Calculation of Differential Inductances of a Tubular Linear PM Actuator

Haiwei Lu, Jianguo Zhu and Youguang Guo
Faculty of Engineering, University of Technology, Sydney, Australia

The inductances of a multi-phase electrical machine are key parameters for the performance analysis of the machine. Commonly, apparent inductances are employed. This, however, is incorrect when the non-linear characteristic of the magnetic core of the machine is considered in dynamic analysis. Instead, the differential inductances should be employed in non-linear analysis. A numerical method for calculating the differential inductances is presented in the paper and it is used for the prediction of the inductances of a tubular linear permanent magnet (PM) actuator. The measurement of the inductances of the prototype is also carried out for the verification of the method.

Key Words: Differential inductance, Numerical method, Permanent magnet, Linear actuator.

1. Introduction

The phase inductances are those of the most important parameters of an electric machine. The machine modeling and performance analysis greatly depend on the valid inductances obtained. Usually, the inductance profiles in an electric machine are complicated functions of machine geometry, winding currents, saturation effect of the magnetic core, and the displacement between the stationary and moving windings or magnets. Accurate dynamic analysis of an electric machine requires the profiles of machine phase windings' self and mutual inductances with respect to the rotor positions so as to capture their variations due to machine movement and load conditions.

Nowadays, numerical solutions are widely used in electrical machine analysis and the machine inductances can be readily obtained by those methods. In general, the apparent inductances are usually employed in most applications. This, however, is incorrect when the non-linear characteristic of the magnetic core is considered in dynamic analysis. For the non-linear analysis, the differential inductances should be used.

This paper presents a numerical method for the calculation of the differential inductances and the method was applied for the prediction of the inductances of a newly developed tubular linear permanent magnet (PM) actuator as a function of the position of the moving armature. The results show the effect of the magnetic saliencies due to both the machine structure and magnetic saturation, as well as the end effect due to the limited length of the actuator. The measurement of inductances of a prototype provides a verification of the proposed method.

Correspondence: Haiwei Lu, Faculty of Engineering, University of Technology, Sydney, PO Box 123, Broadway, NSW 2007, Australia,
email: haiwei@eng.uts.edu.au

2. Apparent and Differential Inductances

An electrical machine can be considered as a system with a set of N-discrete, coupled windings. The flux linkage of each winding is a complicated function of the winding currents, machine geometry, material properties, and the angle, θ, between the stationary and moving windings [1]. For the j-th winding, the flux linkage can be represented as follows,

$$\lambda_j = \lambda_j(i_1, i_2, \ldots, i_j, \ldots i_N, \theta) \tag{1}$$

And the apparent inductance between windings j and k, L_{jk}^{app}, is defined as,

$$L_{jk}^{app} = \lambda_{jk} / i_k \tag{2}$$

where λ_{jk} is the k-th component of λ_j that is produced by the current in the k-th winding i_k. When k=j, L_{jj}^{app} is the apparent self-inductance of the winding j.

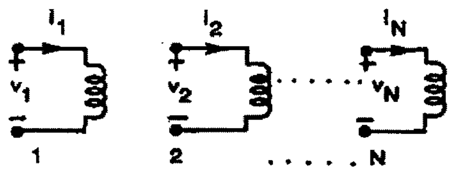


Fig. 1. Electromagnetic device with N-discrete, coupled windings

On the other hand, the differential inductance is defined as the partial derivative of the flux linkage with respect to the corresponding current. The differential mutual inductance between the windings j and k is

$$L_{jk}^{diff} = \partial\lambda_j / \partial i_k \qquad (3)$$

and the differential self-inductance is

$$L_{jj}^{diff} = \partial\lambda_j / \partial i_j \qquad (4)$$

It can be seen that the apparent inductance and differential inductance are identical only under linear or unsaturated condition.

3. Inductances in Dynamic Model

The following equation is commonly used for analyzing the dynamic performance of a multi-phase electromagnetic device.

$$v_j = R_j i_j + d\lambda_j / dt \ , \quad j = 1, 2, ..., N \qquad (5)$$

where R_j is the resistance of phase j, and v_j the phase voltage. According to (1), the derivative of the flux linkage in (5) can be further expressed as,

$$d\lambda_j / dt = \sum_{k=1}^{N} \frac{\partial\lambda_j}{\partial i_k}\frac{di_k}{dt} + \omega\frac{\partial\lambda_j}{\partial\theta} = \sum_{k=1}^{N} L_{jk}^{diff}\frac{di_k}{dt} + \omega\frac{\partial\lambda_j}{\partial\theta} \qquad (6)$$

where $\omega = d\theta/dt$.

Therefore, the differential inductances should be used in the dynamic model of the electromagnetic device.

4. Computation of Differential Inductances

4.1 Energy and current perturbation

One of the methods for computing the differential inductances is to use the finite element (FE) magnetic field solutions in conjunction with energy and current (E/C) perturbation technique.

For a conservative magnetic system with N windings, as shown in Fig. 1, the sum of the magnetic energy, W, and co-energy, W_c, is given by,

$$W + W_c = \lambda_1 i_1 + \lambda_2 i_2 + \cdots + \lambda_N i_N \qquad (7)$$

The magnetic energy in a non-linear magnetic system is illustrated in Fig. 2 for a single coil. The change in the stored magnetic co-energy due to an infinitesimal change in the winding current at a fixed set of flux linkages can be obtained as,

$$dW_c = \lambda_1 di_1 + \lambda_2 di_2 + \cdots + \lambda_N di_N \qquad (8)$$

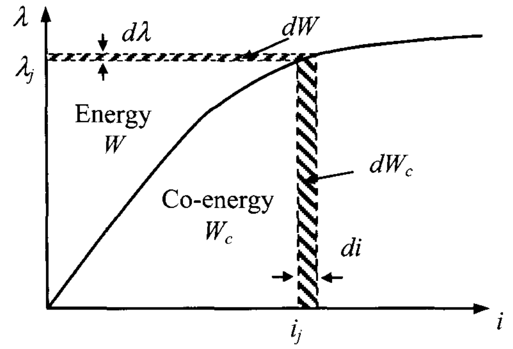


Fig. 2. Magnetic energy in a non-linear system

This differential can also be expressed as,

$$dW_c = \frac{\partial W_c}{\partial i_1}di_1 + \frac{\partial W_c}{\partial i_2}di_2 + \cdots + \frac{\partial W_c}{\partial i_N}di_N \qquad (9)$$

Comparing the terms in (8) and (9) reveals that,

$$\lambda_j = \partial W_c / \partial i_j \ , \quad j = 1, 2, \ldots, N \qquad (10)$$

By differentiating both sides of (10) with respect to the k-th winding current, the differential inductance defined in (3) can be acquired by,

$$L_{jk}^{diff} = \partial^2 W_c / \partial i_j \partial i_k \ , \quad j, k = 1, 2, \ldots, N \qquad (11)$$

In case of self-inductance, (11) can be rewritten as,

$$L_{jj}^{diff} = \partial^2 W_c / \partial^2 i_j \ , \quad j = 1, 2, \ldots, N \qquad (12)$$

4.2 Numerical solution of E/C perturbation

In order to make the E/C technique to be used in numerical solutions, discretization of the above method is required. By applying the central divided difference to (11) and (12), the following can be yielded,

$$\begin{aligned} L_{jk}^{diff} \cong [&W_c(i_j + \Delta i_j, i_k + \Delta i_k) - W_c(i_j - \Delta i_j, i_k + \Delta i_k) \\ &-W_c(i_j + \Delta i_j, i_k - \Delta i_k) \\ &+W_c(i_j - \Delta i_j, i_k - \Delta i_k)] / 4\Delta i_j \Delta i_k \end{aligned} \qquad (13)$$

$$L_{jj}^{diff} \cong [W_c(i_j + \Delta i_j) - 2W_c(i_j) + W_c(i_j - \Delta i_j)] / (\Delta i_j)^2 \qquad (14)$$

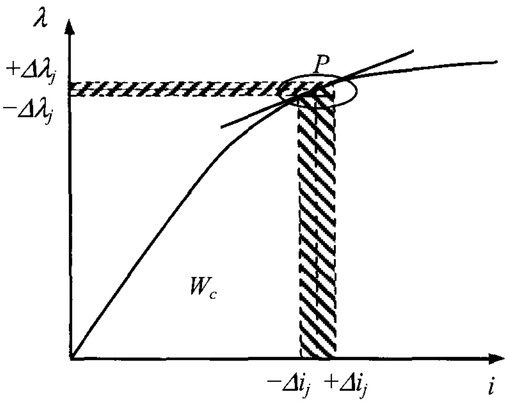
By using (13) and (14), the differential inductances can be readily obtained by computing the co-energy of the machine under the conditions described above. Nonetheless, precision problem might arise when dealing with the machine containing the permanent magnets, and this will lead to the method not to be directly applicable in numerical solutions.

In case of a PM machine, the magnetic field is dominated by that generated by the permanent magnets. The field produced by the winding currents only accounts for a small amount, normally less than 10 %. Therefore, the magnetic energy produced by the winding current is considerably small, especially for the current perturbation. During numerical computation, the incremental energy generated by current perturbation, or the differences between the co-energies, *i.e.* $W_c(i)$, $W_c(i+\Delta i)$ and $W_c(i-\Delta i)$, are overwhelmed by the numerical fault and are not able to be used for the inductances calculation.

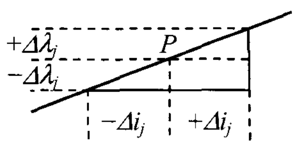
One of the possible resolutions is to eliminate the huge co-energy produced by the permanent magnets before the calculation of the co-energy caused by current perturbations. Consider the current perturbation in a single coil shown in Fig. 3, where P is the operational point set by the permanent magnets. Because of the small value of current perturbation, the non-linear λ-i curve can be segmentally lineariszed near P, as shown in Fig. 3, and the slope of the line is known as the differential permeability of the magnetic core at the operational point P. By applying (14) in conjunction with Fig. 3(b), the following relationship can be obtained:

$$L_{jj}^{diff} = 2\Delta W_c \Big/ \Delta i_j^2 \tag{15}$$

where $\Delta W_c = \Delta\lambda_j \Delta i_j / 2$.



(a)



(b)

Fig. 3. Numerical E/C perturbation

According to the derived results, it can be concluded that the differential inductance can be obtained by finding the co-energy of a linear electromagnetic model, which is obtained by means of linearizing the magnetic core of the non-linear model by setting its permeability with the differential permeability at the operational point P. By the method, the permanent magnets in the system can be eliminated and only the co-energy of the current is to be calculated during the solution.

However, the operational point can be different within different parts of the machine due to the non-uniformity of the magnetic field. For higher accuracy, the non-linear magnetic core must be discretized into a set of linear magnetic regions with different permeabilties at their own operational points.

To apply the method in FE analysis, non-linear field solution is firstly applied to the model at its normal operational conditions including the permanent magnets and winding currents, and the magnetic operational point and the corresponding differential permeability in each non-linear region are found. The system is then re-configured to a linear system by re-defining all the previous non-linear materials to the linear materials with the differential permeability and "switching off" the permanent magnets (setting the coercive force to zero). Finally, a linear field solution is performed to obtain the co-energy produced by a current and hence the differential inductances. Due to the linearity, the amplitude of the current is no longer relevant to the inductances and the method can be applied in either 2D or 3D FE solutions.

5. Inductances of a Tubular Linear PM Actuator

The above numerical E/C method is applied to predict the inductances of a newly developed tubular linear permanent magnet actuator. Fig. 4 shows the schematic structure of the proposed tubular linear actuator [2]. The outer diameter of the actuator is 32 mm and the length of the stator is 40 mm. There are twelve annular windings mounted inside the stator core. Each phase contains four windings connected in series.

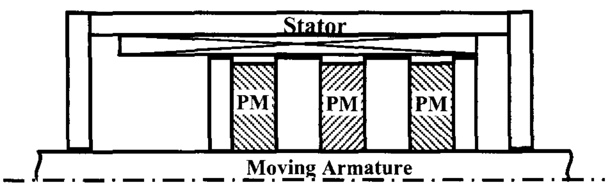


Fig. 4. Schematic structure of the proposed tubular linear PM actuator

An FE model of the motor considering the non-linear properties of magnetic core is established for the analysis of the phase inductances. Factors related to the construction of the prototype are considered during modeling, such as the lamination factor of the magnetic core, the tolerances required for the machining and installation, etc., so that the model could reproduce the reality as much as possible.

5.1 Computed inductances

When computing the inductances, the parts of the actuator that are made up of non-linear magnetic material, such as the stator cylinder and pole pieces on the armature, are discretized by the elements generated by the FE method. Via non-linear solution, the element-based saturation characteristics of these non-linear magnetic parts at the operational point are obtained. Then the model is linearized and the phase inductances including self and mutual inductances are computed via linear solution.

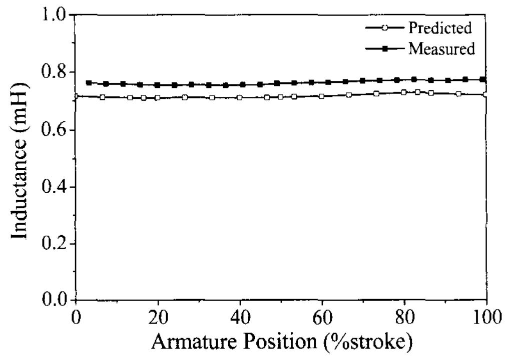
The computed results are shown in Fig. 5. It can be seen that the inductances vary with respect to the position of the armature. The inductances increase when the armature approaches the corresponding phase windings. The magnetic saliency, however, is not remarkable because the magnetic core is not saturated so much.

5.2 Measured inductances

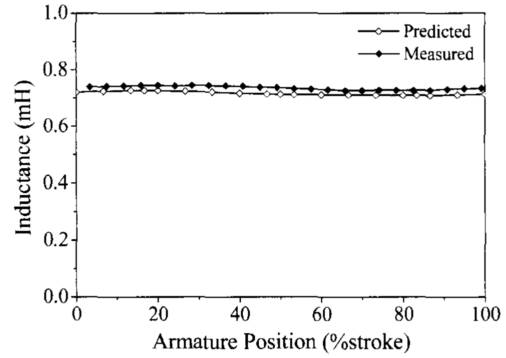
The inductances of the prototype are measured at different armature positions. For the sake of comparison, the results are plotted by solid symbols in the same figure with the predicted values. It can be seen that the predicted inductances are close to the measured values. The variation of the self-inductance caused by the armature position can be clearly seen in both predicted and measured results. The variation of mutual inductances, however, is not as much as predicted.

6. Conclusion

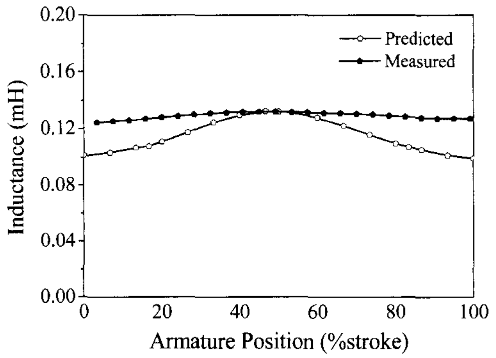
A numerical solution for the differential inductance computation is discussed. The method is successfully implemented based on the FE analysis and is applied to the prediction of inductances of a newly developed tubular linear PM actuator. Comparison between the predicted and the measured value shows the validity of the proposed solution.



(a) Self-inductance of phase *a*



(b) Self-inductance of phase *b*



(c) Mutual inductance between phase *a* and *b*

Fig. 5. Inductances of the proposed linear actuator

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