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Stock Market, Interest Rate and Output: A Model and Estimation for US Time Series Data

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Abstract

In this paper we construct a model of stock market, interest rate and output interaction which is a generalization of the well known 1981 model of Blanchard. We allow for imperfect substitutability between stocks and bonds in the asset market and for lagged portfolio adjustment. The reaction of agents to changes in the stock market is dependent on the state of the economy. We analyze the dynamics of the model and its local stability properties. A discretization in terms of observable variables is derived. Some empirical results for U.S. output, stock price and interest rate data are presented using nonlinear least square estimates. We perform some stochastic simulations of the estimated non-linear model, obtaining distributions of the key economic quantities, their autocorrelation structure and financial statistics which are compared with historical data and RBC models. In addition, following Mittnik and Zadrozny (1993) a VAR with confidence bands for historical data is estimated and cumulative impulse-response functions compared to the model's impulse response functions. We find that the model captures a number of features of the data.

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1 Introduction

The interaction of asset market and output has recently become an important topic in macroeconomic research. A large number of papers have studied the relationship between the asset market and real activity. In this new line of research a considerable body of economic and financial literature has attempted to explain asset price changes using proxies for the changes in macroeconomic fundamentals. Taking contemporaneous or leads of macroeconomic variables as proxies for news on expected returns, future cash flows or as proxy for the discount rate such studies have only been partially successful in explaining asset price movements.

At the same time there are also a large number of papers that study the impact of financial variables on real activity. Recently, in many studies the impact of asset prices or Tobin's Q, interest rate spread and the term structure of the interest rates on real activity have in particular been studied. This is a new and important area of research in empirical macroeconomics since, beside real variables, financial variables appear to be good explanatory variables and predictors of variations in output (Lettau and Ludvigson 2000, 2001, 2002, Stock and Watson 1989, Estrella and Hardouvilis 1991, Estrella and Mishkin 1997).

Researchers nowadays often employ stochastic optimal growth models of RBC (Real Business Cycle) type for studying the relationship of asset market and real activity. Intertemporal decisions are at the heart of the RBC methodology and it is thus natural to study the asset market-output interaction in the context of those models. Some advances have been made by using stochastic growth models to predict asset prices and returns. The asset market implications of the RBC models are, for example, studied in Rouwenhorst (1995), Danthine, Donaldson and Mehra (1992), Lettau (1997), Lettau and Uhlig (1997), Lettau, Gong and Semmler (2001) and Boldrin, Christiano and Fisher (2001). The RBC model with technology shocks as the driving force for macroeconomic fluctuations attempts to replicate basic stylized facts of the stock market such as the excess volatility of asset prices and returns, the spread between asset returns (for example, between equity and risk-free assets)[1] and the Sharpe-ratio as a measure of returns relative to risk.

In this paper we pursue an alternative macroeconomic modeling approach to explain the relationship of stock price, interest rate and aggregate activity. We study a macrodynamic model whose origin is Blanchard (1981) and was further developed by Blanchard (1997). This alternative class of models has also been employed as a baseline model for the study of monetary policy shocks by Mcmillan and Laumas (1988). The Blanchard variant is, however, a perfect foresight model that exhibits

[1] For the latter, see Mehra and Prescott (1985).

saddle path stability and only the imposition of a jump to the stable branch makes the trajectories stable. Here we replace the perfect foresight jump variable technique by gradual adjustments, in particular gradual expectations adjustments based on adaptive expectations. The limiting behavior of our model which admits (amongst other properties) cyclical paths, yields the Blanchard perfect foresight model as a limiting case [2] when the expectations adjust infinitely fast. The model is solved through discrete time approximation and empirically estimated for US time series data.

The remainder of the paper is organized as follows. Section 2 gives a more detailed overview of macroeconomic literature on the stock market and discusses basic stylized facts. Section 3 reviews and presents our generalized variant of the Blanchard model. The implied dynamics are studied in section 4. Section 5 sets out the discretization of the model that is employed and explains the estimation methodology. Section 6 sets out some econometric results employing US time series data by employing nonlinear least squares methodology.[3] Section 7 discusses some stochastic simulations and impulse response analysis of the estimated model. Section 8 provides some conclusions. The mathematical proofs are collected in the appendices.

2 Stylized facts and macromodels

A large number of macroeconometric studies on the stock market and output are based on the consumption based capital asset pricing (CCAP) model. Econometric literature has shown that good predictors of stock prices and returns have proved to be dividends, earnings and growth rate of real output (Fama and French 1988, Fama and French 1989, Fama 1990), and to some extent inflation rates (Schwert 1989). Moreover, financial variables such as interest rate spread and term structure of interest rates have also been significant in predicting stock prices and stock returns (Fama 1990, Schwert 1990). Other balance sheet variables, such as firms' leverage ratio, net worth and liquidity have been successful to a lesser extent (Schwert 1990).

There is another group of macroeconometric studies that departs from the market efficiency hypothesis and adopts the overreaction hypothesis when employing macro variables as predictors for stock prices and stock returns (Schiller 1991, Summers 1986, Poterba and Summers 1988). Moreover, in this tradition the role of shocks, monetary, fiscal and external shocks are seen to be relevant (Cutler, Poterba, Sum-

[2] Further discussion of this type of treatment of saddlepath stability can be found in Flaschel, Franke and Semmler (1997).

[3] Additional estimation results using Smooth Transition Regression (STR) methodology are reported in Chiarella, Semmler and Mittnik (1997).

mers 1989). Although in the long run stock prices may revert to their mean determined by maroeconomic proxies of fundamentals, in the short run speculative forces may be more relevant than prospective yields. This view was, with some success, tested in the mean reversion hypothesis of Poterba and Summers (1988).[4]

For the reverse relation, the impact of financial variables on real activity, there is also a considerable number of recent econometric studies. The early work by Burns and Mitchel (1943) initiated studies on leading indicators to predict changes in real activity. In the more recent business cycle literature the emphasis has been on financial variables. Recent contributions by Stock and Watson (1989), Jaeger (1991) and Plosser and Rouwenhorst (1994) show that financial variables, in particular interest rates (interest rate spread and the term structure of interest rates) as well as stock returns, lead turning points in aggregate activity and are able to capture future development of real activity. [5]

There is also econometric work on the stock market and output interaction in the tradition of Hamilton's regime switching models. The idea of Hamilton (Hamilton, 1989) that output follows two different autoregressions depending on whether the economy is in an expanding or contracting regime is extended to a study of the stock market (Hamilton and Lin, 1996). Connecting to the above work by Schwert it is presumed that time periods of high volatility may interchange with periods of low volatility of stock returns depending on whether the economy is in a recession or expansion. On the other hand, an important factor for the output at business cycle frequency appears to be the state of the stock market. In their version Hamilton and Lin (1996) show some predictive power of the stock market for output and, using a regime change model, the state of the economy as predictor for the volatility of stock returns.

In general, however, it is well recognized that the studies of the interaction of financial and real variables have difficulties in fully capturing the lead and lag patterns in financial and real variables when tested econometrically. To overcome this deficiency, the use of the VAR framework to test for lead and lag patterns has been appealing but the VAR, as the regime change models, do not reveal important structural relations. Dynamic macromodels are needed to provide some rationale for structural relationships and to highlight relevant restrictions on empirical tests.

[4]The overreaction of equity prices in relation to news on fundamentals originates, in this view, in positive feedback mechanisms operating in financial markets. Important contributions have been made that study the social interaction of heterogeneous equity traders, for example the interaction of fundamentalist and chartists (Day and Huang 1990, Chiarella 1992 and Aoki 1997) or arbitrageurs and noise traders (DeLong, Shleifer, Summers, and Waldmann 1990). These are however models with short-run asset price dynamics which are not yet well connected to changes in long-run macro variables.

[5]See, in particular, Estrella and Mishkin (1997).

In contrasting stylized facts and macro models we will focus on the above two types of dynamic macro models which imply some predictions for the asset market-output interaction. We elaborate on stochastic growth models of RBC type and on a variant of an IS-LM version with money market and stock market. Both variants imply some predictions for the interaction of asset market and real activity.

It has been a tradition for the RBC methodology to contrast the historical with the model's times series and to demonstrate to what extend the model's time series can mimic historical data. Models are required to match statistical regularities of actual time series in terms of the first and second moments, cross correlation with output or in terms of impulse-response functions. We thus want to review some stylized facts on macroeconomic fluctuations and asset market against which models can be measured.

In table 1 we present summary statistics of US time series on GNP, consumption, investment, employment treasury bill rate, equity return and the Sharpe-ratio. The latter measure of financial market performance has recently become a quite convenient measure to match theory and facts, since, as a measure of the risk-return trade-off, the Sharpe-ratio captures both excess returns and excess volatility[6]. We employ quarterly data.

Table 1: Stylized Facts on Real Variables and Asset Markets: US Data [7]

Variable	*Std. dev.*	*Mean*
GNP	0.97	
Consumption	0.77	
Investment	2.88	
Employment	0.46	
T-bill	0.86	0.18
Stock-return	7.53	2.17
Equity premium	7.42	1.99
Sharpe-ratio		0.27

The hierarchy of volatility measured by the standard deviation is the usual one for US data. As known from the excess volatility debate (Shiller 1991) the stock return exhibits the strongest volatility. The second strongest volatility is exhibited by investment followed by consumption.

[6]See Lettau (1997), Lettau and Uhlig (1997) and Lettau, Semmler and Gong (2001) where the Sharpe-ratio as measure to match theory and facts in the financial market is employed.

[7]The real variables are measured in growth rates, 1970.1-1993.3. Data are taken from Canova and Nicola (1995)(the exact time series can be found in Citibase (1995); the notations are GNP82, GC82, GIN82, Lhours (man hours employed per week)). Asset market data represent real returns and are from Lettau, Gong and Semmler (2001) and represent 1947.1-1993.3. All data are at quarterly frequency. Asset market units are per cent per quarter. The T-bill rate is the 3 months T-bill rate. The Sharpe-ratio is the mean of the equity premium divided by it's standard deviation.

In addition the equity return carries an equity premium as compared to the risk free interest rate. This excess return was first stated by Mehra and Prescott (1985) as the equity premium puzzle. As can be observed the market return exceeds by far the return from the risk-free rate. As shown in a variety of recent papers[8] the RBC modeling approach insufficiently explains the equity premium and the excess volatility of equity return and thus the Sharpe-ratio. The standard RBC asset market models employ the Solow-residual as technology shocks – as impulse dynamics. For given variance of the technology shock, however, the standard utility functions and no adjustment costs asset market facts are hard to match (for details see Lettau, Gong and Semmler 2001).

In summary, for the actual time series compared to the data from the standard RBC model we observe a larger equity return and stronger volatility of equity prices in contrast to the risk free rate. These two facts are measured by the Sharpe-ratio which basically cannot be matched by standard RBC models.[9] Moreover, it is worth noting that in stochastic growth models there is only a one- sided relationship. Real shocks affect stock prices and returns but shocks to asset prices – or overreaction of asset prices relative to changes in fundamentals – have no effects on real activity. The asset market is always cleared and there are no feedback mechanisms to propagate financial shocks to the real side.

In this paper, we thus employ an alternative framework, a modified macromodel by Blanchard (1981) for studying the stock market-output interaction. Here, there are, in principle, cross effects between asset prices and real activity. Along the line of Tobin (1969) it is presumed that output, through consumption and investment functions, is driven by real activity as well as stock prices. As many studies have recently shown, there appears to be some correlation of output and stock prices through consumption and investment behaviors, although a contemporaneous relation of output and the stock price may be weak. When lags are introduced and Tobin's Q is measured as marginal Q, as some studies do (Abel and Blanchard 1984), the relationship appears to improve.

On the other hand, since the Blanchard macromodel is a rational expectations model shocks to macroeconomic variables cause the stock price to jump whilst keeping the output fixed (rather than allowing it to adjust gradually). Thus because the stock price jumps there is no feedback effect on output. Once the stock price is on the

[8]See, for example, Rouwenhorst (1995), Danthine, Donaldson and Mehra (1992), Boldrin, Christiano and Fisher (2001), Lettau (1997) , Lettau and Uhlig (1997) and Lettau, Semmler and Gong (2001).

[9]Danthinee et al. who study the equity return also state:"To the equity premium and risk free rate puzzles, we add an excess volatility puzzle: the essential inability of the (RBC) models to replicate the observation that the market rate of return is fundamentally more volatile than the national product" (Danthinee et.al. 1992: 531).

stable branch output also then gradually adjusts. [10] The stock price overshoots its steady state value during its jump and then decreases thereafter. Blanchard's macromodel thus predicts that unless unanticipated shocks occur, the stock price moves monotonicly toward a point of rest or if it is there it will stay there. Thus, in fact, only exogenous shocks will move stock prices. This line of research has been econometrically pursued in papers by Summers (1986), Cutler, Poterba and Summers (1989) and McMillan and Laumas (1988). As in other rational expectations models, in its basic version, no feedback mechanisms exist that can lead to an endogenous propagation of shocks and fluctuations.

Based on the Blanchard variant the present paper pursues a modeling strategy for the relationship of asset market and real activity in order to overcome shortcomings of both the RBC model and the rational expectations version of a macromodel. In our model, unlike in the RBC type stochastic growth model, the financial market will impact the real activity and different from the Blanchard model, stock price jumps to their stable path are avoided by positing gradual adjustments of stock prices and output. This, in turn, will give rise to strong endogenous propagation mechanisms and fluctuations of both stock prices and output.

3 A generalized Blanchard model

We follow more or less the notation of Blanchard (1981). Also we focus in this study only on the case in which output prices are fixed.[11] Thus q is the value of the stock market, y is income, g the index of fiscal expenditure so that aggregate expenditure d[12] is given by

$$d = aq + \beta y + g \qquad (\, a > 0, \quad 0 \leq \beta < 1). \tag{3.1}$$

Output adjusts to changes in aggregate expenditure with a delay according to

$$\dot{y} = \kappa_y (d - y) = \kappa_y (aq - by + g), \tag{3.2}$$

[10]Blanchard states: "Following a standard if not entirely convincing practice, I shall assume that q always adjusts so as to leave the economy on the stable path to the equilibrium" (Blanchard 1981:135); see also p. 136 where Blanchard discusses the response of the stock price to shocks, for example, unanticipated monetary and fiscal shocks. For a detailed discussion on policy shocks in the context of the Blanchard model, see McMillan and Laumas (1988).

[11]The inclusion of a slowly varying output price, by assuming some sluggish price adjustment as in Rotemberg and Woodford (1997), would presumably not change the results significantly.

[12]The impact of the stock market on consumption as well as investment spending has been thoroughly studied in recent papers by Lettau and Ludvigson (2000, 2002)

where $b \equiv 1 - \beta$ so that $0 < b \leq 1$ and the speed of output adjustment $\kappa_y > 0$.

From the standard assumption of LM equilibrium in the asset market we can write

$$i = cy - h(m - p) \qquad (c > 0, h > 0), \tag{3.3}$$

where i denotes the short term rate of interest, m and p the logarithms of nominal money and prices respectively.

Real profit is given by

$$\pi = \alpha_0 + \alpha_1 y, \qquad (\alpha_1 \geq 0), \tag{3.4}$$

so that $(x + \alpha_0 + \alpha_1 y)/q$ is the instantaneous expected real rate of return from holding shares where we use x to denote the instantaneous expected change in the value of the stock market. Hence the instantaneous differential between returns on shares and returns on short term bonds (i.e. the instantaneously maturing bond) is given by

$$\epsilon = \frac{x + \alpha_0 + \alpha_1 y}{q} - i. \tag{3.5}$$

A key assumption of Blanchard's approach is that this differential is always zero[13]. This is tantamount to assuming that the two financial assets are regarded as perfect substitutes and that any differential between them is arbitraged away instantaneously. However in our more general treatment we allow for a degree of imperfect substitutability between the two assets and posit that the excess demand for stocks (q^d) is a monotonically increasing function of the instantaneous differential between ϵ and the long run constant equity premium $\bar{\epsilon}$[14]. We further assume that the stock market adjusts to the excess demand with a speed of adjustment that also depends on the differential $(\epsilon - \bar{\epsilon})$. All of these effects can be captured by writing adjustment in the stock market as

$$\dot{q} = \kappa_q(\epsilon - \bar{\epsilon}).(\epsilon - \bar{\epsilon}) \tag{3.6}$$

where $\kappa_q(> 0)$ is the speed of adjustment of the stock market to excess demand for stocks and is itself assumed to be a function of the excess demand. Blanchard assumes that $\kappa_q = \infty$ so that from equations (3.5) and (3.6) we recover

$$\frac{x + \alpha_0 + \alpha_1 y}{q} = i + \bar{\epsilon} \tag{3.7}$$

[13] Note that ϵ may be defined as net of a constant risk premium on equity. Since we want to focus on the equity price and equity premium we subsequently do not consider the term structure of interest rates.

[14] The existence of such a long run constant equity premium is another assumption of our model.

for all time, one of the key assumptions of Blanchard's original analysis[15]. However Beja and Goldman (1980) and Damodaran (1993) advance arguments as to why κ_q should not be set to ∞ and we shall focus here on the implications of this assumption.

The final building block of the model is the same rule for the formation of expectations about the expected change in the value of the stock market. Here we assume the adaptive expectations scheme

$$\dot{x} = \kappa_x(\dot{q} - x), \tag{3.8}$$

where $\kappa_x(> 0)$ is the speed of revision of expectations. The inverse κ_x^{-1} may be interpreted as the time lag in adjustment of expectations. By assuming this time lag to be zero (i.e. $\kappa_x = \infty$) equation (3.8) reduces to the perfect foresight case

$$x = \dot{q}, \tag{3.9}$$

which is also a key assumption in Blanchard's model.

Our most radical departure from the original Blanchard framework is our assumption about the reaction coefficient κ_q, which changes as a function of market conditions. When market conditions are such that q is close to its steady state q_0 (i.e. ϵ is close to $\bar{\epsilon}$), the reaction coefficient κ_q is rather high so that agents are reacting strongly to the return differential. However the high κ_q (coupled with the high κ_x) causes the steady state to be locally unstable and hence leads to a rise (or fall) in the stock market. Agents initially are prepared to go with this general movement in the stock market, however as it proceeds further and further they are conscious that the economy is moving ever further from its steady state (of which they are assumed to have some reasonable idea) and they start to react more cautiously to the return differential. This cautiousness is reflected in a gradual lowering of the value of the coefficient κ_q, which eventually becomes sufficiently low to cause a turn-around in the dynamics that once again become stable towards the steady state. Eventually κ_q returns to former high levels and the possibility of another upward (or downward) stock market movement is established. The behavior of κ_q as a function of the difference in ϵ from its steady state value $\bar{\epsilon}$ is illustrated in Figure 1. We have drawn this function somewhat skewed to the right to indicate greater (less) caution when the share market is below (above) its steady state value. This relation may also exhibit both euphoric (the higher graph) and depressed (the lower graph) states depending on particular news events arriving in the market.

[15]Note that Blanchard's analysis has $\bar{\epsilon} = 0$.

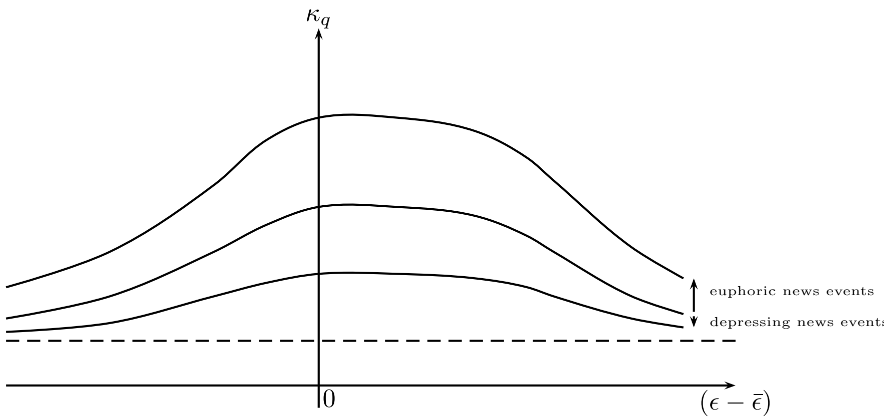


Figure 1: The Behaviour of the Reaction Coefficient κ_q

We thus have fast adaptively formed expectations and a fast adjustment of share prices to the return differential *close to the steady state*. However, *far from the steady state* we assume that agents are aware that the economy is approaching some sort of extreme situation and become increasingly cautious and thus only more and more sluggishly continue to adjust into a direction that they believe cannot continue for much longer.

Consider more closely the functional form $\kappa_q(\epsilon - \bar{\epsilon}) \cdot (\epsilon - \bar{\epsilon})$ with κ_q having the functional form shown in figure 1. Effectively the monotonically increasing function $(\epsilon - \bar{\epsilon})$ is being multiplied by a high value for $(\epsilon \simeq \bar{\epsilon})$ and low values for ϵ far from $\bar{\epsilon}$. Hence the combined functional form has the general shape shown in figure 2. It will be convenient to express the combined functional form in terms of just one function f, which with slight abuse of notation we define according to

$$\kappa_q(\epsilon - \bar{\epsilon}) \cdot (\epsilon - \bar{\epsilon}) = \kappa_q f(\epsilon - \bar{\epsilon}). \tag{3.10}$$

We stress that κ_q on the right-hand side is a constant which we have "pulled out" of the function f in order to make transparent the speed of adjustment at the steady state. The essential features of the function f are its lower slope far from steady state compared to its slope at steady state. It is also possible, depending on the function κ_q, for f to have some turning points and these could lead to a richer dynamic behavior. However in this study we shall concentrate only on the case where f ends up having the slope shown in figure 2.

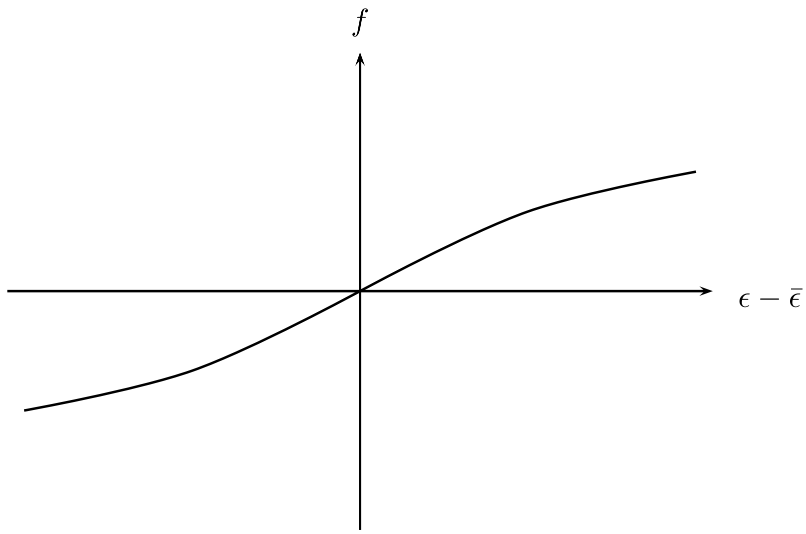


Figure 2: The Function f

Our generalized Blanchard model consists of equations (3.2), (3.6) and (3.8) which we rewrite here as the three-dimensional dynamical system

$$\dot{y} = \kappa_y(aq - by + g), \tag{3.11}$$

$$\dot{q} = \kappa_q f(\frac{x + \alpha_0 + \alpha_1 y}{q} - cy + \delta^{'}), \tag{3.12}$$

$$\dot{x} = \kappa_x(\kappa_q f(\frac{x + \alpha_0 + \alpha_1 y}{q} - cy + \delta^{'}) - x), \tag{3.13}$$

where we write $\delta \equiv h(m-p)$ and $\delta^{'} = \delta - \bar{\epsilon}$. The equilibrium of the system (3.11)-(3.13) is given by

$$\bar{x} = 0, \tag{3.14}$$

and the values $(\bar{y}, \bar{q})$ that solve

$$aq - by + g = 0, \tag{3.15}$$

$$\frac{\alpha_0 + \alpha_1 y}{q} = cy - \delta^{'}. \tag{3.16}$$

For the equilibrium of (3.11)-(3.13), two sets $(\bar{y}, \bar{q})$ are possible and are given by

$$\bar{y} = \frac{\psi \pm \sqrt{\psi^2 - 4bc(g\delta' - a\alpha_0)}}{2bc}, \tag{3.17}$$

$$\bar{q} = \frac{b\bar{y} - g}{a}, \tag{3.18}$$

where $\psi \equiv gc + b\delta' + a\alpha_1$. Provided we assume $\delta > 0$ there will always be at least one positive pair $(\bar{y}, \bar{q})$ which is the equilibrium considered by Blanchard.

The determination of $(\bar{y}, \bar{q})$ is illustrated in figure 3. Quite a number of subcases are possible depending upon what we assume about the sign of α_0, the relationship of $h(m-p)/c$ $(\equiv y_i)$ to $-\alpha_0/\alpha_1$ $(\equiv y_\pi)$, the relationship of b/g to $-\alpha_0/\alpha_1$ and the relationship of g/a to $\alpha_0/h\delta'$. We assume $m - p > 0$ as it seems reasonable that the price level would be less than the nominal stock of money. Note that this assumption also implies that y will be the positive level of output at which the nominal interest rate falls to zero. Blanchard's famous "bad news" and "good news" scenarios revolve around the relationship between y_π and y_i. In figure 3 we will illustrate the 3 cases (a) $y_i > y_\pi$ (the "bad news" case), (b) $y_i < y_\pi$ (the "good news" case) and (c) $y_i = y_\pi$ (the "neutral" case). In cases (a) and (b) we show the second, lower, equilibrium point as being in the positive quadrant though this need not necessarily be the case. In case (b) we have assumed that g is sufficiently large that the two equilibria exist. In this paper we shall focus on the dynamics around the positive equilibria E_g^+, E_b^+ obtained by taking the positive root in (3.17).

Figure 3:

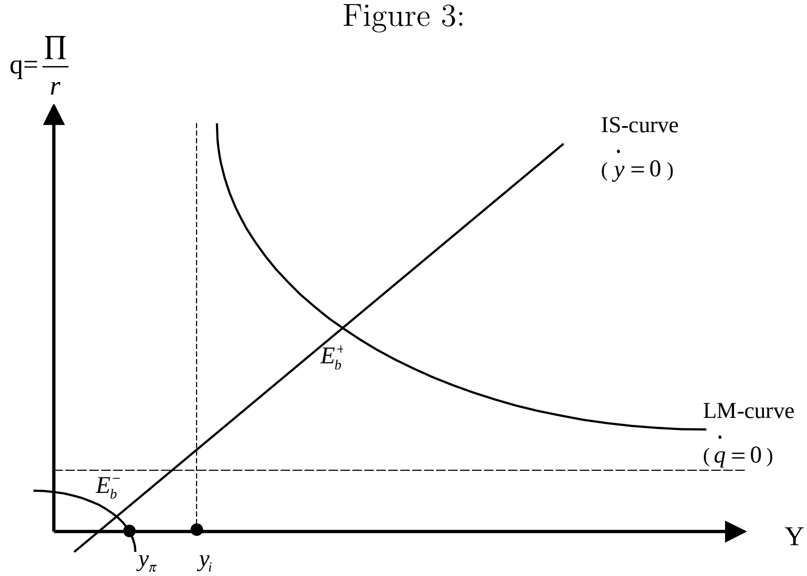


Figure 3(a): A uniquely determined Steady State in Blanchard's Bad News Case

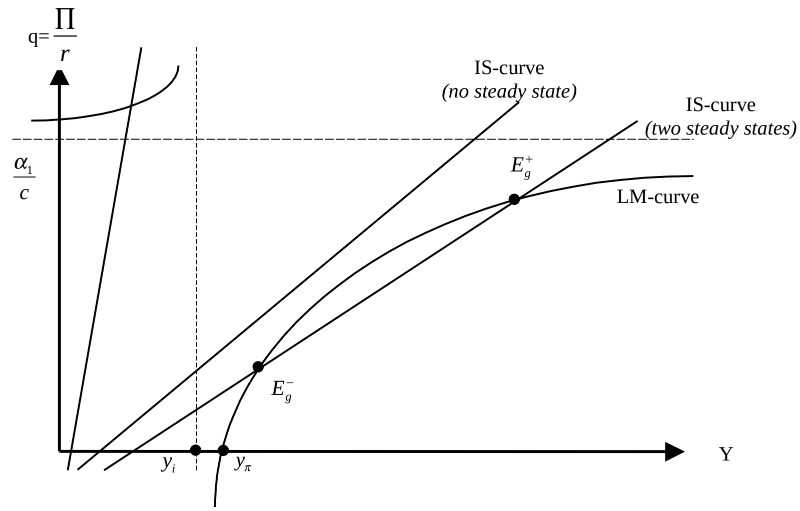


Figure 3(b): Two or No Steady States in Blanchard's Good News Case

Before proceeding to discuss the dynamics of the system (3.11)-(3.13) we first show how the original Blanchard model can be recovered from it. First we assume perfect foresight by letting $\kappa_x \to \infty$ which by (3.13) yields

$$\dot{q} = x. \tag{3.19}$$

Then we assume instantaneous adjustment to excess demand in the stock market by letting $\kappa_q \to \infty$ in (3.12) and also set $\bar{\epsilon} = 0$. Hence

$$\frac{x + \alpha_0 + \alpha_1 y}{q} = cy - h(m - p). \tag{3.20}$$

Combining the last two equations yields the differential equation for q, viz

$$\dot{q} = q[cy - h(m - p)] - \alpha_0 - \alpha_1 y. \tag{3.21}$$

The differential equations (3.11) and (3.21) for y and q constitute the dynamical system studied by Blanchard. In appendix 1 we outline the Jacobian analysis which indicates that the equilibria E_b^+, E_g^+ in figures 3(a) and 3(b) are saddle points in this perfect foresight case. It may be worth noting in passing that if the jump-variable procedure that is used by Blanchard is not adopted then the global dynamics need to be considered. This means taking into account the second equilibrium points E_b^-, E_g^- which can become attractors under certain circumstances, as discussed in Chiarella, Flaschel and Semmler (2001). However we do not undertake this more detailed analysis here as our main purpose is to understand and estimate the three dimensional generalized Blanchard model given by the differential system (3.11)–(3.13).

4 The dynamics of the model

The differential system (3.11)-(3.13) is nonlinear because of the assumed shape of the function f and also because of the quotient $(x + \alpha_0 + \alpha_1 y)/q$. To understand its dynamics we first calculate its Jacobian at an equilibrium point, and this turns out to be

$$J_3 = \begin{bmatrix} -\kappa_y b & \kappa_y a & 0 \\ \lambda\kappa_q\left(\frac{\alpha_1}{q} - c\right) & -\lambda\kappa_q(cy - \delta')/q & \lambda\kappa_q/q \\ \lambda\kappa_x\kappa_q\left(\frac{\alpha_1}{q} - c\right) & -\lambda\kappa_x\kappa_q(cy - \delta')/q & \kappa_x(\lambda\kappa_q/q - 1) \end{bmatrix}. \tag{4.1}$$

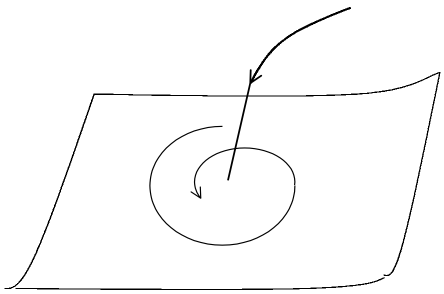
Here we have set $\lambda \equiv f'(0)$ and for notational convenience have omitted the bars indicating equilibrium values. Note also that we have made use of the relation (3.20) to simplify the expression for the elements J_{22} and J_{32}. The characteristic equation of J_3 turns out to be

$$\gamma^3 + A_1\gamma^2 + A_2\gamma + A_3 = 0, \tag{4.2}$$

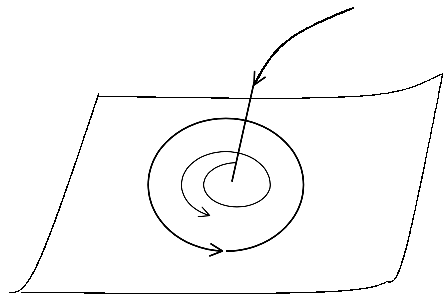
where

$$\begin{aligned} A_1 &= \kappa_y b + \frac{\lambda\kappa_q}{\bar{q}}(c\bar{y} - \delta') - \kappa_y\left(\frac{\lambda\kappa_q}{\bar{q}} - 1\right), \\ A_2 &= -\frac{\lambda\kappa_y\kappa_q}{\bar{q}} + \kappa_x\left[\frac{\lambda\kappa_q}{\bar{q}}(c\bar{y} - \delta') - \kappa_y b\left(\frac{\lambda\kappa_q}{\bar{q}} - 1\right)\right], \\ A_3 &= \frac{\lambda\kappa_y\kappa_q}{q}[b(c\bar{y} - \delta') + a(c\bar{q} - \alpha_1)]. \end{aligned}$$

At the equilibrium E_g^+, E_b^+ it turns out that $A_3 > 0$ which indicates that at these equilibrium points the real parts of the eigenvalues of J_3 have the sign distribution $(-,-,-)$ or $(-,+,+)$. Chiarella, Flaschel and Semmler (2001) show that the parameter κ_x can act as a bifurcation parameter and there exists a value κ_x^* such that the sign distribution is $(-,-,-)$ for $\kappa_x < \kappa_x^*$ and $(-,+,+)$ for $\kappa_x > \kappa_x^*$. Furthermore the conditions of the Hopf–bifurcation theorem are satisfied at κ_x^*. Thus the qualitative behavior around the equilibrium will be as shown in figure 4 under the assumption that the limit cycle born at κ_x^* is stable. For κ_x sufficiently large the dynamics consists locally of a stable and one unstable manifold as shown in figure 3b. For a wide range of parameter values the nonlinearity of the function f acts to turn the locally unstable motion on the unstable manifold into motion stable to a limit cycle as shown in figure 3b. Chiarella, Flaschel and Semmler (2001) demonstrate this result for a stylized form of the function $\kappa_q(\epsilon - \bar{\epsilon})$.



(a): $\kappa_x < \kappa_x^*$.



(b):$\kappa_x > \kappa_x^*$.

Figure 4:

The traditional analysis of perfect foresight models as undertaken by Blanchard (1981) (and many other authors) collapses the two differential equations (3.12)-(3.13) into the one differential equation (3.21) and therefore from the outset loses sight of the fact that the two–dimensional perfect foresight system is in fact the limiting case of a three–dimensional adaptive expectations system. A detailed analysis of how this limiting process works in the case of models of monetary dynamics is given in Chiarella (1986) and in Flaschel, Franke and Semmler (1997). The limiting process is of the same qualitative nature in our generalized Blanchard model as is demonstrated in Chiarella, Flaschel and Semmler (2001). It is also worth noting that the adoption of the three–dimensional viewpoint obviates the need to impose the arbitrary jump–variable technique to ensure that the economy arrives on a stable path from any arbitrary initial conditions. We have cited earlier Blanchard's own comment on the inadequacy of that procedure.

5 Discrete time form for observable variables

In order to estimate the system (3.11)-(3.13) we need to express it as a dynamical system solely in terms of the observable variables y and q. It is possible to derive both a bivariate dynamical system in y and q and a univariate dynamical system in either y or q. We will derive here the discrete time form for a bivariate system in q and y as this will allow us to use observations on both output and the stock market in our estimation procedures.

By eliminating the expectational variable x from the system (3.11) –(3.13) we obtain the bivariate dynamical system in y and q

$$\begin{aligned} \dot{y} &= \kappa_y(aq - by + g), & (5.1)\\ \ddot{q} &= -\frac{\phi_1}{\phi_3}\dot{y} + \frac{(\kappa_x - \phi_2)}{\phi_3}\dot{q} - \frac{\kappa_x}{\phi_3}\phi(y, q, \dot{q}). & (5.2)\end{aligned}$$

After some straight forward manipulations we find that the two dimensional dynamical system (5.1)–(5.2) may be reduced to the third order differential equation (representing a univariate process),

$$\dddot{y} = -\kappa_y\left[b + a\frac{H^{(1)}}{H^{(3)}}\right]\dot{y} + \frac{(\kappa_x - H^{(2)})}{H^{(3)}}[\kappa_y by + \ddot{y}] - \frac{a\kappa_y\kappa_x}{H^{(3)}}G(y, \dot{y}, \ddot{y}). \qquad (5.3)$$

The functions ϕ, G, $H^{(i)}$ are defined as

$$\begin{aligned} \phi(y, q, \dot{q}) &= -\alpha_0 - \alpha_1 y + q[cy - \delta' + f^{-1}(\dot{q}/\kappa_q)], & (5.4)\\ G(y, \dot{y}, \ddot{y}) &= \phi\left(y, \frac{b}{a}y + \frac{1}{a\kappa_y}\dot{y} - \frac{g}{a}, \frac{b}{a}\dot{y} + \frac{1}{a\kappa_y}\ddot{y}\right), & (5.5)\\ H^{(i)}(y, \dot{y}, \ddot{y}) &= \phi_i\left(y, \frac{b}{a}y + \frac{1}{a\kappa_y}\dot{y} - \frac{g}{a}, \frac{b}{a}\dot{y} + \frac{1}{a\kappa_y}\ddot{y}\right), \quad (i = 1, 2, 3) & (5.6)\end{aligned}$$

where ϕ_i denotes the partial derivative of ϕ with respect to its i^{th} argument.

In our empirical study and in our numerical simulations we take

$$f(x) = \bar{f}\ tanh(\lambda x), \qquad (\lambda > 0, \bar{f} > 0). \qquad (5.7)$$

The expressions for ϕ, G and H^i implied by equation (5.7) are given below.

For an empirical estimation we can discretize (5.1)–(5.2) and (5.4)–(5.6) by using the standard discretizations [16]

$$\begin{aligned} \dot{z}(t) &= \frac{z(t) - z(t - \Delta t)}{\Delta t}, & (5.8)\\ \ddot{z}(t) &= \frac{z(t) - 2z(t - \Delta t) + z(t - 2\Delta t)}{(\Delta t)^2}, & (5.9)\\ \dddot{z}(t) &= \frac{z(t) - 3z(t - \Delta t) + 3z(t - 2\Delta t) - z(t - 3\Delta t)}{(\Delta t)^3}. & (5.10)\end{aligned}$$

[16]Since the differential equations(5.2), (5.3)will be estimated with the addition of noise terms we are in fact dealing with the discretization of stochastic differential equations (see Kloeden and Platen (1995)).The discretization used here corresponds to the simple Euler-Maruyama scheme. In a separate study we have used the higher order Milstein scheme, but this does not appear to alter greatly the results; see Chiarella, Semmler and Zhu (2002)

Employing (5.8) the discrete time form of (5.1) can be written as

$$y_t = y_{t-h} + h\kappa_y(aq_{t-h} - by_{t-h} + g), \tag{5.11}$$

where the step size $h = \Delta t$.

The discrete-time form of (5.2) can be derived by using the discretisation (5.9), thus

$$q_t = 2q_{t-h} - q_{t-2h} - h^2\frac{\phi_1}{\phi_3}\dot{y} + h^2\frac{\kappa_x - \phi_2}{\phi_3}\dot{q} - \frac{\kappa_x h^2\phi(y_{t-h}, q_{t-h}, \dot{q})}{\phi_3}, \tag{5.12}$$

where again $\dot{y}, \dot{q}$ can be approximated by (5.8). Thus we set

$$\dot{y} = \frac{y_{t-h} - y_{t-2h}}{h} \tag{5.13}$$

and

$$\dot{q} = \frac{q_{t-h} - q_{t-2h}}{h}. \tag{5.14}$$

Using the form (5.7) for f it turns out that

$$\phi(y, q, \dot{q}) = -\alpha_0 - \alpha_1 y + q\left[cy - \delta' + \frac{1}{2\lambda}ln\left(\frac{\kappa_q\bar{f} + \dot{q}}{\kappa_q\bar{f} - \dot{q}}\right)\right], \tag{5.15}$$

and

$$\begin{aligned} \phi_1(y, q, \dot{q}) &= -\alpha_1 + cq, && (5.16)\\ \phi_2(y, q, \dot{q}) &= cy - \delta' + \frac{1}{2\lambda}ln\left(\frac{\kappa_q\bar{f} + \dot{q}}{\kappa_q\bar{f} - \dot{q}},\right) && (5.17)\\ \phi_3(y, q, \dot{q}) &= \frac{q}{\lambda}\frac{\kappa_q\bar{f}}{(\kappa_q\bar{f} - \dot{q})(\kappa_q\bar{f} + \dot{q})}. && (5.18) \end{aligned}$$

Use of (5.13)–(5.18) in (5.12) gives us the discrete time form of the stock price equation. Note that in the bivariate model (5.11), (5.12) the output equation (5.11) is linear with one lag whereas the stock price equation (5.12) is nonlinear with two lags. The univariate model (5.3) can be discretized in a similar way using (5.8)–(5.10) giving then rise to a nonlinear difference equation in y with three lags. A related nonlinear difference equation for the stock price, q, is more tedious to derive and will here be left aside.

Since in a univariate representation of our model, as in (5.3), or in a dynamic equation for stock price, q, some information will be lost, we rather prefer to pursue an estimation of the bivariate system (5.11)–(5.12) for the observable variables y and q.

6 Empirical results for US time series data

We estimate the parameters of the nonlinear bivariate system (5.11)–(5.12) by employing again NLLS estimation. For the US data discussed in Section 2, estimation results are reported below.[17] We employ for our estimations monthly data on real stock price and real output.

We directly estimate the parameters of the discrete time nonlinear bivariate system (5.11)–(5.12) with the number of lags constrained to what arise by using the Euler-Maruyama scheme. The estimated parameters, obtained from the BP- filtered[18] data, are reported in Table 2.

Table 2: Parameter Estimates, US: 1960.01-1993.10, Detrended Data [19]

Economic Structure	*Speeds of Adjustment*	*Government Policy*
$a = 0.122$	$\kappa_y = 0.185$	$g = 0.000$
$b = 0.370$	$\kappa_q = 0.240$	$\delta = -6.670$
$\alpha_0 = 0.065$	$\kappa_x = 1.120$	
$\alpha_1 = 6.620$		
$c = 1.568$		
$\lambda = 0.036$		
$\bar{f} = 0.205$		

It is noticeable from Table 2 that all parameters have the predicted sign, except δ. Note, however, that this may be due to the fact that δ is taken as a constant. Also the estimates of the speeds of adjustment have the expected positive sign. One can observe the hierarchy in the speed of adjustments that also other studies would suggest. In particular the slow output adjustment compared to the speed of stock price adjustment seems to match empirical facts.

In model (5.11)-(5.12) the term $\delta = h(m-p)$ is fixed. Since historically real balances

[17] The above model (5.11)-(5.12), however, constrains the lag structure. There are many frameworks within which nonlinearities in economic time series can be tested with longer lag structure. Threshold models may be useful for this purpose, see Tong (1990), and Granger and Teräsvirta (1993). We, therefore have also used a more data based methodology and let the data determine the type of nonlinearities and the lag structure. In Chiarella, Semmler and Mittnik (1997) we report for U.S. data the results of a regime change model of Smooth Transition Regression type with an unconstrained lag structure. Moreover, there are also estimation results reported for the above model (5.11)-(5.12) for a European data set.

[18] The Band-Pass filter developed and applied by Baxter and King (1995) has been employed in order to detrend the data.

[19] We employ monthly data which are taken from the Hamilton and Lin (1996) data set. As real stock price we take the Standard & Poor's Composite index deflated by the consumer price. For the output variable we take the monthly production index. All variables here are detrended by the BP-filter. The standard errors for the parameters could not be computed since the Hessian matrix was not positive definite.

might substantially vary we also undertake, for the BP-filtered data, estimations by including real balances as exogenous variable. We use the time series of real balances to form $\delta_t = h(m_t - p_t)$ as an exogenous sequence.[20] In addition, we can account for the long run equity premium $\bar{\epsilon}$. The results with real balances and equity premium are presented in table 3.

*Table 3: Parameter Estimates, US: 1960.01-1993.10, Detrended Data**

Economic Structure	*Speeds of Adjustment*	*Government Policy*
$a = 0.122$	$\kappa_y = 0.285$	$g = 0.000$
$b = 0.370$	$\kappa_q = 1.998$	
$\alpha_0 = 0.397$	$\kappa_x = 1.798$	
$\alpha_1 = 0.05$		
$c = 0.400$		
$h = 0.100$		
$f = 0.025$		
$\bar{\epsilon} = 0.035$		

$*$ In this variant the estimation is undertaken with real balances as a time series and a term for the equity premium.

In the variant reported in table 3, the effect of the equity premium is picked up in the parameter $\bar{\epsilon} = 0\ .035$. The terms for the real balances, h, and the equity premium now have the correct signs. Note that the parameters (a and b) for the output equation do not change for the variants of tables 2 and 3.

In terms of the mean square prediction error (MSPE) for the model reported in tables 2 and 3 we obtain the results of table 4.

Table 4: MSPE for the two variants

	variant 1	*variant 2*
MSPE for stock price	17.455	15.4
MSPE for output	0.545	0.545

The MSPE improves for the stock price as one moves from the variant 1 to variant 2 where in the variant 2 a time series for the real balances is employed and a term for the risk premium is implicitly estimated. We want to note, however, that we cannot measure the size of the risk premium directly from our coefficient $\bar{\epsilon}$, since we are

[20]The data for the time series of $m_t - p_t$ are obtained from Citibase (1995).

using detrended data.[21] Note that the MSPE for output does not change since the estimated parameters for the output equation is independent of the specification of the stock price equation.

7 Stochastic simulations and impulse response functions

In order to evaluate further the model's match with the data we, first, perform some simulation experiments with the estimated non-linear model and second, estimate a VAR and compare the impulse response functions obtained from the data with those from the model. The main aim of the simulations is to see how well the estimated model can reproduce the stylized facts on real and financial times series data as presented in section 2 . We use the estimated parameters reported in Table 3 for the variant 2 referred to in section 6, namely the estimation that used the historical time series for real balances. In section 7.1 we use the estimated parameters in a stochastic version of the original set of differential equations (5..12)- (5..14) for y, q and x. Here we focus on the correlation and autocorrelation features and the financial statistics such as the volatility of asset prices and returns, equity premium and the Sharpe-ratio of the model. In section 7.2 we study the impulse-response functions from a VAR estimation of the data and compare these with the responses of the model to similar shocks.

7.1 Stochastic simulations

We suppose that external noise processes are impinging on both the output market and the stock market. We capture the resulting non-linear stochastic dynamics by writing stochastic differential equation versions of (5..12)-(5..14) as

$$dy = \kappa_y(aq - by + g)dt + s_y dw_y, \tag{7.1}$$

$$dq = \kappa_q f(\xi)dt + s_q dw_q, \tag{7.2}$$

$$dx = \kappa_x(\kappa_q f(\xi) - x)dt + \kappa_x s_q dw_q, \tag{7.3}$$

where we set $\xi \equiv (x + \alpha_0 + \alpha_1 y)/q - cy + \delta'$; dw_y, dw_q are assumed to be increments of independent Wiener processes and s_y, s_q denote the standard deviations per unit

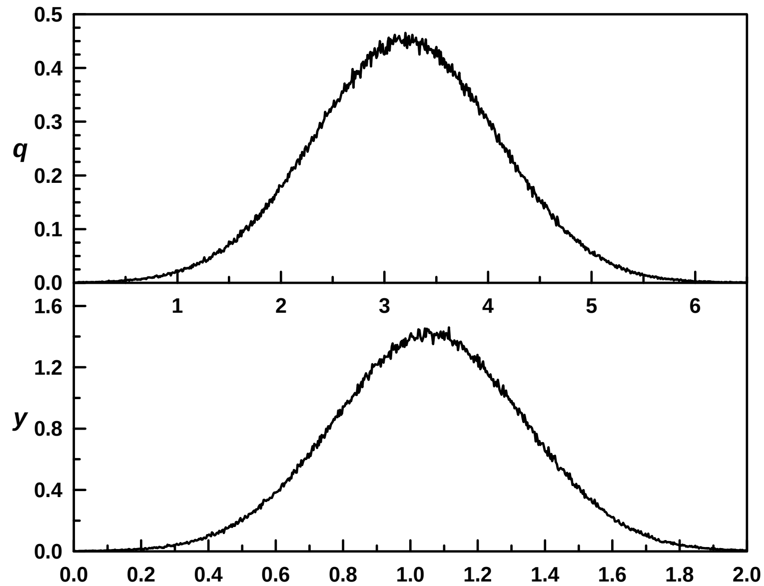
[21] We also undertook estimations for first differenced data but when we performed the estimations by including the real balances as exogenous variable the estimations always became unstable, so we abandoned this approach. The instability of the estimates is presumably due to the fact that first differencing of the time series, particularly for the stock price, makes the time series very volatile.

time (here annualised) of the direct changes in y and changes in q over dt due to the external noise processes.

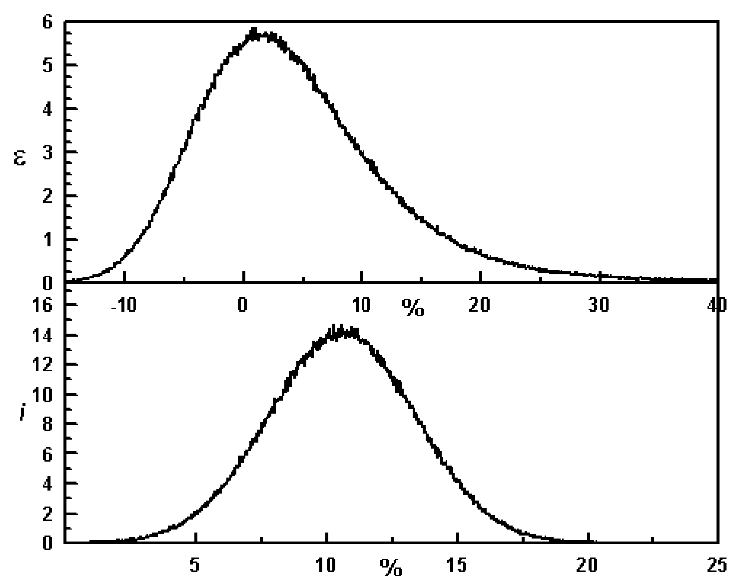
We have used the estimates in table 3, together with the historical time series for real balances and generated 1,000,000 paths for dw_y and dw_q over a period of 35 years taking $dt =$ one month . Along each path we also calculate the interest rate and equity premium according to equations (3.3) and (3.5) respectively. In our simulations we have taken the standard deviations of the external noise processes to be

$$s_y = 0.022 \qquad \text{and} \qquad s_q = 0.154. \tag{7.4}$$

We stress that s_y and s_q do not correspond to the standard deviation of the y and q distributions since the external noises feed through equations (7.1)-(7.3) which are interlinked, here in a nonlinear manner. In fact, from the simulation, we obtained different values of standard deviation from the q and y distributions with the corresponding annualised values being $\sigma_y = 0.0473$ and $\sigma_q = 0.150$. They are of the same order and with a similar ratio compared with the annualised values calculated from the historical time series of y and q respectively namely $\sigma_y = 0.051$ and $\sigma_q = 0.20$.



(a): Distribution of q and y at final time



(b): Distribution of the premium and interest rate at final time

Figure 5:

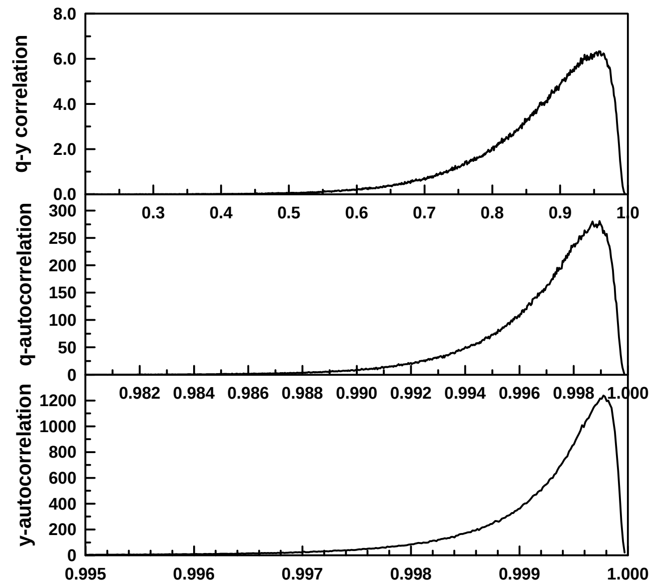


Figure 6: Distribution of $q-y$ correlation q autocorrelation and y autocorrelation

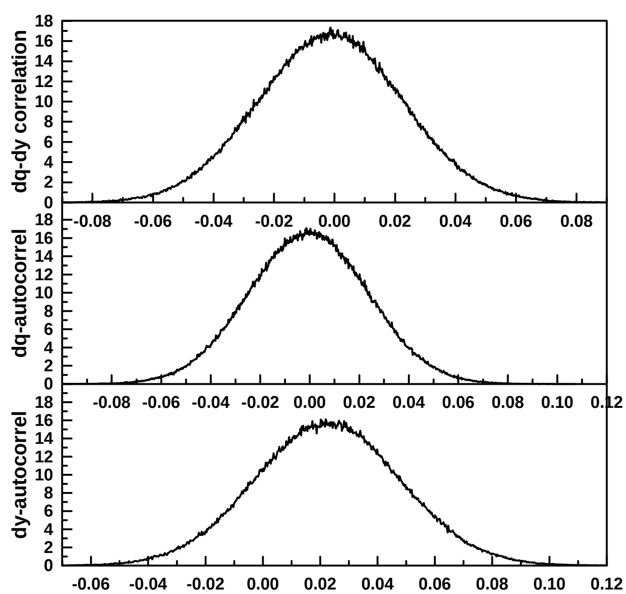


Figure 7: Distribution of correlation between the changes in q and y, the autocorrelation for changes in q and changes in y

From the 1,000,000 simulations we calculated a number of statistics. In figure 5 (a)-(b) we plot the distribution of y, q, i and ϵ at final time. The distribution of y, q and ϵ seem to be centered around reasonable values, however the nominal interest rate seems to be centered around rather high values, perhaps reflecting the high interest rates experienced during the 1980s.

Figure 6 displays the distribution of the $q - y$ correlation as well as the y autocorrelation and q autocorrelation, calculated along the 1,000,000 simulated paths. Figure 7 displays the correlation between the changes in q and y, and the auto-correlation of changes in q and changes in y.

Table 5 gives some comparative statistics on the performance of our estimated nonlinear model, the baseline RBC model and a modified RBC model by Boldrin, Christiano and Fisher (2001).

*Table 5 : Financial statistics of the different models**

Statistic	*Data*	$SSMM$	$BLRBC$	$BCFRBC$
$\sigma_y(\%p.a.)$	5.20	4.73	2.11	1.97
$\sigma_q(\%p.a.)$	20.04	15.0	0.40	18.40
$EP(\%p.a.)$	6.63	3.50	0.001	6.63
SR	0.34	0.45	0.002	0.36
$\rho(y,q)$	0.002	0.95	–	0.16
$\rho(y)$	0.834	0.99	–	–
$\rho(q)$	0.877	0.99	–	–
$\rho(\Delta y, \Delta q)$	-0.019	0.00	–	–
$\rho(\Delta y)$	0.182	0.02	0.02	0.36
$\rho(\Delta q)$	0.208	0.00	–	–

* σ_x denotes the standard deviation of the variable x, annualized, in percent; EP the equity premium; SR the Sharpe-ratio, in percent; $\rho(x, y)$ the correlation between x and y (both variables detrended) and $\rho(x)$ as well $\rho(\Delta x)$ denote autocorrelations. The sign "– " means not available or applicable.

In the Data column of the table 5, σ_y and σ_q as well as $\rho(x)$ and $\rho(\Delta x)$ are computed from monthly data, using the Hamilton and Lin (1996) data set; EP and SR are taken from Boldrin, Christiano and Fisher (2001);[22] $\rho(\Delta x)$ means autocorrelation of first differences of the detrended data. The column $SSMM$ represents results from the stochastic simulations of the macro model (7..1)-(7..3). The simulated results are obtained from 1,000,000 replications of an Euler-Maruyama discretisation of (7.1)-(7.3). For the simulations a ratio of s_y/s_q was chosen as input such that the resulting output of the ratio σ_y/σ_q corresponded roughly to the σ_y/σ_q as obtained for the actual time series, reported in column 2. Based on the σ_q as obtained from

[22] To what extent the Sharpe-ratio, SR, may be time varying, i.e. vary over the business cycle, is explored in Woehrmann, Semmler and Lettau (2001). For our purpose it suffices to presume a constant Sharpe-ratio.

the simulated series, the quarterly σ_q is roughly 0.075 or 7.5 % . This was used for computing the Sharpe-ratio, SR, of our simulations. The computed SR turns out to be 0.45 and seems a bit too high when compared to that for the data. Yet, we note that SR =0.45 can only be used as an indicator of the Sharpe-ratio, since our indicator of the equity premium of 3.5% is not an actual equity premium from non-stationary actual time series, but rather obtained from the estimated and simulated time series which were both detrended. Since, however, the standard deviation σ_q =15% and the EP=3.5% are obtained from the 1,000,000 simulations, the value of SR obtained can be interpreted as a reasonably good indicator of the SR.

The column $BLRBC$ represents results for the baseline RBC model. We use here the statistics for the baseline RBC model as reported by Boldrin, Christiano and Fisher (2001). As can be observed the basic statistics for the asset price — the standard deviation of the equity price is only $\sigma_q = 0.40$ percent in column 4 — cannot be matched at all even if a technology shock with standard deviation of $\sigma_y = 2.11$ percent is used as input in the computation of asset prices in the context of the baseline RBC model. Therefore , also the EP and the SR come out much too small.

The column $BCFRBC$ reports the statistics from an modified RBC model which takes into account habit formation in the utility function, adjustment cost of capital and a two sector model. The statistics are also annualized and in percentages. The modified model is more successful as far as the financial statistics, EP and the SR, are concerned but as the results in Boldrin, Christiano and Fisher (2001) show, the simulated improved model fails along some real dimensions. Note that their $\rho(\Delta y)$ are computations from growth rates and therefore have an interpretation different from those in the $SSMM$ column. Note also that their results on the standard deviations of the actual time series σ_y and σ_q are different from those for $SSMM$, since they employ a different time period and they use growth rates. Their simulated results are obtained from 500 replications, whereas as stated above we have used 1,000,000 replications.

7.2 VAR and Impulse-Response functions

Another way to study how the model matches the data is to compare impulse-response functions from historical data and from our nonlinear model. First, we undertake the VAR estimation with first differenced data and then study the impulse-response functions for the impact effect of shocks as well as the cumulative impulse-response functions, which give us the level effects.

In our model the interest rate is determined implicitly when the money supply – in our case real balance – is given. In the subsequent VAR we will, however, directly employ the interest rate.[23] Thus, the variables included in the VAR model are monthly industrial production (PR), monthly T-bills (TB) and stock prices (ST), with PR and ST entering in first differences (DPR and DST).[24] For the sample period from 1961:01 to 1993:06 the Akaike information criterion suggests a lag length of two. We employ Cholesky decomposition to orthogonalize the residuals with the order of the variables being as listed above. By doing so, we assume that stock prices respond immediately to all shocks to the system; T-bills respond immediately to own shocks and shocks to production, but only with delay to shock in stock prices; and only own shocks have simultaneous effects on production.

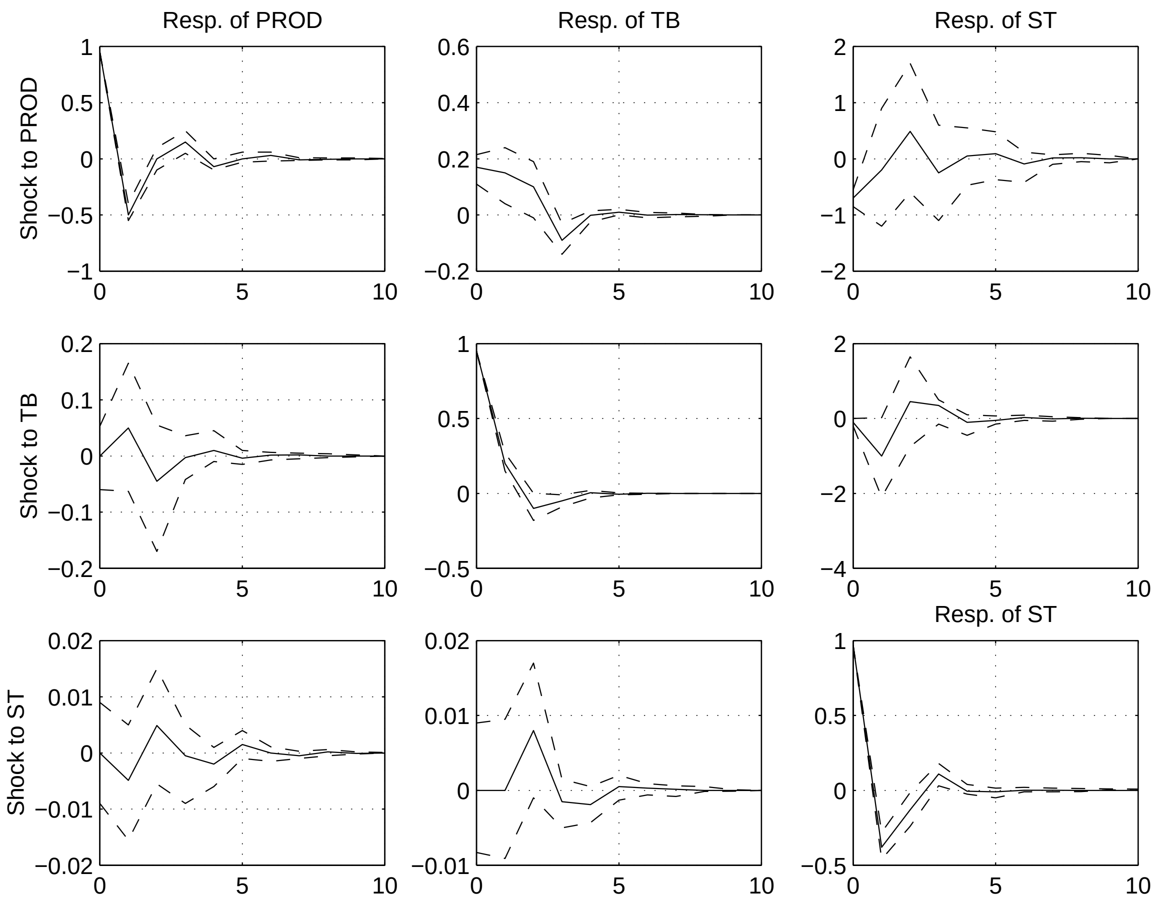


Figure 8: Impulse-response for first differences

Figure 8 shows the nine unit-impulse-response functions (solid lines) implied by

[23]It appears to us a better procedure to employ the interest rate instead of the money supply, since the latter may, as has been shown in many papers, empirically exhibit a very unstable relationship to the interest rate.

[24]The source of the data is the same as employed for the estimations reported in tables 2 and 3. The monthly T-Bill rate is also from the Hamilton and Lin (1996) data set.

the estimated VAR model. To judge the significance of these responses we computed asymptotic two-standard-deviation confidence bands (dashed lines) following Mittnik and Zadrozny (1993). The estimated impulse responses are as follows. A positive shock to DPR has only short-run effects on DPR, which become significantly negative after two periods and then die out; the short-run reaction of TB is significantly positive but vanishes after three periods; the simultaneous response to DST is negative, while lagged responses are insignificant. A shock to TB affects TB itself positively for one lag, but the effect disappears beyond the second period. The initial response of DST to the interest rate shock is, as one would expect, negative. It is (marginally) significant, but then practically disappears after lag one. There are responses of DPR to the interest rate shock but they appear as not significant.[25] Finally, a positive shock to the stock returns (DST) is followed by a significantly negative return in the following period which is about a quarter of the size of the shock, whereas higher-order responses are practically zero. The responses of DPR and DTB to DST are also not significant.

[25] It is well known in the empirical literature on the impact of the interest rate on output that the output reacts to interest rate changes only with a delay. Therefore, the number of lags underlying our VAR model may not sufficiently represent a lag structure that is needed to see an impact of the interest rate on output.

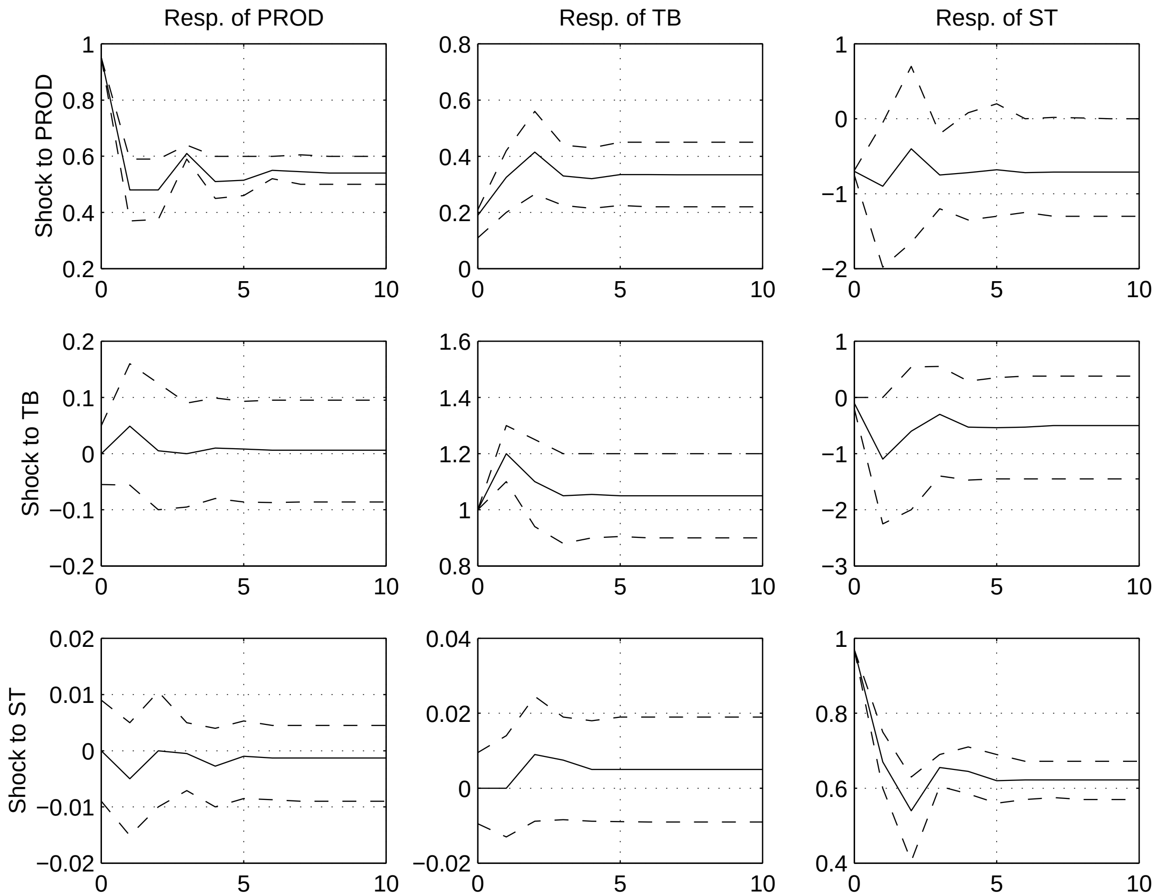


Figure 9: Cumulative impulse-responses

Figure 9 displays the cumulative impulse-responses or so-called step responses. For the differenced variables DPR and DST the step responses indicate the effects in terms of their levels, PR and ST. The results suggest that a one-time shock to production has a significant positive long-run effect on production, which is about 55% of the original shock. The response in stock prices is negative, but only marginally significant. A shock to TB does not appear to affect production significantly. The stock prices, again, as one would expect, react negatively to the shock in the TB. For lags zero and one we have (marginal) significance. A positive shock to stock prices has lasting positive and significant affects on stock prices with about 60% of the original shock persisting in the long-run; the responses of production are insignificant.

Altogether, we see that shocks in differences and levels exhibit strong autoregressive effects. Although the cross-effects from output to the other variables as well as a cross-effect of the interest rate to stock price and output appears to be observable, the cross-effects from stock price to the other variables are weak or insignificant. As in other empirical studies have shown, the shock to the stock price do not appear to effect output significantly.[26] A similar result of no lasting effect of asset price volatility on output is also shown in Lettau and Ludvigson (2000).[27]

An impulse-response study can also be undertaken by employing our dynamic model (3.11-(3.13). We report results from the model's impulse-responses when the estimated parameters of table 3 are employed.

[26]Note, however, that in the context of linear impulse-response functions we cannot distinguish between the possibly different effects of large and small shocks, for example, of the stock price on output. To properly study such effects, nonlinear impulse-response functions would have to be employed. In our context we think of the above employed impulse -response functions as tools to study the linearized behavior of our model about an equilibrium. There may exist a transmission mechanism, for example, exerted through the credit market as suggested by the work on the financial accelerator, that may generate a strong effect of asset price shocks on output, if the asset price shocks are large, but which cannot be captured in linear impulse -response studies.

[27]Lettau and Ludvigson (2000) use an VECM to estimate the stock price effect on consumption and find no lasting effect of stock prices on consumption but only a transitional effect.

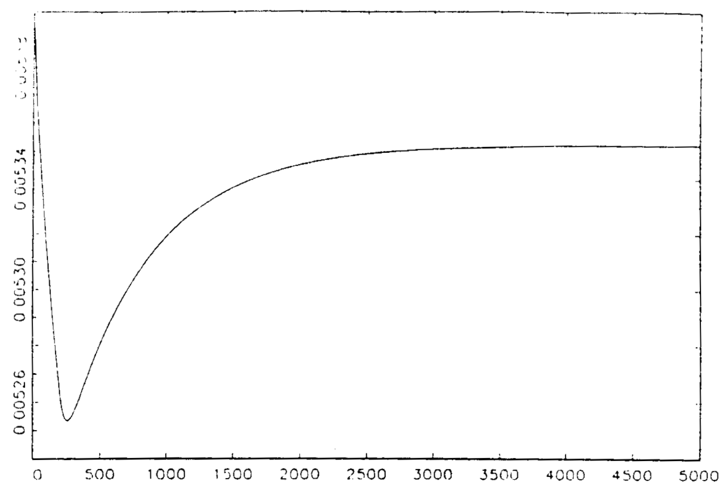


Figure 10: Stock price response to interest rate shocks, nonlinear model, US

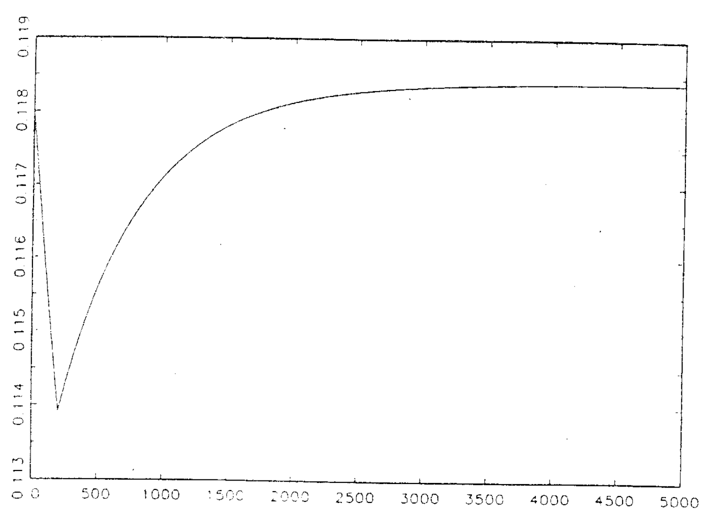


Figure 11: Output response to interest rate shocks, nonlinear model

In Figures 10 and 11 the response of the stock price and output are depicted for shocks to the equity market. In the context of our model the shock to the equity price is set up in such a way that it reflects a shock to the interest rate which, in our set-up gradually affects the equity price. In the model simulation we employ persistent shocks for a number of periods.[28] As can be observed the features of the impulse - response functions obtained from the above VAR for first differences,

[28]Given our small step size, we have chosen persistent shocks of 100 periods' duration in order to make the effect on the stock price visible.

Figures 8 and, in particular, the cumulative impulse-response functions representing level effects, Figure 9, are matched by the system simulation employing estimated parameters in our nonlinear model. In particular, the positive interest rate shock moves the stock price down but also the output falls.

8 Conclusions

In the paper we have generalized a well-known model of the real and stock market interaction originating in the work by Blanchard (1981). In contrast to the perfect foresight-jump-variable model by Blanchard we allow for imperfect asset substitution between stocks and bonds in the asset market and for gradual portfolio adjustment. We model expectations as adaptive with perfect foresight being a limiting case and analyze the type of dynamics that can arise in the full three-dimensional system, and contrast that with the Blanchard (1981) limit case of perfect foresight. The model we have studied can also be viewed as an alternative to RBC models with an asset market. In order to empirically apply our continuous time model we use the Euler-Maruyama scheme to obtain a discrete time approximation of the solution path as well as for the estimation of the discretized continuous time model. A discretization in terms of observable variables is proposed and an estimation procedure for a nonlinear bivariate system in stock price and output suggested. A direct estimation of our proposed bivariate model is undertaken using nonlinear least squares. The results of the latter procedure suggest the existence of nonlinearities in the real and stock market interaction. In the context of our model we can also make some inference on the equity premium and the Sharpe-ratio. We have performed some simulation experiments on a stochastic version of our estimated nonlinear model and compared the resulting statistics with those obtained from the RBC model. In addition, following Mittnik and Zadrozny (1993) a VAR with confidence bands for historical data is estimated and cumulative impulse-response functions compared to the model's impulse response functions. Overall the stochastic version of our estimated nonlinear model performs reasonably well on most of the measures we have discussed. Finally, we want to note that our approach could be further developed to study the effects of shocks, for example, monetary policy or exchange rate shocks on the interest rate, stock price and output in the context of the more fully developed nonlinear dynamic macromodels of the type discussed in Chiarella and Flaschel (2000) and Chiarella, Flaschel, Groh and Semmmler (2000).

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Appendix

A Stability Analysis of the Blanchard Model

The Jacobian of the differential equation system (3.11) and (3.21) at an equilibrium point $(\bar{y}, \bar{q})$ is easily calculated to be

$$J_2 = \begin{pmatrix} -\kappa_y b & \kappa_y a \\ c\bar{q} - \alpha_1 & c\bar{y} - h\delta' \end{pmatrix}.$$

Thus the determinant of the Jacobian is given by

$$|J_2| = -\kappa_y[b(c\bar{y} - h\delta') + a(c\bar{q} - \alpha_1)].$$

In the cases considered in figure 2 it is always the case that $c\bar{y} - h\delta > 0$. At the equilibrium E_b^+ in figure 2(a) we have $c\bar{q} - \alpha_1 > 0$ hence $|J_2| < 0$ at this equilibrium which is thus a saddle point.

At the equilibrium E_g^+ in figure 2(b) the fact that the slope of $\dot{y} = 0$ is less than the slope of $\dot{q} = 0$ can be expressed algebraically as

$$\frac{b}{a} > \frac{\alpha_1 - c\bar{q}}{c\bar{y} - h\delta}.$$

This latter condition implies that $|J_2| < 0$ at the equilibrium E_g^+ which is also a saddle point.

B The Characteristic Equation of the Generalized Blanchard Model

Consider first of all the calculation of $|J_3|$, where J_3 is defined in equation (4.1) of the main text. By an elementary row operation we find that

$$\begin{aligned} |J_3| &= \begin{vmatrix} -\kappa_y b & \kappa_y a & 0 \\ \lambda\kappa_q\left(\frac{\alpha_1}{q} - c\right) & -\lambda\kappa_q(cy - \delta')/q & \lambda\kappa_q/q \\ 0 & 0 & -\kappa_x \end{vmatrix} \\ &= -\kappa_x[\lambda\kappa_y\kappa_q b(cy - \delta')/q - \lambda\kappa_y\kappa_q a(\alpha_1/q - c)] \\ &= -\lambda\kappa_y\kappa_q\kappa_x[b(cy - \delta')/q - a(\alpha_1/q - c)] \\ &= \frac{-\lambda}{q}\kappa_q\kappa_x\kappa_y[b(cy - \delta') + a(cq - \alpha_1)] \\ &= -\lambda\kappa_q\kappa_q|J_2|/q, \end{aligned}$$

where $|J_2|$ is given in appendix 1.

Using the analysis of the equilibrium points E_g^+, E_b^+ given in appendix 1, we can assert that at these equilibrium points

$$|J_3| > 0,$$

which indicates that the real parts of the eigenvalues of J_3 have the sign distribution $(-,-,-)$ or $(-,+,+)$.