PROPAGATION IN PHOTONIC CRYSTAL WAVEGUIDES

L C Botten[1], A A Asatryan[1], T N Langtry[1], C M de Sterke[2] and R C McPhedran[2]

[1]Department of Mathematical Sciences
University of Technology, Sydney
PO Box 123, Broadway, NSW 2007
Ph: (02) 9514 2247, Fax: (02) 9514 2260
Lindsay.Botten@uts.edu.au

[2]School of Physics
The University of Sydney
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Abstract

An essential component in the design and fabrication of large scale integrated optical circuits is the photonic crystal (PC) waveguide, a device which can channel light, with minimal loss, around tight bends using band gap effects. In this paper, we present a rigorous formulation for the coupling and propagation of waves in a PC crystal waveguide and show that, for sufficiently long guides, it is possible to generate simple, exact quasi-analytic forms for the energy properties of the guides.

1. Introduction

Photonic crystals, with their capacity to inhibit propagation for ranges of frequencies in some or all directions, will likely form the building blocks of components that will figure in future optical integrated circuits [1]. The devices in such circuits will be linked by waveguides (eg, Fig. 1) and significant work is being devoted to understanding propagation in guides, and reflection losses that occur at bends. Much of this effort has been computational and has failed to yield significant insight into the processes involved. To date, the modelling of waveguide propagation has been limited to coupled mode theory [2] which has provided a qualitative description of the transmission properties of guides, although the limits of its applicability are not clear.

Closely related is the issue of coupling radiation into and out of the guide. The solution of this problem requires a comprehensive knowledge of the modes of the guide and to date there is no literature that solves the full circuit problem involving coupling into the guide, propagation and coupling out of the guide. The paper outlines a rigorous theory which accurately and efficiently models the process and can be extended to handle the interfacing of guides to other devices.

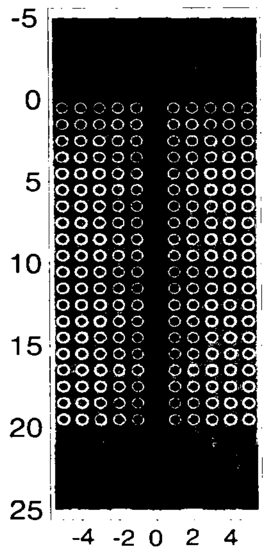

Fig. 1 Intensity in a 21 layer guide (plane-wave illumination), wavelength $\lambda / d = 3.2$, radius $a/d = 0.3$, index $\nu = 3$.

2. Theoretical Formulation

2.1 Overview

The key step is the computation of Bloch modes of the crystal — full solutions of the wave equation for an infinite lattice — which provide a complete basis for the solution of field problems. We consider a rectangular symmetric crystal in which the component layers are gratings of dielectric cylinders. Between the layers, the modes are expanded as a superposition of down ($\mathbf{f}_-$) and up ($\mathbf{f}_+$) travelling plane waves, where the $\mathbf{f}_\mp$ are

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vectors of plane wave coefficients. Bloch modes occur in forward-backward pairs and are solutions of the eigenvalue equation

$$\mathscr{T}\begin{pmatrix}\mathbf{f}_-\\ \mathbf{f}_+\end{pmatrix} = \mu\begin{pmatrix}\mathbf{f}_-\\ \mathbf{f}_+\end{pmatrix},\ \text{where } \mathscr{T} = \begin{pmatrix}\mathbf{T}-\mathbf{RT}^{-1}\mathbf{R} & \mathbf{RT}^{-1}\\ -\mathbf{T}^{-1}\mathbf{R} & \mathbf{T}^{-1}\end{pmatrix}$$

is the inter-layer translation operator and $\mathbf{R}$ and $\mathbf{T}$ are the reflection and transmission matrices for the component grating layers — computed using multipole techniques[3]. The forward and backward propagating states, respectively $\left(\mathbf{F}_-^T\ \ \mathbf{F}_+^T\right)^T$ and $\left(\mathbf{F}_+^T\ \ \mathbf{F}_-^T\right)^T$, corresponding to the eigenvalues $\Lambda = \mathrm{diag}\left(\mu_j\right)$ and Λ^{-1}, are highlighted by the diagonalisation of $\mathscr{T} = \begin{pmatrix}\mathbf{F}_- & \mathbf{F}_+\\ \mathbf{F}_+ & \mathbf{F}_-\end{pmatrix}\begin{pmatrix}\Lambda & 0\\ 0 & \Lambda^{-1}\end{pmatrix}\begin{pmatrix}\mathbf{F}_- & \mathbf{F}_+\\ \mathbf{F}_+ & \mathbf{F}_-\end{pmatrix}^{-1}$, the translation operator. Here, $\mathbf{F}_\mp$ are matrices comprising the columns of eigenvectors $\mathbf{f}_\mp$ from the solution of the eigenvalue equations above. This theory underpins the study of propagation in a finite slab which can be modelled with two key parameters: the propagation constants Λ above, and the reflection matrix for a semi-infinite crystal $\mathbf{R}_\infty = \mathbf{F}_+\mathbf{F}_-^{-1}$ [4].

2.2 Properties of a Single Guide

We model a single guide as a cavity of width h between two identical semi-infinite crystals characterised by their $\mathbf{R}_\infty$. Consistency conditions for up- and down-travelling waves in the horizontal cavity then yield the dispersion equation $\det\mathbf{G}(k,\alpha_0) = 0$ where $\mathbf{G} = \mathbf{I} - \mathbf{R}_\infty\mathbf{P}\mathbf{R}_\infty\mathbf{P}$. Here, the propagation matrix $\mathbf{P} = \mathrm{diag}\left[\exp\left(i\,\chi_p h\right)\right]$ characterises the phase variation of the plane wave fields across the cavity; $\chi_p = \sqrt{k^2 - \alpha_p^2}$, $\alpha_p = \alpha_0 + 2\pi p/d$ (for a lattice of period d), and the real solutions α_0 of the dispersion equation correspond to propagating modes (Fig. 2). These values allow the separation of adjacent maxima in field plots (π/α_0) to be calculated. However, the evanescent modes, which are needed to form the complete basis required in the calculation of the energy properties, cannot be generated in this way — due to implicit assumptions of quasiperiodicity (i.e. real α_0) in our calculation of scattering matrices.

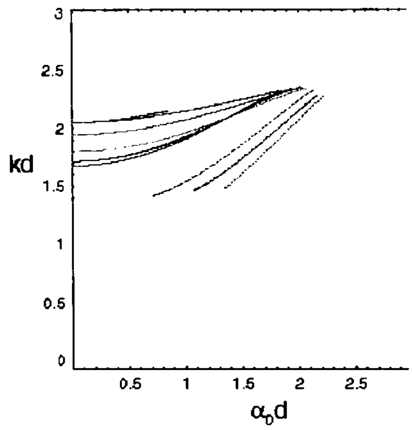

Fig. 2. Dispersion curves for a guide of width h/d=5.0 (violet), 3.0 (blue), 1.0 (cyan), 0.7 (green), 0.4 (red). Other parameters as in Fig. 1.

2.3 A periodic array of waveguides

The computation of a complete set of modes can be achieved by considering a periodic array of waveguides which, if sufficiently separated and operated in a bandgap, gives rise to negligible cross-talk. We follow the treatment of Sec. 2.1 and proceed to compute the Bloch modes of this periodic waveguide structure, formulating both the eigenvalue matrix Λ and $\mathbf{R}_\infty$ for this structure. Within an L-layer crystal, the field is expanded in forward and backward propagating Bloch modes, while, in free space, in terms of plane waves. Fields are matched at the upper and lower interfaces and we de-

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rive expressions for vectors of reflected ($\mathbf{r} = \mathbf{R}\delta$) and transmitted ($\mathbf{t} = \mathbf{T}\delta$) plane wave coefficients involving the reflection and transmission scattering matrices

$$\mathbf{R} = \left(\mathbf{R}_\infty - \mathbf{P}^L\mathbf{R}_\infty\mathbf{P}^L\right)\left(\mathbf{I} - \mathbf{R}_\infty\mathbf{P}^L\mathbf{R}_\infty\mathbf{P}^L\right)^{-1} \quad \text{and} \quad \mathbf{T} = \left(\mathbf{I} - \mathbf{R}_\infty^2\right)\mathbf{P}^L\left(\mathbf{I} - \mathbf{R}_\infty\mathbf{P}^L\mathbf{R}_\infty\mathbf{P}^L\right)^{-1}$$

where $\mathbf{P} = \mathbf{F}_-\Lambda\mathbf{F}_-^{-1}$ is the interlayer propagation constant. These forms are strikingly similar to the standard Airy formulae for the slab Fabry-Perot interferometer

$$r = \frac{r_{12} + r_{23}e^{2i\beta}}{1 + r_{12}r_{23}e^{2i\beta}} = \frac{\rho\left(1 - e^{2i\beta}\right)}{1 - \rho^2 e^{2i\beta}} \quad \text{and} \quad t = \frac{t_{12}t_{23}e^{i\beta}}{1 + r_{12}r_{23}e^{2i\beta}} = \frac{\left(1 - \rho^2\right)e^{i\beta}}{1 - \rho^2 e^{2i\beta}}$$

with cross layer propagation constant β and interface Fresnel reflection coefficient ρ. In a guide of sufficient length supporting only a single propagating mode, the inverses that arise in both $\mathbf{R}$ and $\mathbf{T}$ are rank-1 perturbations that may be analytically inverted by the Sherman-Woodbury formula. This leads to the analytic form for the transmitted flux $\mathcal{E}_T = |v_-\delta|^2\left(1 - |\rho|^2\right)^2 / |1 - \rho^2\mu^{2L}|^2$ where μ is the sole propagating eigenvalue and ρ is the (1,1) element of $\mathbf{F}_-^{-1}\mathbf{F}_+$ — a close relative of $\mathbf{R}_\infty$. Fig. 3 compares the model transmittance with that computed for a finite crystal of dimension 11×11 cylinders ($a/d = 0.3$, $\nu = 3$) with a unit width channel of length $11d$ cut through the centre of the crystal. The blue curve, which was computed using a full multipole theory[5], is for the finite structure with a point source located in the centre of the channel 0.5 d from the channel entrance, while the red curve was computed using the expression for $\mathcal{E}_T$ above. As is evident from Fig. 3, the discrepancies between actual and modelled positions of the transmission maxima and minima are remarkably small (less than 0.3%). Fig. 4 displays the field intensity in mid-channel of the photonic crystal waveguide of similar geometry but with length $L = 21d$ (as in Fig. 1). The dashed curve is the actual data from the multipole theory calculation while the solid curve is from the asymptotic model, the spatial dependence of which for a monomodal guide can be shown to be $g(n) = |\mu^n - \rho\mu^{2L-n}|^2$. There is outstanding agreement in the location of the maxima and minima, and also in the fringe visibility given by $v = 2|\rho|/\left(1 + |\rho|^2\right)$.

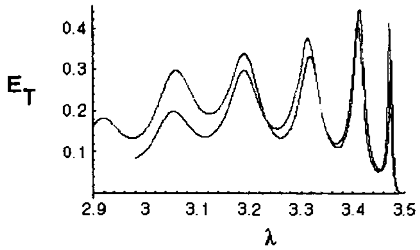

Fig. 3 Transmittance comparison.

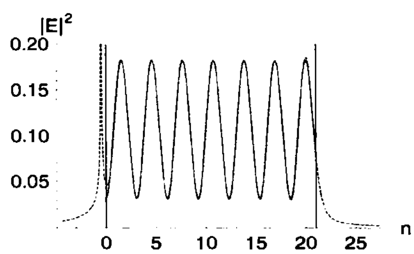

Fig. 4 Mid-channel intensity plot.

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