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# A Web-based Decision Support System for Linear Bilevel Problems

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**Abstract.** Advanced web and database technology support remote data access and communication. It has opened new opportunities for decision support system builders to develop web-based DSS. This paper presents a web-based decision support system for linear bilevel problems. An extended Kuhn-Tucker method for linear bilevel programming is implemented in this system. It can compute a global optimal solution for linear bilevel programming problems.

***Keyword*:** Web intelligent, linear bilevel programming, Kuhn-Tucker method, Decision support systems

## 1. Introduction

Information system researchers and technologists have built and investigated Decision Support Systems (DSS) for more than 35 years [1]. Web technology and database technology allow organizations to make decisions in a distributed environment that supports remote data access and communication [2]. The computer facilities for utilizing DSS have been moving towards a more widespread use of the Internet with its graphical user interface, the web [3,4].

Bilevel programming (BLP) was motivated by the game theory of Von Stackelberg [5,6] in the context of unbalanced economic markets. The majority of research on BLP has centered on the linear version of the problem. There have been nearly two dozen algorithms, such as, the *K*-th best approach [7,8], Kuhn-Tucker approach [9,10,11], complementarity pivot approach [12], penalty function approach [13,14], proposed for solving linear BLP problems since the field being caught the attention of researchers in the mid-1970s.

A popular way to solve a linear BLP problem is that a BLP problem is transferred into a nonlinear programming problem using Kuhn-Tucker conditions. The reformulation of the linear BLP problem is a standard mathematical program and relatively easy to solve because all but one constraint is linear. Omitting or relaxing the constraint leaves a standard linear program that can be solved by using simplex algorithm. This is the case of the algorithms proposed by Bard and Falk [9], and, Fortuny-Amat and McCarl [15,16].

However there exists a limitation for the branch and bound algorithm. This is how to solve a linear BLP problem when the upper-level's constraint functions are in arbitrary linear form. Our previous work presented a new definition of solution and related theorem for linear BLP, thus solved the fundamental deficiency of existing linear BLP theory [17]. In [18], we proposed an extended *K*th-best approach for linear BLP. In [19], we completed theoretical foundation of

Kuhn-Tucker approach and developed an extended Kuhn-Tucker approach. In [20], we proposed an extended branch and bound algorithm for linear BLP. This paper proposes a web-based linear bilevel decision support system (WLBLDSS). Following the introduction, this paper overviews the extended Kuhn-Tucker method in Section 2. WLBLDSS architecture and implementation are addressed in Section 3. A conclusion and future work are given in Section 4.

## 2. The extended Kuhn-Tucker Method for linear Bilevel Programming

A linear BLP problem is given by Bard [5]: For $x \in X \subset R^n$, $y \in Y \subset R^m$, $F : X \times Y \to R^1$, and $f : X \times Y \to R^1$,

$$\min_{x \in X} F(x, y) = c_1 x + d_1 y \tag{1a}$$

$$\text{subject to } A_1 x + B_1 y \le b_1 \tag{1b}$$

$$\min_{y \in Y} f(x, y) = c_2 x + d_2 y \tag{1c}$$

$$\text{subject to } A_2 x + B_2 y \le b_2, \tag{1d}$$

where $c_1$, $c_2 \in R^n$, $d_1$, $d_2 \in R^m$, $b_1 \in R^p$, $b_2 \in R^q$, $A_1 \in R^{p \times n}$, $B_1 \in R^{p \times m}$, $A_2 \in R^{q \times n}$, $B_2 \in R^{q \times m}$.

Let $u \in R^p$, $v \in R^q$ and $w \in R^m$ be the dual variables associated with constraints (1b), (1d) and $y \ge 0$, respectively. We presented and proved the following theorem in [19].

**Theorem 1** A necessary and sufficient condition that $(x^*, y^*)$ solves the linear BLP problem (1) is that there exist (row) vectors $u^*$, $v^*$ and $w^*$ such that $(x^*, y^*, u^*, v^*, w^*)$ solves:

$$\min F(x, y) = c_1 x + d_1 y \tag{2a}$$

$$\text{subject to } A_1 x + B_1 y \le b_1 \tag{2b}$$

$$A_2 x + B_2 y \le b_2 \tag{2c}$$

$$u B_1 + v B_2 - w = -d_2 \tag{2d}$$

$$u(b_1 - A_1 x - B_1 y) + v(b_2 - A_2 x - B_2 y) + w y = 0 \tag{2e}$$

$$x \ge 0, y \ge 0, u \ge 0, v \ge 0, w \ge 0. \tag{2f}$$

Theorem 1 provides theoretical foundation for the extended branch and bound algorithm. The basic idea of the algorithm is to suppress the complementarity term and solve the resulting linear sub-problem. At each iteration, a check is made to see if (2e) is satisfied. If so, the corresponding point is in the inducible region and hence a potential solution to (1). If not, a branch and bound scheme is used to implicitly examine all combinations of complementarity slackness [20].

## 3. WLBLDSS Architecture and Implementation

### 3.1 WLBLDSS Architecture

Figure 1 shows a framework of WLBLDSS. It is consists of a web server which resides a method base, a database sever which interacts with the web sever using an ODBC connection and clients which access the web server through Internet. The web sever manages the system and provides services.

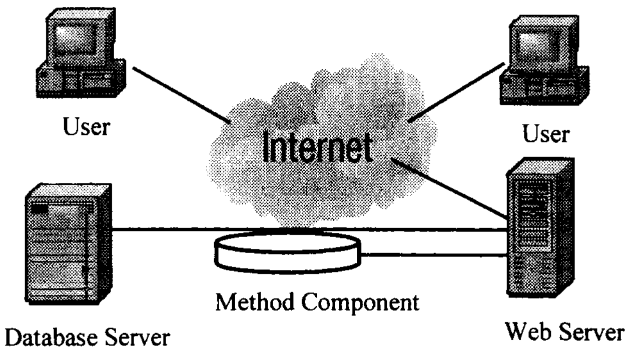


Figure1 Architecture of WLBLDSS

### 3.2 Client Side and Server Application Design

The interface design represents the central component for successful information delivery between a user and WLBLDSS. Interface design for web-based information systems is probably more a science than an art. We integrate intelligent knowledge about BLP together with advanced Javascript technology to design efficient. The web server (IIS 4.0) manages all the pages, traces user information and provides simultaneously services to multiple users through sessions, applications and coking facilities. Using the server side application program, the web server can manage and implement client tasks.

### 3.3 Method Design

A web-based linear bilevel DSS intends to provide necessary computerized assistance to different decision makers and various applications. The extended branch and bound algorithm can solve a wider class of linear bilevel problems can than current capabilities permit [19, 20].

### 3.4 WLBLDSS Database Design

The strategy of design the WLBLDSS database is sharing data information among users. The relational data mode technology is used to database design and management of WLBLDSS.

There are three main entities in WLBLDSS database: users, resource (problem) data and solution data.

## 4. Conclusions and Future Work

The WLBLDSS is developed as an online decision-making tool to support linear bilevel decision-making. The system is interactive, flexible and easy to use for various linear bilevel decision problems. We hope to get feedback from our users and to make the system widely available.

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