joint Tx and Rx I/Q imbalance estimation and compensation method. The system has a bandwidth of 200 MHz, and the fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) size $N_{FFT} = 256$. The data symbols are mapped onto subcarriers with 1024QAM so that over Gigabit per second data rate can be achieved. The CFO is selected as $\Delta f = 1$ kHz. With FFT, the estimation and compensation described in previous sections can all be performed in the discrete frequency domain. The frequency domain training signals $A(f)$ and $B(f)$ are therefore replaced by the training sequences $A[k]$ and $B[k]$ where $0 \le k \le N_{FFT} - 1$ is the subcarrier index. $A[k]$ can be selected as any binary sequence for simplicity and $B[k] = A[k]$ for $1 \le k \le \frac{N_{FFT}}{2} - 1$ and $B[k] = -A[k]$ for $\frac{N_{FFT}}{2} + 1 \le k \le N_{FFT} - 1$. Since the direct current (DC) component of the second training signal is not defined in (6), the Tx and Rx I/Q imbalances at DC are interpolated using subcarriers $k = 1$ and $N_{FFT} - 1$.

In the discrete frequency domain, the phase factor estimation and expectation operation are performed as follows. Suppose that the discrete frequency domain version of $u(f)$ is denoted as $u_i[k]$ for the ith transmission frame. The phase factor is estimated as

$$e^{j\varphi_i} = \frac{\sum_{k=0}^{N_{FFT}-1} u_i[k]}{\left|\sum_{k=0}^{N_{FFT}-1} u_i[k]\right|}. \quad (14)$$

The expectation is implemented using a lowpass filter with a forgetting factor μ. For example, if the discrete frequency domain version of $E\left[e^{-j\varphi} u(f)\right]$ is denoted as $\tilde{u}_i[k]$ at the ith transmission frame, it is updated as

$$\tilde{u}_i[k] = \tilde{u}_{i-1}[k] + \mu\left\{e^{-j\varphi_i} u_i[k] - \tilde{u}_{i-1}[k]\right\} \quad (15)$$

Though it is not part of the I/Q imbalance estimation algorithm, the expectation $E\left[e^{j2\varphi}\right]$, denoted as p_i for the ith transmission frame, is also evaluated using the same updating method

$$p_i = p_{i-1} + \mu(e^{j2\varphi_i} - p_{i-1}) \quad (16)$$

in order to investigate how the forgetting factor impacts on the I/Q imbalance estimation convergence speed and performance.

The I/Q imbalance parameters are selected as follows. For the transmitter, the phase imbalance is -5 degree and the amplitude imbalance is 1 dB. To simulate the frequency dependency, finite impulse response (FIR) filters $[0.01, 1, 0.01]$ and $[0.01, 1, 0.2]$ are applied to the I and Q channels respectively. For the receiver, the phase imbalance is 10 degree and the amplitude imbalance is -1 dB. The FIR filters applied to the I and Q channels are $[1.02, 0.04, -0.03]$ and $[1.03, -0.02, 0.012]$ respectively.

The simulation results are now described as follows. Firstly, Fig. 2 shows the phase factor expectation evaluated using (14) and (16) with an initial value 1 under 30 dB SNR for $\mu = 0.01$ and 0.001. We see that with relatively larger μ the magnitude of the phase factor expectation decreases rapidly and is then stabilized with residual values between 0.03 and 0.054 (see the inset plot), whereas with relatively smaller μ the magnitude of the phase factor expectation decreases slowly but the residual values (less than 0.01) are also smaller. A smaller phase factor expectation indicates more accurate estimation of the I/Q imbalance. This is demonstrated in Figs. 3 and 4, which show that with $\mu = 0.01$ the image rejection ratio (IRR), defined as the signal power versus image power, for both Tx and Rx can be improved to above 40 dB whereas with $\mu = 0.001$ the IRRs can be further improved to above 50 dB. At higher SNRs, the IRRs will be even better. Since the I/Q imbalance varies slowly

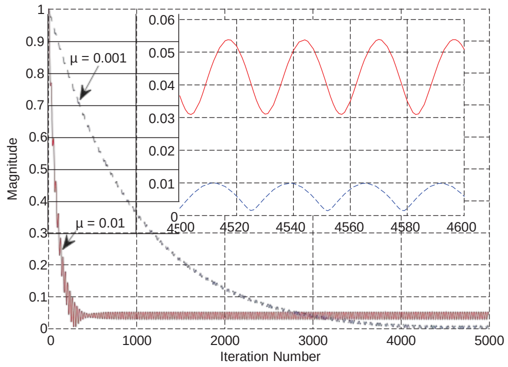

Fig. 2. Phase factor expectation for different forgetting factors under SNR = 30 dB.

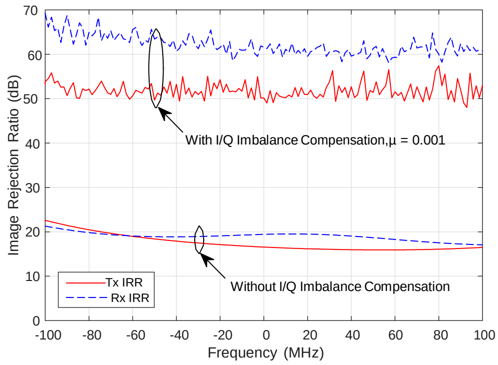

Fig. 4. Snapshots of IRR after I/Q imbalance compensation with $\mu = 0.001$ under SNR = 30 dB.

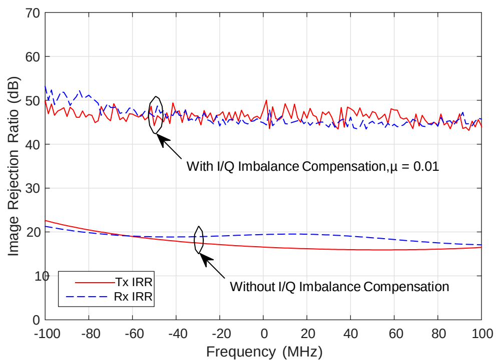

Fig. 3. Snapshots of IRR after I/Q imbalance compensation with $\mu = 0.01$ under SNR = 30 dB.

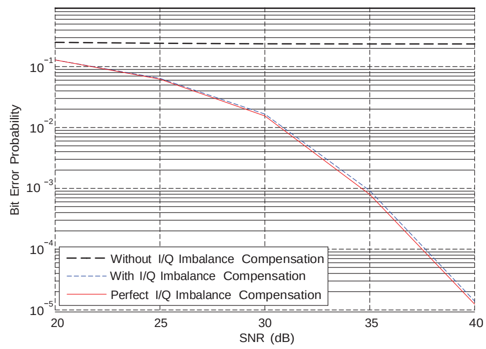

Fig. 5. BER performance (uncoded) with and without I/Q imbalance compensation for 1024QAM ($\mu = 0.001$ is used for I/Q imbalance estimation).

under normal operation conditions, using a smaller μ for I/Q imbalance estimation will be preferable. The IRRs without I/Q imbalance compensation are also shown in these figures for comparison. We see that the IRRs for Tx and Rx with I/Q imbalances are only about 20 dB which is insufficient for high data rate links with high order modulation.

Finally, the bit-error-rate (BER) performance (uncoded) is evaluated with and without I/Q imbalance compensation, and the results are presented in Fig. 5. The BER curve with perfect I/Q imbalance compensation (i.e., assuming perfect knowledge of the I/Q imbalances) is also shown. It is validated that without I/Q imbalance compensation high order modulation is impossible. The BER curve with I/Q imbalance compensation using the estimated I/Q imbalances almost coincides with the one using perfect I/Q imbalance compensation, which indicates that the estimation accuracy is sufficient for supporting 1024QAM without performance loss.

V. Conclusions

We have shown that the phase rotation introduced by carrier frequency offset can be exploited to easily separate the transmitter and receiver I/Q imbalances. With the help of a feedback link, transmitter I/Q imbalance pre-compensation and receiver I/Q imbalance compensation can be performed respectively to meet stringent image rejection requirement for higher order modulation. The frequency domain training sequences have very simple design and can be easily incorporated into the transmission frame structure. The proposed joint transmitter and receiver I/Q imbalance estimation method will be very useful for future high speed multichannel microwave backhaul systems.

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