PHOTONIC BANDGAPS IN WOODPILE STRUCTURES

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Abstract

Three-dimensional photonic band gap structures are the ultimate goal for photonic crystals as, unlike one and two-dimensional crystals, they provide for total confinement. One such configuration is the woodpile structure, consisting of crossed layers of dielectric rods with a stacking sequence, repeating itself every four layers. Our study of propagation in a woodpile involves the formulation of plane wave scattering matrices for the structure, derived recursively from the scattering matrices of its component layers, using a Rayleigh method, in which the field quantities are written as multipole expansions. For each layer, there is dispersion in only one direction and thus the 2D-diffraction problem is characterised by a family of 1D problems, each associated with incidence parameters corresponding to the diffracted orders of the orthogonal grating, leading to scattering matrices that are block diagonal or some permutation thereof. The theory is applied to deduce transmission spectra and band diagrams for woodpile photonic crystals.

Introduction

Photonic crystals are periodic, lossless lattices that are the optical analogues of electronic crystals or semiconductors, in that their energy spectra contain bands of frequencies for which light cannot propagate within the lattice [1]. With the development of a lossless medium that is impervious to light, it is possible to control the flow of light in ways hitherto not possible. This may ultimately lead to the development of integrated optical circuits that will revolutionise communications technology, providing the capacity to transmit and route signals optically without the need to ever convert the photons to electrons and back, as is the case with the use of present electronic switching technology.

Scattering matrices for woodpile structures

The woodpile structure consists of layers of cylindrical rods with a stacking sequence, which repeats itself every four layers. Within each layer, the rods are parallel and separated by a distance d. The distance between successive layer centres is h and the rod axes in adjacent layers are orthogonal. To obtain a periodicity of four layers in the stacking direction, rods separated by one intermediate layer are offset by a distance of $d/2$ in the direction perpendicular to the rod axes.

First, consider a single grating consisting of identical parallel cylindrical rods of radius a whose axes are separated by d. In the chosen Cartesian coordinate system, the z axis is vertical and the cylinder axes are parallel to the x axis and lie in the xy plane. Fields above and below the grating are expressed as expansions of plane waves, comprising transverse electric (TE) and transverse magnetic (TM) components of the electric field, with respect to the vertical z axis. These TE/TM components are combined

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as block matrices according to the equations $\mathcal{J}_D^{\pm} = \left[\left(\mathbf{E}_D^{\pm}\right)^T \quad \left(\mathbf{F}_D^{\pm}\right)^T \right]^T$ and $\mathcal{J}_I^{\pm} = \left[\left(\mathbf{E}_I^{\pm}\right)^T \quad \left(\mathbf{F}_I^{\pm}\right)^T \right]^T$, where $\mathbf{E}_I^{\pm} = \left[E_{I,s}^{\pm} \right]$ and $\mathbf{F}_I^{\pm} = \left[F_{I,s}^{\pm} \right]$ respectively denote the TE and TM components of the incoming plane wave electric field, while $\mathbf{E}_D^{\pm} = \left[E_{D,s}^{\pm} \right]$ and $\mathbf{F}_D^{\pm} = \left[F_{D,s}^{\pm} \right]$ respectively denote the TE / TM components of the outgoing electric field.

Now let $\mathcal{R}_a$ and $\mathcal{R}_b$ respectively denote reflection matrices for incidence from above and below the layer and let $\mathcal{T}_a$ and $\mathcal{T}_b$ denote the corresponding transmission matrices. These matrices will be functions of the incidence direction, the lattice geometry and the refractive index of the rods. It is clear that a relationship of the form

$$\begin{bmatrix} \mathcal{J}_D^- \\ \mathcal{J}_D^+ \end{bmatrix} = \begin{bmatrix} \mathcal{T}_a & \mathcal{R}_b \\ \mathcal{R}_a & \mathcal{T}_b \end{bmatrix} \begin{bmatrix} \mathcal{J}_I^- \\ \mathcal{J}_I^+ \end{bmatrix}$$

will hold. Using a Rayleigh multipole method [2], we have been able to obtain explicit expressions for the reflection and transmission matrices [3].

To obtain scattering matrices for the woodpile structure, we derive combined reflection and transmission scattering matrices for a crossed pair and then use an algorithm [2, 3] to form a stack of such pairs. For a crossed pair, we note that the bottom grating has cylinders aligned with the x' axis in a secondary coordinate system $x' = y$, $y' = -x$ and $z' = z$. This enables us to derive scattering matrices for the bottom layer with respect to the system (x', y', z'). After some intricate permutations of their entries, we can express these matrices in the primary coordinate system. It is then straightforward [2, 3] to stack these one-dimensional gratings in the woodpile fashion.

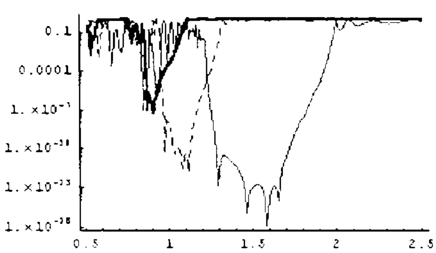

Figure 1: Transmittance for a 20 layer silicon woodpile, with $d = 0.711\,\mu$m and $h = 0.21\,\mu$m. The cylinder radii are 0.035, 0.05 and 0.1 μm, represented by the thick, dashed and thin curves respectively.

The Bloch Method

Crucial to the characterization of field propagation in a photonic crystal is the elaboration of its eigenstates or Bloch modes, which form a complete basis in which to expand all field quantities. These modes are derived via plane wave representations of the field immediately above and below any layer. It can be shown that the modes of the crystal are the eigenstates of the inter-layer translation operator $\mathbf{T}$.

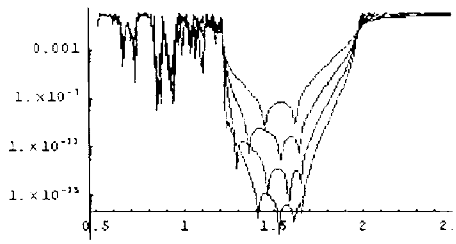

Figure 2: The woodpile of Fig. 1, with cylinder radius 0.1 μm, showing 12, 16, 20 and 24 layers.

It is the propagating states $\mu = e^{-i\mathbf{k}_0 \cdot \mathbf{e}_3}$ that are of greatest interest. Here, $\mathbf{e}_3$ is a vector that characterizes the periodic stacking of the

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composite layers, μ is an eigenvalue of $\mathbf{T}$ and $\mathbf{k}_0$ is the Bloch vector [4], which defines the quasiperiodicity that characterizes the given mode. The importance of the propagating states lies in their capacity to transmit energy over arbitrary distances within the crystal. Band gaps are characterised by the complete absence of propagating states, thus removing the mechanism of energy trans-mission through the crystal.

Results

Figure 1 shows the transmittance for a 20-layer woodpile for vertical incidence, with the electric field perpendicular to the axes of the top layer of cylinders. The large drop in transmittance near the wavelength of 1.4 μm signals the possibility of a photonic band-gap. It is possible that the gap may not persist at all incidence angles, but in this case the gap is a true one, as can be seen from the band diagram of Figure 3. Notice the typical decreasing strength of the band gap and shift in the point of maximum transmission to lower wavelengths as the rod diameter decreases.

The transmission scattering matrix for a crystal of ℓ layers may be derived, following [4]. The asymptotic behaviour of this expression with increasing ℓ is governed by its dominant eigenvalue and the field intensity decays as $|\mu|^{2\ell}$ within a band gap. This is clearly shown in Figure 2, where each additional group of four layers causes the transmittance to diminish by a constant factor.

In Figure 3, we show a projected band diagram for an infinite woodpile (as in Figure 2), with a complete band gap evident. The vertical axis is proportional to the wavenumber, while the horizontal axis traverses the irreducible part of the projection of the Brillouin zone ([5], p 37).

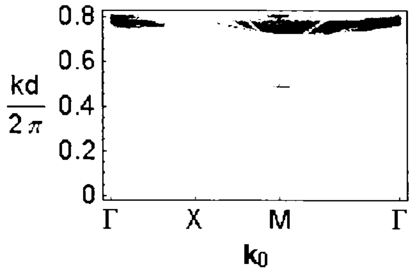

Figure 3: Two dimensional projection of the band structure of an infinite woodpile crystal, with $d = 0.711\,\mu$m, $h = 0.21\,\mu$m, and cylinder radius of $0.1\,\mu$m.

References

1. Joannopoulos, J. *et al*, 1995. *Photonic crystals: Molding the flow of light*. Princeton UP.
2. Botten, L. C. et al. 2000. Electromagnetic Scattering and Propagation through Grating Stacks of Metallic and Dielectric Cylinders for Photonic Crystal Calculations, Part 1: Method. *J. Opt. Soc. Am. A*, 17, 2165.
3. Smith, G. H. et al 2002. Cylinder gratings in conical incidence with applications to modes of air-cored photonic crystal fibers. *Submitted to Phys. Rev. E.*
4. Botten, L. C. et al 2001. Photonic band structure calculations using scattering matrices. *Phys. Rev. E* 64, 46603-.
5. Kittel, C. 1996. *Introduction to solid state physics*, John Wiley and Sons, New York.

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