EFFECTS OF DISORDER IN PHOTONIC CRYSTAL WAVEGUIDES

T N Langtry[1], L C Botten[1], C M de Sterke[2], A A Asatryan[1], and R C McPhedran[2]

[1]Department of Mathematical Sciences
University of Technology, Sydney
PO Box 123, Broadway, NSW 2007
Ph: (02) 9514 2261, Fax: (02) 9514 2248
Tim.Langtry@uts.edu.au

[2]School of Physics
The University of Sydney
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Abstract

Waveguides may be formed in photonic crystals (operated in a band gap) by removing a line of scatterers. The utility of photonic crystals in the future development of optical circuits will be dependent on fabrication quality and, particularly, the regularity of the crystal. The paper investigates their sensitivity to fabrication defects through a study of the effects of disorder in the radii and refractive indices of the scatterers.

Introduction

High efficiency waveguides can be formed in a photonic crystal for frequencies inside a band gap by removing lines of scatterers (Fig. 1) [1]. In this paper we model fabrication defects in two-dimensional waveguides based on a finite cluster of dielectric cylinders, arranged in a square lattice with lattice constant d, by introducing random perturbations in the parameters characterising the material and optical properties of the guide. The guide is excited by a line source parallel to the cylinders, located close to the entry of the guide. The resulting field intensity is calculated using a multipole method yielding high accuracy [2]. Quantitative results characterising the effects of fabrication defects are obtained by Monte Carlo simulation [3].

Corresponding studies of straight and bent waveguides of equivalent length indicate that in both cases such structures exhibit remarkable tolerance to defects for quite high levels of disorder in both the radii of the cylinders and their refractive indices (ie, variations of up to 20% in either parameter) (Fig. 2). Beyond these levels of disorder we observe a transition to a regime in which the field decays exponentially and thus is localised. There is a strong similarity between the features of straight and bent guides.

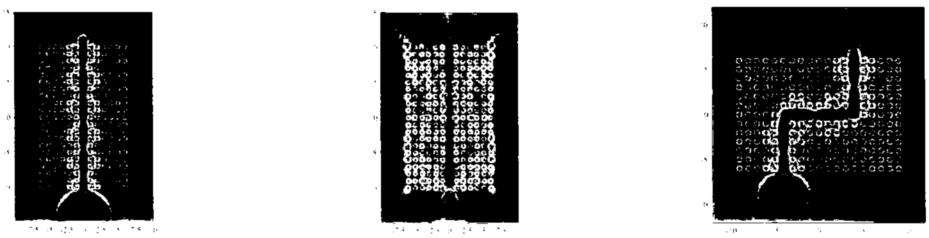

Figure 1. Plots of electric field intensity for a straight guide (252 cylinders) with no disorder and with radius $0.3d$ at wavelengths: (left) $\lambda = 3.3d$ (in the band gap), and (middle) $\lambda = 5.0d$ (outside the gap). At right: guide with bends (226 cylinders with radius $0.3d$) at $\lambda = 3.3d$. The length of this guide (measured down the centre of the channel) is equal to the length of the straight guide.

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Outline of multipole method

In this study we consider in-plane propagation with the $\vec{E}$ field parallel to the cylinders. The wave equation is solved for a point source $\nabla^2 G(\mathbf{r}) + k^2 n^2(\mathbf{r}) G(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{c}_s)$, where G is the Green function due to a source at $\mathbf{c}_s$. The exterior field may be represented by a Wijngaard expansion, valid outside the rods:

$$G(\mathbf{r}) = 1/(4i) H_0^{(1)}(k\,|\,\mathbf{r} - \mathbf{c}_s\,|) + \sum_q \sum_{m=-\infty}^{\infty} B_m^q H_m^{(1)}(k\,|\,\mathbf{r} - \mathbf{c}_q\,|) e^{im \arg(\mathbf{r} - \mathbf{c}_q)}$$

where the sums are over rods (q) and cylindrical harmonics (m), the first term represents the real source, and the remaining terms represent scattering by all cylinders. A local expansion is also valid in an annulus outside each (ℓ'th) rod:

$$G(\mathbf{r}) = \sum_{m=-\infty}^{\infty} [A_m^\ell J_m(k\,|\,\mathbf{r} - \mathbf{c}_\ell\,|) + B_m^\ell H_m^{(1)}(k\,|\,\mathbf{r} - \mathbf{c}_\ell\,|)] e^{im \arg(\mathbf{r} - \mathbf{c}_\ell)}.$$

These expressions for G must be consistent, and Graf's addition theorem is used to compare them given their use of different origins. We thus find $A_m^\ell = K_m^\ell + \sum_{q \neq \ell} \sum_m S_{mp}^{\ell q} B_p^q$ or, in matrix form, $\mathbf{A} = \mathbf{SB} + \mathbf{K}$, where $\mathbf{K}$ is associated with the source and $\mathbf{SB}$ is due to scattering by all cylinders $q \neq \ell$. Another relation between $\mathbf{A}$ and $\mathbf{B}$ is needed to solve this system; this is obtained from the boundary conditions at each cylinder boundary, in particular $\mathbf{B} = \mathbf{RA}$. Combining these equations to obtain a linear system in the $\mathbf{B}$ coefficients yields the equation $(\mathbf{I} - \mathbf{RS})\mathbf{B} = \mathbf{RK}$. This system is then solved to find the B_m^ℓ and thus the entire field. The method is efficient and accurate even for disordered structures.

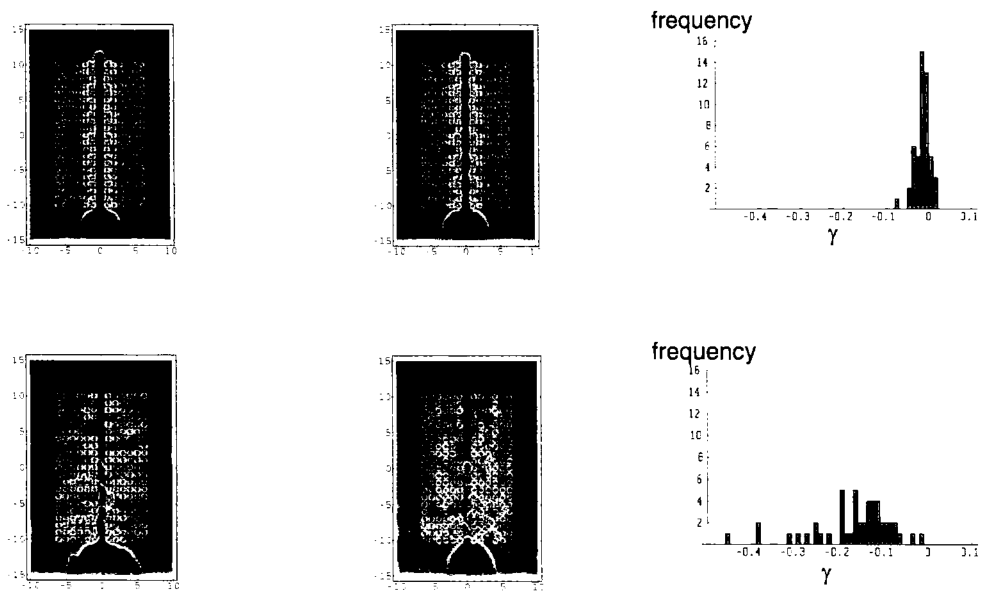

Figure 2. Straight guide with disordered radius: perturbations of radius are uniformly distributed on $[0, \delta_a d)$ for $\delta_a = 0.03$ (top) and $\delta_a = 0.10$ (bottom), with 50 realisations in each simulation. In each row there are two individual crystal realisations,

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and a histogram of the attenuation coefficients γ of the realisations for the intensity along the centre of the guide.

Results of Monte Carlo simulations

Quantitative results characterising the effects of disorder on a waveguide were obtained by a Monte Carlo method. Ensembles of 50 realisations of a crystal with disordered cylinder radii and refractive indices were analysed using the multipole method: for each ensemble, disorder was introduced by uniformly distributed perturbations of selected parameters. The results are comparable for levels of disorder in each parameter (ie, radius and refractive index) that are equivalent, relative to the size of that parameter. Figure 2 displays results for perturbations of radius in the straight guide: the perturbations are uniformly distributed on $[0,\delta_a d)$ for $\delta_a = 0.03$ (top) and $\delta_a = 0.10$ (bottom). Each row contains density maps for the intensity of the electric field for two individual crystal realisations, and a histogram (including all realisations) of the attenuation coefficients γ (the slope of a straight line fitted to the log of intensity along the centre of the guide). It is clear that there is still strong guiding at the $\delta_a = 0.03$ level, with little attenuation. However, at $\delta_a = 0.10$ we observe exponential decay of the intensity—an indication of Anderson localisation—with the attenuation coefficients now clearly shifted away from 0: the channel is effectively closed.

Figure 3 plots the logarithm of the mean intensity (over all realisations) along the centre of the guide, for both the straight guide (black) and the bent guide (red in first segment of guide, green in second and blue in third). The vertical lines indicate the start and end of the guide, and the source is positioned at 0 on the horizontal axis. The figure indicates that guiding is preserved for quite high levels of disorder (eg $\delta_a \leq 0.06$) and that in this regime, at least to first order, the beat length (and hence the propagation constant) for the bent and straight guides are equivalent. For larger δ_a we observe exponential decay (on average) of the intensity along the guide, again characteristic of Anderson localisation.

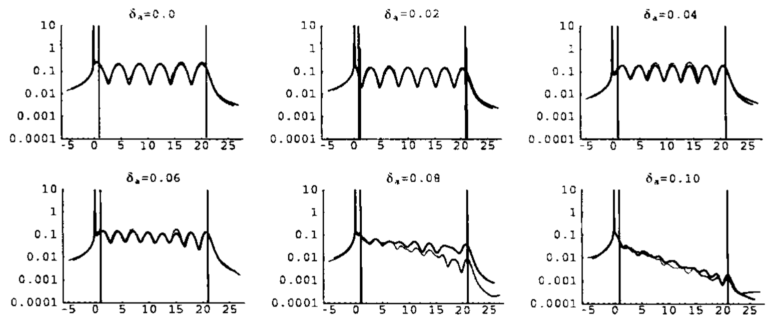

Figure 3. Comparison of logarithm of mean intensity of straight and bent guides with disordered radius at the same levels of randomisation: centre of guide.

References

1. Parker, G. and Charlton, M., 2000. *Physics World*, 8:29.
2. Botten, L.C. *et al*, 2000. *J Opt Soc Amer A*, 17:2165.
3. Langtry, T.N. *et al*, Monte Carlo modelling of imperfections in two-dimensional photonic crystals. *Math Comput Simul*, to appear.

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