CROSS-DISCIPLINARY TEACHING OF MATHEMATICS

LEIGH N WOOD, GLYN MATHER, LES KIRKUP AND PETER LOGAN

In this paper we describe findings from the literature on cross-disciplinary teaching of mathematics to undergraduates, as part of a review of cross-faculty teaching of science subjects at the University of Technology, Sydney. Disciplinary differences in teaching styles can inform difficulties faced by engineering and business students. The necessity for varied teaching approaches was illuminated by differences in learning styles among students. For mathematics, a common theme is optimising presentation to engineering students, with general agreement on the need for relevance; a concept which is also raised regarding teaching statistics to business students. Innovations include the development of courses based on interdisciplinary teaching approaches, with discussions focusing on course revisions.

Introduction

Teaching across disciplines can offer many benefits as well as impediments. For staff and students, there is the opportunity to experience new ways of thinking and for the transfer of knowledge, but conversely a lack of enthusiasm or cooperation can raise barriers to the delivery of good quality teaching. This paper is centred on investigations into cross-disciplinary teaching in general, and such teaching in the framework of providing mathematics and statistics for engineering and business students.

There is a range of scenarios for cross-disciplinary teaching. At one extreme, it could be seen as mathematicians, statisticians, engineers and business experts together cooking up spicy dishes with ingredients from their different cultures. They serve the banquet to engineering and business students within an open plan kitchen/dining/living room. They all savour each dish, and discuss the components that have been blended together and analyse the cooking techniques. The other extreme involves each specialist coming out of their individual, sequestered rooms to produce their own single course in a designated kitchen after limited prior discussion with other staff. The meal is served in a formal dining room and the students gulp the food down silently, then sit passively waiting for the next specialist to come out of their room to dish up the next course.

Our recent research into cross-disciplinary teaching arose out of concerns about the effectiveness of the teaching of science subjects at UTS to students of other faculties. It involved a number of strands, including interviews with students and academics (reported on at the EMAC conference earlier this year [13, 25]). We also undertook a literature review to inform the research.

We have focused here on a selection of themes from the literature, with an emphasis on teaching mathematics to engineering students and statistics to business students. Writings on disciplinary differences provide some clues about the cultures of teaching staff in particular. Most deal with science teaching approaches and styles, versus those for the social arena, but it is clear students face differing teaching styles which are mirrored by their own variations in learning styles. We looked at literature on engineering mathematics and discussions concerning statistics for business students, and reviewed reports on innovative course design based on interdisciplinary collaboration.

Disciplinary differences

There seems to be wide acceptance of the notion that differences exist between disciplines in their broad cultural characteristics, teaching (or cooking) approaches and styles, alongside differences in the ingredients that each discipline adds to the mix. The primary focus of research has been on differentiating between broad divisions, with some forays into categorising individual disciplines. Some of the literature is of limited utility here, since there is often little distinction made between theoretical sciences (such as mathematics) and vocational sciences (such as engineering and business). Nonetheless, it is clear that students taking cross-disciplinary subjects or team-taught subjects will probably be exposed to differing disciplinary approaches and cultures, which may pose obstacles for their learning experience.

Bradbeer [6] presented an excellent exposition of the concepts in summing up the arguments of many authors and comparing their conclusions. He started with the theories of Kolb [14; cited in 6], probably the best known of the researchers exploring the concept of disciplinary differences, and ranged through to Nulty and Barrett, Becher and others. Kolb [14] began his explorations in terms of his notion of experiential learning, which essentially asserts that learning depends on information transfer and transformation. He suggested that learning styles can be broken down into four basic types, centred on people's approach to dealing with information, and these form the foundation of his categorisation of disciplines. The categories were linked to psychological types, using the Myers-Briggs personality type theory. Bradbeer [6] noted that, "Personality types and learning styles do seem to be related in complex ways … [and] we can conclude that different disciplines both process and structure knowledge in different and distinctive ways." (p. 384)

In terms of groupings, Bradbeer presented a scheme based on Nulty and Barrett's approaches [19], Becher's concept of "academic tribes" (first presented in 1989 – see also [3]) and the classification systems of other theorists. When they are all overlaid, there is congruence in the categorisation of mathematics, described as hard/pure, abstract/reflective; and of engineering, which is regarded as a science-based profession (hard/applied, abstract/active). In these schema, statistics is placed under mathematics. Business is grouped with engineering as both being science professions (however, it should be noted that some business courses may tend more towards the social professions and therefore might differ markedly).

On the subject of disciplinary culture, Bradbeer [6] summed up Becher's theory of academic tribes as being in alignment with other theorists, founded on viewing, "… academic disciplines as simultaneously structures and organisations with their own very distinctive cultures; sufficiently distinctive to be plausibly regarded as tribes …" [6, p. 388] Becher [3] stated that, "The cultural aspects of disciplines and their cognitive aspects are inseparably intertwined. … disciplinary practices can be closely matched with the relevant characteristics of their associated domains of enquiry." (p. 153)

Where there is some dispute within the literature is on the extent to which these disciplinary differences affect student learning – of fundamental importance since it is they who have to consume the results. There is a general underlying assumption that the student who chooses a particular career is already formed in that mould; that is, an engineering student is already an engineer. Bradbeer [6], for instance, stated that, "Students learn in different ways and many have unconsciously chosen disciplines that are more accommodative of their personal learning styles." (p. 382) This seems of dubious reliability

without more research – it may be true of some students, but relies on their having a sophisticated understanding of the chosen profession.

With time, the student may grow into the discipline, and Nulty and Barrett [19] investigated this possibility through a comparative study of Australian first and final year students. They found that first year students had similar learning styles irrespective of their discipline, and they also found little evidence of disciplinary styles in third year students. To explain this result, they postulated that most courses involve a range of subjects from different disciplines and so there is in fact little opportunity for the disciplinary culture of the major to be instilled in the students. They pointed out that one of the implications from these results is that teachers will need to accommodate the fact that their students will have a range of learning styles and approaches.

In addition, the existence of differences does not necessarily mean the outcomes are fixed, since students can adapt their learning approaches to circumstances. Despite the identification of disciplinary groupings of learning styles, Kolb, "… did not claim that particular learning styles and specific disciplines were necessarily linked, but rather that some learning styles were more adapted to some disciplinary knowledge structures than others. So it would be possible to learn something … of a discipline using a learning style that did not exactly match the discipline's knowledge structure." [6, p. 392]

Teaching mathematics to engineers

There was broad agreement that mathematics is an essential ingredient in engineering courses. O'Kane [20] presented a provocative view of the importance of mathematics for engineers:

> … during the past three hundred years a handful of mathematicians of the highest stature, not engineers, progressively created and refined the fundamental concepts of engineering discourse, and these concepts have entered the engineering curriculum with a delay of roughly one hundred years. This promethean achievement was not a response to immediate engineering demand, and is the ultimate reason, now largely forgotten, for the presence of mathematics in our engineering schools. (pp. 362-363)

The mathematicians consistently maintained that mathematics should be taught by mathematicians. According to Varsavsky [23]: "There appears to be general agreement that mathematics is best taught by mathematicians and in many institutions mathematicians have the professional responsibility to develop and deliver mathematics subjects for engineers." (p. 344) Others also iterated the point (such as [2, 7, 10]).

Kümmerer [14] presented a general discussion on the range of approaches used to teach mathematics to engineers. In some courses, mathematics is taught as a kind of machine to produce the answer to a problem. Another style is to make things 'user friendly' so that only the essentials are presented, which can lead to oversimplification. Alternatively, mathematics may be viewed as simply something to be endured before the student starts physics or engineering. Kümmerer proposed instead that understanding is crucial (his emphasis): "The most efficient way to learn mathematics **is to understand mathematics.** … Therefore, as a primary goal a course on higher mathematics should enable students to handle mathematics on their own. … I want to emphasize that even when there is limited time and differing needs, mathematics is best taught with understanding." [14, p. 324]

Bajpai *et al.* in 1975 [1] suggested a range of improvements to the then current teaching methods, most of which are still relevant thirty years later. They include: using a modelling approach; specialist engineering mathematics lecturers (whilst being based in the

mathematics department); motivation, that is, clarifying for students the important role played by mathematics in engineering; an integrated approach towards mathematics techniques; presenting a comparison of mathematical methods; and presenting more relevant examples. They also argued for greater enthusiasm from teachers: "For, if the teacher can find a challenge and an excitement in teaching mathematics to engineering students, there is a chance that the students may find a challenge and an excitement in learning." (pp. 361-2)

There was a wide range of other recommendations for appropriate teaching methods in this context. For example, small group work was supported by writers as being beneficial since it resembles the students' future working lives: "These changes train engineers to carry out mathematical tasks in an environment which is much closer to that which they will encounter in their professional practice." [26, p. 596]

The need for relevance was highlighted by many writers as being important in assisting students with learning mathematics. To make a mathematics course seem relevant to engineering students – and hence worth an investment of time – the subject has to be made to seem valuable for their own specialisation and future cases. It is also beneficial for the educator to be familiar with the other subjects the students are doing, so examples can be used from these. As Kümmerer [15] pointed out, "In most cases a translation into the other language is necessary before students can recognize that one is discussing the same subject." (p. 331) Courses need to be tailored to the career ahead, such as in terms of materials that students perceive are applicable to their profession; realistic problems will further assist with promotion of the value of numerical methods [1, 10, 18].

The use of computer and online teaching can offer benefits to the mathematics educator. Various advantages to using computer packages were presented in the literature, including:

- Students enjoy using software applications; for instance, at Wollongong University: "… during the implementation of projects and computer labs … [it was] observed that students showed that they had an overwhelming enthusiasm in attending and using computers with algebra/numerical packages." [26, p. 597]
- Computer packages which incorporate visualisation and animation can help promote understanding of mathematics, through reducing its apparent abstract nature [4].
- Colgan [8] presented a case study on the introduction of MATLAB into mathematics teaching for engineering students and found that it offered an avenue for the introduction of applied engineering examples.
- Varsavsky and Carr [24] developed computer-based resources to provide additional materials, which also gave the opportunity for development of new learning styles. The resources were a positive addition to their course, but they asserted it is still too early to fully estimate the worth of computer-based learning in mathematics.

In terms of content, a number of writers made reference to the core curriculum in mathematics for engineering students at European tertiary institutions developed by SEFI (Société Européenne pour la Formation des Ingénieurs). SEFI established a Mathematics Working Group (MWG) in 1982, "… with the aim of furthering developments in mathematical education, assessing the impact of computer technology and monitoring the needs of industry."[2, p. 223] The MWG continues to advocate a mix of teaching strategies: "The SEFI-MWG recognizes that the design of courses with an optimum mix of teaching strategies is becoming much more difficult and expensive in terms of human

resources and equipment but believes that the successful core curriculum of the 1990s should include all such elements."[2, p. 227] Some Australian universities also follow this curriculum, for instance, the Swinburne University of Technology uses the SEFI model as well as textbooks developed for the SEFI syllabus [10].

Statistics for business students

This subject was a fruitful line of enquiry and revealed some interesting correlations with views on teaching engineering students. McAlevey and Sullivan [17], for instance, asserted that there is a need for using real-life problems since, "Students are best motivated by exposure to real applications, problems, cases and projects." (p. 426) This is affirmed by Parr and Smith [21], who presented an argument for using realistic case studies. These discussions resemble those about using real-life examples and applications for engineering students; in fact Kolb [14; cited in 6] placed business with engineering in his classification of disciplines. Parr and Smith [21] also considered the case-study approach as helpful for developing problem-solving skills and providing integration with other subjects.

Student perceptions of statistics courses were also a common topic for investigation. McAlevey and Stent [16] undertook a survey at Otago University, of business students' perceptions of the notion of good teaching in reference to statistics courses. In other studies, students of science and mathematics have given weight to the clarity of teacher explanation, and placed an emphasis on preparation by the lecturer and the organisation of course material. This survey indicated that the business students preferred similar modes of delivery. Surprisingly, the need for real applications was ranked low by the students. McAlevey and Stent's explanation for this was that an introductory statistics course was probably too early in their learning, that is, the use of applications at that point can obscure the actual statistics.

Zanakis and Valenzi [27] surveyed students at a university in Florida, USA, this time with an emphasis on anxiety, and looked at how student attitudes changed during a revised statistics course. They hypothesised that there is a certain incidence of anxiety – similar to mathematics anxiety, which is commonly recognised as a problem – as well as a certain level of fear about using a computer for analysis. The results of the survey showed that certain improvements did occur over the course, however, they also found that there was a reduced interest or belief in statistics. Their explanation for this was that, although anxiety was reduced, statistics was still perceived as being very difficult; in addition, the course had not convinced students of the worth and career relevance of statistics.

Another course restructure focused on alleviating mathematics and computer anxiety. Zeis' team [28] developed a course in business statistics, "… designed to 'put the horse before the cart', data collection and management are presented before analysis and inference." (p. 83) They also did surveyed the students, in this case to look at learning outcomes and found that there was clearly improvement in reported skill levels.

Interdisciplinarity

Bradbeer [6] defined the different strands of the concept 'interdisciplinary':

> … multidisciplinary study [is where] the student simultaneously studies several disciplines, each taught by specialists. No explicit attempt is made by the teachers to relate their disciplines to each other … Interdisciplinarity involves a curriculum where teaching and learning occur in the overlap areas between disciplines. Often this involves elements of one discipline being taught by a teacher of

another. Students are expected not just to be able to move from one discipline to another but also to be able to synthesise insights from the various disciplines studied. (p. 382)

Thus multidisciplinarity could be seen as courses being served in an open plan area, but each dish remains firmly within the culture of the specialist. Interdisciplinarity, on the other hand, involves truly multicultural servings with blended flavours and ingredients.

There has been a great deal written on interdisciplinary innovations in teaching, most of it couched in the framework of 'our course'. It makes for interesting reading; unfortunately, it is difficult to extrapolate useful techniques. The writings are predominantly based on descriptions of either new courses or courses that have been revised in the light of new information on teaching techniques; or student dissatisfaction with previous versions of the course; or in an attempt to reduce isolation and fragmentation of subjects (see for instance [5, 9, 11, 22]). In general, articles focused on multidisciplinary approaches – that is, specialists presenting stand-alone sections – rather than taking a genuine interdisciplinary view. The articles all reported gains for students – and promotion of collaboration with other disciplines – but these assessments were usually based on student opinion rather than an objective assessment of gains in learning (exceptions are [9, 11], where they administered various tests at the end of the course).

Fitchett [12] explained the impetus for interdisciplinarity as the need to present mathematics and science in terms of how it is practiced by professionals. He theorised that in order to learn about processes and not just content, students must learn to integrate knowledge, rather than compartmentalising it. Deeds *et al.* [9] approached development of a new interdisciplinary course at Drury University (USA) in a similar spirit, and chose integration as the preferred mode to promote understanding of the subject matter, retention of material, and comprehension of the connections between science and the everyday world. Their prescription for successful ventures in this arena involved three critical elements, that is, "… building connections between departments, broadening the number of participants, and ensuring support from the administration." [9, p. 181]

Another example was an interdisciplinary science course at Alfred University (USA), as presented by Boersma *et al.* [5]. They described their rationale as wanting students to, "… see the interrelationships between different scientific disciplines and between science and mathematics because it has been shown that students often learn science better when an interdisciplinary approach is taken …" (p. 398)

Conclusions

Interdisplinarity is being promoted as providing significant gains in student learning. At this stage, it is difficult to ascertain how successful new courses really are. Disciplinary cultural differences between academics can pose obstacles to communication, both with each other and with students. For the students, crossing over between cultures can prove difficult and, without appropriate institutional support or other coping mechanisms of their own, they may leave university altogether. There are many ways the mathematics educator can assist student success, the first step being to recognise that each one will bring different internal and external circumstances with them (including variations in learning styles).

The major consideration must remain the student – what ingredients and styles are appropriate for their consumption – since there is no point in offering subjects that are palatable to only a few (tomato sauce does not seem to fit with wonton soup, but Worcestershire sauce might work). Current trends are towards fostering multiculturalism in

many fields, but the needs of the consumer – and their palates – should remain of paramount concern.

Acknowledgements

<div align="center">

References

</div>

1. Bajpai, A., Mustoe, L., & Walker, D. (1975). Mathematical Education of Engineers: Part 1. A critical appraisal. *International Journal of Mathematical Education in Science & Technology, 6*(3), 361-380.
2. Barry, M., & Steele, N. (1993). A Core Curriculum in Mathematics for the European Engineer: An Overview. *International Journal of Mathematical Education in Science & Technology, 24*(4), 223-229.
3. Becher, T. (1994). The Significance of Disciplinary Differences. *Studies in Higher Education, 19*(2), 151-161.
4. Blyth, W. F. (2001). Animations using Maple in First Year. *Quaestiones Mathematicae, Suppl. 1*, 73-82.
5. Boersma, S., Hluchy, M., Godshalk, G., Crane, J., DeGraff, D., & Blauth, J. (2001). Student-Designed, Interdisciplinary Science Projects – Placing Students in the Role of Teacher. *Journal of College Science Teaching, 30*(6), 397-402.
6. Bradbeer, J. (1999). Barriers to Interdisciplinarity: Disciplinary discourses and student learning. *Journal of Geography in Higher Education, 23*(3), 381-396.
7. Clayton, B. (1995). Mathematics for Engineers: An integrated approach. In L. Mustoe & S. Hibberd (Eds.), *Mathematical Education of Engineers* (pp. 29-39). Oxford: Clarendon Press.
8. Colgan, L. (2000). MATLAB in First-Year Engineering Mathematics. *International Journal of Mathematical Education in Science & Technology, 31*(1), 15-25.
9. Deeds, D. G., Allen, C. S., Callen, B. W., & Wood, M. D. (2000). A New Paradigm in Integrated Math and Science Courses – Finding common ground across disciplines. *Journal of College Science Teaching, 30*(3), 178-183.
10. Easton, A., & Steiner, J. (1996). *A Core Curriculum in Mathematics for the Australian Engineer.* Paper presented at the Second Biennial Australian Engineering Mathematics Conference, Sydney, Australia: Institution of Engineers, Australia.
11. Elliott, B., Oty, K., McArthur, J., & Clark, B. (2001). The Effect of an Interdisciplinary Algebra/Science Course on Students' Problem Solving Skills, Critical Thinking Skills and Attitudes Towards Mathematics. *International Journal of Mathematical Education in Science & Technology, 32*(6), 811-816.
12. Fitchett, S. (2002). *Using Environmental Science to Bridge Mathematics and the Sciences.* Paper presented at the Second International Conference on the Teaching of Mathematics (ICTM2), Crete. Available on CD-ROM.
13. Kirkup, L., Wood, L., Mather, G. & Logan, P. (2003) Teaching Mathematics and Physics to Engineers: Reflections from the chalk face. In *EMAC 2003 Proceedings* (pp. 97-102). Sydney.
14. Kolb, D. (1984). *Experiential Learning: Experience as the source of learning and development*. Englewood Cliffs, NJ: Prentice Hall. Cited in J. Bradbeer (1999). Barriers to Interdisciplinarity: Disciplinary discourses and student learning. *Journal of*

Geography in Higher Education, 23(3), 381-396.

15. Kümmerer, B. (2001). Trying the Impossible – Teaching mathematics to physicists and engineers. In D. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level – An ICMI Study* (pp. 321-334). Dordrecht: Kluwer Academic Publishers.

16. McAlevey, L. G., & Stent, A. F. (1999). Undergraduate Perceptions of Teaching a First Course in Business Statistics. *International Journal of Mathematical Education in Science & Technology, 30*(2), 215-225.

17. McAlevey, L. G., & Sullivan, J. C. (2001). Making Statistics More Effective for Business? *International Journal of Mathematical Education in Science & Technology, 32*(3), 425-438.

18. McGregor, R., & Scott, B. (1995). A View on Applicable Mathematics Courses for Engineers. In L. Mustoe & S. Hibberd (Eds.), *Mathematical Education of Engineers* (pp. 115-129). Oxford: Clarendon Press.

19. Nulty, D. D., & Barrett, M. A. (1996). Transitions in Students' Learning Styles. *Studies in Higher Education, 21*(3), 333-345.

20. O'Kane, J. (1995). Engineers and Their Mathematicians – Foundations for a debate on mathematics in the engineering curriculum. In L. Mustoe & S. Hibberd (Eds.), *Mathematical Education of Engineers* (pp. 361-378). Oxford: Clarendon Press.

21. Parr, W. C., & Smith, M. A. (1998). Developing Case-Based Business Statistics Courses. *The American Statistician, 52*(4), 330-337.

22. Sjöberg, A. (1996). *Project-Oriented Courses in Scientific Computing with Applications from Natural Sciences and Engineering.* Paper presented at the Second Biennial Australian Engineering Mathematics Conference, Sydney, Australia: Institution of Engineers, Australia.

23. Varsavsky, C. (1995). The Design of the Mathematics Curriculum for Engineers: A joint venture of the mathematics department and the engineering faculty. *European Journal of Engineering Education, 20*(3), 341-345.

24. Varsavsky, C., & Carr, A. (1997). *Designing an Interactive Electronic Book for First Year University Mathematics*, available at www.atcminc.com/mPUBLICATIONS/EP/EPATCM97/ATCMPO14/paper.html

25. Wood, L.N., Mather, G., Logan, P. & Kirkup, L. (2003) Teaching Mathematics and Physics to Engineers: Reflections from the back row. In *EMAC 2003 Proceedings* (pp. 295-300). Sydney.

26. Worthy, A. (1996). *Mathematical Education of Engineers – Towards the year 2000.* Paper presented at the Second Biennial Australian Engineering Mathematics Conference, Sydney, Australia: Institution of Engineers, Australia.

27. Zanakis, S. H., & Valenzi, E. R. (1997). Student Anxiety and Attitudes in Business Statistics. *Journal of Education for Business, 73*(1), 10-17.

28. Zeis, C., Shah, A., Regassa, H., & Ahmadian, A. (2001). Statistical Components of an Undergraduate Business Degree: Putting the horse before the cart. *Journal of Education for Business, November/December*, 83-88.

Leigh N Wood
University of Technology, Sydney
Australia
<Leigh.Wood@uts.edu.au>

Glyn Mather
University of Technology, Sydney
Australia
<Glyn.Mather@uts.edu.au>

Les Kirkup
University of Technology, Sydney
Australia
<Les.Kirkup@uts.edu.au>

Peter Logan
University of Technology, Sydney
Australia
<Peter.Logan@uts.edu.au>